Programmable adiabatic demagnetization: State preparation by simulated cooling

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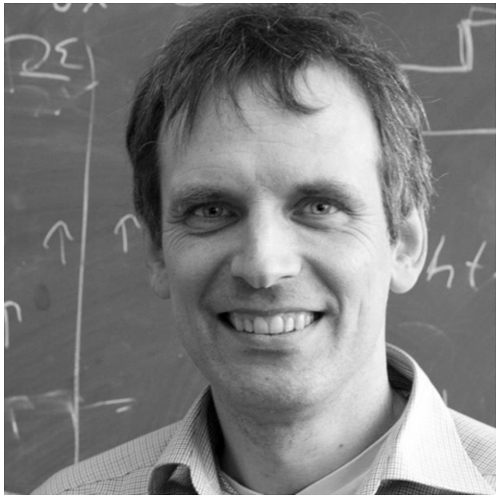


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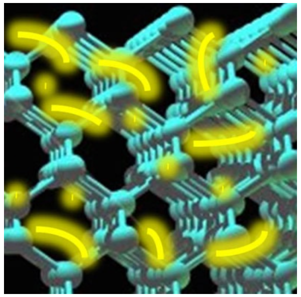
Mark Rudner

University of Washington

arXiv:2210.17256

Simulating Physics on Computers

N-particle QM



$$i\hbar\partial_t\psi(\boldsymbol{r}_1,\dots,\boldsymbol{r}_N,t) = \left[-\frac{\hbar^2}{2m}\nabla_1^2 - \cdots - \frac{\hbar^2}{2m}\nabla_N^2 + V(\boldsymbol{r}_1,\dots,\boldsymbol{r}_N)\right]\psi(\boldsymbol{r}_1,\dots,\boldsymbol{r}_N,t)$$

“Curse of dimensionality”

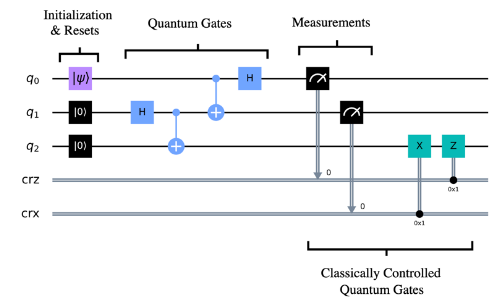
$$\widehat{H}_{spins} = \sum_{i,\alpha=x,y,z} h_i^\alpha\sigma_i^\alpha + \sum_{i,j,\alpha,\beta} J_{ij}^{\alpha\beta}\sigma_i^\alpha\sigma_j^\beta + \cdots$$

$\sigma_i^{\alpha=x,y,z}$: Pauli matrices

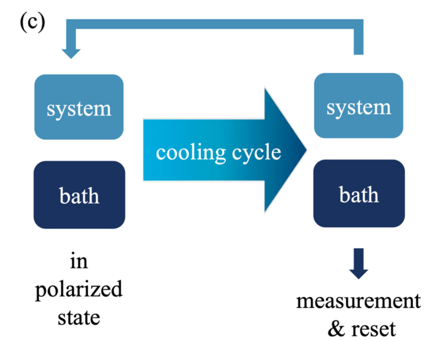
Discrete models: digital computer friendly

Outline

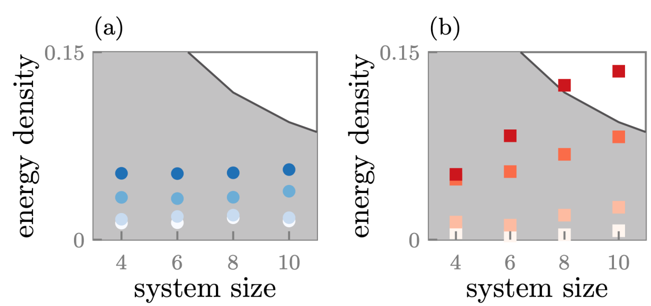
- How to find ground states on quantum simulators?



- Simulated adiabatic demagnetization protocol

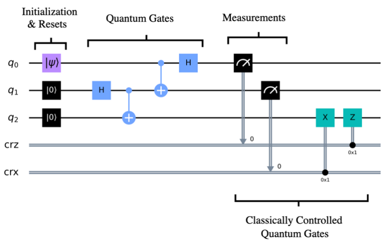


- Cooling trivial and topological excitations



Ground States on Quantum Computers

Q: How to prepare the ground state of a given Hamiltonian on a quantum simulator?



