Exploring Hermitian and Non-Hermitian Topology in Synthetic Dimensions

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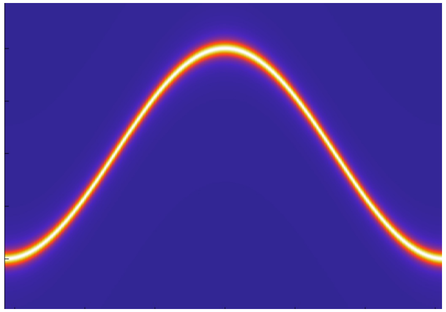
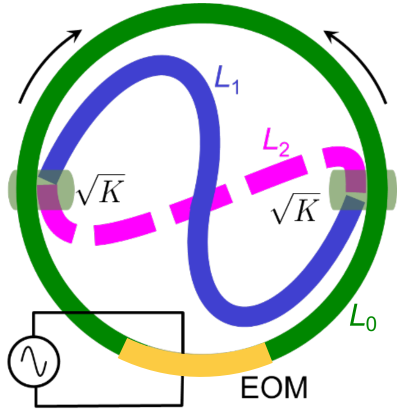
Outline

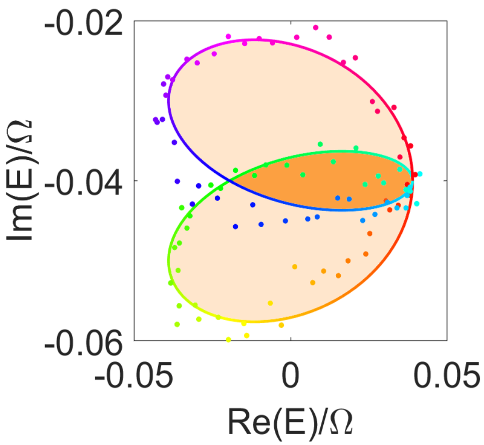
Background



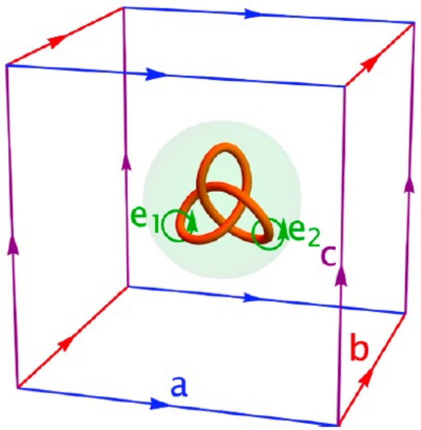
Hermitian Topology: experiments



Non-Hermitian Topology: experiments



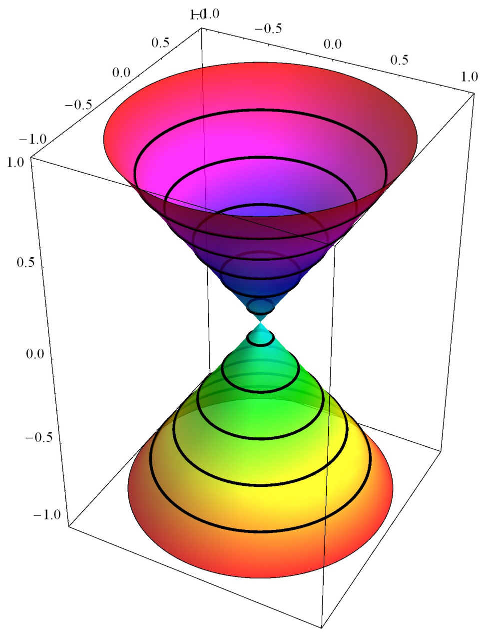
Non-Hermitian Topology: theory



Topological physics is much richer in higher dimensions

Weyl-point physics in three dimensions

$$H = k_x\sigma_x + k_y\sigma_y + k_z\sigma_z + E_0 I$$



A Four-Dimensional Generalization of the Quantum Hall Effect

Shou-Cheng Zhang and Jiangping Hu

We construct a generalization of the quantum Hall effect, where particles move in four dimensional space under a $SU(2)$ gauge field. This system has a macroscopic number of degenerate single particle states. At appropriate integer or fractional filling fractions the system forms an incompressible quantum liquid. Gapped elementary excitation in the bulk interior and gapless elementary excitations at the boundary are investigated.

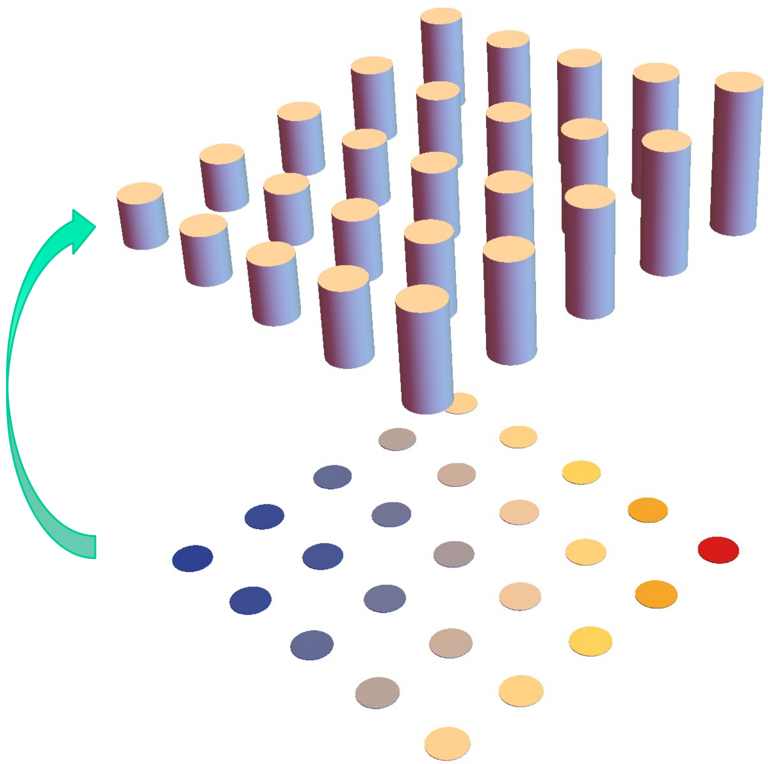
The concept of synthetic dimension

Explore higher-dimensional physics in lower dimensional physical systems

3D physics

Add one synthetic dimension

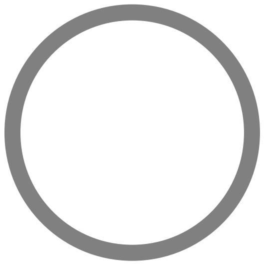
2D representation



Tsomokos et al, PRA 82, 052311 (2010);
Boada et al, PRL 108, 133001 (2012);
Jukic and Buljan, PRA 87, 013814 (2013).

For a review on synthetic dimension in photonics
L. Yuan et al, Optica 5, 1396 (2018).

A single ring resonator



$\vdots$

$\omega_0 + 2\Omega$

$\omega_0 + \Omega$

ω_0

$\omega_0 - \Omega$

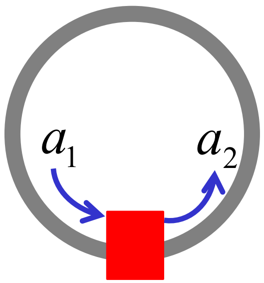
$\omega_0 - 2\Omega$

$\vdots$

In the absence of group velocity dispersion (GVD) in the ring waveguide, the ring supports a set of resonances with equally spaced resonant frequencies.

$\Omega = 2\pi/T$ where T is the round trip time

Gauge potential for light



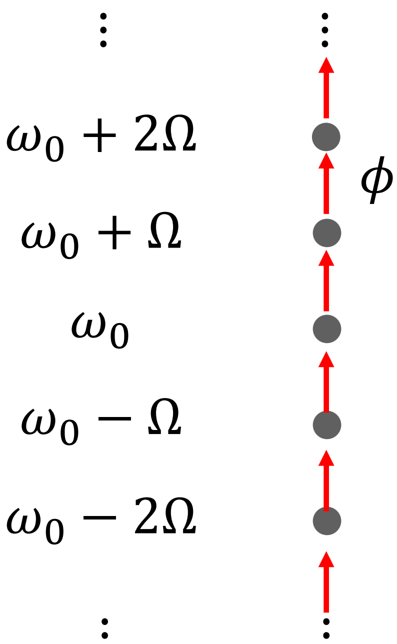
$$a_2 = a_1 e^{i\alpha sin(\Omega_{\mathrm{m}} t + \phi)}$$

$$\Omega_m = \Omega$$

Modulation frequency

Mode spacing

Modulation resonantly couples different modes together



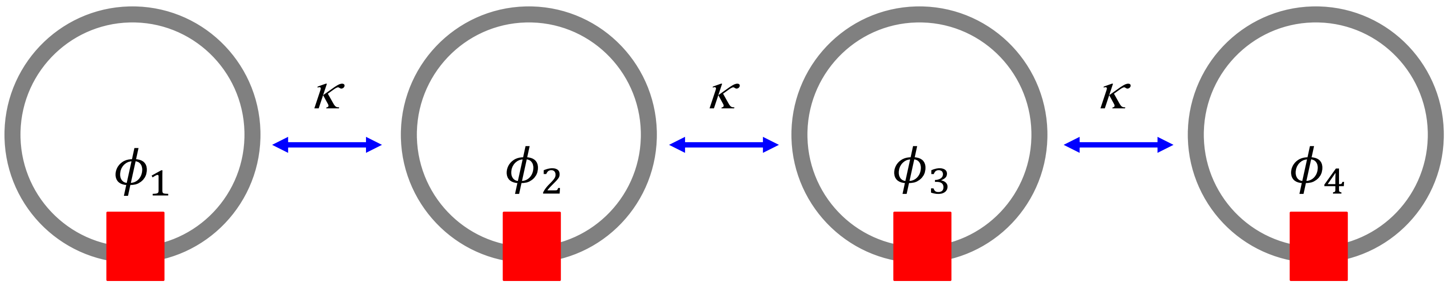
$$H = V \sum_n a_n^\dagger a_{n+1} e^{-i\phi} + a_{n+1}^\dagger a_n e^{i\phi}$$

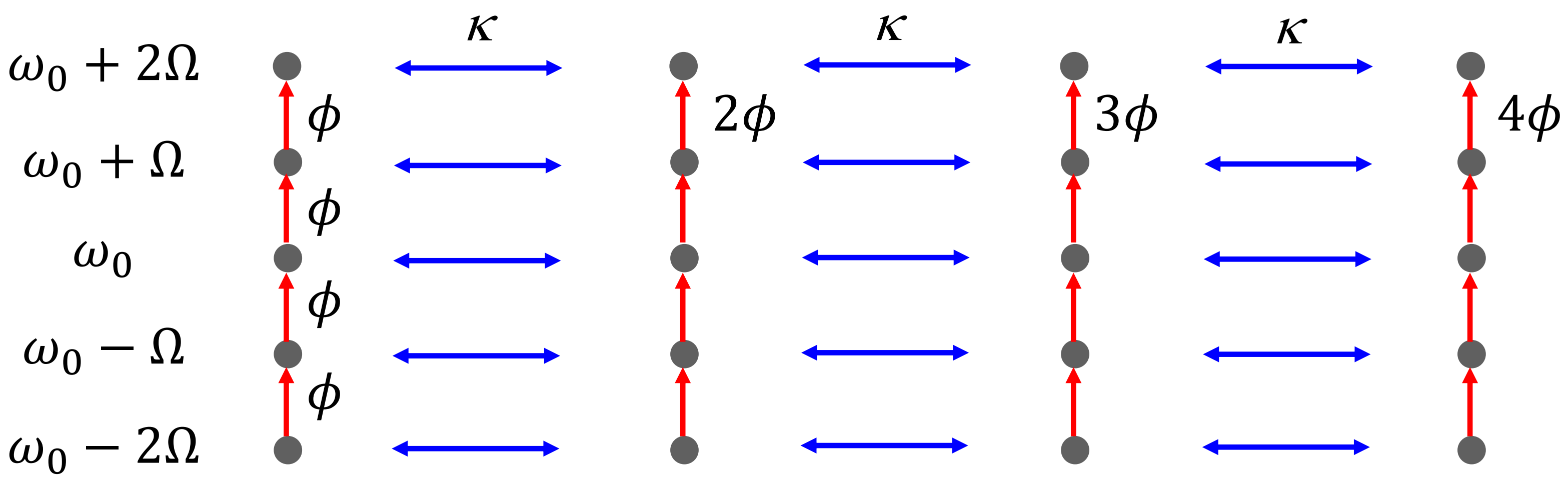
- 1D physics in 0D structure
- ϕ is the gauge potential which breaks reciprocity

K. Fang, Z. Yu and S. Fan, Nature Photonics 6, 782 (2012)

Tight-binding lattice in synthetic space

Array of coupled resonator

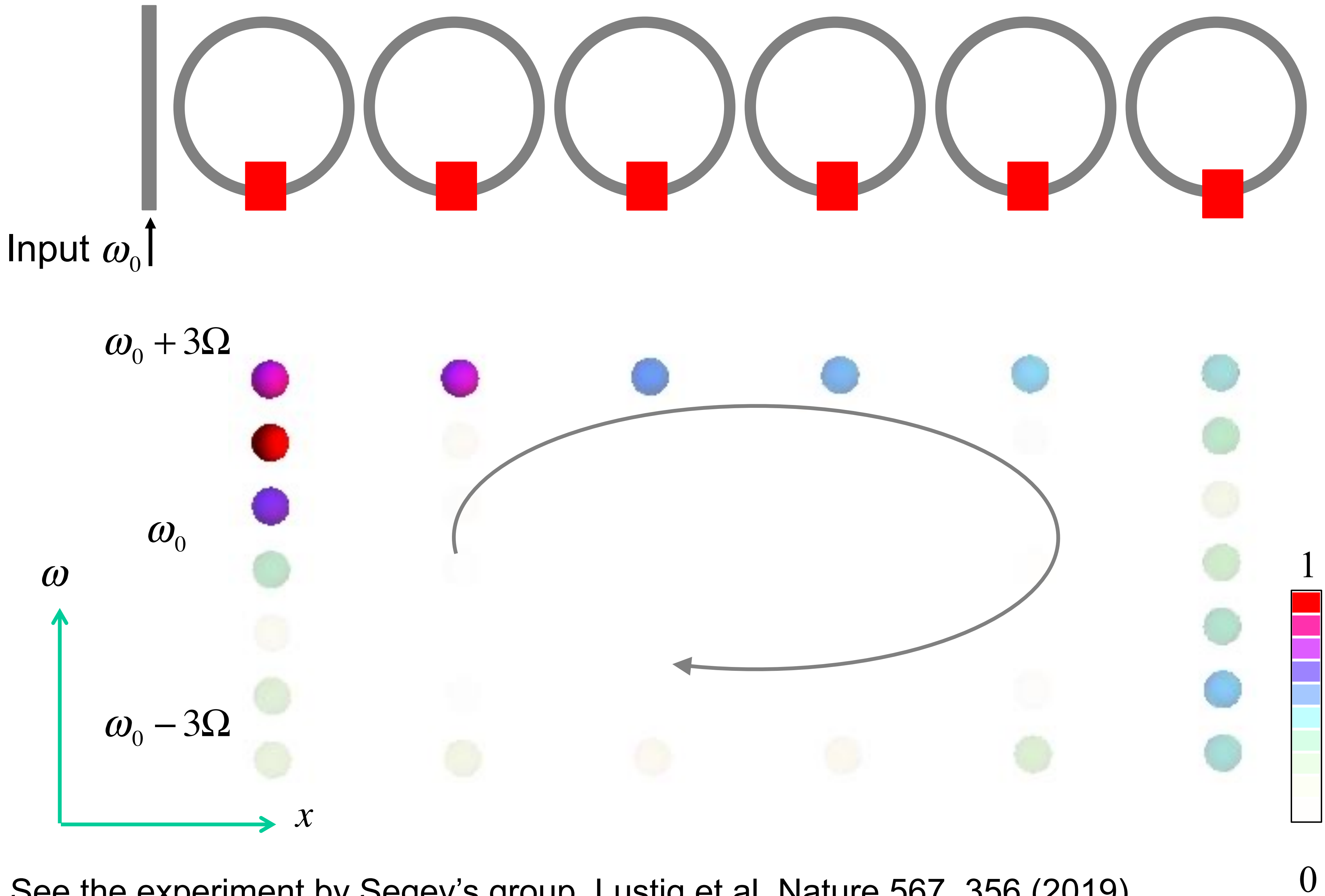




A two-dimensional space, having a real-space axis and a frequency axis with applied gauge field in the synthetic space.

L. Yuan, Y. Shi and S. Fan, Optics Letters 41, 741 (2016)
See also, Ozawa et al, PRA 93, 043827 (2016).

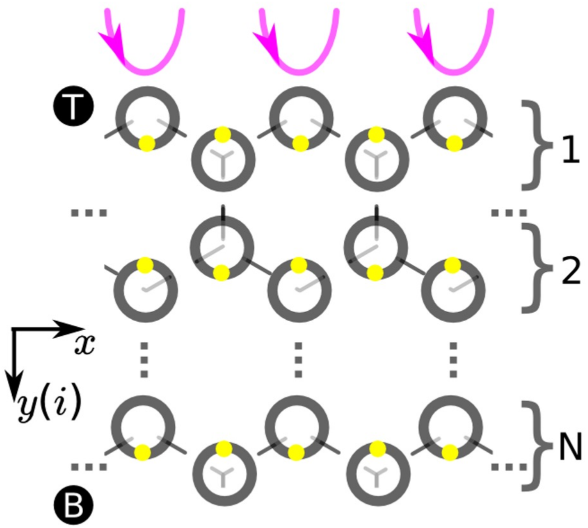
One-way edge mode in the synthetic space



See the experiment by Segev's group, Lustig et al, Nature 567, 356 (2019).

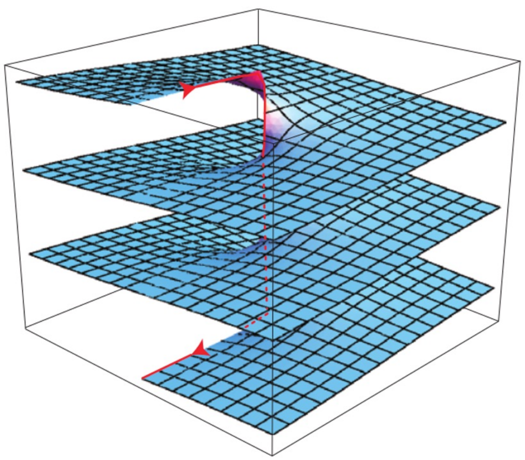
Many non-trivial topological phenomena in synthetic space

Weyl semimetal



Lin et al, Nature Communications 7, 13731 (2016).

3D topological insulator

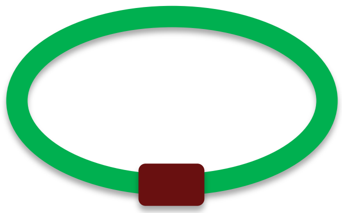


Lin et al, Science Advances 4, eaat2774 (2018).

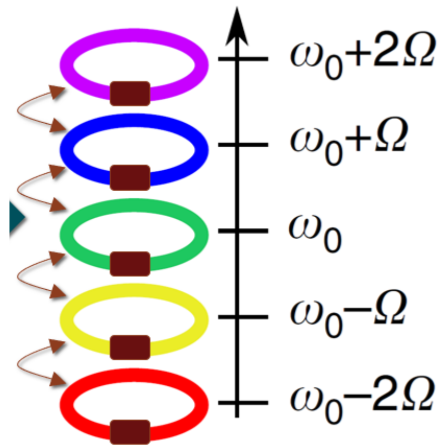
Higher-order topological insulator

Dutt et al, Light: Science and Applications, 9, 131 (2020).

