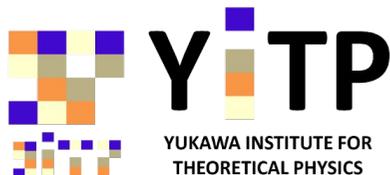


# Past and future of numerical relativity

**Masaru Shibata** (born: 04.02.1966)

Max Planck Institute for Gravitational Physics at Potsdam

Yukawa Institute for Theoretical Physics

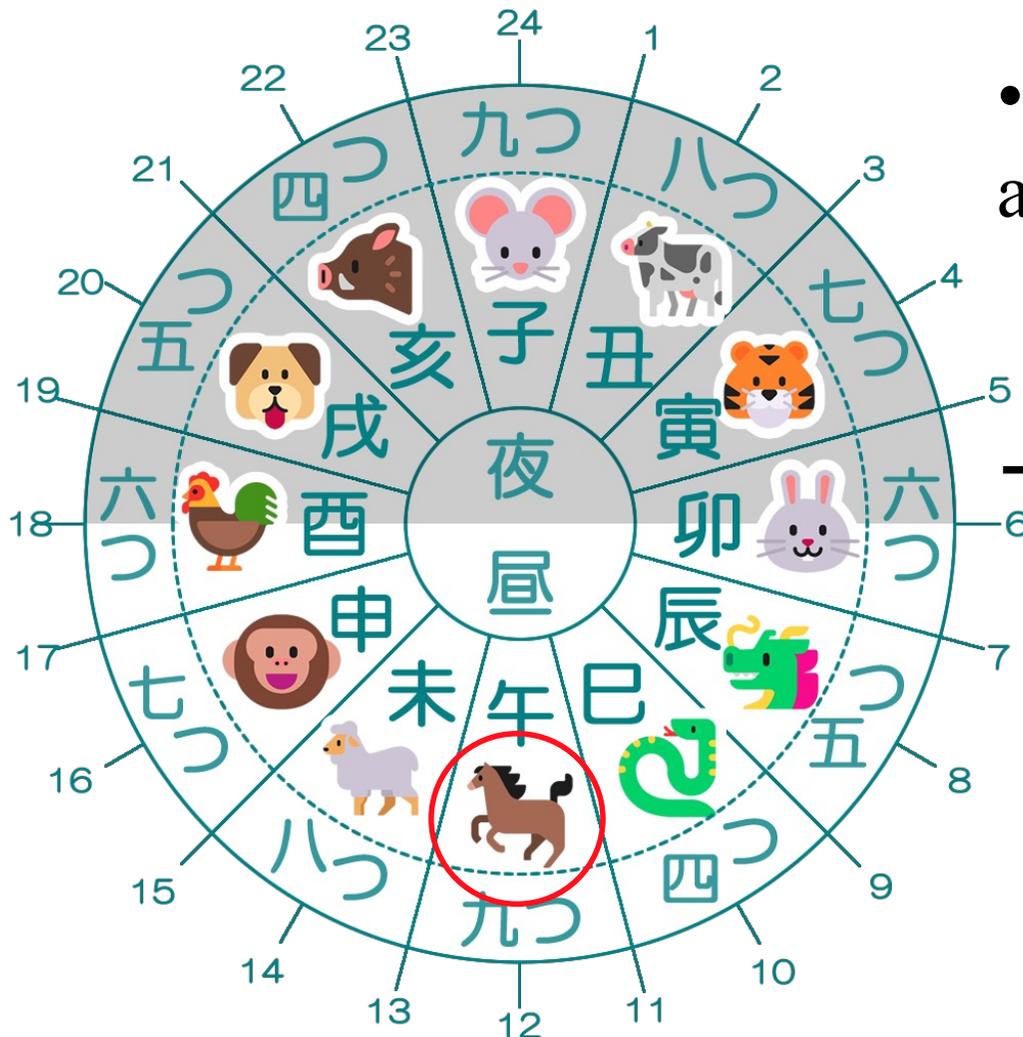


06.02.2026



# Why we celebrate 60 yrs old in Japan?

- In the Chinese astrological calendar, there are 12 yrs for each of which animals are different



- In addition, there is another cycle of 5 species; Wood, Fire, Water, Gold, and Soil  
→ The period is  $12 \times 5 = 60$  yrs

**This year is “fire-horse”.**



開運

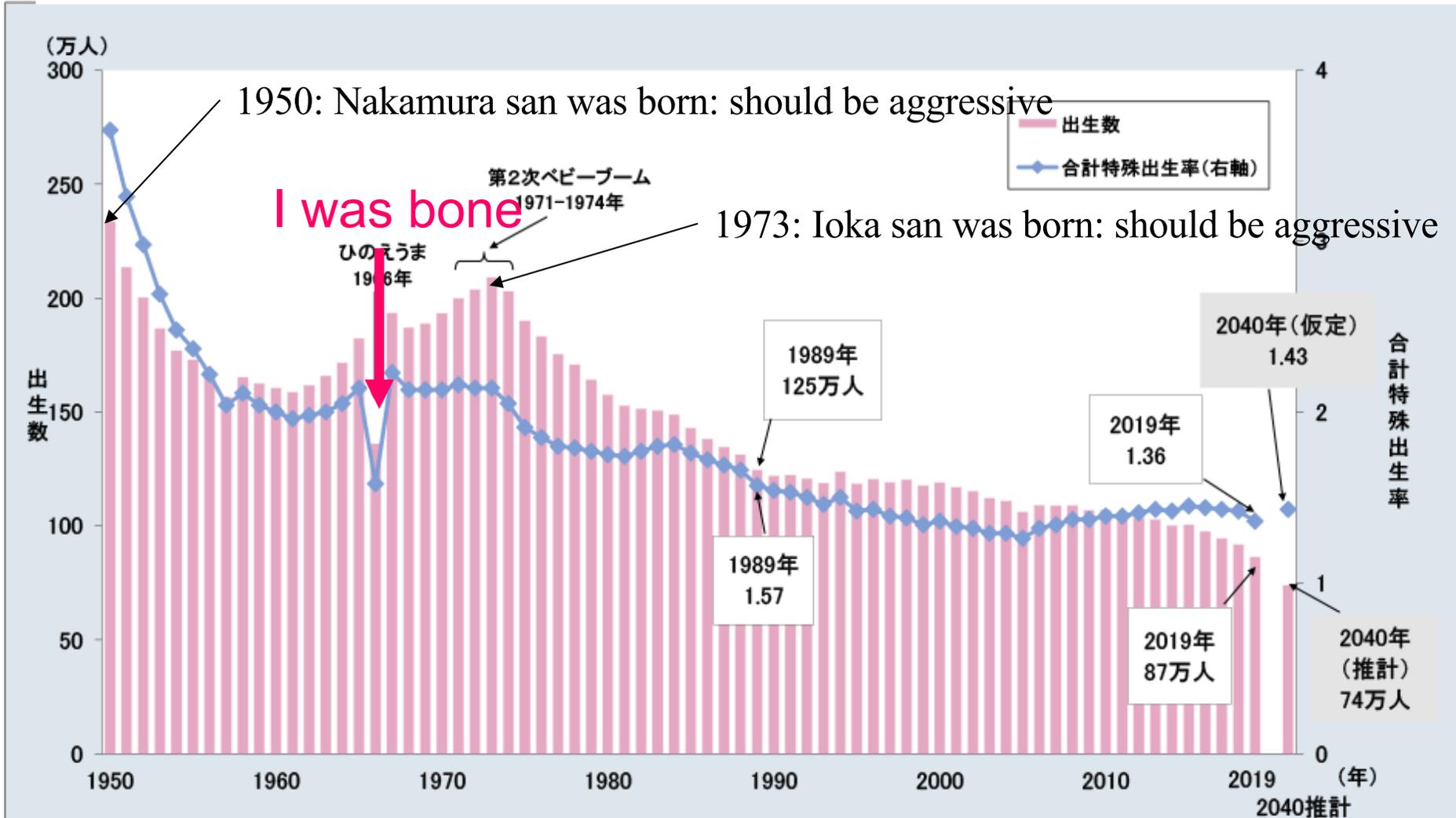
令和  
丙午

毘沙門堂門跡



# I am a minority in Japan, and a lucky person

## Evolution of the birth rate in Japan



資料：2019年までは厚生労働省政策統括官付参事官付人口動態・保健社会統計室「人口動態統計」（2019年は概数）、2040年の出生数は国立社会保障・人口問題研究所「日本の将来推計人口（平成29年推計）」における出生中位・死亡中位仮定による推計値。

# Outline

Numerical relativity also has a history of ~60 years

1. 1960s and 70s
2. 1980s
3. 1990s
4. 21 century
5. Issues for the future
6. Summary

I showed photos of some of impressive persons  
but they are omitted in this file

