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# Density-Matrix Renormalization Group Study of Extended Kitaev-Heisenberg Model

参考 : Kazuya Shinjo, *et. al.*, ArXiv e-prints (2014),  
arXiv:1410.4790 [cond-mat.str-el]

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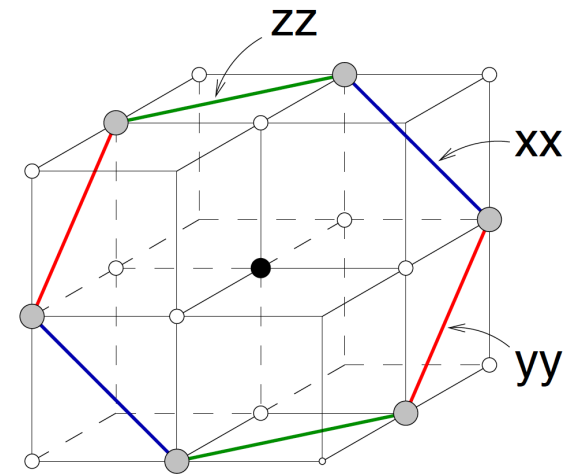
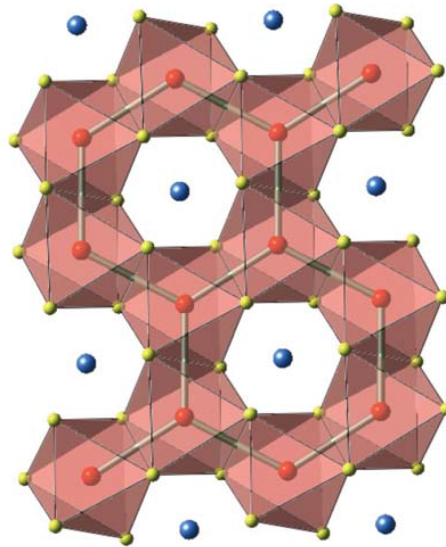
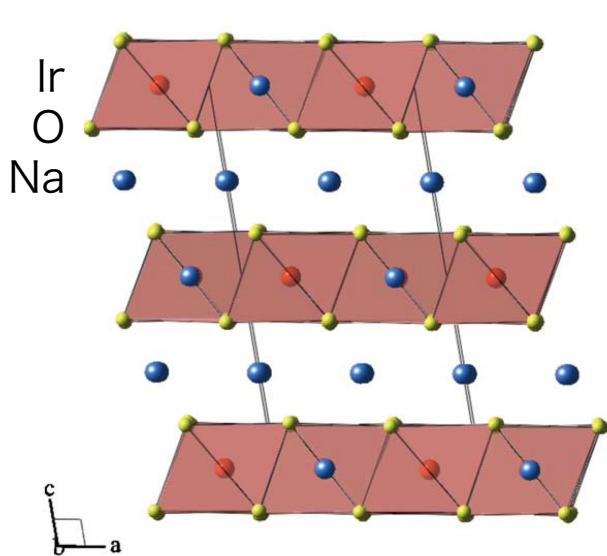
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# Kitaev-Heisenberg model and $\text{Na}_2\text{IrO}_3$



[Y. Singh and P. Gegenwart, Phys. Rev. B **82**, 064412 (2010)]

[G. Jackeli and G. Khaliullin, Phys. Rev. Lett. **102**, 017205 (2009)]

Kitaev-Heisenberg model

$$\mathcal{H}_{ij}^{(\gamma)} = \underline{2KS_i^\gamma S_j^\gamma} + \underline{JS_i \cdot S_j}.$$

Kitaev term    Heisenberg term

# Extended Kitaev-Heisenberg model and its phase diagram

$$\hat{\mathcal{H}} = \sum_{\Gamma} \sum_{\langle lm \rangle \in \Gamma} \hat{\mathcal{H}}_{lm}$$