Band structure in synthetic space

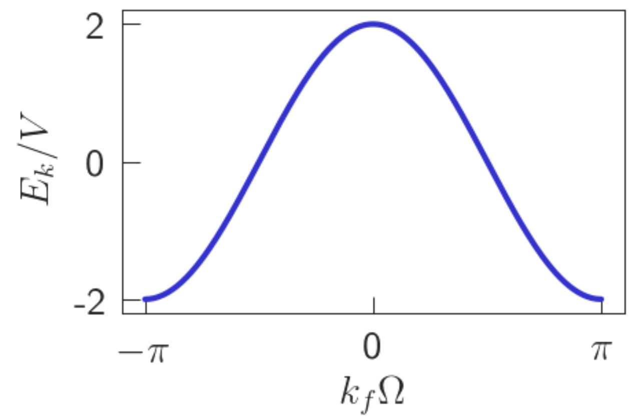


$$T_{EOM} = e^{i\alpha\cos(\Omega t+\phi)}$$



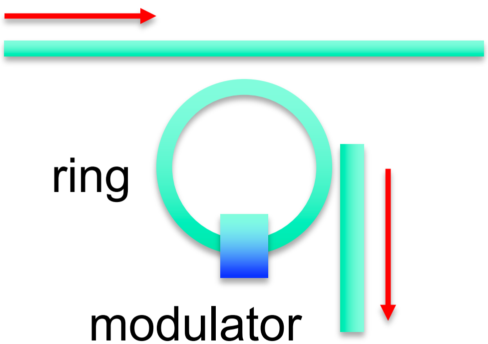
$$H = V\sum_n a_n^\dagger a_{n+1} + a_{n+1}^\dagger a_n$$

$$E_{k_f} = 2V\cos k_f\Omega$$



Quasi-momentum in the synthetic frequency dimension k_f $\equiv$ time t

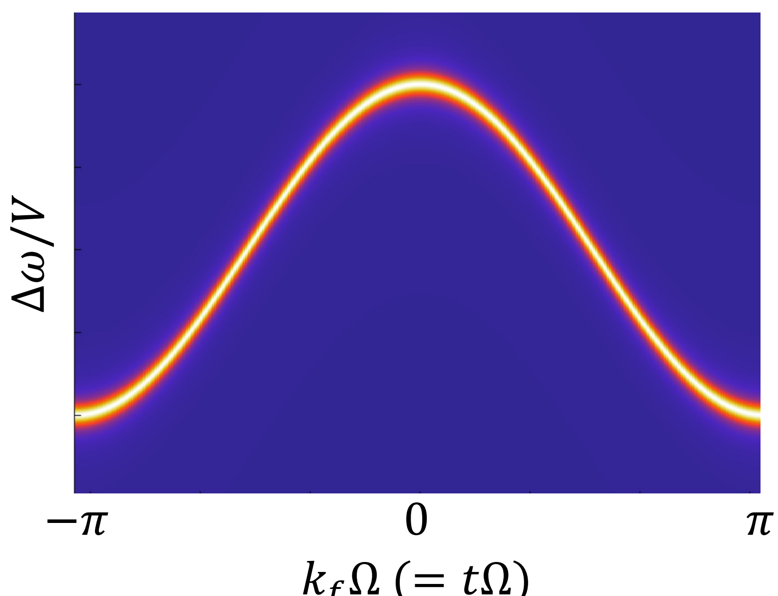
Band structure from time-dependent transmission



$$V(t) = V_1 \cos \Omega t$$

$$T(\omega, k_f) = \frac{\gamma^2}{[\omega - E(k_f)]^2 + \frac{\gamma^2}{4}} = \frac{\gamma^2}{\Delta\omega^2 + \frac{\gamma^2}{4}} = T(\Delta\omega, t)$$

Theoretical transmission $(\gamma = V/10)$



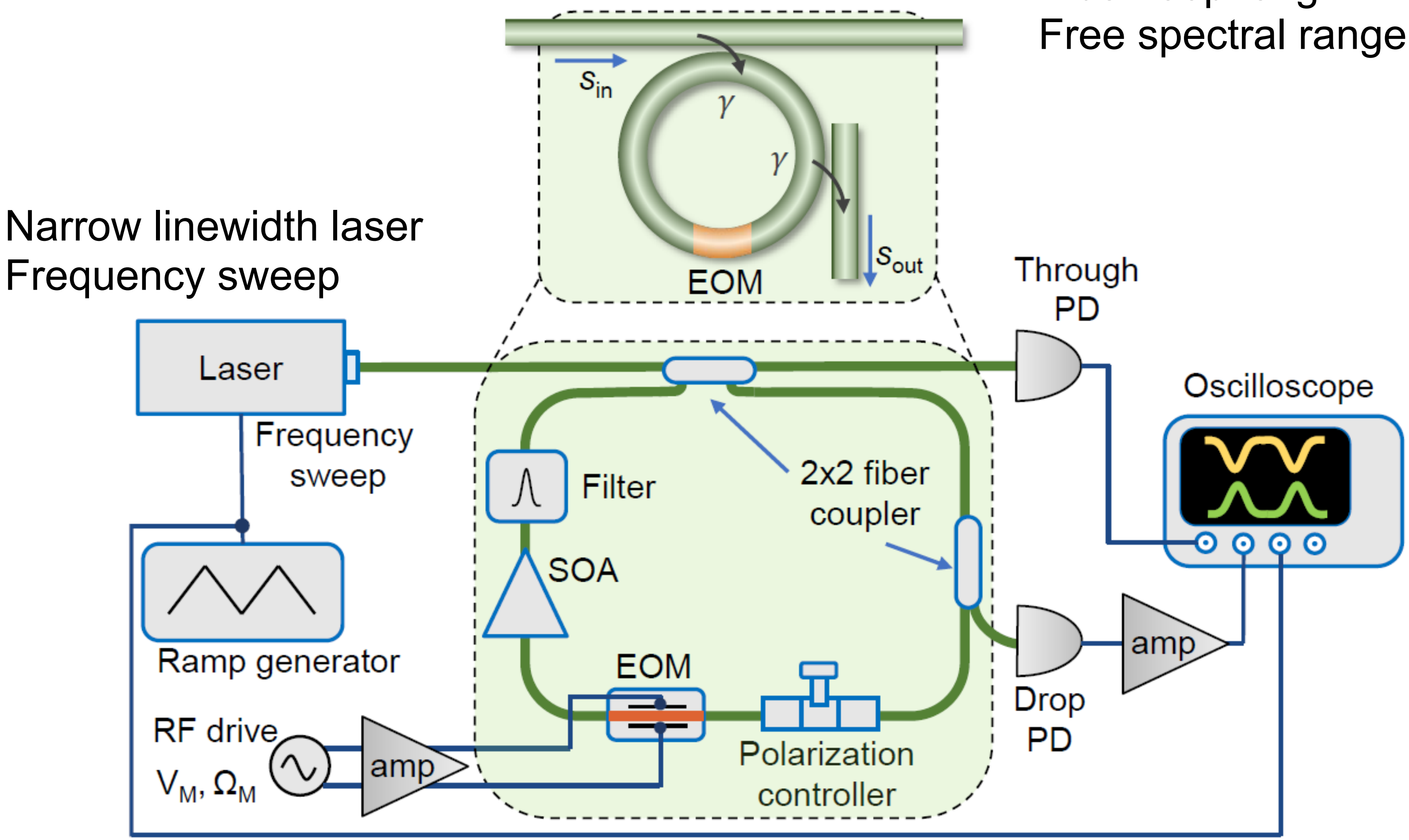
$\gamma = V/10$

To determine the band structure, one needs to measure the transmission of the system as a function of time (t or k_f), for various detunings $\Delta\omega$.

Experimental setup

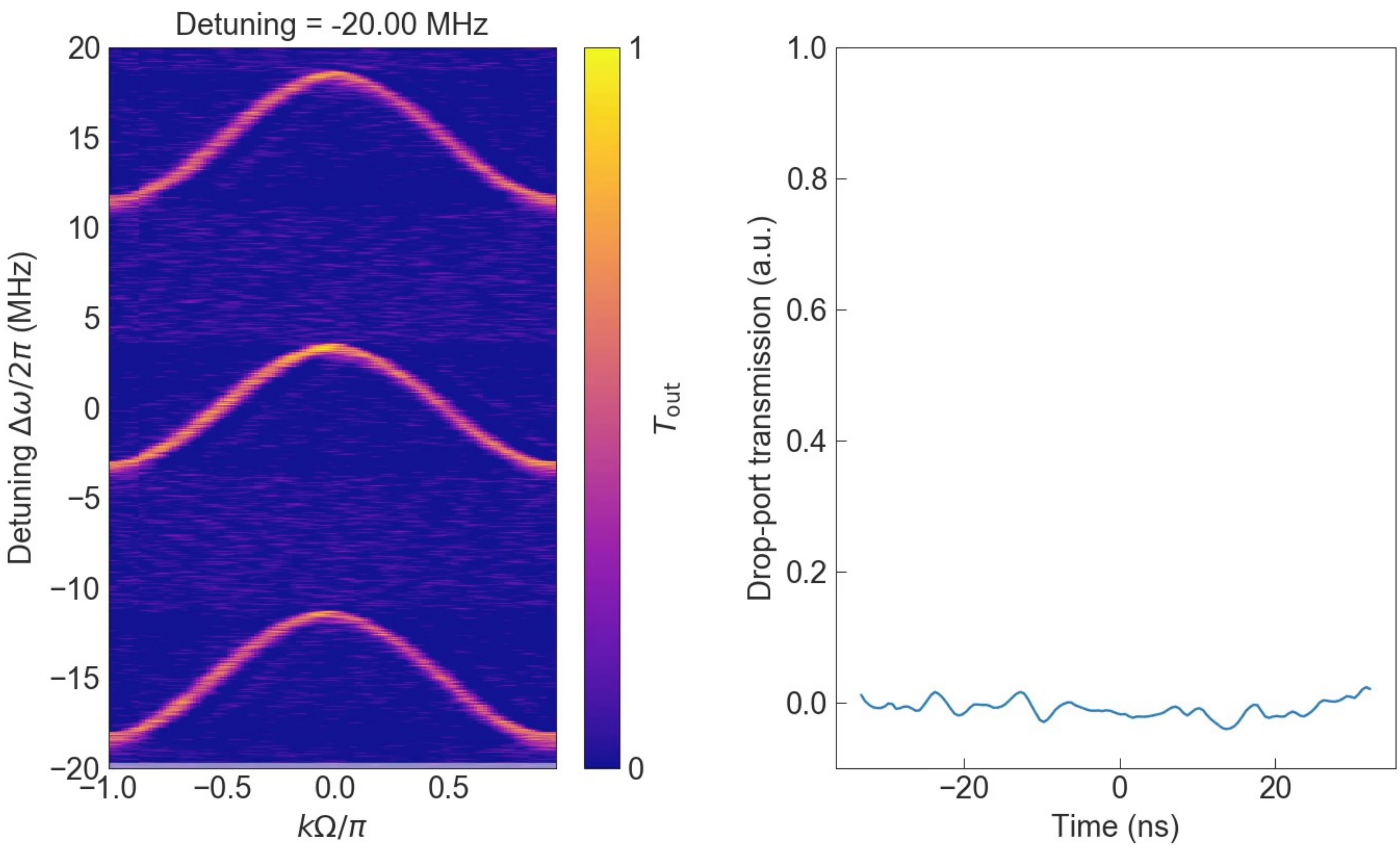
Fiber loop length ~13m
Free spectral range: 15.1MHz

Narrow linewidth laser
Frequency sweep



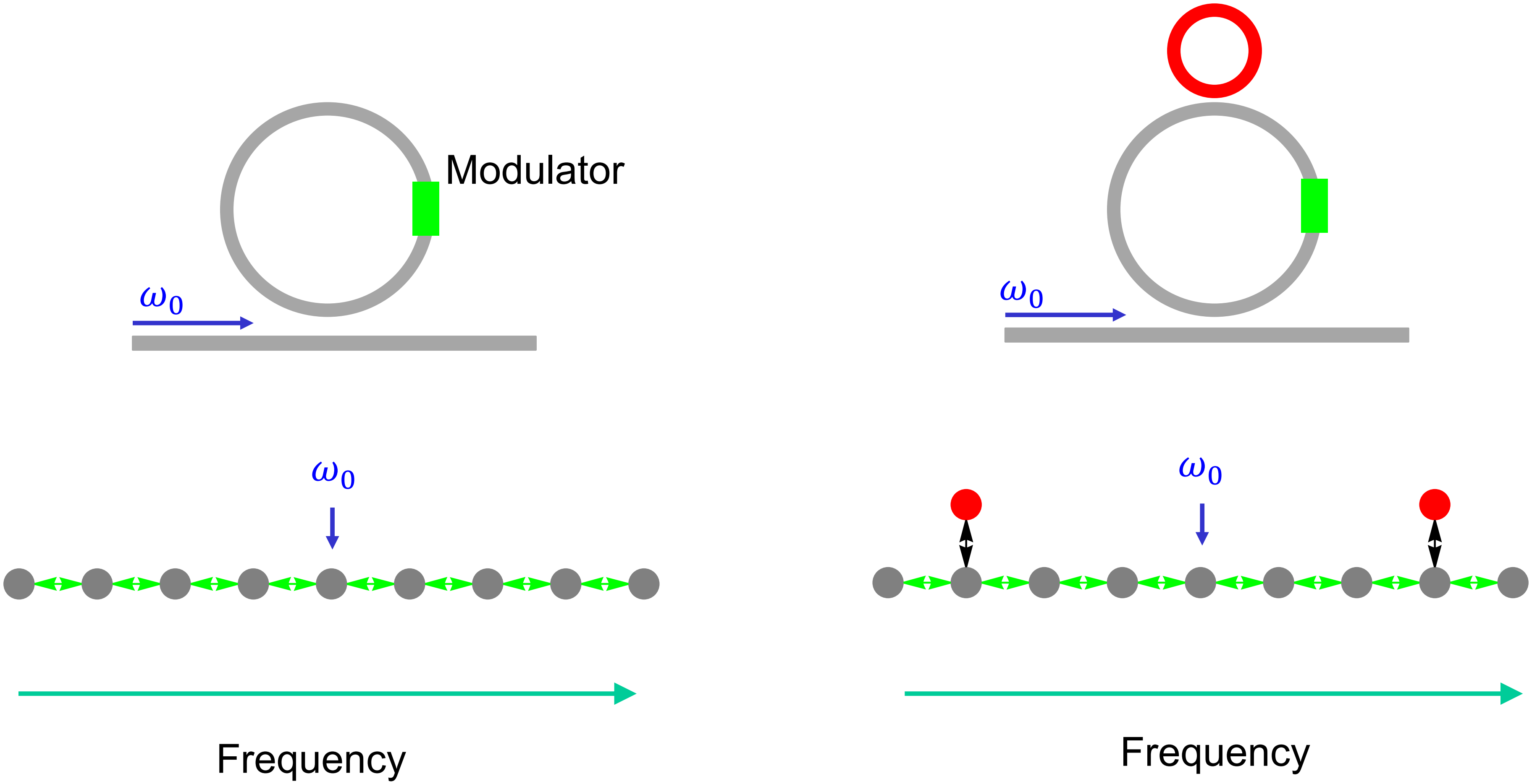
$LiNbO_3$ electro-optic modulator,
modulation frequency up to 2 GHz,
loss compensated with semiconductor optical amplifier

Modulation with a single frequency Ω equal to free spectral range



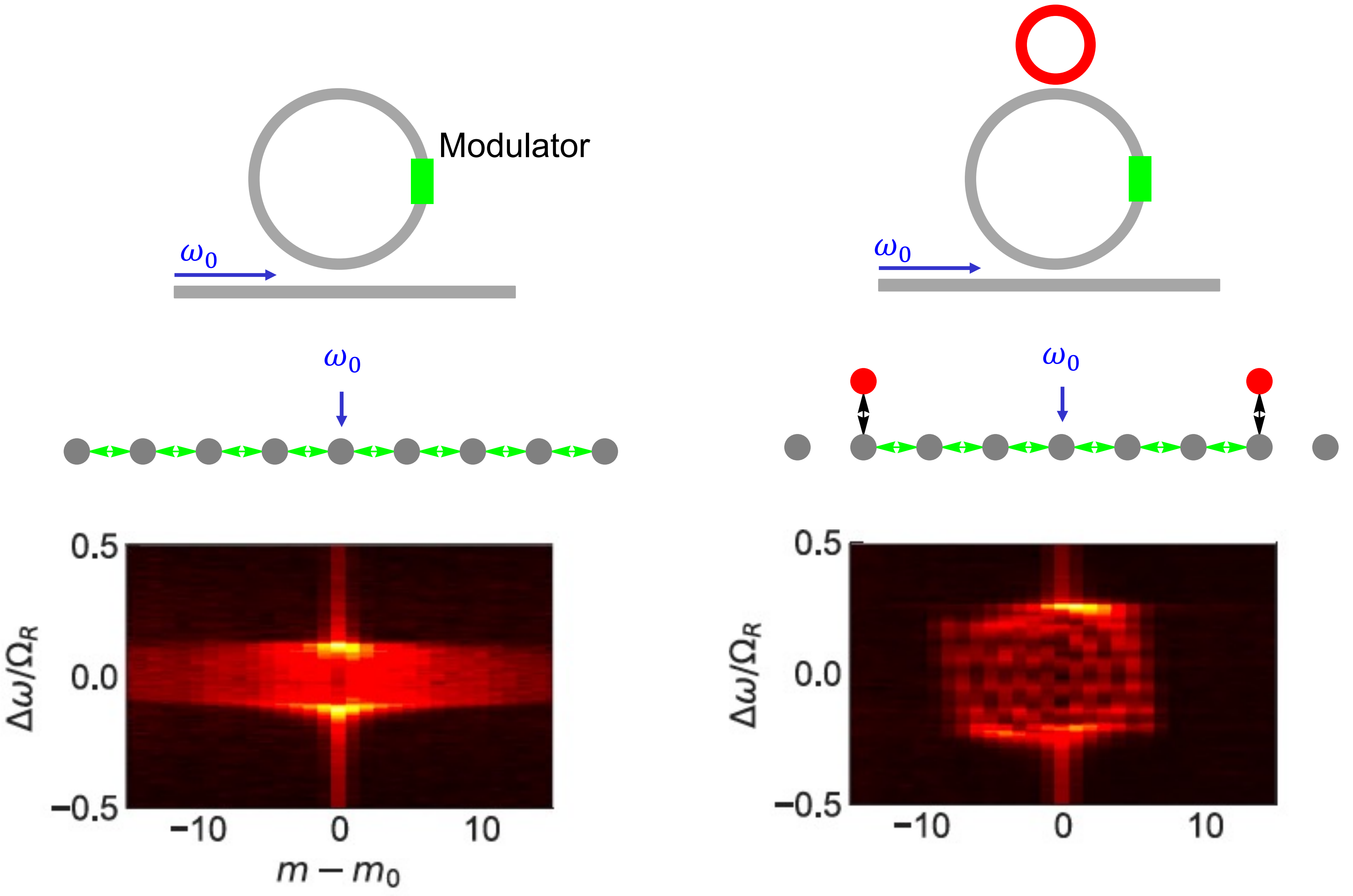
A. Dutt, M. Minkov, Q. Lin, L. Yuan, D. A. B. Miller, S. Fan, Nature Communications 10, 3122 (2019).

