ブラックホール時空で運動する弦のカオスの普遍性

棚橋典大 [九州大学 マス･フォア･インダストリ研究所]

based on

Chaos of Wilson Loop from String Motion near Black Hole Horizon
橋本幸士、村田佳樹、棚橋典大 [arXiv:1803.06756]

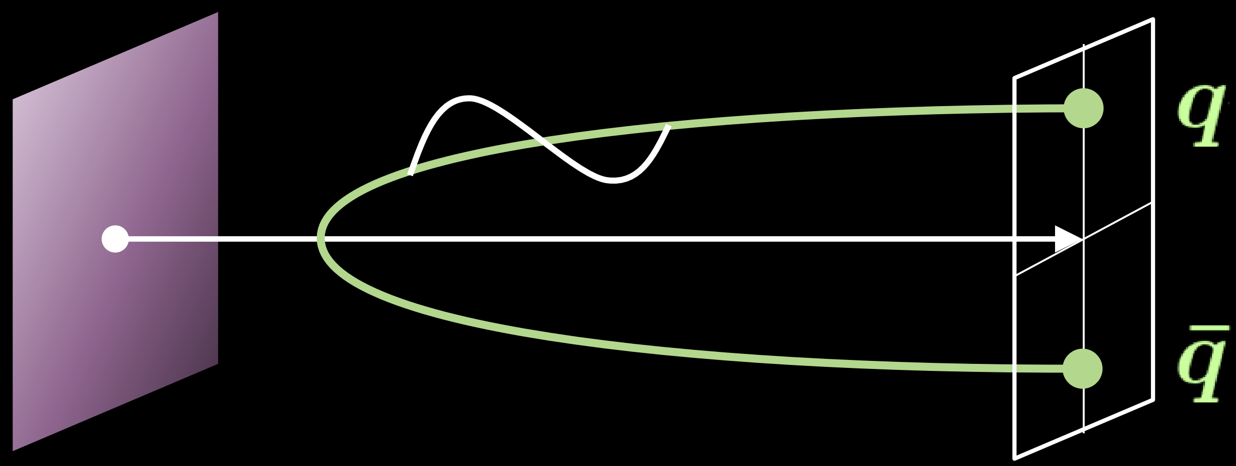
Universality in Chaos of Particle Motion near Black Hole Horizon
橋本幸士、棚橋典大 [arXiv:1610.06070]

Setup:

Fundamental String moving **near AdS BH horizon**

= Dynamics of "quark-anti quark pair" at finite temperature

Maldacena '98
Rey & Yee '98



Results:

✓ **String motion become chaotic** due to BH gravity

⇔ **Force between the quarks** becomes chaotic when $T \neq 0$.

✓ **Lyapunov exponent** λ is smaller than **surface gravity** κ

$$\lambda \leq \kappa = 2\pi T/\hbar$$

[Maldacena-Shenker-Stanford '15]

CONTENTS

CLASSICAL CHAOS

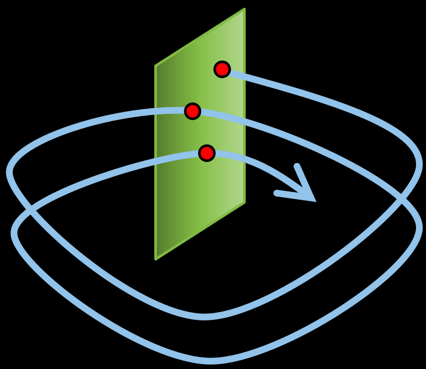
◆ Diagnostics of classical chaos

- Poincaré plot

  = Section of orbits in phase space

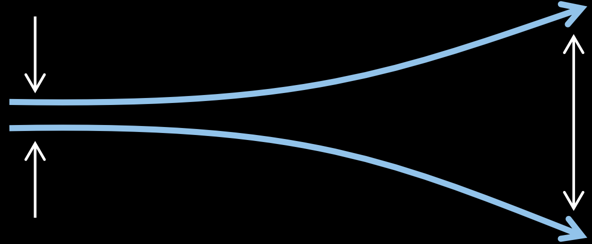
  non-chaotic → regular shaped plot

  chaotic → scattered plot



- Lyapunov exponent λ = Separation growth rate of nearby orbits

$d(0)$

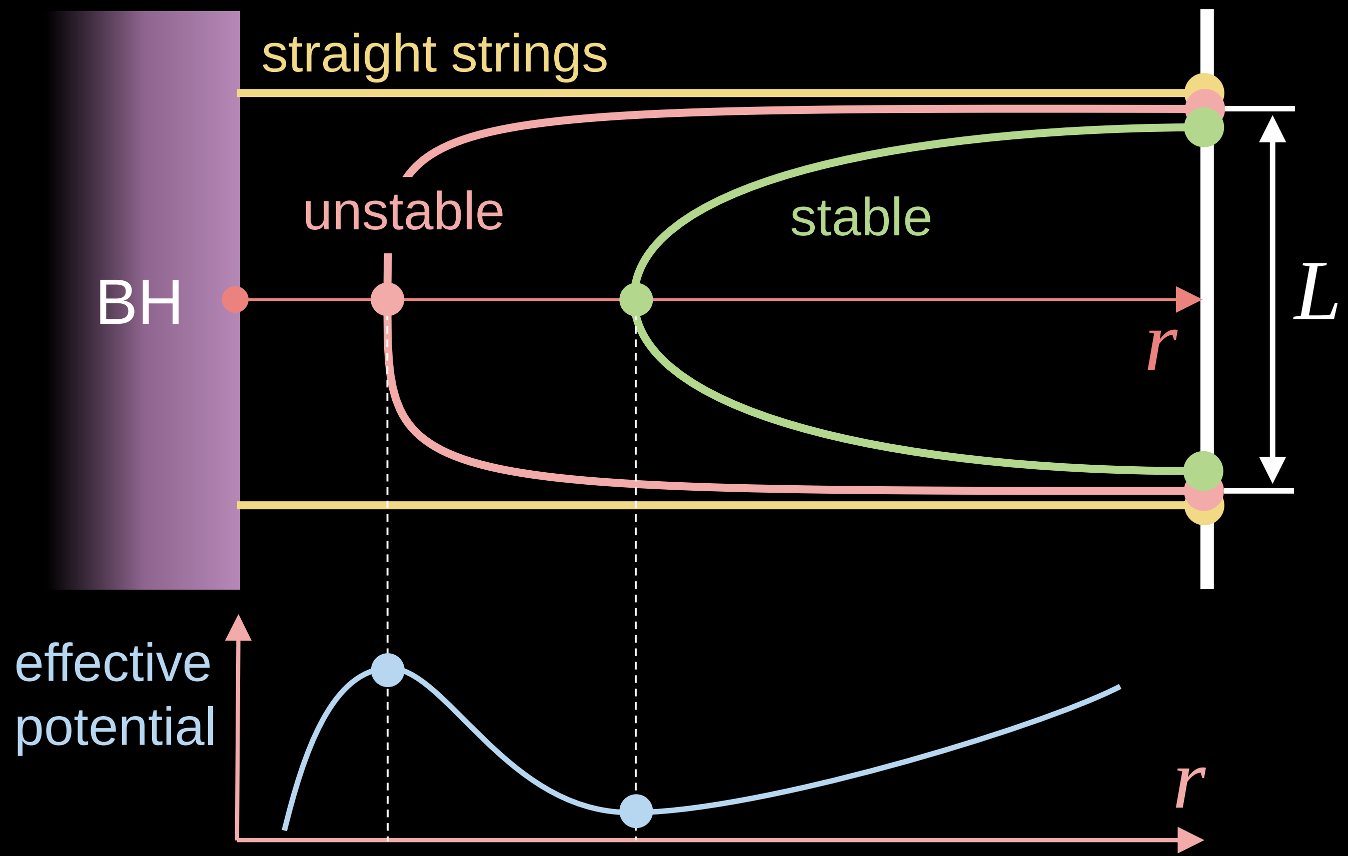


$$d(t) \sim d(0) \exp(\lambda t)$$

✓ We will focus on the string motion for a while.

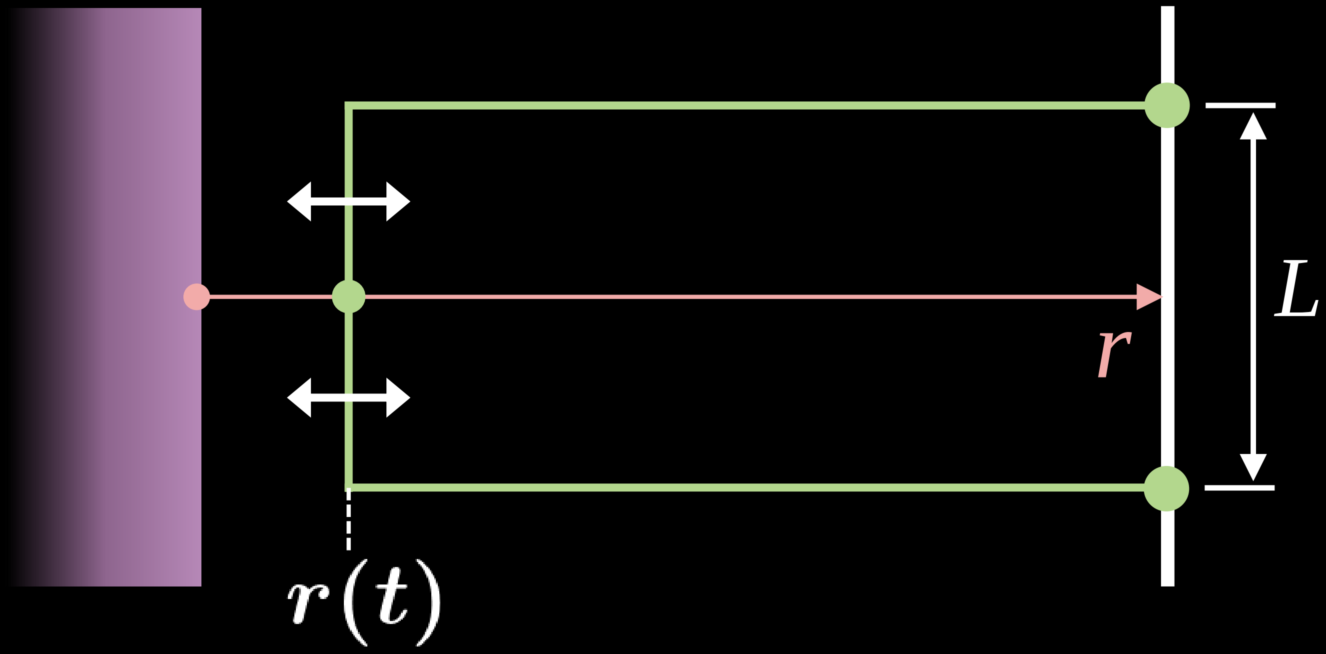
STRING IN AdS

Three shapes of static Nambu-Goto string in AdS



STRING IN AdS

"Square-shaped string" approximation



$$\mathcal{L} \simeq -L\sqrt{r^4(t)f\big(r(t)\big) - \frac{\dot{r}^2(t)}{f\big(r(t)\big)}} + 2\big(r(t) - r_H\big) \quad \left(f(r) = 1 - \frac{r_H^4}{r^4}\right)$$

