Lieb-Schultz-Mattis theorem for 1d quantum magnets with magnetic space group symmetries

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In collaboration with

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(RIKEN)

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Linhao Li
(ISSP)

arXiv:2211.?????

Outline

- **Introduction:** non-featureless spectrum and conventional LSM thm
- **LSM thm for magnetic space groups (MSGs)**
  - ➢ Symmetry-twisting method
  - ➢ Lattice homotopy
- **LSM constraints from MSGs in 1d**
- **Summary**

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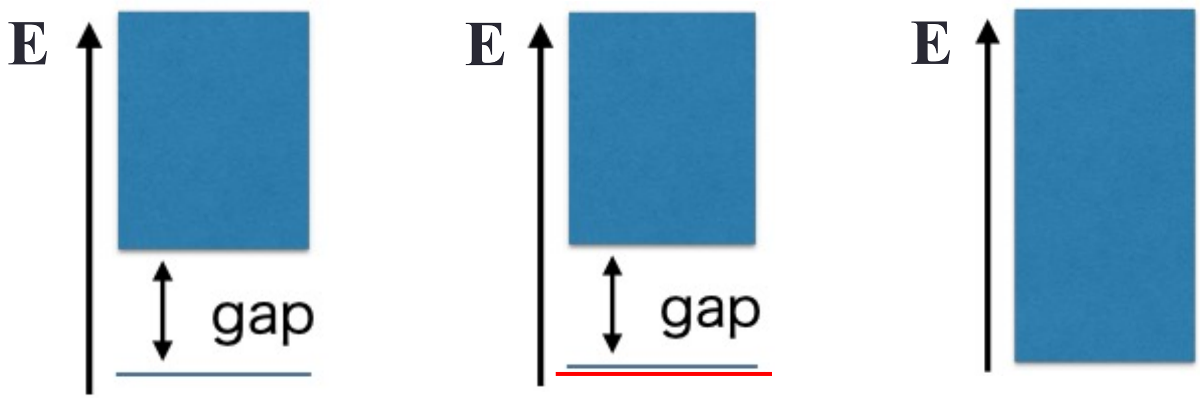
The question

Given a quantum system w/ a Hamiltonian, it's natural to ask

- whether there is a spectral **gap**
- what are the properties of ground state(s) / low-energy states

…

energy
spectrum



w/ degeneracy

gapless/critical

The question

Given a quantum system w/ a Hamiltonian, it's natural to ask

- whether there is a spectral **gap**
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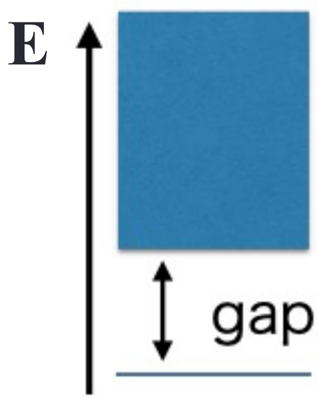
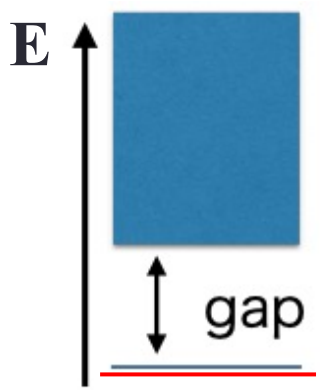
…