Creating boundary in frequency synthetic space

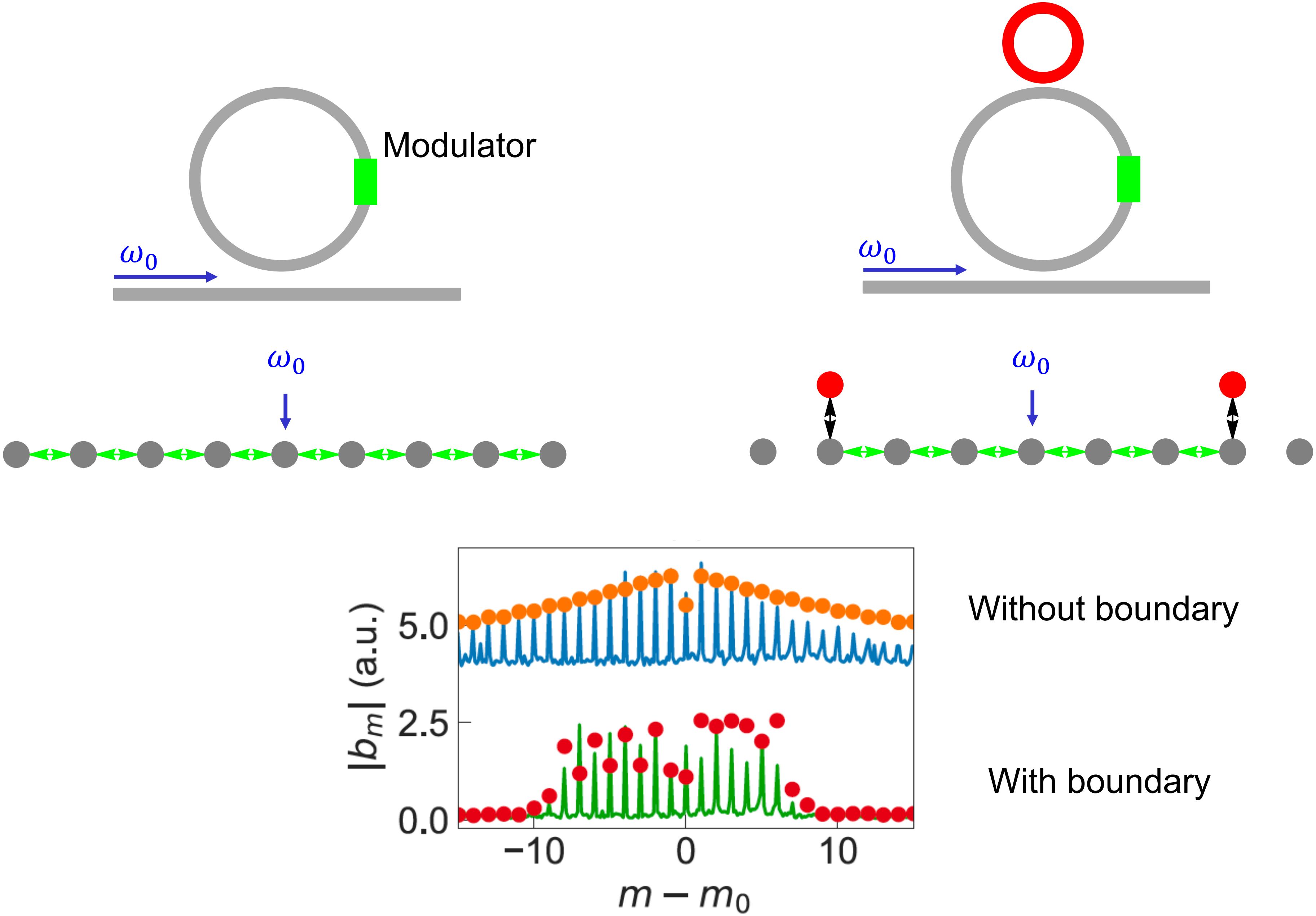


A. Dutt, L. Yuan, K. Y. Yang, K. Wang, S. Buddhiraju, J. Vuckovic, and S. Fan, Nature Communications 13, 3377 (2022).

Experimentally demonstrated boundary

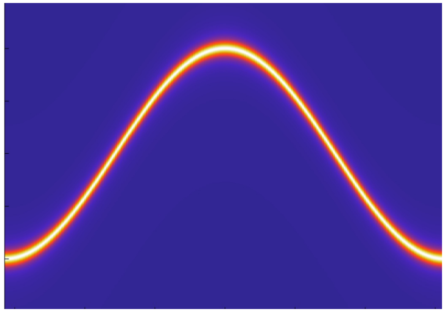


Fabry-Perot Oscillation Along Frequency Dimension

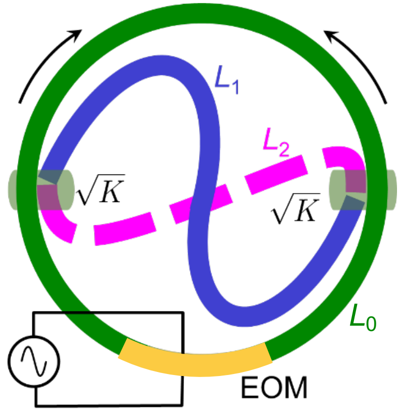


Outline

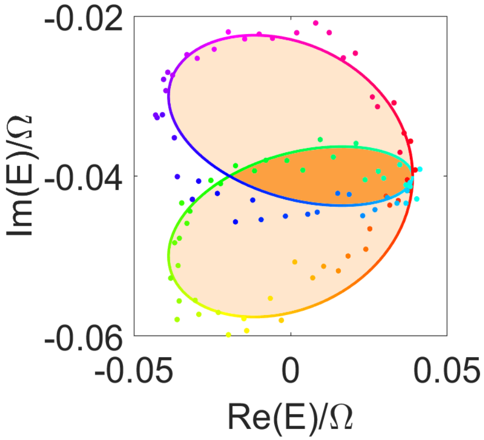
Background



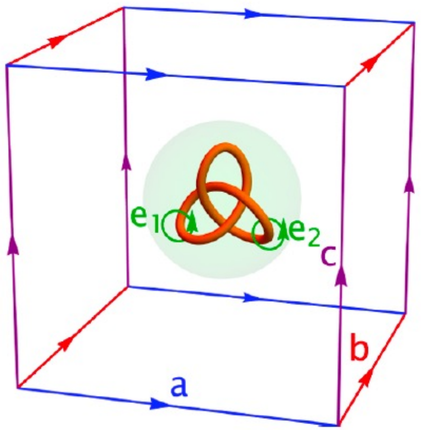
Hermitian Topology: experiments



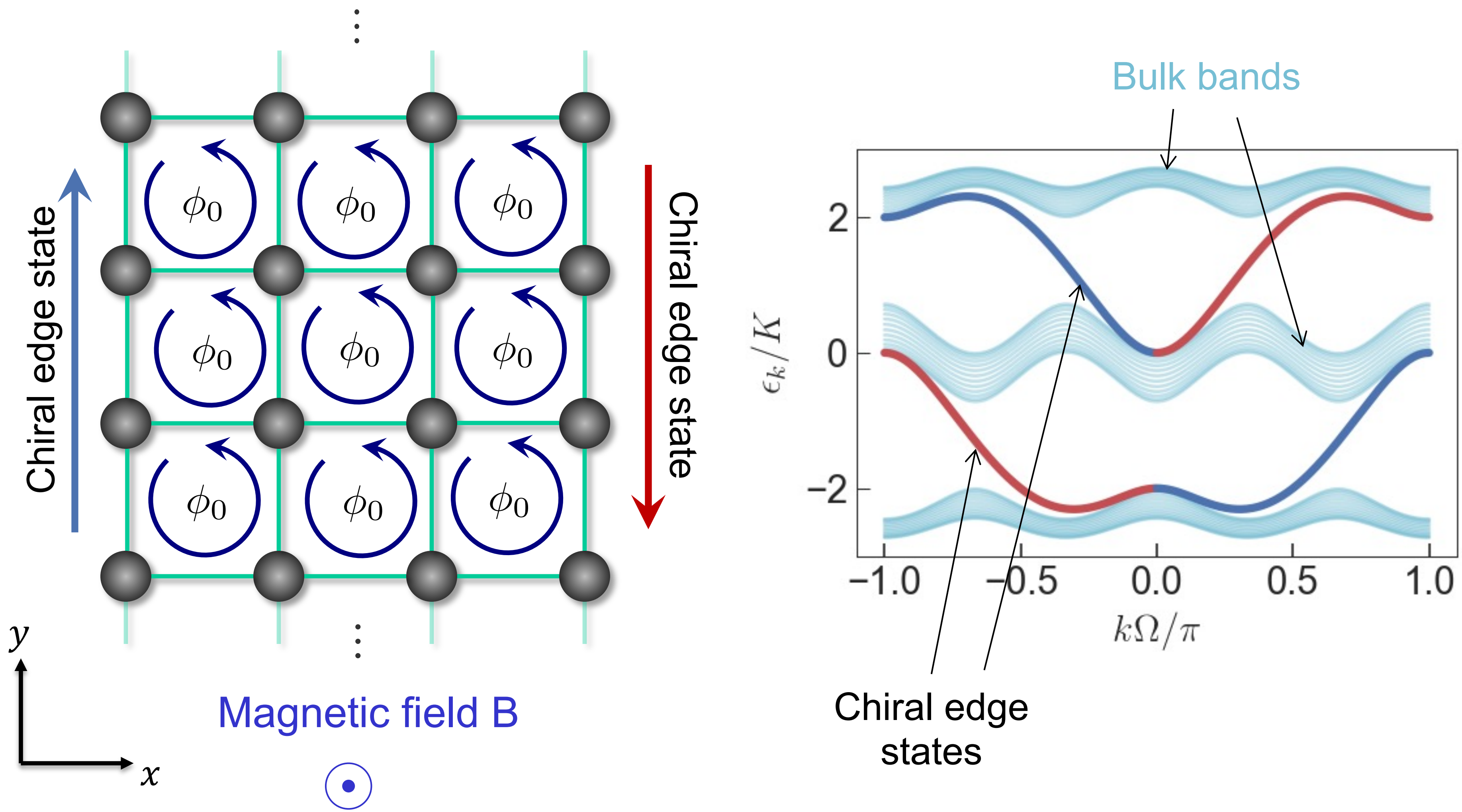
Non-Hermitian Topology: experiments



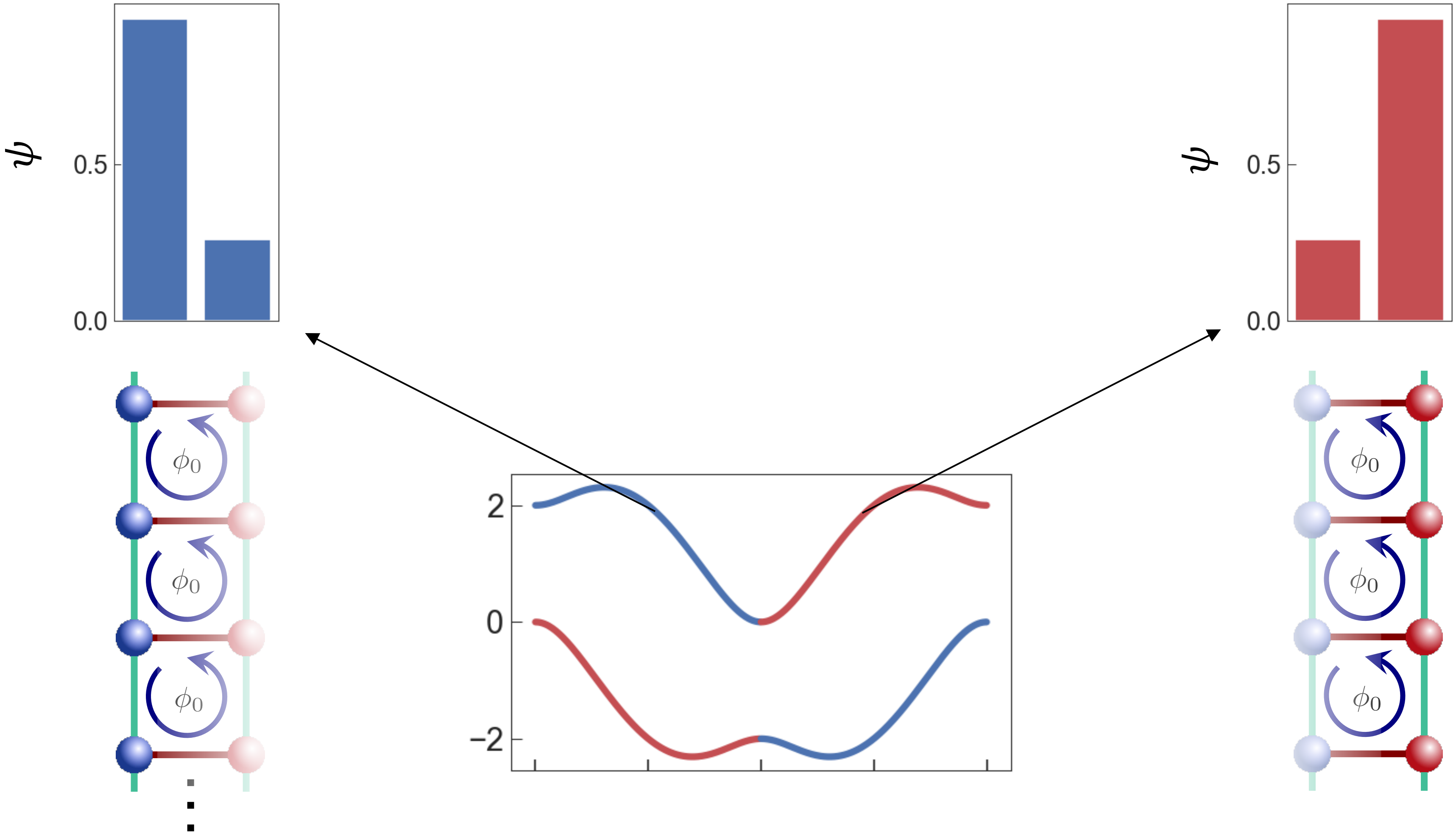
Non-Hermitian Topology: theory



Quantum Hall systems



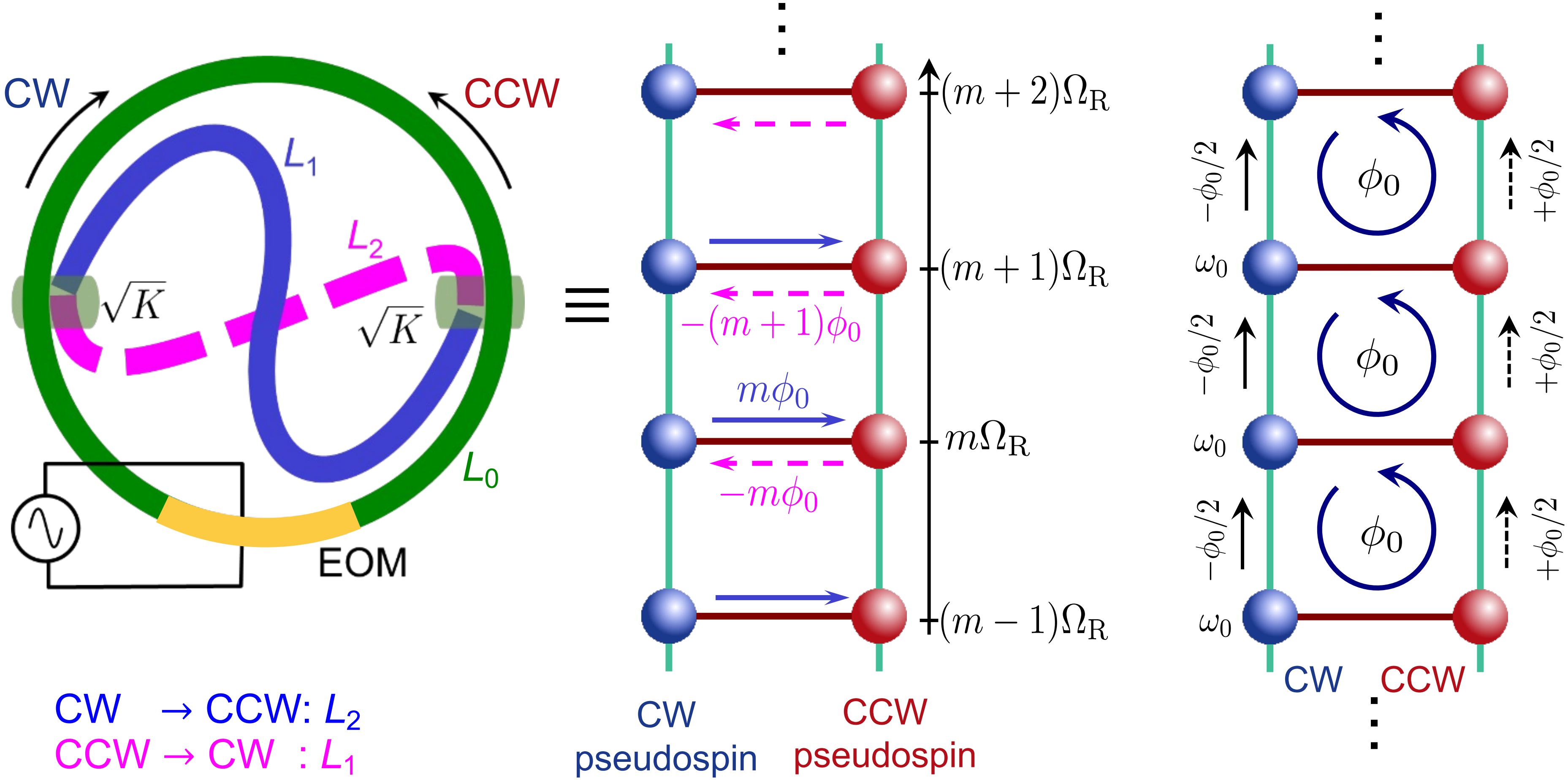
Chiral two-legged Hall ladder



Entire bulk sites removed → no bulk bands but chiral edge states preserved

Theory: D. Hugel and B. Paredes, Physical Review A 89, 023619 (2014).
Demonstrated in cold-atom systems with real-space lattices.

