Conformality of supersymmetric gauge theory

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We show that the 4-dimensional $\mathcal{N}=1,2$ supersymmetric gauge theories are conformal provided that the anomalous breaking of chiral symmetry is unobservable. In supersymmetric gauge theories, the trace anomaly which controls the scale evolution of the coupling constant and the axial anomaly of the R-symmetry belong to the same multiplet, so that the irrelevance of the latter leads to the conformality. For the $\mathcal{N}=1$ case, this also automatically resolves the "anomaly puzzle" since both anomalous violations are unphysical. We also show that the anomaly mediated supersymmetry breaking does not happen for the same reason. We then conclude that any extensions of the standard model which restore the supersymmetry below the grand unification scale are excluded.

Supersymmetry (SUSY) has been the target of extensive debates since its conception [1–10] thanks to its rich phenomenology as regards the extension of the standard model (SM), as well as the theoretical and mathematical interests, especially in the formulation of string theories. One of the most interesting feature of SUSY was that the nonrenormalization theorem [11–13], which forbids radiative corrections to some selected interactions. The most appreciated advantage was the cancellation of the quadratic divergences to the mass of the Higgs boson, which leads to the resolution of the hierarchy problem of the SM [14].

The current situation of SUSY in 2025 is however not good. The minimal supersymmetric extension of the SM (MSSM) requires a SUSY breaking scale close to the Higgs boson mass, since the dynamics of SUSY must trigger the spontaneous symmetry breaking of the electroweak $SU(2)_L \times U(1)_Y$ gauge symmetry. Due to the null results of SUSY search in accelerator experiments [15–22], the separation between these two scales is getting larger and larger, which is also called the little hierarchy problem [24, 25]. To avoid this issue, supersymmetric models sacrificing the minimality have recently been discussed [26–29], but other strong experimental constraints on the flavor off-diagonal interactions [30], CP violations [23, 31, 32], and R-parity (baryon or lepton number) violating interactions [33–39], are also left as unnatural points.

On the theoretical side, models with multiple supersymmetric charges have been extensively discussed, and numerous successes have been achieved, such as the exact confining solution of the $\mathcal{N}=2$ supersymmetric Yang-Mills theory [40, 41]. As fundamental aspects, the scale evolution of the coupling constant and the infrared behavior were deeply investigated since the early era [42, 43]. The $\mathcal{N}=4$ theory is known to be conformal [44–46], but its duality with the type IIB string theory in the large N_c limit was one of the epoch-making discovery in mathematical physics [47–49]. As for $\mathcal{N}=2$, it is believed that the supersymmetric gauge theories evolve according to the perturbative one-loop contribution and also due to the nonperturbative instanton effect at low energy scale [50, 51]. The $\mathcal{N}=1$ case, which was mostly discussed in phenomenology, also knew an intense theoretical debate due to the "anomaly puzzle". In SUSY, the axial anomaly of the $U(1)_R$ current j^μ_R and the trace of the energy-momentum tensor T^μ_μ belong to the F-component of a same supermultiplet (the so-called anomaly multiplet) [52]

$$\mathcal{F}_X = \frac{2}{3}T^\mu_\mu + i\partial_\mu j^\mu_R, \tag{1}$$

so that the one-loop finiteness of the axial anomaly certified by Adler-Bardeen theorem [53–59] imposes the same property to the trace anomaly which controls the running gauge coupling. However, explicit calculations show that the latter has higher order corrections [42, 60–63], and this apparent puzzle was discussed in many previous works [64–73].

As we saw above, the role of the quantum anomaly is crucially important in supersymmetric gauge theories. Recently, the explicit violation of the chiral (axial) $U(1)$ symmetry in nonabelian gauge theory was shown to be irrelevant by the present author [74, 75]. The flow of the argument is as follows. The anomalous breaking of chiral symmetry is due to the nontrivial topology of the gauge field (or instanton configuration), and the corresponding topological charge is probed by the integral

$$\begin{aligned}
&\frac{\alpha_s}{8\pi}\int d^4x\, F_{\mu\nu,a}\tilde{F}^{\mu\nu}_a \\
&= \frac{ig_s\alpha_s}{24\pi}\int d^3\vec{x}\, f_{abc}\epsilon_{ijk}A_{ia}(\vec{x})A_{jb}(\vec{x})A_{kc}(\vec{x})\Big|^{t=+\infty}_{t=-\infty} \\
&= \Delta n,
\end{aligned} \tag{2}$$

