

Unphysical topological charge in nonabelian gauge theory (short version for particle physics phenomenology)

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Based on

N. Yamanaka, arXiv:2212.10994 [hep-th]

N. Yamanaka, arXiv:2212.11820 [hep-ph]

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Topological charge

$$\frac{\alpha_s}{8\pi} \int d^4x F_{\mu\nu,a} \tilde{F}_a^{\mu\nu} = \int d^4x \partial_\mu K^\mu$$

$$\tilde{F}_a^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}^a$$

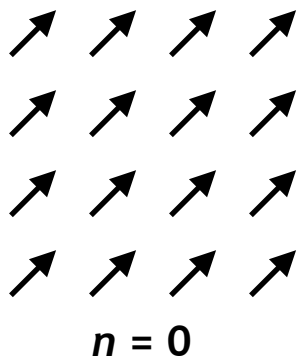
$$K_\mu \equiv \frac{\alpha_s}{8\pi} \epsilon_{\mu\nu\rho\sigma} \left[A_a^\nu F_a^{\rho\sigma} - \frac{g_s}{3} f_{abc} A_a^\nu A_b^\rho A_c^\sigma \right]$$

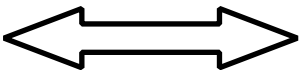
$$= \frac{ig_s \alpha_s}{24\pi} \int d^3\vec{x} f_{abc} \epsilon_{ijk} A_{ia}(\vec{x}) A_{jb}(\vec{x}) A_{kc}(\vec{x}) \Bigg|_{t=-\infty}^{t=+\infty}$$

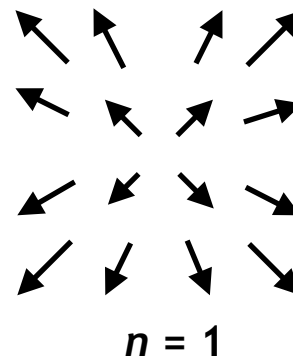
$$= \Delta n \quad \text{Integer!}$$

Integral of total derivative, but nonzero!

$\Rightarrow \Delta n =$ change of **winding number** of gauge configurations




 Change of winding #



(3-dimension
in reality)

Theta-term and Strong CP problem

QCD vacuum is a coherent superposition of vacua with different winding number
(topological charge)

$$|\theta\rangle = \sum_n e^{-in\theta} |n\rangle$$

A correlator in the path integral formulation is written as

$$\begin{aligned} \langle\theta|O_{\text{phys}}|\theta\rangle &= \sum_{n,m} e^{-i(n-m)\theta} \langle m|O_{\text{phys}}|n\rangle \\ &= \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\bar{\psi} O_{\text{phys}} e^{i \int d^4x \mathcal{L}_{\text{QCD}} + i\theta \frac{\alpha_s}{8\pi} \int d^4x F_{\mu\nu a} \tilde{F}_a^{\mu\nu}} \end{aligned}$$

(example of one flavor QCD)

If θ is not zero, $\mathcal{L}_\theta = \theta \frac{\alpha_s}{8\pi} F_{\mu\nu}^a \tilde{F}_a^{\mu\nu}$ appears effectively in Lagrangian
(so-called θ -term)

There is in principle no symmetry argument to forbid θ -term

Strong CP problem

θ -term : $\mathcal{L}_\theta = \theta \frac{\alpha_s}{8\pi} F_{\mu\nu}^a \tilde{F}_a^{\mu\nu}$

\Rightarrow A natural interaction term of QCD, **θ should be $O(1)$**

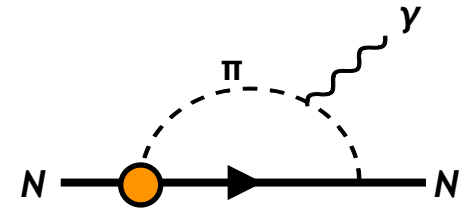
Neutron EDM (also atoms) is strongly constrained by experiments

$$|d_n| < 1.8 \times 10^{-26} \text{ e cm}$$

C. Abel et al., Phys. Rev. Lett. 124, 081803 (2020).

Converting to θ via chiral EFT analysis,

$$|\theta| < 1 \times 10^{-10} \quad \Rightarrow \text{Why so small??}$$



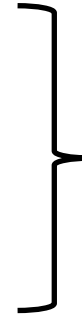
R. G. Crewther et al., Phys. Lett. B 88 (1979) 123.

