Unobservability of magnetic monopole

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(Dated: March 5, 2025)

We show that the magnetic monopole (and the dyon) is unphysical by proving that it always involves the unphysical gauge field component. Thanks to their topological nature, monopoles are not affected by continuous radiative corrections, so their perturbativity is guaranteed, and therefore they are unobservable. We then discuss the phenomenological consequences. So far, it was believed that the inflation must occur after the generation of $U(1)$ gauge theory through spontaneous symmetry breaking so as to avoid the overclosure of the Universe by the monopoles. Our conclusion therefore removes this constraint, since they have no physical effect. We list some scenarios which can be resurrected through our discussion, notably those related to the Pati-Salam model. We also demonstrate that the unobservability of the monopole and its condensation, which is the criterion of color confinement, are not inconsistent.

I. INTRODUCTION

In 1931, Dirac showed the possibility of the existence of magnetic monopoles by generalizing Maxwell's equations [1, 2]. One of them, the Bianchi identity, is extended as

$$\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial^\mu F^{\rho\sigma} = -k^\nu, \tag{1}$$

where $F^{\mu\nu}$ is the field strength tensor of a $U(1)$ gauge theory and k^μ is the magnetic monopole current. Numerous experimental measurements have been done so far, but the magnetic monopoles have never been discovered [3–11].

In the early 70's, the Abelian monopole was generalized to the nonabelian case by Wu and Yang [12, 13]. Shortly after, in 1974, Georgi proposed the grand unified theory (GUT) [14, 15], while 't Hooft and Polyakov demonstrated the existence of a monopole solution in $SU(2)$ gauge theory [16–19]. Their research showed that monopoles arise when a nonabelian gauge symmetry breaks down and smaller gauge groups containing $U(1)$ appear. Soon after that, it was realized that magnetic monopoles appear after the spontaneous symmetry breaking of GUT [20–29], and hence became problematic in cosmology because such objects would overclose the Universe while they have never been observed in experiments. This issue is called the "monopole problem". To resolve this problem, the idea of inflation which dilutes cosmological relics was decisive [30–35]. It was actually a strong requirement that the inflation had occurred after the transition of GUT.

The monopole problem prohibits new physics beyond the standard model (BSM) that would produce a (or multiple) $U(1)$ gauge theory at the low temperature. On the other hand, monopoles which originate from TeV scale BSM physics could be candidates for dark matter (DM) [36–41].

Magnetic monopoles are also important in the context of color confinement which is believed to be an essential property of asymptotically free nonabelian gauge theory, although this phenomenon has not been proved analytically. A criterion of the confinement is the magnetic monopole condensation [42–47] which is based on the dual Meissner effect. In the conventional superconductor picture, the condensation of Cooper pairs excludes the magnetic field from the interior of the superconductor. On the other hand, in the dual superconductor picture, the condensation of magnetic monopoles forms electric flux tubes between colored particles. In some theoretical models, the monopole condensation has been analytically derived [48–51], and also strongly suggested in QCD from lattice results [52, 53].

Recently, it has been claimed that the topological charge of the nonabelian gauge theory is not observable [54, 55]. Since the magnetic monopoles are also topological defects, one would expect them to be unphysical as well. The aim of this paper is to demonstrate this conjecture. If they are unobservable, there is no need to dilute them through inflation, thereby removing an important constraint on GUT. For instance, the Pati-Salam model was affected by the monopole problem [56–60]. If the monopole is unphysical, it is possible to conceive scenarios where a (or multiple) $U(1)$ gauge symmetry appears in low energy physics. In Fig. 1, we display the cosmological chronology of some of them as well as the conventional one. We also have to show the consistency of this unphysicalness with the monopole condensation, but this is actually possible.

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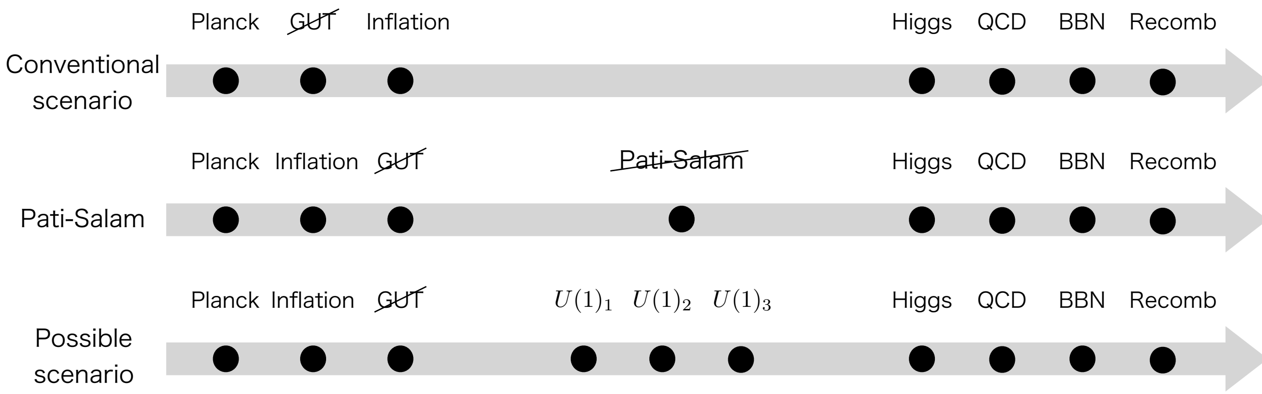

