Partial supersymmetry breaking and dispersive Majorana modes in quantum wires

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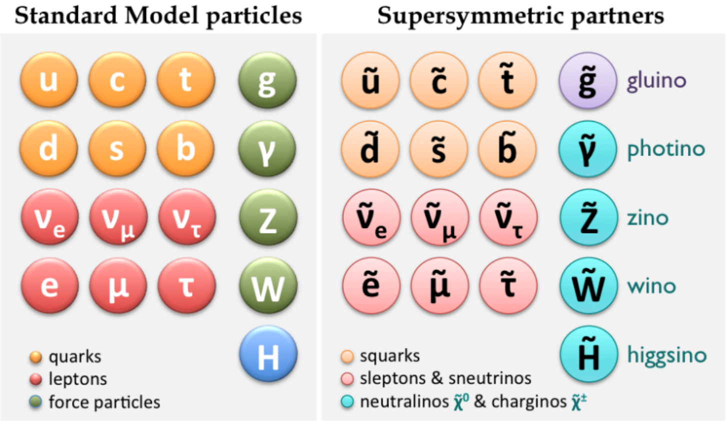
Supersymmetry (SUSY)

Loosely speaking, supersymmetry is a symmetry between bosons and fermions. In particle physics, supersymmetric extensions of the Standard Model predicts a partner particle for each known particle.

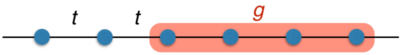


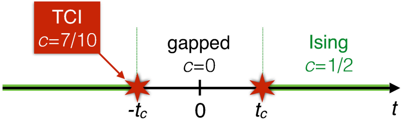
SUSY in Majorana chains

- Interacting Majorana chain with emergent space-time SUSY: tricritical Ising model (via Jordan-Wigner mapping)

$$H = \imath t \sum_j \gamma_j \gamma_{j+1} + g \sum_j \gamma_j \gamma_{j+1} \gamma_{j+2} \gamma_{j+3},$$

Rahmani, Zhu, Franz, Affleck, PRL 115, 166401 (2015); PRB 92, 235123 (2015)

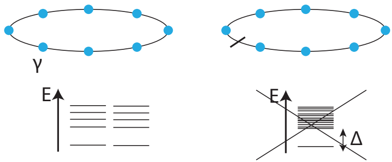




- Translational invariant Majorana models with emergent quantum-mechanical SUSY

$$H = \sum_{i=1}^{N} H_i, \qquad Q = \sqrt{\frac{H}{2}}\, T(1+P)$$

Hsieh, Halász, Grover PRL 117, 166802 (2016)

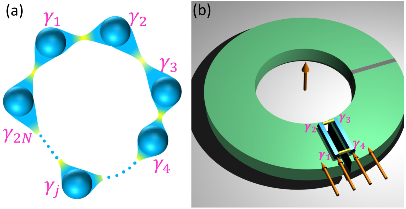


- Closed Majorana chains (with closed PH gap) with emergent quantum-mechanical SUSY

$$H = i \sum_{j=1}^{2N} t_j \gamma_j \gamma_{j+1}, \qquad \prod_{j=1}^{N} t_{2j-1} = \prod_{j=1}^{N} t_{2j},$$

$$Q_{A,B} = \gamma_{A,B} \sqrt{H}$$

Huang, Shimasaki, Nitta, PRB 96, 220504(R) (2017)



Quantum Mechanical Supersymmetry (QM SUSY)

- QM SUSY: Symmetry of the many-body Hamiltonian which connects states with even and odd fermion parity (i.e., bosons and fermions)
- Supercharge Q which satisfies the superalgebra

$$\{Q, Q^\dagger\} = QQ^\dagger + Q^\dagger Q = 2\mathcal{H}_{\text{SUSY}}$$
$$\{P, Q\} = 0$$

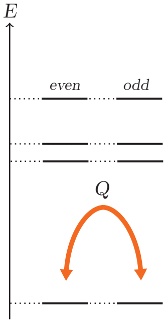
with P the parity operator.

