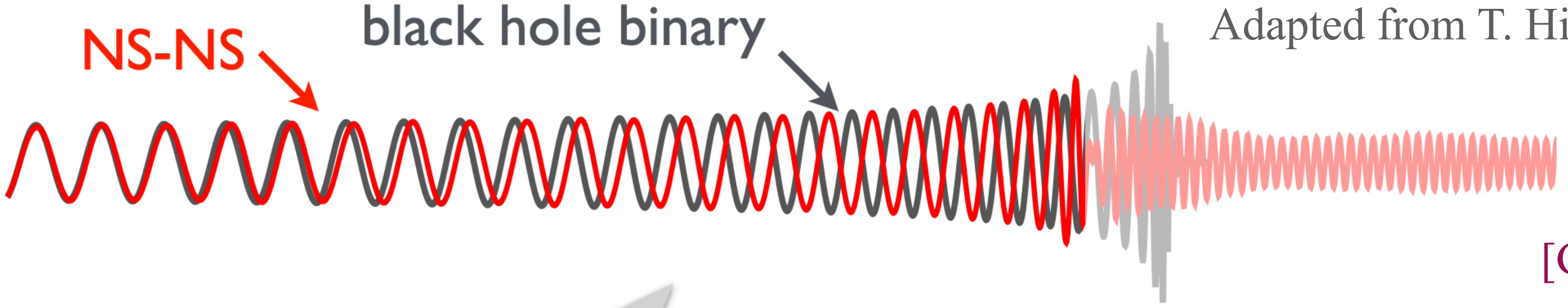
Dynamical tides of coalescing neutron star binaries: Impacts of stratification and solid-crust elasticity

Yong Gao (高勇)

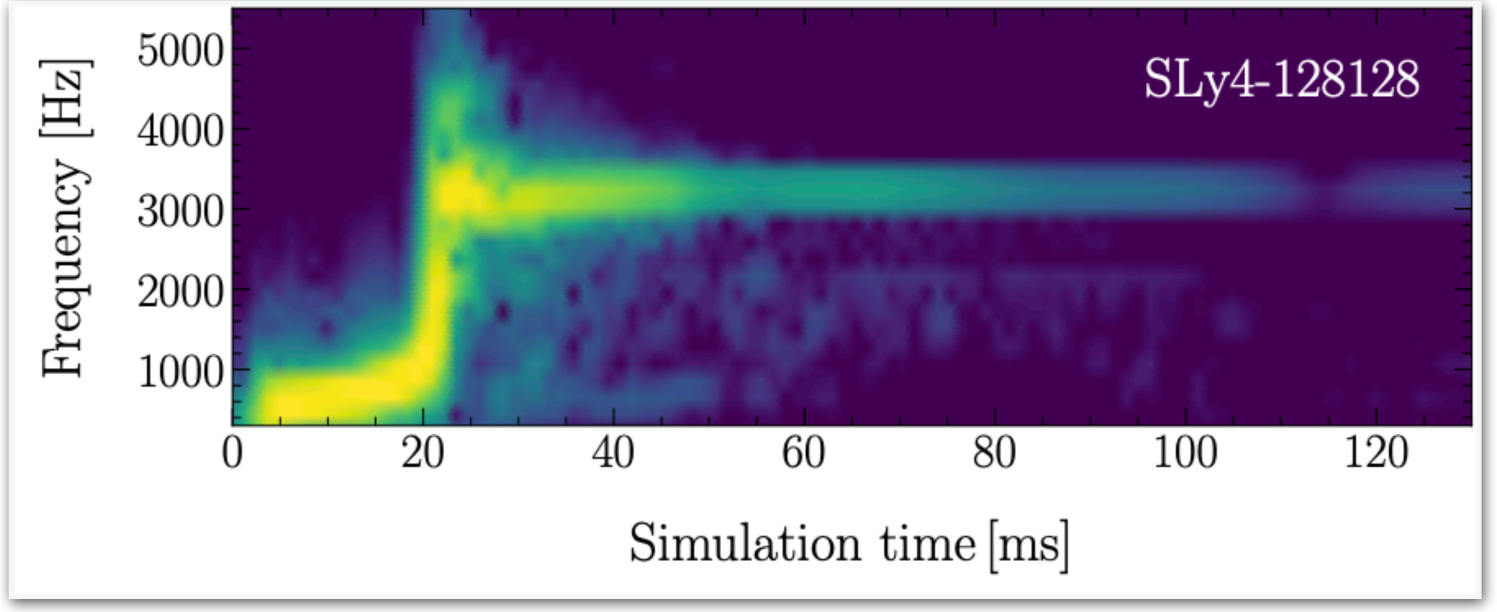
Max Planck Institute for Gravitational Physics, Potsdam

February 6, 2026



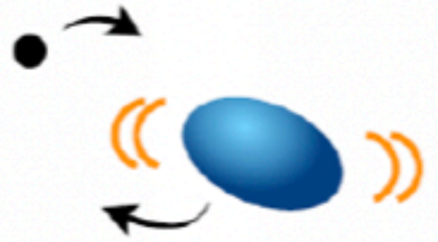
Adapted from T. Hinderer

[Gao+2026]

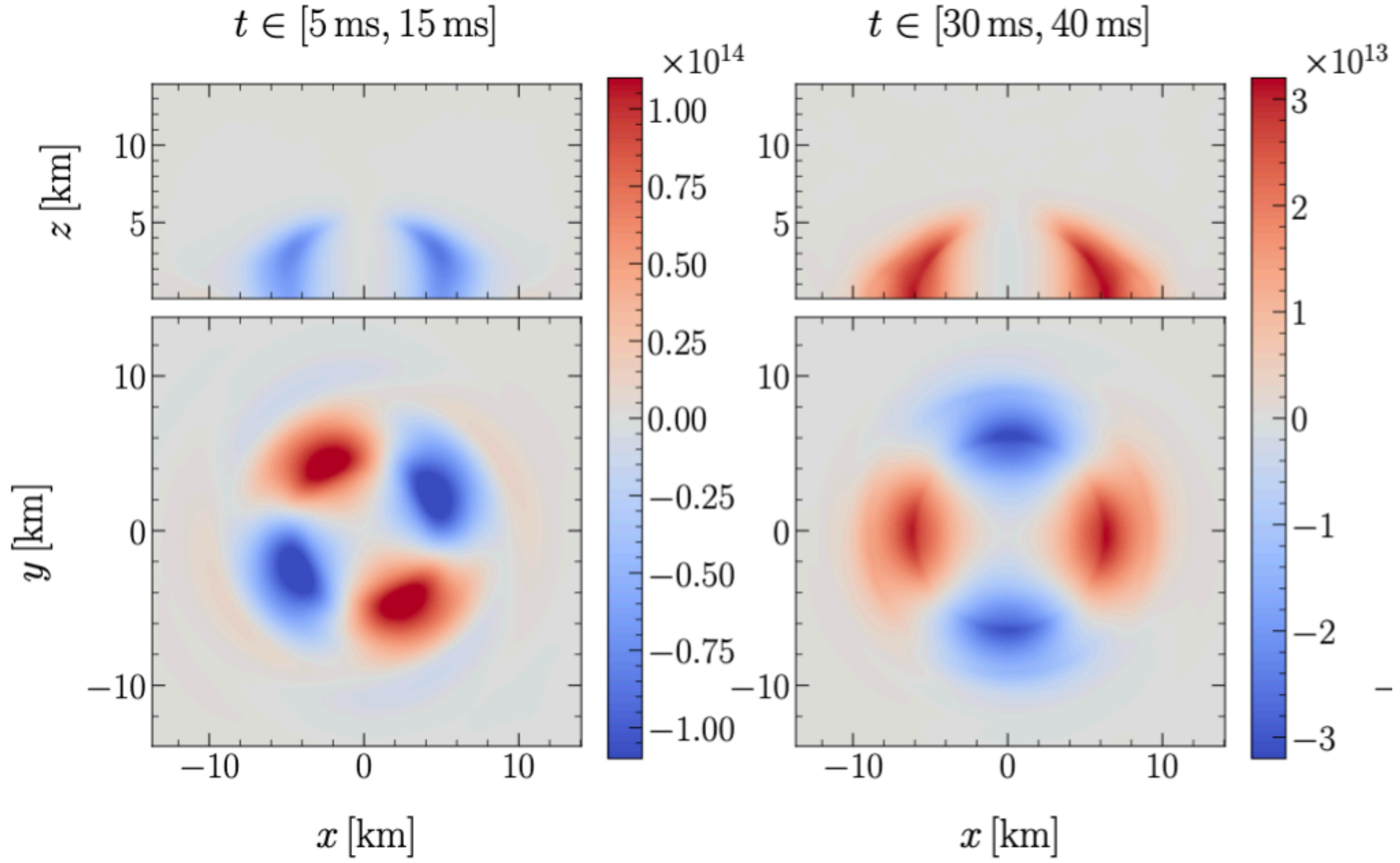


~same signal with point particle

Various tidal effects

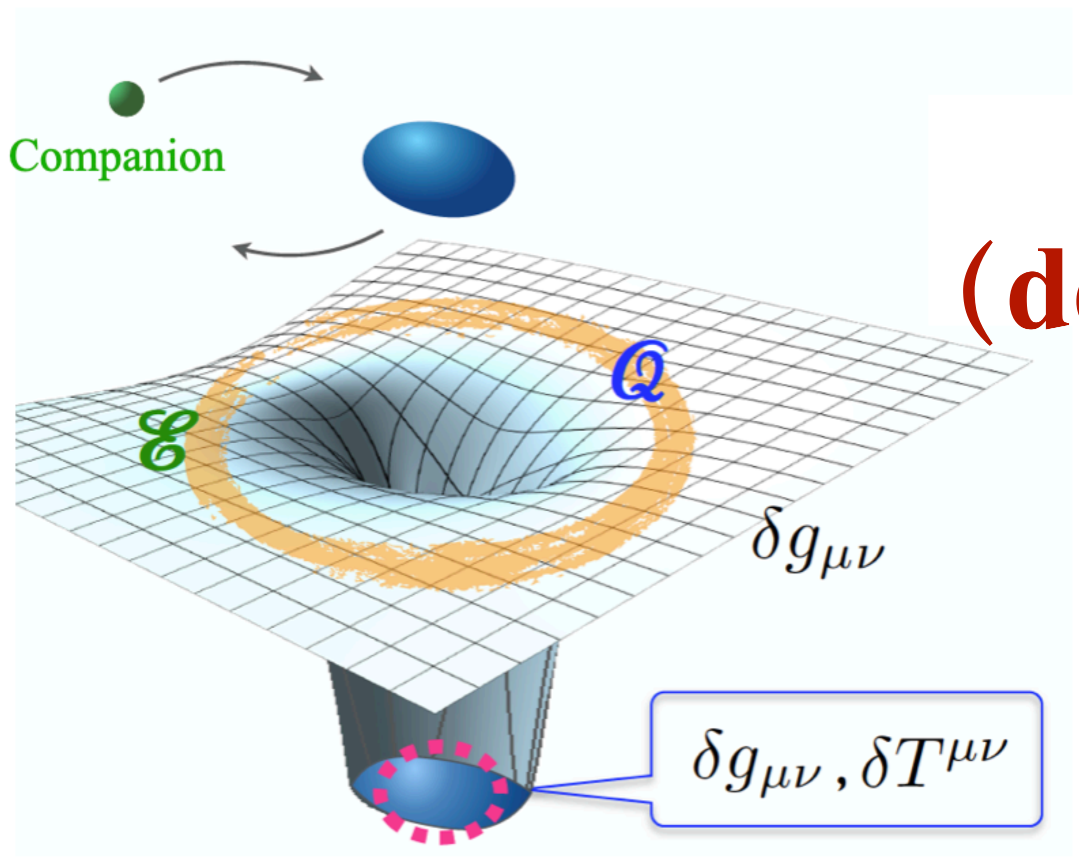


Resonant or non-resonant excitation of various oscillation modes



Asteroseismology of NS interior

Adiabatic tides versus dynamical tides

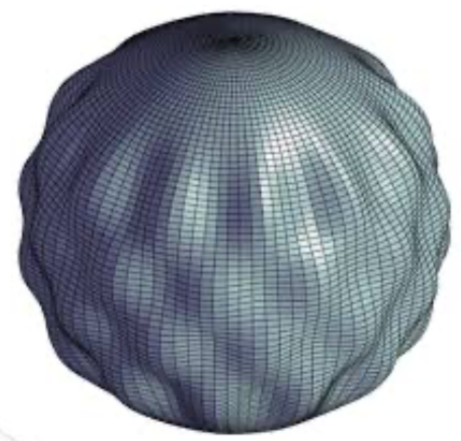


Credit: Tanja Hinderer

($\omega_\alpha \gg m\Omega_{\rm orb}$, (deformation of the NS)

$$Q_{ij} = -\lambda \mathcal{E}_{ij} \quad \lambda = \frac{2}{3} k_2 R^5$$

($\omega_\alpha \simeq m\Omega_{\rm orb}$ resonance, internal fluid flows)



$$\vec{\xi}(\mathbf{r}, t) = \sum a_\alpha(t) \vec{\xi}_\alpha(\mathbf{r}) e^{i\omega t}$$

$$\ddot{a}_\alpha + \omega_\alpha^2 a_\alpha = \frac{GM' W_{lm} Q_\alpha}{a^{l+1}} e^{-im\Phi(t)}$$

