



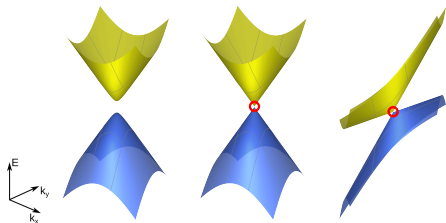
# Understanding chiral crossings with local and global symmetry constraints

Moritz M. Hirschmann

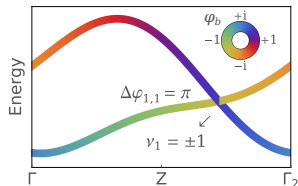
Max Planck Institute for Solid State Research, Germany

November 10, 2022

# Topological band crossings

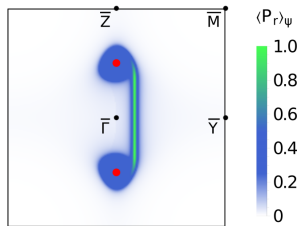


Two-fold screw rotation  $\tilde{C}_2^z$



Implications of band crossings

- Top. invariants  
Chern number, chirality
- Surface states
- Effects on transport properties

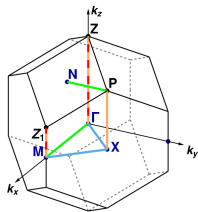


# Symmetries relate Weyl points

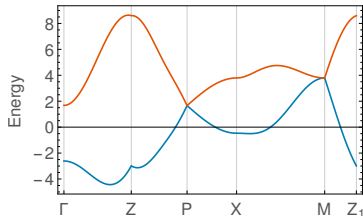
Symmetry  $D$  on chirality  $\nu_{\mathbf{k}}$

- Rotations and time reversal:  $\nu_{\mathbf{k}} = \nu_{D\mathbf{k}}$
- Mirror and Inversion:  $\nu_{\mathbf{k}} = -\nu_{D\mathbf{k}}$

Yet, time reversal  $\nRightarrow$  four Weyl points.

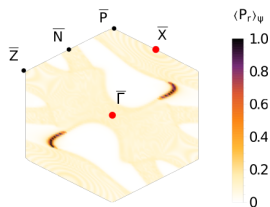


three Weyl points



SG 80 ( $I4_1$ ) spinless

two Weyl points



SG 119 ( $I\bar{4}m2$ ) spinful

# Outline

## 1. Local constraint

Relation between rotation eigenvalues and the chirality

## 2. Global constraint

Enforced band exchange by (non)symmorphic symmetry

## 3. Applications of global and global constraints

Double Weyl points in TaAs

Quasi-symmetry-protected quadruple Weyl points

Fourfold crossing with Chern number  $\nu = 5$

Topological nodal planes

Chiral nodal lines



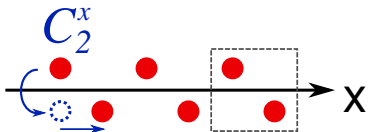




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# Global constraint I



$$(C_n^z(x, y, \frac{m}{n}))^n = \pm T(0, 0, m) = \pm \exp(imk_z)$$

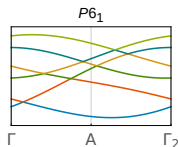
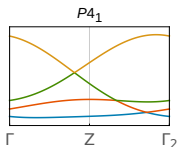
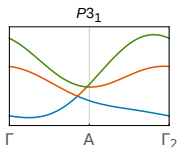
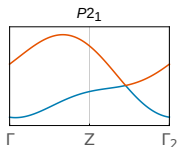
$$\Rightarrow \lambda_{C_n} = (\pm 1)^{1/n} \exp\left(i \frac{2\pi p + mk_z}{n}\right)$$

$$\equiv \exp(i\phi(k_z))$$

with band crossings at  $\{k_c\}$

$$\varphi_b(k_z) = \phi(k_z) + \sum_{k_c \leq k_z} \Delta\varphi_{b,c}$$

BZ periodicity  $\varphi_b(-\pi) = \varphi_b(\pi) \Rightarrow \sum_{c_b} \Delta\varphi_{b,c_b} + 2\pi \frac{m}{n} = 0 \pmod{2\pi}$