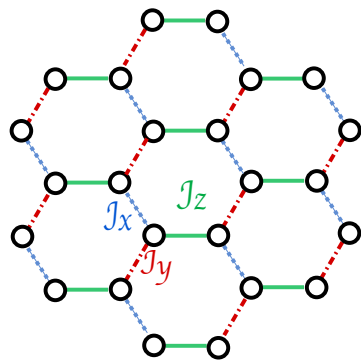
$$\hat{\mathcal{H}}_{lm} = K S_l^\gamma S_m^\gamma + J \left( S_l^\alpha S_m^\alpha + S_l^\beta S_m^\beta \right)$$

$$+ I_1 \left( S_l^\alpha S_m^\beta + S_l^\beta S_m^\alpha \right)$$

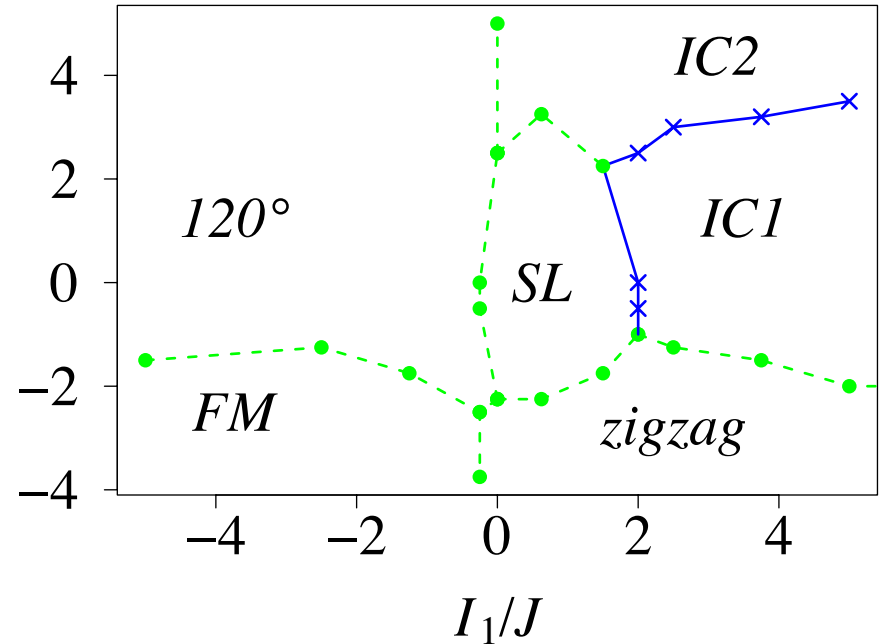
$$+ I_2 \left( S_l^\alpha S_m^\gamma + S_l^\gamma S_m^\alpha + S_l^\beta S_m^\gamma + S_l^\gamma S_m^\beta \right)$$

$I_2/J$

where  $\Gamma$  represents a combination of  $(\alpha, \beta, \gamma) = (x, y, z)$ ,  $(z, x, y)$ , and  $(y, z, x)$  on the  $\mathcal{J}_z$ ,  $\mathcal{J}_y$  and  $\mathcal{J}_x$  bond