Two-legged synthetic Hall ladder

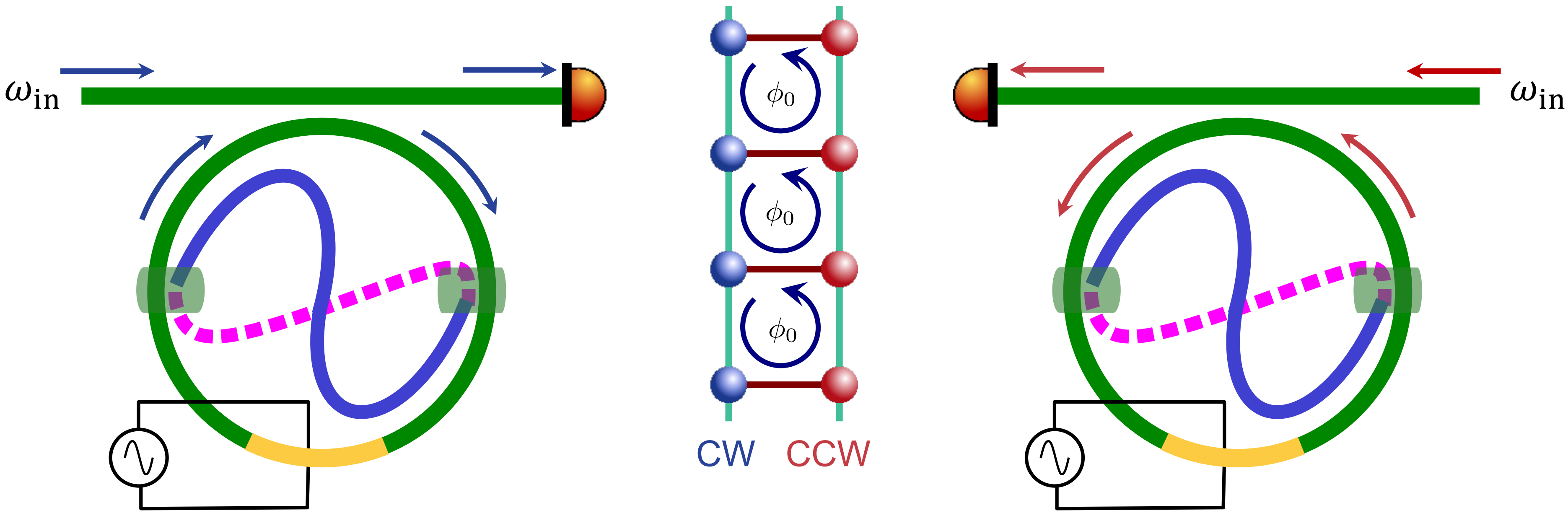


$$\Delta\phi = \beta_m(L_1 - L_2) = \frac{2\pi m}{L_0}\Delta L$$

- Realize effective magnetic field in synthetic space

A. Dutt, Q. Lin, L. Yuan, M. Minkov, M. Xiao and S. Fan, Science 367, 59 (2020).

Spin-resolved band structure measurement



Measure clockwise (CW) transmission, yields band structure projected on CW pseudospin by sweeping ω_{in}

Measure counter-clock-wise (CCW) transmission, yields band structure projected on CCW pseudospin

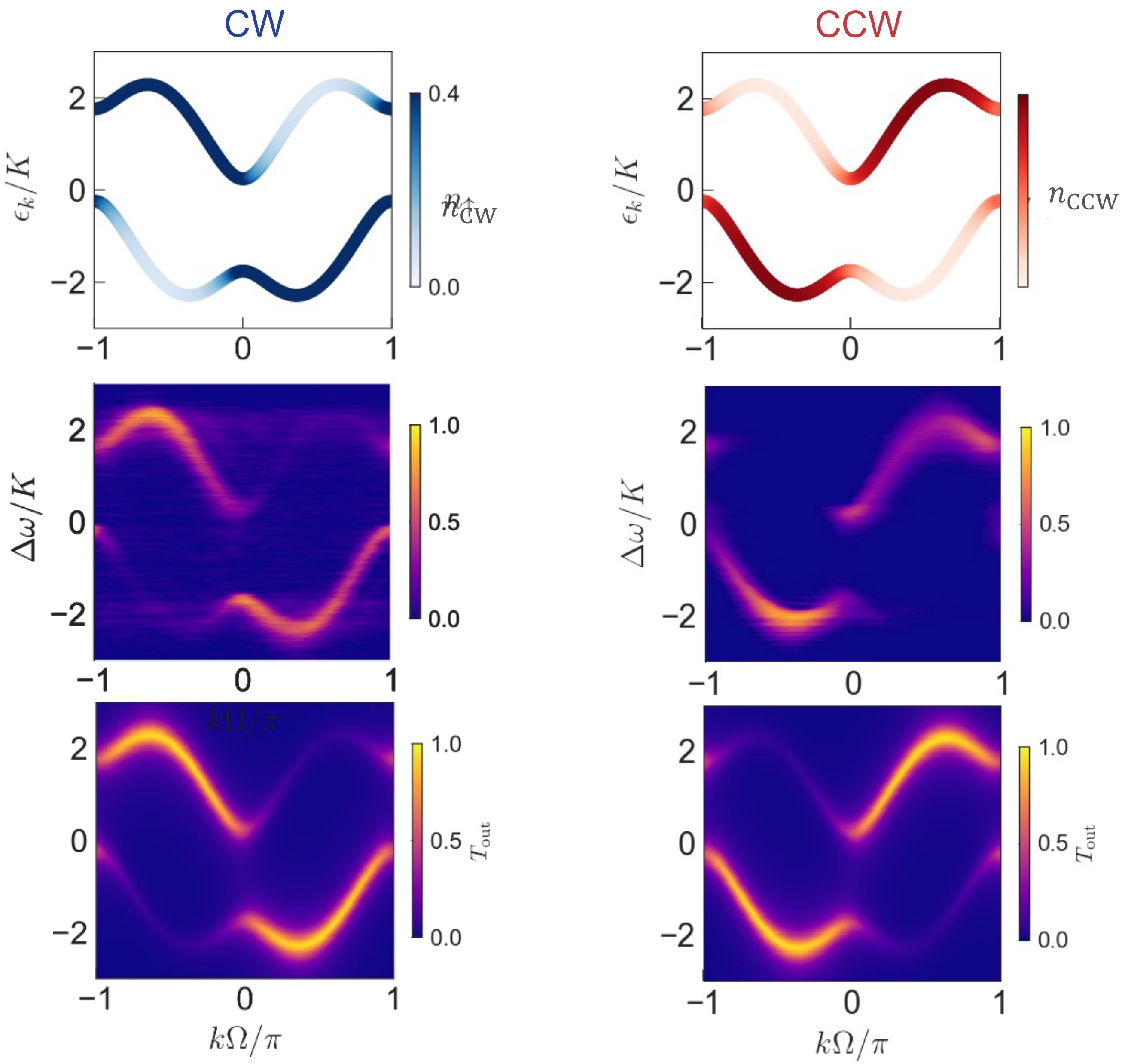
”Spin”: CW or CCW modes

Chiral bands and spin-momentum locking

Theoretical spin-projected bands

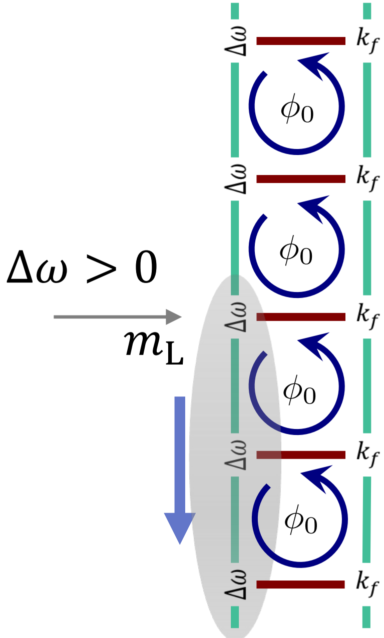
Experimentally measured bands

Floquet simulations

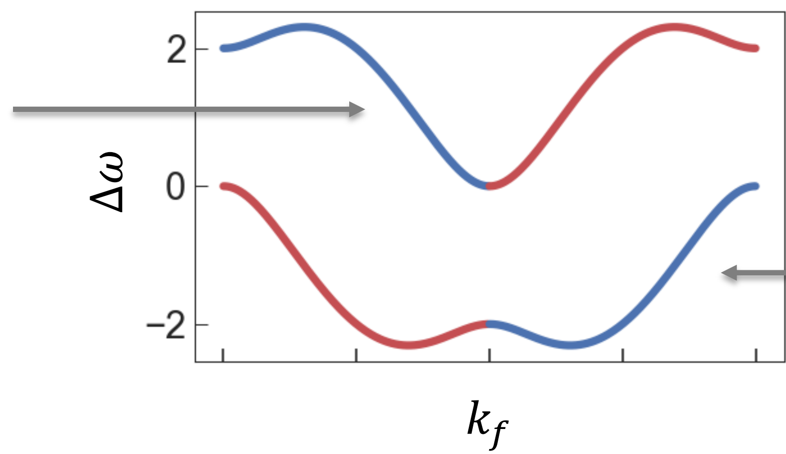


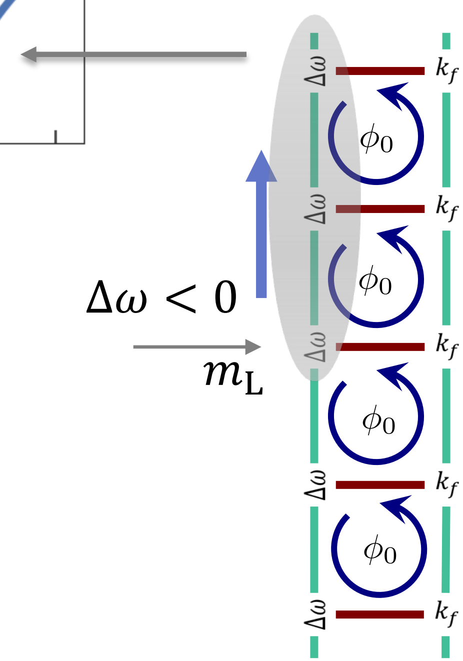
$J/K = 2$
$\phi_0 = 3\pi/4$

Chiral currents



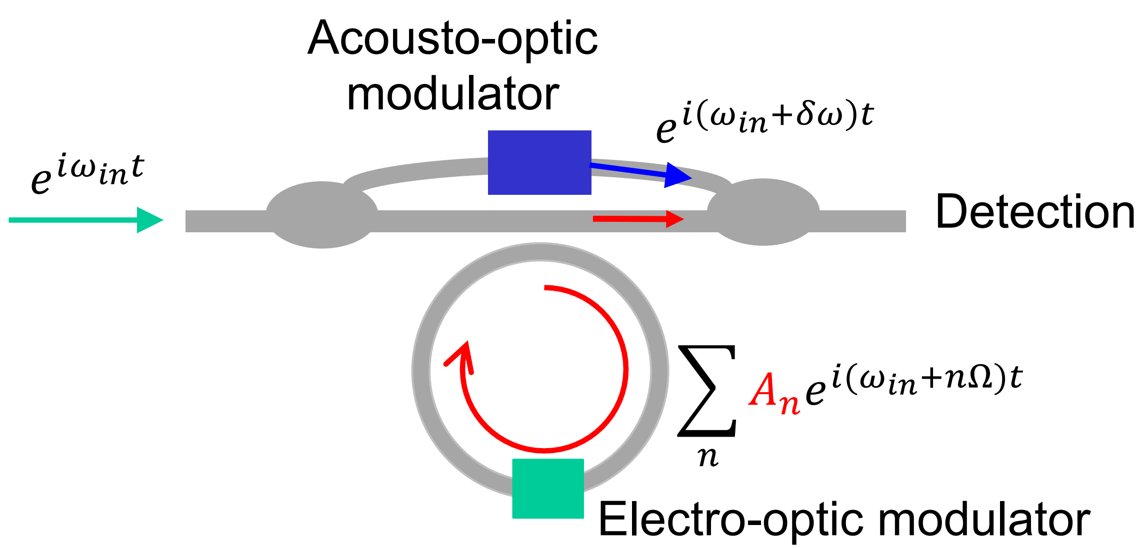
Chiral current: $j_C = \sum_{m>m_L} P(m) - \sum_{m<m_L} P(m)$





To demonstrate one-way transport, we measure the output frequency spectrum

Heterodyne measurement of the spectrum



Aim of the measurement

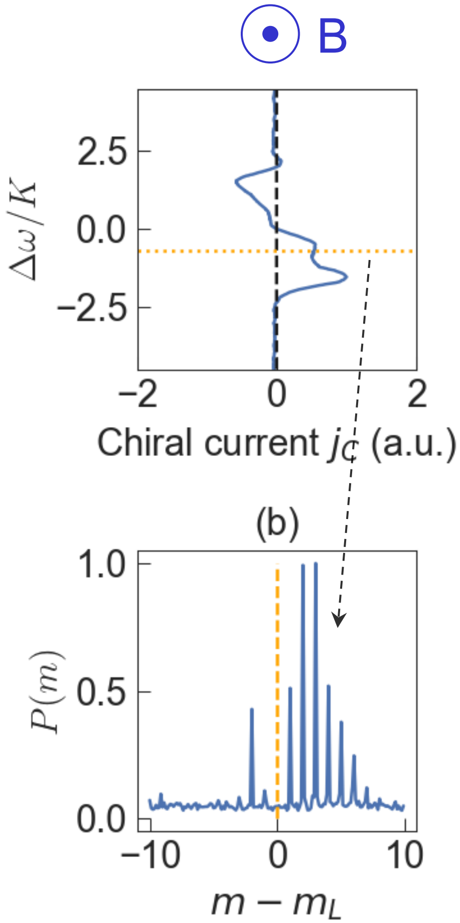
$$I(t) = \left| \sum_n A_n e^{i(\omega_{in}+n\Omega)t} + e^{i(\omega_{in}+\delta\omega)t} \right|^2 \sim \sum_n A_n e^{i(n\Omega-\delta\omega)t} + c.c$$

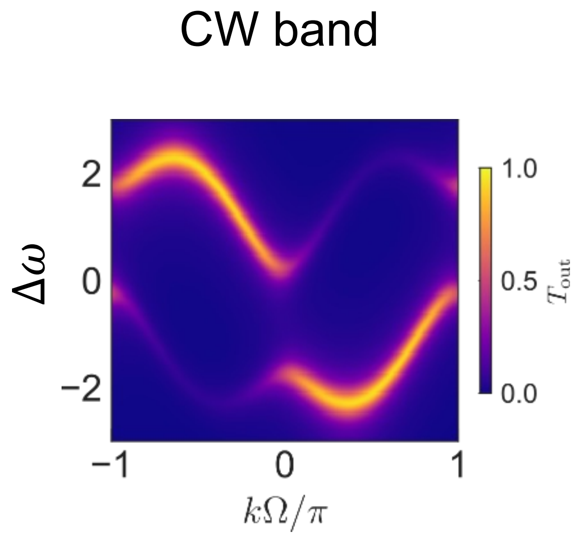
Output of the modulated cavity

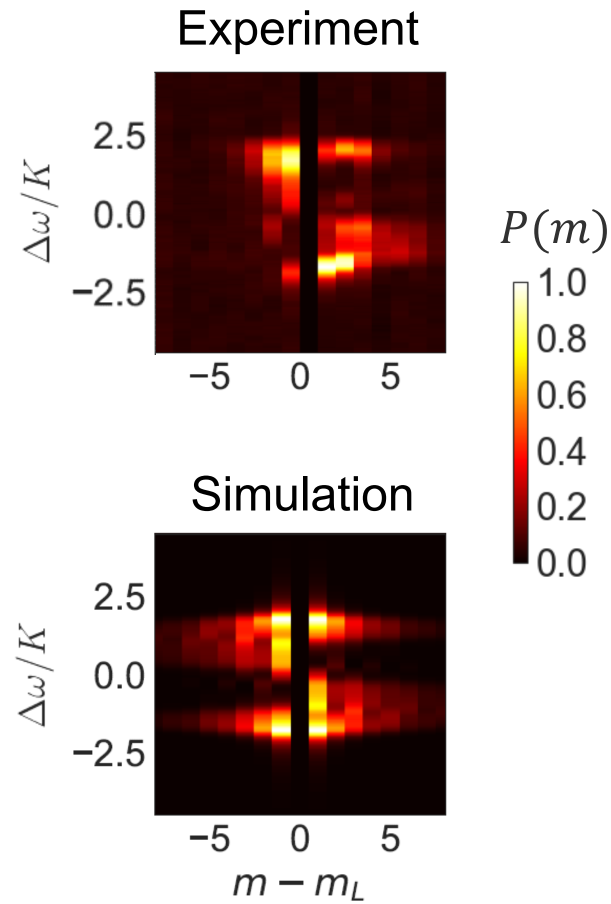
Small part of the input signal, frequency-shifted

Detected intensity

Chiral current measurement results







A. Dutt, Q. Lin, L. Yuan, M. Minkov, M. Xiao and S. Fan, Science 367, 59 (2020).

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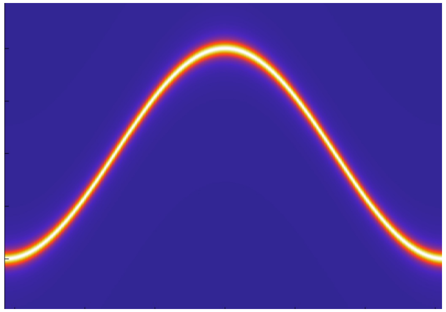
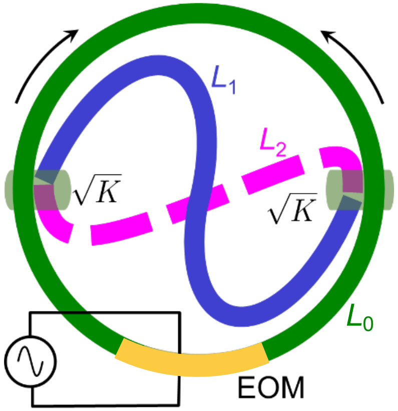
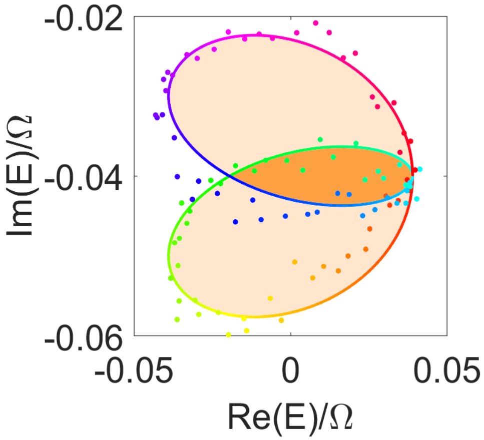


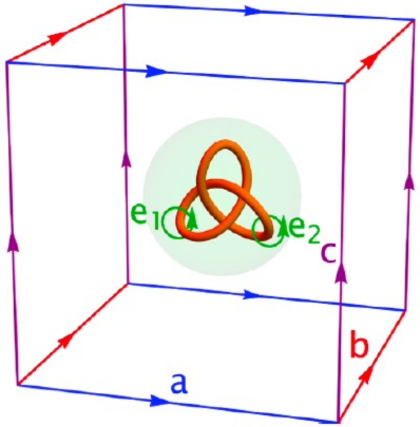
Hermitian Topology: experiments



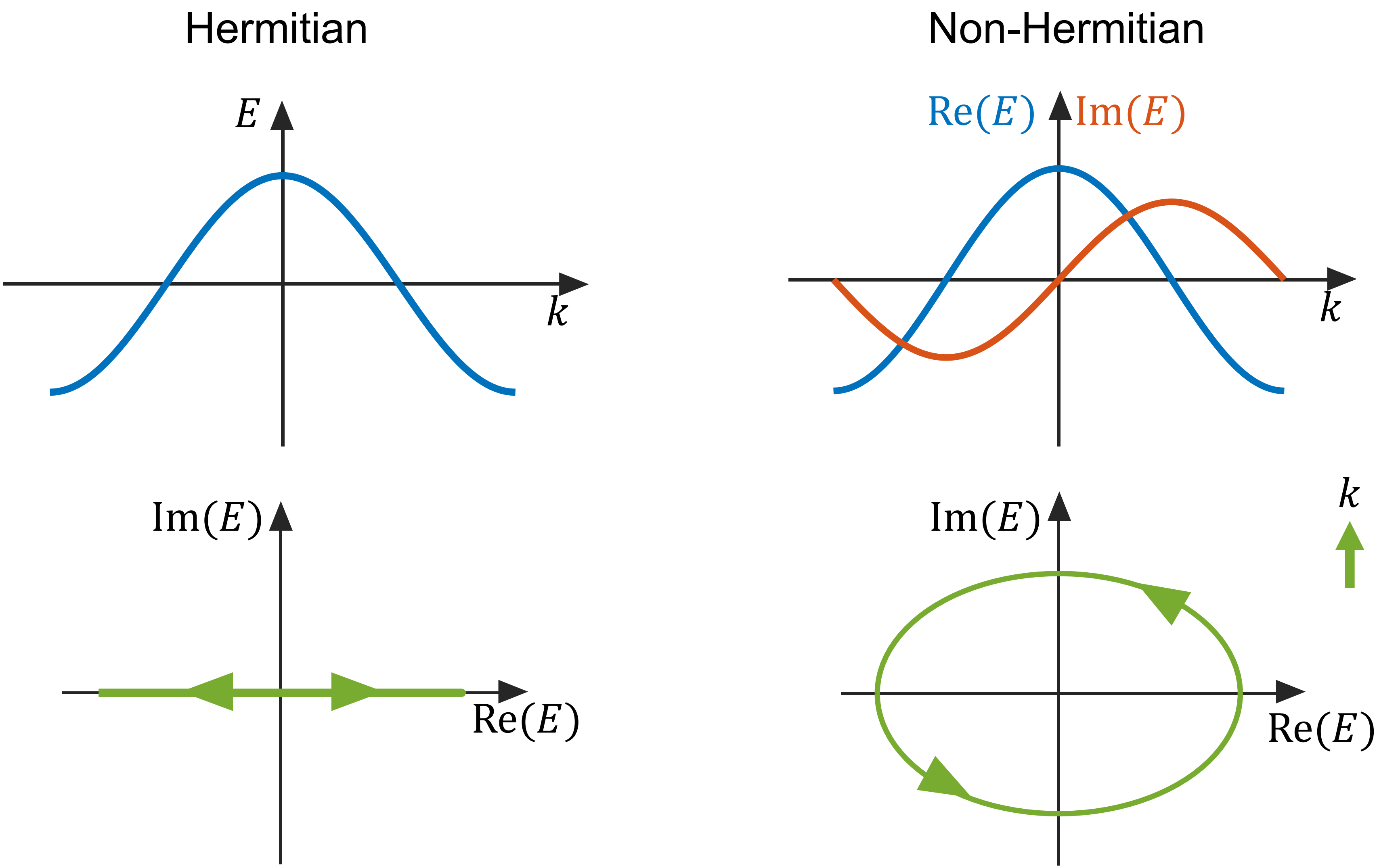
Non-Hermitian Topology: experiments



Non-Hermitian Topology: theory

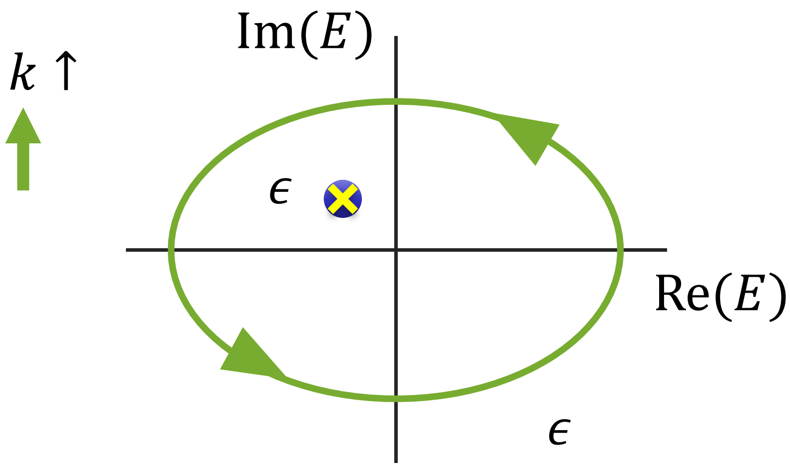


Band structure of Hermitian versus non-Hermitian systems in one dimension



Point-gap topology

Topology of a single non-Hermitian band with respect to a given reference frequency ϵ