# Global constraint II

If all  $c_b$  are two-band crossings:

$$\begin{aligned} b = 1 & \quad \sum_{c_1} \Delta\varphi_{1,c_1} = -2\pi \frac{m}{n} \pmod{2\pi} \\ b = 2 & \quad \sum_{c_2} \Delta\varphi_{2,c_2} - \sum_{c_1} \Delta\varphi_{1,c_1} = -2\pi \frac{m}{n} \pmod{2\pi} \\ & \quad \Leftrightarrow \sum_{c_2} \Delta\varphi_{2,c_2} = -2 \cdot 2\pi \frac{m}{n} \pmod{2\pi} \\ & \quad \vdots \\ \text{any } b & \quad \sum_{c_b} \Delta\varphi_{b,c_b} = -b \cdot 2\pi \frac{m}{n} \pmod{2\pi}. \end{aligned}$$

# Outline

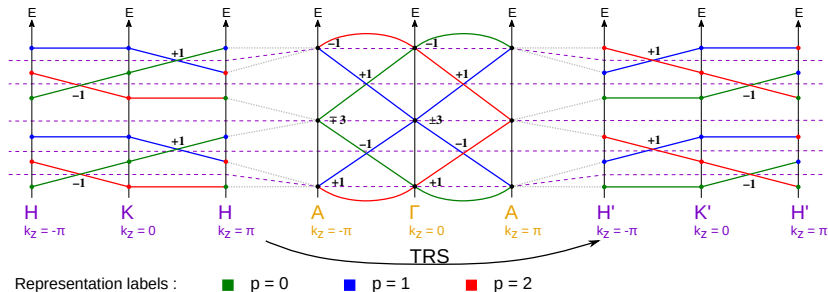
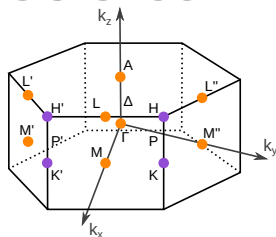
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# Application to band structures I

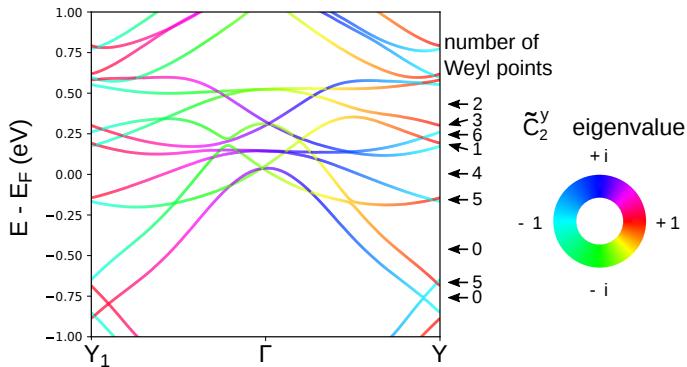
Chern number  $\nu$  of a band crossing

$$\nu = \Delta\varphi \frac{n}{2\pi} \pmod n$$

Eigenvalues and Weyl points for SG 144 ( $P3_1$ )  
(simplest arrangement)



# Application to band structures II



For  $C_2^y(0, \frac{1}{2}, \frac{1}{2})$  the global constraint is:

$$\sum_{c_b} \Delta\varphi_{b,c_b} = b \cdot \pi \pmod{2\pi}$$

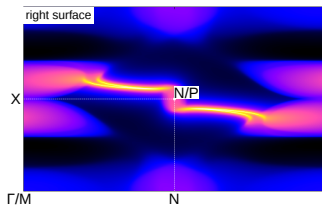
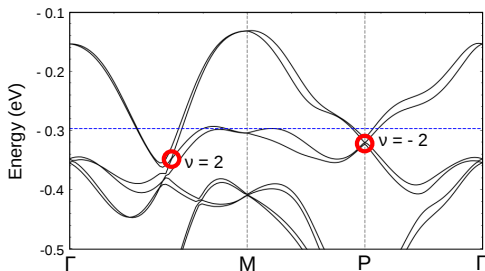
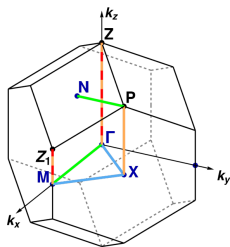
every second band exhibits an odd number of Weyl points



# Double Weyl points in TaO<sub>2</sub>

Oxygen deficient NbO<sub>2</sub> and (proposed) TaO<sub>2</sub>

- SG 80  $I4_1$
- $C_4^z T$  creates Kramers pairs at  $P$
- At  $P$  the only symmetry is  $C_2^z$
- $\Delta\varphi = 0, \nu = 0 \pmod{2}$  : double Weyl point