- SUSY is a "fermionic" symmetry (unlike unitary symmetries)

$$\{\mathcal{H}_{\text{SUSY}}, Q\} = 0$$
$$Q^2 = (Q^\dagger)^2 = 0$$

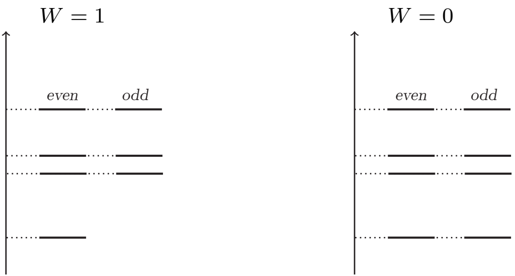
- Extended QM SUSY: many supercharges $\mathcal{Q}_i$ which satisfies the superalgebra

$$\{\mathcal{Q}_i, \mathcal{Q}_j^\dagger\} = 2\delta_{ij}\mathcal{H},$$
$$\{\mathcal{Q}_i, \mathcal{Q}_j\} = \{\mathcal{Q}_i^\dagger, \mathcal{Q}_j^\dagger\} = 0,$$
$$\{P, \mathcal{Q}_i\} = 0.$$



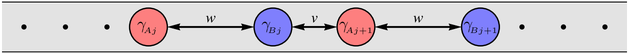
Spontaneous breaking of QM SUSY

- Unbroken QM SUSY:
  $Q\left|0\right\rangle = Q^\dagger\left|0\right\rangle = 0 \qquad \Rightarrow e^{\imath(\epsilon Q+\bar{\epsilon}Q^\dagger)}\left|0\right\rangle = \left|0\right\rangle$
- Spontaneously broken QM SUSY:
  $Q\left|0\right\rangle \neq 0$ or $Q^\dagger\left|0\right\rangle \neq 0 \Rightarrow e^{\imath(\epsilon Q+\bar{\epsilon}Q^\dagger)}\left|0\right\rangle \neq \left|0\right\rangle$
- Witten index:
  difference between the # of bosons and # of fermions
  $W = \mathrm{Tr}(-1)^F =$ (# even − # odd eigenstates)



- Spontaneously broken unitary symmetry $\rightarrow$ Goldstone mode (boson)
  Spontaneously broken supersymmetry $\rightarrow$ Goldstino (fermion)
- Extended SUSY:
  Witten's no-go theorem prohibits partial SUSY breaking
  Loophole: central charges

Bipartite Majorana chain



- Chain of Majorana modes

$$\mathcal{H}_{\text{eff}} = \imath \sum_j \left(w\gamma_{Aj}\gamma_{Bj} + v\gamma_{Bj}\gamma_{Aj+1} \right)$$

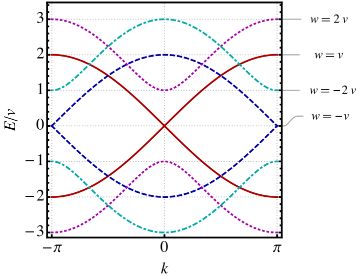
- In momentum space

$$\mathcal{H}_{\text{eff}} = \sum_k [c_k^\dagger, c_{-k}] \cdot \mathbf{H}_{\text{eff}}(k) \cdot \boldsymbol{\tau} \cdot \begin{bmatrix} c_k \\ c_{-k}^\dagger \end{bmatrix}$$

where $\mathbf{H}_{\text{eff}}(k) = (0, v \sin k, v \cos k - w)$

- Dispersion

$$E_k = |\mathbf{H}_{\text{eff}}(k)| = \sqrt{w^2 + v^2 - 2wv \cos k}$$

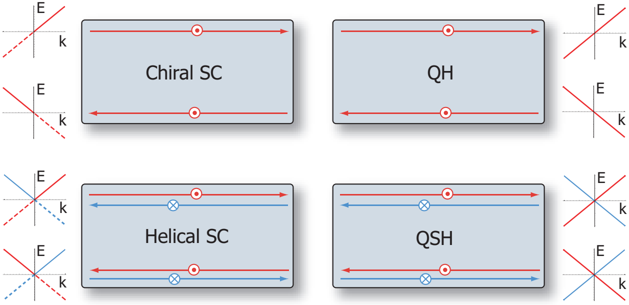


- Topological invariant $\mathcal{M}_{\text{eff}} = |w| - |v|$
- In the continuum limit $\rightarrow$ 1D Dirac equation

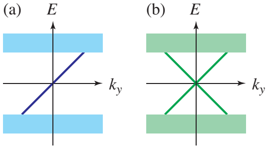
$$H = vk\,\tau_y + \left(mv^2 - \frac{v}{2} k^2 \right) \tau_z$$

- Hybridization of 0D Majorana modes $\Rightarrow$ 1D Majorana mode in a 1D system