where F and $\tilde{F}$ are the strength of nonabelian gauge field and its dual, respectively, with the gauge coupling constant $\alpha_s \equiv g_s^2/4\pi$. The last equality yields the shift of the gauge winding number Δn which is always an integer number. We see that this integral contains a triple product (contraction with ϵ_{ijk}) of the spatial components of the gauge field, including the unphysical longitudinal component which, according to Becchi-Rouet-Stora-Tyutin (BRST) symmetry [76, 77], originates from the local gauge symmetry $A^\mu_a(x) \to A^\mu_a(x) + \partial^\mu \chi_a(x) + O(g_s)$. This implies that the topological charge is also unobservable. We also note that higher order terms of g_s do not contribute to the chiral anomaly up to the renormalization of fields because Adler-Bardeen theorem forbids higher order corrections to the topological charge operator. This unobservability may be derived in two different ways [74, 75].

The irrelevance of the topological charge immediately affects the anomalous nonconservation of the axial $U(1)$ symmetry. The physical contribution to the chiral Ward-Takahashi identity [78] then becomes

$$\sum_\psi^{N_f} \Big[\partial^\mu(\bar\psi\gamma_\mu\gamma_5\psi) + 2m_\psi \bar\psi i\gamma_5\psi\Big]_{\lambda\neq 0} = -\frac{N_f\alpha_s}{8\pi} F_{\mu\nu,a}\tilde F^{\mu\nu}_a\Big|_{\Delta n=0}, \quad (3)$$

where the subscripts of the left-hand side $\lambda \neq 0$ means that we removed the chiral Dirac zero-modes, which are the unphysical contribution equivalent to the topological charge according to Atiyah-Singer index theorem [79, 80], and that of the right-hand side $\Delta n = 0$ means that the topological charge was removed from the gauge configuration. The above identity now has only a local effect so that the global axial $U(1)$ symmetry is not violated by the anomaly operator in the observable space (up to fermion masses of the Lagrangian), which actually plays the most important role in our discussion.

In this work, we show that nonabelian gauge theories with global supersymmetry (also called "rigid") are actually conformal, based on the unobservability of the topological charge (2) and the irrelevance of the axial $U(1)$ anomaly stated by the presented author (for the abelian case, there is no topological charge, so the problem does not exist). The essential point of our discussion is to show that the two main effects, the nonperturbative instanton and the perturbative R- and trace anomalies are not relevant. In this work, we focus on the $\mathcal{N} = 1, 2$ SUSY in 4-dimension, and we do not explicitly consider the supergravity. For the $\mathcal{N} = 1$ case, we find a consistent resolution of the anomaly puzzle. We also show that the anomaly mediated SUSY breaking (AMSB) cannot occur. We eventually outline the most important implications to the phenomenology of particle physics.

The unobservability of the topological charge and the irrelevance of the axial $U(1)$ anomaly immediately leads to the irrelevance of the topological instantons [81–84]. The instanton of the Yang-Mills theory is the classical solution of the self-(anti)dual equation

$$\tilde F^a_{\mu\nu} = \pm F^a_{\mu\nu}. \quad (4)$$

With this solution, the Euclidean Yang-Mills Lagrangian, given by

$$S_{\rm YM} = \frac{1}{4}\int d^4x \Big[\pm F^a_{\mu\nu}\tilde F^a_{\mu\nu} + \frac{1}{2}\big(F^a_{\mu\nu} \mp \tilde F^a_{\mu\nu}\big)^2\Big], \quad (5)$$

is minimized. Equation (4) has a known solution,

$$A^{(\rm cl)}_{\mu a} = 2\eta_{a\mu\nu}\frac{x_\nu}{x^2+\rho^2}, \quad (6)$$

where $\eta = \epsilon_{a\mu\nu}$ $(\text{for}\, \mu,\nu = 1,2,3), -\delta_{a\mu}$ $(\text{for}\, \nu = 4), \delta_{a\nu}$ $(\text{for}\, \mu = 4)$, and ρ is an arbitrary parameter denoting the instanton size. This single instanton solution has a topological charge, and it is interpreted as the quantum tunneling between vacua with different winding numbers. Since the topological charge is not observable, the instantons, and dilute instanton gas configurations, cannot be distinguished from the trivial vacuum. This means that effects associated with the number of instantons are all irrelevant at the level of observables.