Winding number

$$\nu := \int_0^{2\pi} \frac{\mathrm{d}k}{2\pi i} \frac{\mathrm{d}}{\mathrm{d}k} \ln[E(k) - \epsilon]$$

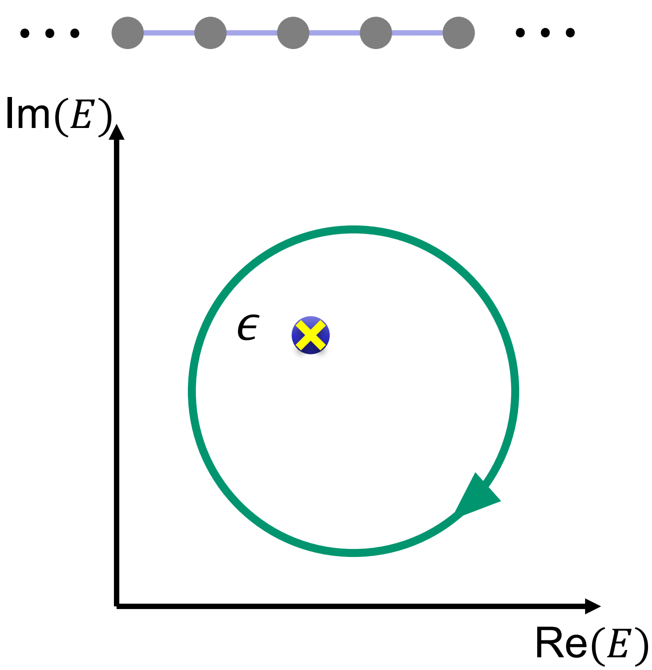
$\nu = 1$ if ϵ is in the loop

$\nu = 0$ if ϵ is outside the loop

K. Kawabata, K. Shiozaki, M. Ueda, and M. Sato, Phys. Rev. X **9**, 041015 (2019).
Z. Gong, et al., Phys. Rev. X 8, 031079 (2018).
N. Okuma, et al., Phys. Rev. Lett. 124, 086801 (2020).

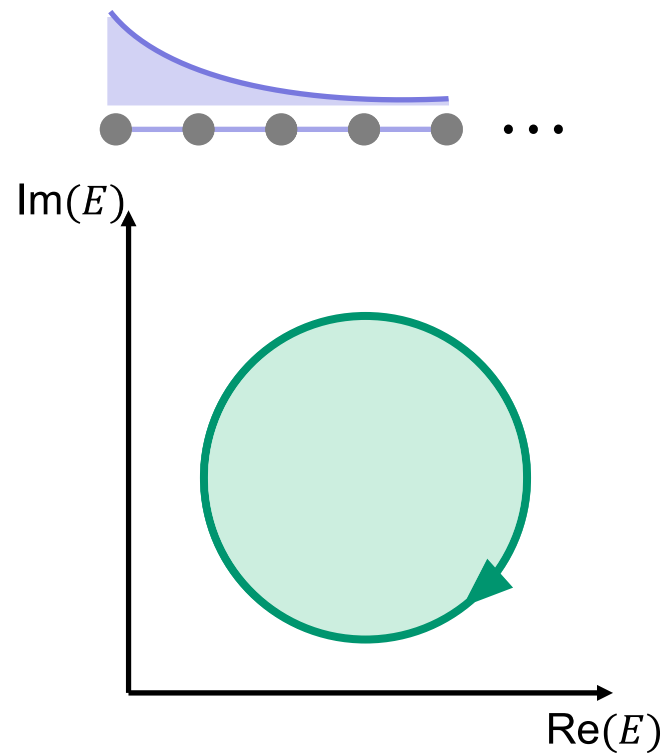
Implication of point-gap topology: non-Hermitian skin effect

Infinite (periodic) structure



$v = -1$ at ϵ

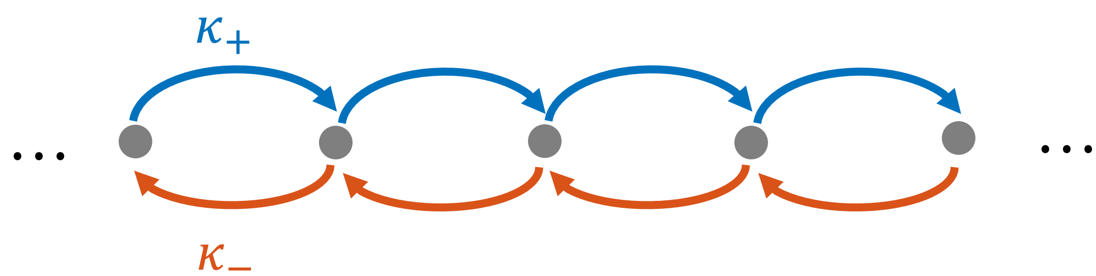
Semi-infinite structure



There are exactly $|v|$ edge states at every such ϵ

Theory: Z. Gong, et al., Phys. Rev. X 8, 031079 (2018); N. Okuma, et al., Phys. Rev. Lett. 124, 086801 (2020).
Experiment: S. Weidemann, et al., Science 368, 311 (2020); L. Xiao, et al., Nat. Phys. 16, 761 (2020);
T. Helbig, et al., Nat. Phys. 16, 747 (2020).

Hatano-Nelson model



$$\hat{H} = \kappa_+ \hat{a}^\dagger_{n+1}\hat{a}_n + \kappa_- \hat{a}^\dagger_n \hat{a}_{n+1}$$

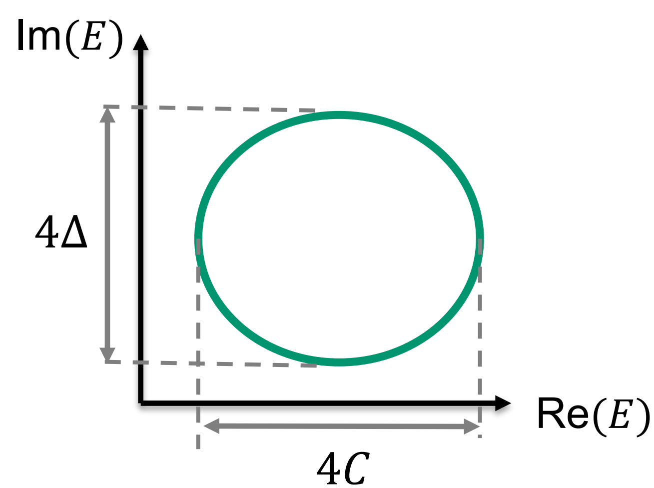
$\kappa_+ = C + \Delta$ $\kappa_- = C - \Delta$ $C, \Delta \in \mathbb{R}$

Direction-dependent coupling amplitude

Complex-valued dispersion

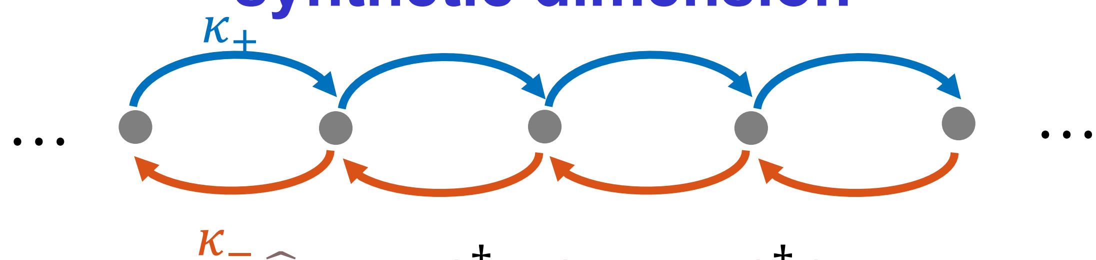
$$E(k) = 2C\cos k + 2\Delta i \sin k$$

Sign of $C\Delta$ determines the handedness of the loop



N. Hatano and D. R. Nelson, Phys. Rev. Lett. 77, 570 (1996).

Implementation of the Hatano-Nelson model in a synthetic dimension



$$\hat{H} = \kappa_+ \hat{a}_{n+1}^{\dagger}\hat{a}_n + \kappa_- \hat{a}_n^{\dagger}\hat{a}_{n+1}$$

$$\kappa_\pm = C \pm \Delta$$

Hermitian part

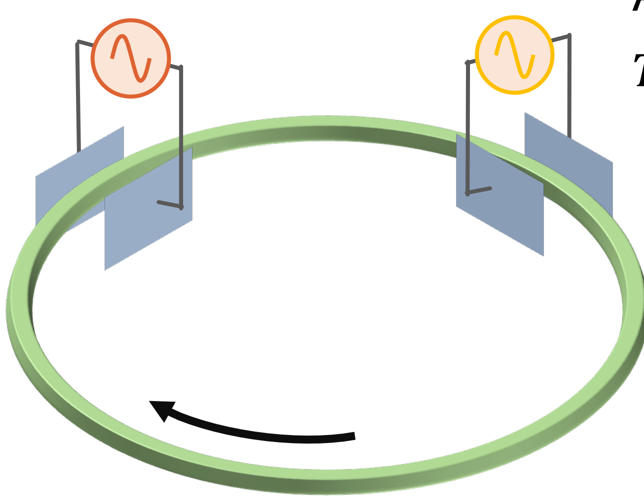
Phase modulation

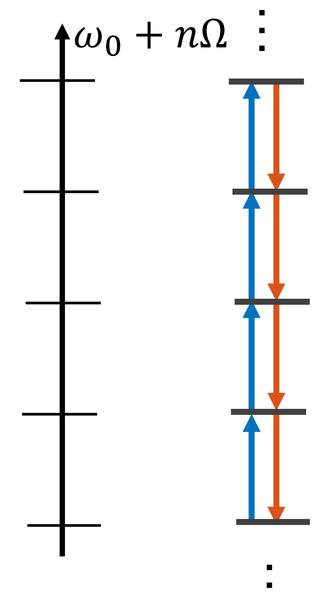
$$T_{Ph} = \exp(-i2C\cos\Omega t)$$

Anti-Hermitian part

Amplitude modulation

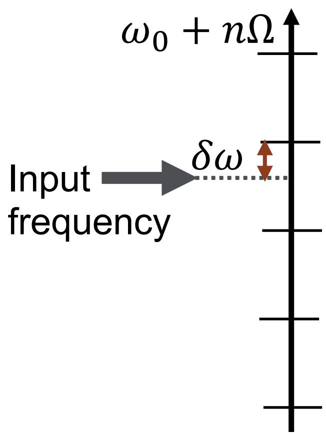
$$T_{Am} = 1 + 2\Delta\sin\Omega t$$

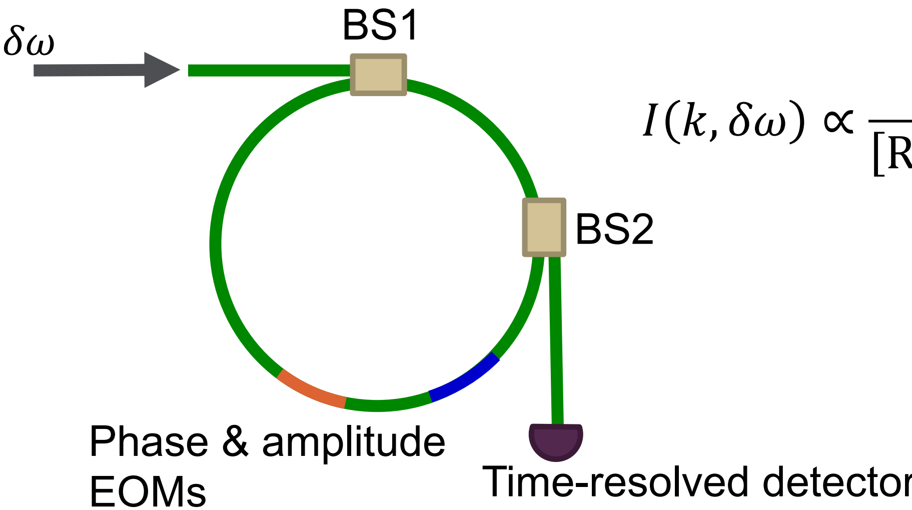




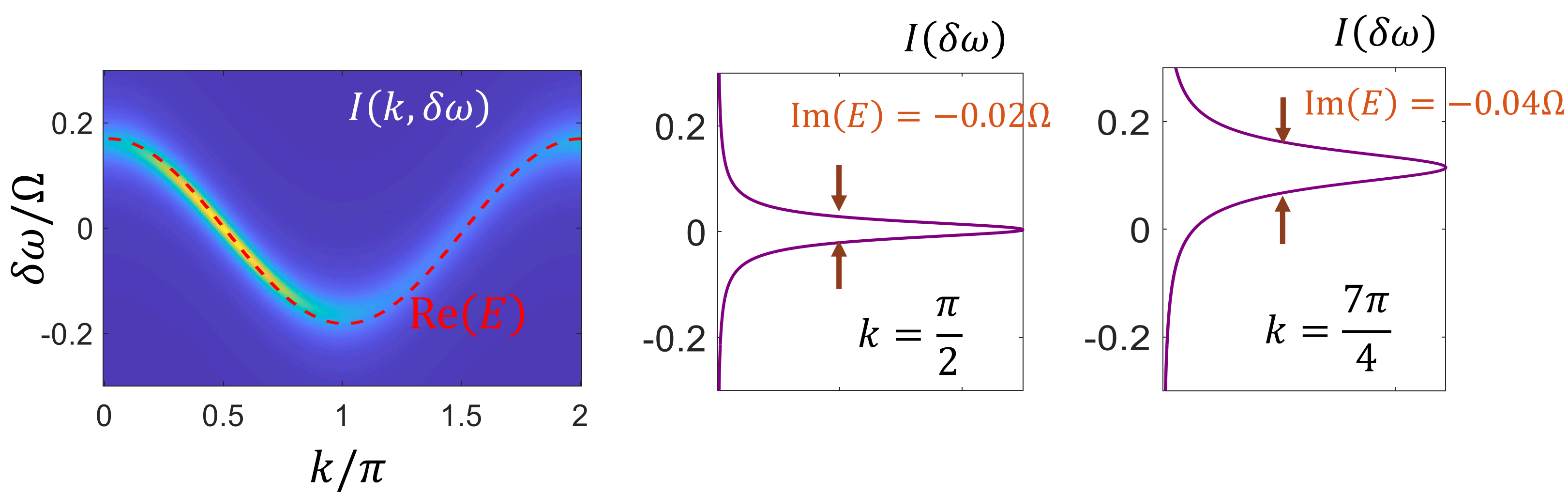
K. Wang*, A. Dutt*, K. Y. Yang, C. C. Wojcik, J. Vučković, and S. Fan, Science 371, 1240 (2021).

Complex band structure measurement in frequency synthetic space

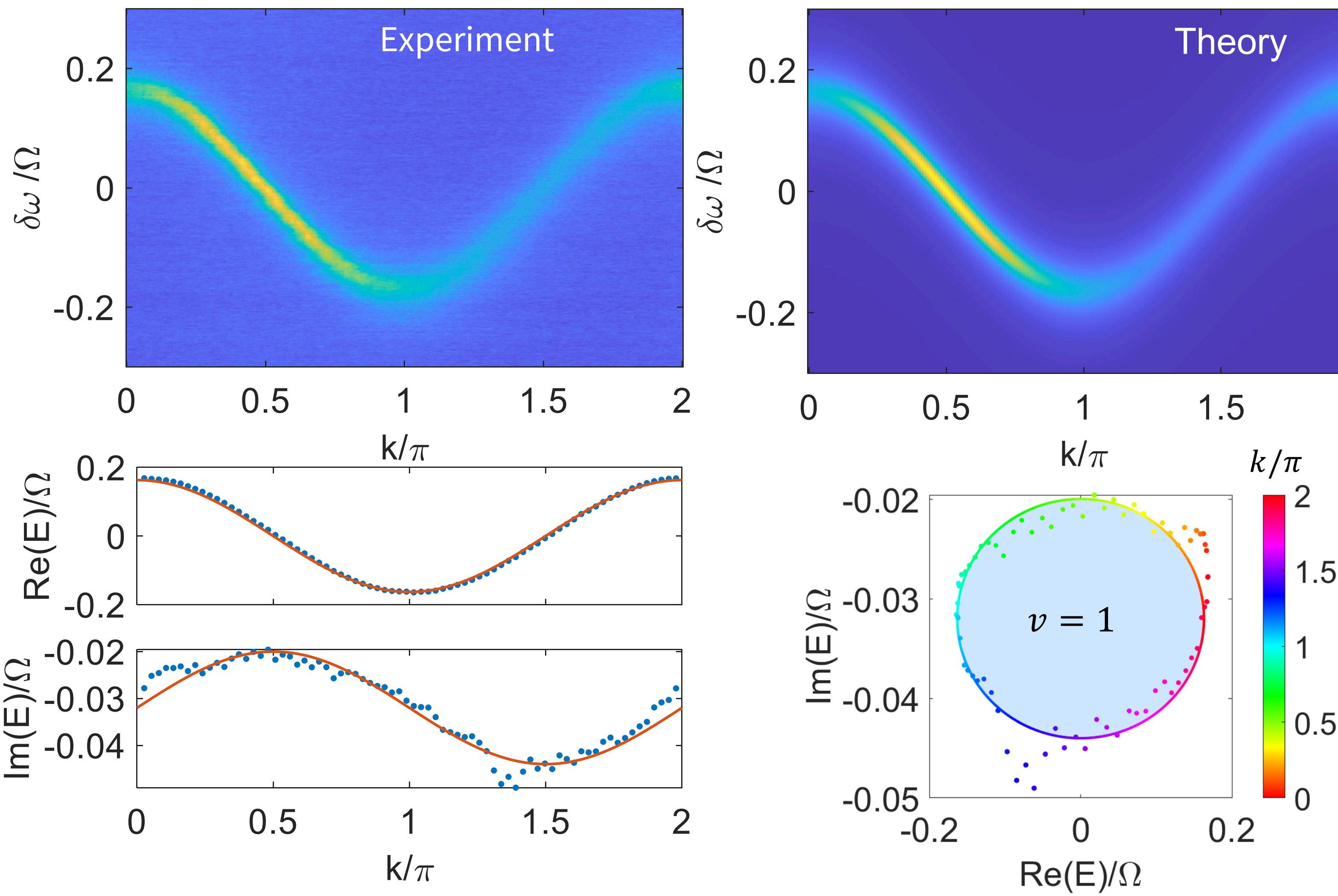




$$I(k, \delta\omega) \propto \frac{1}{[\mathrm{Re}(E) - \delta\omega]^2 + [\mathrm{Im}(E)]^2}$$