Chiral and helical 1D Majorana fermions



Qi, Hughes, Raghu, Zhang, PRL 102, 187001 (2009)



Sato, Fujimoto, J. Phys. Soc. Jpn. 85, 072001 (2016)

QM SUSY in Majorana chains

- The Majorana chain exhibits SUSY with supercharges

$$\textcircled{1} \qquad Q_M = \sqrt{\frac{\mathcal{H}_{\text{SUSY}}}{2}} (\tilde{\gamma}_A + \imath\tilde{\gamma}_B)$$

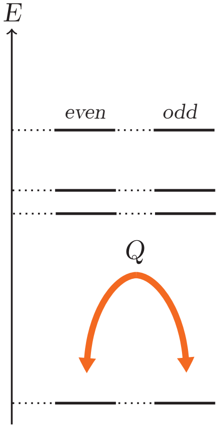
$$\textcircled{2} \qquad Q_T = \sqrt{\frac{\mathcal{H}_{\text{SUSY}}}{2}} T(1+P)$$

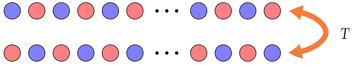
$$2\mathcal{H}_{\text{SUSY}} = QQ^\dagger + Q^\dagger Q = \{Q, Q^\dagger\}$$

$\{P, Q\} = 0$, $Q^2 = (Q^\dagger)^2 = 0$
with $\mathcal{H}_{\text{SUSY}} = \mathcal{H}_{\text{eff}} + \text{const} > 0$

① Huang, Shimasaki, Nitta, PRB 96, 220504(R) (2017)
② Hsieh, Halász, Grover PRL 117, 166802 (2016)

- Spontaneously broken SUSY: Witten index is zero
$W = \text{Tr}(-1)^F = (\#\text{even} - \#\text{odd eigenstates}) = 0$
- Goldstino → 1D Majorana fermion





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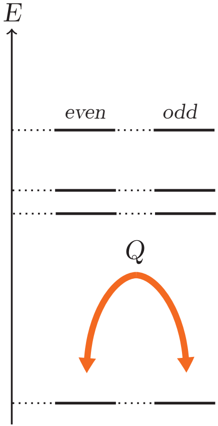

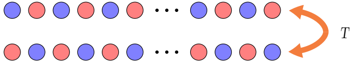

Extended QM SUSY and partial SUSY breaking

- The two $\mathcal{N} = 2$ superalgebras can be combined
- The Majorana chain exhibits *extended* SUSY $\mathcal{N} = 4$ with central charges. The $\mathcal{N} = 4$ superalgebra is given by

$$\begin{aligned} \{\mathcal{Q}_i, \mathcal{Q}_j^\dagger\} &= 2\delta_{ij}\mathcal{H}_{\text{SUSY}} + \mathcal{Z}_{ij}, \\ \{\mathcal{Q}_i, \mathcal{Q}_j\} &= \{\mathcal{Q}_i^\dagger, \mathcal{Q}_j^\dagger\} = 0, \\ \{P, \mathcal{Q}_i\} &= 0, \end{aligned}$$

- Supercharges:

$$\mathcal{Q}_1 = \sqrt{\frac{\mathcal{H}_{\text{SUSY}}}{2}}\, d_M(1+P), \qquad \mathcal{Q}_2 = \sqrt{\frac{\mathcal{H}_{\text{SUSY}}}{2}}\, T(1+P),$$

- Central charges:

$$\mathcal{Z}_{ij} = \mathcal{H}_{\text{SUSY}} \begin{bmatrix} -(1+P(-1)^{d_M^\dagger d_M}) & \{d_M(1+P), T^\dagger\} \\ \{d_M^\dagger(1-P), T\} & 0 \end{bmatrix},$$

  where $d_M = (\widetilde{\gamma}_A + \imath\widetilde{\gamma}_B)/2$.
- Partially broken SUSY

$$\begin{aligned} \mathcal{Q}_2 \left|0\right\rangle &= \sqrt{2\mathcal{H}_{\text{SUSY}}}\, \left|1\right\rangle, \qquad \mathcal{Q}_2^\dagger \left|1\right\rangle = \sqrt{2\mathcal{H}_{\text{SUSY}}}\, \left|0\right\rangle, \\ \mathcal{Q}_1 \left|0\right\rangle &= \mathcal{Q}_1 \left|1\right\rangle = \mathcal{Q}_1^\dagger \left|0\right\rangle = \mathcal{Q}_1^\dagger \left|1\right\rangle = \mathcal{Q}_2^\dagger \left|0\right\rangle = \mathcal{Q}_2 \left|1\right\rangle = 0. \end{aligned}$$