When the gauge theory contains fermions as in supersymmetric theories, their chiral Dirac zero-modes interact with the instanton and anomalous nonconservation of axial $U(1)$ symmetry is induced through the multi-fermion 't Hooft vertex [87–90]. For instance, the partition function of the $\mathcal{N} = 1$ supersymmetric Yang-Mills theory becomes

$$Z_{\rm SYM} = \int \mathcal{D}A \prod_i (\mathcal{D}\psi_i \mathcal{D}\bar\psi_i) e^{-S_{\rm YM} - \frac{i}{2}\int d^4x \sum_i \bar\psi_i \not{D}\psi_i} = \prod_{i,l}(\lambda_{il})\int \mathcal{D}A\, e^{-S_{\rm YM}}, \quad (7)$$

where i is the label of all fermions coupled to the gauge theory with $\not{D}$ the Dirac operator in the appropriate representation and couplings, and λ_{li}'s are the corresponding l-th eigenvalues. The non-zero mode ($l \neq 0$) contribution mutually cancels with superpartners in SUSY. While the presence of zero modes $\lambda_{i0} = 0$ naively leads to $Z_{\rm SYM} = 0$, they indeed become finite when combined with $2N_c$ gaugino [69, 91] or N_c quark zero-mode propagators $S_{i0}(x,y) = \frac{\psi_{i0}(x)\psi^\dagger_{i0}(y)}{\lambda_{i0}}$ per instanton number. This multi-fermion correlation is just the 't Hooft vertex, and it was so far believed to be relevant when interacting with the instanton, but its effect cannot be observed as well since the number of chiral zero-modes and the unobservable topological charge are the same.

The $\mathcal{N} = 1, 2$ supersymmetric gauge theories have so far been known to not be conformal due to the perturbative anomalous effect and also due to the nonperturbative instanton. As regards the latter, the derivation of the gaugino condensate is known [92–102]. The running of the gauge coupling was also believed to be affected by instantons at low energy scale, and it was so far extensively studied [42, 43, 51, 99, 103–115]. For the $\mathcal{N} = 2$ case, it was particularly interesting that we could match the prepotential, the factorized superfield function of the exact low energy effective Lagrangian [40, 41], with the gauge coupling whose scale is given by the expectation value of the scalar field in the flat direction. The prepotential is actually the superfield version of the 't Hooft interaction, and it is composed of an infinite series of power corrections for which the order is given by the instanton number. However, as we saw above, the instanton and the 't Hooft vertex are not observable, so they can neither violate the scale invariance in the physical space, nor contribute to the scale evolution of the coupling constant.

According to our result, the perturbative anomalous effect is also irrelevant, but showing it needs more detailed discussions. As already mentioned, the chiral anomaly of the $U(1)_R$ symmetry and the trace anomaly belong to the same supersymmetric representation. Their corresponding currents are part of the vector supermultiplet (the so-called Ferrara-Zumino supercurrent) [52]

$$\mathcal{J}^\mu = j_R^\mu + \theta S^\mu + T^\mu_\nu \theta\sigma_\nu\bar\theta + \cdots , \tag{8}$$

where $\theta, \bar\theta$ are Grassmann spinor variables expressed in the Weyl spinor representation (which we use from now on), and S^μ is the supersymmetric current. In $\mathcal{N}=1,2$ supersymmetric gauge theories, not only the R- and scale symmetries, but also the special supersymmetry (also called S-supersymmetry, generated by the current $x^\nu\gamma_\nu S^\mu$) has quantum anomaly. For the $\mathcal{N}=1$ case, matter chiral superfields Φ may also be present with their own independent Konishi anomaly [116], given in terms of superfields and the supercovariant derivative D as

$$\bar D^2(\Phi^* e^{2V}\Phi) = \frac{t_2(\Phi)}{8\pi^2} W_a W_a, \tag{9}$$

where $V \equiv V_a t_a$ is the gauge vector superfield, $(W_a t_a)_\alpha \equiv -\frac{1}{4}\bar D^2 e^{-2V} D_\alpha e^{2V}$ (α is the undotted spinor index), and $t_2(\Phi)$ is the Dynkin index of the representation of Φ. The right-hand side includes the chiral, scale symmetries, and the $\mathcal{S}$-supersymmetry. According to our discussion given above, all those anomalies must be irrelevant and the corresponding violations of global symmetries do not occur, since the usual SUSY (generated by the spatial integration of S^0, also called Q-supersymmetry), which has no anomaly, transforms each element of Eq. (8) into another (it has recently been pointed out that the R-symmetry anomaly triggers the anomaly of Q-supersymmetry [117–120]).