# Quadruple Weyl

Twofold quadruple Weyl points  $\nu = \pm 4$  in cubic point groups

T. Zhang, R. Takahashi, C. Fang, S. Murakami, *Physical Review B*, 102, 125148 (2020)

$$H(\mathbf{k}) = (d_0 + d_1(k_x^2 + k_y^2 + k_z^2)) \sigma_0 \\ + a_0 k_x k_y k_z \sigma_z + \begin{pmatrix} 0 & a_1(k_x^2 + k_y^2 e^{-i2\pi/3} + k_z^2 e^{-i4\pi/3}) \\ H.C. & 0 \end{pmatrix}$$

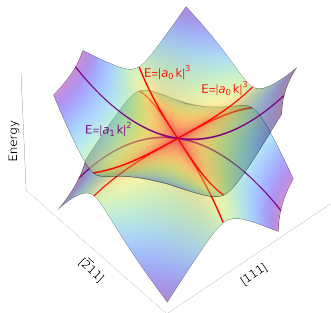
for  $a_0, a_1 \in \mathbb{R} \Rightarrow H(\mathbf{k})$  belongs to point group 432.

Symmetry eigenvalues:

$\lambda_{C_4}$  do not exchange:  $\nu = 0 \pmod{4}$

$\lambda_{C_3}$  exchange:  $\nu = \pm 1 \pmod{3}$

Lowest possible Chern number:  $\nu = \pm 4$   
(Quadruple Weyl)



# Quasi-symmetric quadruple Weyl

What happens in a cubic point group without fourfold rotation?

One finds the same Hamiltonian:

$$H(\mathbf{k}) = (d_0 + d_1(k_x^2 + k_y^2 + k_z^2)) \sigma_0 \\ + a_0 k_x k_y k_z \sigma_z + \begin{pmatrix} 0 & a_1(k_x^2 + k_y^2 e^{-i2\pi/3} + k_z^2 e^{-i4\pi/3}) \\ H.C. & 0 \end{pmatrix}$$

now with  $a_1 \in \mathbb{C}$ .

But this is not enough to break the  $C_4$  symmetry, needs  $k^n$  with  $n > 3$

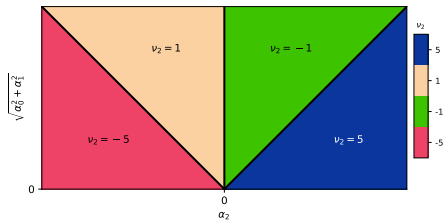
→  $C_4$  is a quasi-symmetry of the low-energy model.

$$U(C_4^z) = \begin{pmatrix} 0 & e^{i(\arg(a_1)+2\pi/3)} \\ e^{-i(\arg(a_1)+2\pi/3)} & 0 \end{pmatrix},$$

$$U(C_4^z)^\dagger H(k_x, k_y, k_z) U(C_4^z) = H(k_y, -k_y, k_z).$$

As before:  $\lambda_{U(C_4^z)}$ :  $\nu = 0 \pmod{4} \rightarrow \nu = \pm 4$

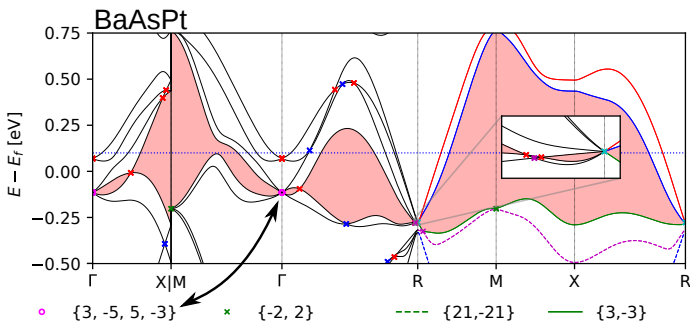
# Crossing with chirality $\nu = 5$



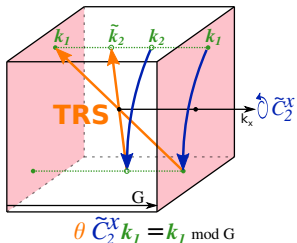
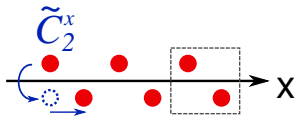
Cubic point group PG 23

- $C_3 : \nu = 1 \pmod 3$   
 $\rightarrow \nu \in \{\dots, -5, -2, 1, 4, \dots\}$
- $C_2 : \nu = 1 \pmod 2$   
 $\rightarrow \nu \in \{\dots, -5, -3, -1, 1, 3, 5, \dots\}$