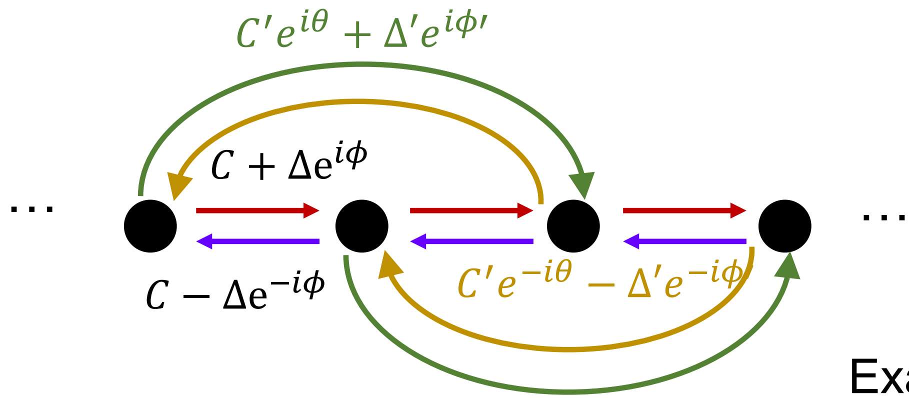


Demonstrating the Hatano-Nelson model



K. Wang*, A. Dutt*, K. Y. Yang, C. C. Wojcik, J. Vučković, and S. Fan, Science 371, 1240 (2021).

Implementation of long-range coupling



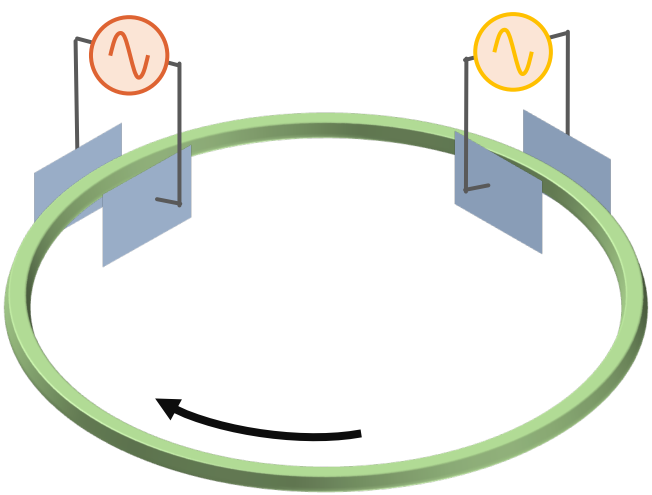
Example: $\{m\} = \{1,2\}$

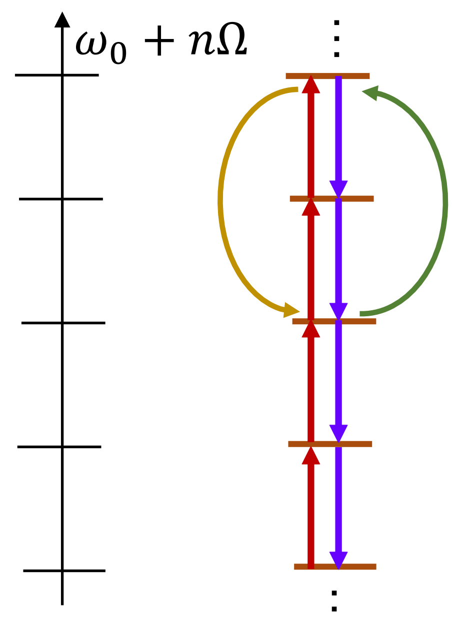
Phase modulation

$$T_{Ph} = \exp[-i2C\cos\Omega t - \boldsymbol{i2C'\cos(2\Omega t + \theta)}]$$

Amplitude modulation

$$T_{Am} = 1 + 2\Delta\sin(\Omega t + \phi) + \boldsymbol{2\Delta'\sin(2\Omega t + \phi')}$$

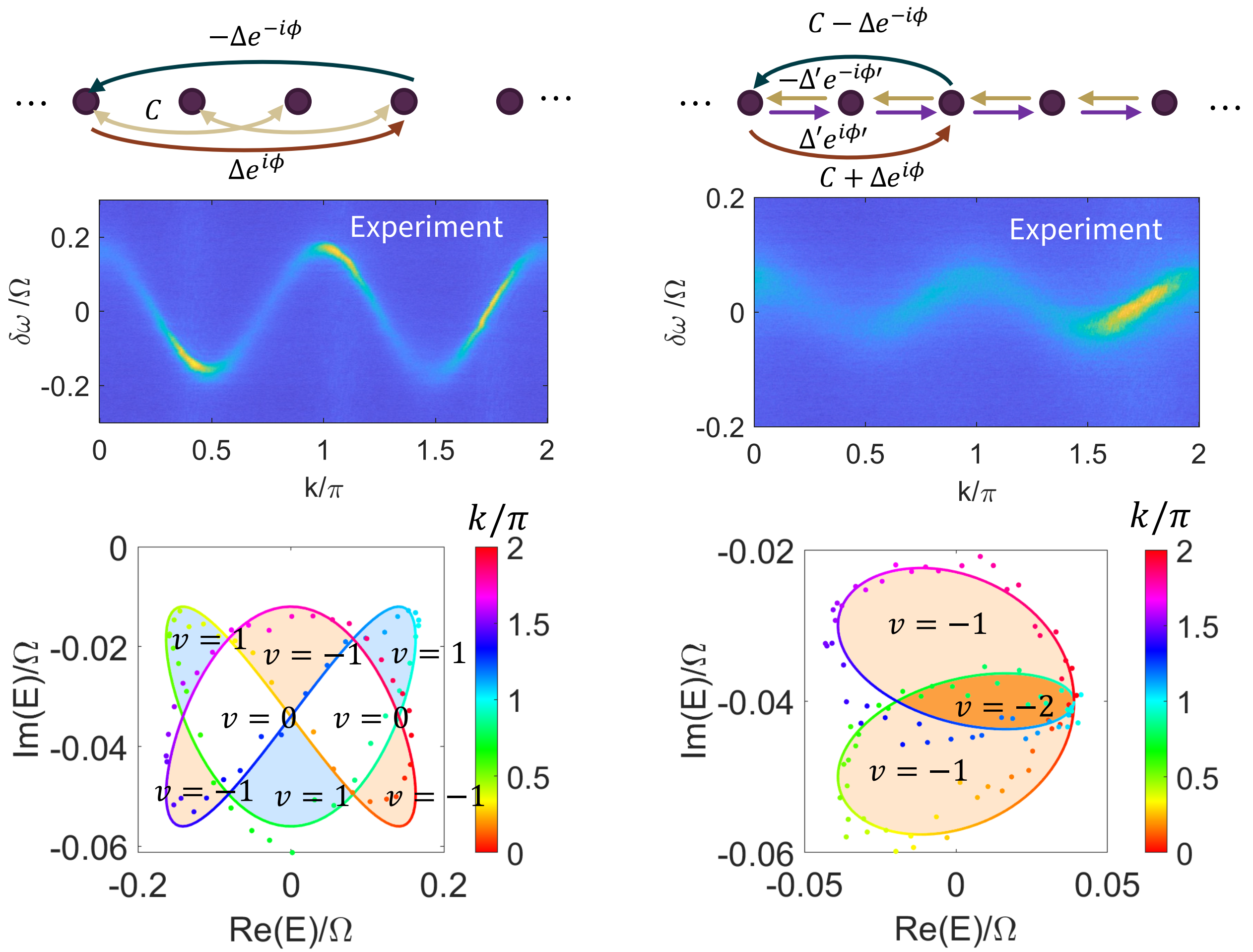




K. Wang*, A. Dutt*, K. Y. Yang, C. C. Wojcik, J. Vučković, and S. Fan, Science 371, 1240 (2021).

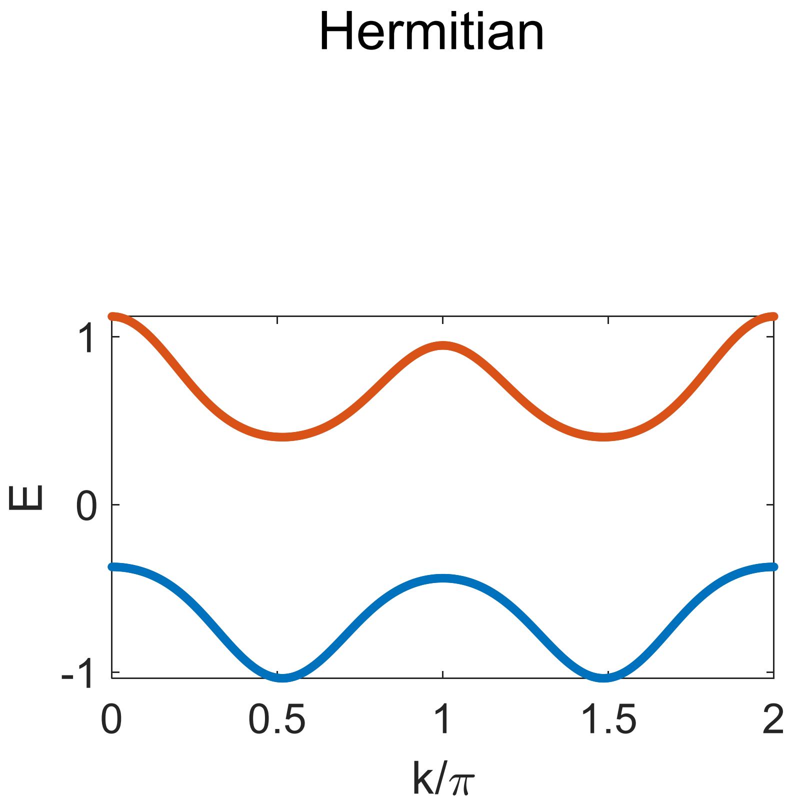
Experimental demonstration of complex winding of a single non-Hermitian band

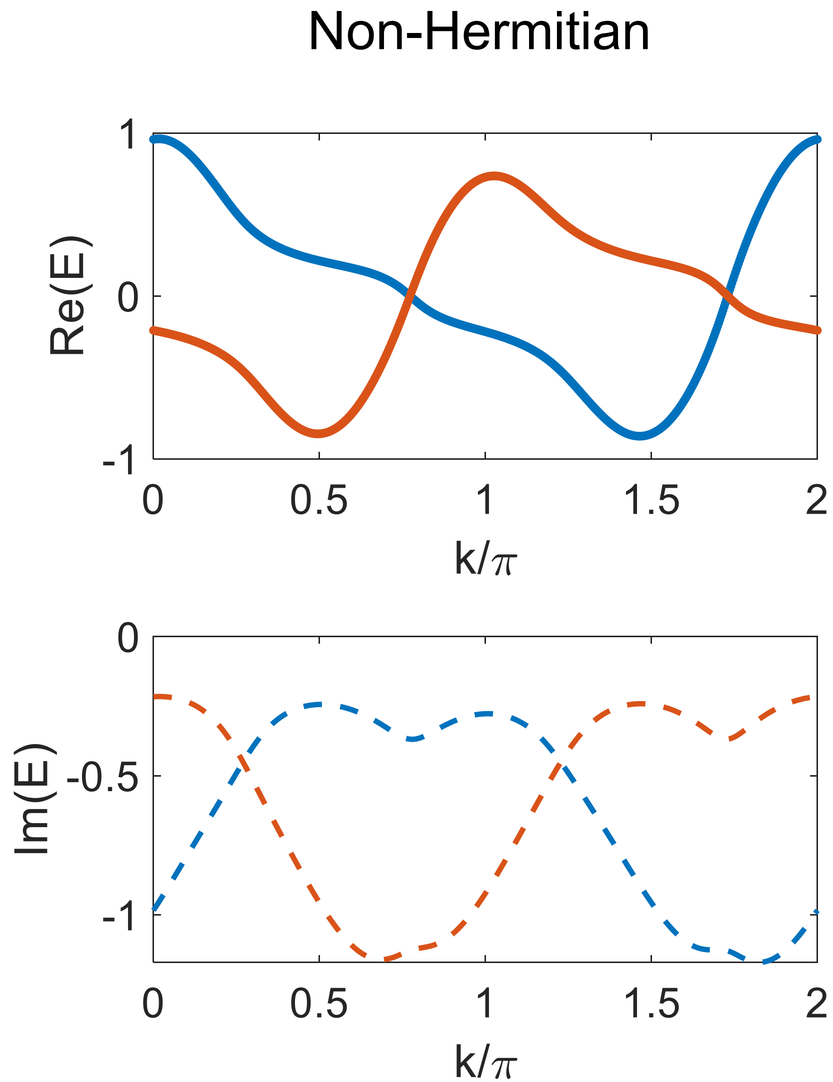
Longer-ranges of coupling can give rise to more nontrivial windings



K. Wang*, A. Dutt*, K. Y. Yang, C. C. Wojcik, J. Vučković, and S. Fan, Science 371, 1240 (2021).

Multiple energy bands of Hermitian versus non-Hermitian systems in one dimension

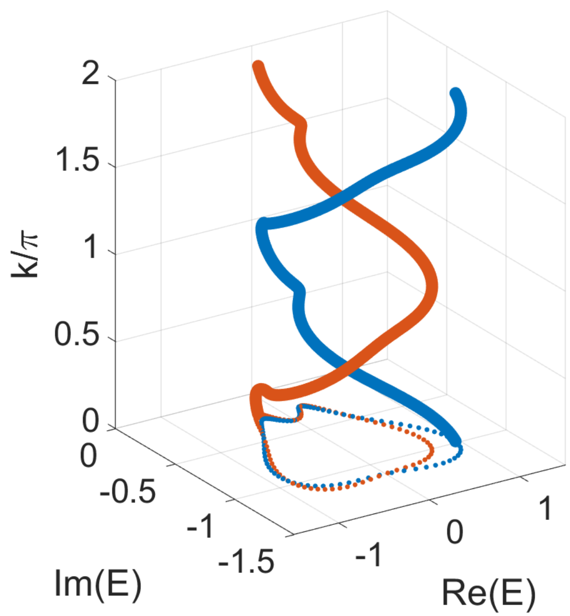




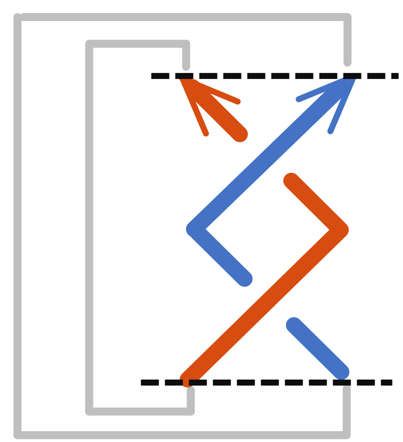
Braiding of non-Hermitian energy bands

$$E(k = 0) = E(k = 2\pi)$$

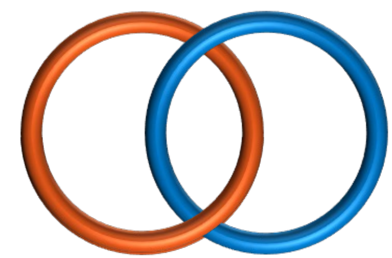
Non-Hermitian band structure
in (Re(E), Im(E), *k*) – space



Closure of the braid forms a knot/ link

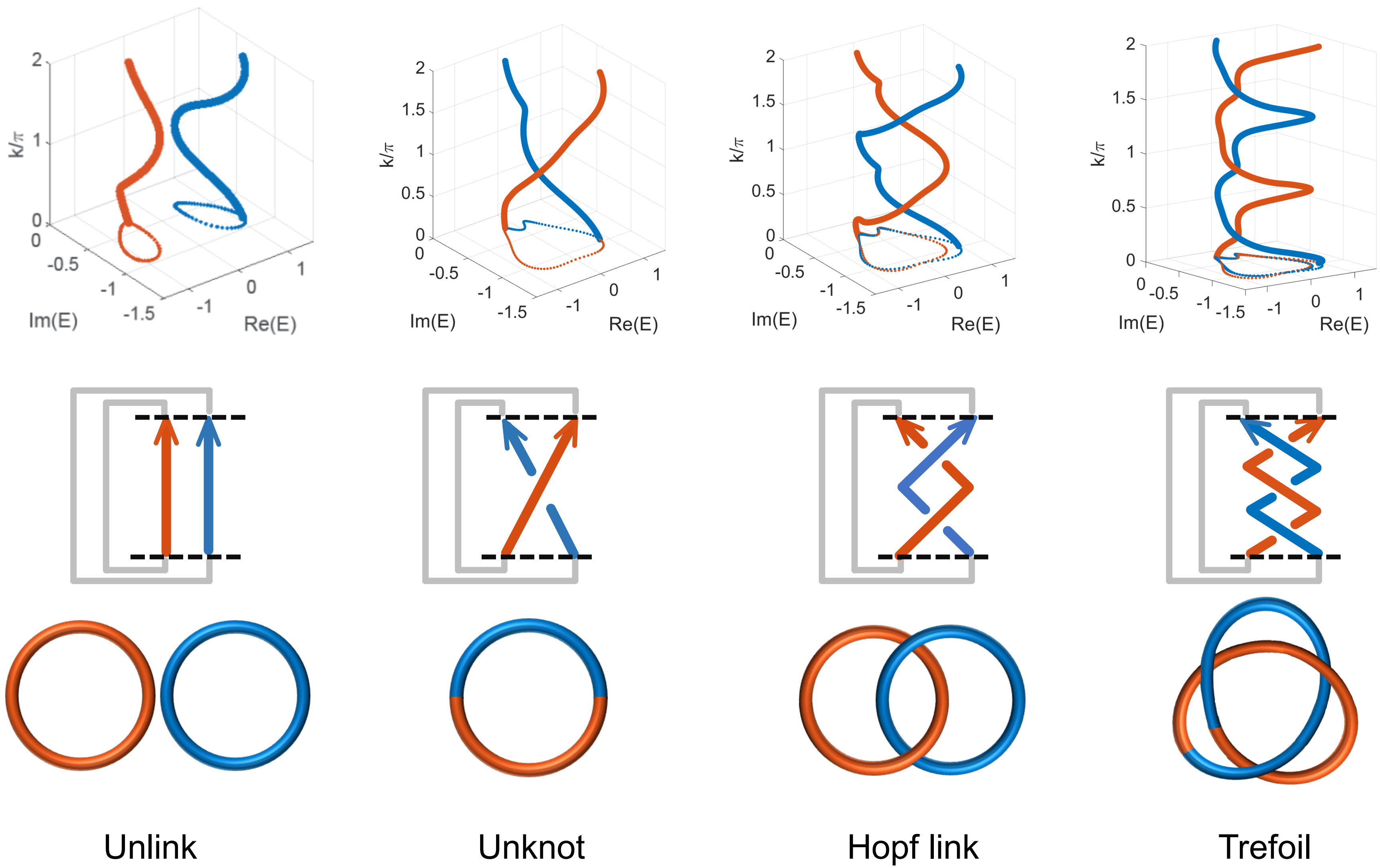


Braid diagram



Knot diagram

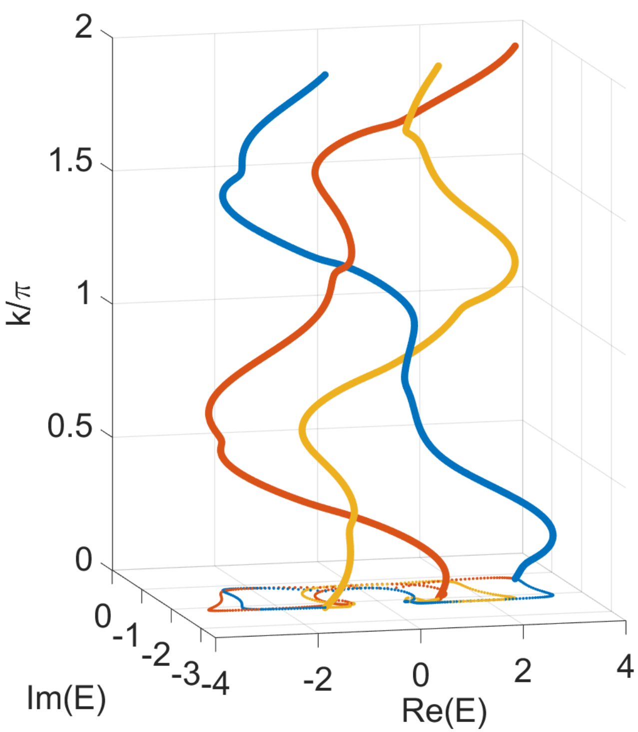
Examples of two-band braids and the corresponding knots/links