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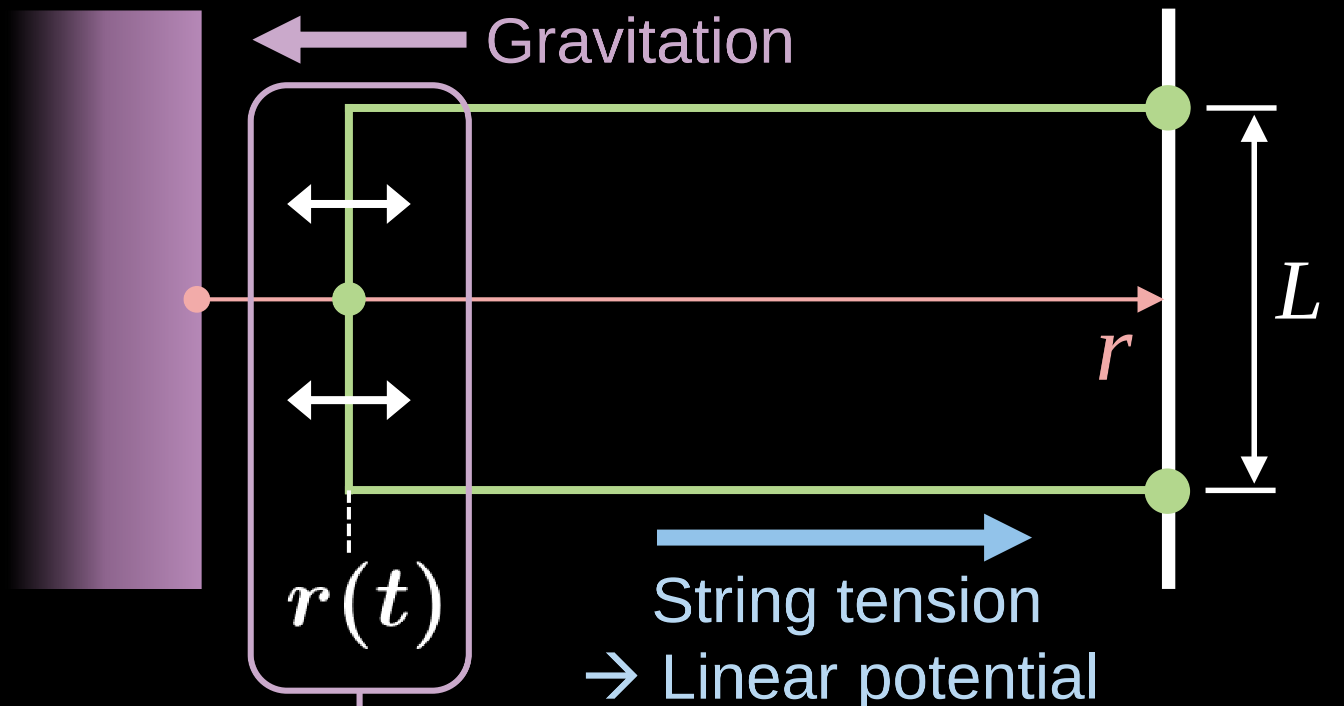

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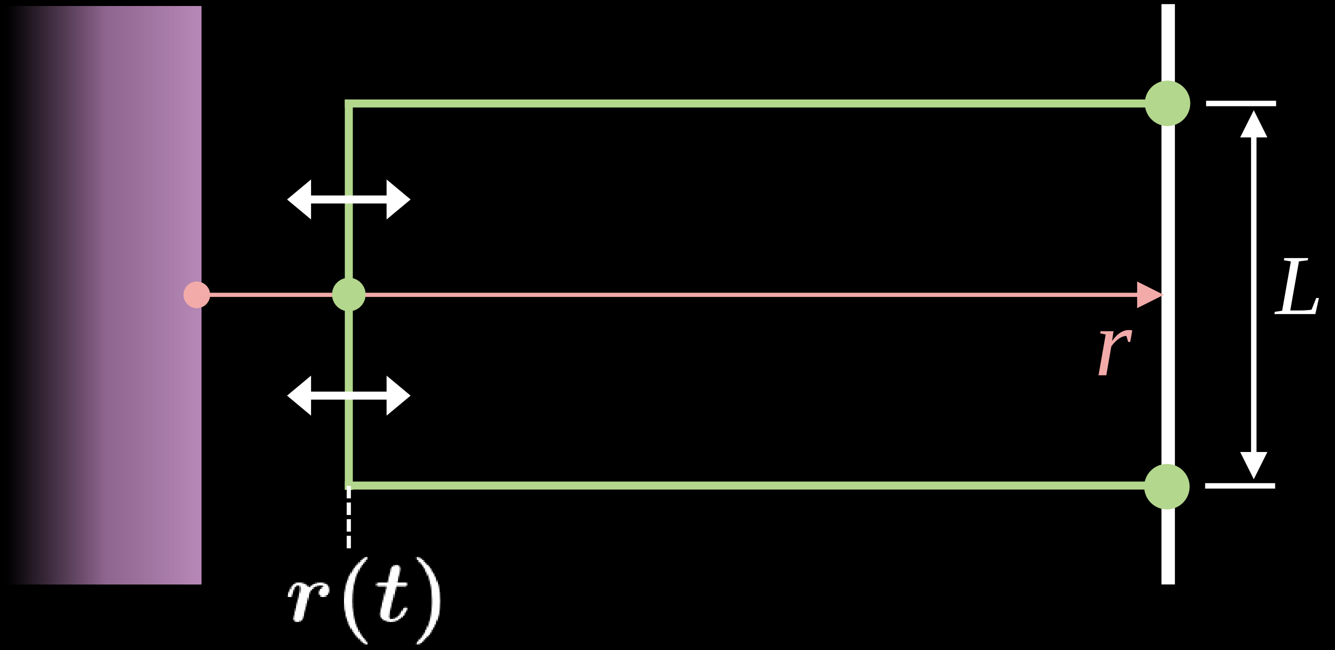
STRING IN AdS

"Square-shaped string" approximation

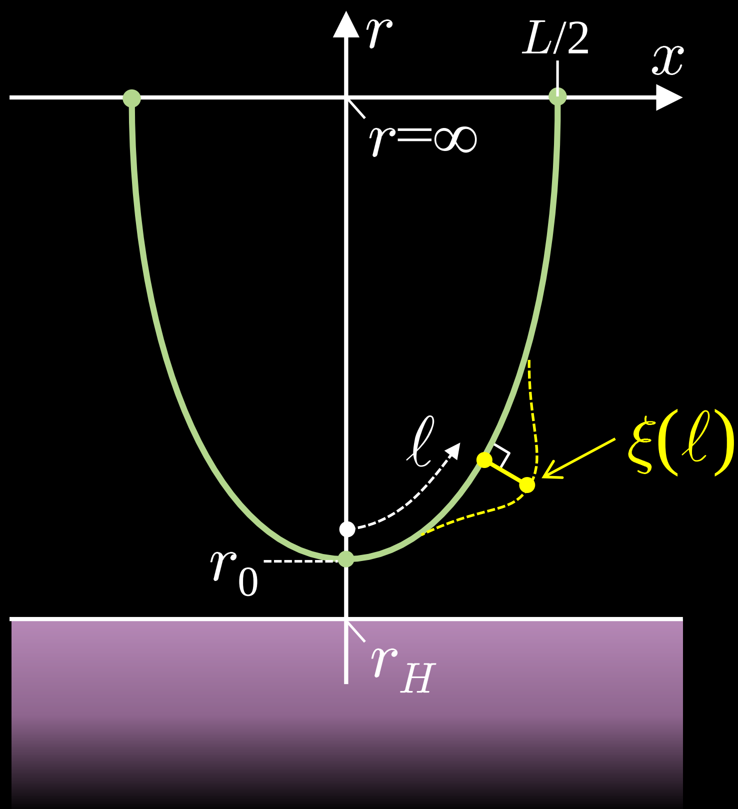


When the string is on the potential maximum near horizon,

$$\mathcal{L} \simeq \frac{1}{2r_H^5 L^2}\left[\dot{r}^2 + \kappa^2 \left(r(t) - r_*\right)^2\right] \Rightarrow r(t) \sim r_* + e^{\kappa t}$$

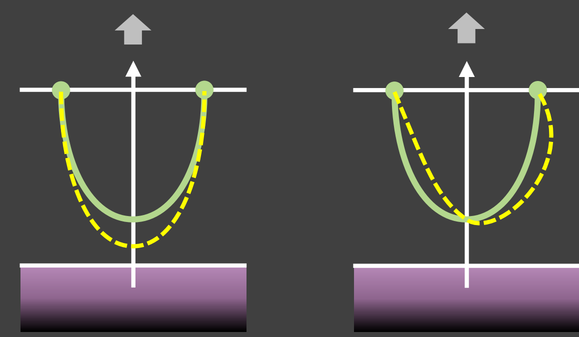
$$\Rightarrow \lambda \lesssim \kappa = 2\pi T/\hbar$$