$\rightarrow$  Lowest possible chiralities  $\nu = 1, -5$



# Nodal planes



Symmetries:

- two-fold screw rotation  $\tilde{C}_2^x$
- time reversal  $\theta = i\sigma_y \mathcal{K}$

Kramers theorem applies to  $\theta \tilde{C}_2^x$  because

- $(\theta \tilde{C}_2^x)^2 = +T_{(1,0,0)} = e^{ik_x} = -1$  at  $k_x = \pi$
- invariant planes:  $k_x = 0$  and  $k_x = \pi$

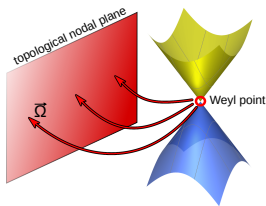
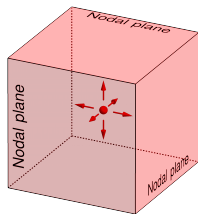
$$(\theta \tilde{C}_2^x)^2 = -1$$

$\Rightarrow$  two-fold degenerate plane  $k_x = \pi$

# Trio of topological nodal planes

Necessary symmetries: Yu, Z. M., et al. (2019). *Physical Review B*, 100(4), 041118.

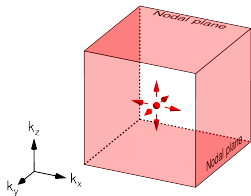
- $\theta\tilde{C}_2^x, \theta\tilde{C}_2^y, \theta\tilde{C}_2^z$   
⇒ three intersecting nodal planes
- $\theta^2 = -1$ , single Kramers-Weyl point at  $\Gamma$  with  $\nu_{\text{Weyl}} = \pm 1$
- Multiplicity of any other Weyl point  $> 1$ .  
⇒ nodal planes compensate the Weyl point at  $\Gamma$  with  $\nu_{\text{NP}} = \mp 1$ .



Properties of nodal plane trios:

- No surface states
- Vanishing anomalous Hall effect

# Topological nodal plane duo

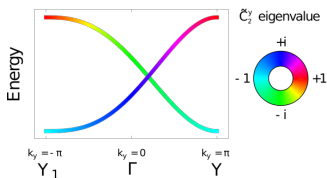


Symmetries:

- $\theta\tilde{C}_2^x, \theta\tilde{C}_2^z \Rightarrow$  two nodal planes
- Time-reversal symmetry  $\theta$  is broken.
- Two-fold screw rotation  $\tilde{C}_2^y$

$\tilde{C}_2^y$  representation:

- $\tilde{C}_2^y$ -invariant lines: rotation axes
- $(\tilde{C}_2^y)^2 = T_{(0,1,0)}(i\sigma_y)^2 = -e^{ik_y}$   
 $\Rightarrow$  symmetry eigenvalues  $\pm ie^{ik_y/2}$

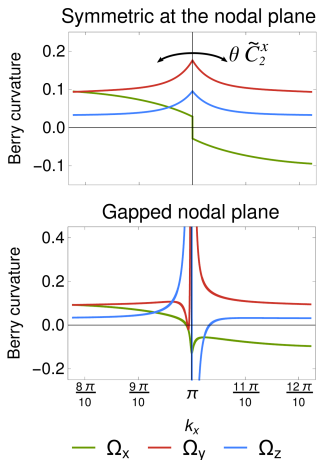
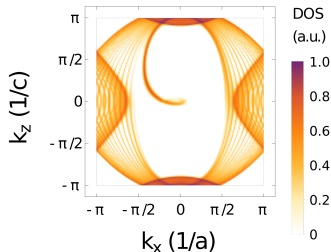