Noisy **I**ntermediate-**S**cale **Q**uantum (**NISQ**) era ***Preskill, 2018***

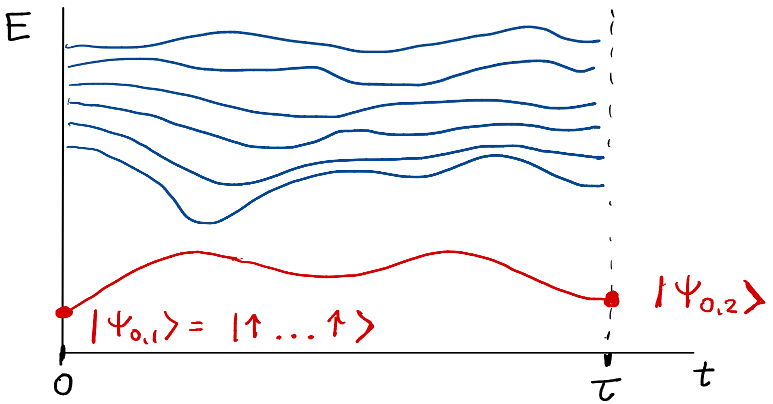
- **Adiabatic Preparation**

$$H(t) = \left(1 - \frac{t}{\tau}\right) H_1 + \frac{t}{\tau} H_2$$

H_1: “Simple” Hamiltonian

e.g. $\boldsymbol{H_1 = -h \sum_j \sigma_j^z}$

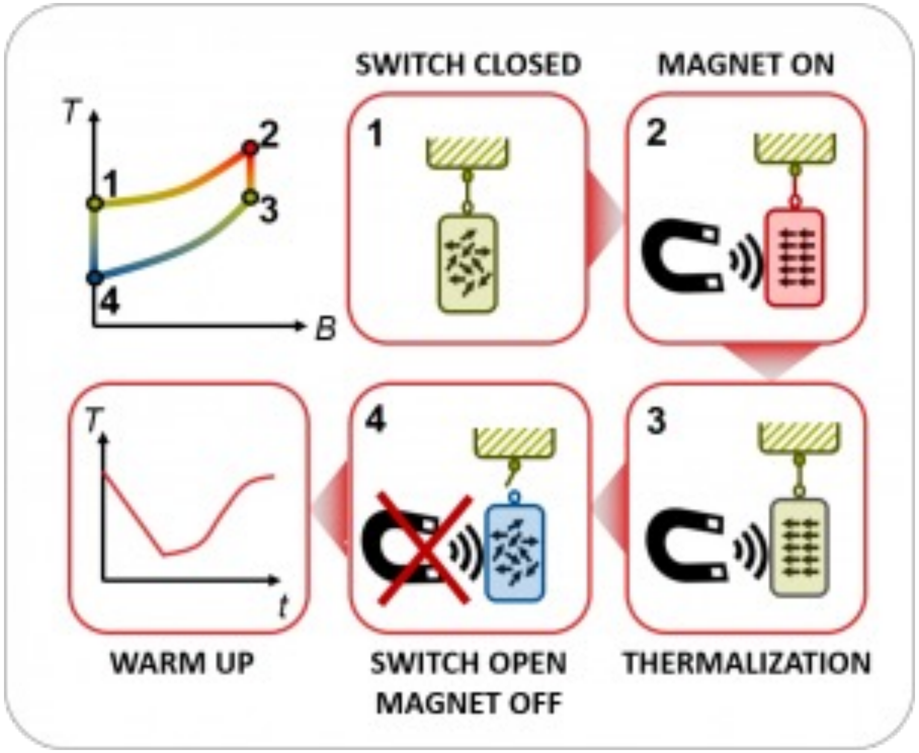
H_2: Hamiltonian of interest



- **Variational Quantum Eigensolvers**: prepare a wavefunction $|\psi\left(\lambda_1, \ldots, \lambda_{N_p}\right)\rangle$, minimize $\langle\psi|H|\psi\rangle$ over $\lambda_1, \ldots, \lambda_{N_p}$

Variational Quantum Eigensolver review: Tilly et al. (2021)

Adiabatic Demagnetization

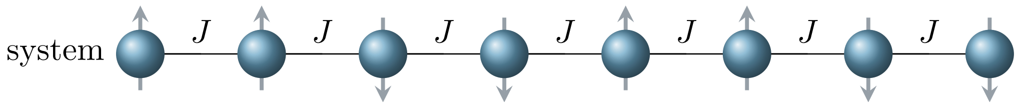


cryodynamics.de

Decrease *B(t)* adiabatically $\Longrightarrow$ decrease T

Simulated Adiabatic Cooling

Hamiltonian H_S

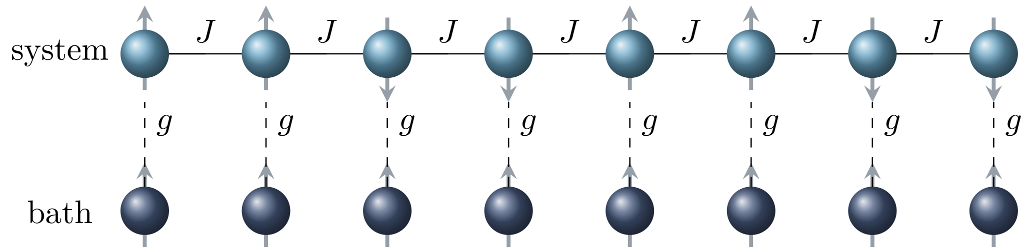


Anne Matthies, Mark Rudner, Achim Rosch, EB, **arXiv:2210.17256**
see also S. Polla et al. (2021)

Simulated Adiabatic Cooling

$$H(t) = H_S + H_B(t) + H_g(t)$$

$$H_B(t) = -B(t)\sum_j \sigma_j^z$$



$$H_g(t) = -g(t)\sum_j \sigma_j^y s_j^y$$

Initial state:

$$|\psi(0)\rangle = |\psi\rangle_S \otimes |\uparrow \cdots \uparrow\rangle_B$$

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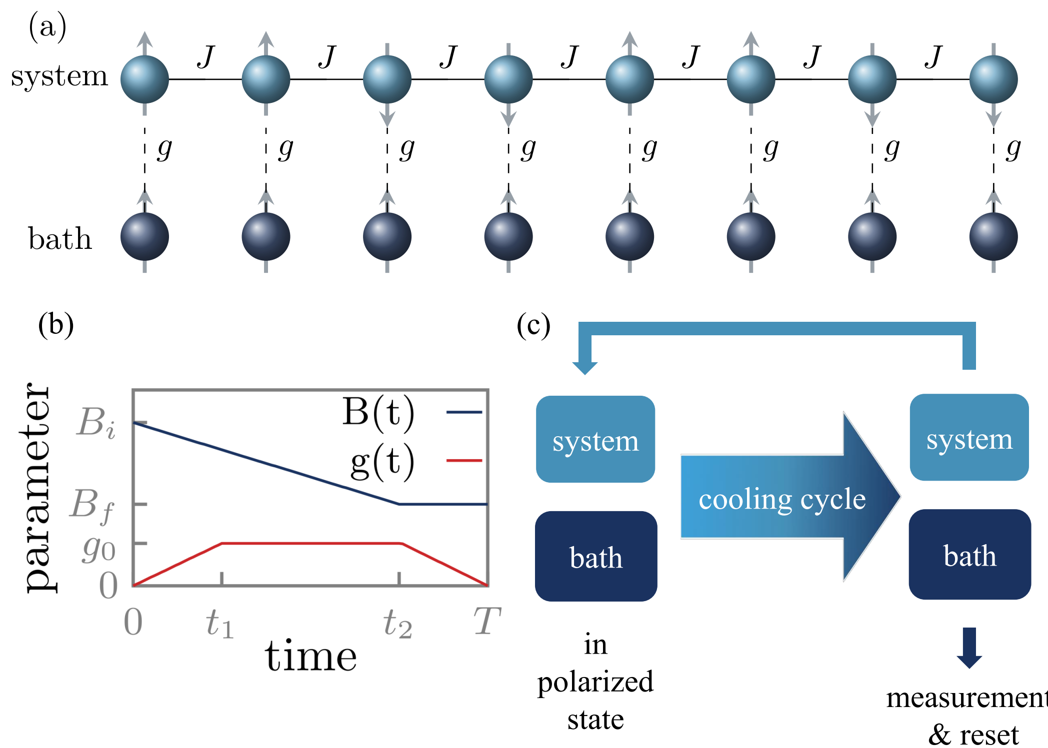

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Simulated Adiabatic Cooling

Why does this cycle cool?

Adiabatic demagnetization:

As $B(t)$ decreases, the entropy of the bath spins **increases**

$\Rightarrow$

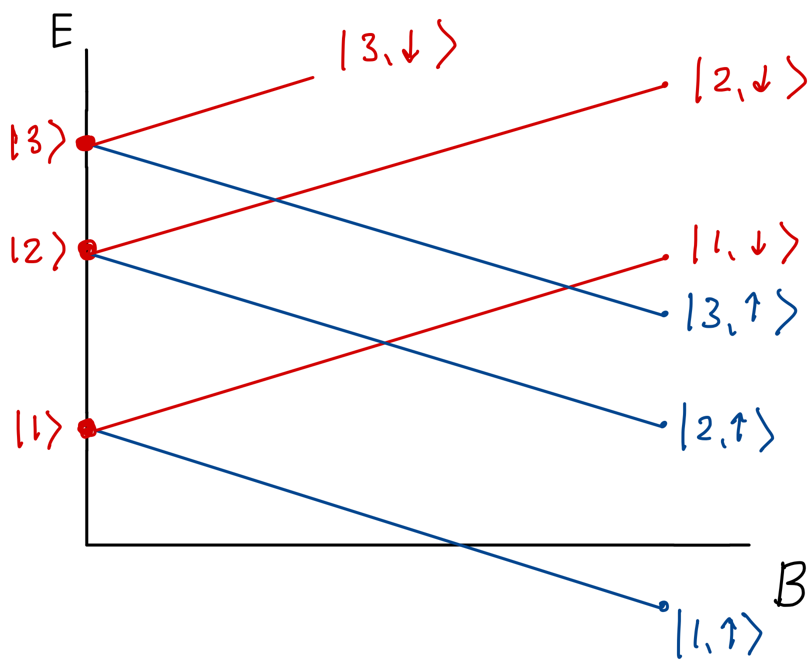
Entropy conserved

$$S_S(t) + S_B(t) = const$$

Entropy of the system **decreases**

Microsopic picture:
Imagine system with
3 states, single bath spin

No system-bath coupling ($g = 0$)



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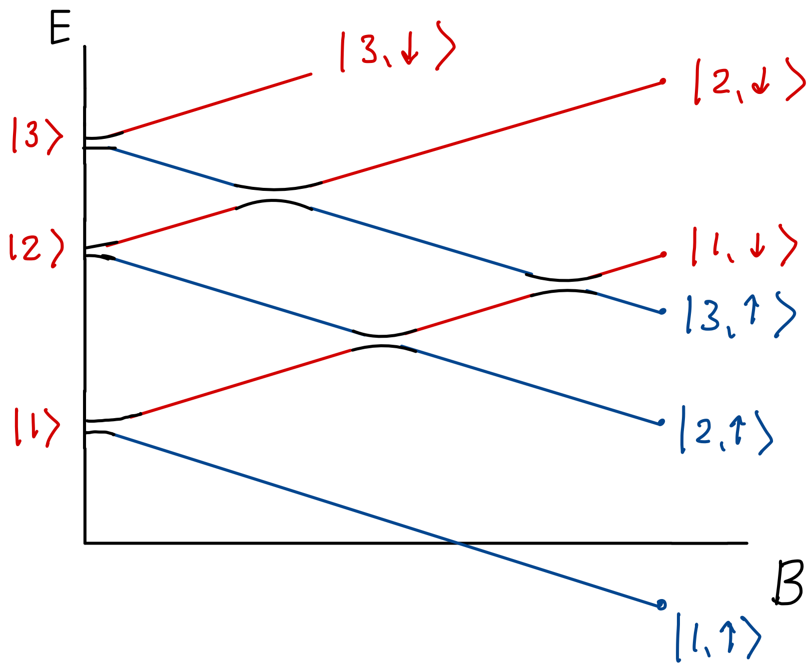
Microsopic picture:
Imagine system with 3 states, single bath spin

$$g \neq 0$$

Steady state (many cycles):