Perturbative string motion



Truncate $\xi(\ell)$ up to lowest 2 modes

$$\xi(t,\ell) = c_0(t)e_0(\ell) + c_1(t)e_1(\ell)$$



Then expand $\mathcal{L}$ up to 3rd order in $c_i(t)$

ex.) For $r_0 = 1.1\, r_H$,

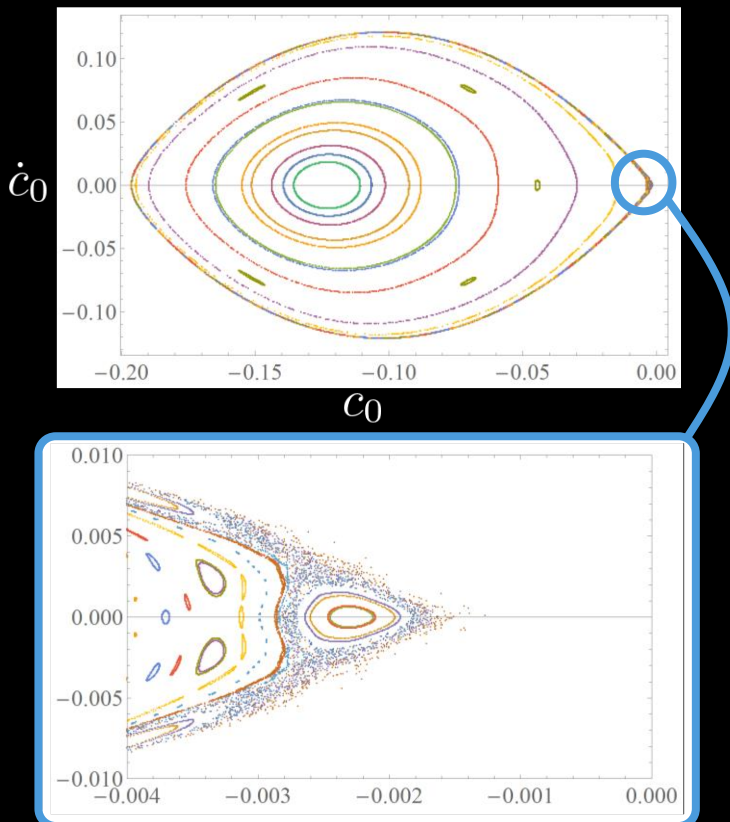
$$\frac{\mathcal{L}}{\mathcal{T}} = \sum_{n=0,1} \left(\dot{c}_n^2 - \omega_n^2 c_n^2\right) + 7.11 c_0^3 + 35.3 c_0 c_1^2$$

$$+ 4.66 c_0 \dot{c}_0^2 + 1.32 c_0 \dot{c}_1^2 - 7.57 \dot{c}_0 c_1 \dot{c}_1$$

$$(\omega_0^2 = -1.40,\ \omega_1^2 = 7.57)$$

Perturbative string motion

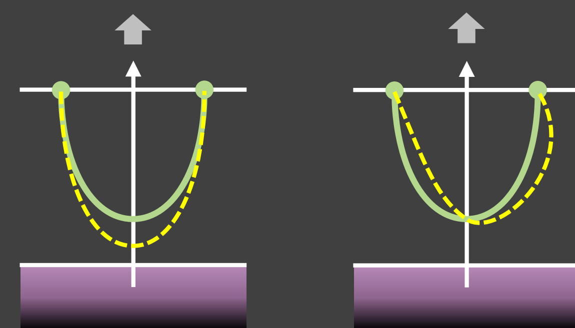
Poincaré plot



- Chaotic (scattered plot) in near-horizon region
- $\lambda \sim 0.04 \times 2\pi T / \hbar$

Truncate $\xi(\ell)$ up to lowest 2 modes

$$\xi(t,\ell) = c_0(t)e_0(\ell) + c_1(t)e_1(\ell)$$



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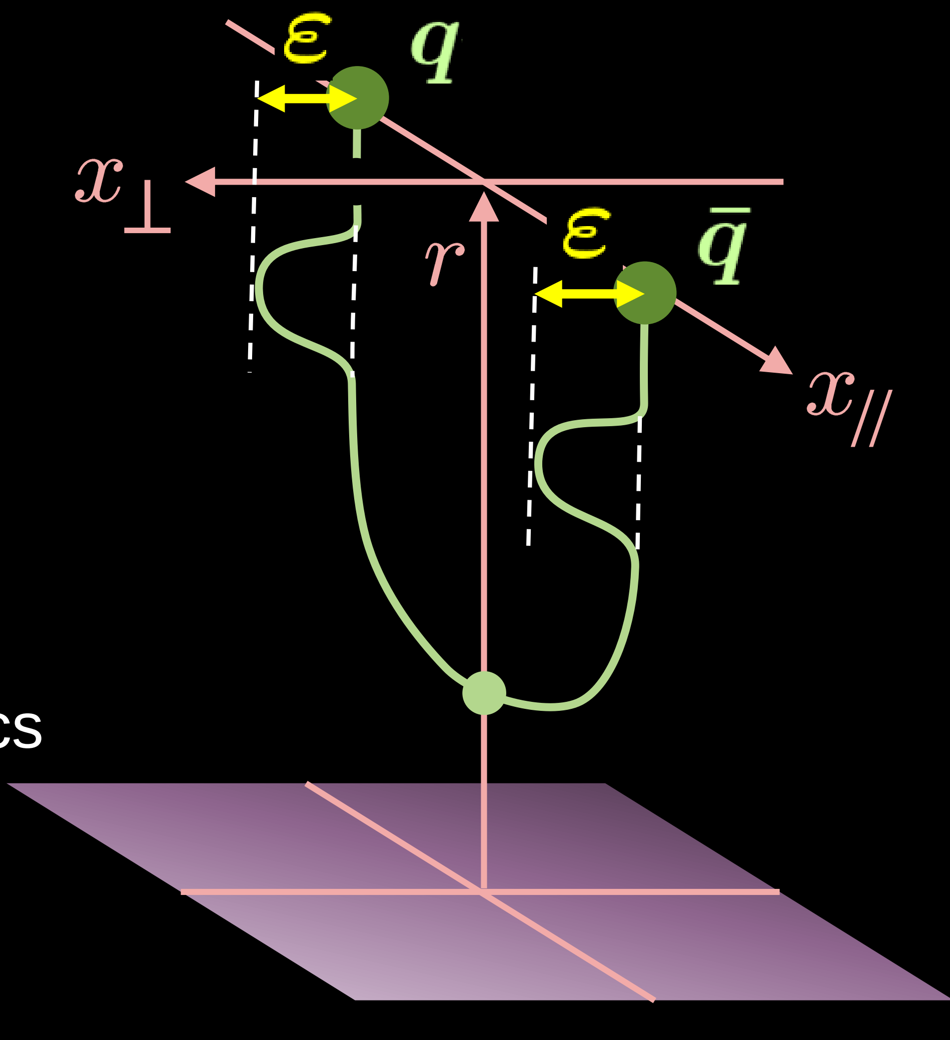
CONTENTS

Chaos Bound for AdS String

Nonlinear dynamics of string

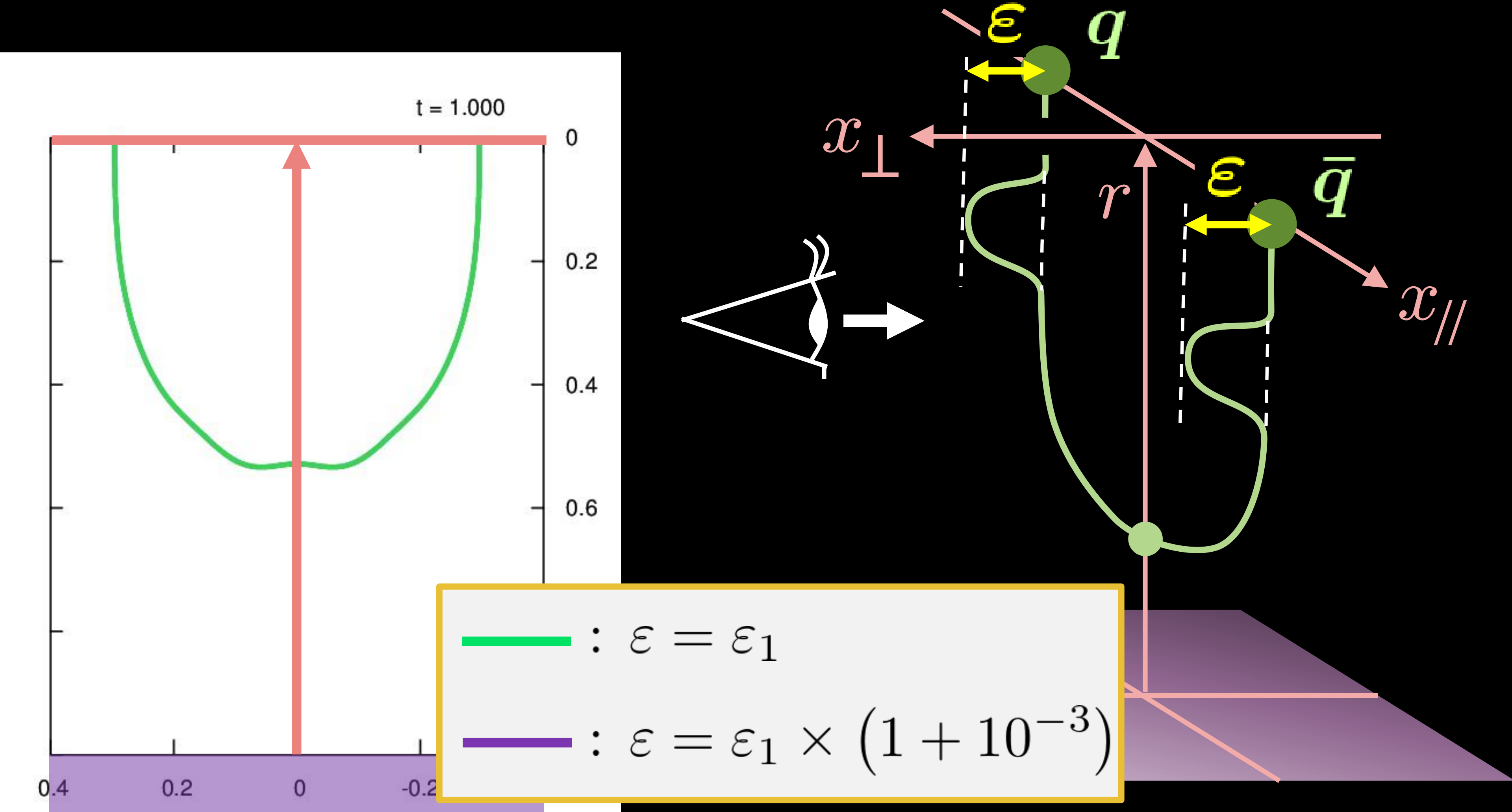
Shake the string end points
by amplitude ε
→ Nonlinear string motion