- Witten's no-go theorem prohibits partial SUSY breaking
  Loophole: central charges

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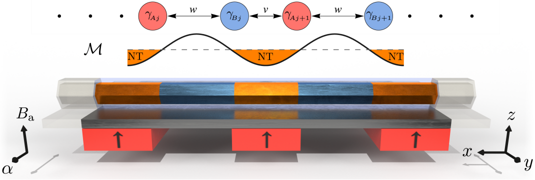
Array of quasi-Majorana modes induced by modulated fields



- Oreg-Lutchyn model with spatially-modulated fields

$$H = \left(\frac{p^2}{2m} + \frac{\alpha}{\hbar}\sigma_y p - \mu \right)\tau_z + \mathbf{b}(x)\cdot\boldsymbol{\sigma} + \Delta(x)\tau_x$$

- local Majorana mass $\mathcal{M} = \sqrt{\mu(x)^2 + \Delta(x)^2} - |b(x)|$
  quasi-Majorana modes localize at the nodes $\mathcal{M} = 0$
- Effective theory $\rightarrow$ Chain of Majorana modes

$$\mathcal{H}_{\text{eff}} = \imath \sum_j \left(w\gamma_{Aj}\gamma_{Bj} + v\gamma_{Bj}\gamma_{Aj+1} \right)$$

finite overlaps:

$w = -\imath \langle\gamma_{Aj}| H |\gamma_{Bj}\rangle \propto e^{-L_{AB}/\xi_{\text{M}}}$

$v = -\imath \langle\gamma_{Bj}| H |\gamma_{Aj+1}\rangle \propto e^{-L_{BA}/\xi_{\text{M}}}$

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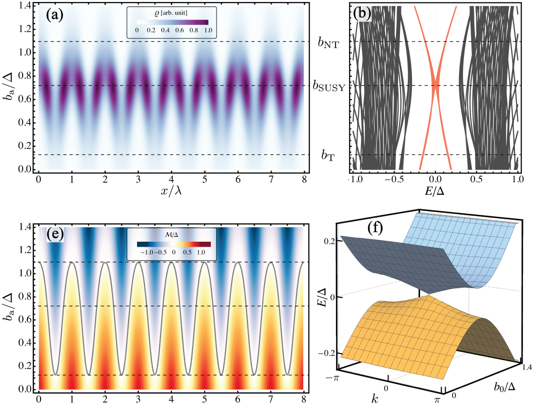
Energy dispersion and LDOS (numerical calculations)

- Magnetic field calculated with FEM: $|\mathbf{b}_{\text{nm}}(x)|^2 \approx |b_{\text{nm}}|^2\,(2 - 2\cos(2\pi x/\lambda))$
- The local Majorana mass $\mathcal{M}(x) \approx \sqrt{\mu^2 + \Delta^2} - |b_{\text{nm}}|\sqrt{(2 - 2\cos(2\pi x/\lambda))}$
- LDOS at zero energy: localized modes
- Energy dispersion E_k: Dispersive subgap level



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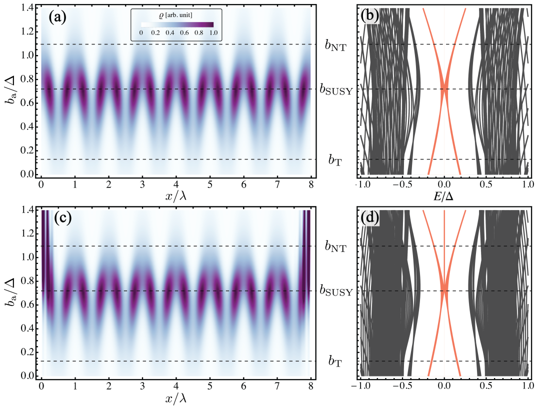
Edge modes of edge modes

- Open boundaries in the globally nontrivial phase
- Global topological invariant $\mathcal{M}_{\text{eff}} = |w| - |v|$
- LDOS at zero energy: localized modes accumulating at the boundary