Taxonomy of NS oscillation modes

- ~~**Pressure (p) modes:** drive by pressure.~~
- **Fundamental (f) modes:** the first (nodeless) p-mode.
- **Gravity (g) modes:** driven by buoyancy (thermal/ composition gradients or discontinuity in density).
- **Interface (i) modes:** driving by discontinuity in shear modulus.
- **Shear (s) and torsional (t) modes:** driven by elastic force in the crust
- ~~**Spacetime (w) modes:** akin to BH QNMs, need dynamical spacetime (non-existent in Newtonian)~~

Possible observational consequences

Dephasing in GWs

$$E_{\rm mode}^{\rm max} \simeq 3\times 10^{49} \times q\left(\frac{2}{1+q}\right)^{5/3} \times R_{12}^2 M_{1.35}^{-2/3}\left(\frac{f_\alpha}{200\ \mathrm{Hz}}\right)^{\frac{1}{3}}\left(\frac{|Q_{nl}|}{0.01}\right)^2 \mathrm{erg}$$

$$\frac{\Delta\Phi}{2\pi} \approx -\frac{t_D}{t_{\rm orb}}\frac{E_{\rm mode}^{\rm max}}{|E_{\rm orb}|}$$

[Shibata 1993,Lai 1994,Kokkotas+1995]

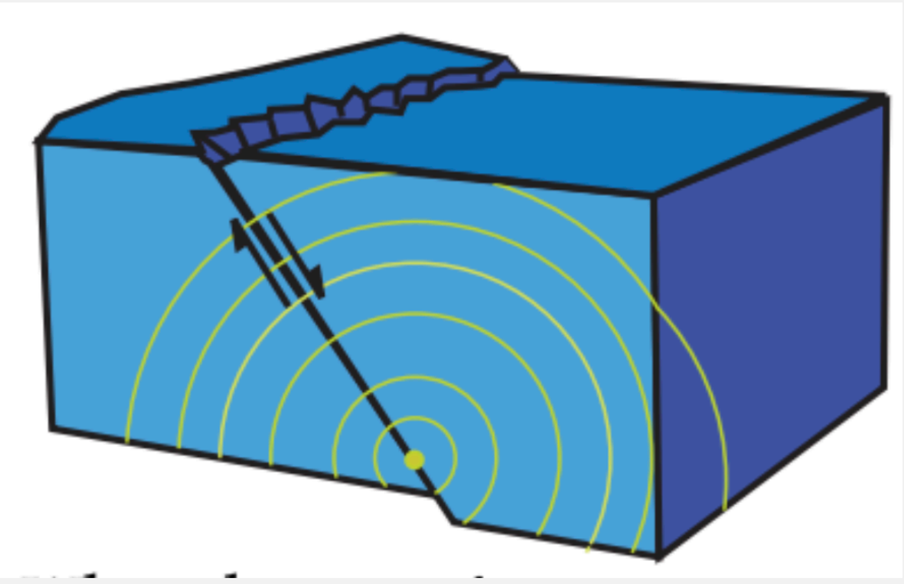
$$= -0.1 M_{1.35}^{-4} R_{12}^2 \frac{2}{q(1+q)}\left(\frac{f_\alpha}{200\ \mathrm{Hz}}\right)^{-2}\left(\frac{|Q_{nl}|}{0.01}\right)^2$$

- **Which oscillation mode is more likely to contribute to the GW phase shift and to induce crustal overstrain, or efficiently channel energy into the crust?**

Coupling to the magnetosphere: magneto-elastic oscillations and possible EM emissions??

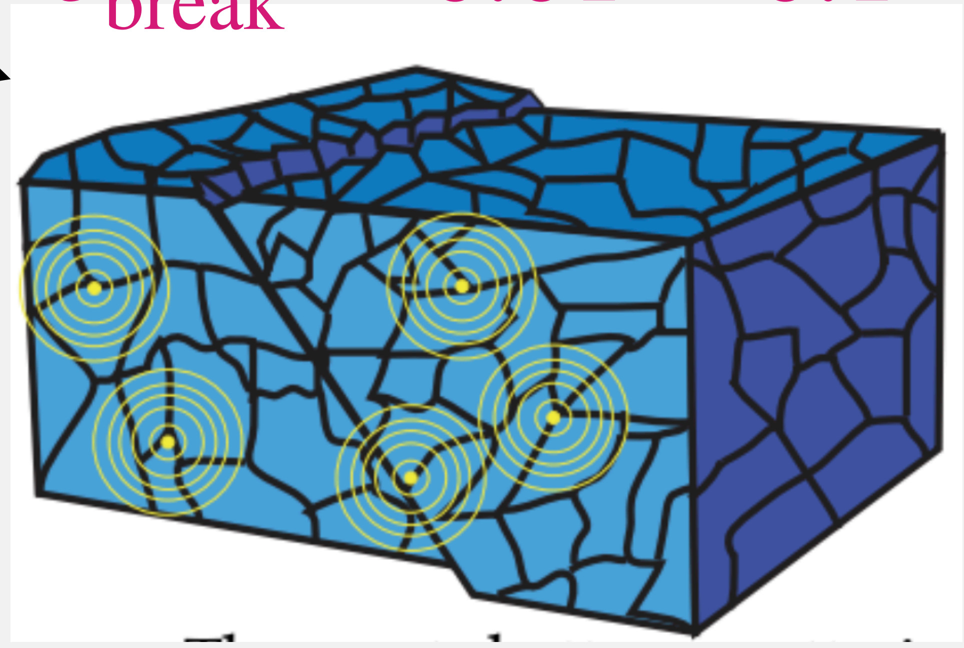
Credit: David Tsang

[Tsang+2012, Kuan+2022,2023, Suvorov+2024, Most+2024]

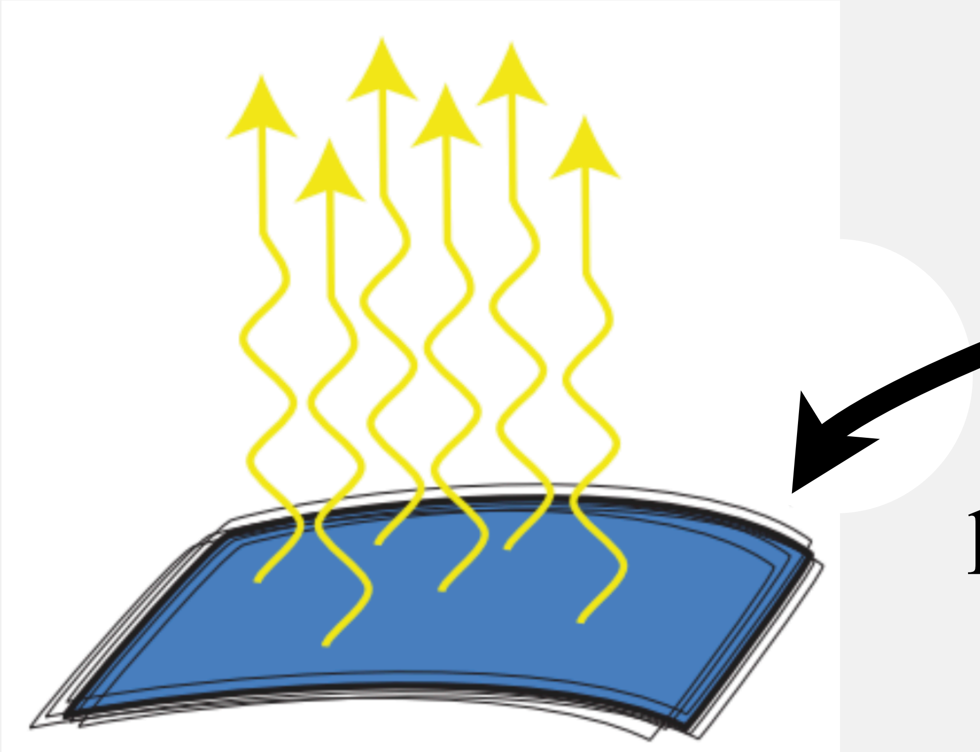


Crust fractures due to overstrain

$\sigma_{\rm break} \sim 0.01 - 0.1$



The crust breaks (star quake), scattering mode energy into high frequency



Couple to the magnetosphere, perturbation in magnetic field, accelerate particles, and lead to **precursor emission**

Motivation and what we have done (Gao+2026)

- The answer critically rely on the **matter description** and **relativistic formulation of oscillations and tidal response**
- **Matter description:** **Stratification** from ordinary baryons to possibly exotic particles such as hyperons or quarks, as well as by distinct phase structures, such as **solid** and superfluid. **Entangled, and can give interesting coupling effects**
- **Relativistic formulation:** mode oscillations and tidal coupling. Fully linear-GR calculations **guarantee the mode orthogonality**, relativistic tidal coupling is vital to **determine the energy budget** ($E_{\rm mode} \propto Q_{n\ell}^2 \omega_{\rm gw}^4 / \omega_\alpha^2$)

- Using nuclear-theory based, unified equation of state ([Xia+2018, Niu+2025]), with solid crust and composition stratification
- Applying relativistic perturbation theory to study the mode spectrum and tidal coupling, **give a first-step picture of the tidal resonance problem**

Mode spectrum of non-rotating NS ($l = m = 2$)

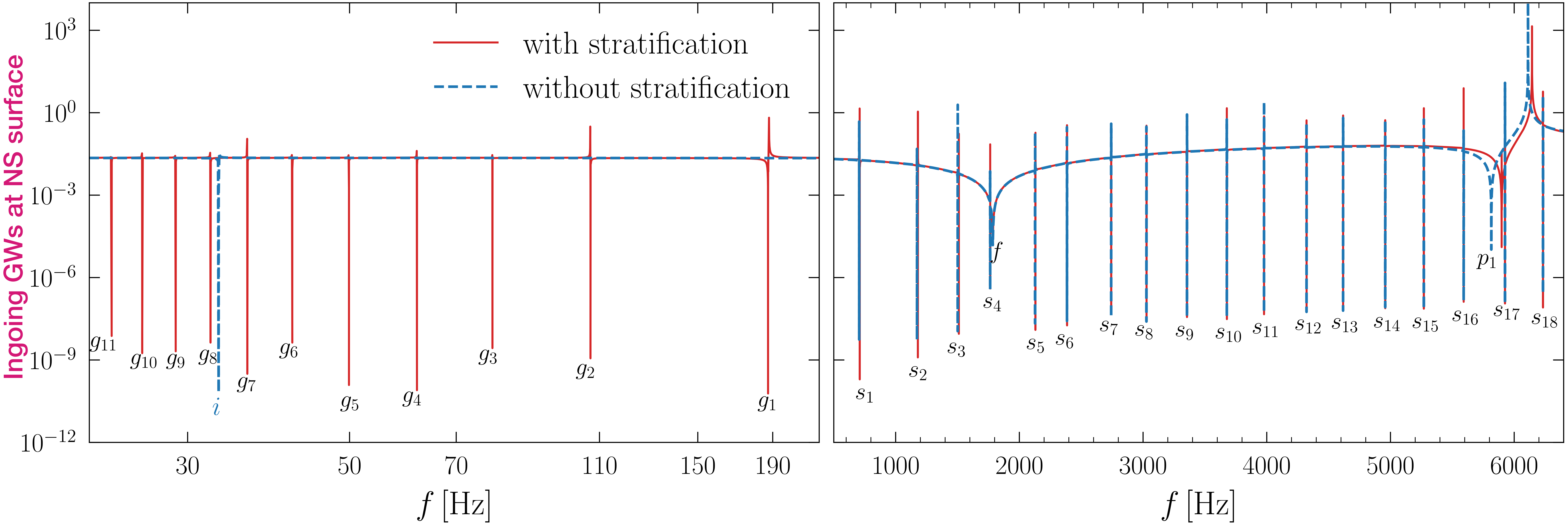
$$\Psi = A_{\rm in}\Psi^+ + A_{\rm out}\Psi^-$$