➢Easy for *free* systems, but in general hard for *interacting* systems

energy
spectrum





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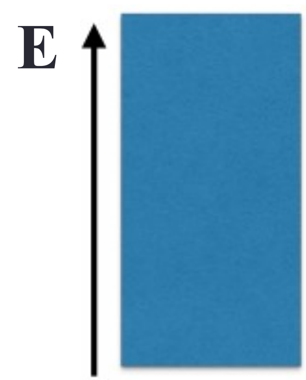
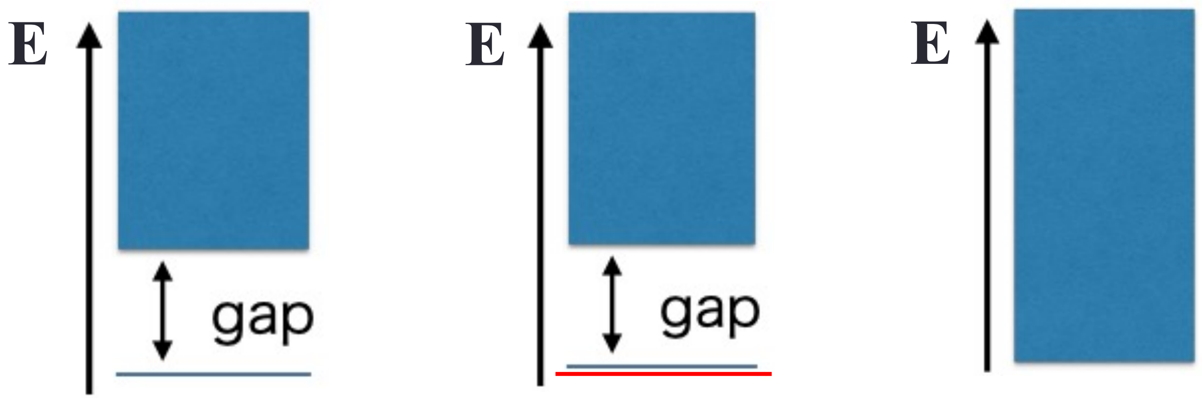


gapless/critical

The question

Let’s be more specific:

energy
spectrum



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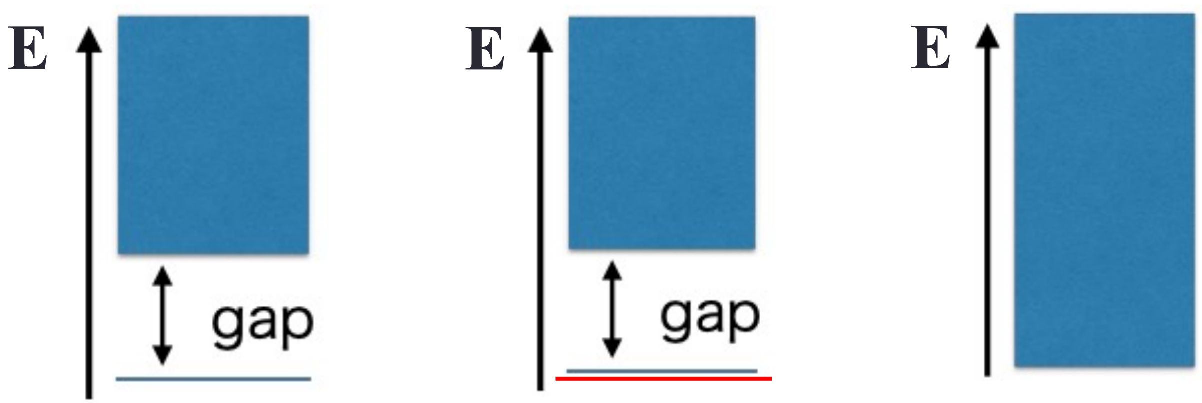
- Knowing just the **microscopic DOF** and the **symmetries** of the Hamiltonian, what low-energy properties can we determine?