Shake the quark — $\overline{\text{quark}}$ pair
→ Nonlinear flux tube dynamics



Chaos Bound for AdS String

Nonlinear dynamics of string for slightly different ε



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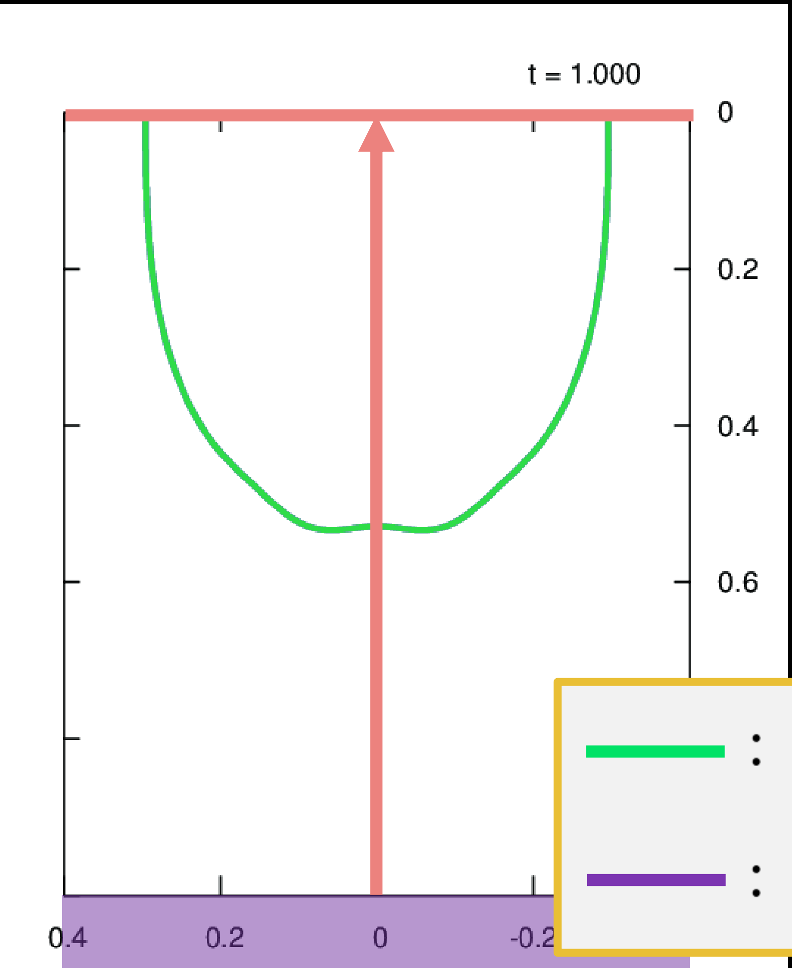
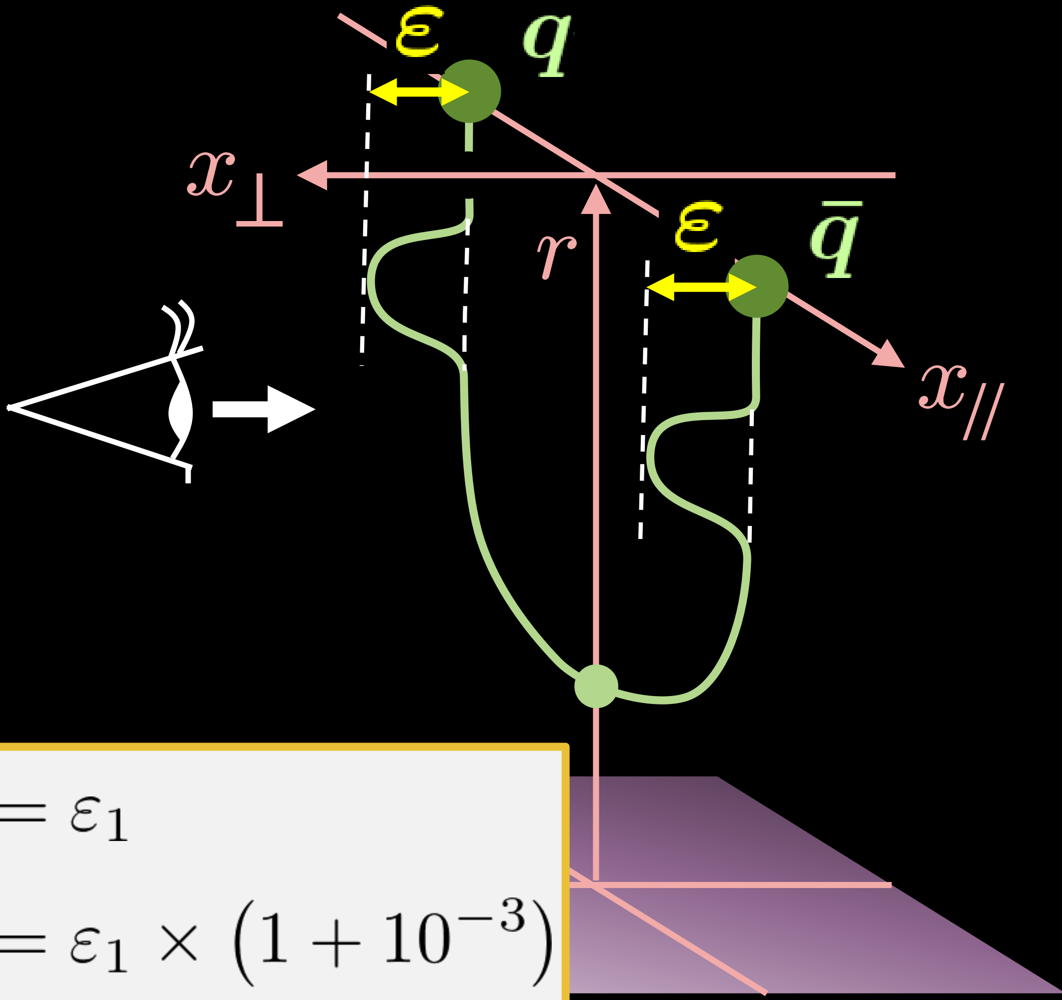




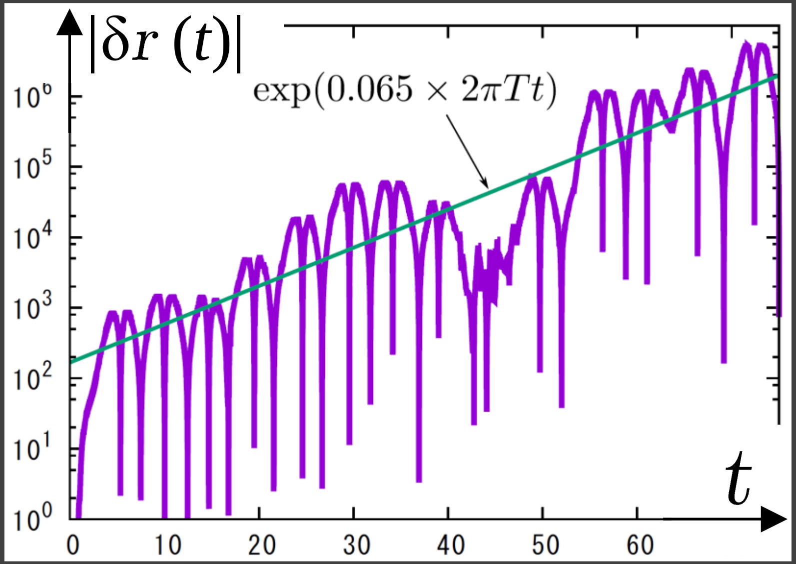
: $\varepsilon = \varepsilon_1$

: $\varepsilon = \varepsilon_1 \times (1 + 10^{-3})$

Chaos Bound for AdS String

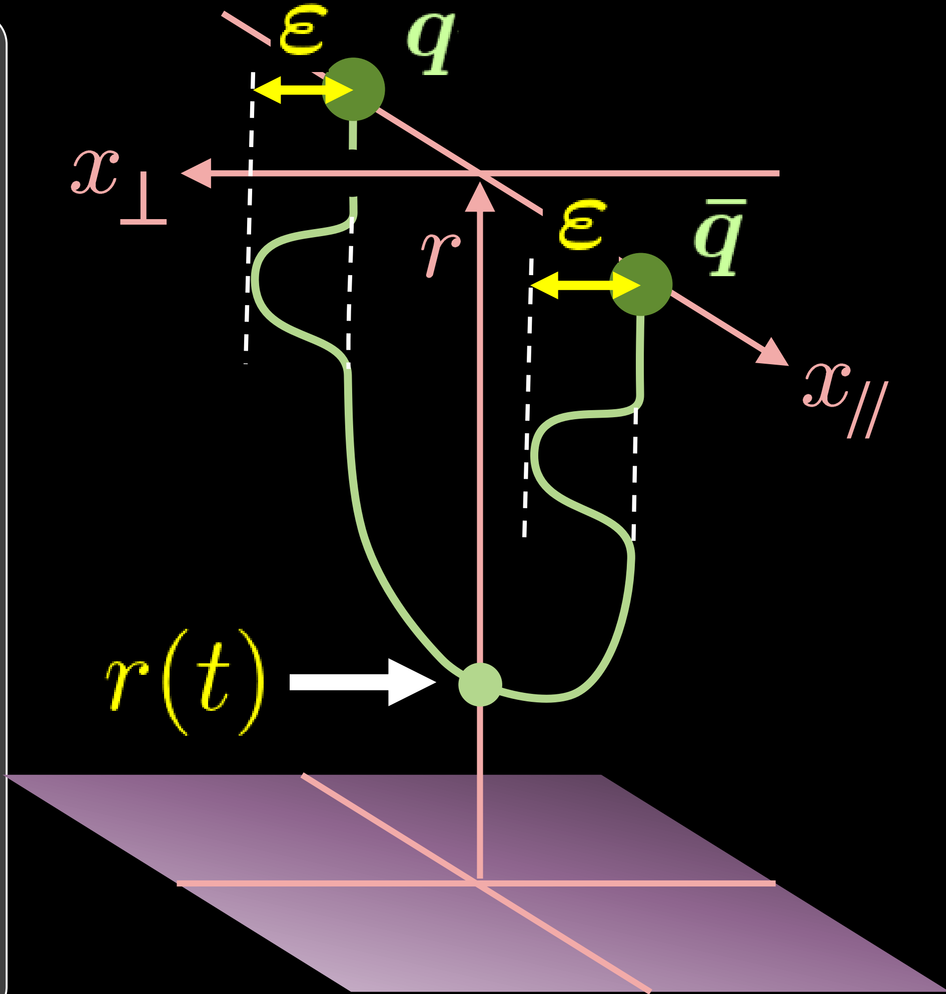
Nonlinear dynamics of string for slightly different ε

String position difference $\delta r(t)$



$$|\delta r(t)| \sim e^{\lambda t},$$

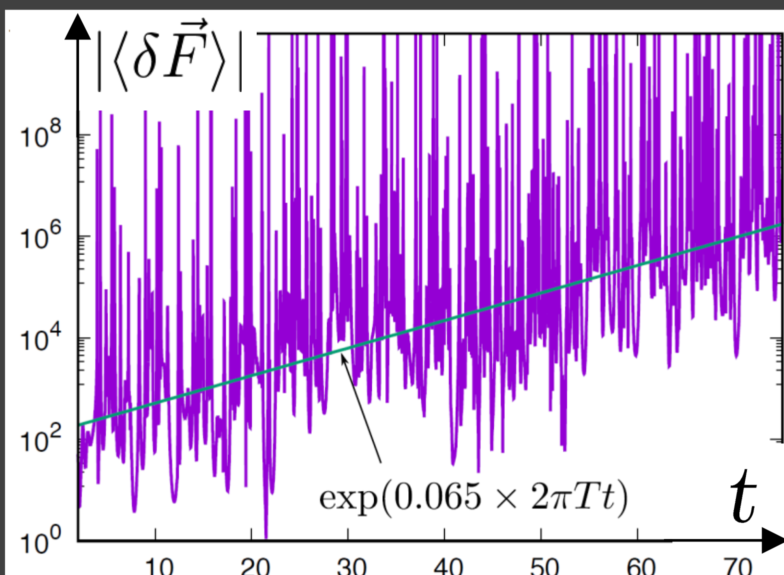
$$\lambda \sim 0.065 \times 2\pi T/\hbar$$