# 1 1960s and 70s: first topics of NR

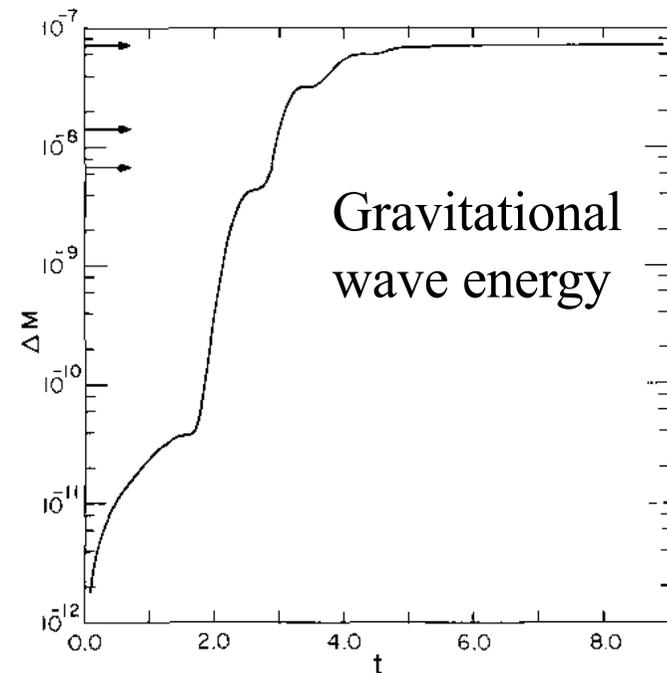
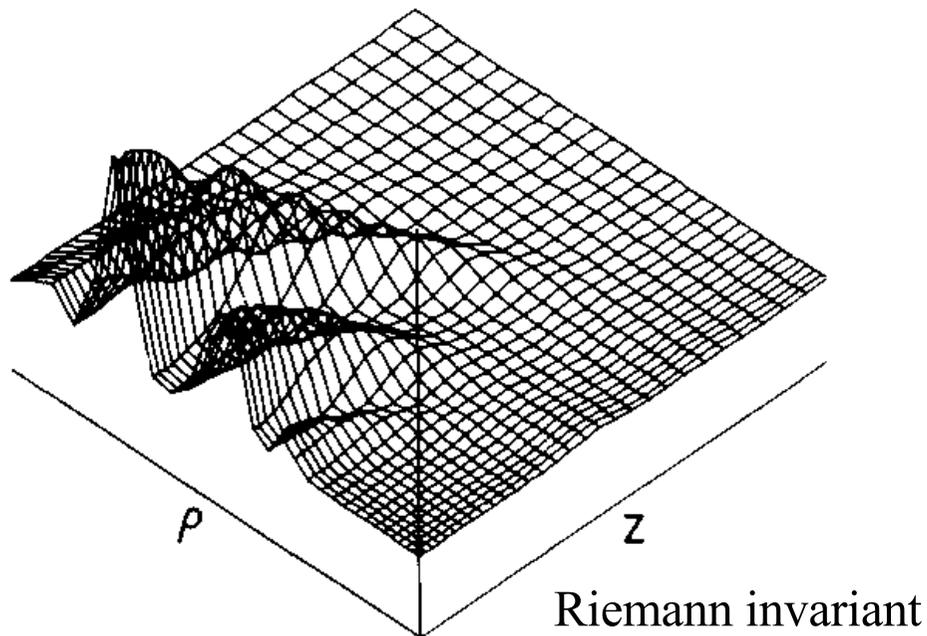
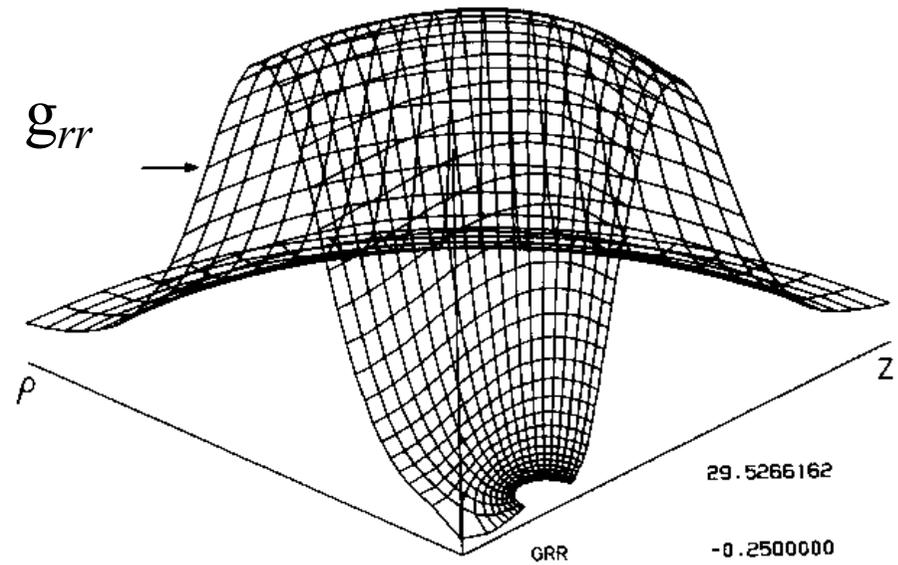
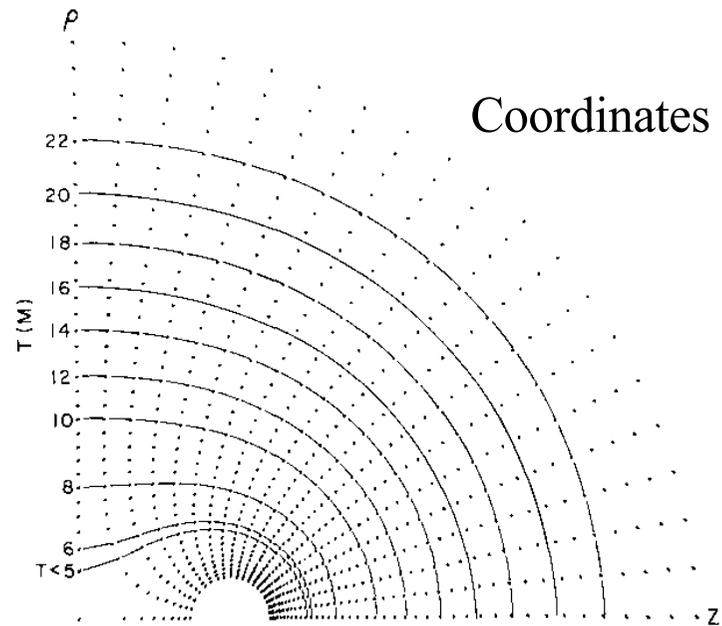
1. 1 D problem: Stellar core collapse\*

2. Black hole head on collision

- ADM formalism (1962)
- Stellar core collapse by Lagrange gauge (May & White 1966, Fernandez & Misner 1964)
- Initial data of head on collision of BHs: Wheeler, Lindquist, Misner, Gibbons, York, ....
- **Maximal slice** is founded to have a singularity avoidance property (Estbrook et al. 1973)
- **Smarr** (1977): Head on collision of two BHs  
→ First non-spherical simulation in NR; heralded multi-dimensional numerical relativity

\* NR with cylindrically symmetric spacetime was also performed in 1970s

# L. Small (1977): Amazing work at that time



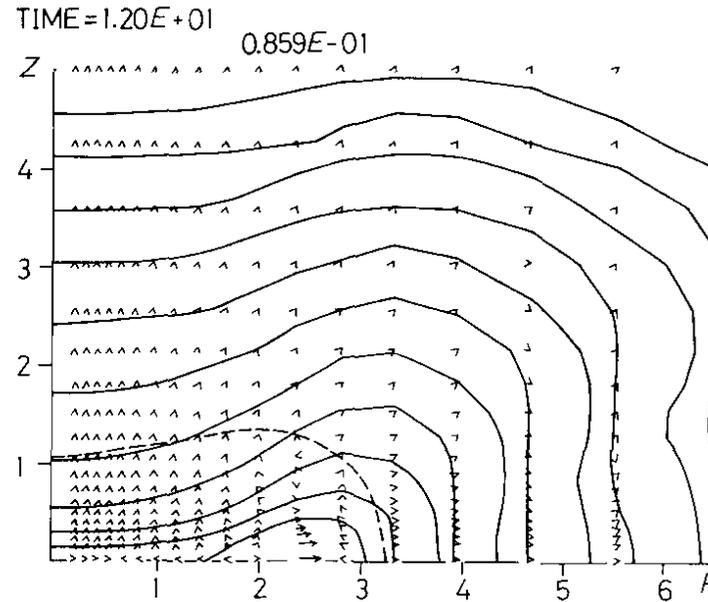
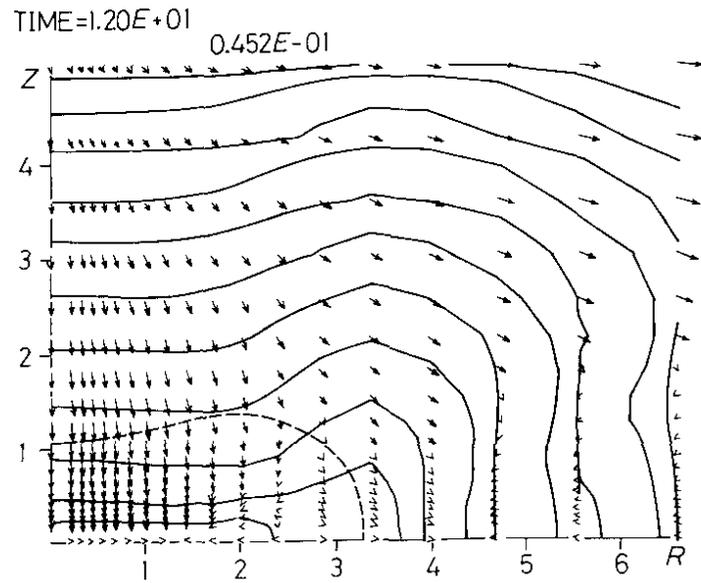
## Other important progresses in 1970s

- **Smarr & York (1978)** proposes **minimal distortion** gauge: Basic idea for simulating systems with rotation
- **Apparent horizon** is the useful tool for studying BH thanks to Hawking
- York's review paper on the “standard 3+1” formalism (1979); useful because no textbook was present
- 2+1+1 formalism in numerical relativity (Maeda+1979): formalism based on **dimensional reduction**
- These illustrate that **original** ideas in the formulation were valuable in NR at that time

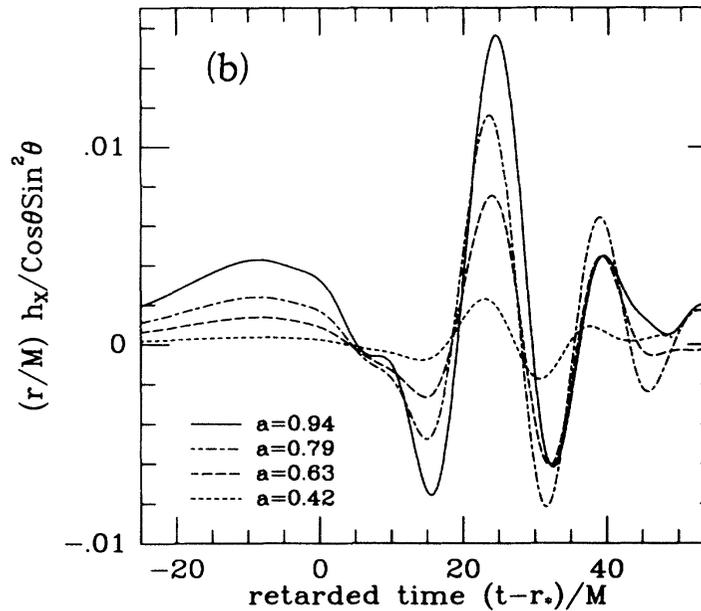
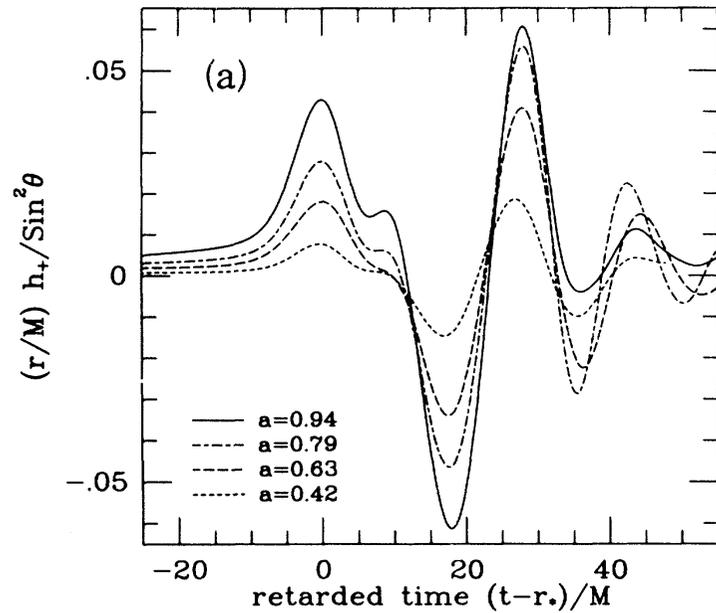
## 2 1980s: Era of axisymmetric NR

- **Nakamura** performed the first NR simulation for stellar collapse to a BH in 1981: 2+1+1 formalism
- **Bardeen & Piran** developed radial and isothermal gauges in 1983
- **Stark & Piran** performed rotation collapse to a BH and extracted gravitational waves for the first time in 1985: **radial gauge + mixed slicing**
- **Shapiro & Teukolsky** developed an axisymmetric code with collisionless particle as the matter part
- **Works of Nakamura & Stark-Piran contain a lot of original ideas in the formulations and gauge conditions**

# Results of Nakamura, Stark, Piran



Nakamura  
PTP 1981



Stark & Piran  
PRL 1985

June 2017

With boss of mafia?



Bodyguard?

Humanshield?



## Prelude toward 3D simulation in 80s

- **Teukolsky** derived general solutions of “Teukolsky wave”, which are useful for checking the codes (1982)
- **Nakamura** developed a primitive version of BSSN formalism in 1987, focusing on the propagation of linear gravitational waves (Teukolsky waves)
- **He found that the standard ADM is not suitable for NR**; but surprisingly researchers in other countries did not pay attention to his work until late 1990s
- **Bona** and **Masso** proposed harmonic formalism and harmonic (harmonic-like) slicing in the late 1980s

Why not NSBS or SNBS? BSSN was named by **Alcubierre** later; According to him, SNBS sounds something like supernovae, so he thought that it would not be appropriate

### 3 1990s: Phase change

- I became a graduate student in 1989; so the description of history from now is based on my experience (and prejudice)
- People started seriously considering gravitational-wave astronomy from the late 1980s
- In the middle of 1990s, the LIGO project was approved
- ✓ Last October we had Kip Thorne in AEI and he gave a lecture on the history

# Early 1990s

- For data analysis of gravitational waves, accurate templates of gravitational waves are crucial
- Promising source at that time was binary neutron star merger, because 3 NS-NS binaries which will merger within the Hubble time was already observed in our Galaxy → predicted event rate: 1—10/Myrs; horizon distance is  $\sim 200$  Mpc/yrs (e.g., Phinney 1991)
- **3D numerical relativity simulation is necessary for modeling** → **Binary black hole grand challenge project** was established in US in 1990s
- Why BH-BH? According Kip, BH mass is  $\sim 10$  times larger → Horizon distance is  $\sim 1000$  times deeper and should be of more events: He was right....