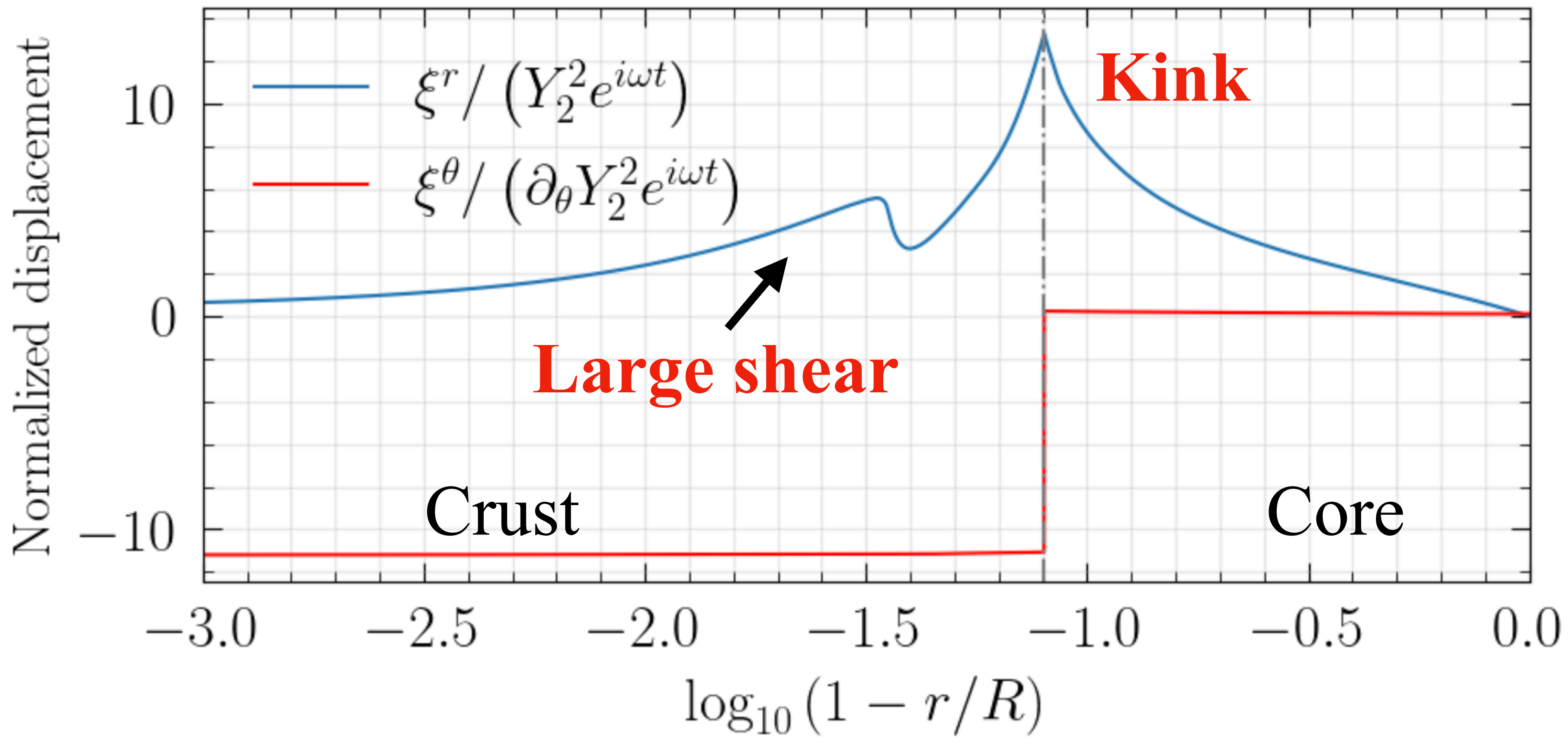
[Detweiler & Lindblom 1983, 1985, Andersson+1995]

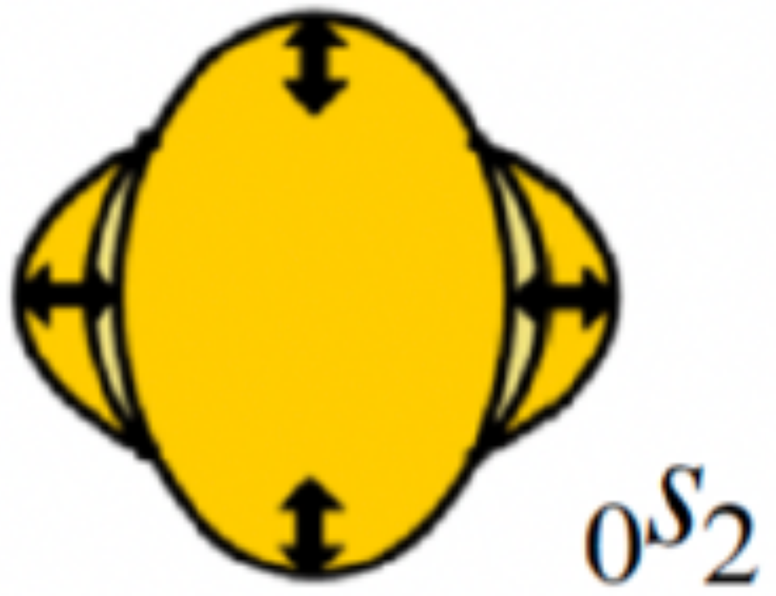
[Carter & Quintana 1972, Finn 1992, Yoshida 2002, Kruger+2014, 2024]

Quasinormal modes: $A_{\rm in} = 0$

Master equations in Mathematica: https://gravyong.github.io/links/

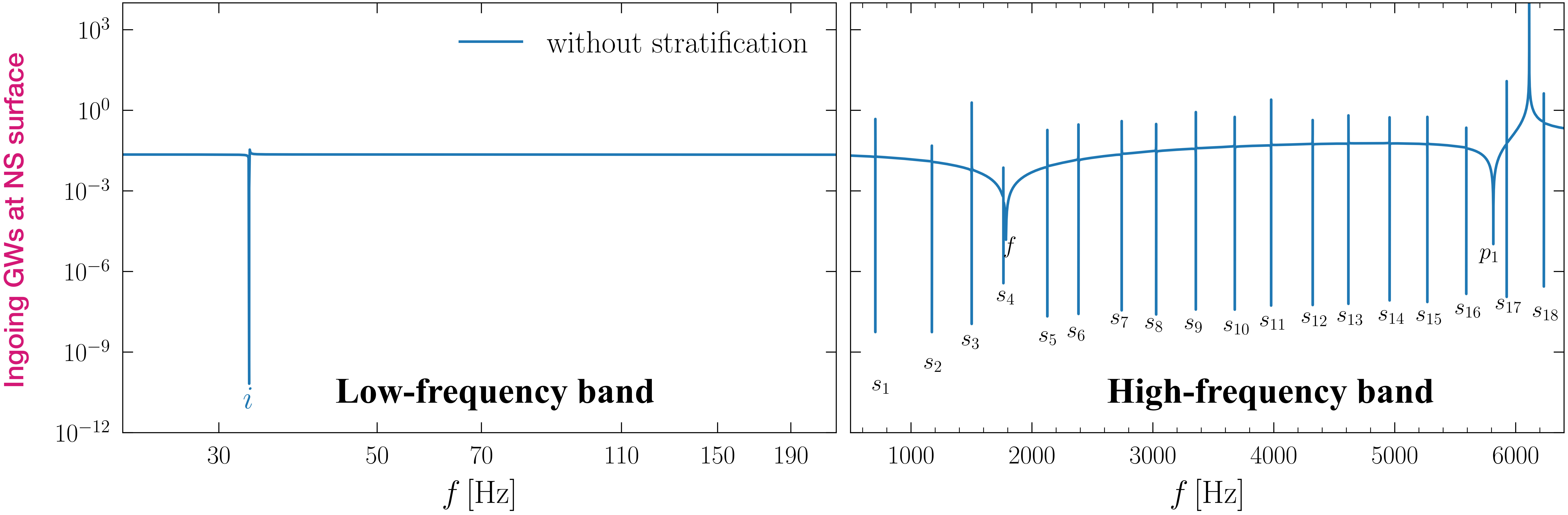


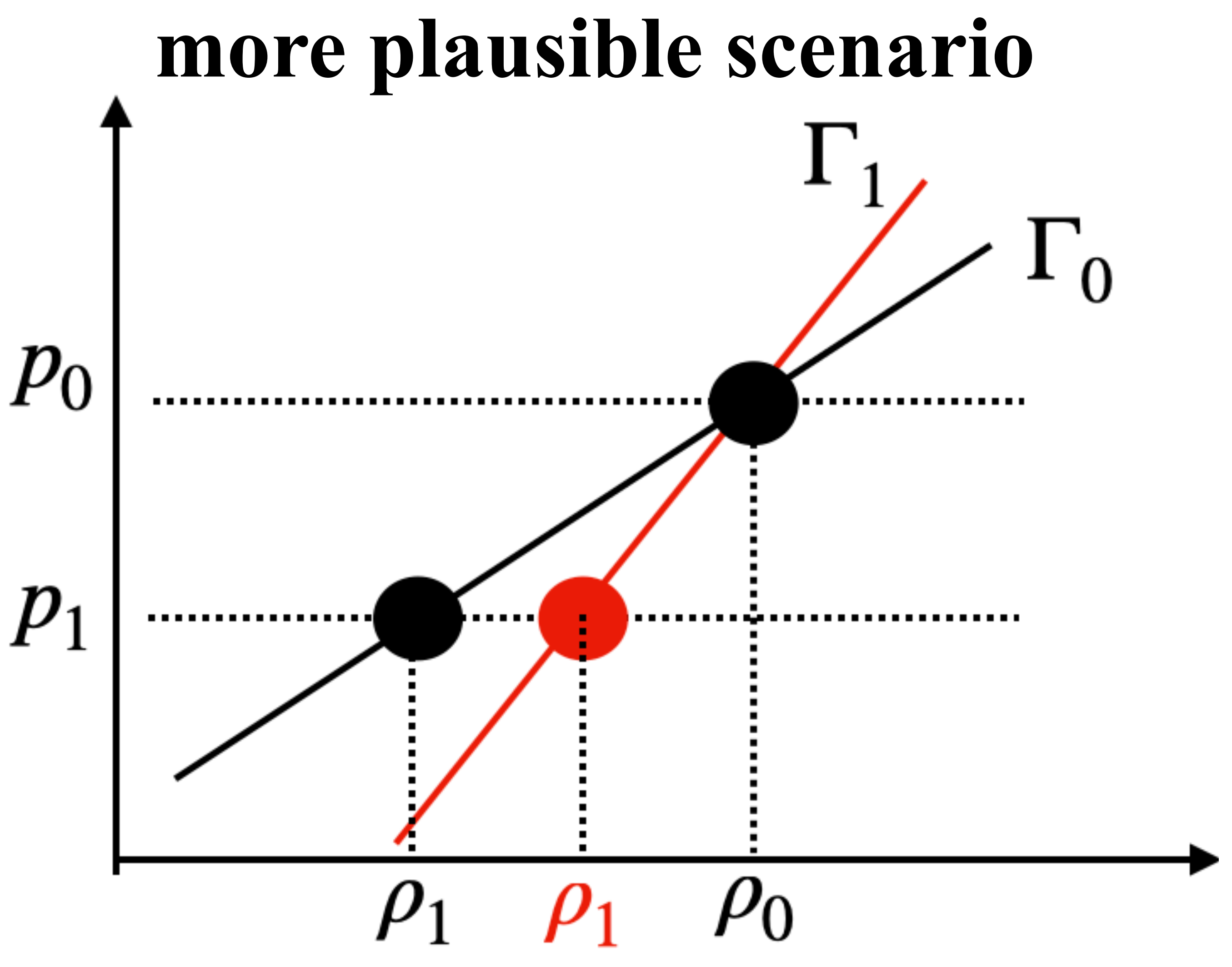


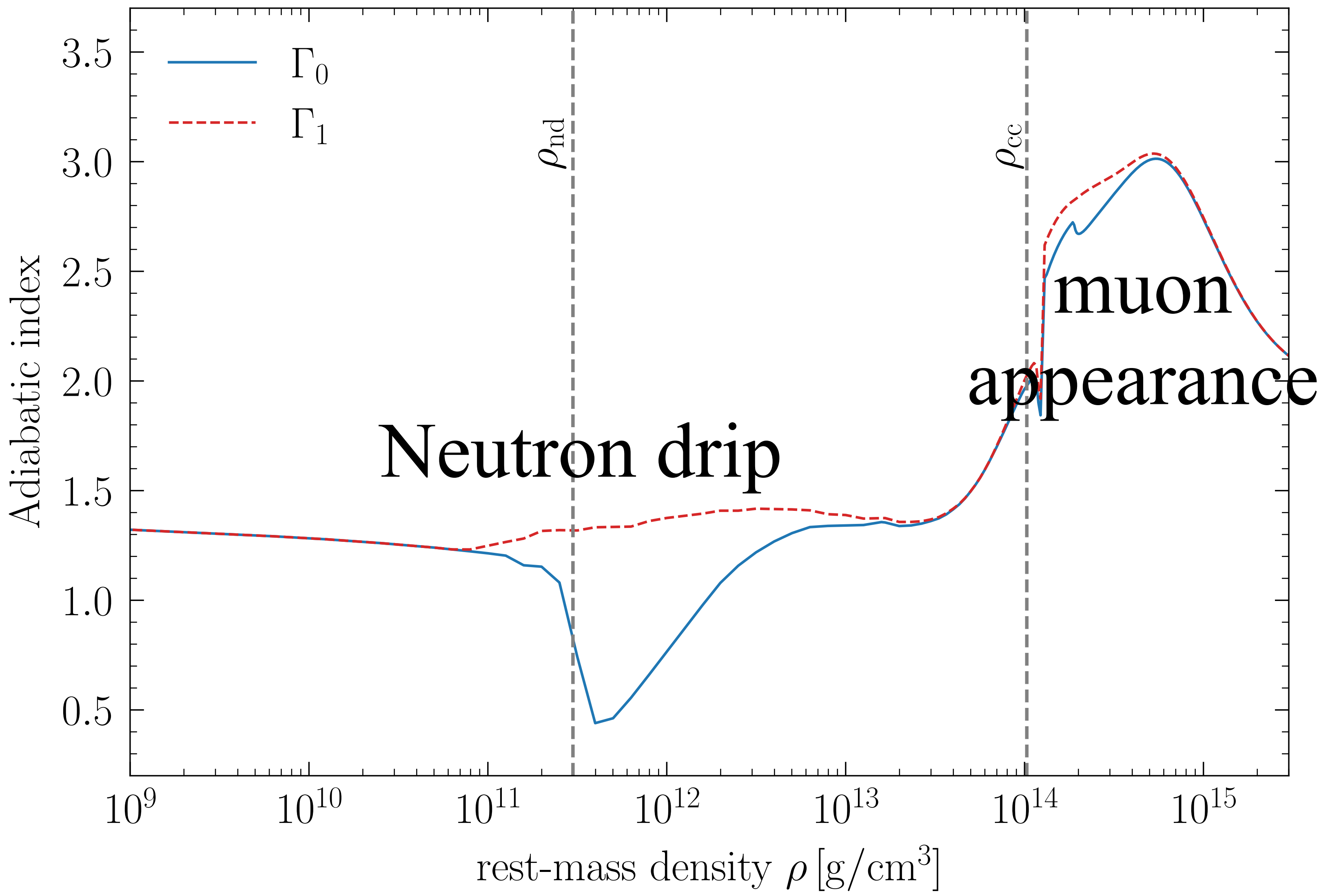


Only confined into crust

$$f \simeq 10^3 \left(\frac{R - R_{\rm cc}}{\lambda}\right)\left(\frac{1\ \rm km}{R - R_{\rm cc}}\right)\left(\frac{v_{\rm s}}{10^8\ \rm cm/s}\right) \rm Hz$$