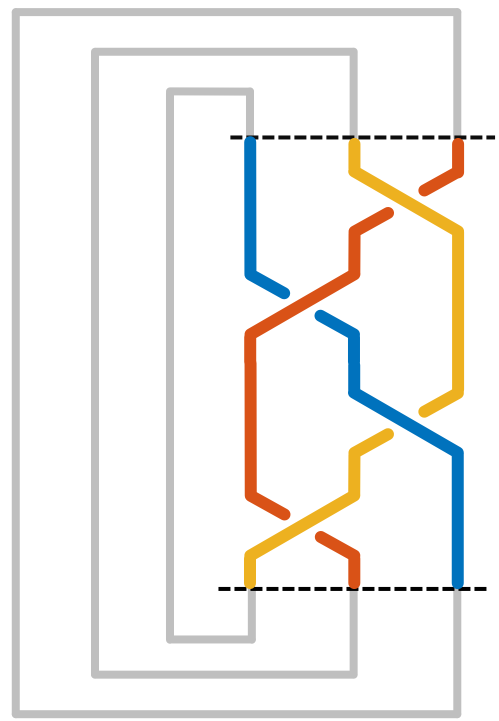


N separable non-Hermitian bands in one dimension can be classified by the braid group $\mathbb{B}_N$

$$\pi_1[UConf_N(\mathbb{C})] = \mathbb{B}_N$$

An example with three energy bands ($N = 3$)





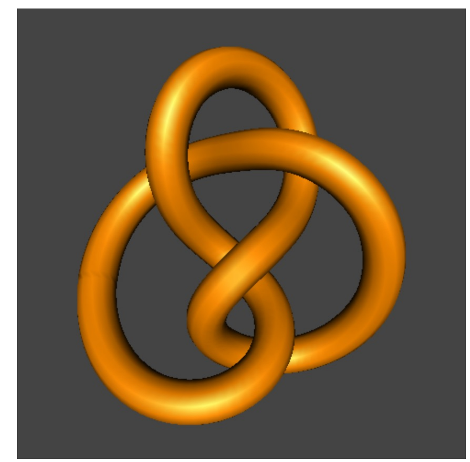


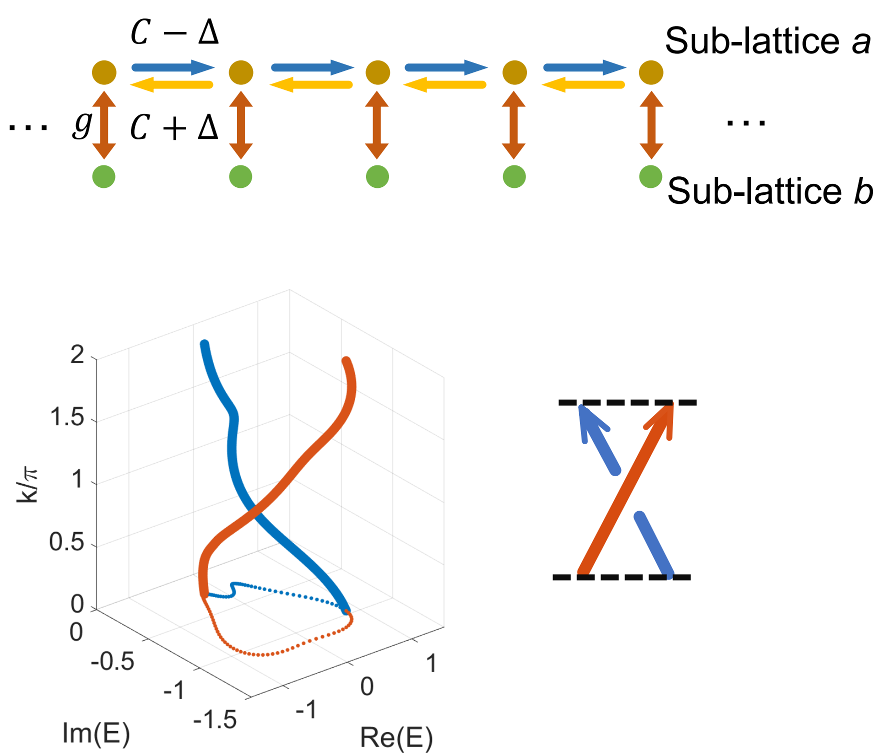
Figure-eight knot

C. C. Wojcik, X.-Q. Sun, T. Bzdušek, and S. Fan, Phys. Rev. B 101, 205417 (2020).

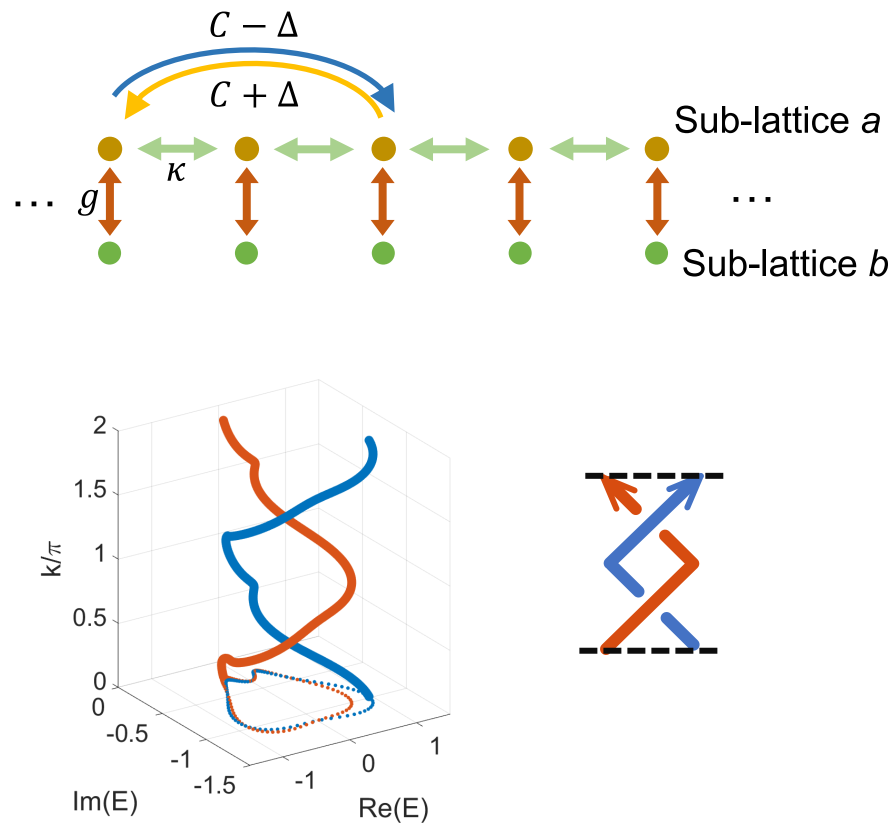
A model Hamiltonian for realizing two-band braids

$$\hat{H}^{(m)} = \sum_n g\, \hat{a}_n^\dagger \hat{b}_n + g\, \hat{b}_n^\dagger \hat{a}_n - i\gamma\, \hat{a}_n^\dagger \hat{a}_n + \kappa\, \hat{a}_{n+1}^\dagger \hat{a}_n + \kappa\, \hat{a}_n^\dagger \hat{a}_{n+1} + (C-\Delta)\, \hat{a}_{n+m}^\dagger \hat{a}_n + (C+\Delta)\, \hat{a}_n^\dagger \hat{a}_{n+m}$$

$m = 1$



$m = 2$



K. Wang, A. Dutt, C. C. Wojcik, and S. Fan, Nature 598, 59 (2021)

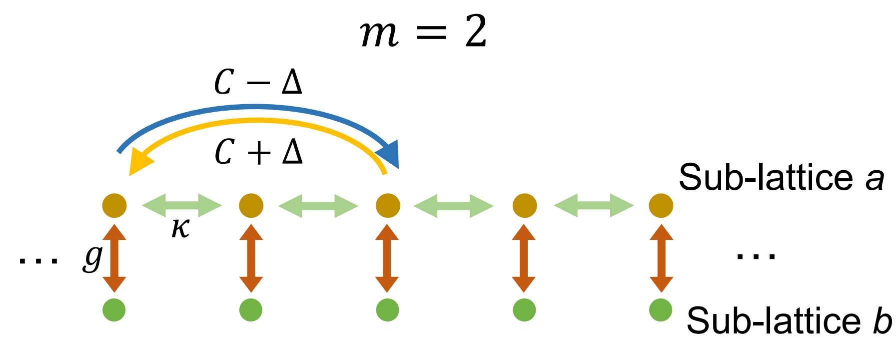
Experimental realization of the model Hamiltonian

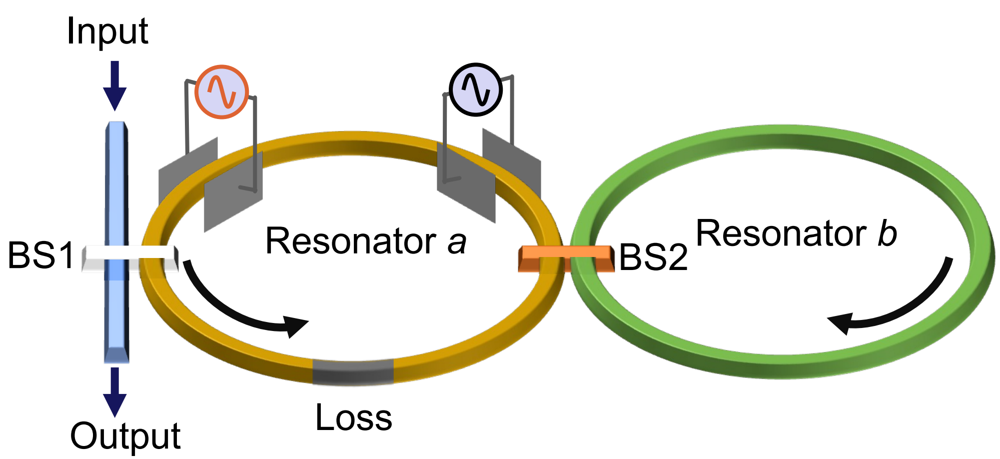
Phase modulation

$$T_{Ph} = \exp[-i(2\kappa \cos \Omega t + 2C \cos m\Omega t)]$$

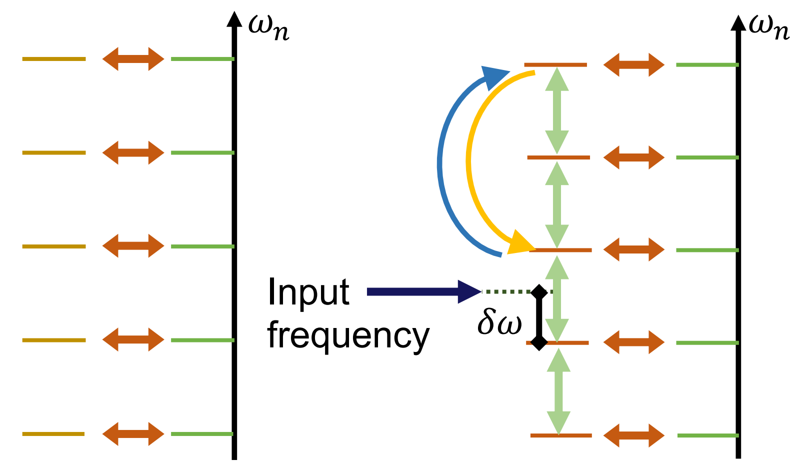
Amplitude modulation

$$T_{Am} = 1 + 2\Delta \sin m\Omega t$$

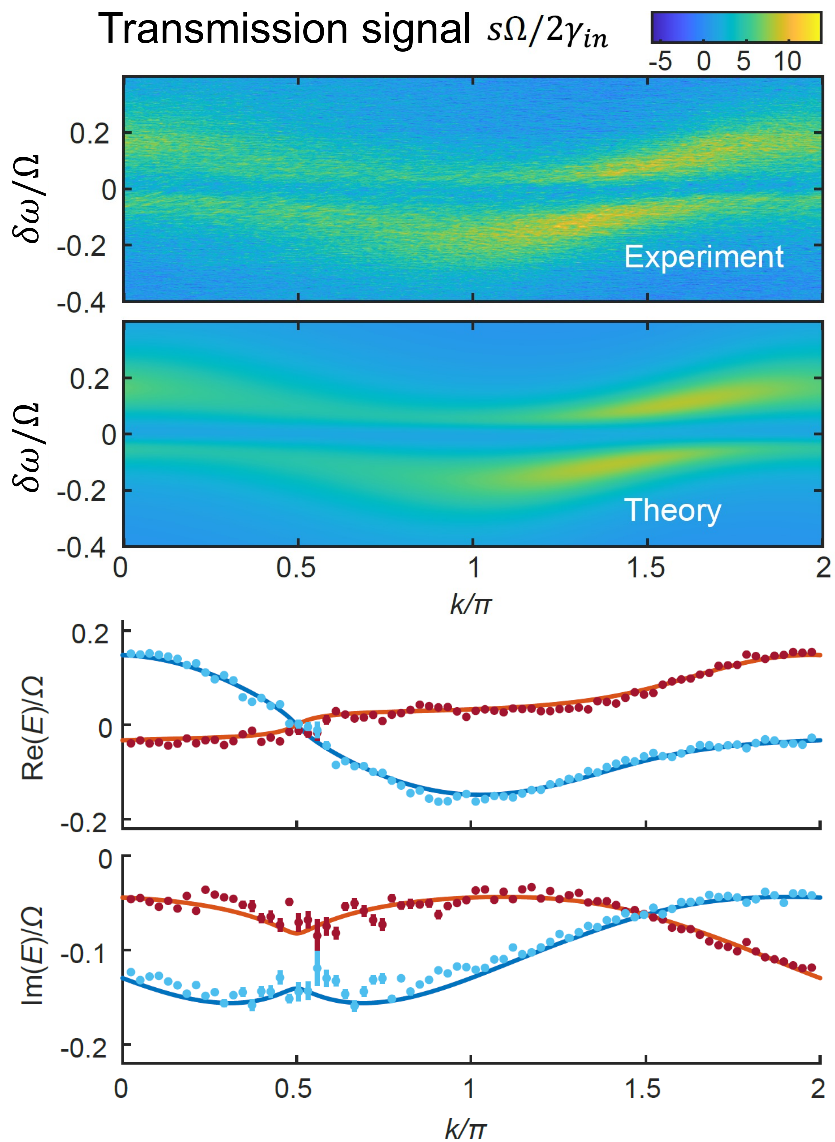


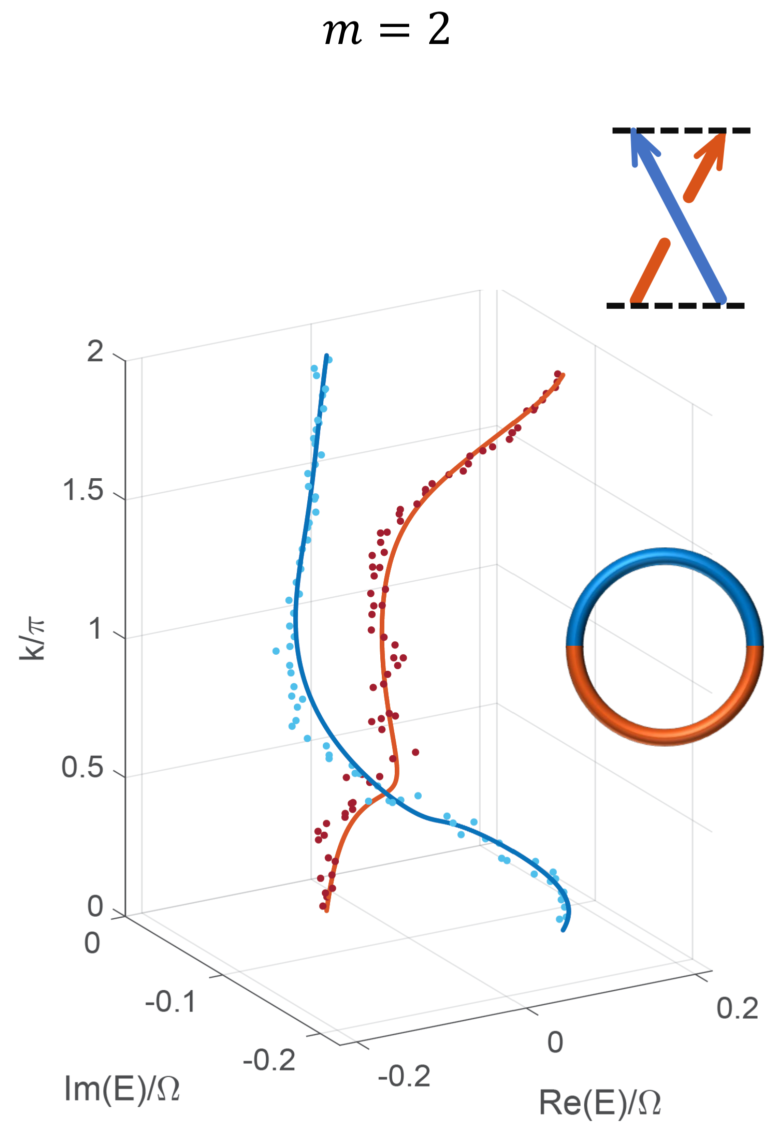


Frequency modes



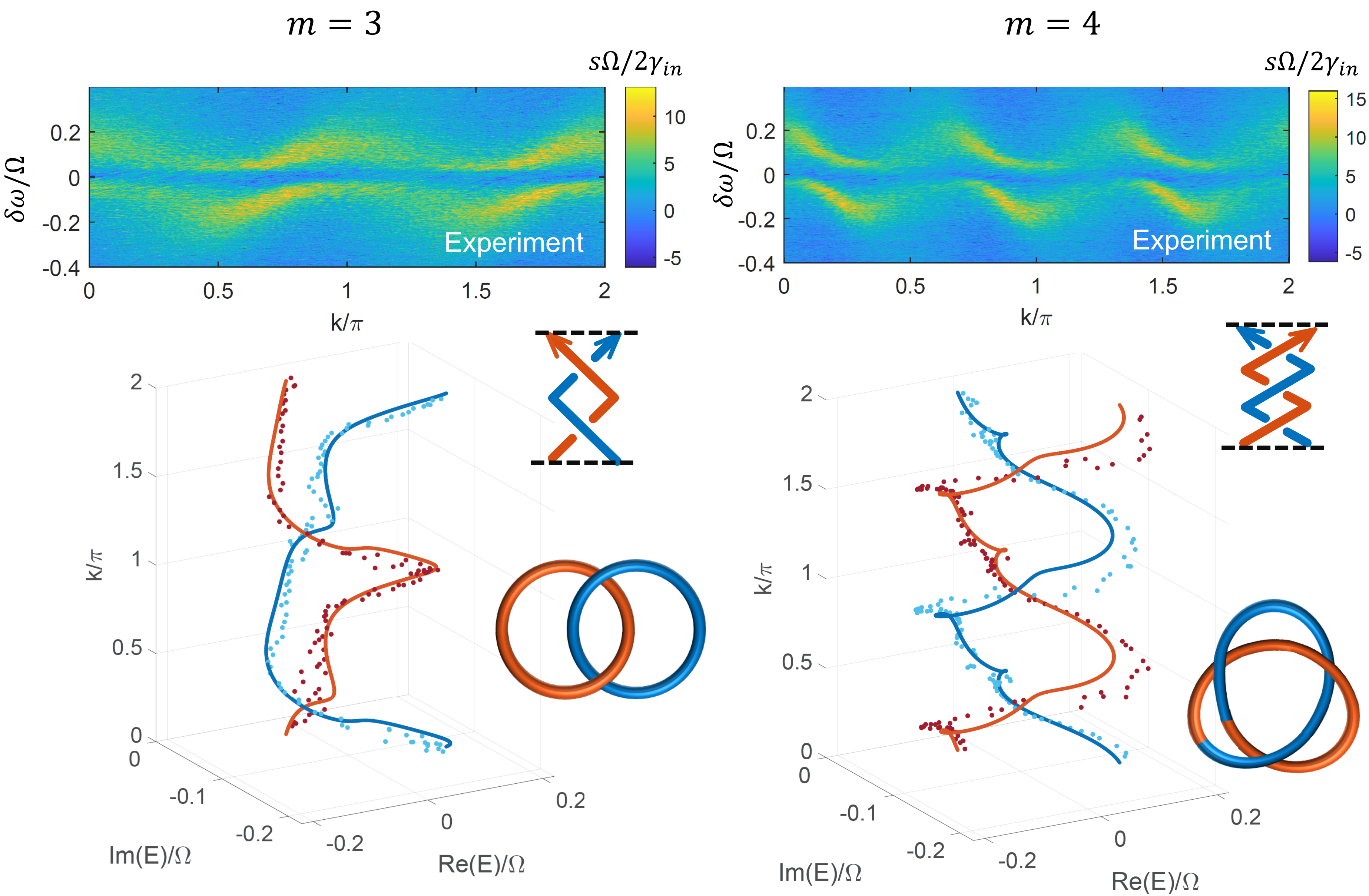
Experimental realization of a two-band complex-energy braid