FIG. 1. Possible scenarios for the cosmological timeline. "Planck" denotes the early Universe described by Planck scale physics. Slashed "GUT" and slashed "Pati-Salam" signify the times at which symmetries of GUT and Pati-Salam model break down, respectively. In the conventional scenario (top timeline), the inflation must occur after the GUT transition to dilute the density of magnetic monopoles. We suggest some scenarios with high scale inflation, where the gauge symmetry breaks at a low energy scale such as the Pati-Salam model (middle timeline) or more radical ones (bottom timeline), where multiple $U(1)$ gauge symmetries (given by the notations "$U(1)_1$", "$U(1)_2$" and "$U(1)_3$") are generated between the GUT scale and the SM scale denoted as "Higgs". "QCD", "BBN" and "Recomb" mean the times at which the QCD transition, the Big Bang nucleosynthesis and the recombination occur, respectively.

This paper is organized as follows. In the next section, we analytically derive the unobservability of the magnetic monopole. In Section II, we then explore the phenomenological consequences of our finding to GUT and BSM physics. We also show in Section III the compatibility of our result with the magnetic monopole condensation, and finally conclude in the last section.

II. UNPHYSICAL MAGNETIC MONOPOLE

A magnetic monopole may be generated if the gauge group G is broken down to H and when the second homotopy group $\pi_2(G/H)$ is non-trivial. In this section, we use the example of the $SU(2)$ model by 't Hooft and Polyakov [16–19], where $\pi_2(SU(2)/U(1)) = \mathbb{Z}$, to show that magnetic monopoles are unphysical. The Lagrangian we consider is

$$\mathcal{L} = |(D_\mu\phi)^a|^2 - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{\lambda}{4}\left(|\phi^a|^2 - v^2\right)^2, \quad (2)$$

where ϕ is the scalar field transforming in the adjoint representation. Here, the covariant derivative D_μ and the gauge field strength $G^a_{\mu\nu}$ are defined by

$$(D_\mu\phi)^a = \partial_\mu\phi^a - gf^{abc}A^b_\mu\phi^c, \quad (3)$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu, \quad (4)$$

where A^a_μ is the nonabelian gauge field and f^{abc} is the structure constant of the $SU(2)$ group with a, b, c the indices of the adjoint representation. In the vacuum, the scalar field is minimized at the vacuum expectation value (VEV) $|\phi| = v$ leaving a tangential $U(1)$ degree of freedom. The Lagrangian expanded around $|\phi| = v$ then becomes

$$\mathcal{L} = g^2v^2\left[A^a_\mu A^{a\mu} - \frac{1}{v^2}\left(A^a_\mu\phi^a\right)\left(A^{b\mu}\phi^b\right)\right] + \cdots, \quad (5)$$

where the ellipses denote the other terms of Eq. (2). The gauge fields orthogonal to the VEV of ϕ obtain a mass gv. We then end up with a massless electromagnetic gauge boson.

After the breakdown of $SU(2)$ gauge symmetry, we obtain the following electromagnetic field tensor

$$F_{\mu\nu} = \frac{\phi^a}{v}G^a_{\mu\nu}. \quad (6)$$

The following magnetic charge is then generated:

$$\begin{aligned} M &= \lim_{r\to\infty}\oint \boldsymbol{B}\cdot d\boldsymbol{S} \\ &= -\frac{1}{2}\int_V \epsilon^{ijk}\partial_i F_{jk} dV, \end{aligned} \quad (7)$$

where the indices i, j, k denote the spatial coordinates.