Phase diagram when  $K/J = -25$

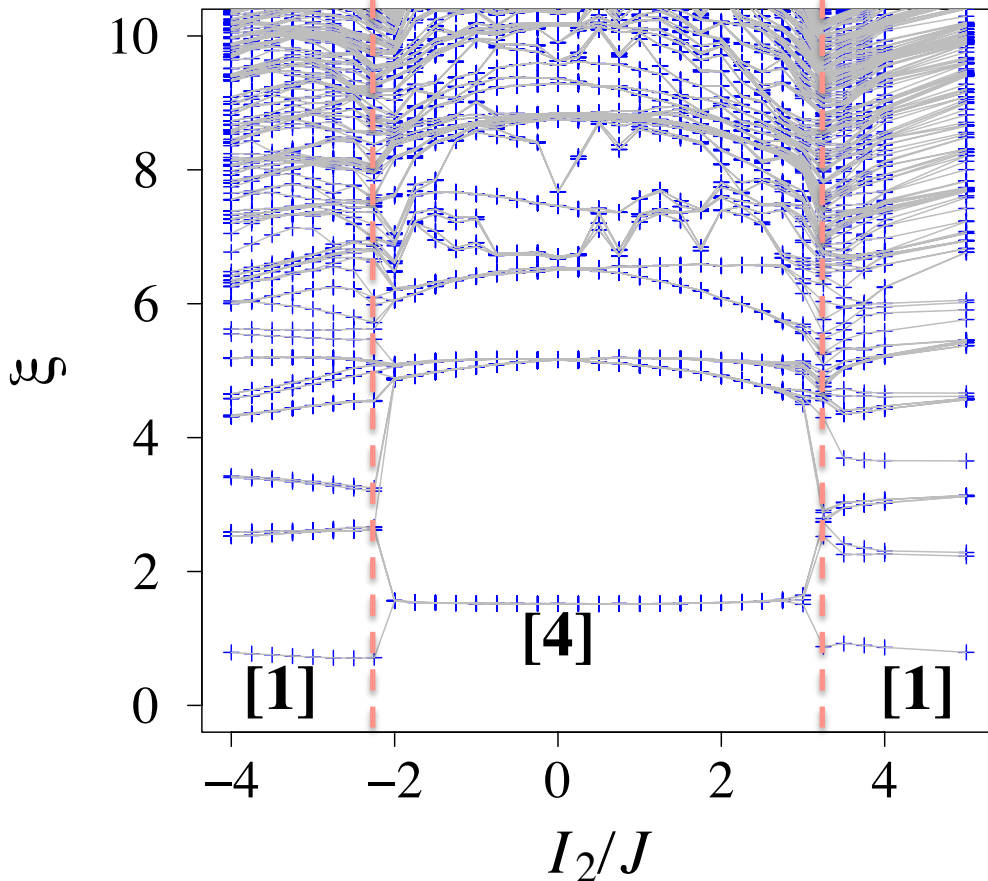


Blue : 1st order  
Green : 2nd order

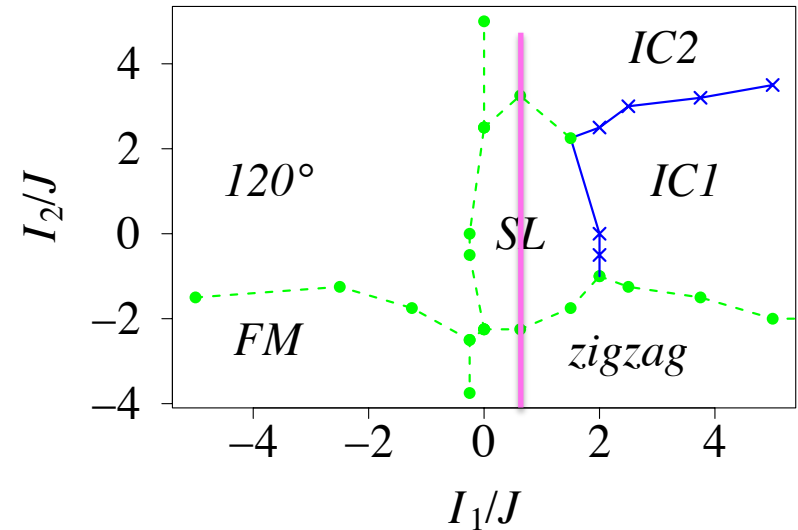
$I_1$  : coming from crystal structure of  $\text{Na}_2\text{IrO}_3$   
 $I_2$  : coming from trigonal distortion

# Entanglement spectrum (E. S.)

$$I_1/J = 0.63$$



$$K/J = -25$$

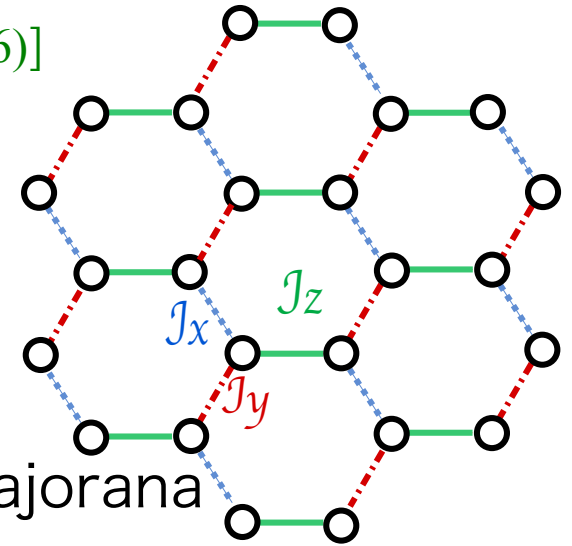


- ✓ Kitaev spin liquid shows degeneracy of E. S.
- ✓ Phase boundaries between the Kitaev spin-liquid and the magnetically ordered phases is determined by examining the Schmidt gap defined as  $\Delta\xi = \xi_1 - \xi_2$