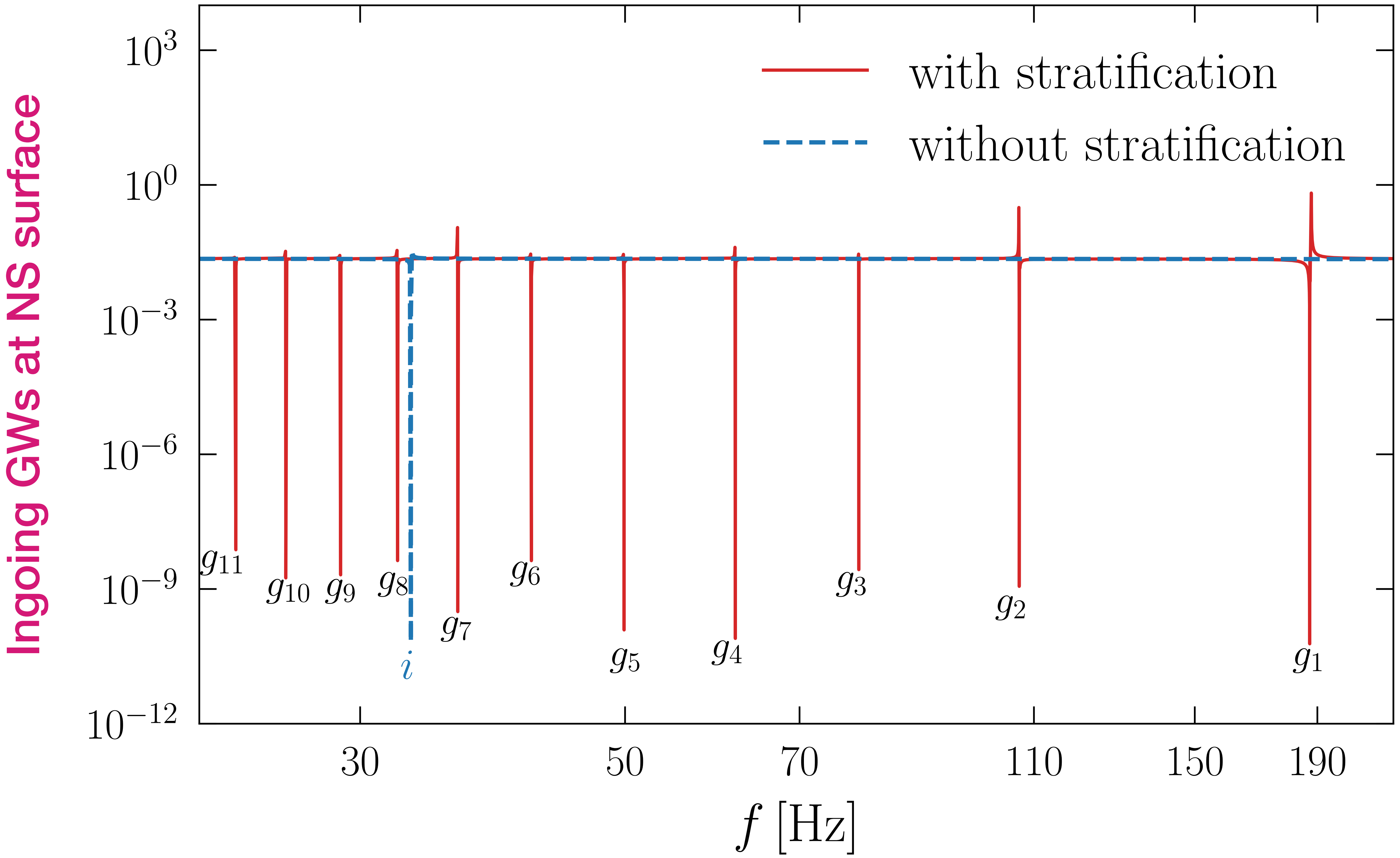


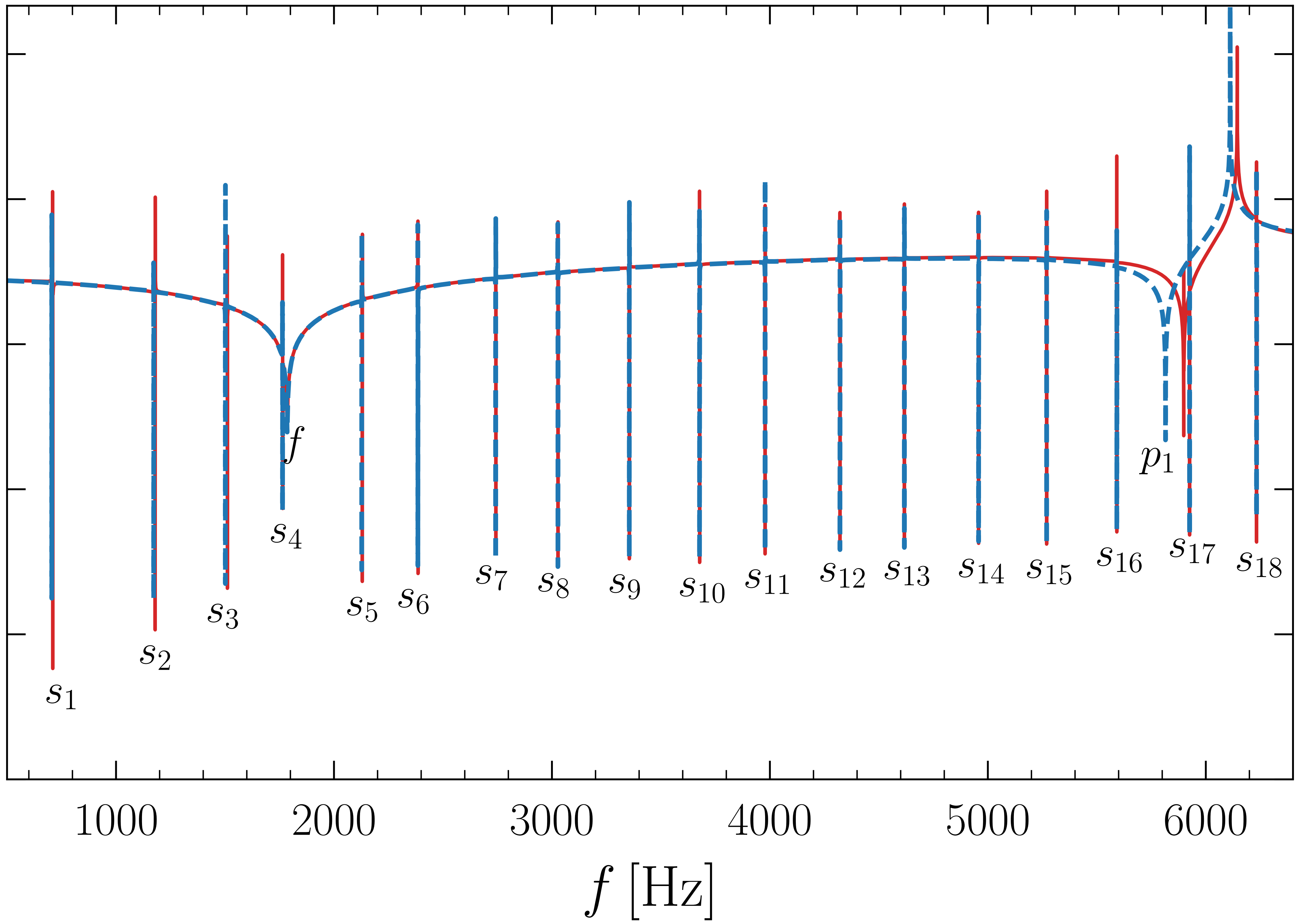




- Only core *g*-modes, crust *g*-modes not appear
- Canonical i-mode disappear for stratified model