Let us now inspect the consistency with the anomaly puzzle. In the superfield formalism, the quantum anomaly is expressed as

$$\bar D^{\dot\alpha}\mathcal{J}_{\alpha\dot\alpha} = D_\alpha X, \tag{10}$$

where $\mathcal{J}_{\alpha\dot\alpha} \equiv -2\sigma^\mu_{\alpha\dot\alpha}\mathcal{J}_\mu$. The chiral superfield X is actually the anomaly multiplet, whose F-term contains the chiral and trace anomalies, as seen in Eq. (1). It has been pointed out that the redefinition of X by $X' = X + \frac{1}{2}\bar D^2\Xi$ does neither change the Q-supersymmetry, nor the space time translation due to $T^{\mu\nu}$, which is just the supersymmetric version of the improvement of the symmetry generators [121]. The introduction of this new real vector superfield Ξ then modifies the relation between the R- and trace anomalies,

$$\begin{aligned} \frac{2}{3}T^\mu_\mu &= \mathrm{Re}[\mathcal{F}_X] + \frac{1}{3}\mathcal{D}_\Xi, \\ \partial_\mu j^\mu_R &= \mathrm{Im}[\mathcal{F}_X], \end{aligned} \tag{11}$$

where $\mathcal{D}_\Xi$ is the D-term of Ξ. This extra term due to Ξ brings a new degree of freedom to the trace anomaly, and it was claimed to be the resolution of the anomaly puzzle [71–73]. However, if we admit that the axial anomaly is not anymore relevant, the R-transformation becomes possible without affecting observables, and consequently makes the definition of the axial ($\partial_\mu j^\mu_R$) and trace (T^μ_μ) anomalies ambiguous by mixing the two (since the R-transformation is a complex phase rotation for the F-term). Since a change of the basis can transform the original T^μ_μ to $\partial_\mu j^\mu_R$ while the axial $U(1)_R$ anomaly is irrelevant, the trace anomaly must also be unphysical as well (and so does the S-supersymmetry anomaly), so that the $\mathcal{N}=1$ gauge theory is conformal. Another consistent way to understand this is to consider holomorphic transformation parameters, for which a quantity that transforms under conformal symmetry behaves as $\phi \to e^{a+ib}\phi$ and the change of the basis yields $e^{c+id}\phi \to e^{ac-bd+i(bc+ad)}\phi$ still keeps the same physics while the real and imaginary parts look completely different. We then conclude in this part that the anomaly puzzle does not actually exist, since the $U(1)_R$ and trace anomalies cannot be both observed, even if they might not have the same coefficients.

We also comment on the renormalization of the $\mathcal{N}=1$ supersymmetric gauge theory using the Wilsonian cutoff scheme which was discussed in Ref. [70]. In the holomorphic basis, the $\mathcal{N}=1$ Yang-Mills Lagrangian must be of the form

$$\begin{aligned} \mathcal{L}_{\mathrm{SYM}} &= \frac{1}{16}\int d^2\theta \left(\frac{1}{g_s^2} + i\frac{\theta_s}{8\pi^2}\right) W_a(V) W_a(V) \\ &\quad +(\text{h.c.}), \end{aligned} \tag{12}$$

where θ_s is the topological θ-angle. Here we explicitly wrote the argument of W in terms of the vector superfield V, because the above expression will not become holomorphic if we use the "canonical" normalization $g_s V$ to form W since g_s is real, while the latter is more appropriate for the Wilsonian renormalization group evolution. To keep the holomorphicity in the path integral formulation in converting to the canonical normalization, the anomalous Jacobian appears from the measure as

$$\mathcal{D}V \exp\Bigg(-\frac{1}{16}\int d^4x d^2\theta \Big(\frac{1}{g_s^2} + i\frac{\theta_s}{8\pi^2} - \frac{2t_2(V)}{8\pi^2}\ln g_s \Big) \times W_a(g_sV)W_a(g_sV) + (\text{h.c.}) \Bigg). \quad (13)$$

This is just the supersymmetric extension of Fujikawa's method [122, 123], and a similar Jacobian also arises from the rescaling (including the complex phase) of the matter superfields for $\mathcal{N}=1$ [124], while the Jacobian cancels in the $\mathcal{N}=2$ case. By again admitting our result, the exponent of the Jacobian, which is fully originating from the chiral $U(1)_R$ and trace anomalies, has no observable effects and can therefore be omitted. We could indeed show that the path integral formulation with the Wilsonian renormalization scheme does not affect the holomorphicity, the conformality, and the anomaly puzzle.