$\Rightarrow$  uncompensated, odd number of Weyl points

$\Rightarrow$  **The nodal plane must be topological.**

# Topological nodal plane duo

Fermi arc:  
single Weyl  $\rightarrow$  Nodal plane





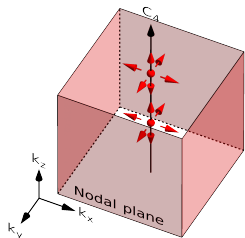
# Space groups with nodal planes

	possible topological nodal plane				enforced topological nodal plane				
Paramagnets	4.8 [t]	17.8 [t]	18.17 [t]	19.26 [T]	20.32 [t]	26.67	29.100	31.124	33.145
	36.173	76.8 [t]	78.20 [t]	90.96 [t]	91.104 [t]	92.112 [T]	94.128 [T]	95.136 [t]	96.144 [T]
	113.268	114.276	169.114 [t]	170.118 [t]	173.130 [t]	178.156 [t]	179.162 [t]	182.180 [t]	185.198
	186.204	198.10 [T]	212.60 [T]	213.64 [T]					
~Ferromagnets	4.9 [t]	11.54	14.79	17.10 [t]	18.18 [t]	18.19 [t]	19.27 [T]	20.34 [t]	26.68
	26.69	29.101	29.102	31.125	31.126	MnSi	33.147	36.174	36.175
	51.294	51.296	52.310	52.311	53.327		54.342	54.344	55.357
	55.358	56.369	56.370	57.382	57.383	57.384	58.397	58.398	59.409
	59.410	60.422	60.423	60.424	61.436	62.446	62.447	62.448	63.463
	63.464	64.475	64.476	90.98 [t]	90.99 [t]	92.114 [T]	92.115 [T]	94.130 [T]	94.131 [t]
	96.146 [T]	96.147 [T]	113.269	113.271 [t]	Nd <sub>5</sub> Si <sub>4</sub>	114.279 [t]	127.390	127.393	128.402
	128.405	129.414	129.417	130.426		135.486	135.489	136.498	136.501
	137.510	137.513	138.522	138.525		169.115 [t]	170.119 [t]	173.131 [t]	176.147
	178.158 [t]	179.163 [t]	179.164 [t]	182.181 [t]	182.182 [t]	185.199	185.200	186.205	186.206
193.258	193.259	194.268	194.269						
~Antiferromagnets	3.5 [t]	3.6 [t]	4.10 [t]	16.4 [t]	16.5 [t]	16.6 [T]	17.11 [t]	17.13 [t]	17.14 [T]
	17.15 [T]	18.20 [t]	18.21 [T]	18.22 [T]	18.24 [t]	19.28 [T]	19.29 [t]	20.36 [t]	21.42 [t]
	21.44 [t]	25.61	CoNb <sub>3</sub> S <sub>6</sub>	25.65	26.71	26.72	26.76	27.82	27.85
	27.86	28.94		28.96	28.98	29.104	29.105	29.109	30.118
	30.119	30.120	30.122	31.128	31.129	31.133	32.139	32.142	32.143
	33.149	33.150	33.154	34.161	34.162	34.164	35.169	35.171	36.178
	37.184	37.186	75.4 [t]	75.6 [t]	76.11 [t]	77.16 [t]	77.18 [t]	78.23 [t]	81.36 [t]
	81.38 [t]	89.92 [t]	89.93 [t]	89.94 [T]	90.100 [T]	90.102 [t]	91.109 [T]	91.110 [T]	92.116 [T]
	92.117 [t]	93.124 [t]	93.125 [T]	93.126 [T]	94.132 [T]	94.134 [t]	95.141 [T]	95.142 [T]	96.148 [T]
	96.149 [t]	99.168	99.170	100.176	100.178	101.184	101.186	102.192	102.194
	103.200	103.202	104.208	104.210	105.216	105.218	106.224	106.226	111.256
	111.257	111.258	112.264	112.265	112.266	113.272	113.274	114.280	114.282
	115.288	115.290	116.296	116.298	117.304	117.306	118.312	118.314	168.112 [t]
171.124 [t]	172.128 [t]	177.154 [t]	180.172 [t]	181.178 [t]	183.190	184.196	195.3 [T]	207.43 [T]	
208.47 [T]	215.73	218.84							

# Nodal plane duo in SG 94

SG 94  $P4_22_12$  with spin-orbit coupling

- Two nodal planes at  $k_x = k_y = \pi$
- Along Z- $\Gamma$ -Z: hour-glass crossing
- rotation  $4_2$ :  $n = 4, m = 2$



Consider the lowest band  $b = 1$  on the fourfold rotation axis:

$$\text{global constraint: } \sum_{c_1} \Delta\varphi_{1,c_1} = -m \frac{2\pi}{n} \pmod{2\pi}$$