Interplay between elasticity and composition gradient

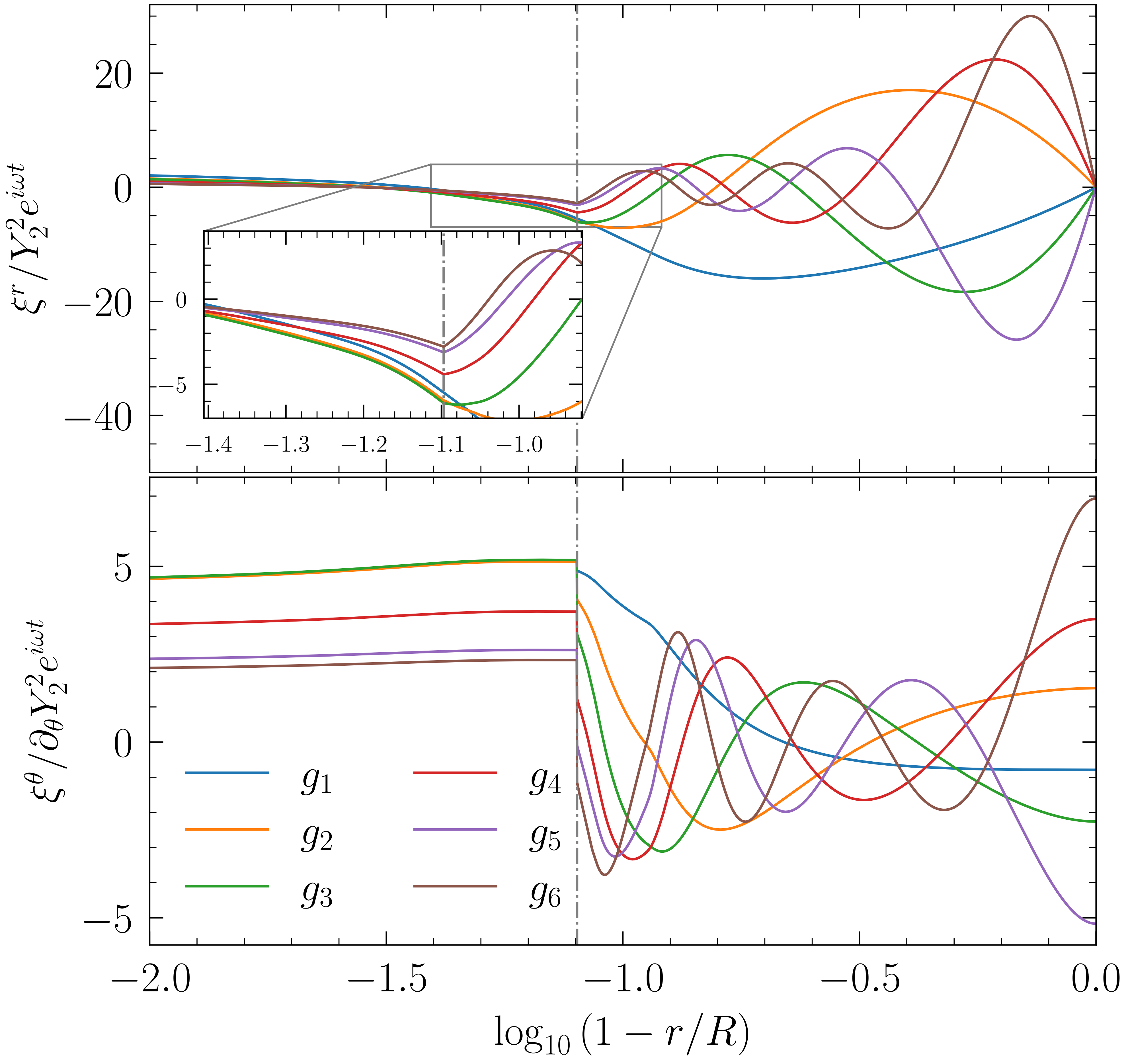




Interplay of elasticity and stratification—disappearance of *i*-mode

Restoring force of low-frequency mode = buoyancy + elastic force

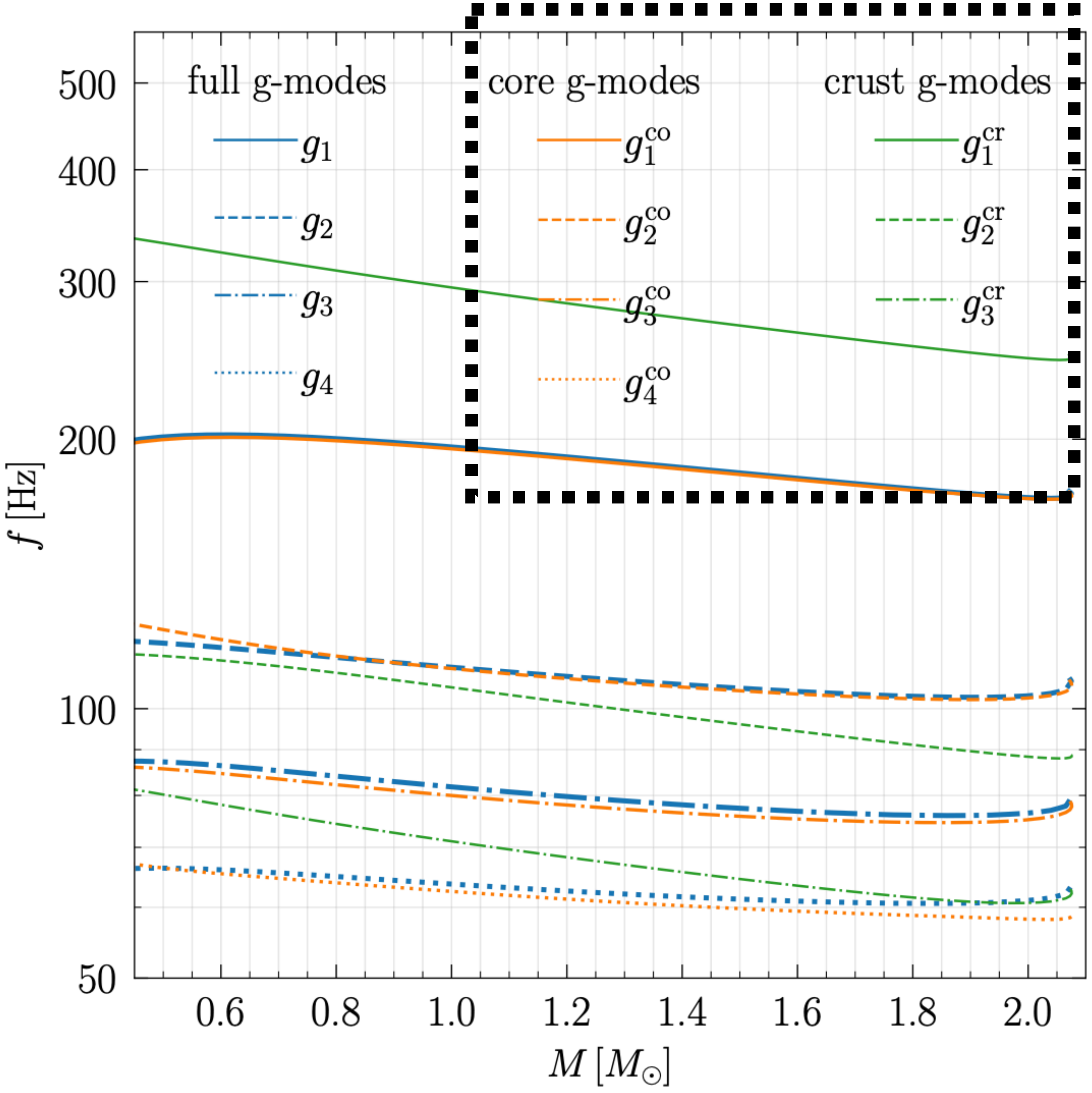
gravito-interface mode dominated by *g*-mode



- Replaced by compositional gravity modes with mixed gravity–interfacial character, driven primarily by strong buoyancy in the outer core.
- The precursor or even crust melting driving by interface mode is not physical anymore (*[Tsang+2012, 2013, Neil+2023, 2024, Zhou+2022, Pan+2022]*)

Interplay of elasticity and stratification—mode penetration into crust

Remove elasticity by hand



Criterion for penetration:

$$(\epsilon + p)\omega^2\xi^\theta \gg \frac{\mathrm{d}\delta\pi_r^\theta}{\mathrm{d}r} \sim \mu\frac{\mathrm{d}^2\xi^\theta}{\mathrm{d}r^2} \sim \mu k^2\xi^\theta$$

Horizontal acceleration of mode

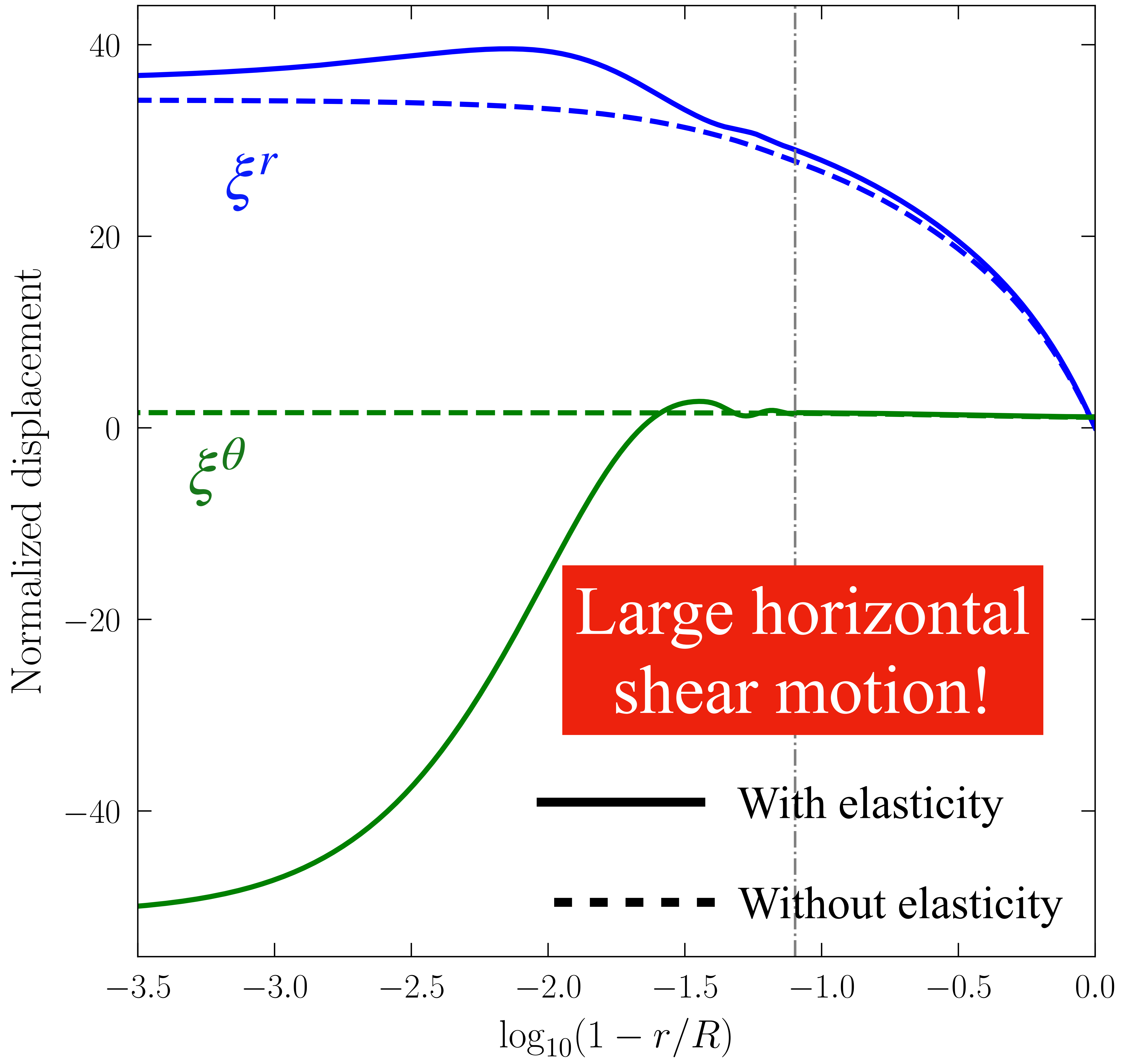
Resistance of the solid crust due to shear force

$$f_{\mathrm{crit}} = \left(\frac{\mu}{\epsilon + p}\right)^{1/2} k \approx 100\left(\frac{v_s}{10^8\ \mathrm{cm\ s^{-1}}}\right)\left(\frac{R}{\lambda}\right)\,\mathrm{Hz}$$

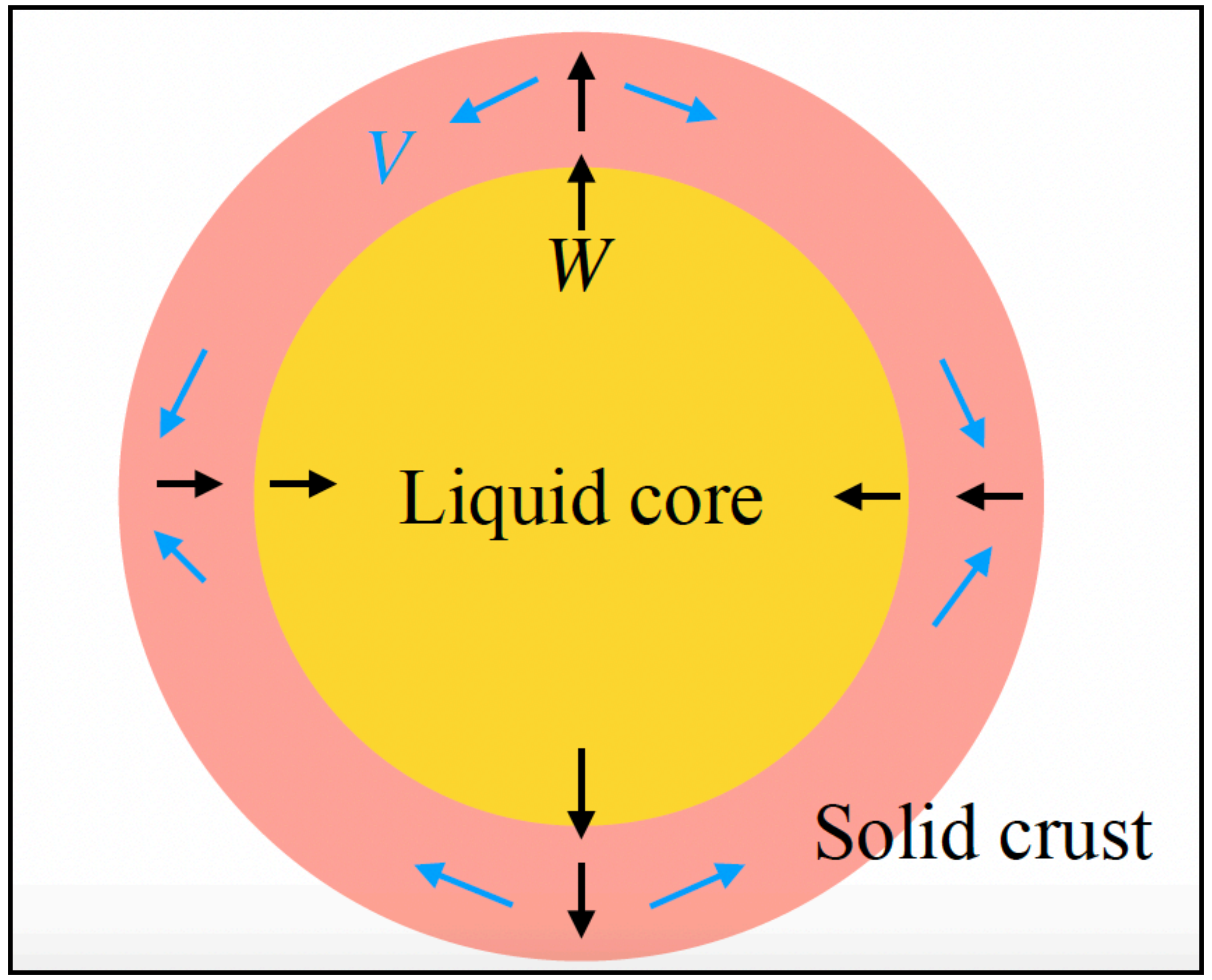
- Core g-mode, $\lambda \sim R, f_{\mathrm{crit}} \sim 100\,\mathrm{Hz}$, amplitude is suppressed
- Crust g-mode, $\lambda \sim 0.1R, f_{\mathrm{crit}} \sim 1000\,\mathrm{Hz}$, **cannot penetrate, disappear from the spectrum**

How about *f*-mode?