So far we have seen that the R- and trace anomalies are both unphysical, but we should also address the explicit operator level realization. As we mentioned in the beginning of this work, the chiral anomaly $\partial_\mu j^\mu_R \propto F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$ is irrelevant because the topological charge (2) always contains the unphysical longitudinal gauge component. For the trace anomaly, however, it is less obvious since $T^\mu_\mu \propto F^a_{\mu\nu}F^{\mu\nu}_a$, even if we use the equations of motions to erase the fermions and scalars (note that the chiral Dirac zero-modes may be converted to the topological charge). Since the R-transformation is just a complex phase rotation of the F-term ($\mathcal{F}_X = aF^a_{\mu\nu}F^{\mu\nu}_a + ibF^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$), the two operators must have the same structure. A good candidate is the self-(anti)dual relation (4), for which we have $F^a_{\mu\nu}F^{\mu\nu}_a = F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$. In such a case, the trace anomaly integrated over space-time becomes unobservable since it also contains the unphysical longitudinal gauge mode. Interestingly, the instanton (6) which is the solution of the self-(anti)dual equation (4) also has a special conformal symmetry, so this totally fits the expected conformality.

Now let us inspect the AMSB [125, 126] within our result. This mechanism consists of breaking SUSY in some hidden sector and communicating it via the quantum anomaly to a SUSY gauge theory of another decoupled sector. If we suppose that the hidden sector such as the supergravity contains a background superfield Φ with vacuum expectation value like $\langle\Phi\rangle \equiv 1 + \langle\mathcal{F}_\Phi\rangle\theta^2$ (the so-called "compensator"), the redefinition of other dynamical superfields of the hidden sector may absorb $\langle\Phi\rangle$. It was then stated that this change of basis affects the decoupled sector via the renormalization, i.e. the trace anomaly, since the renormalization scale parameter is also holomorphically modified by $\langle\Phi\rangle$. This then induces the mass of the gaugino and the trilinear scalar coupling proportional to $\langle\mathcal{F}_\Phi\rangle$ in $\mathcal{N}=1$ theory [125, 126]. Again, if we admit our result, the trace anomaly has no observable effects, so the AMSB does not happen.

When the SUSY gauge theory contains mass parameters in the superpotential (like the μ-term in MSSM), the conformality will be explicitly broken, and a scale dependence will be introduced in the theory. This is however a very local effect in the scale axis, since at a scale M larger than the mass parameter m the conformality breaking effects are suppressed by powers of $1/m$, and at $M < m$ the superfields affected by m just decouple from the theory. In another word, the explicit mass parameters act as boundary conditions in the scale axis, where theories with different number of superfields are connected.

Since SUSY, in particular $\mathcal{N}=1$, has been discussed very extensively in phenomenology, its conformality greatly affects many conclusions of past research. The most notable one must be the running couplings of gauge interactions and the grand unified theory (GUT) [127]. The SUSY $SU(5)$ GUT [128–131] was known to have a very good unification of gauge couplings at very high energy scale, around 10^{16}GeV, and it was one of the most important driving force of the phenomenological study of SUSY. Since the conformality forbids the running coupling, SUSY extensions of the SM are totally inappropriate for GUT, and they are most probably inadequate for becoming the candidates of theory beyond the SM. It is also not hard to extend our study to other SUSY gauge theories in $d \neq 4$. Although more inspections are needed, this discussion will definitely affect the superstring theory as well.

We note that although quantum dynamics of SUSY gauge theory is not observable, it still keeps its value as calculational techniques if the study of a theory corresponding to it via duality has a demand [47, 132, 133]. For instance, the $\mathcal{N}=4$ theory is known to be conformal, but its instanton physics has been studied [134]. If we admit our result, this effect has no observable consequences, but these inspections may remain valuable in the context of the development of calculational techniques and mathematics.

We also add some conceptual remarks concerning the conformality which forbids the quantum anomalies. The trace anomaly is in principle the source of all quantum effects and interactions, so its irrelevance means that the dynamics is strictly classical and deterministic in SUSY theory and that the quantum information will not be processed, so that "nothing happens". Thanks to that, approximate SUSY may be used in other fields of research [135–141] as a fixed point which makes an expansion around it possible.

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