## **Problem of NR in 1990s**

- **Most of people did not know suitable formalisms in 3D numerical relativity**
- We did not know **suitable gauge conditions in 3D numerical relativity** → My main focus was to find suitable gauges and methods to impose them
- **Computational resources were poor**; simulation in 1995 Shibata-Nakamura paper (Phys. Rev. D 52, 5428) was performed with (59, 59, 29) grids
- **Adaptive mesh refinement techniques** had not been developed yet in numerical relativity
- People believed that the **excision of BH horizon** (Unruh, Seidel-Suen) is the unique way to evolve BHs (but **this was wrong!**)

## Late 1990s

- Grand challenge itself did not yield spectacular results
- This was the reason why Kip developed a large NR group in Caltech since 2001 → later SXS collaboration

## Late 1990s

- Grand challenge itself did not yield spectacular results
- This was the reason why Kip developed a large NR group in Caltech since 2001 → later SXS collaboration
- However, **the Grand challenge project became seeds for cultivating the next generation talents**
- During Grand challenge they tried several formalisms in NR but nothing remains useful now (this is why Frans Pretorius chose harmonic formulation)
- In Japan we (I) made entirely unique progress
- We started from evolution of non-linear gravitational waves using the original version of BSSN (SN)  
→ SN formalism works even for **non-linear problems**
- The next step is to perform a simulation of NS mergers

## My activity in late 1990s

- Formalism=I knew well, Shibata-Nakamura (BSSN) formalism is robust (this was the advantage at that time)
  - Slicing=Maximal slicing should be robust
  - Spatial gauge=Minimum distortion gauge is likely to be robust
  - However, to impose them we need to solve **elliptic type equations**, which we want to avoid because solving them is super time-consuming
  - **My guess**: “Approximate” maximal slicing + “approximate” minimum distortion gauge would be OK
- \* Why NS-NS? We did not know how to handle BH at that time

# Equations of Maximal slicing and Minimum distortion gauge

$$D_k D^k \alpha = 4\pi\alpha(E + S_k^k) + \alpha \tilde{A}_{ij} \tilde{A}^{ij},$$

$$\begin{aligned} \tilde{D}^i \left( \tilde{D}_i \tilde{\beta}_j + \tilde{D}_j \tilde{\beta}_i - \frac{2}{3} \tilde{\gamma}_{ij} \tilde{D}_k \tilde{\beta}^k \right) + 6\tilde{D}^i \phi \left( \tilde{D}_i \tilde{\beta}_j + \tilde{D}_j \tilde{\beta}_i - \frac{2}{3} \tilde{\gamma}_{ij} \tilde{D}_k \tilde{\beta}^k \right) \\ - 2\tilde{A}_{ij} \tilde{D}^i \alpha - \frac{4}{3} \alpha \tilde{D}_j K = 16\pi\alpha J_j. \end{aligned}$$

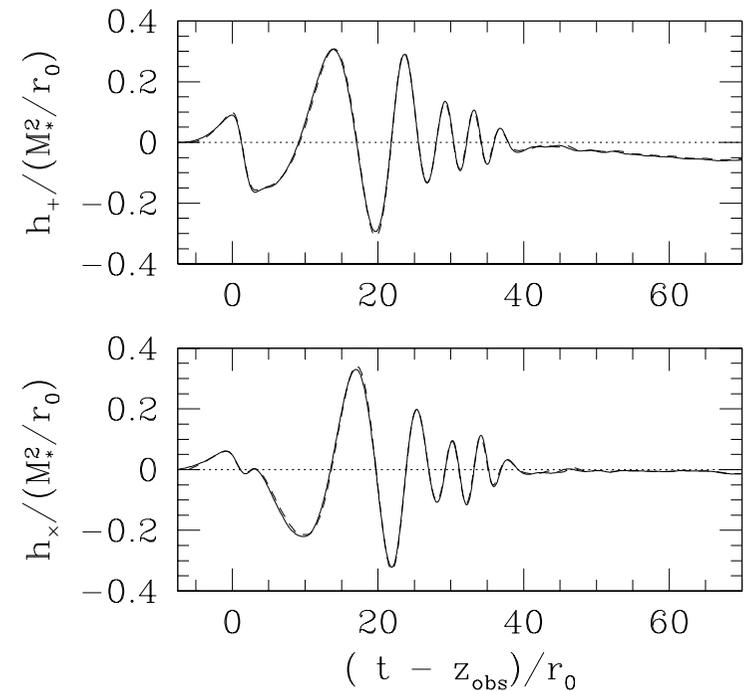
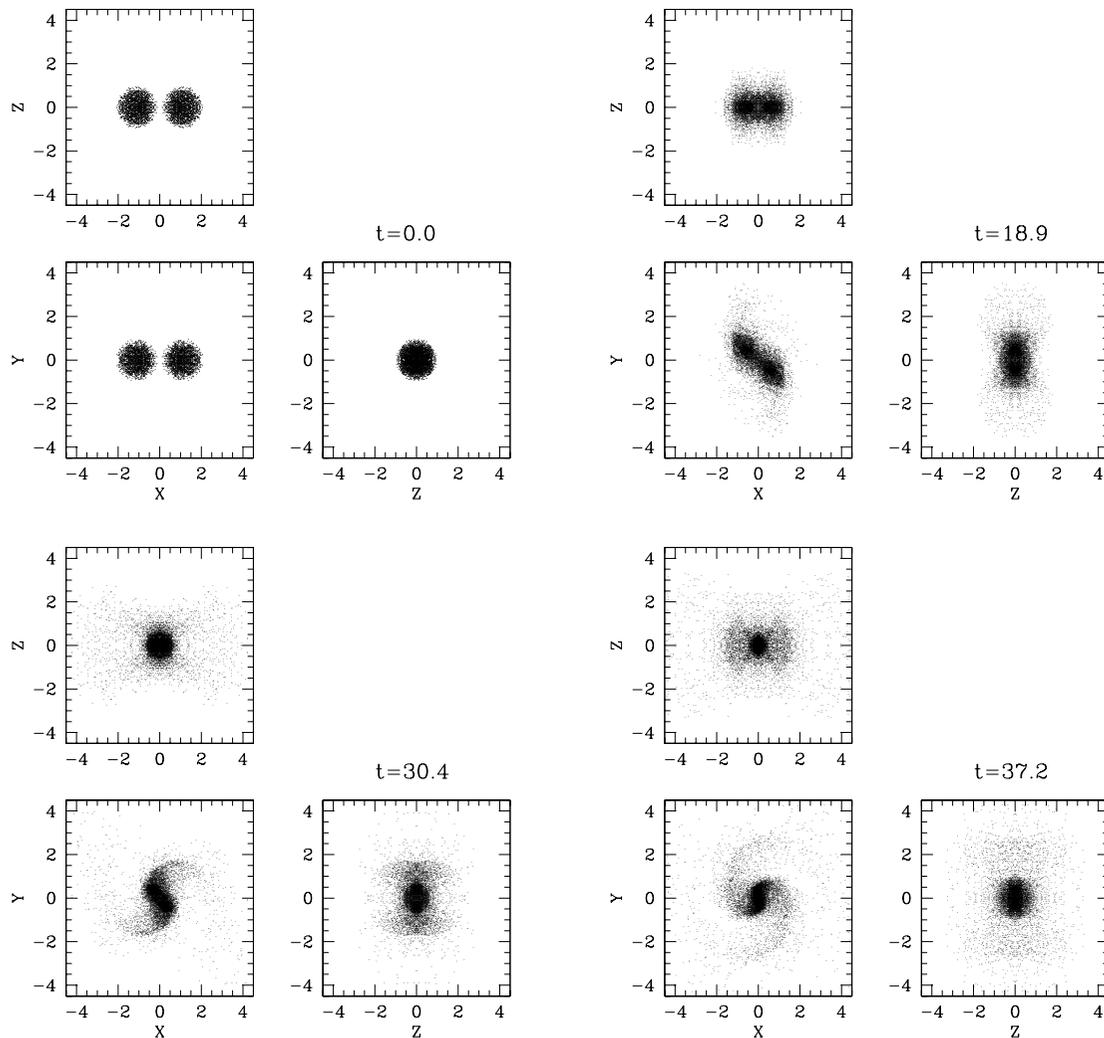
# First merger simulation (Prog. Theor. Phys. **101**, 1199, 1999)

- 1998: Try merger of clusters of *collisionless particles* with “approximate” maximal slicing + simplified version of minimum distortion gauge
- Lapse Elliptic equation  $\rightarrow$  parabolic equations with a couple of iteration:  $t_{dyn} \gg \Delta t$
- Spatial gauge  $\rightarrow$  Simplified elliptic equation
- Works well!

$$\partial_\lambda \ln \alpha = D_k D^k \ln \alpha + (D_k \ln \alpha)(D^k \ln \alpha) - 4\pi(E + S_k^k) - \tilde{A}_{ij} \tilde{A}^{ij} - \frac{1}{3} K^2,$$

$$\delta_{ij} \Delta \beta^i + \frac{1}{3} \beta^k{}_{,kj} - 2\tilde{A}_{ij}(\tilde{D}^i \alpha - 6\alpha \tilde{D}^i \phi) - \frac{4}{3} \alpha \tilde{D}_j K = 16\pi \alpha J_j.$$