\Rightarrow Strong CP problem

Resolutions proposed for the Strong CP problem

Conventional resolutions:

- Massless up quark
- Spontaneous CP breaking
- Axion mechanism



Need new physics
beyond standard model

Recently, several intrinsic QCD resolutions were also proposed:

- CP conserving correlations under instanton background

W.-Y. Ai, J. S. Cruz, B. Garbrecht, and C. Tamarit, Phys. Lett. B **822**, 136616 (2021).

- Color field screening by finite θ

Y. Nakamura and G. Schierholz, Nucl. Phys. B **986**, 116063 (2023).

- **Unphysical topological charge**

NY, arXiv:2212.10994 [hep-th]; arXiv:2212.11820 [hep-ph].

We propose a new approach:
the strong CP problem is resolved if the topological charge is unphysical.

Objective:

We show that the topological charge of nonabelian gauge theory is unphysical, and inspect the phenomenological consequences.

Unphysical topological charge

Topological charge is a triple product of gauge fields

$$\frac{ig_s\alpha_s}{24\pi} \int d^3\vec{x} f_{abc}\epsilon_{ijk} A_{ia}(\vec{x}) A_{jb}(\vec{x}) A_{kc}(\vec{x}) \Big|_{t=-\infty}^{t=+\infty}$$

Time is frozen, restricted to 3-dimensional space

ϵ_{ijk} means  \Rightarrow Covers all 3-dimensional directions!

\Rightarrow Also covers the **unphysical gauge freedom direction!**
($\vec{A}_a \rightarrow \vec{A}_a + \vec{\nabla}\chi_a$)

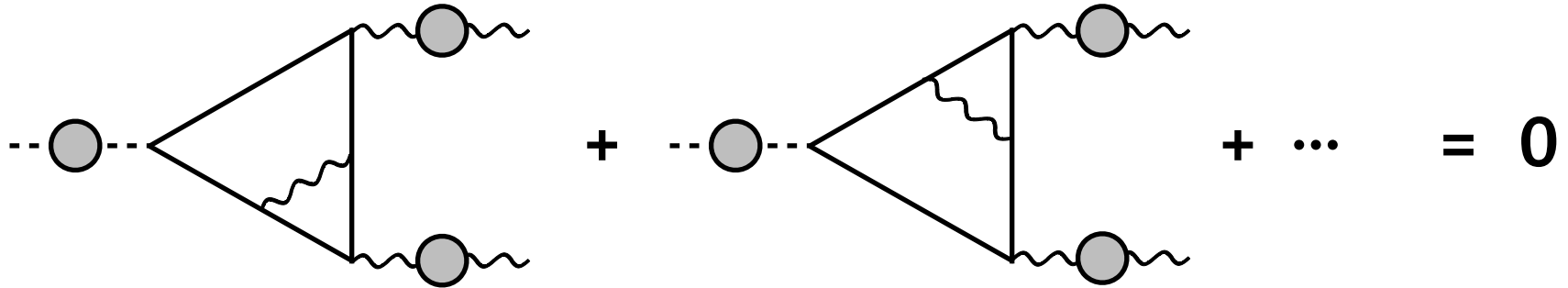
➡ Topological charge is unphysical !!

This only holds in perturbation theory ... Is it really OK ??