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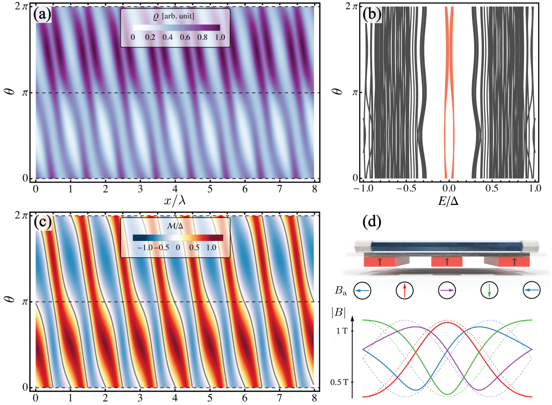
Adiabatic pumping of 0D Majorana modes

- By applying a rotating magnetic field $b_{\text{a}}[\cos\theta\,\hat{\mathbf{x}}+\sin\theta\,\hat{\mathbf{z}}]$. produces a sliding field

$$\mathcal{M}(x)\propto\sin\left(2\pi x/\lambda+\theta\right)$$

- $\theta\rightarrow\theta+\pi$: adiabatic pumping of one 0D Majorana state
- $\theta\rightarrow\theta+2\pi$: adiabatic pumping of two 0D Majorana states (one fermion)
- Analogy to Thouless pumps of cold atoms: local Majorana mass $\sim$ optical lattice



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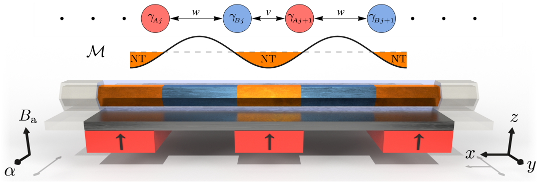
Experimental considerations



- Hierarchy of length scales

  (length of the wire) $\gg$ (spatial periodicity of the field) $\gtrsim$ (Majorana localization length)

- Conditions for SUSY and massless Majorana fermions

$$L_{\mathrm{AB}} \approx L_{\mathrm{BA}}$$

- The SUSY point appears at a single point of parameter space. However, electronic interactions may pin the Majorana modes to zero energy in a larger window [see Domínguez, Cayao, San-Jose, Aguado, Yeyati, Prada, npj Quantum Materials 2, 13 (2017)]
- The transition between a dip $G = 0$ to a quantized peak $G = 2e^2/\hbar$ in the zero-bias conductance may signal the onset of SUSY [see also Flensberg, PRB 82, 180516 (2010)]

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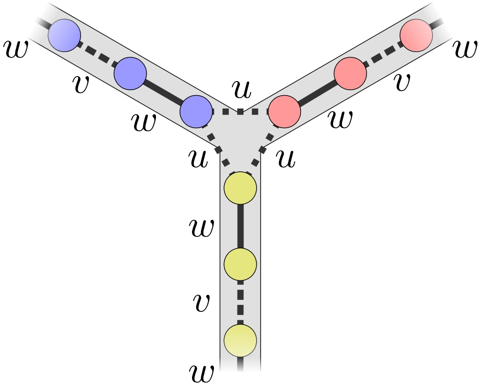
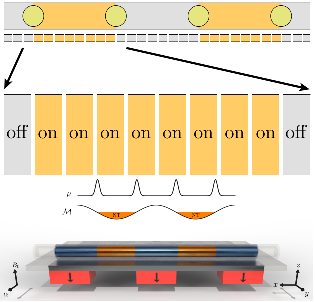
Trijunctions

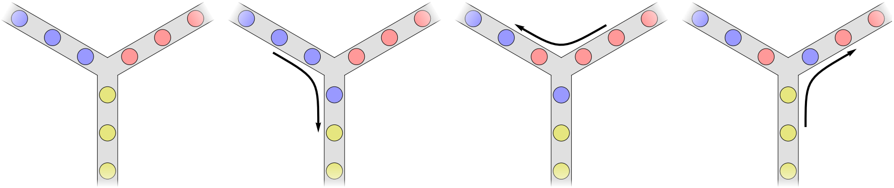
- T-shaped or Y-shaped trijunctions with lattices of $2N$ Majorana modes on each branch
- Nanomagnets, piano keyboard, or in planar Josephson junctions

$$\mathcal{H}_{\text{eff}} = \imath u \left(\gamma_{1,1}\gamma_{2,1} + \gamma_{2,1}\gamma_{3,1} + \gamma_{3,1}\gamma_{1,1}\right) + \imath \sum_{m=1}^{3} \left(\sum_{n=1}^{N} w \gamma_{m,2n-1}\gamma_{m,2n} + \sum_{n=1}^{N-1} v \gamma_{m,2n}\gamma_{m,2n+1} \right)$$