$$\rho \to |\psi_0\rangle\langle\psi_0|$$



Simulated Adiabatic Cooling

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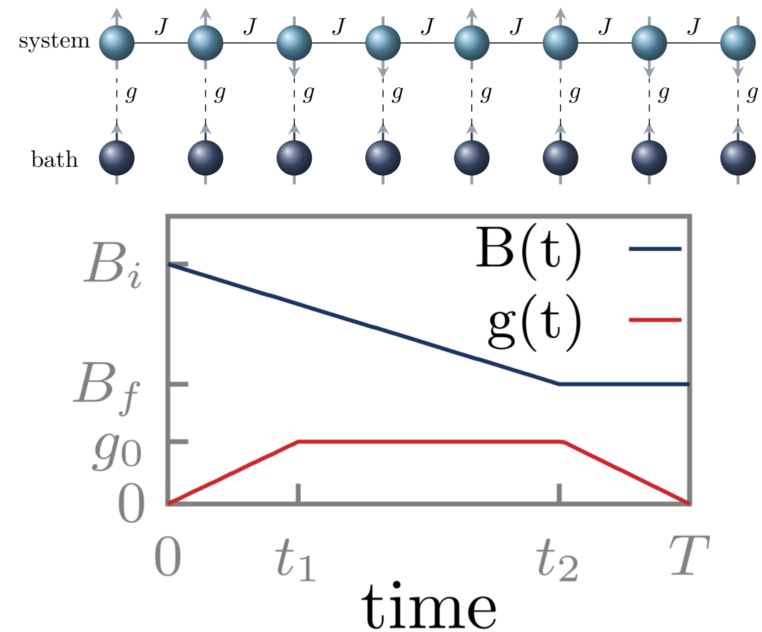


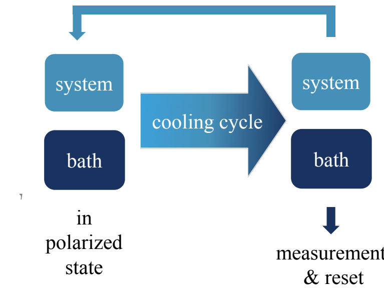
Perturbation theory in $g(t)$
(Assuming:
theormalizing system,
slow time evolution)

$$\langle \dot{H}_S \rangle \approx -4\pi g(t)^2 \int d\omega\, n_B(\omega)\omega \chi_S''(\omega)\delta(\omega - 2B(t))$$

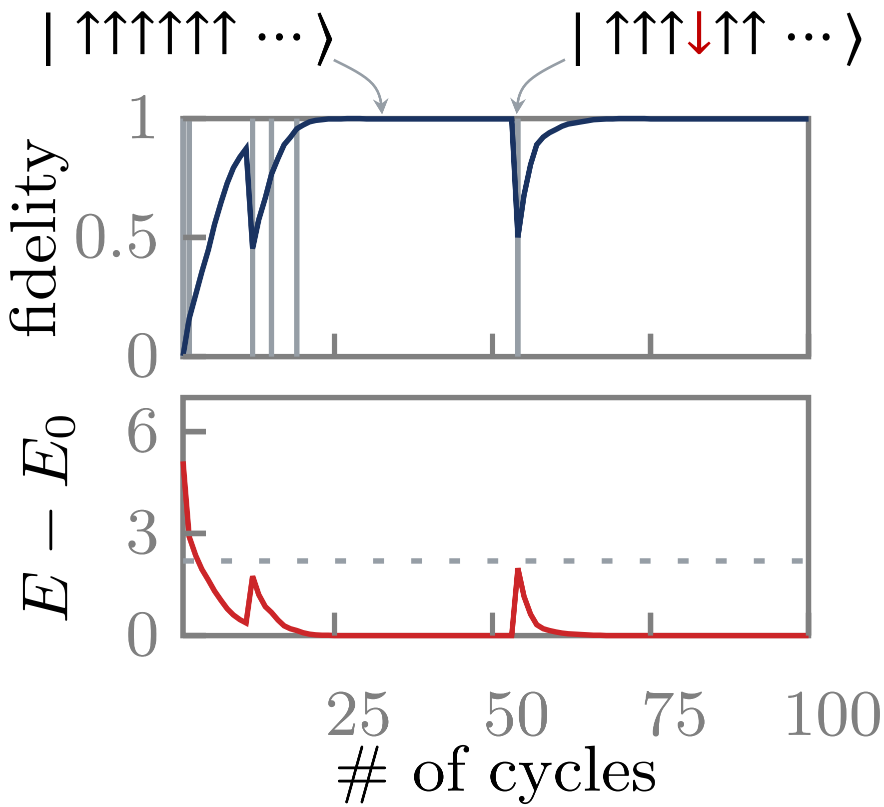
$$\chi_S''(t) = \sum_j i\theta(t)\langle [s_j^y(t), s_j^y(0)] \rangle$$

Simulated Adiabatic Cooling

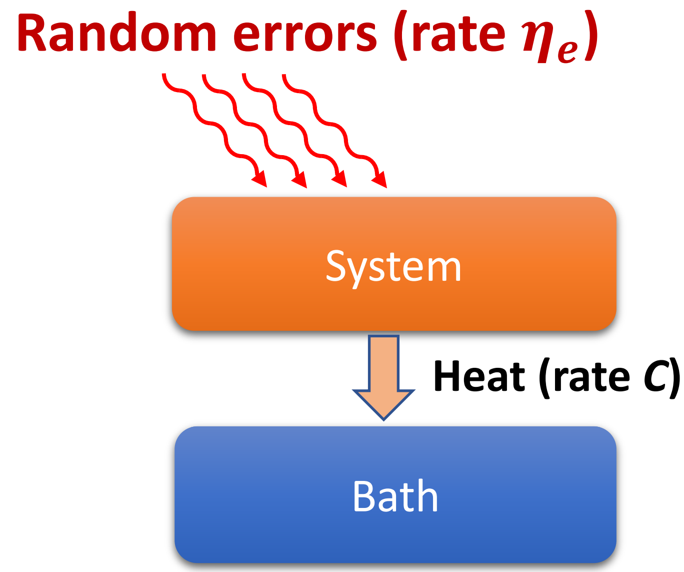
H_S: Quantum Ising model



Measured bath state:



Noise



Steady state $\hat{\rho}$ with $E = \langle \hat{H}_S \rangle = \mathrm{Tr}\left[\hat{\rho}\hat{H}_S\right] > E_0$

Energy density of steady state: $\dfrac{\partial e}{\partial t} = \dfrac{1}{L}\left\langle \dfrac{\partial \hat{H}_S}{\partial t} \right\rangle = -C(e - e_0) + \eta_e = 0$

$$e - e_0 \propto \eta_e$$

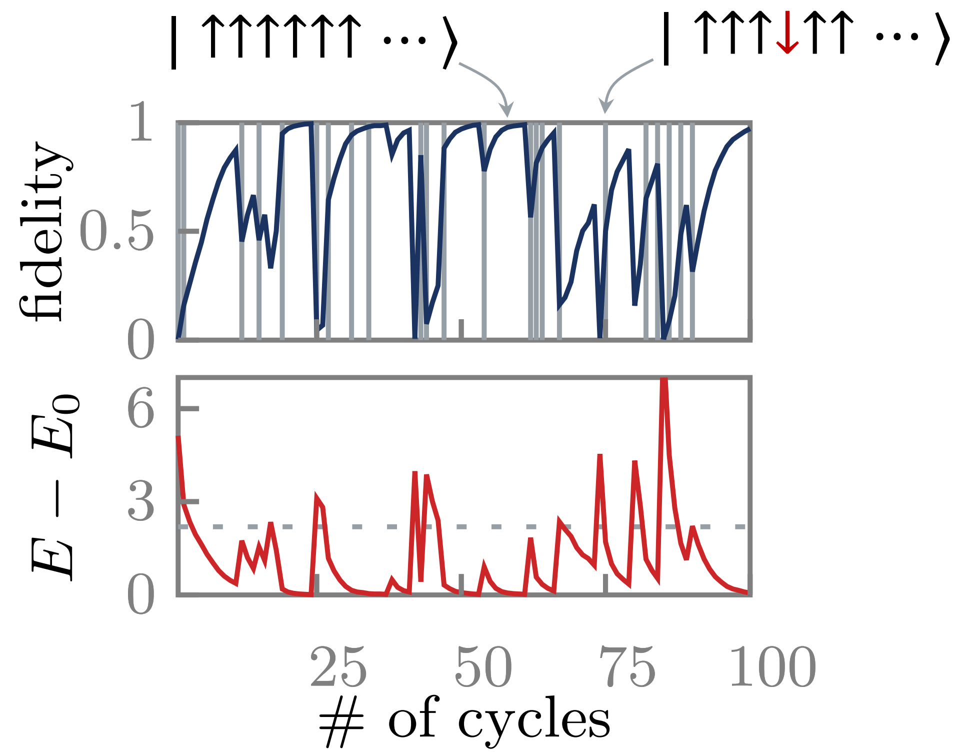
“Thermalizing” Hamiltonain:

$\rho \to \rho_{eq} \propto e^{-\hat{H}_S/T}$ for $C, \eta_e \to 0$ with fixed $\dfrac{C}{\eta_e}$

Noise

Depolarizing noise rate $\eta_e = 2 \cdot 10^{-2}$

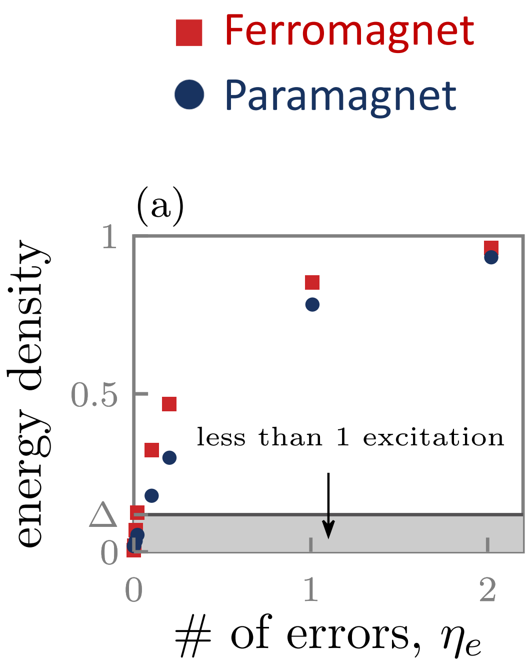
Measured bath state:



Improve energy by post selection:

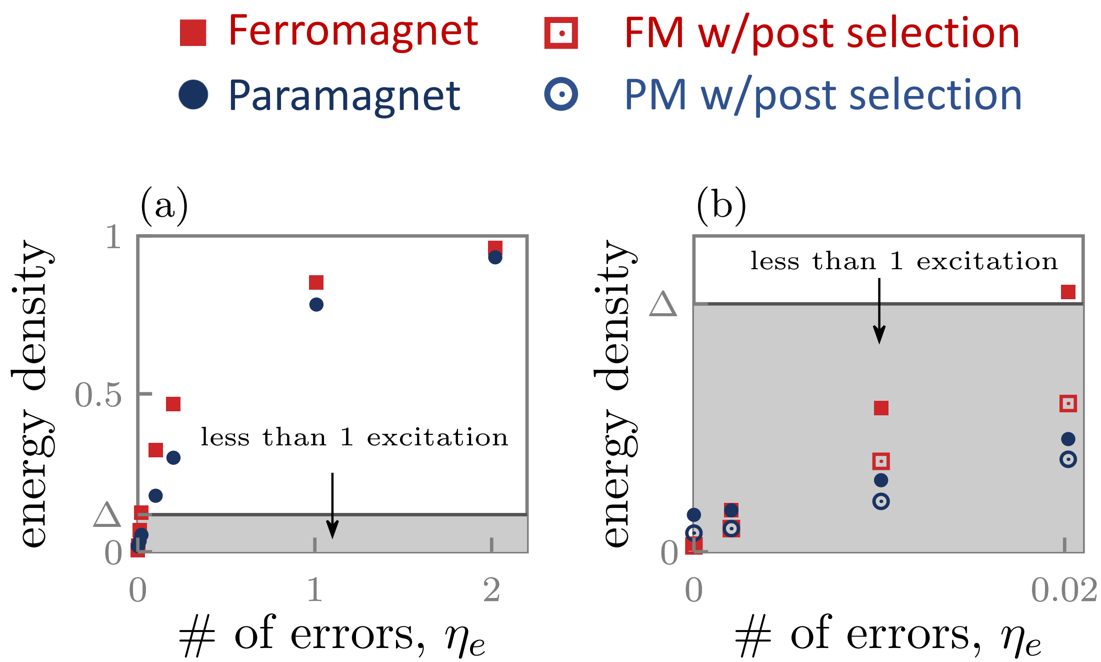
Measure system only when bath measurement outcome is $|\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow \cdots\rangle$

Noise



η_e: average number of depolarizing errors (phase/bit flip) per spin per cycle

Noise

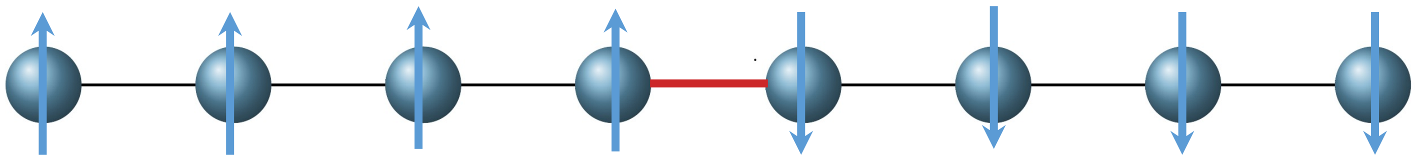


η_e: average number of depolarizing errors (phase/bit flip) per spin per cycle

- **FM is more difficult to cool**
- **Post-selection decreases energy**

Topological vs. Trivial excitations

FM: Domain wall excitations



Cannot be removed by local coupling to bath!

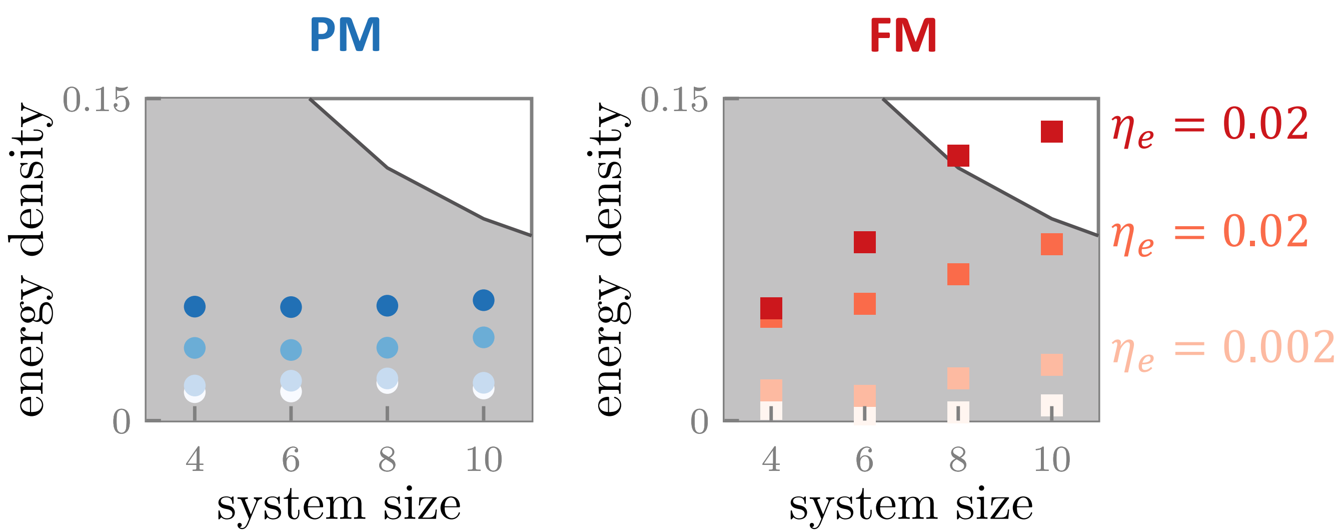
Energy of steady state: $\frac{\partial e}{\partial t} = \frac{1}{L}\left\langle \frac{\partial \hat{H}_S}{\partial t} \right\rangle = -C(e - e_0)^2 + \eta_e = 0$