Chaos Bound for AdS String

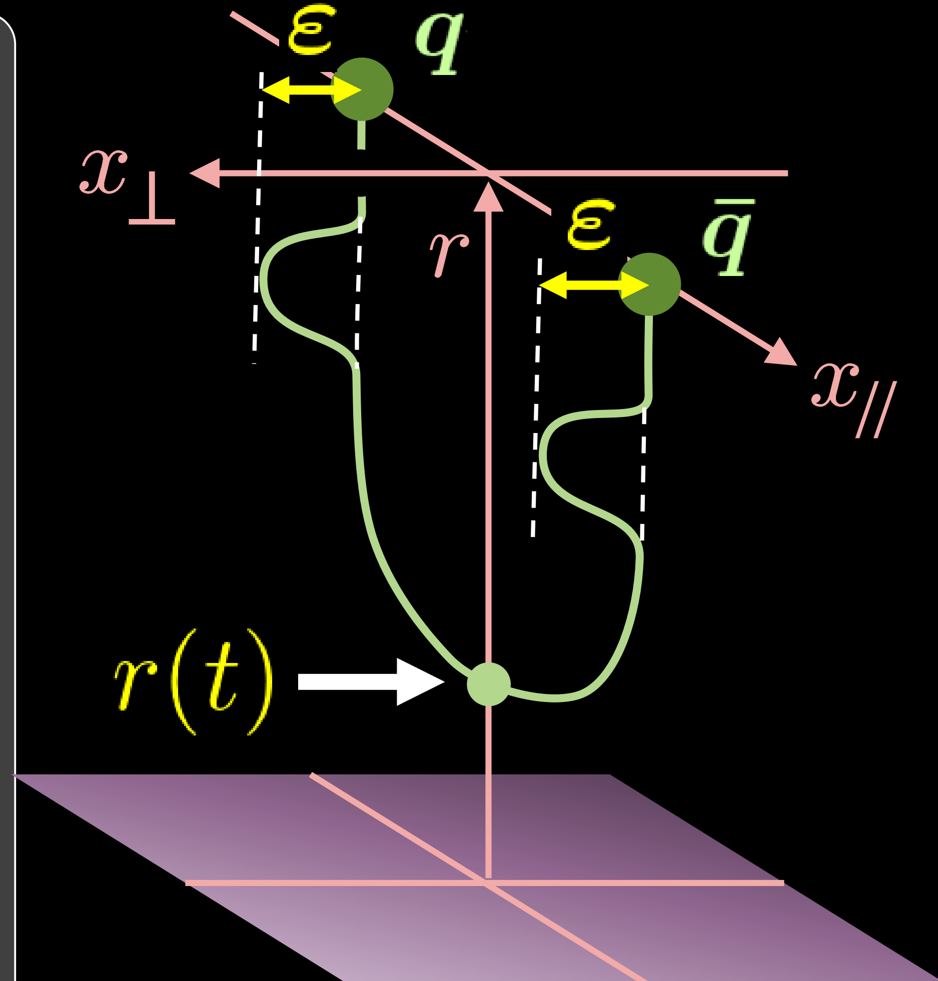
Nonlinear dynamics of string for slightly different ε

Inter-quark force $\langle \vec{F}(t) \rangle \propto \partial_r^3 \vec{X}(t,r)$



$$|\langle \delta \vec{F} \rangle| \sim e^{\lambda t},$$

$$\lambda \sim 0.065 \times 2\pi T/\hbar$$



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Maldacena '98
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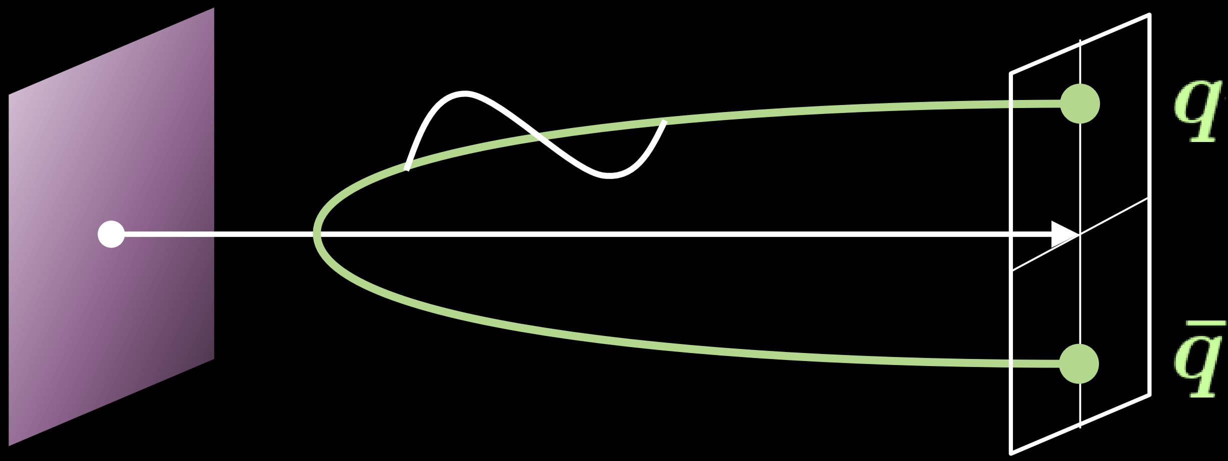


Results:

✓ **String motion become chaotic** due to BH gravity

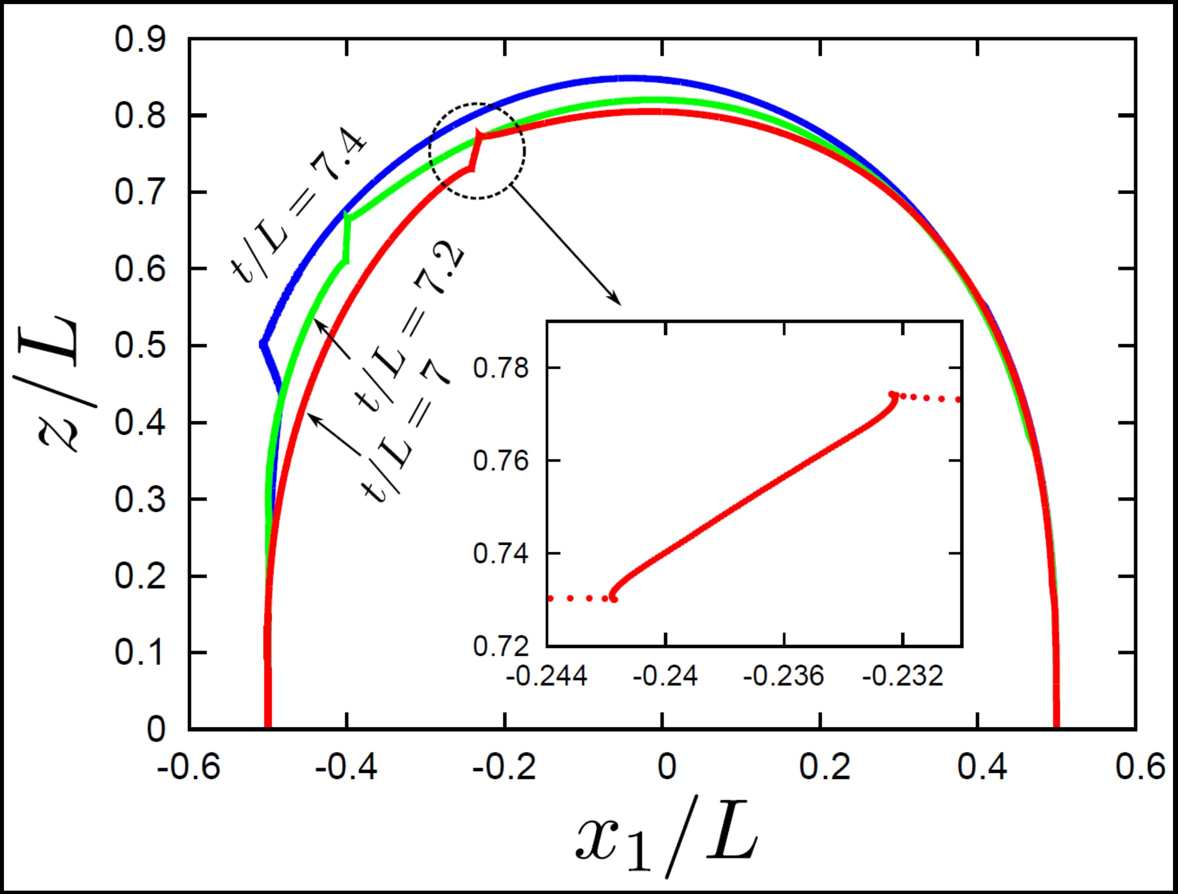
⇔ **Force between the quarks** becomes chaotic when $T \neq 0$.