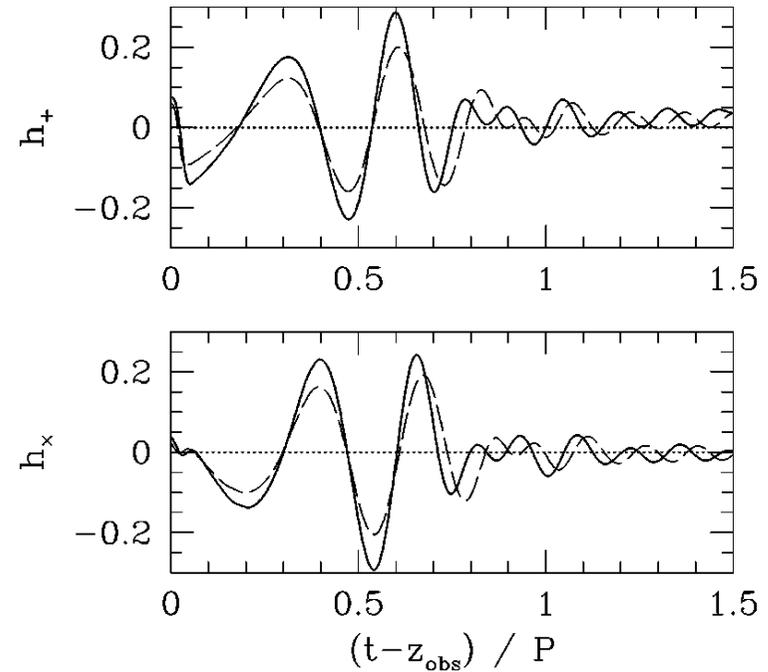
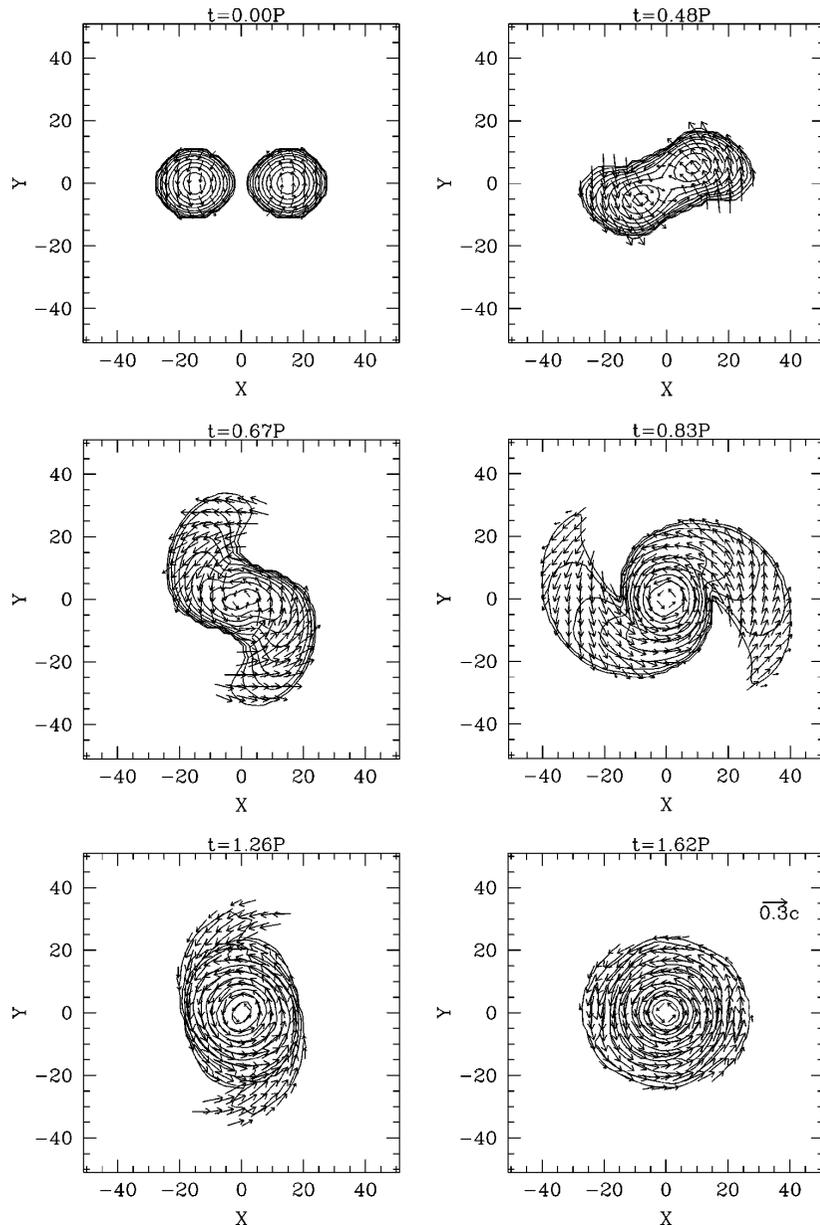
# Results: the first successful 3D NR (1999, unfortunately, only few people refers to...)



Gravitational waves

OK, NS-NS simulation can be done!

# First simulation of NS-NS (MS, PRD 1999, 2000)



Before this work, only few people referred to my works but after this work, people started referring to my work and inviting well-known conferences

# My citation history

Review affiliation  
Help colleagues find you.

REVIEW

Add photo  
Complete your profile.

ADD

Add co-authors  
We have co-authors suggestions.

ADD

masaru shibata

Max Planck Institute for Gravitational Physics & Kyoto University  
Verified email at aei.mpg.de - [Homepage](#)

FOLLOW

Cited by

All

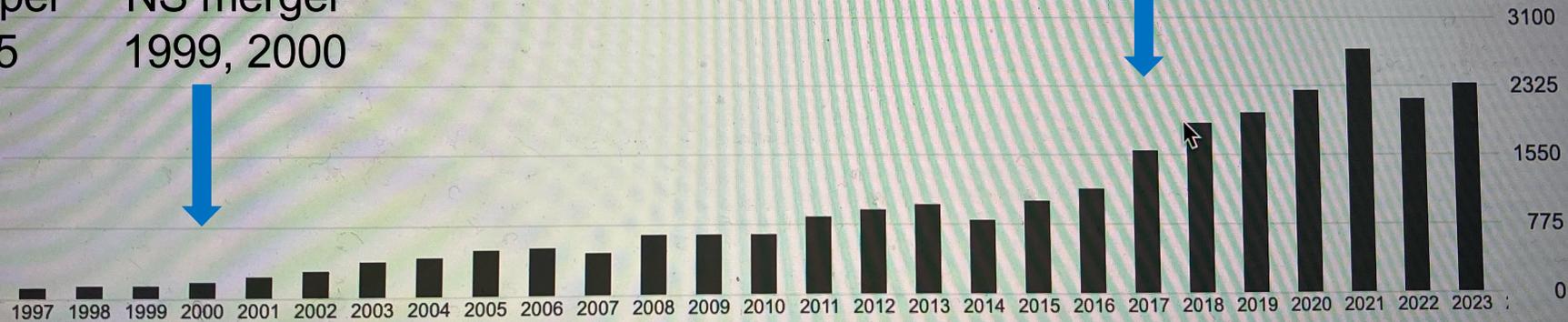
×

Citations per year

GW170817

SN paper  
1995

NS merger  
1999, 2000



Physical and Quantum Gravity 28 (9), 094011

Production of all the r-process nuclides in the dynamical ejecta of neutron star mergers  
Tanajima, Y Sekiguchi, N Nishimura, K Kiuchi, K Kyutoku, M Shibata  
Astrophysical Journal Letters 789 (2), L39

710

2014

Mass ejection from the merger of binary neutron stars  
Mitsuda, K Kiuchi, K Kyutoku, H Okawa, Y Sekiguchi, M Shibata, ...  
Physical Review D—Particles, Fields, Gravitation, and Cosmology 87 (2), 024001

700

2013

Modeling GW170817 based on numerical relativity and its implications

477

2017

Public access

0 articles

not available

Based on funding mandates

## 4 21 century

- Prelude of BH-BH simulation
  1. **Alcubierre & Bruegmann** developed **hyperbolic gauges**, which can approximately satisfy maximal slicing and minimum distortion, in 2003
  2. **Bruegmann** et al. performed one orbit simulation for BH-BH in 2004; BSSN is used: at that time the tool for BH-BH was already prepared but they did not perform
  3. **Garfinkle** proposed generalized harmonic formalism in 2002 → motivation of **Pretorius**
  4. Note: before 2005, people believed that **BH excision** is the way to be used, but later, we understood that this was not the case if we use the BSSN formalism

## 2005: Breakthrough year “for BH evolution”

- **Pretorius:** Generalized harmonic formulation
- **Campanelli et al.:** BSSN + hyperbolic gauge
- **Baker et al.:** BSSN + hyperbolic gauge
- ✓ 4<sup>th</sup>-order finite difference and hyperbolic gauge condition are the keys
- ✓ **In BSSN, no excision is necessary**
- ✓ Note that 3D NR simulation (NS-NS merger) became already feasible in 1999. Thus, 2005 is not the year of the breakthrough of “3D NR”
- ✓ BH-NS can be easily performed: The first work was done by Shibata & Uryu in 2006

## After 2005

- **BH-BH**: Production phase for a variety of parameters
- **NS mergers**: Need to take into account more physics and more grid resolutions in particular for MHD
- The same for **supernovae** simulations in GR (Ott, Sekiguchi, Moesta, Kuroda....)

## BH-BH: current status

- More than 1000 accurate simulations have been performed, in particular, by the SXS collaboration for a variety of binary parameters
- The waveform data have contributed significantly to developing GW template through EOB and frequency space templates and to GW detection and data analysis
- 2015: GW150914; NR contributed to the first detection
- New direction: extend to elliptic orbits and high mass ratio
- In near future, templates purely by numerical relativity will be available; we may not have to rely on EOB, PN-based waveforms in the GW data analysis

# Neutron star mergers

- 2005~: Simulation with realistic EoS using hybrid approach (cold part + Gamma-thermal heating, Shibata et al.)
- 2010~: Simulation with tabulated EoS (Duez et al., Sekiguchi et al., Fourcart et al...)
- 2011: First simulation with neutrino transfer (Sekiguchi et al.)
- 2014: First MHD simulation with reasonable resolution (Kiuchi et al.)
- 2015: Simulation with neutrino heating (Sekiguchi et al., Radice et al.)
- 2017: GW170817; NR contributed to interpreting EM counterparts

# Other important works associated with NS mergers from the viewpoint of NR

- 1993: **Lai, Rasio, Shapiro**: late inspiral GW has the information of NS EoS
- 2008: **Flanagan & Hinderer**: Clearer formulation
- 2010: **Metzger et al.**, kilonova paper
- 2011: **Nakar & Piran**: Radio afterglow paper
- ✓ Accurate late inspiral simulation is the key
- ✓ **Mass ejection should be explored in NR**: electron fraction & velocity of the ejecta have important information; EoS and neutrino transfer are important
- ✓ GW170817 verified that their papers are important;  
**NR became an important field of astronomy**

# 5 Future issues (NR with matter)

I will talk based on my prejudice

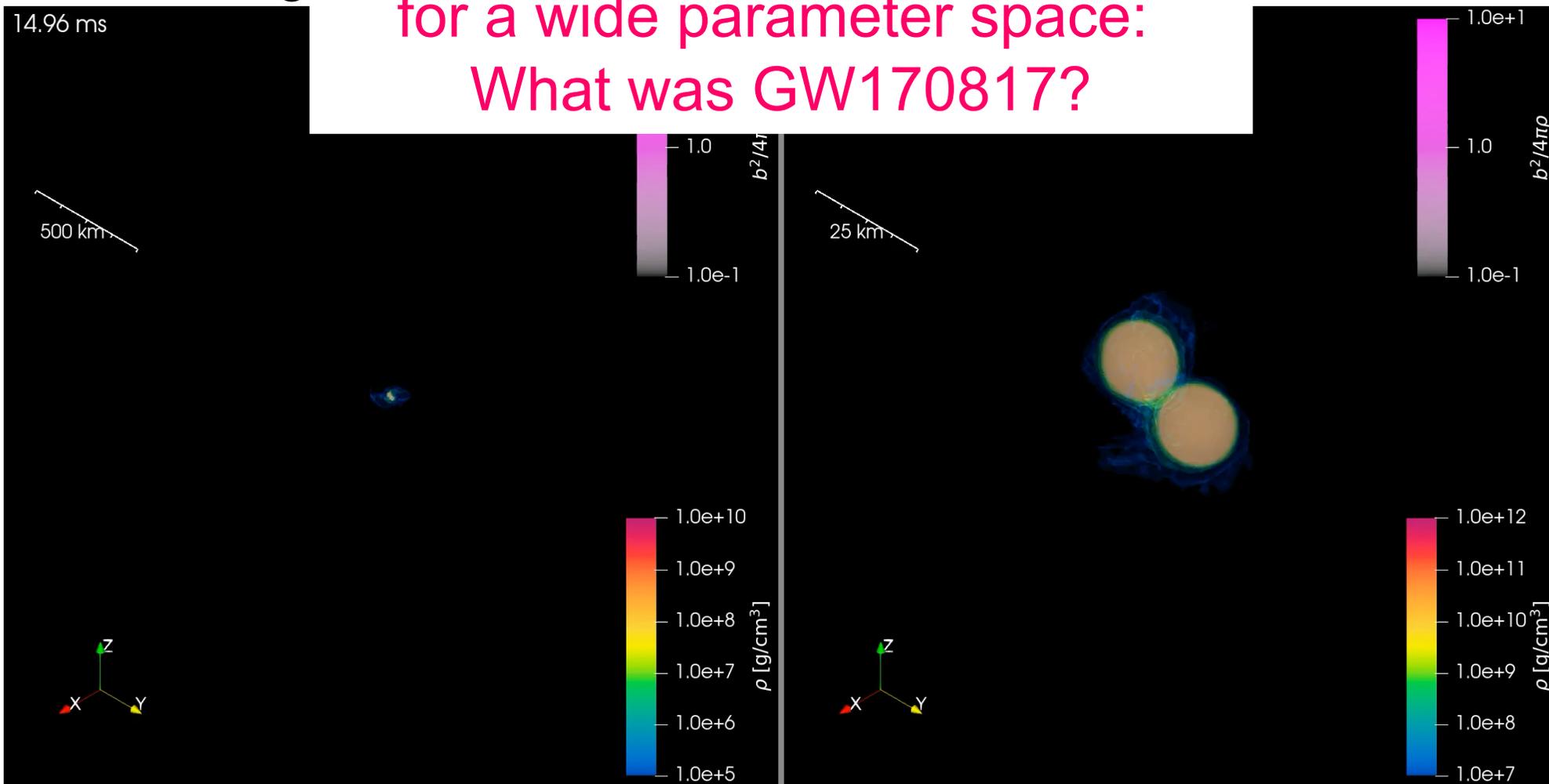
- More long-term self-consistent simulations for NS-NS
- Understanding MHD physics and dynamo (Most...)
- GR simulations with better neutrino transfer (KK)
- To incorporate neutrino oscillation (Wu, HR, Just...)
- To develop **photon radiation transfer** codes to apply to the phenomena associated with SMBHs
- Exploring collapsar in 3D; long-term, well resolved MHD simulations are keys (Aloy, Moesta, SF)
- Tidal disruption in full GR until mass ejection and hopefully until jet launch (Piran)
- Develop more efficient codes (GPU; Talks by Radice, Most, Moesta, Musolino...)