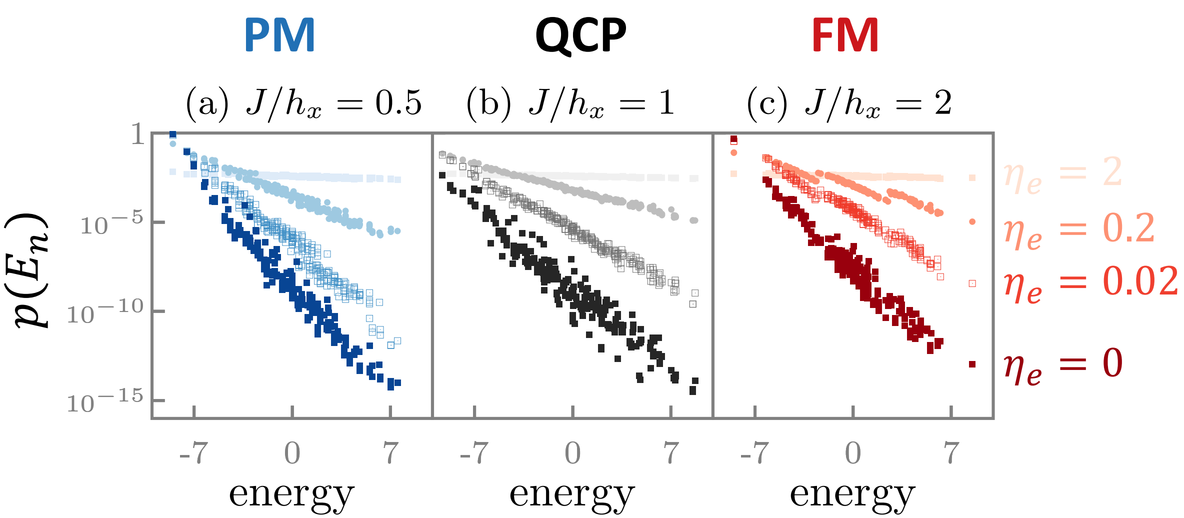
$$e - e_0 \propto \sqrt{\eta_e}$$

Finite $d = 1$ system (# of excitations = O(1)):

$$e - e_0 \propto \eta_e L$$

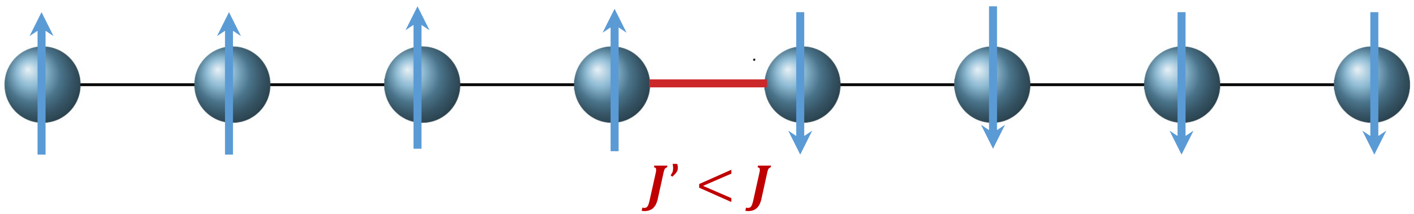
Topological vs. Trivial excitations



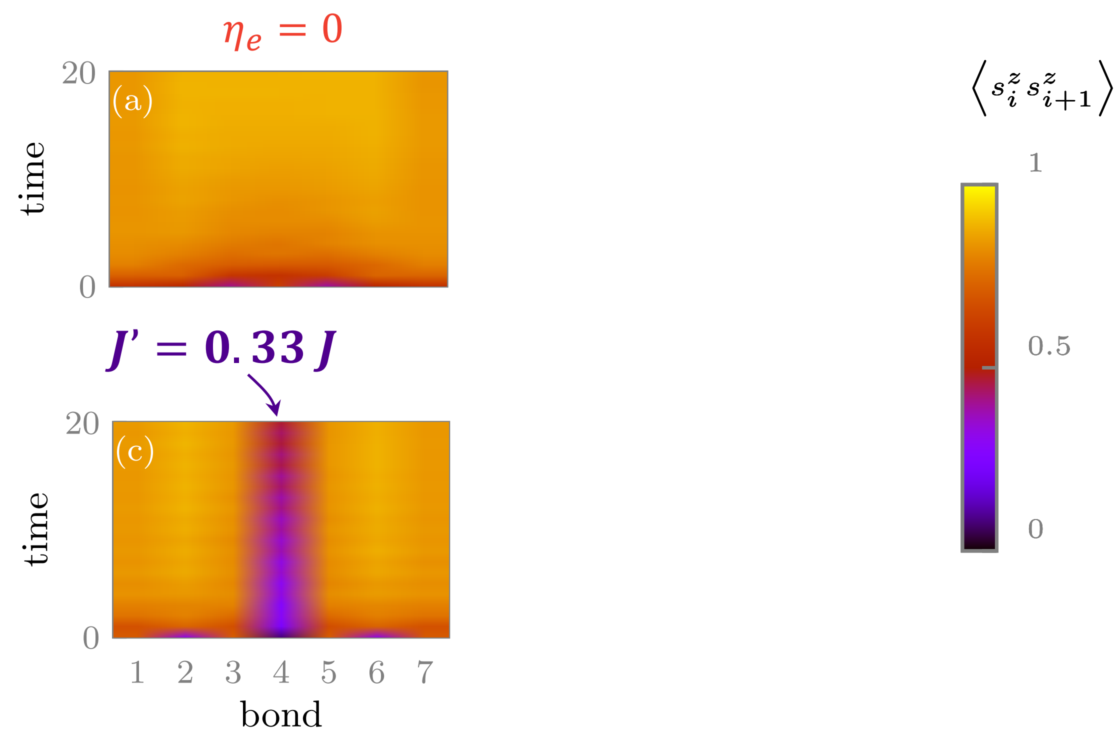


Topological vs. Trivial excitations

"Trapping" topological excitations



Reduce the number of *mobile* excitations



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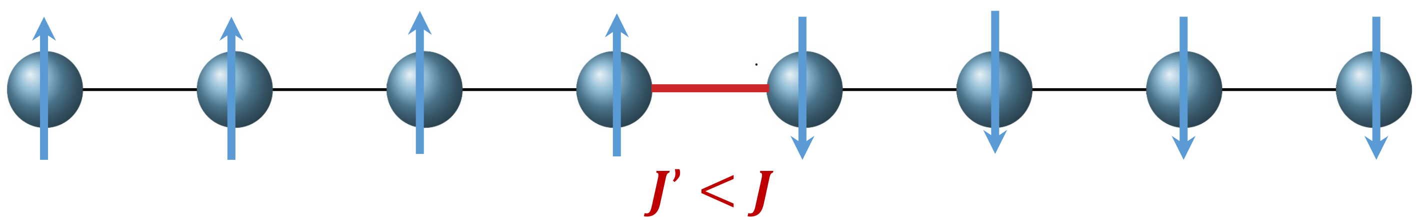