Adler-Bardeen theorem

Radiative corrections do not contribute to the chiral anomaly up to renormalization of external fields

⇒ Works for nonabelian gauge theories, even with nonrenormalizable interaction



S. L. Adler and W. A. Bardeen, Phys. Rev. **182**, 1517 (1969);
D. Anselmi, Phys. Rev. D **91**, 105016 (2015).

Chiral anomaly = topological charge does not change at any scale, even if external fields are “dressed” by nonperturbative effects (’t Hooft anomaly matching)

⇒ **Topological charge is strictly perturbation finite!**

⇒ **We can FULLY use the argument of unphysical gauge component!**

(To be precise, it is possible to show the irrelevance of Gribov copies)

Ward-Takahashi identity (WTI) of BRST symmetry

Let us also show the unphysicalness of topological charge using WTI

Assume the following BRST algebra

$$\left\{ \begin{array}{l} [Q_B, A(x)] = iC(x) \\ \{Q_B, \bar{C}(y)\} = B(y) \\ \{Q_B, C(x)\} = 0 \\ [Q_B, B(x)] = 0 \end{array} \right.$$

The following WTI then holds:

$$\begin{aligned} & \langle 0 | \{Q_B, T[A(x), \bar{C}(y)]\} | 0 \rangle \\ &= \langle 0 | T[A(x)B(y) - iC(x)\bar{C}(y)] | 0 \rangle = 0 \end{aligned}$$

Here operators A and \bar{C} may be chosen arbitrarily.

It is possible to derive the unphysicalness of topological charge by choosing suitable operators (next page).

WTI for topological charge

Consider the following (topological) BRST quartet

$$\left\{ \begin{array}{l} K_\mu = \frac{\alpha_s}{4\pi} \epsilon_{\mu\nu\rho\sigma} \left[A_a^\nu \partial^\rho A_a^\sigma + \frac{1}{3} g_s f_{abc} A_a^\nu A_b^\rho A_c^\sigma \right] \\ C_\mu = \frac{\alpha_s}{4\pi} \epsilon_{\mu\nu\rho\sigma} (\partial^\nu c_a) (\partial^\rho A_a^\sigma) \\ \bar{C}_\mu = \frac{\alpha_s}{8\pi} g_s f_{abc} \epsilon_{\mu\nu\rho\sigma} (\partial^{-2} \partial^\nu \bar{c}_a) A_b^\rho A_c^\sigma \\ B_\mu = \frac{\alpha_s}{8\pi} g_s f_{abc} \epsilon_{\mu\nu\rho\sigma} \left[(\partial^{-2} \partial^\nu B_a) A_a^\rho A_b^\sigma + (\partial^{-2} \partial^\nu \bar{c}_a) F_b^{\rho\sigma} c_c \right] \end{array} \right. \quad \leftarrow \text{Topological current}$$

T. Kugo, Nucl. Phys. B **155**, 368 (1979).

$$\leftarrow \text{We have freedom to choose it}$$

$$F_a^{\mu\nu} \equiv \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g_s f_{abc} A_b^\mu A_c^\nu$$

After taking total derivatives and taking $|x - y| \rightarrow \infty$,
we obtain the following “topological” WTI

$$\sum_{|\Omega\rangle \neq |0\rangle} \langle 0 | \partial_\mu K^\mu(x) | \Omega \rangle \langle \Omega | \partial_\nu B^\nu(y) | 0 \rangle = 0 \quad (\text{ghost term cancels})$$

The only surviving operator is $F\tilde{F}$
thanks to Adler-Bardeen theorem!

$$\propto \sum_{|\Omega\rangle \neq |0\rangle} \langle 0 | F\tilde{F}(x) | \Omega \rangle \langle \Omega | F\tilde{F}(y) | 0 \rangle = 0$$

$|\Omega\rangle$: vacua belonging
to different
topological sectors

\Rightarrow Effect of $F\tilde{F}$ vanishes at the level of observables even if amplitude is finite!

Operator product expansion (OPE)

Is it possible to probe the topological sector of physical states?

⇒ What is the value of $\langle \text{phys}' | F \tilde{F} | \text{phys} \rangle \equiv \langle 0 | F \tilde{F}(x) \phi(x') | 0 \rangle$?