# Merger, mass ejection, and jet from NS-NS

1.25-1.65  $M_{\odot}$

Simulations should be performed  
for a wide parameter space:  
What was GW170817?

et al. PRL 2025

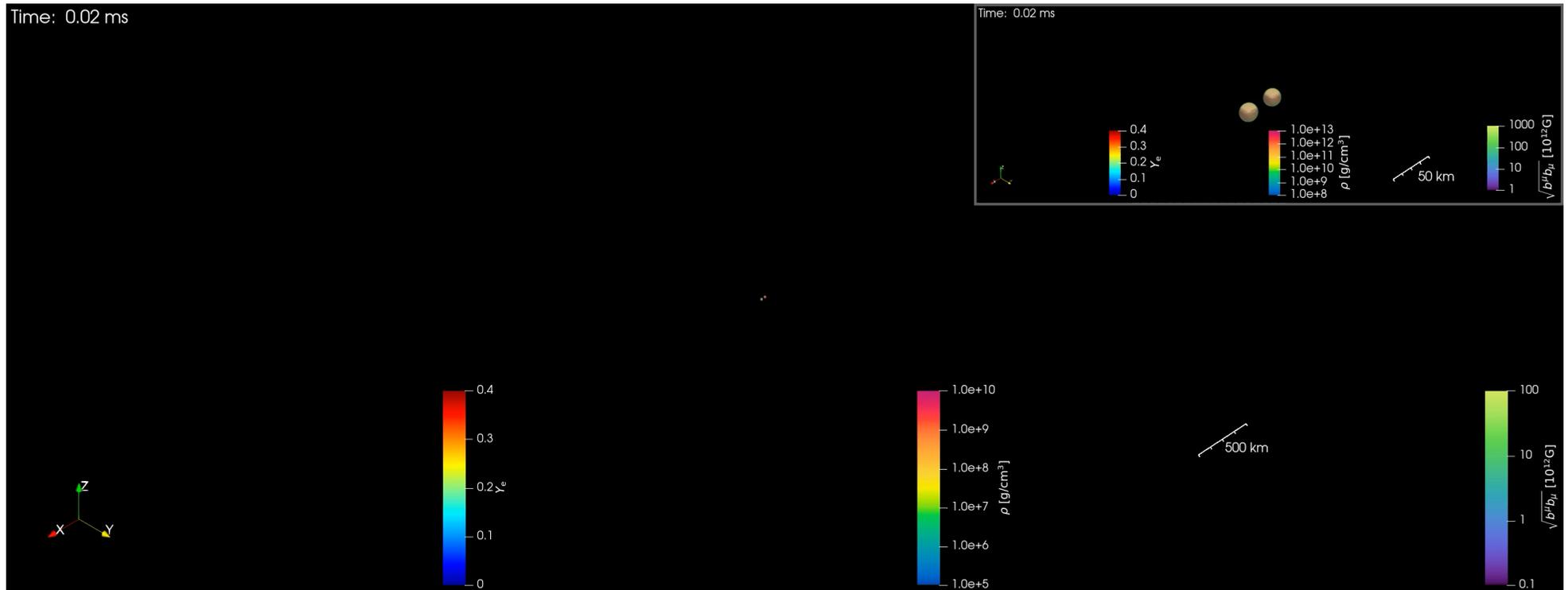


For large scale

For small scale

Credit: Kota Hayashi

# Dynamo in NS-NS merger

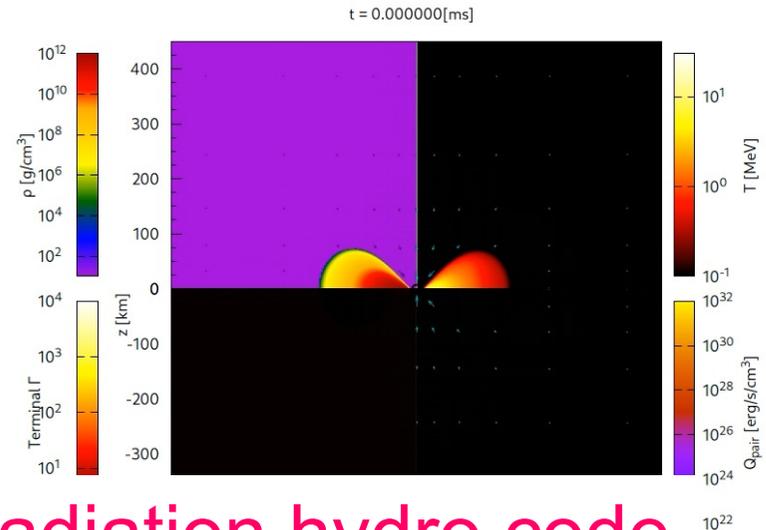
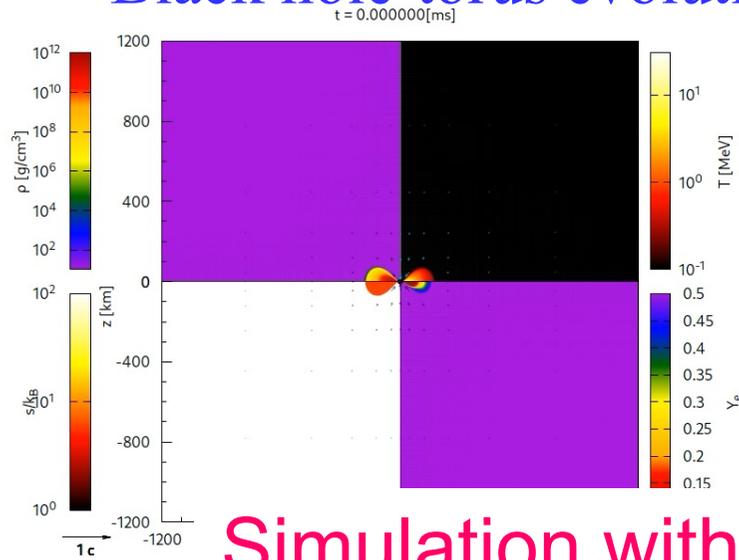


Kiuchi, Reboulet-Salze et al. 2024 (see also Gurierez+ 2026)

**More resolved, longer simulations are needed**

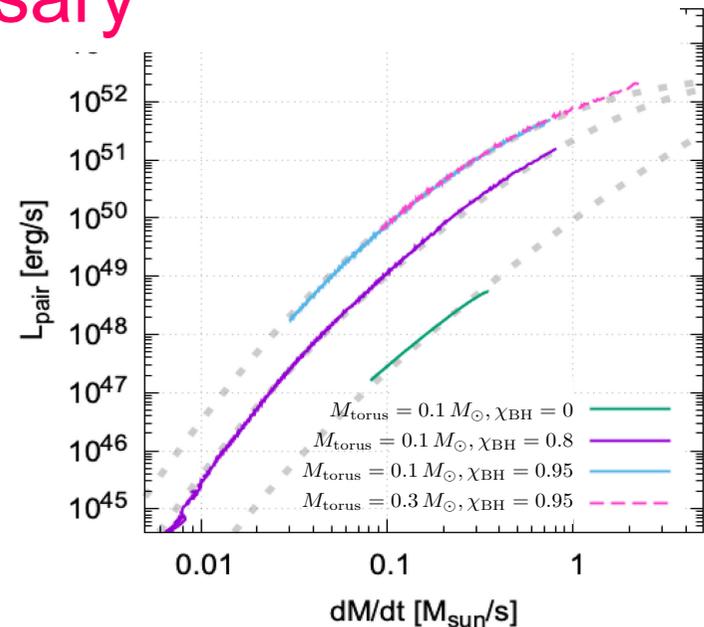
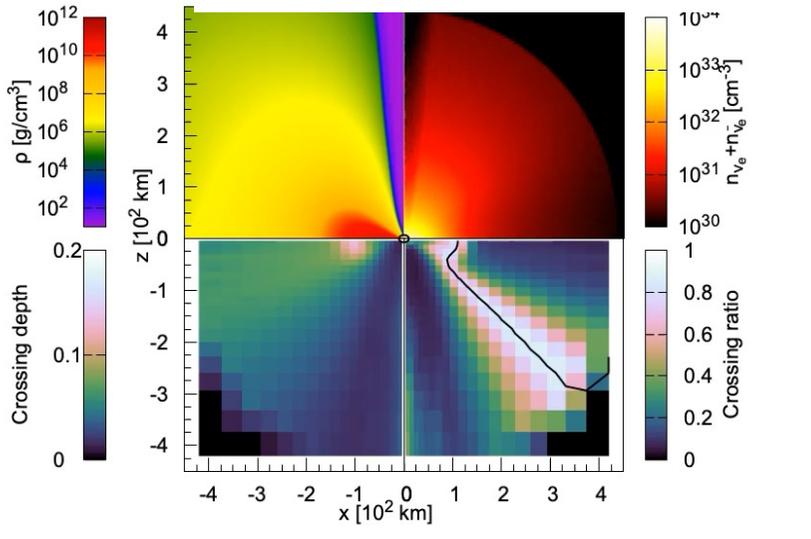
# Monte-Carlo radiation hydrodynamics in GR

## Black hole-torus evolution



Simulation with better radiation hydro code is necessary

Inst

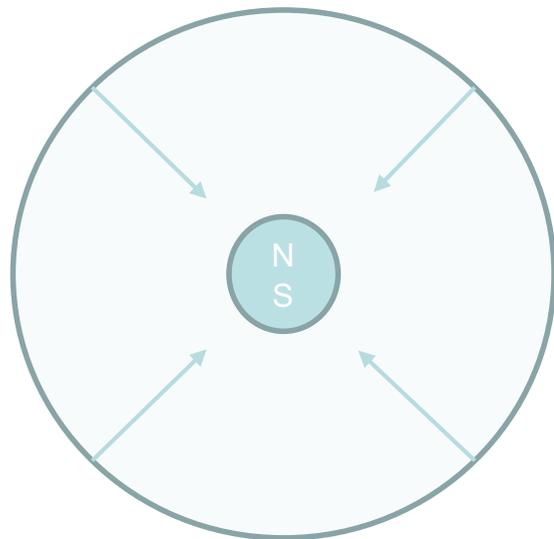


# Application to collapsars

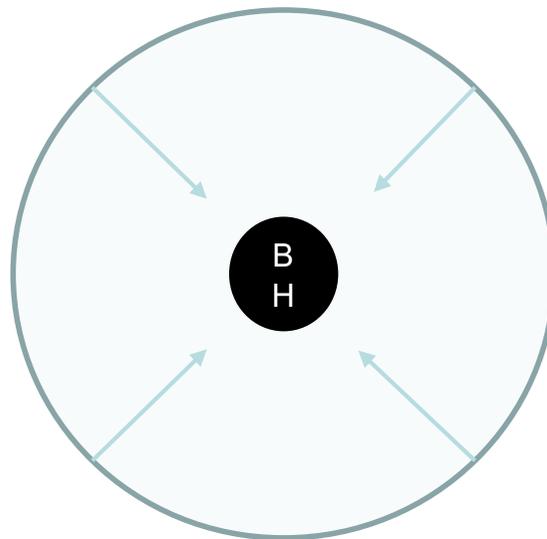
Naïve qualitative scenario is

**GR simulation for  $> 10\text{sec}$  is necessary**

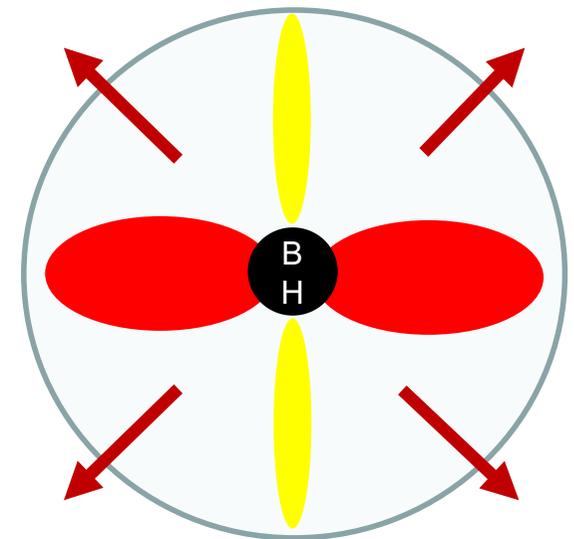
2. Proto neutron star formation
3. Further infall  $\rightarrow$  black hole formation
4. Accretion onto black hole + formation of disk
5. Jet from vicinity of the black hole + explosion