The energy of the static and noninteracting magnetic monopole, according to the Lagrangian (2), is given by the following Hamiltonian

$$\begin{aligned} H = \int d^3x \Big[&|(D_0\phi)^a|^2 + \frac{1}{2}\left|G^a_{0i}\right|^2 + |(D_i\phi)^a|^2 \\ &+ \frac{1}{4}\left|G^a_{ij}\right|^2 + \frac{\lambda}{4}\left(|\phi^a|^2 - v^2\right)^2\Big]. \end{aligned} \quad (8)$$

In order for the energy of the state not to diverge, $|(D_i\phi)^a|$ needs to dump faster than $r^{-\frac{3}{2}}$ at long distance. However $\partial_i\phi^a$ behaves as r^{-1} due to the angular dependent terms of the 3-dimensional derivative in the spherical coordinate $\vec{\partial} = \vec{e}_r\frac{\partial}{\partial r} + \frac{\vec{e}_\theta}{r}\frac{\partial}{\partial\theta} + \frac{\vec{e}_\varphi}{r\sin\theta}\frac{\partial}{\partial\varphi}$. Therefore,

$gf^{abc}A_i^b\phi^c$ must cancel out $\partial_i\phi^a$ to the order of $r^{-\frac{3}{2}}$ and we see that $\partial_i\phi^a$ equals $gf^{abc}A_i^b\phi^c$ at $r\to\infty$. We are now ready to show that Eq. (7) is unphysical. The condition $D_i\phi=0$ implies that

$$A_i = \frac{1}{v^2 g} f^{abc}\phi^b(\partial_i\phi^c), \quad (9)$$

$$G_{ij} = \frac{1}{v^2 g} f^{abc}(\partial_i\phi^b)(\partial_j\phi^c). \quad (10)$$

Now we substitute the above G_{ij} to Eq. (6) then to Eq. (7), and obtain

$$M = -\frac{1}{2gv^3}\int_V \epsilon^{ijk} f^{abc}(\partial_i\phi^a)(\partial_j\phi^b)(\partial_k\phi^c)dV. \quad (11)$$

By again using $D_i\phi=0$ at $r\to\infty$, the integrand gets the asymptotic form

$$\epsilon^{ijk} f^{abc} f^{alm} f^{bno} f^{cpq} A_i^l A_j^n A_k^p \phi^m\phi^o\phi^q, \quad (12)$$

at long distance. From this derivation, it becomes clear that the monopole solution must necessarily include the above combination of three gauge field operators with the Levi-Civita tensor ϵ^{ijk}, which covers all three-dimensional directions. Since the time dependence is eliminated for isolated systems, the gauge fields coupled to noninteracting monopoles have just three degrees of freedom. When combined with ϵ^{ijk}, at least one of the gauge fields has an unobservable longitudinal gauge component, which makes the *magnetic monopole unphysical*. The same conclusion applies for dyons which carry both electric and magnetic charges. It is also a trivial task to extend this discussion to other gauge groups by noting that the gauge field A_μ^a belongs to the gauge group G, the VEV v to G/H, and finally the $U(1)$ gauge field strength $F_{\mu\nu}$ to the $U(1)$ part of H, which were introduced in the beginning of this section.

Topologically, the magnetic monopole is labeled by an integer number which cannot be affected by continuous radiative corrections, and any arbitrary higher-order contributions are prohibited. Therefore, the perturbativity (leading order perturbative finiteness) is ensured, much like the Adler-Bardeen theorem which applies to the chiral anomaly [61, 62]. The unobservability of the longitudinal gauge field component is a crucial consequence of the Becchi-Rouet-Stora-Tyutin (BRST) symmetry [63, 64], which is actually warranted in perturbation theory and is therefore exactly the required property for our demonstration. By being perturbative, the field amplitude becomes infinitesimal [65], thus avoiding the Gribov ambiguity [66, 67]. We emphasize that the above discussion is not refuting the mathematical derivation of the magnetic monopole solution by 't Hooft and Polyakov [16, 17]. Our statement is that the monopole solution is not present in the physical space, like the Faddeev-Popov ghost, and therefore cannot be observed. This approach was also used in proving the unphysicalness of the topological charge (see Refs. [54, 55] for details).

III. IMPLICATIONS FOR GUTS AND NEW PHYSICS

When the symmetry of GUT spontaneously breaks dowm leaving $U(1)$ gauge symmetries, monopoles are generated [20–27]. From the traditional understanding, their nondetection is due to the dilution of the monopole density by the cosmological inflation [30–35]. That is, this GUT transition should have occurred before inflation. However, if monopoles are unphysical and thus unobservable, as we have shown in the previous section, there is no more necessity to consider them. From now on, there are no restrictions in the time order between the GUT transition and the inflation. Furthermore, we can conceive scenarios with additional generation of $U(1)$ gauge theories at arbitrary scales. Also, models that identify the DM as magnetic monopoles are prohibited.

According to the above discussion the following unification scenarios are possible (see Fig. 1):

1. GUT transition before inflation,
2. Inflation before GUT transition,
3. Inflaton is contained in GUT,
4. Step-by-step unification.