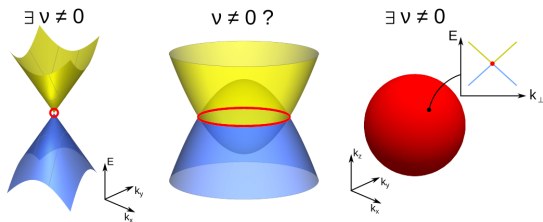
Total chirality  $\nu_{Z-\Gamma-Z}$  on the fourfold rotation axis:

$$\text{add local constraint: } \nu_{Z-\Gamma-Z} \equiv \sum_{c_1} \nu_{c_1} = \sum_{c_1} \frac{n}{2\pi} \Delta\varphi_{1,c_1} = -m \pmod{4}$$

$\nu_{Z-\Gamma-Z} = 2 \pmod{4}$  cannot be compensated by generic Weyl points

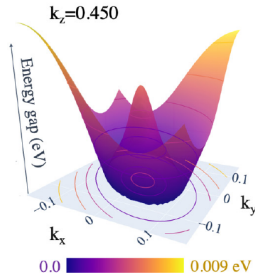
$\Rightarrow$  The nodal plane duo exhibits a charge of  $\nu_{\text{nodal plane}} = -\nu_{Z-\Gamma-Z} \neq 0$ .

# Application: Chiral nodal lines I



Question: Are chiral nodal lines possible?

- Line protected by mirror symmetry  
 $\rightarrow \nu = 0$
- Can there be a stable line with  $\nu \neq n\mathbb{Z}$   
enclosing a  $C_n$  rotation axis?



# Application: Chiral nodal lines II

Surface *including* line:

$$\nu_{\text{line}} + \nu_{\text{axis}} = \Delta\varphi \frac{n}{2\pi} \pmod{n}$$

$\Rightarrow \nu_{\text{line}} = 0 \pmod{n}$ , thus it cannot be protected by rotation symmetry.

Surface *excluding* line:

$$\nu_{\text{axis}} = \Delta\varphi \frac{n}{2\pi} \pmod{n}$$

Model of a chiral nodal line:

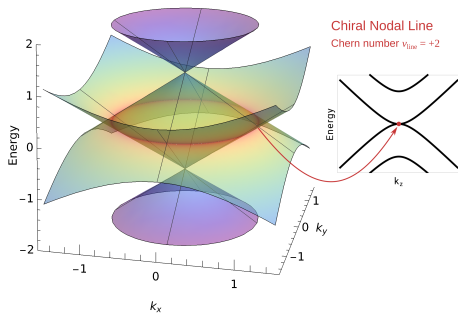
$$W(\mathbf{k}, \epsilon) = \mathbf{k} \cdot \boldsymbol{\sigma} + \epsilon\sigma_0$$

$$A(\mathbf{k}) = k_z\sigma_1$$

$$H(\mathbf{k}) = \begin{pmatrix} W(\mathbf{k}, \epsilon) & A(\mathbf{k}) \\ A(\mathbf{k})^\dagger & W(\mathbf{k}, -\epsilon) \end{pmatrix}$$

$$U_{\text{line}} = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & -2 \end{pmatrix}$$