✓ **Lyapunov exponent** λ is smaller than **surface gravity** κ

$$\lambda \leq \kappa = 2\pi T/\hbar$$

[Maldacena-Shenker-Stanford '15]

◆Cusp formation on the string

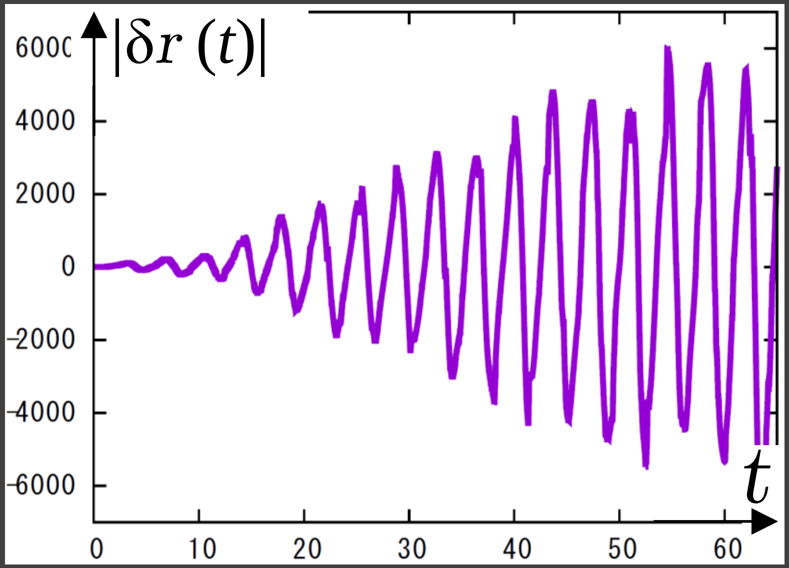


[Ishii, Murata ‘15]

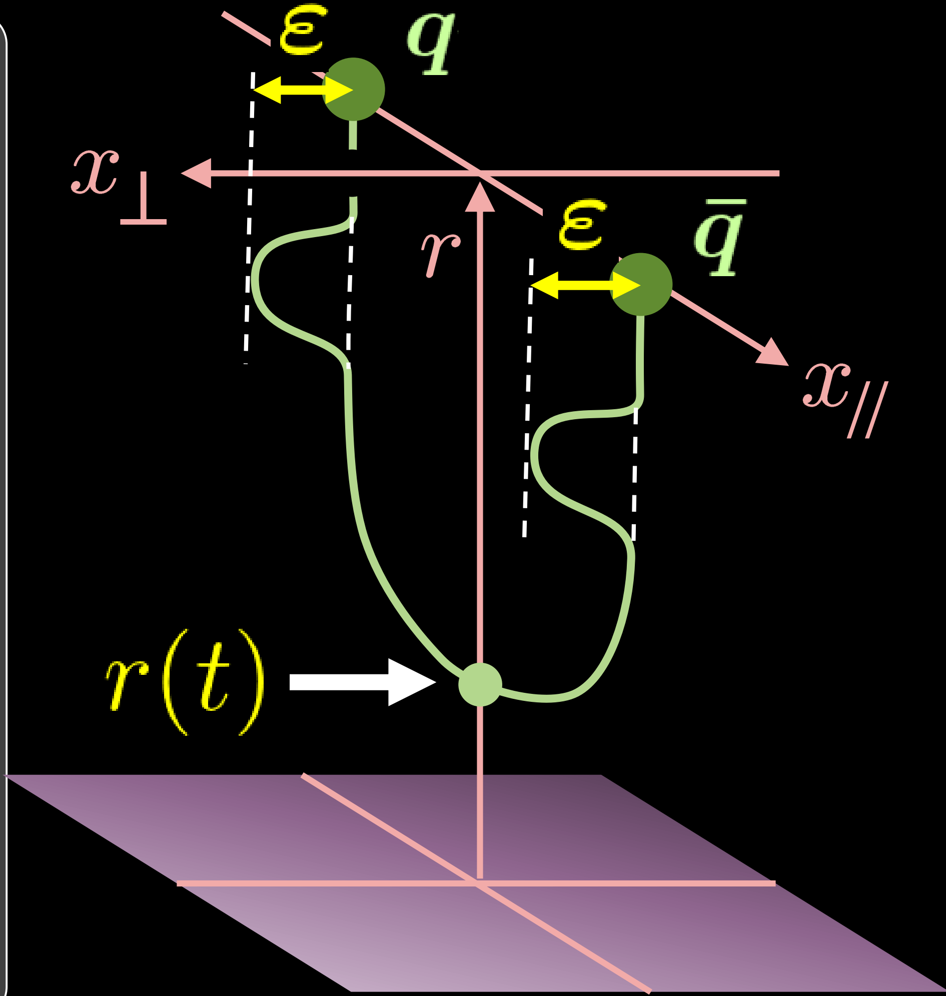
Chaos Bound for AdS String

Nonlinear dynamics of string for slightly different ε

When the string is
not close to the horizon,



λ grows just by power-law,
not exponentially.



◆ Motivation:

A bound on chaos in QFT at temperature T :

$$\lambda \leq 2\pi T/\hbar$$

[Maldacena-Shenker-Stanford '15]

◆ Probing the effect of temperature T to chaos in QFT.

■ We study effect of temperature to chaos in classical gravity.

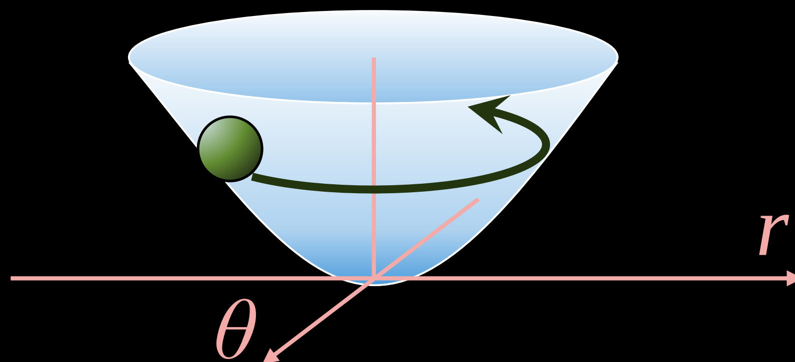
Use BH surface gravity $\kappa = 2\pi T/\hbar$ instead.

■ To probe effect of κ, we look at trajectories very close to BH.

Chaos Bound for Particle near BH

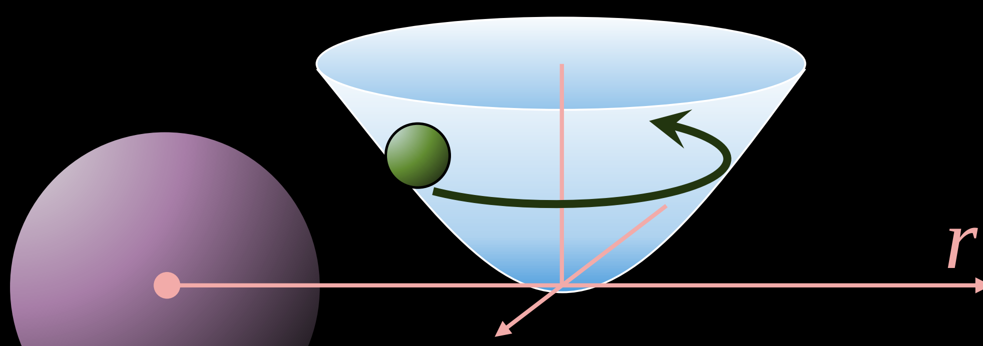
- To realize a particle moving very close to BH horizon,

  1. put a particle in a trapping harmonic potential



(← no chaos)

  2. take it close to a BH horizon



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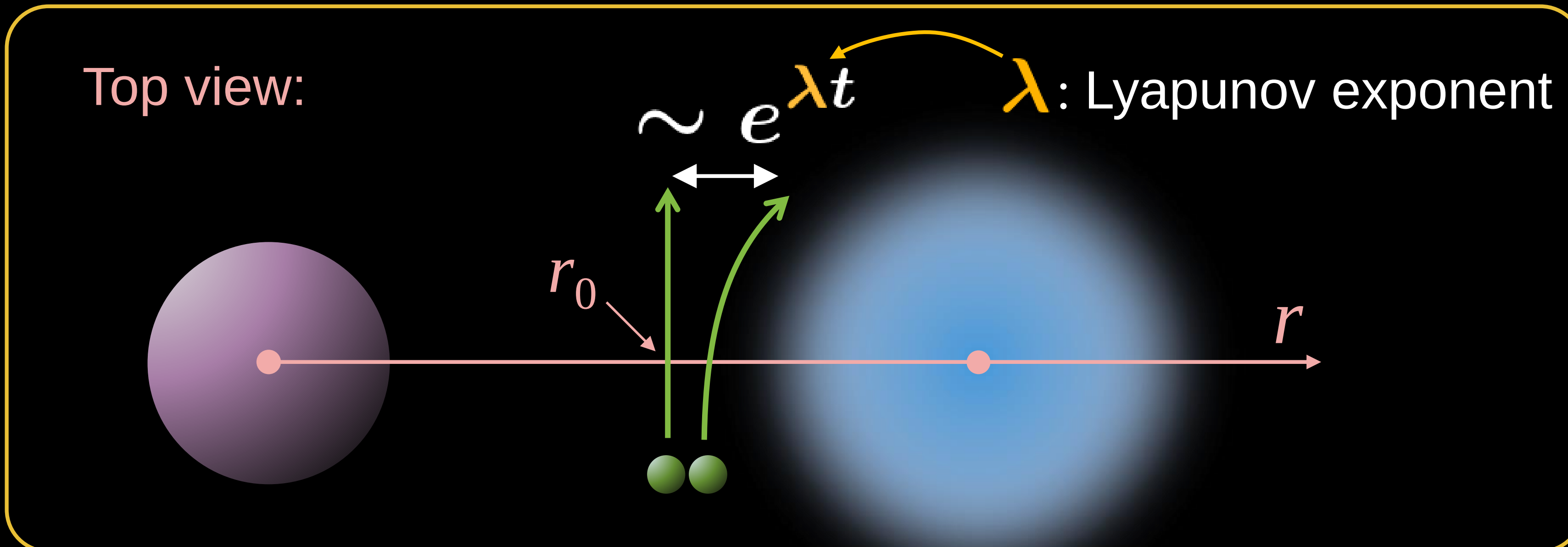

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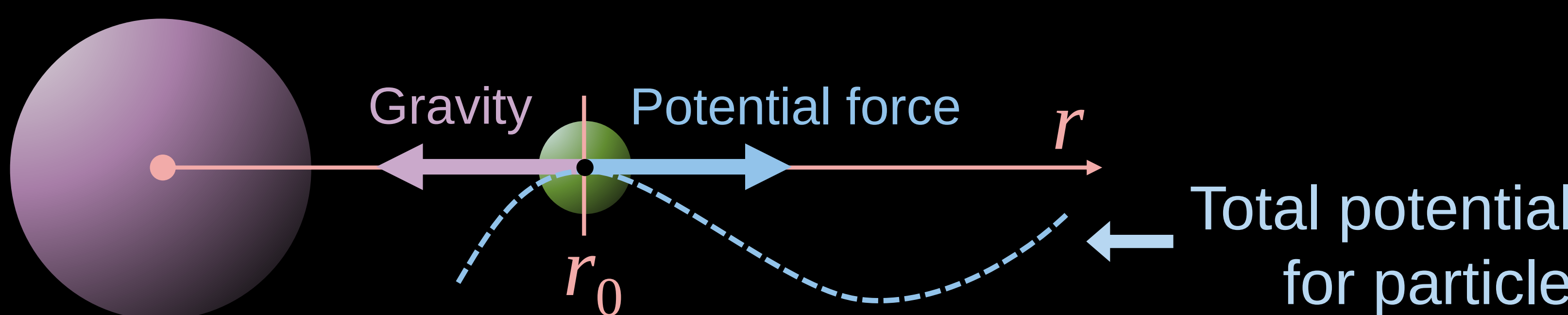
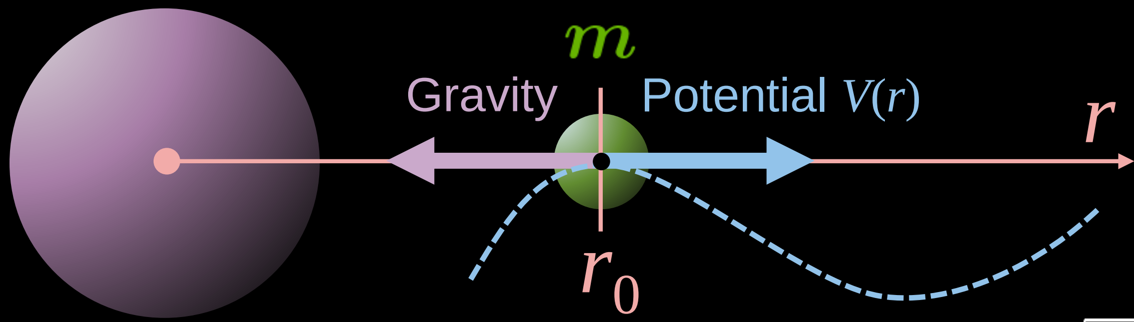