$t < \text{a few } 100\text{ms}$



$t \sim 1 \text{ s}$



$t > 1 \text{ s}$

# A self-consistent scenario for jet launch (I believe)

1. BH formation, and subsequently disk grows
2. MRI turbulence in the disk leading to an equipartition

$$\rightarrow \frac{B^2}{8\pi} \sim 0.05 \rho_{\text{disk}} c_s^2 \rightarrow B^2 \sim \rho_{\text{disk}} c_s^2$$

3. Matter and magnetic field falls into the BH  $\rightarrow$  typical size of the magnetic field is the magnitude in the disk

4. If the ram pressure of the infalling matter into the BH decreases below  $\frac{B^2}{8\pi}$ , i. e.,  $\frac{B^2}{8\pi} > \rho_{\text{fall}} v_{\text{fall}}^2$ , a jet is launched by Blandford-Znajek mechanism

5. That is,  $\rho_{\text{disk}} c_s^2 > \rho_{\text{fall}} v_{\text{fall}}^2 \rightarrow \rho_{\text{fall}} \sim 10^{-3} \rho_{\text{disk}}$

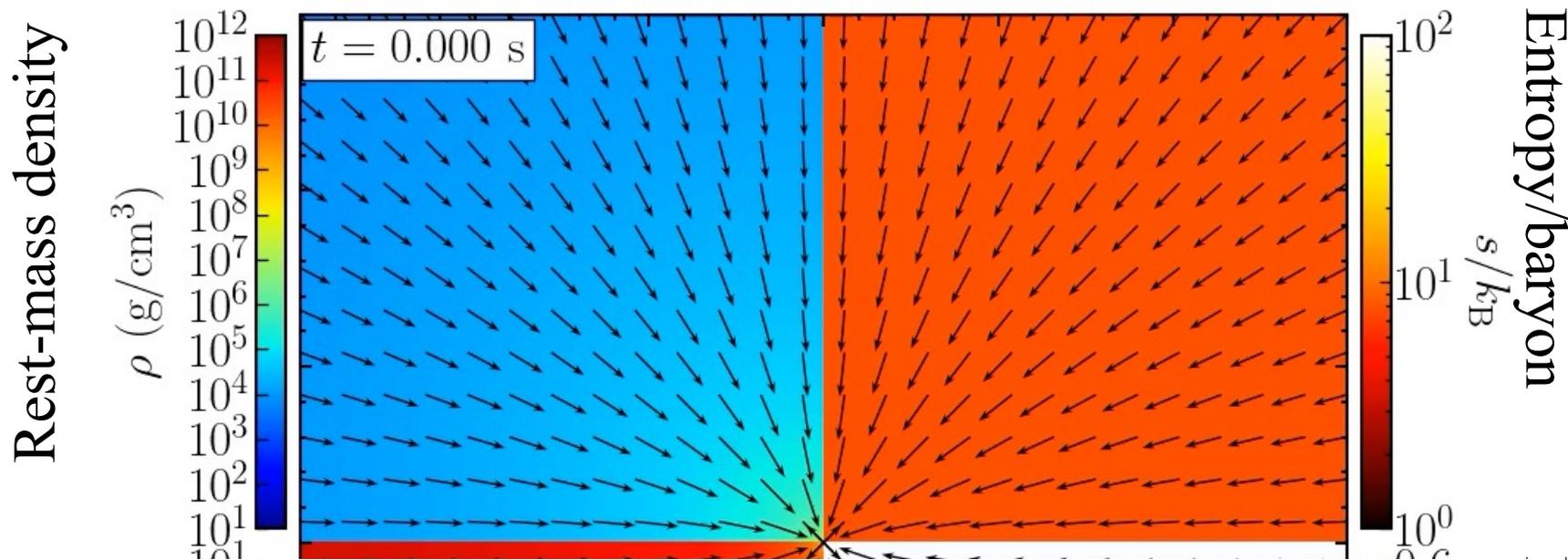
6. Typical magnitude of magnetic field strength

$$B \sim 10^{14} \text{ G} \left( \frac{\rho_{\text{disk}}}{10^{10} \text{ g cm}^{-3}} \right)^{1/2} \left( \frac{c_s}{10^9 \text{ cm s}^{-1}} \right): \text{ suitable for BZ!}$$

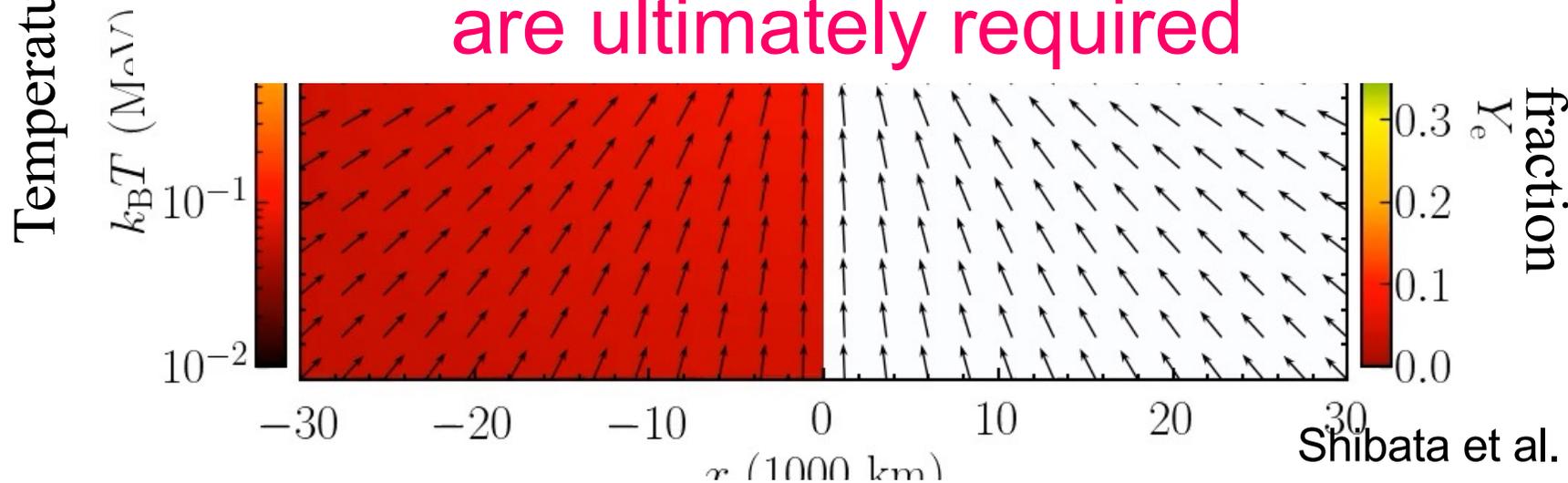
Start from 16 solar mass BH + infalling matter + **toroidal field**;