$$[H(\mathbf{k}), U_{\text{line}}] = 0$$



# Summary

- Local constraint

$$\nu = \Delta\varphi \frac{n}{2\pi} \pmod n$$

- Global constraint

$$\sum_{c_b} \Delta\varphi_{b,c_b} = -b \cdot 2\pi \frac{m}{n} \pmod{2\pi}$$

- Applications

Double Weyl points in TaAs

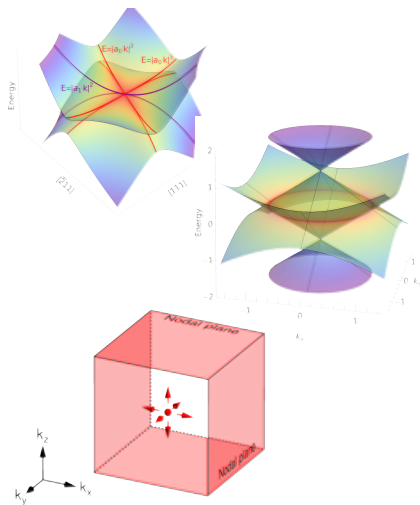
Quasi-symmetry-protected quadruple

Weyl points

Crossings with Chern number  $\nu = 5$

Topological nodal planes

Chiral nodal lines



# Summary - Thank you!

- Local constraint

$$\nu = \Delta\varphi \frac{n}{2\pi} \pmod{n}$$

- Global constraint

$$\sum_{c_b} \Delta\varphi_{b,c_b} = -b \cdot 2\pi \frac{m}{n} \pmod{2\pi}$$

- Applications

Double Weyl points in TaAs

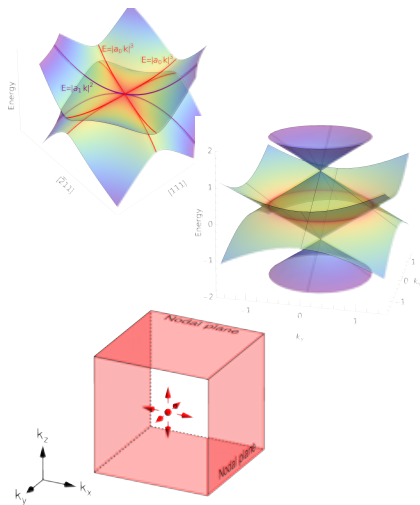
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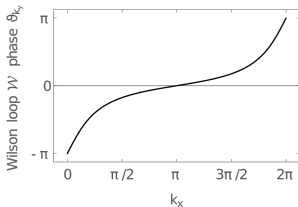
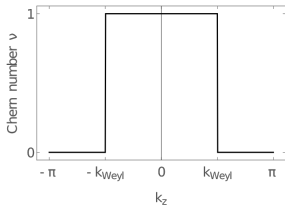
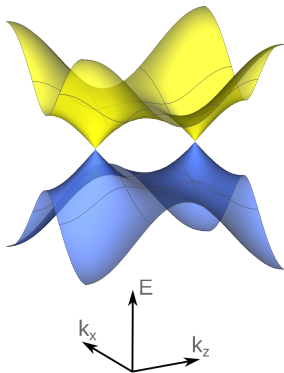
Chiral nodal lines





# Topology in a Weyl semimetal

- Chern number  $\nu$  : Obstruction to a smooth gauge for eigenstates.
- Wilson loop : Its eigenvalues are the Wannier charge centers.

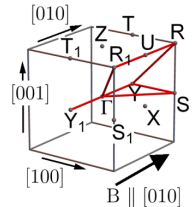
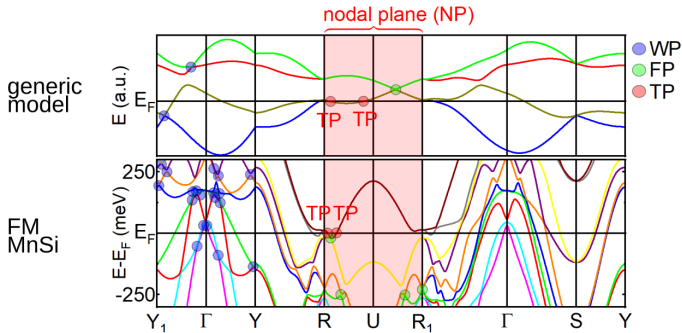




# Band structure of MnSi

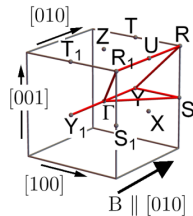
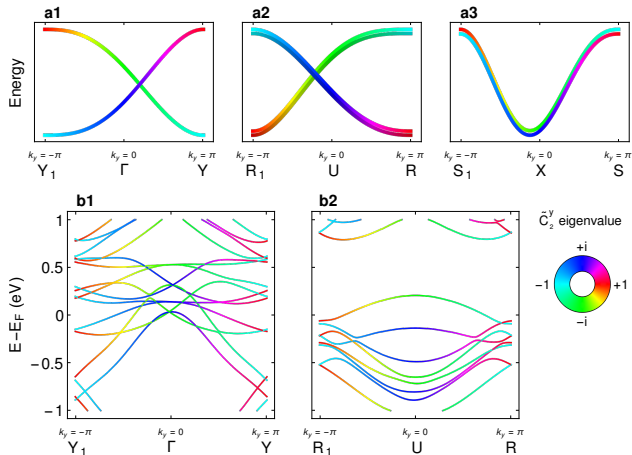
Topological nodal planes in ferromagnetic MnSi (mSG 19.27  $P2'_12_12'_1$ )

- Several twofold (WP) and fourfold (FP) Weyl points
- Band degeneracy protected at the Fermi surface: **Topological protectorate (TP)**



# Symmetry eigenvalues in MnSi

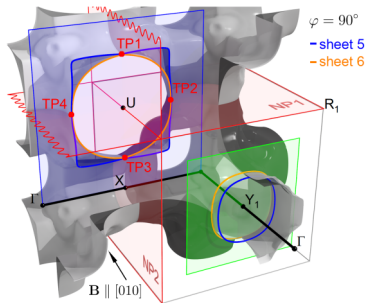
Does the picture of the eigenvalue winding argument hold?