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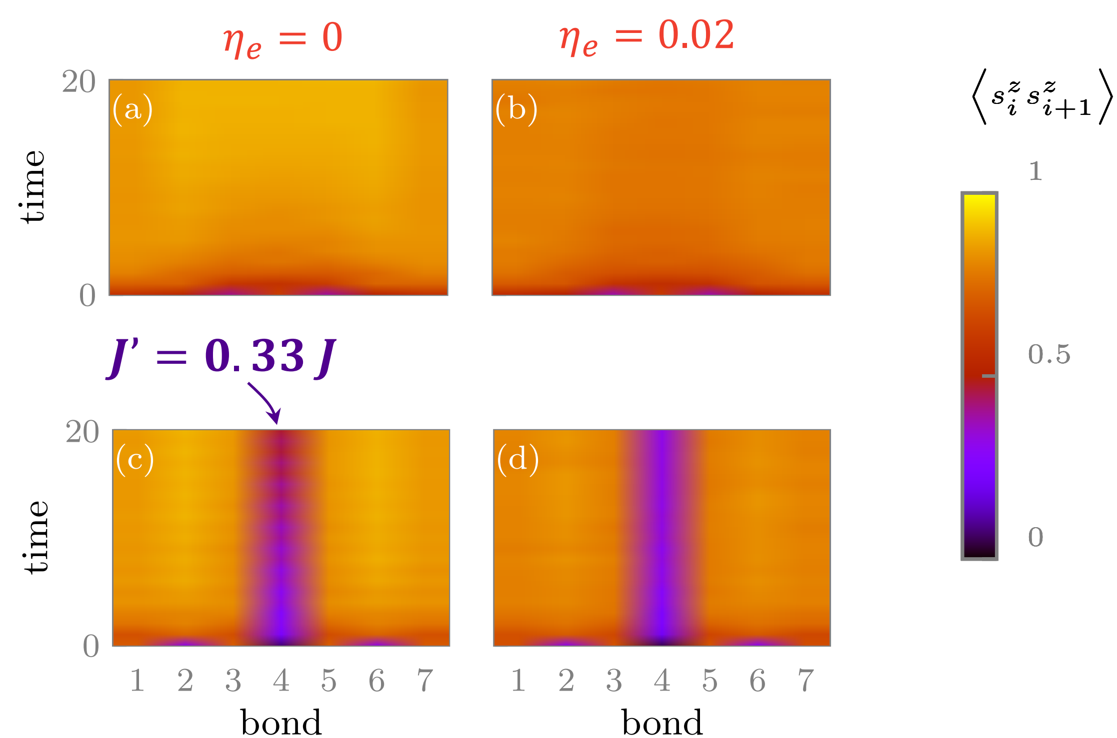
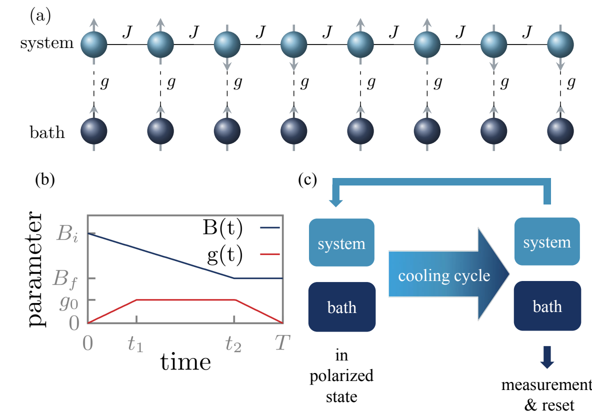


Summary

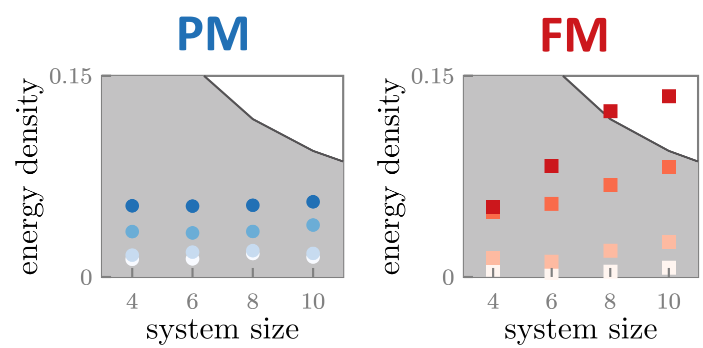
- ***Simulated adiabatic cooling: Robust method to find low-energy states of generic Hamiltonians.***



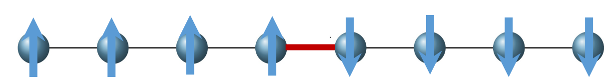
- ***Topological excitations are harder to cool:***

$$e - e_0 \propto \eta_e^{1/M}$$

M: # of excitations that can be removed locally



- ***Reduce # of mobile topological excitations by trapping***



Thank you.