See also Aloy's and Moesta's presentations

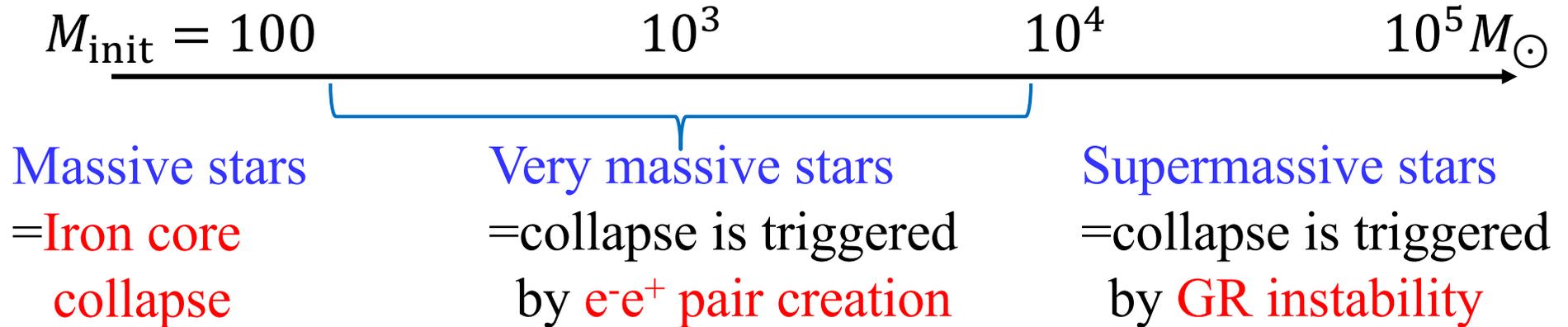
$\Rightarrow 0.01c, 0.1c, 1c$



**3D long-term well-resolved simulations are ultimately required**



# Understanding the fates of all *very/super-massive* stars

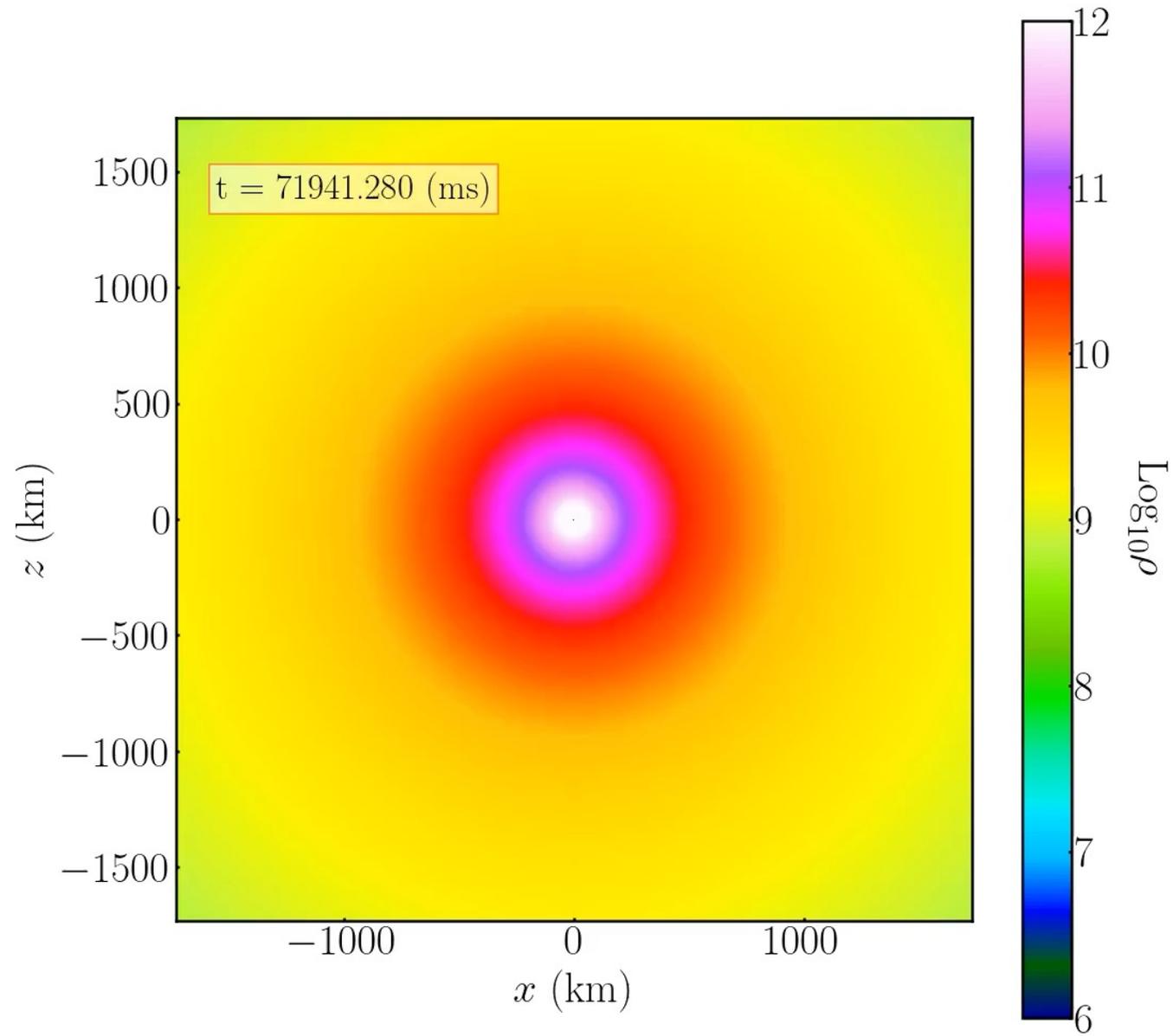


$R/M_{\text{core}}$ at the onset of collapse		
~1000	~2000—3000	<1000

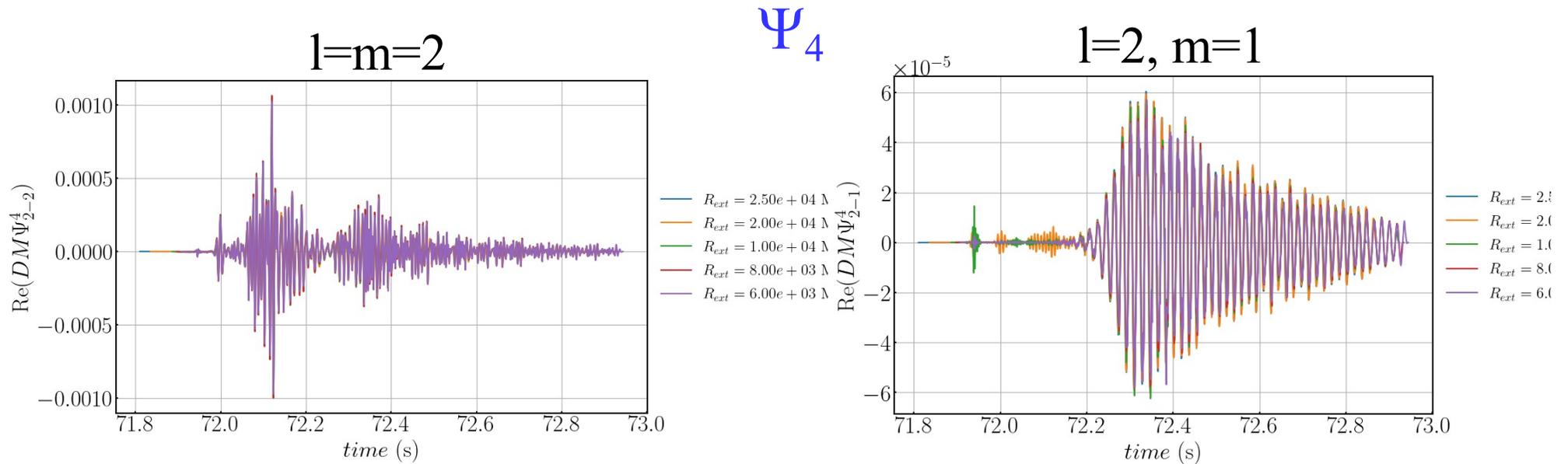
Maximum angular momentum  $\propto \sqrt{MR}$

Fate of the rapidly rotating stars			
BH or NS + stellar-mass disk	Explosion	BH + <b>heavy disk</b>	BH + small-mass disk

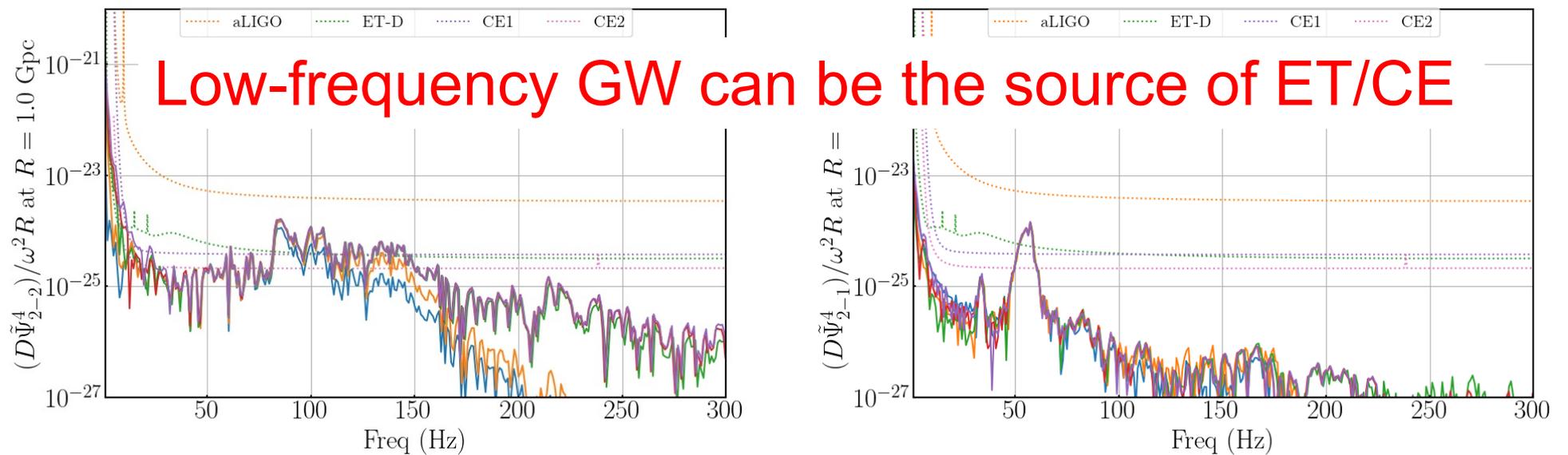
# 3D collapse simulation: *preliminary* by Lam et al.