Operator product expansion:

Operators separated by finite distances may be rewritten as a sum of local operators O_i

$$F \tilde{F}(x) \phi(y) = \sum_{\underline{O_i \neq F \tilde{F}}} C_i O_i \left(\frac{x+y}{2} \right) \quad \phi : \text{arbitrary operator}$$

Important point:

Adler-Bardeen theorem forbids generation of single topological charge density operator

⇒ We lose the information of topological sector for finitely distanced and correlated operators

We must therefore separate with infinite distance (or isolate them to not make them interact)

➡ Finitely separated operators do not change topology (correlated)

Generalized topological WTI

To avoid OPE, we must isolate operators by infinite distances (or uncorrelate)

The topological WTI for arbitrary Green's function:

$$\sum_{|\Omega\rangle \neq |0\rangle} \langle 0 | \phi(x') | 0 \rangle \langle 0 | \underline{F \tilde{F}(x)} | \Omega \rangle \langle \Omega | F \tilde{F}(y) | 0 \rangle \langle 0 | \phi(y') | 0 \rangle = 0$$

ϕ : arbitrary scalar BRST singlet function

⇒ Factorized into topological WTI thanks to operator product expansion

The topological WTI with higher power of $F\tilde{F}$ for arbitrary Green's function:

$$\sum_{\Omega_1, \dots, \Omega_{2n-1}} \langle 0 | \phi(x') | 0 \rangle \langle 0 | F \tilde{F}(x_1) | \Omega_1 \rangle \langle \Omega_1 | F \tilde{F}(x_2) | \Omega_2 \rangle \times \dots \\ \times \langle \Omega_{2n-2} | F \tilde{F}(y_2) | \Omega_{2n-1} \rangle \langle \Omega_{2n-1} | F \tilde{F}(y_1) | 0 \rangle \langle 0 | \phi(y') | 0 \rangle = 0$$

➡ Topological sectors and θ are unphysical !

**Now let us
include fermions**

Chiral Ward-Takahashi identity

The well-known chiral (or anomalous) WTI:

$$\partial_\mu J_5^\mu \equiv \partial_\mu \sum_\psi^{N_f} \bar{\psi} \gamma^\mu \gamma_5 \psi = -2 \sum_\psi^{N_f} [m_\psi \bar{\psi} i \gamma_5 \psi] - \frac{N_f \alpha_s}{8\pi} F_{\mu\nu, a} \tilde{F}_a^{\mu\nu}$$

Right-hand side has $F\tilde{F}$, we saw that its integral is unphysical

What is the unphysical part of the quark?

Atiyah-Singer's theorem:

$$\text{ind}(\not{D}) = -\frac{\alpha_s}{8\pi} \int d^4x F_{\mu\nu, a} \tilde{F}_a^{\mu\nu}$$

(number of chiral Dirac zero-modes = topological charge)

Since the topological charge is unphysical,

chiral Dirac zero-modes of quarks are also unphysical !

$U(1)_A$ symmetry is physical

We remove the unphysical topological charge and chiral Dirac zero-modes from the known chiral WTI to obtain the “physical chiral WTI”:

$$\sum_{\psi}^{N_f} \left[\partial^{\mu} \bar{\psi} \gamma_{\mu} \gamma_5 \psi + 2m_{\psi} \bar{\psi} i \gamma_5 \psi \right]_{\lambda \neq 0} = -\frac{N_f \alpha_s}{8\pi} F_{\mu\nu, a} \tilde{F}_a^{\mu\nu} \Big|_{\Delta n=0}$$

Remove only chiral zero-modes ($\lambda=0$)

Right-hand side vanishes at zero momentum inflow since no topological charge (just a total divergence, without nontrivial effect at long distance)

\Rightarrow Global conservation of “physical” $U(1)_A$ current!
(up to current quark mass m_{ψ})

\Rightarrow “Physical” chiral ($=U(1)_A$) rotation does not change topological charge !

$\Rightarrow U(1)_A$ symmetry is physical at the “physical” Lagrangian level !

$U(1)_A$ of QCD is only broken spontaneously, restores at $T > T_c$

(Consistent with lattice calculations)

Chiral anomaly is physical

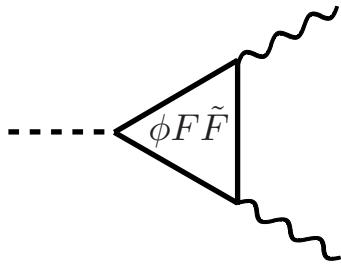
We saw that $U(1)_A$ is not explicitly broken

Is then the chiral anomaly unphysical?