So far, only Scenario 1 was allowed due to the constraint from the nonobservation of physical monopoles. Traditional approaches have rejected Scenarios 2 and 3 because they cannot explain the null experimental detection, but if these objects cannot be observed, such settings could become viable candidates which have their own interests. In Scenario 2, the inflation may be a phenomenon triggered by the Planck scale physics. The advantage of Scenario 3 is that the GUT Lagrangian itself contains the inflaton. There, the mass scale of inflaton is slightly higher than that of the unification scale, so the GUT transition occurs right after the inflation. It is interesting to note that Scenarios 2 and 3 can generate sufficient baryon number so as to explain the matter abundance of the current Universe [68–70]. The GUT scale of Scenario 1, 2 and 3 must be higher than 10^{16} GeV according to the constraint of proton decay experiment [71].

Furthermore, we can consider Scenario 4 where the SM gauge group $SU(3)_C\times SU(2)_L\times U(1)_Y$ is progressively unified at low energy scales. For instance, in the Pati-Salam model [56–60], the gauge group is set as $SU(4)\times SU(2)_L\times SU(2)_R$, unifying the strong interaction and the hypercharge of the SM. Since this is a subgroup of $SO(10)$ [72, 73], considering $SO(10)$ GUT allows for a stepwise unification of interactions. Historically, the Pati-Salam model faced the monopole problem, necessitating the symmetry breaking at a high energy scale in the early Universe (Scenario 1). Other model [74–84] were also constrained by the same reason. However, if monopoles are unphysical, storylines where the gauge symmetry breaks at lower energy scales can be conceived. Futhermore, this motivates us to consider that the dark

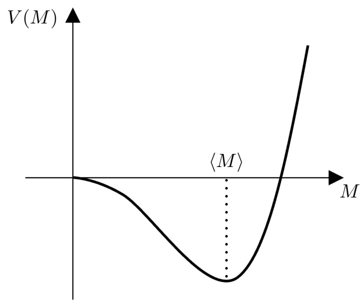

FIG. 2. Potential triggering the magnetic monopole condensation. It occurs when V has a minumum at a nonzero value $\langle M \rangle$.

photon [37, 85], atomic dark matter [86], the $U(1)$ symmetries used in the Froggatt-Nielsen model [87–91], or abelian gauge charges appearing in other BSM physics originate from a unified theory.

IV. MONOPOLE CONDENSATION

Color confinement is believed to occur when magnetic monopoles condense in the vacuum. However, we concluded in this work that these objects are actually unphysical. In this section, we show that the unobservability and the condensation of magnetic monopoles are not inconsistent.

To condense the monopole in the vacuum, its potential must have a minimum at a nonzero value as displayed in Fig. 2. Such potential can be written as

$$V(M) = \sum_{n \geq 2} C_n M^n. \tag{13}$$

Here, C_n are coefficients chosen so as to give the minimum of the potential at nonzero M, and the summation does not include $n = 1$ because it is unphysical as we have discussed above. Let us briefly review the physical meaning of the VEV of a field. The VEV points to the configuration at which several interaction terms counterbalance so that the creation and annihilation reach equilibrium. Therefore, the fact that the potential of Eq. (13) generates a VEV means that the monopoles are interacting with each other. This indicates that there is at least two interacting objects in the space. Since the monopole is never generated by quantum corrections as for the case of the topological charge (Adler-Bardeen theorem [61, 62]), the potential of Eq. (13) does not actually contain the topological information about the monopole so its condensation can happen even if it is unphysical. This suggests that the condensation of magnetic monopoles can be rewritten as VEVs of other nontopological operators.

V. CONCLUSION

In this paper, we argued, using the example of the $SU(2)$ model of 't Hooft and Polyakov, that magnetic monopoles (and dyons) are unobservable. We have therefore resolved the monopole problem, and their nondetection in experiments is no longer problematic. Previously, the scenario suggesting that the inflation diluted the monopole density was the most accepted one. However according to our discussion, other interesting scenarios that were previously denied have now been allowed. In some of them, the inflation occurs before the GUT transition and thereby appropriate baryon number can be generated by the GUT.

Furthermore, it is possible to envision scenarios where $U(1)$ gauge symmetries emerge at lower energy scales. We have actually resurrected the stepwise gauge unification, such as the Pati-Salam model, which is one of the subgroups of $SO(10)$. At the scale where the symmetry of the Pati-Salam model breaks down, the proton decay does not occur, making it consistent with observations. We are also now allowed to discuss the unification of BSM models involving $U(1)$ gauge theory, such as the dark photon.

We also showed that the condensation and the unobservability of the magnetic monopole are not inconsistent with each other thanks to the topological nature, and the physics of color confinement is not affected by our finding.

ACKNOWLEDGMENTS

This work was supported by Daiko Foundation. We thank Yukihiro Kanda for useful discussions and comments.

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