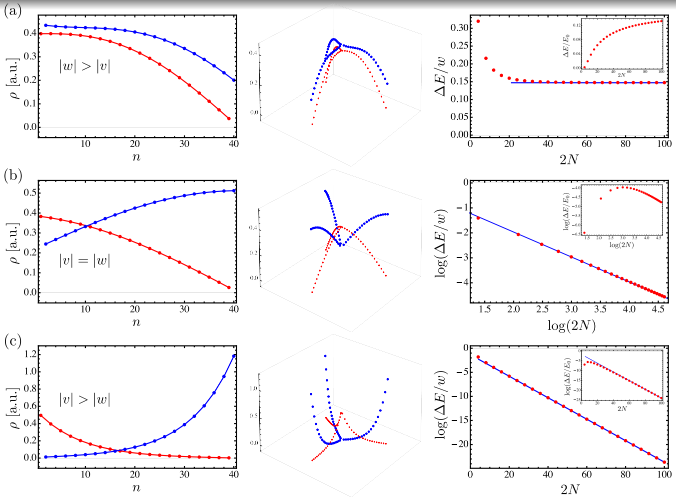


Braiding



See also Hegde *et al.*, Ann. Phys. **423**, 168326 (2020).

Topological phases and energy scaling

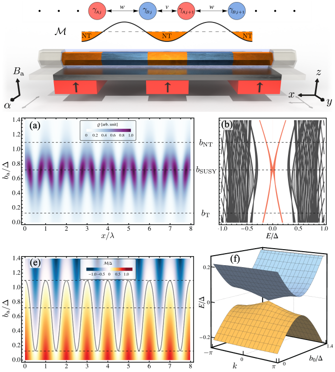


- $|w| > |v|$, trivial phase, $\Delta E/w \to E_0$ for $N \to \infty$
- $|w| = |v|$, SUSY, $\Delta E/w \propto 1/N$ for $N \to \infty$
- $|w| < |v|$, nontrivial phase, $\Delta E/w \propto \mathrm{e}^{-2N/\xi_\text{eff}}$ for $N \to \infty$

Marra, Inotani, Nitta, arXiv:2207.10103

Conclusions

- Spatially-modulated fields can induce dispersive 1D Majorana fermions in 1D nanowires by the hybridization of 0D Majorana modes
- Real-world realization of centrally-extended QM-SUSY and partial breaking of the extended SUSY algebras
- Dispersive 1D Majorana fermion is the Goldstino of the spontaneously, partially broken SUSY
- Zero-bias dip to peak transition as a signature of SUSY
- Adiabatic pumping of Majorana modes analogous to Thouless pumps of (non-Majorana) fermions
- Implementation of novel braiding protocols?



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email to:
`pmarra@ms.u-tokyo.ac.jp`

Pseudohelicity

- Let's define the Majorana pseudospin operator

$$\mathbf{T} = \frac{1}{2}\boldsymbol{\tau} = \frac{1}{2}(\tau_x, \tau_y, \tau_z)$$

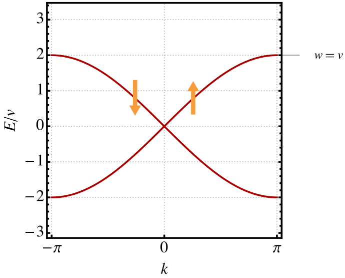
- The expectation value of $\mathbf{T}$ on a state $E_k > 0$ is

$$\langle \mathbf{T} \rangle = \left(0,\ \frac{v \sin k}{2E_k},\ \frac{v \cos k - w}{2E_k}\right) = \frac{\mathbf{H}(k)}{2E_k}$$

- two Majorana modes with opposite pseudospins $\langle \mathbf{T} \rangle = \frac{1}{2}\operatorname{sgn}(v \sin k)\hat{\mathbf{y}}$ at $k \to 0, \pi$ for $v = \pm w$.
  e.g., $\langle \mathbf{T} \rangle = \pm\frac{1}{2}\hat{\mathbf{y}}$ at $k \to 0^{\pm}$ for $v = w > 0$

- Pseudohelical, but not necessarily helical (the effective Hamiltonian does not act on the spin degrees of freedom)

- The expectation values $\langle T_x \rangle$ and $\langle T_y \rangle$ coincide with the Majorana polarization P_{M_x} and P_{M_y} (Sticlet, Bena, Simon, PRL 108, 096802, 2012)



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