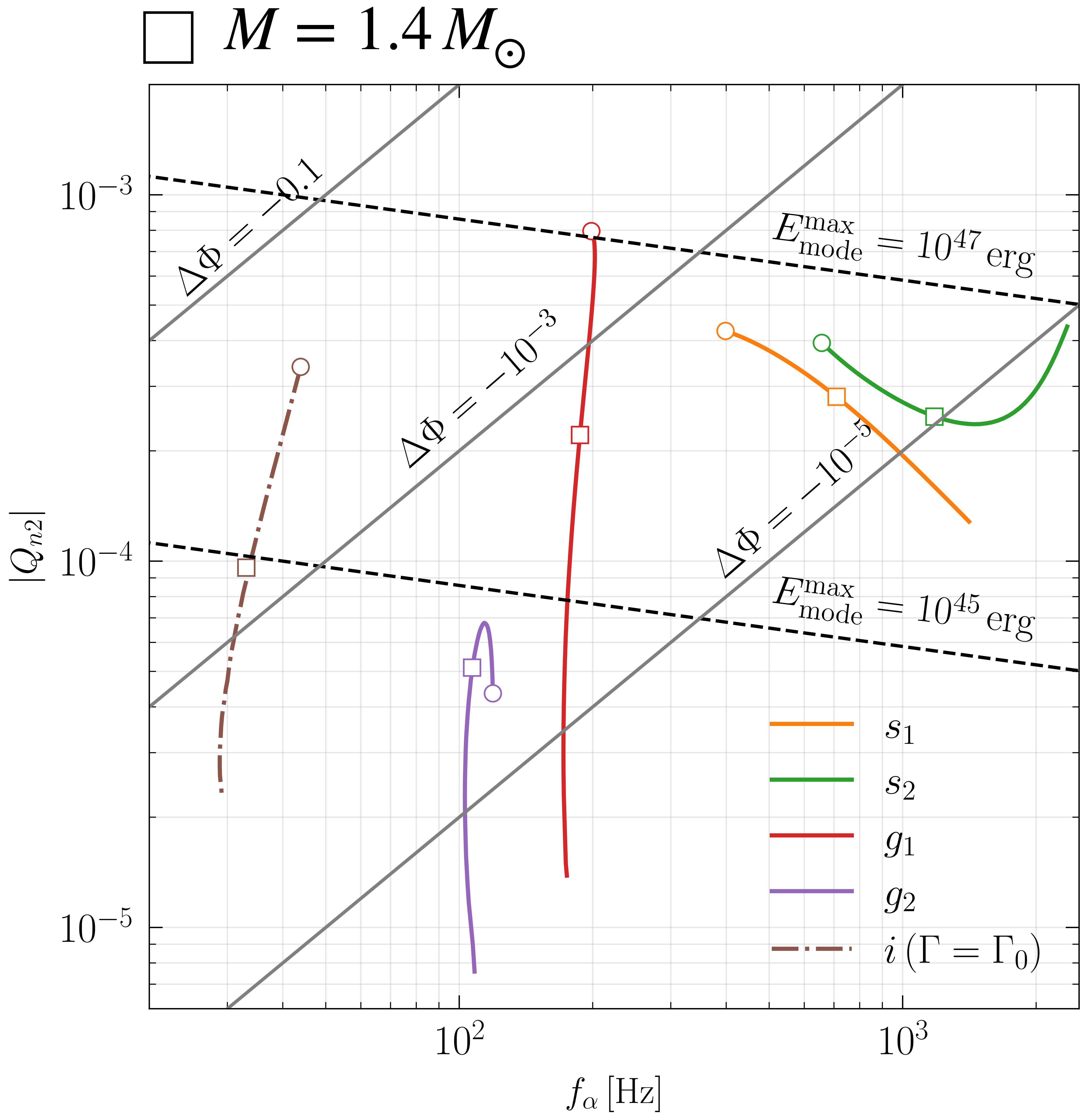
$f \sim 2000 - 3000\,\text{Hz} \gg f_{\text{crit}}$

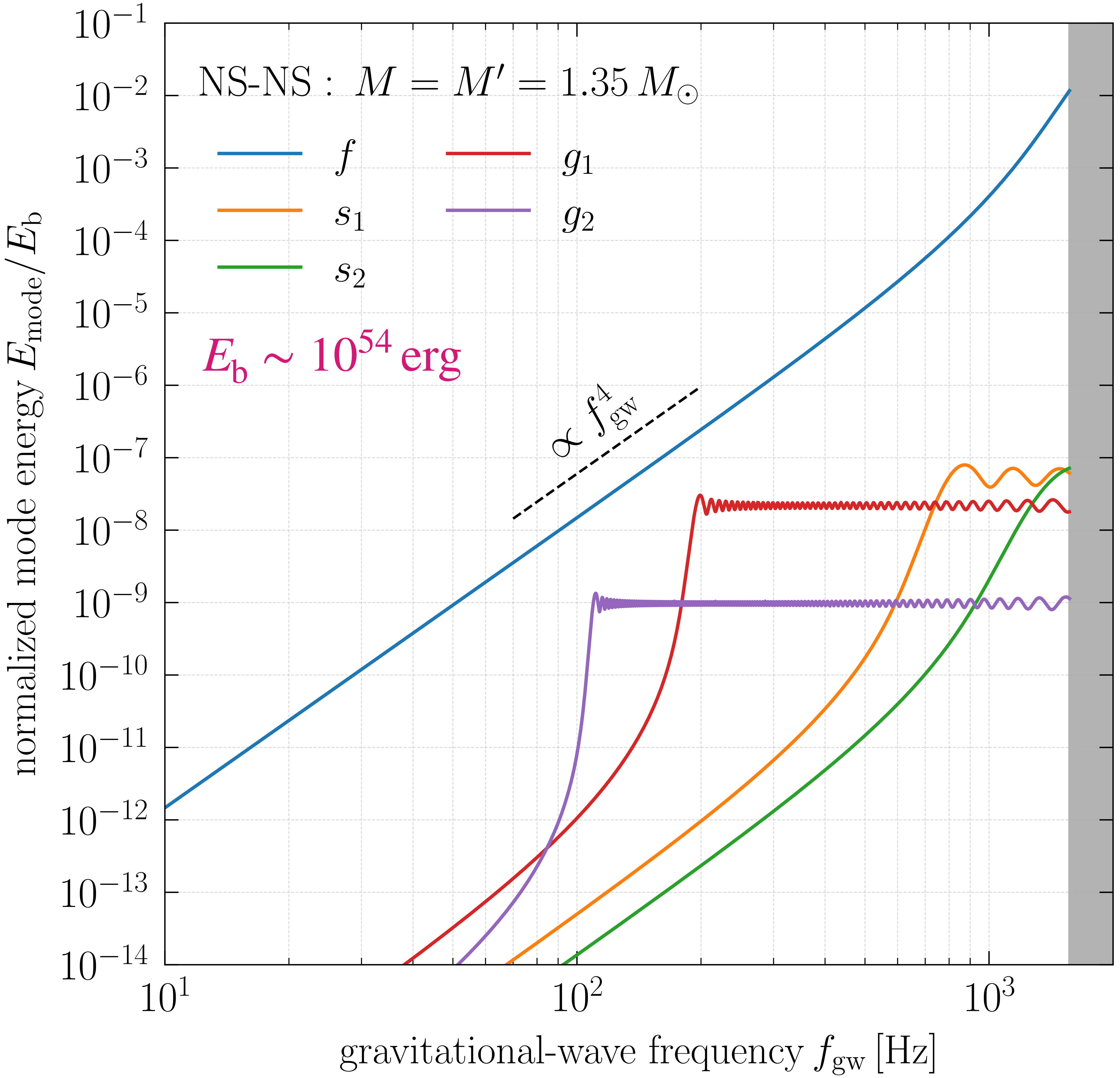


- For pure fluid case, it's not possible that *f*-mode induces large shear motions in the crust region
- Once includes solid crust, the elastic force play a role