⇒ No, the **chiral anomaly can be physical**,

only its 4-dimensional integral (=topological charge) is unphysical

Local anomaly interaction



⇒ Physical!

Anomaly only violates $U(1)_A$ locally!

(If we assign a charge to (pseudo)scalar ϕ , conservation law is OK)

⇒ η' decay, axion, CP-odd Higgs ... are physical!

Topological charge

$$\int d^4x F \tilde{F}$$

⇒ Unphysical!

This looks like a global violation of $U(1)_A$, but does not contribute to observables!

Consistency with U(1) problem

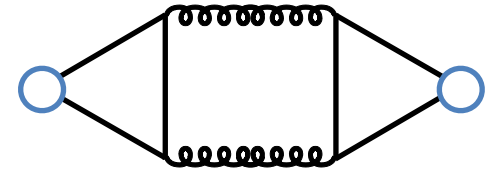
U(1) problem: η' is heavy (960MeV), no $U(1)_A$ Nambu-Goldstone mode?

Conventionally, resolved by increasing η' mass by the **topological susceptibility**

$$\left(\frac{\alpha_s}{8\pi}\right)^2 \int d^4x e^{ik \cdot x} \langle 0 | F_{\mu\nu,a} \tilde{F}_a^{\mu\nu}(x) F_{\rho\sigma,b} \tilde{F}_b^{\rho\sigma}(0) | 0 \rangle$$

$$= \sum_h \langle 0 | F \tilde{F} | h(k) \rangle \frac{i}{k^2 - m_h^2} \langle h(k) | F \tilde{F} | 0 \rangle$$

$|h(k)\rangle$: hadronic state generated by $F\tilde{F}$



However,

in zero momentum limit $k=0$, $\langle 0 | F \tilde{F} | h(k=0) \rangle \propto \langle 0 | F \tilde{F} | \Omega \rangle$

$\Rightarrow \langle 0 | F \tilde{F} | h(k=0) \rangle$ and topological susceptibility at $k=0$ are unphysical!

In the chiral limit, NG mode with $k^2=0$ has no mass shift \Rightarrow Massless!

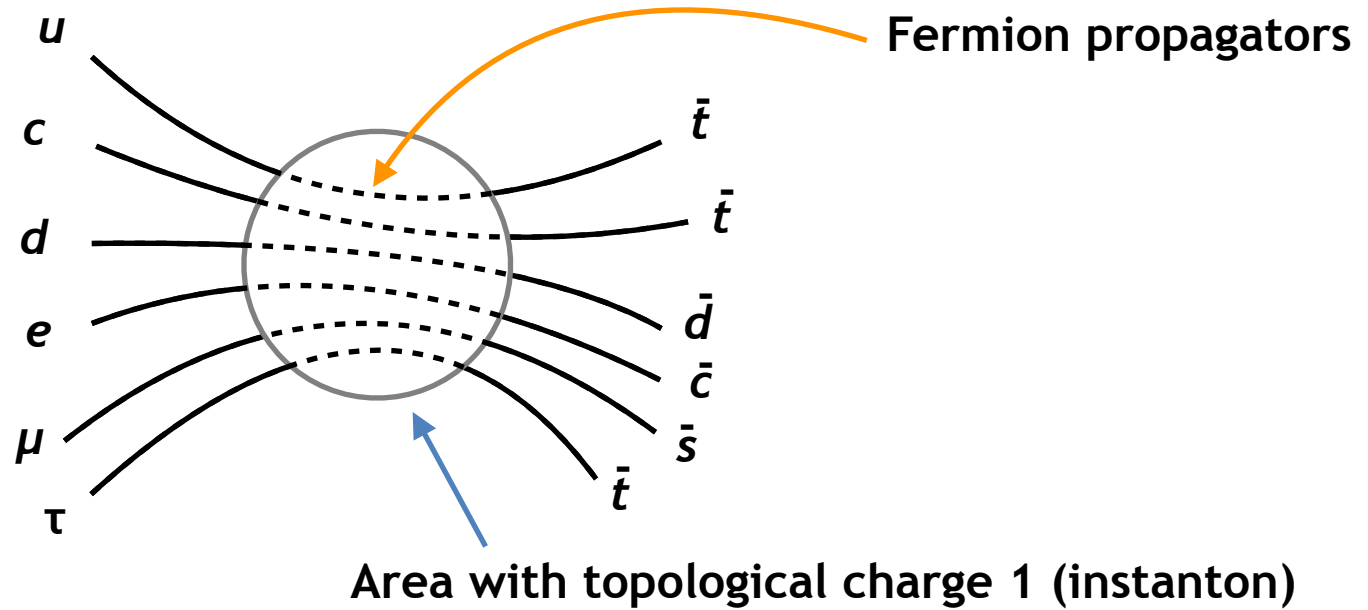
Why does η' have a large mass in real QCD?