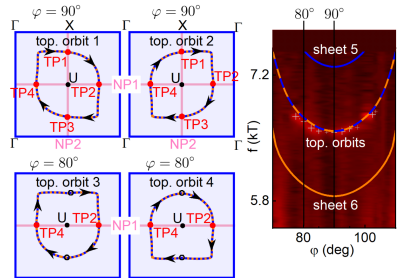
# Topological nodal planes in MnSi

Ferromagnetic MnSi (mSG 19.27  $P2'_12_12'_1$ )

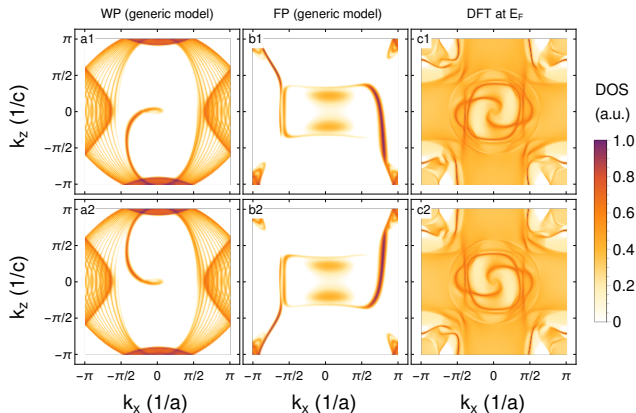
Fermi surface sheets



de Haas-van Alphen spectroscopy provides evidence for two gapless nodal planes



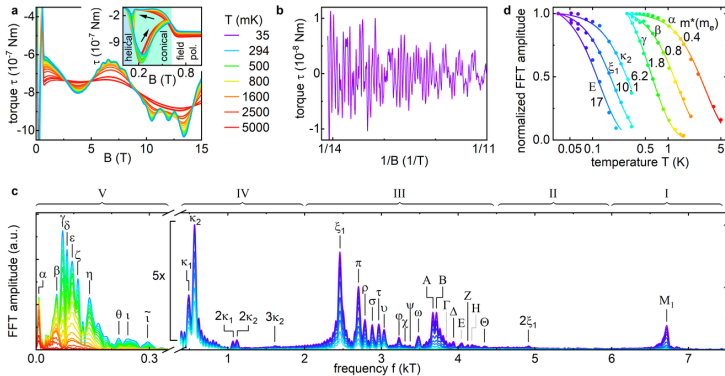
# Surface states from DFT for MnSi



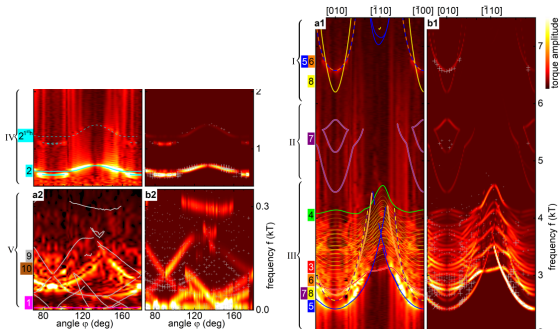
- Generic tight-binding model: Characteristic surface states
  - ferromagnetic MnSi: Several overlapping bands, many Weyl points
- ⇒ Need for simpler systems to study topological nodal planes.

# De Haas-van Alphen data

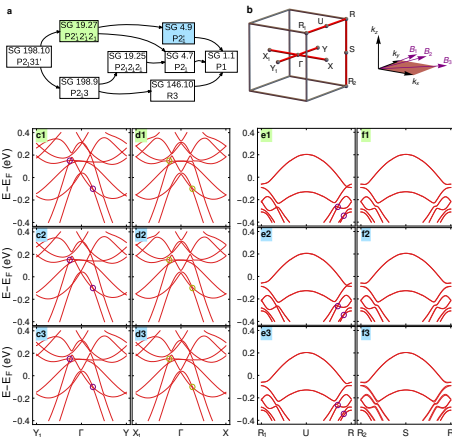
- Oscillations measured using magnetic torque
- Several orbits passing the nodal plane
- Focus on the single frequency  $M_1$ .



# Angle-resolved spectrum



# Band structure for rotated fields



# Berry curvature of the generic model