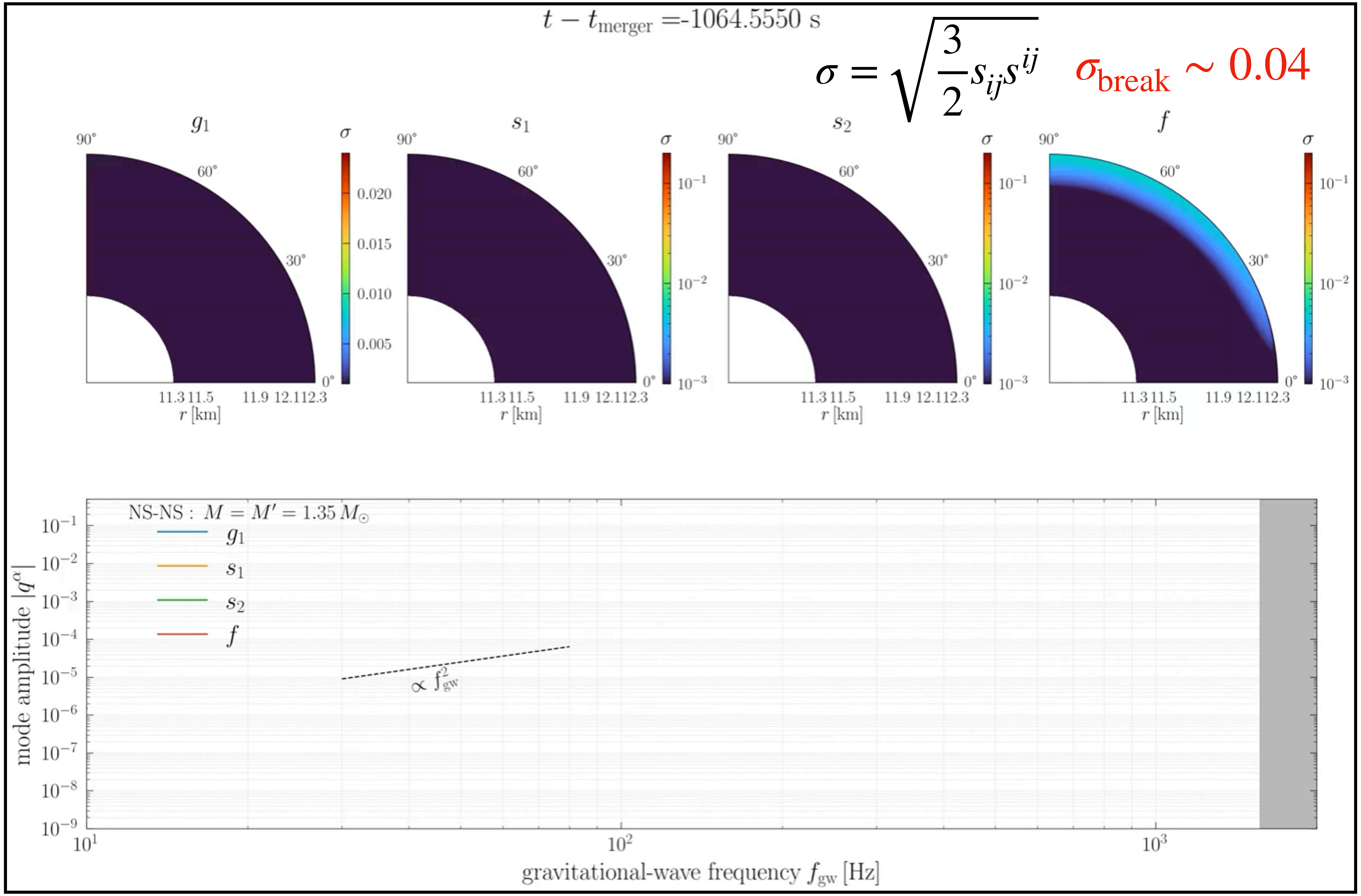


Dynamical tides for different modes



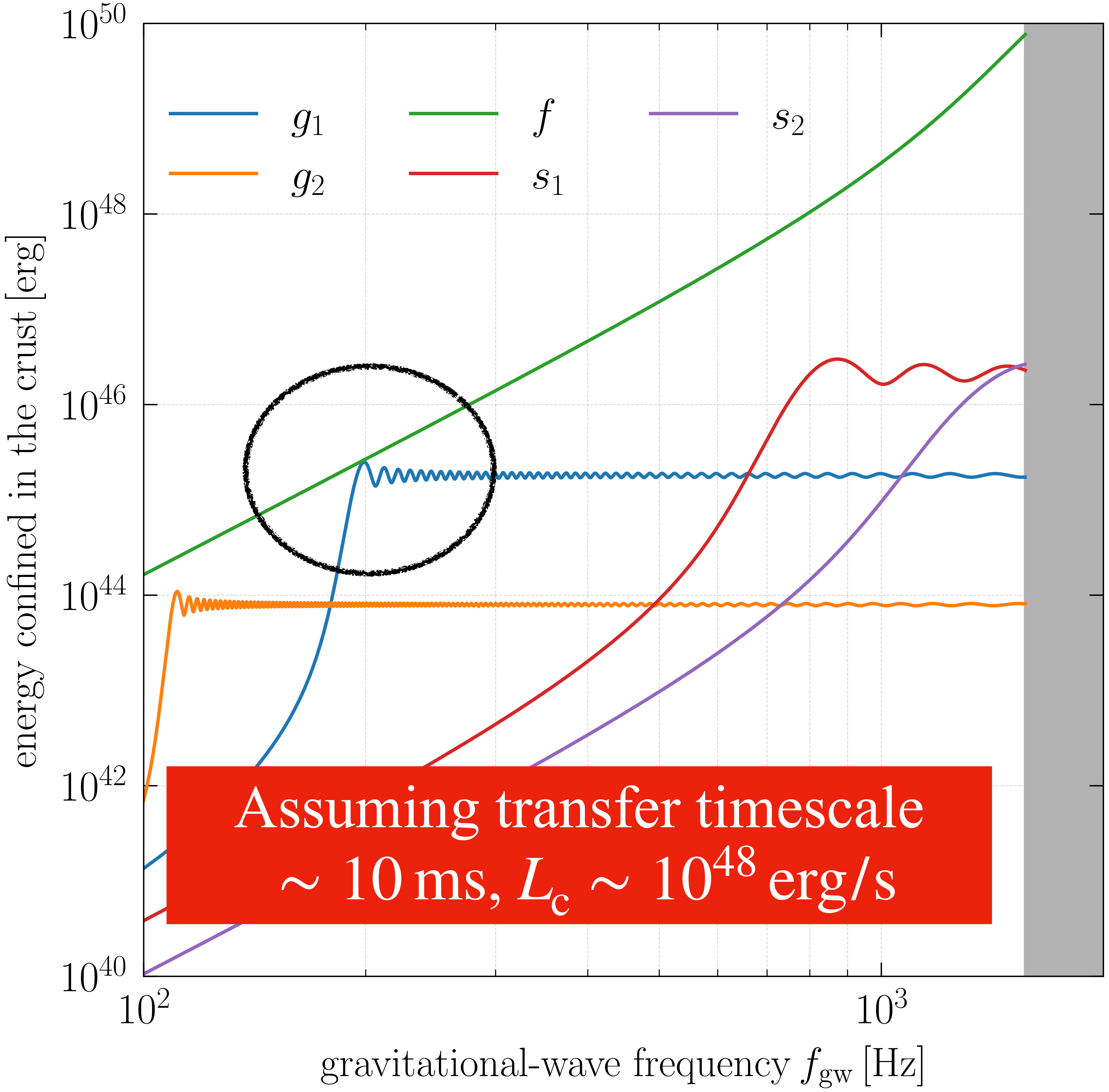


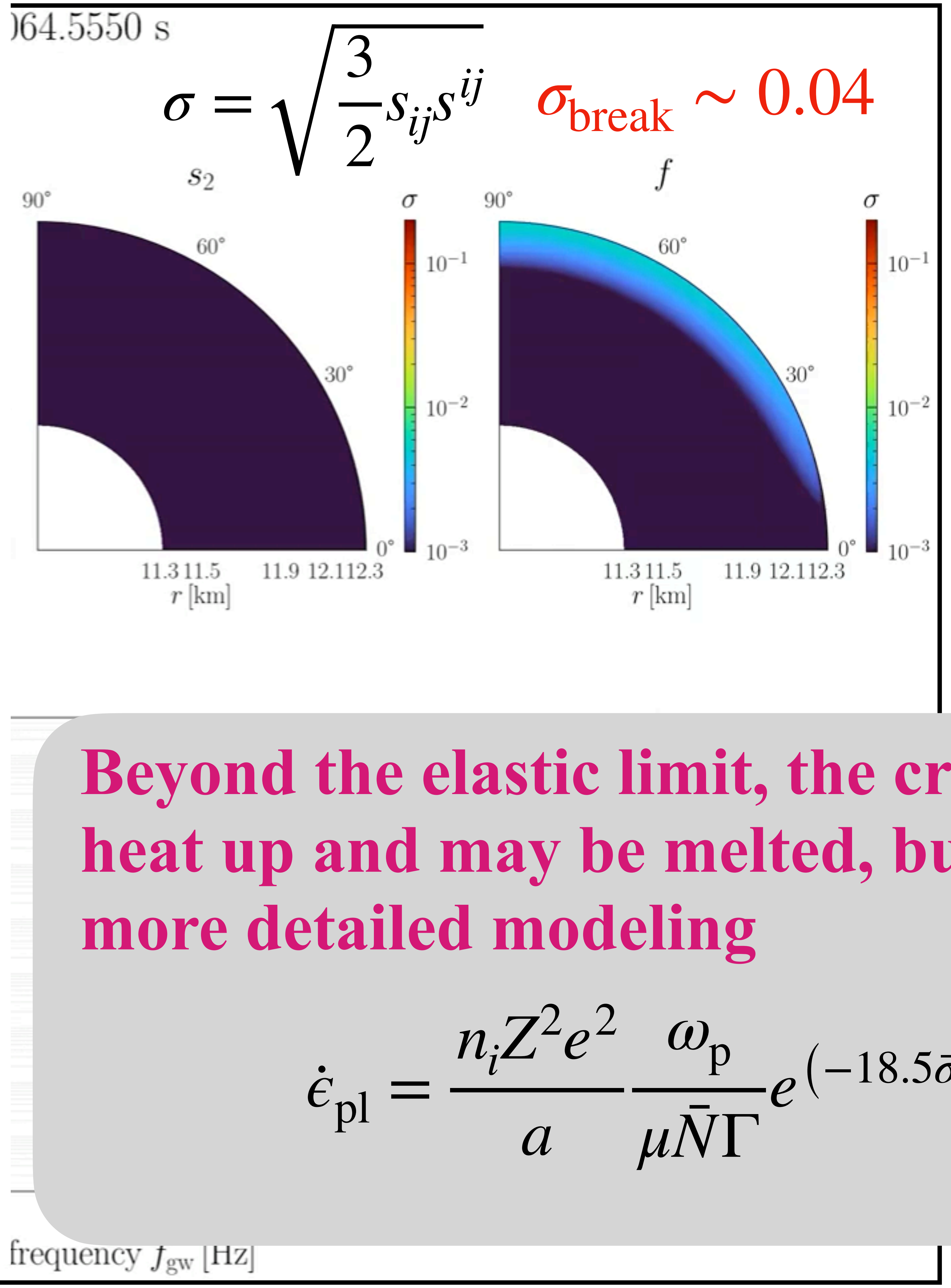
Accumulations of stress and overstrain of the crust



Direction of the companion

Accumulations of stress and overstrain of the crust





Direction of the companion

Beyond the elastic limit, the crust can heat up and may be melted, but needs more detailed modeling

$$\dot{\epsilon}_{\rm pl} = \frac{n_i Z^2 e^2}{a} \frac{\omega_{\rm p}}{\mu \bar{N} \Gamma} e^{(-18.5\bar{\sigma}_b + \bar{\sigma}\bar{N})\Gamma}$$

Gravity sector: Formulation of relativistic tidal response

- Most studies adopted the Newtonian version or relativistic analogy to Newtonian ones

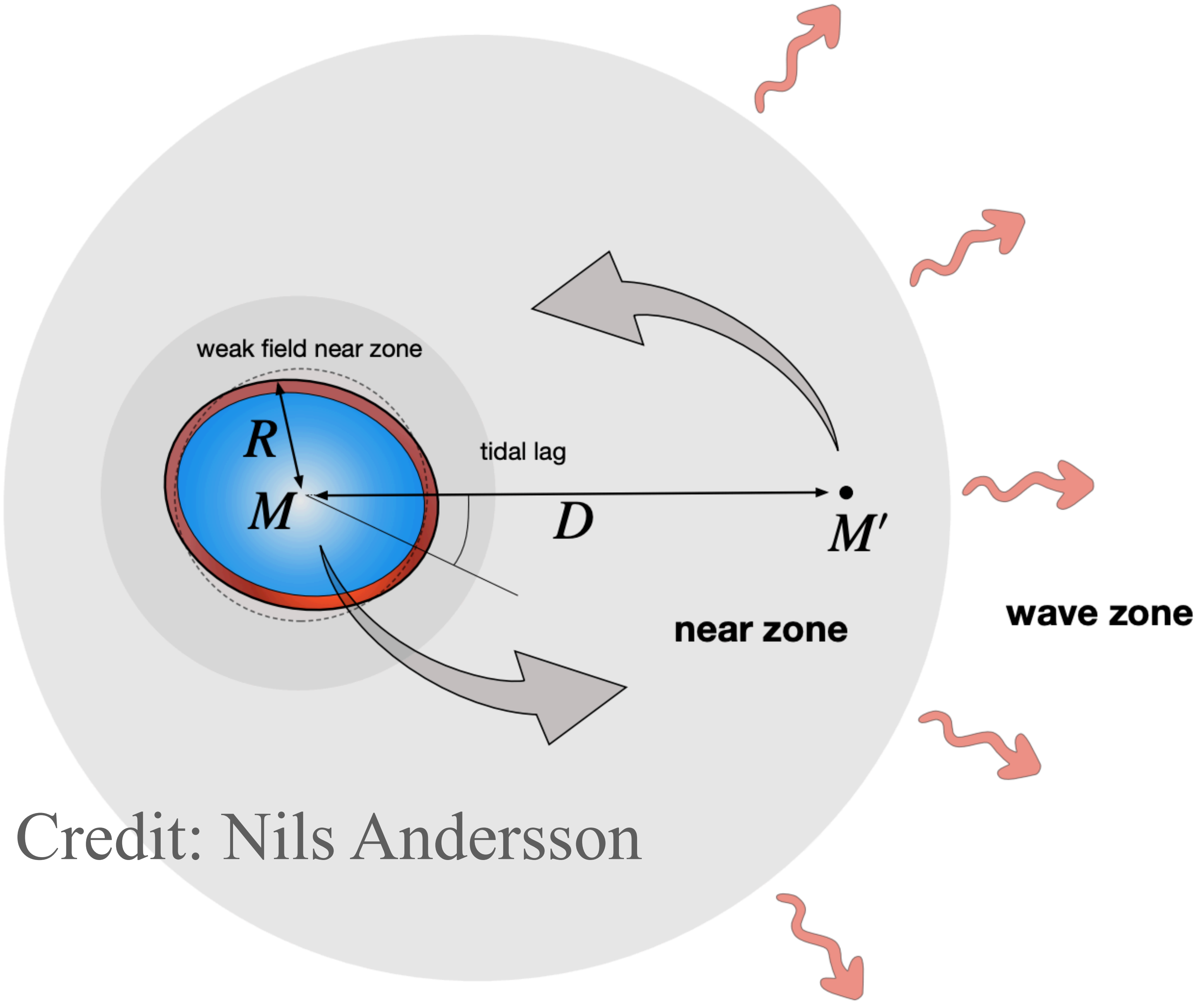
[Kuan+2021, Counsell+2024, 2025, Gao+2025…]

$$\ddot{a}_\alpha + \omega_\alpha^2 a_\alpha = \frac{GM'W_{lm}Q_{n\ell}}{a^{l+1}} e^{-im\Phi(t)} \qquad Q_{n\ell} = \frac{1}{MR^\ell} \int (\epsilon + p)\bar{\xi}^i_\alpha \nabla_i \left(r^\ell Y^m_\ell\right) \sqrt{-g}\, \mathrm{d}^3x$$

- Project the Newtonian tidal potential onto relativistic eigenfunctions, with corrections on volume and inertial mass

- **Good approximation for *f*-mode, but may not be accurate enough for other low-frequency modes, can be several orders of difference by slightly different definitions, leading to unreliable** $E_{\mathrm{mode}} \sim \left|Q_{n\ell}\right|^2 \omega_{\mathrm{gw}}^4/\omega_\alpha^2$

Formulation of relativistic tidal response



Credit: Nils Andersson

$$S_{\rm eff} = \int d\tau \left[-m - \frac{1}{2} E_{ab} Q^{ab} + \dots \right] \qquad \tilde{Q}^{ab}(\omega) = -\frac{1}{2}\tilde{F}(\omega)\tilde{E}^{ab}(\omega)$$

- The homogeneous solution outside of an oscillating NS can be represented analytically by Mano-Suzuki-Takasugi (MST) solution [MST 1996]