# Gravitational waveform and amplitude at 1 Gpc

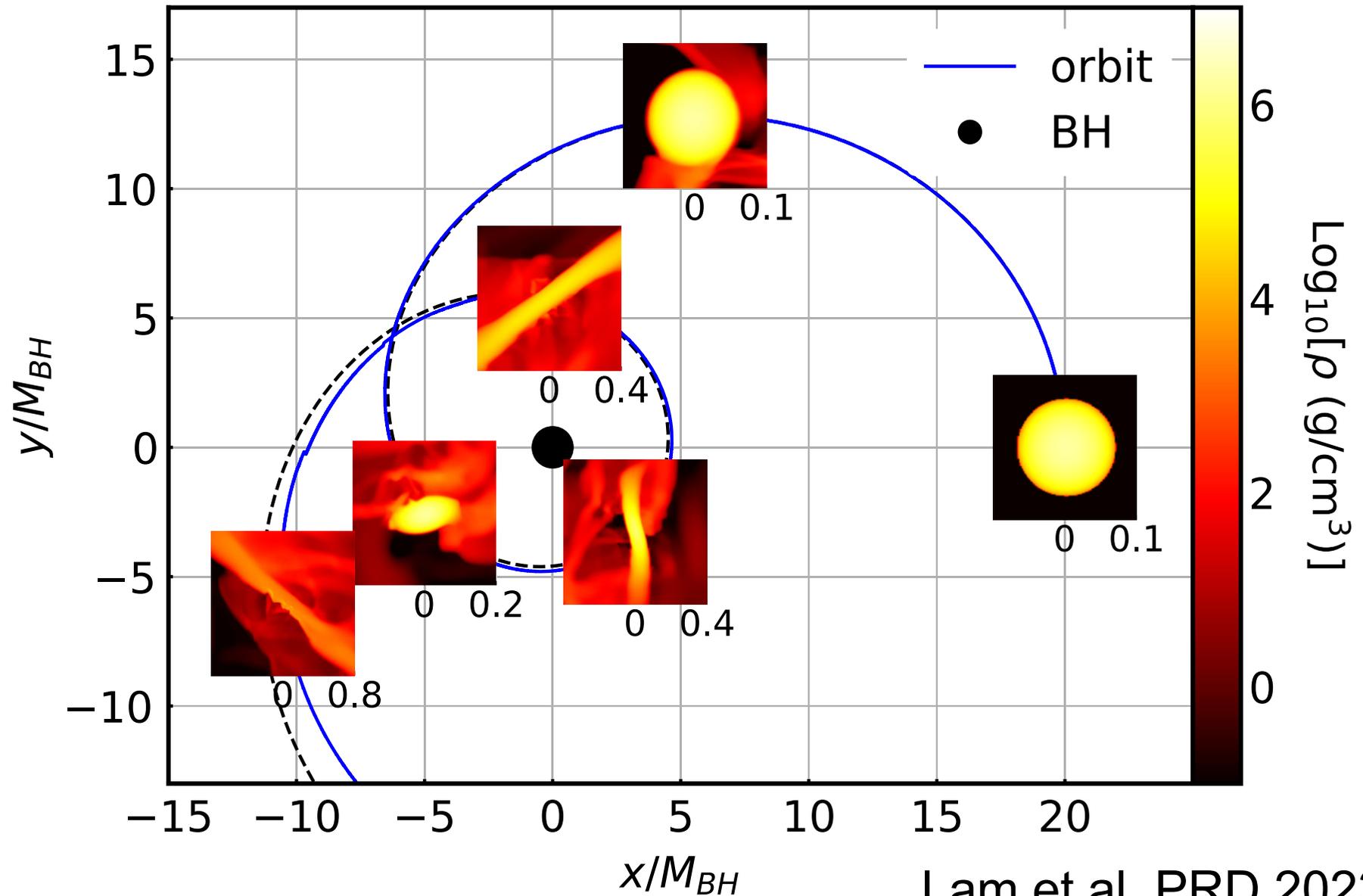


## Fourier spectrum



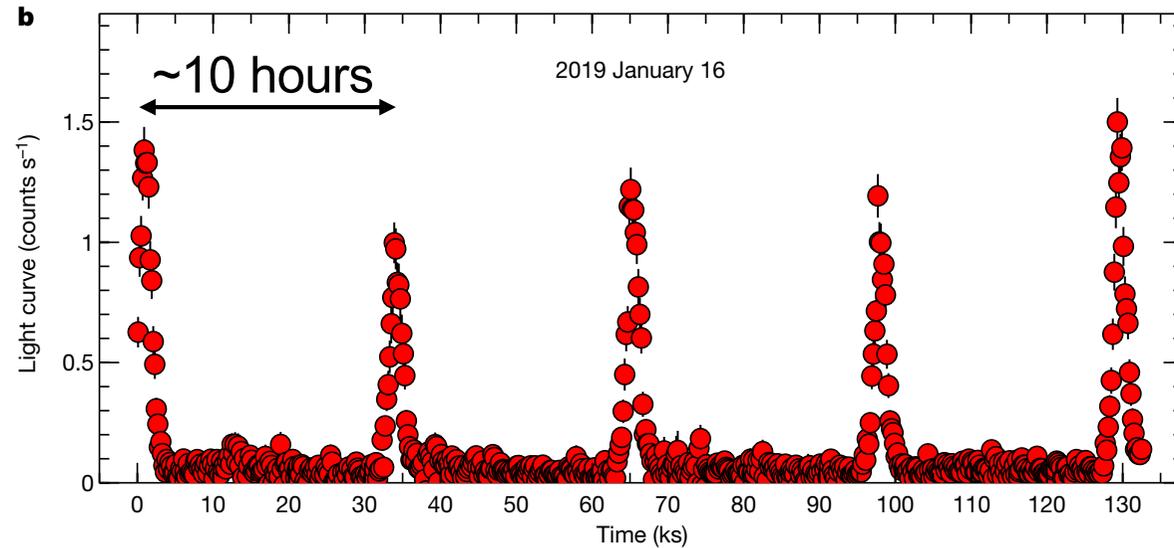
# Tidal disruption by SMBH in full GR

See Tsvi's presentation



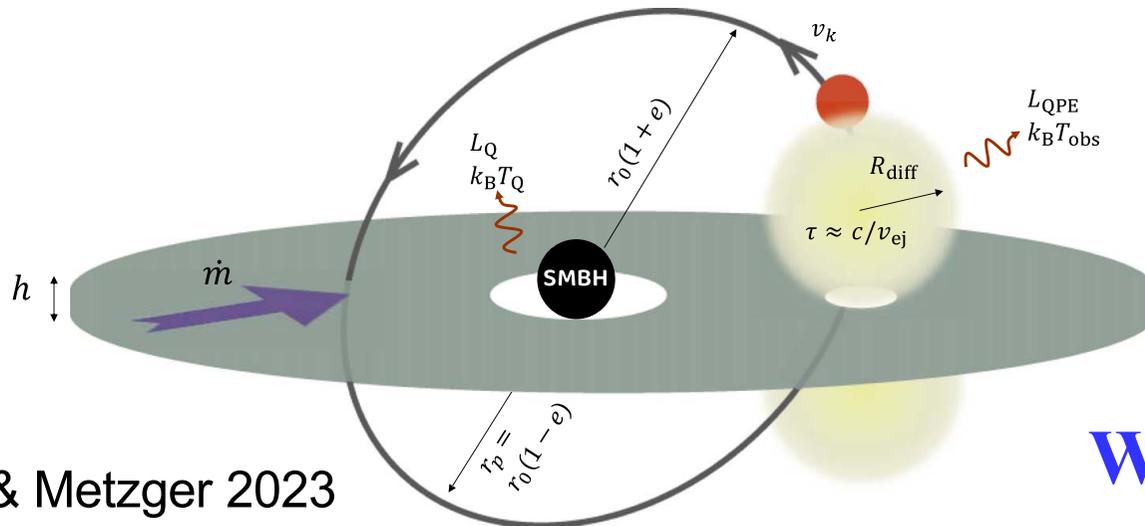
Lam et al. PRD 2023

# Modeling QPE/Exploring BH+disk collision



Xray observation

Miniutti et al, Nature 2019

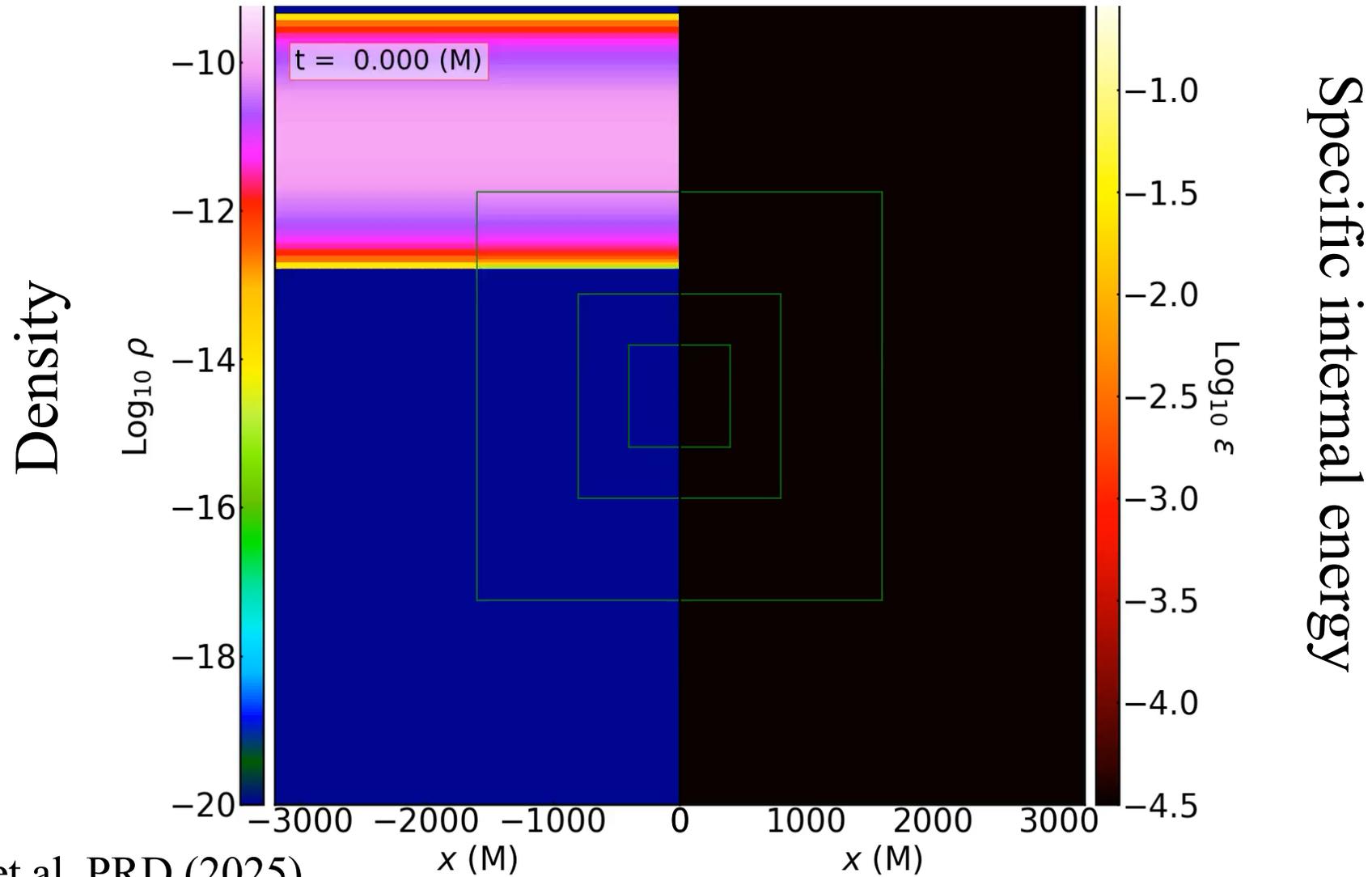


Itai & Metzger 2023  
TDE+collision=QPE?

What happens if BH  
collides with disk?

# BH + disk collision

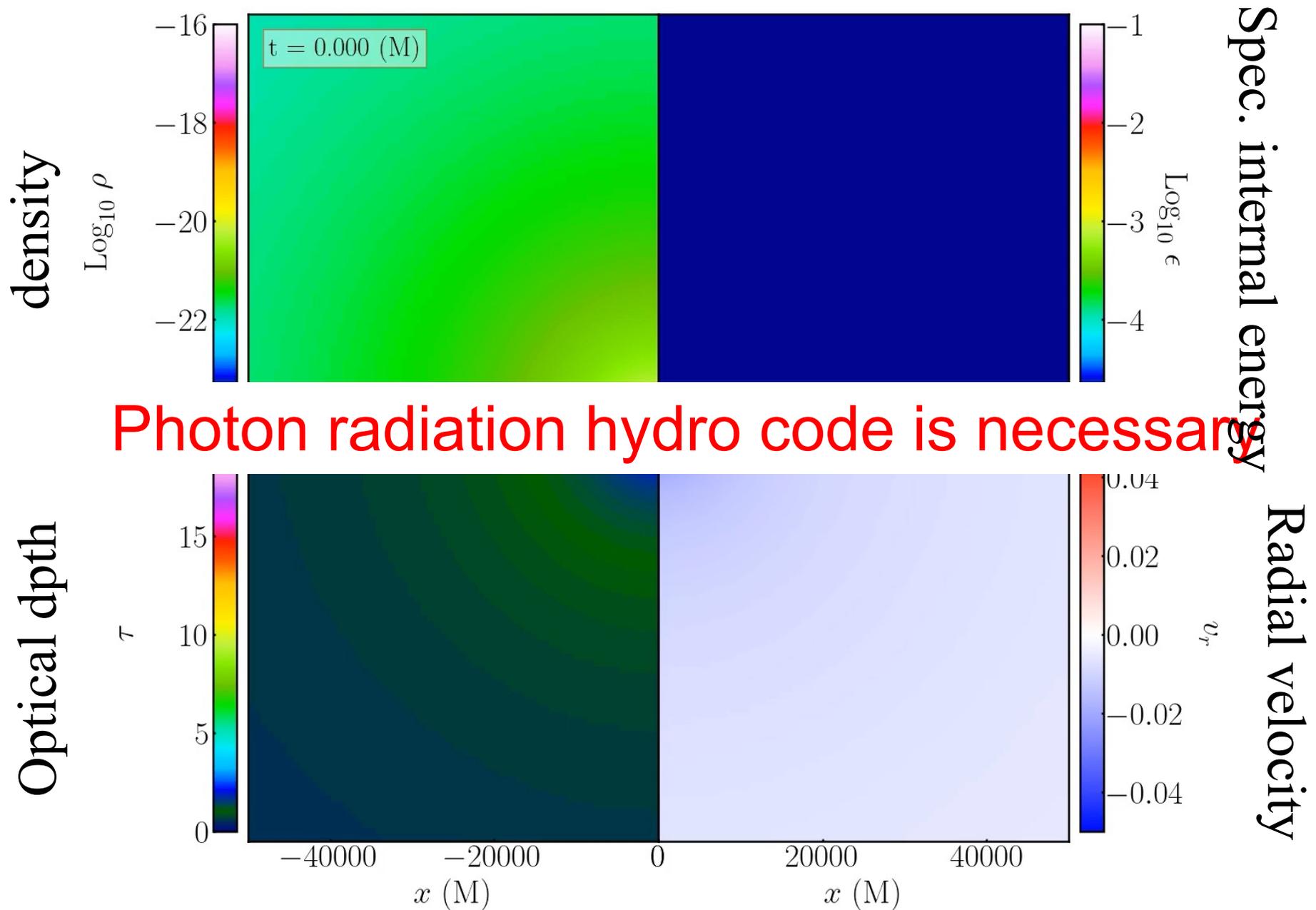
- Future issue: **What are the observed features?**  
**Need photon radiation transfer**



# Formation and evolution of little red dots?

- Basically, observed only at high redshifts
- Compact (point source)
- Red: typical temperature  $\sim 5000\text{K}$
- But with blue component (V shape spectrum)
- Emission lines of H, He, and others
- No/weak X-ray, radio
- Typical luminosity  $\sim 10^{44-45}$  erg/s
- ✓ What is this?
- ✓ A popular model=supermassive black hole + envelope
- ✓ How is such a system formed? What are observational signals?  $\rightarrow$  Relativistic computation may be the key

# Hyper collapsar: viscous hydro simulation in GR



# Summary

- Numerical relativity has become a mature field
- The efforts of our predecessors (now older than 60 yrs old) were quite significant
- NR will be effectively utilized in the interpretation of astronomical observational data
- It is useful for inferring aspects that cannot be directly observed
- ✓ Original ideas for the **application of NR** will be important (e.g., Most's talk); pay attention not only to mergers & supernovae, but to others, because we can do relatively easily

**Thank you for your attention!**