The mass shift is probably due to nonzero quark mass which brings finite momentum inflow to topological susceptibility. This probably has a momentum dependence (to be checked with lattice).

Implications to particle physics phenomenology

't Hooft vertex (conventional)

Consider the following multi-fermion amplitude in the standard model



Conventionally, amplitude of this process is finite in the chiral limit if

- * all fermions have zero-modes
- * all fermions are combined so as to form $U(1)_{B+L}$ anomaly

This gives rise to a (nonlocal) contact 12-fermion interaction ('t Hooft vertex)

⇒ Baryon and lepton numbers may be generated!

(Previous understanding)

Implication to baryogenesis

't Hooft vertex is generated by the propagation of zero-modes

⇒ 't Hooft vertex is unphysical !

(it may exist, but does not contribute to observables)

Important phenomenological consequences:

⇒ No instanton/sphaleron induced baryogenesis

⇒ Standard leptogenesis does not work (lepton # does not lead to baryon #)

⇒ We need explicit local baryon # violating interactions

such as Grand unification, leptoquarks, R-parity violation, ...

A comment on lepton number violation:

Sterile neutrino, $0\nu\beta\beta$ decay are not mandatory for baryogenesis

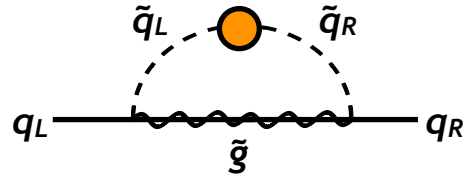
What do CP phases of quark mass become?

$\mathcal{L}_{\text{odd}} = -m_{\text{odd}} \bar{\psi} i \gamma_5 \psi \quad \Rightarrow \text{Introduce complex phase to quark mass}$

Generated in new physics beyond standard model (and also in standard model)

Example of supersymmetry:

Squarks have CP-odd transition



Physical particles are in mass eigenstates : real mass

CP-odd mass may be “rotated away” by $U(1)_A$ transformation:

$$m_{\text{even}} \bar{\psi} \psi + m_{\text{odd}} \bar{\psi} i \gamma_5 \psi \rightarrow m \bar{\psi}' \psi'$$

If θ -term is physical, chiral rotation cannot erase both θ and m_{odd}

From our discussion, θ -term is unphysical \Rightarrow **Chiral rotation removes m_{odd} !**

m_{odd} does not decouple with increasing BSM scale, but it is unphysical:

Our result (unphysical m_{odd}) leads to **correct decoupling of BSM scale**

Modification of the BSM phenomenology after our work

- **Modification procedure:**

 - Neglect θ -term**

 - Neglect CP-odd mass of quarks**

- **Comparison with axion mechanism:**

 - No axions**

 - Induced θ -term is unphysical**

- **Impact on particle physics phenomenology:**

 - The only source of CP violation of SM is the CP phase of CKM matrix**

 - BSM contribution all decouples with increasing BSM energy scale
(scales as power of $1/\Lambda_{\text{BSM}}$)**

 - Baryogenesis is impossible in SM**