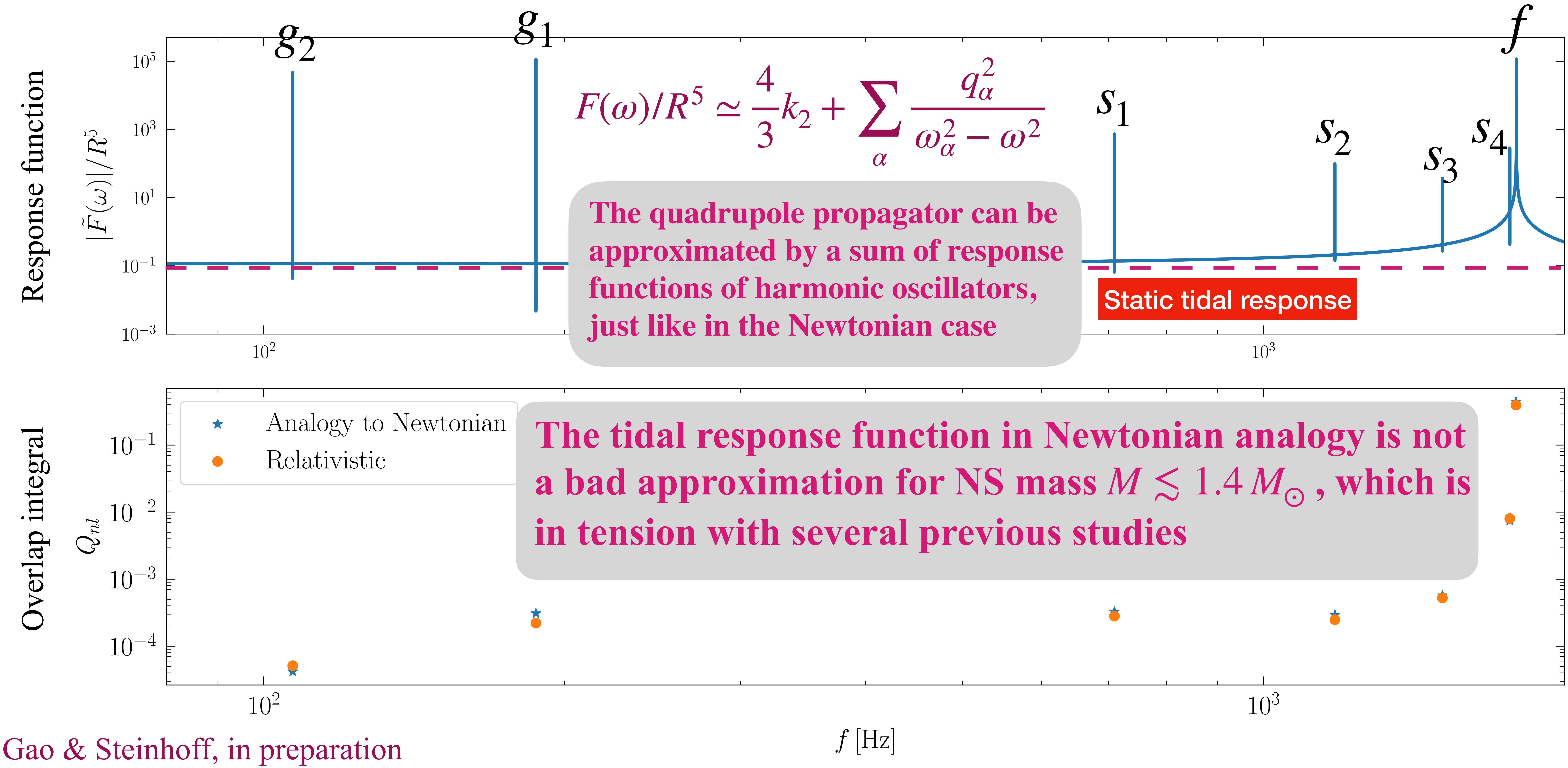
$$X = A_1 X_N^{\nu} + \epsilon^4 A_2 X_N^{-\nu-1} \qquad \epsilon = 2M\omega$$

$$\frac{d^2X}{dr_*^2} + \left[\left(1 - \frac{2M}{r}\right) \frac{l(l+1) - \frac{6M}{r}}{r^2} + \omega^2 \right] X = S[X]$$

Effective source from tidal field

$$\frac{3G}{4M^5}\tilde{F} = -\frac{428 A_2}{7 A_1} \left\{ 1 - \epsilon^2 \left[\frac{33054269}{9437400} - \frac{107}{105} \log \frac{2\omega}{\mu_0} \right] \right\} - \frac{56}{107} \left\{ 1 + \epsilon^2 \left[\frac{13138723}{18874800} - \frac{107}{105}\gamma_E \right] \right\} + \mathcal{O}\left(\epsilon^4\right)$$

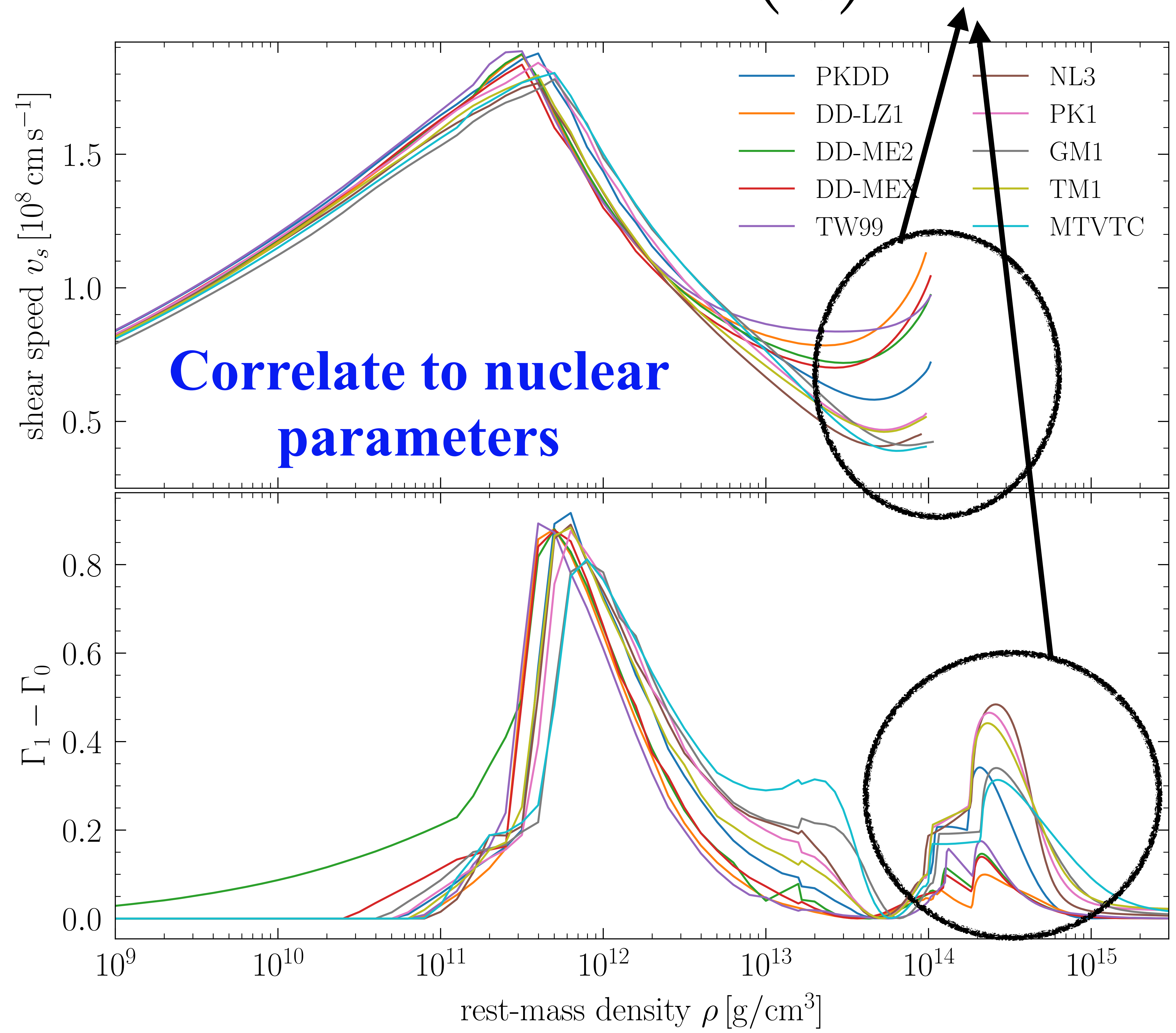
Relativistic tidal response



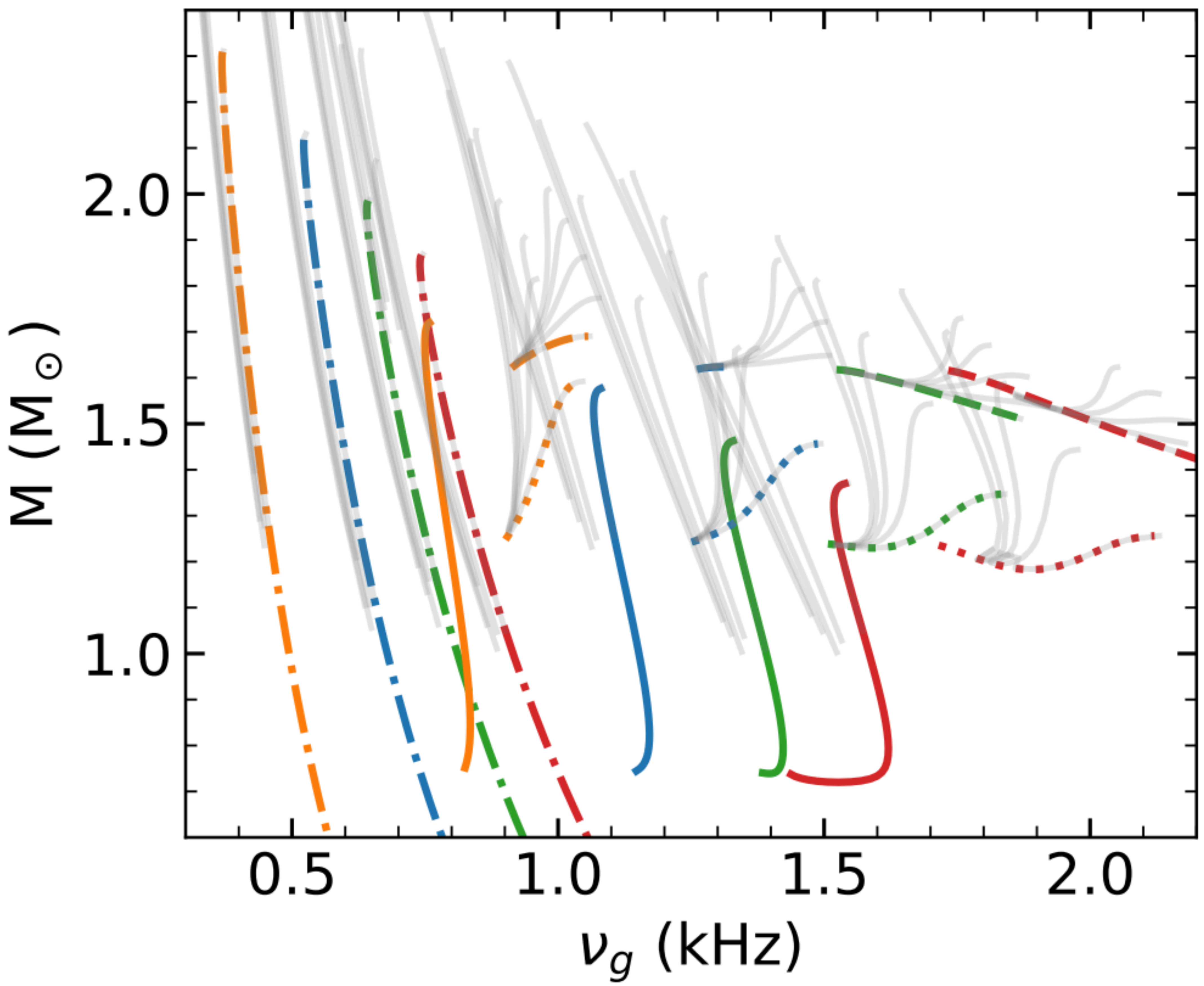
Gao & Steinhoff, in preparation

Matter sector: nuclear parameters and exotic phases

$$x_p \sim \frac{1}{b} \quad b = 3\pi^2 n \left(\frac{\hbar c}{4S}\right)^3 \approx 22 \left(\frac{n}{n_0}\right)\left(\frac{30\text{MeV}}{S}\right)^3$$



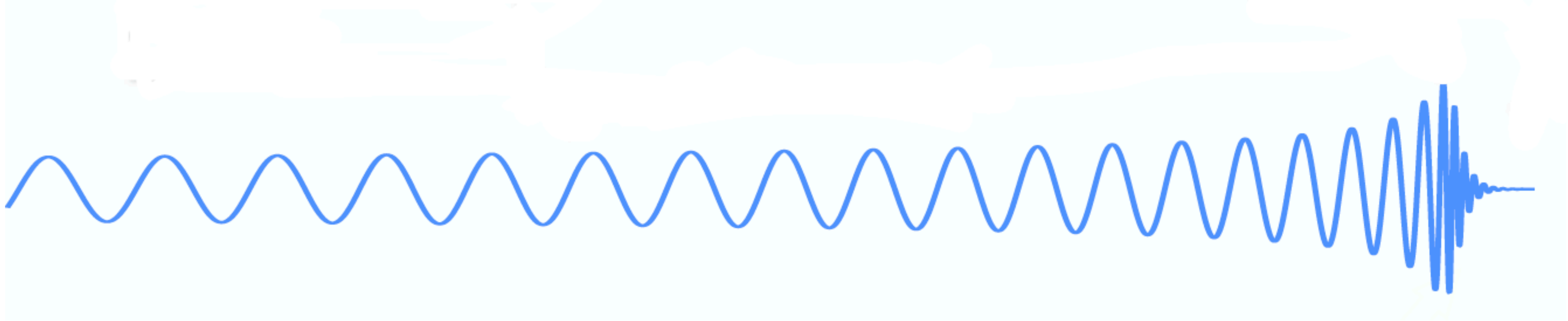
Exotic phase structures and new class of g-modes



[Zhao+2022]

Summary

- Mode spectrum—interplay between elasticity and stratification (vanishing of canonical i-mode, mode penetration into crust and induced shear for f and g)
- Dynamical tides— *f*-mode always dominate the energy contribution, potentially break the crust, and g-mode is marginal (depends on nuclear parameters but important to channel energy into precursor)
- Relativistic formulation—The tidal response is formulated in perturbative GR by using MST solutions
- Future— dependence on nuclear parameters, exotic phases, simulations of magneto-elastic oscillations and EM precursor, tidal resonance of low-frequency band modes…



Thank you for listening!