# Kitaev honeycomb lattice model

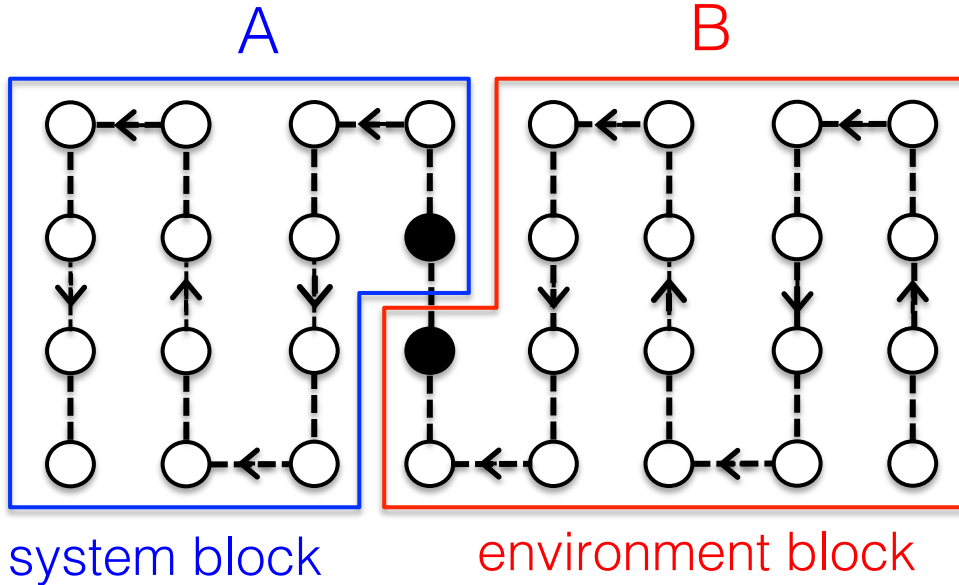
[A. Kitaev, Ann. Phys. 321, 2 (2006)]

$$\begin{aligned}\hat{\mathcal{H}}_{ij}^\gamma &= -J_\gamma S_i^\gamma S_j^\gamma \\ &= \begin{cases} -J_x S_i^x S_j^x & (\text{at } \mathcal{J}_x\text{-bond}) \\ -J_y S_i^y S_j^y & (\text{at } \mathcal{J}_y\text{-bond}) \\ -J_z S_i^z S_j^z & (\text{at } \mathcal{J}_z\text{-bond}) \end{cases}\end{aligned}$$



- ✓ ground state: spin liquid with gapless Majorana excitation
- ✓ When  $|J_z| \gg |J_{x,y}|$ , the model can be mapped to toric-code model.
- ✓ If time reversal symmetry is broken, fermions acquires an energy gap and vortices obey non-Abelian anyon  
→ Topological quantum computation

# Density-matrix renormalization group (DMRG)



Schmidt decomposition

$$\begin{aligned}
 |\psi\rangle &= \sum_{i,j} \psi_{i,j} |i\rangle_A |j\rangle_B \\
 &= \sum_{a=1}^m p_a |p_A^a\rangle |p_B^a\rangle
 \end{aligned}$$

打ち切り次数  $m$  が大きいほど、  
正確になる

Entanglement entropy

$$S_E = - \sum_i p_i \ln p_i = \sum_i \xi_i e^{-\xi_i}$$

Entanglement spectrum

$$\xi_i = - \ln p_i \quad [\text{H. Li and F. D. M. Haldane, Phys. Rev. Lett. } \mathbf{101}, 010504 \text{ (2008)}]$$