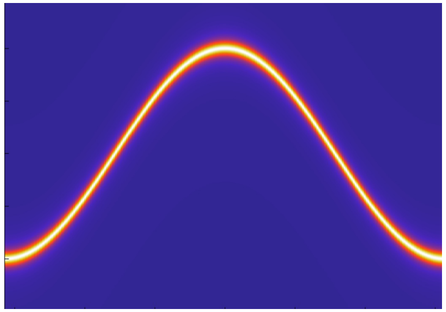
K. Wang, A. Dutt, C. C. Wojcik, and S. Fan, Nature 598, 59 (2021)

Demonstration of higher-order braids

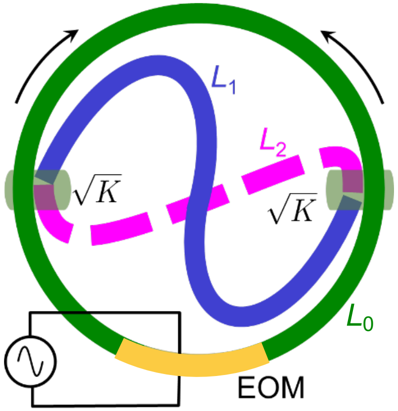


Outline

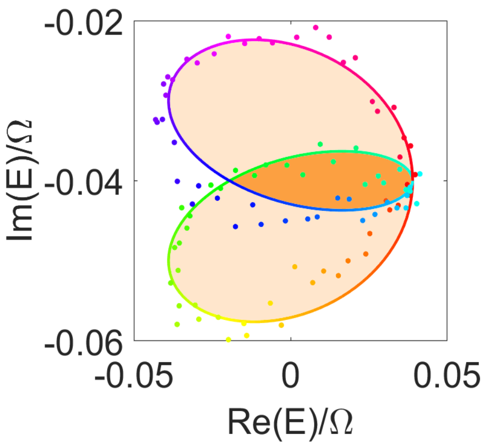
Background



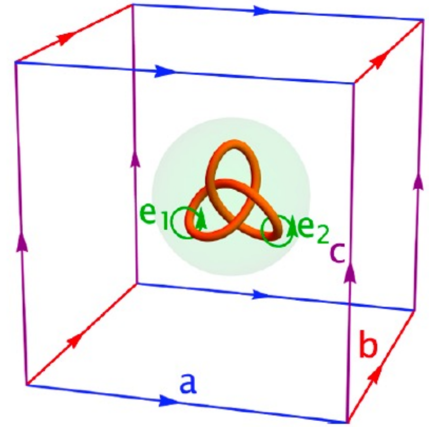
Hermitian Topology: experiments



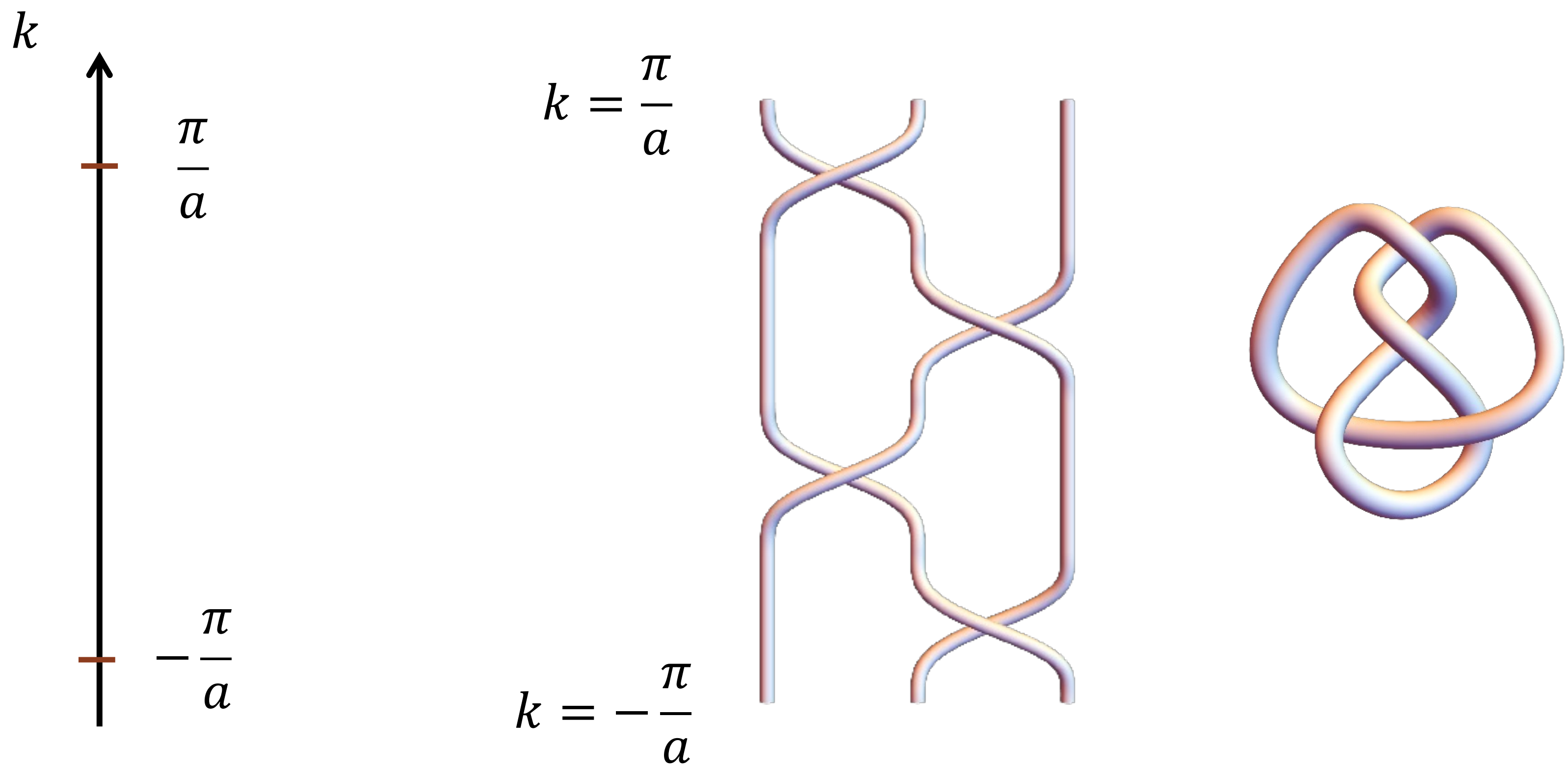
Non-Hermitian Topology: experiments



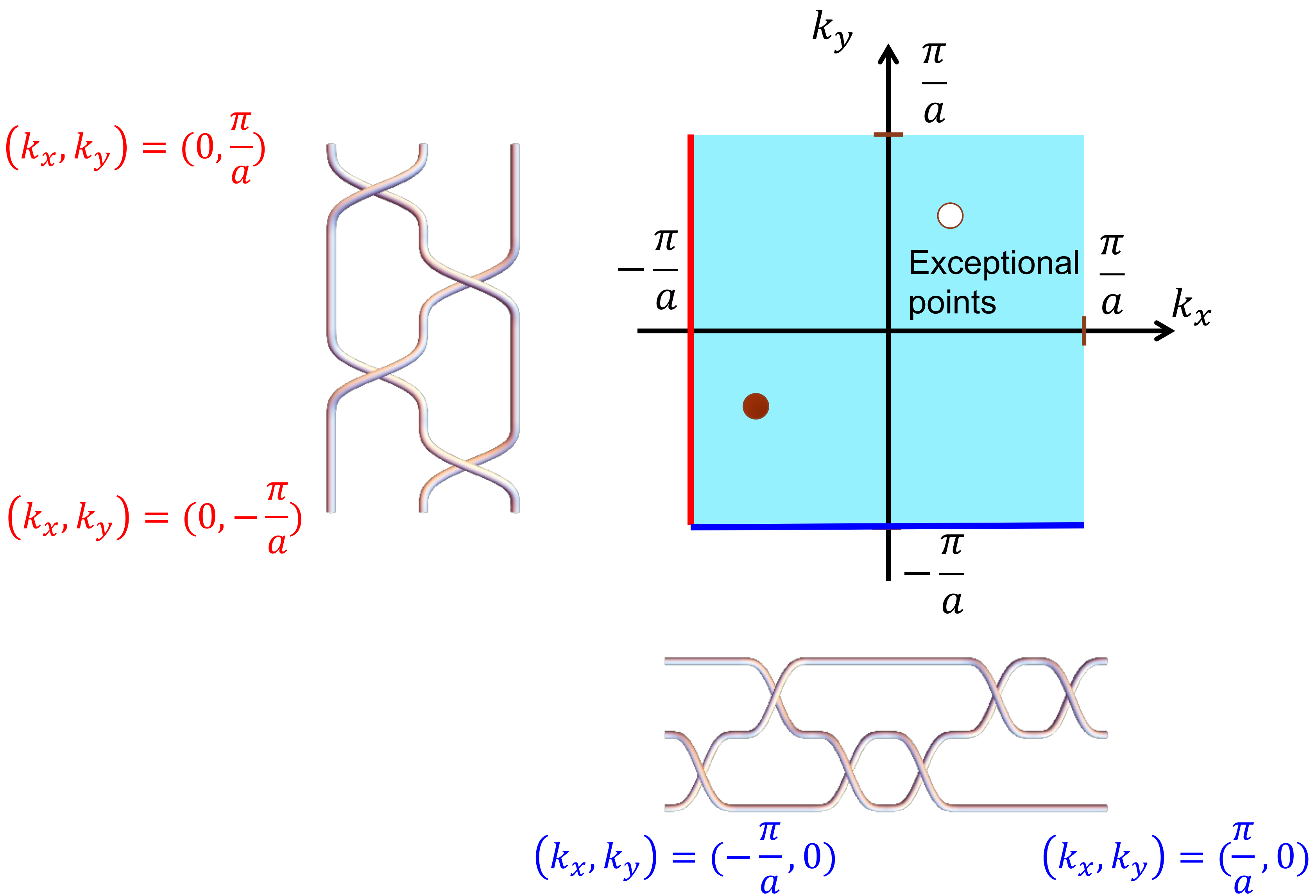
Non-Hermitian Topology: theory



Eigenvalue topology in one dimension



Eigenvalue topology in two dimensions



Classification of N-band eigenvalue topology

Class of homotopy

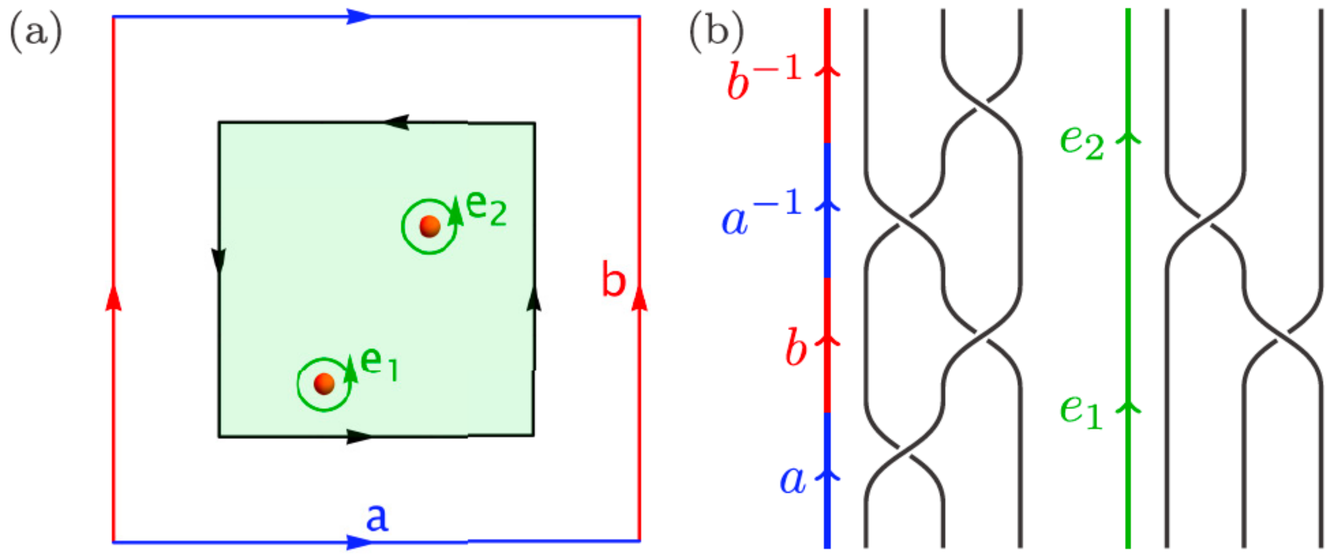
Group homomorphism

$$[T^d - \Delta, UConf_N(C)] = Hom(\pi_1(T^d - \Delta), B_N)$$

- d-dimensional Brillouin zone
- Exceptional contour
- N-band
- Fundamental group of $T^d - \Delta$
- N-strand braid group

C. Wojcik, K. Wang, A. Dutt, J. Zhong and S. Fan, Physical Review B 106, L161401 (2022).

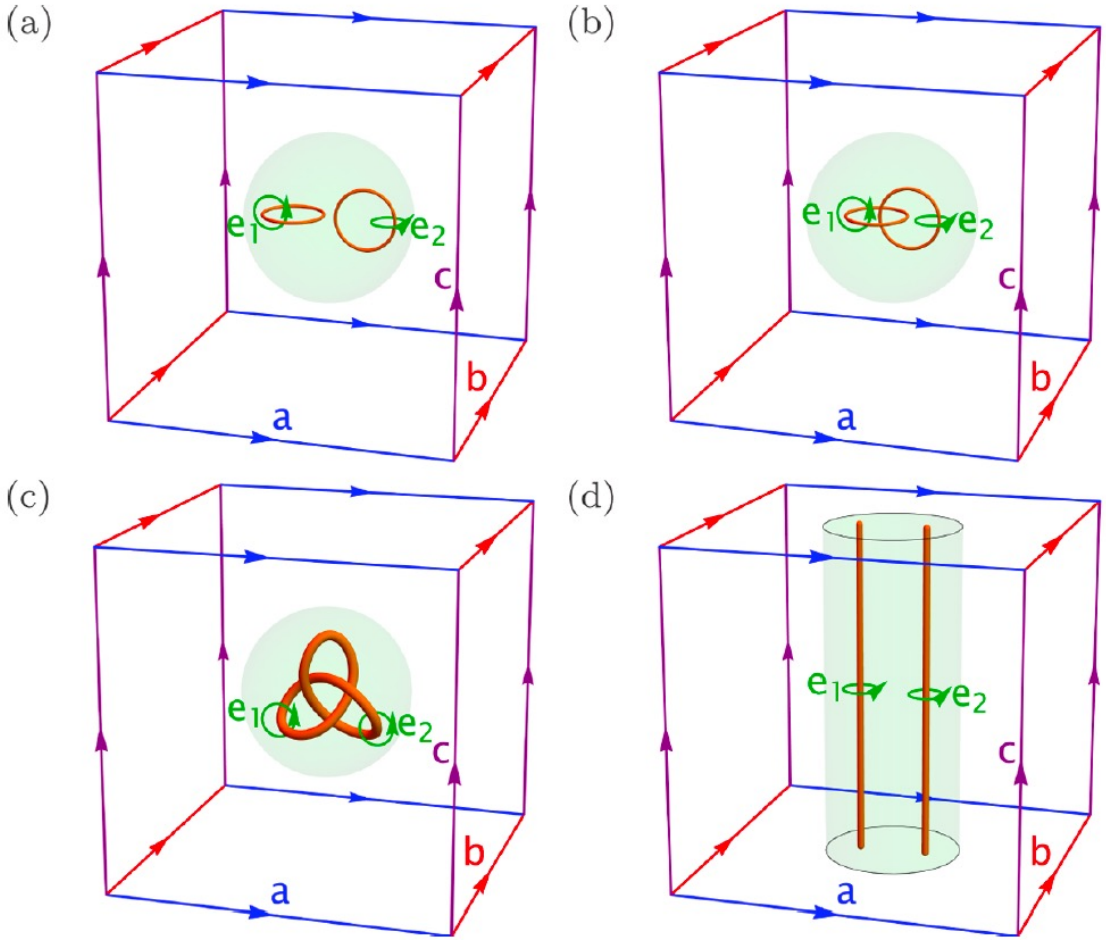
2D example



$$Hom(\pi_1(T^2 - \Delta), B_N) = \{a, b, e_1, e_2, \ldots, e_k \in B_N : [a, b] = e_1 e_2 \cdots e_k\}$$

C. Wojcik, K. Wang, A. Dutt, J. Zhong and S. Fan, Physical Review B 106, L161401 (2022).

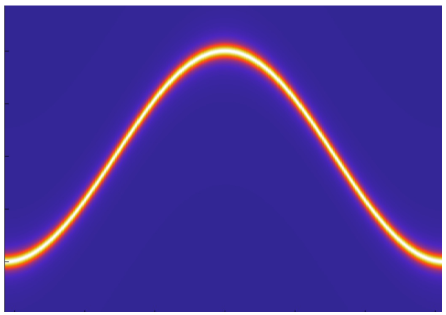
3D examples



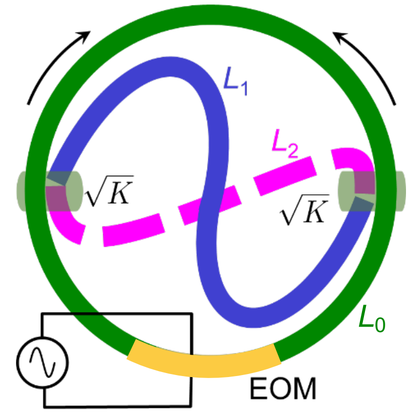
C. Wojcik, K. Wang, A. Dutt, J. Zhong and S. Fan, Physical Review B 106, L161401 (2022).

Summary

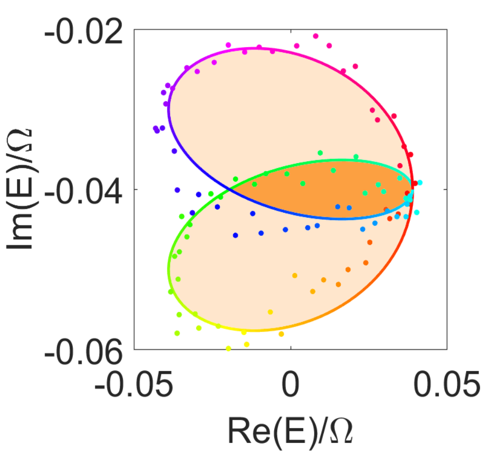
Background



Hermitian Topology: experiments



Non-Hermitian Topology: experiments



Non-Hermitian Topology: theory