Chaos Bound for Particle near BH

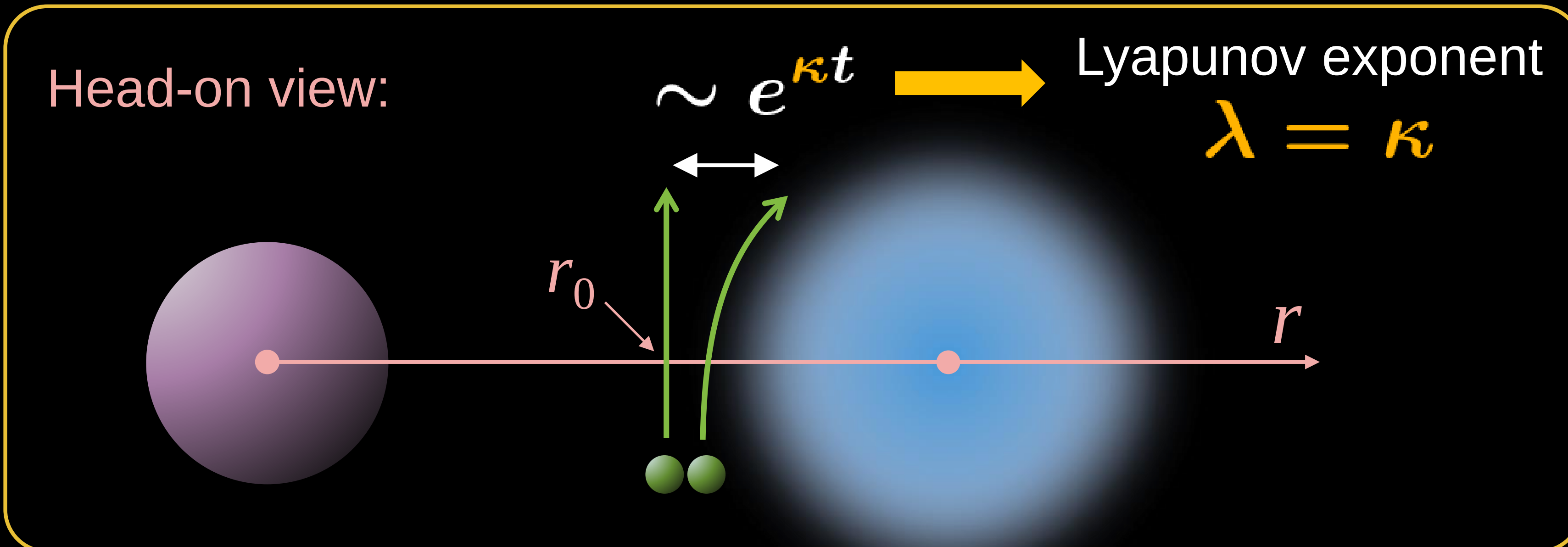


$$\mathcal{L} = -m\sqrt{-g_{\mu\nu}\dot{X}^{\mu}\dot{X}^{\nu}} - V(X) \simeq -m\sqrt{f(r) - \frac{\dot{r}^2}{f(r)}} - V(r)$$

- Slow radial velocity near $r = r_0$
- Linear approximation for $V(r) \sim (\text{slope}) \times (r - r_{\text{horizon}})$
- Near-horizon limit $r_0 \to r_{\text{horizon}}$

$$\mathcal{L} \simeq C(m, \kappa, \text{slope of } V) \times \left[\dot{r}^2 + \kappa^2 (r - r_0)^2\right]$$

Chaos Bound for Particle near BH



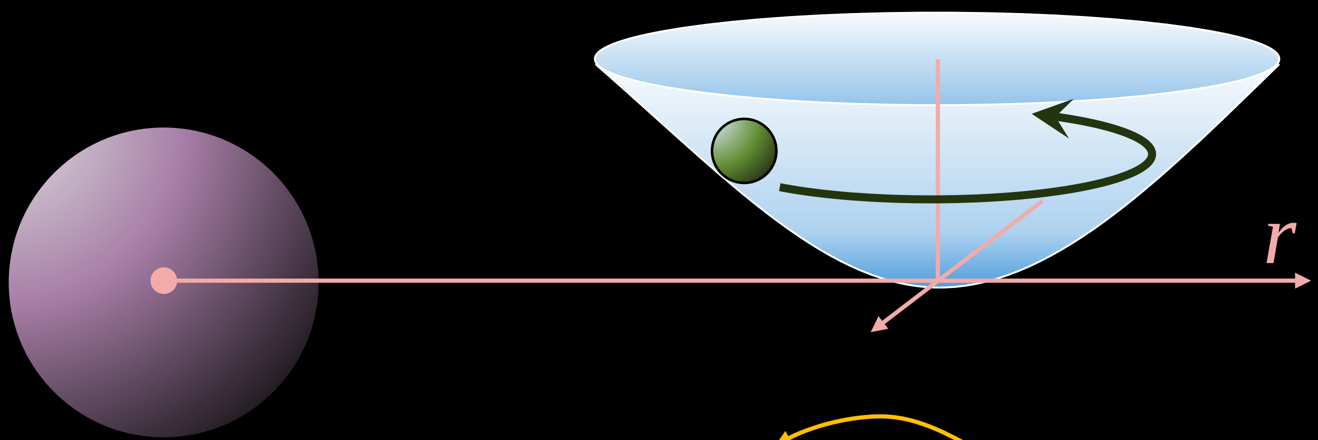
- Linear approximation for $V(r) \sim (\text{slope}) \times (r - r_{\text{horizon}})$
- Near-horizon limit $r_0 \to r_{\text{horizon}}$

$$\mathcal{L} \simeq C(m, \kappa, \text{slope of } V) \times \left[\dot{r}^2 + \kappa^2 (r - r_0)^2\right]$$

✓ A generic trajectory obeys $\lambda \leq \kappa = 2\pi T/\hbar$

Chaos Bound for Particle near BH

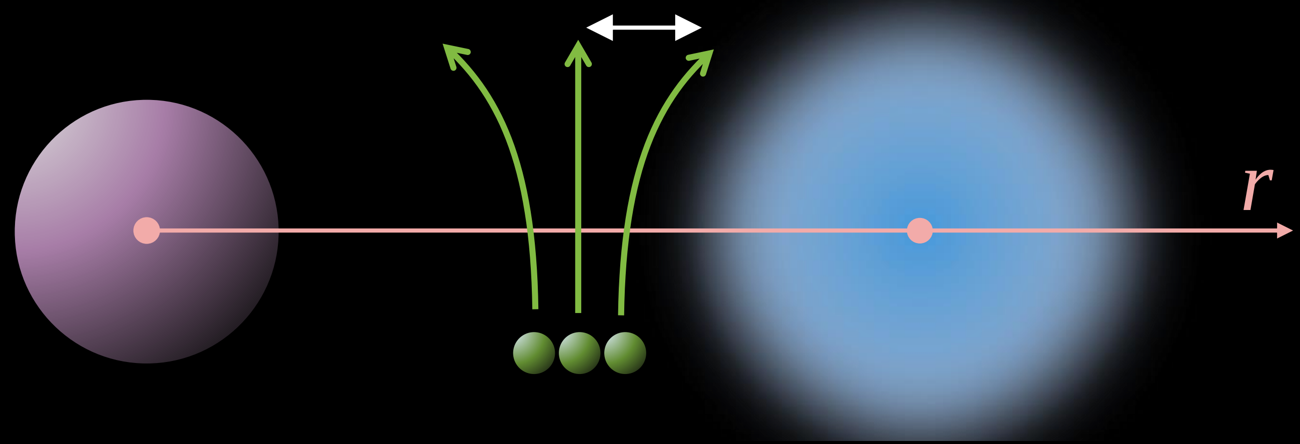
Particle in harmonic potential near a BH



Top view:

$\sim e^{\lambda t}$

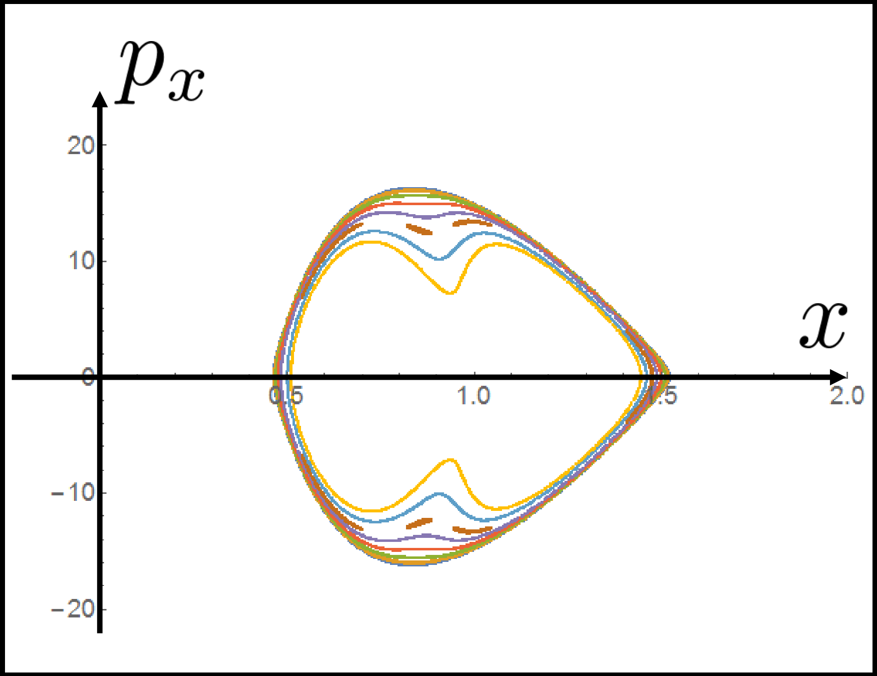
λ: Lyapunov exponent



Poincaré plot at $y = 0$

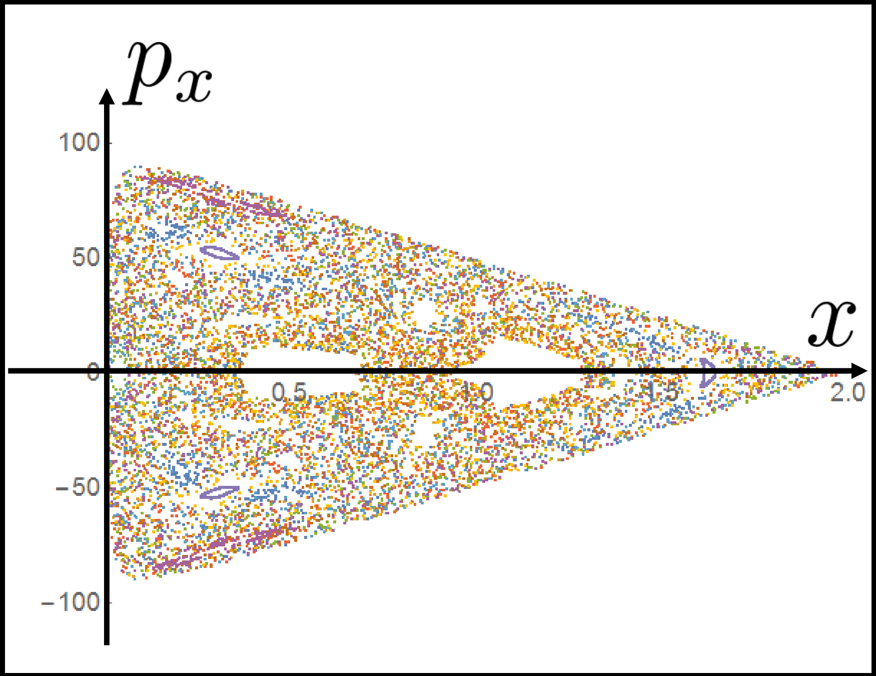
$$\mathcal{L} = -\sqrt{f(x) - \frac{\dot{x}^2}{f(x)} - \dot{y}^2 - \frac{\omega^2}{2}\left[(x - x_c)^2 + y^2\right]} \qquad \left(f(x) \equiv 2\kappa x \right)$$

Particle near Potential Minimum



Regular KAM tori, no chaos

Particle near BH Horizon



Lyapunov exponent $\lambda \sim 0.2\,\kappa$

Satisfies the bound $\lambda \leq \kappa$

Numerical check

$$\mathcal{L} = -\sqrt{f(x) - \frac{\dot{x}^2}{f(x)} - \dot{y}^2} - \frac{\omega^2}{2}\left[(x - x_c)^2 + y^2\right] \qquad \left(f(x) \equiv 2\kappa x \right)$$

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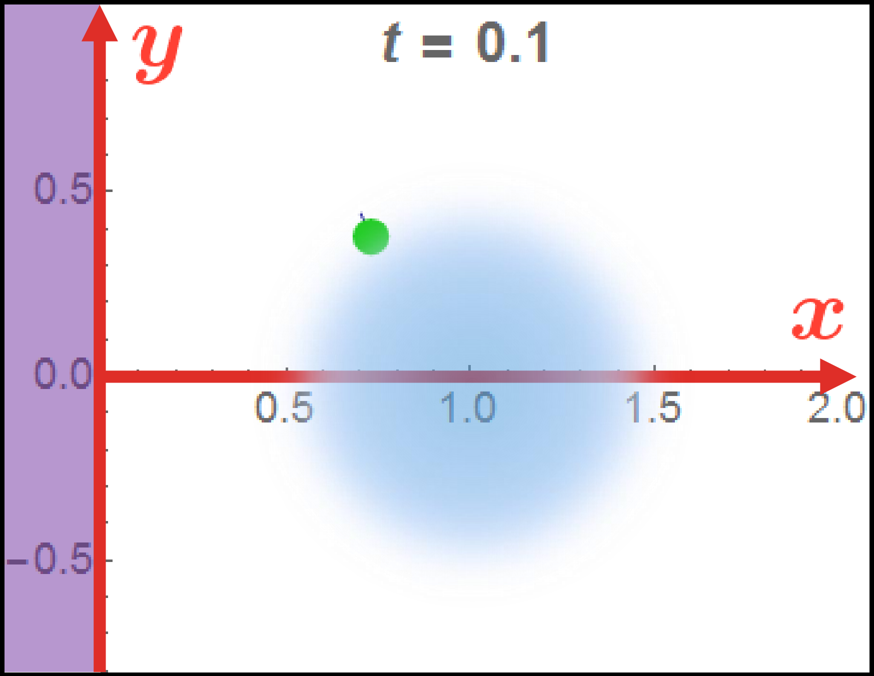
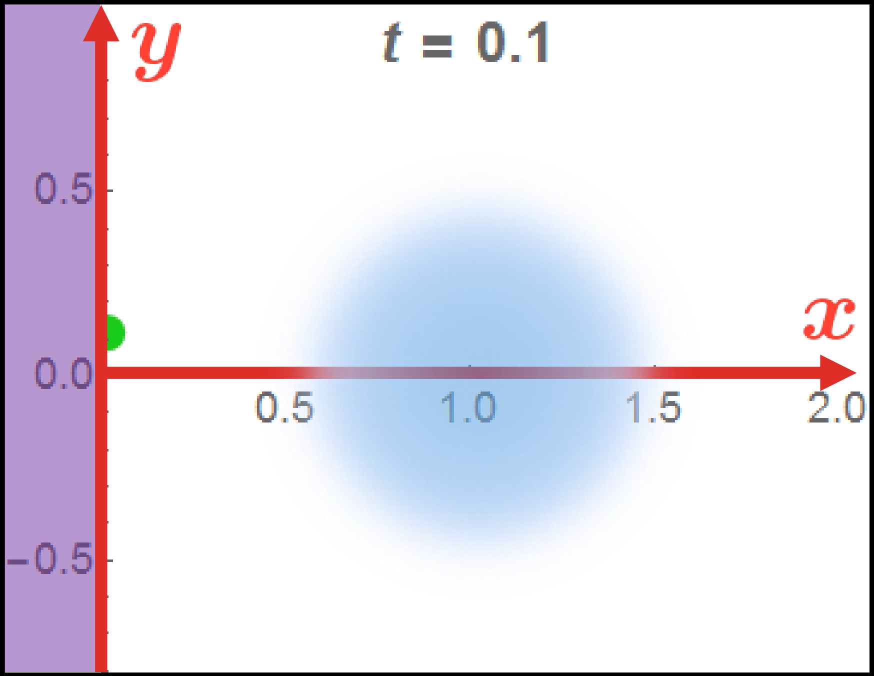
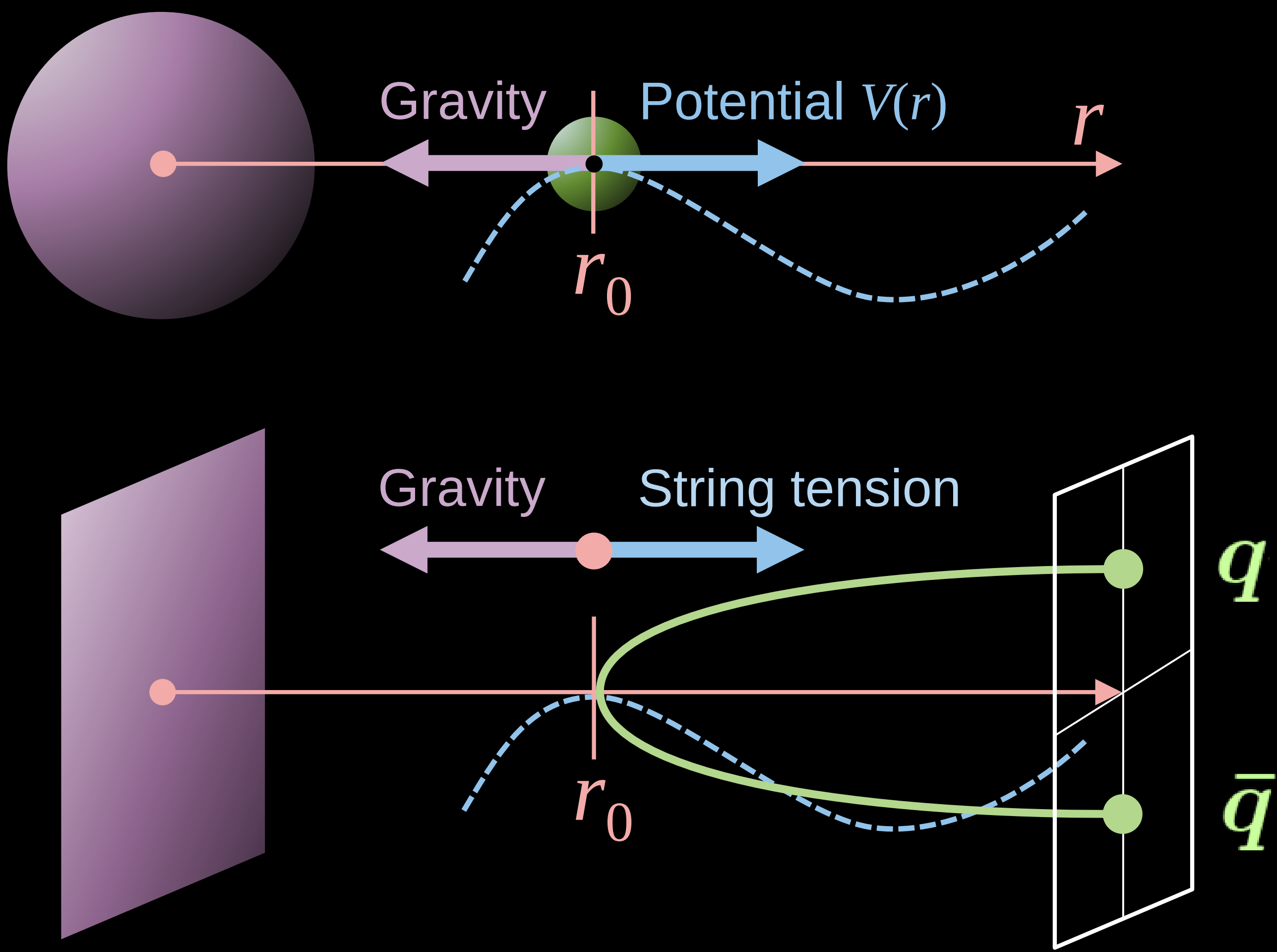


Particle near BH Horizon



Chaos Bound for AdS String



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