energy spectrum



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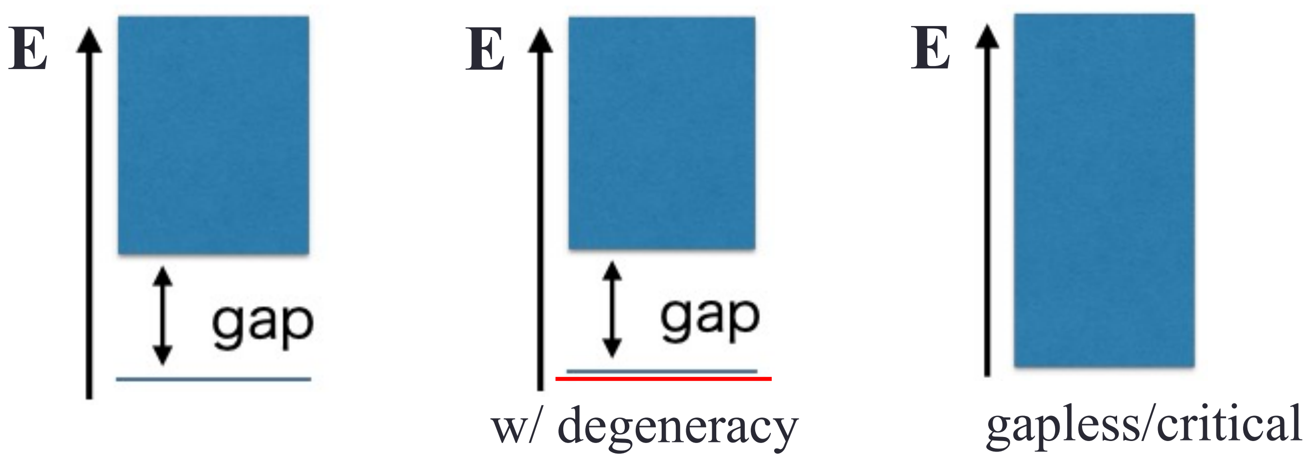

featureless

non-featureless

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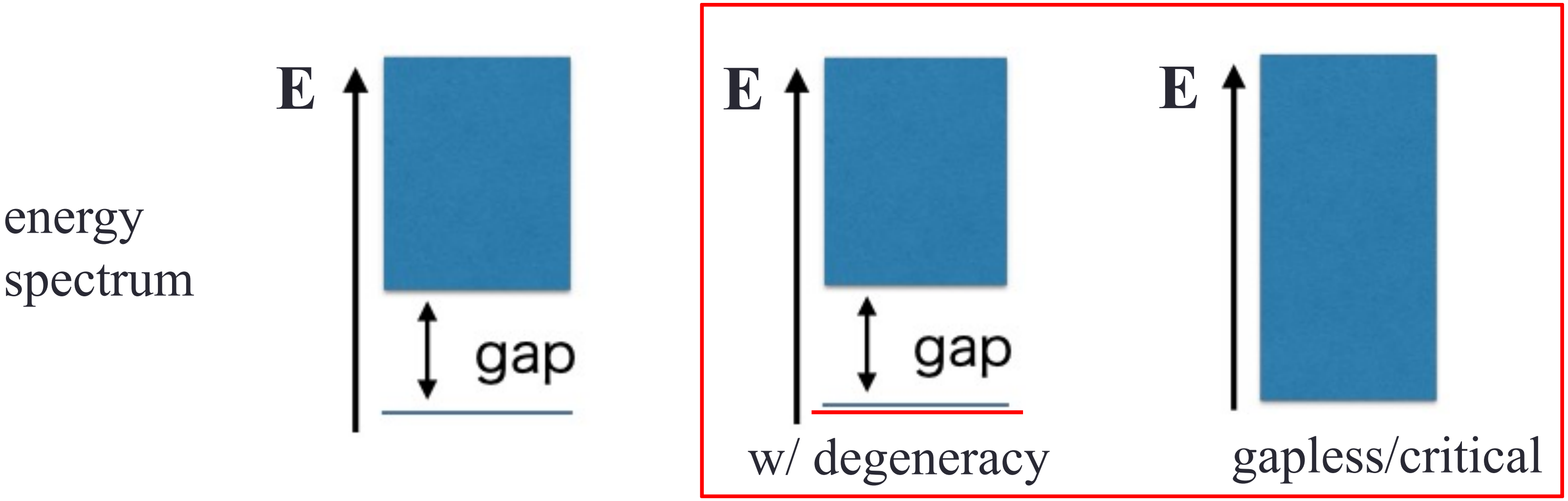
➢ It determines when the system must be **non-featureless**.



featureless

non-featureless

1d spin chains w/ spin-rotation and translation symm

$$H_{\mathrm{HAF}}^{s=1} = \sum_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$

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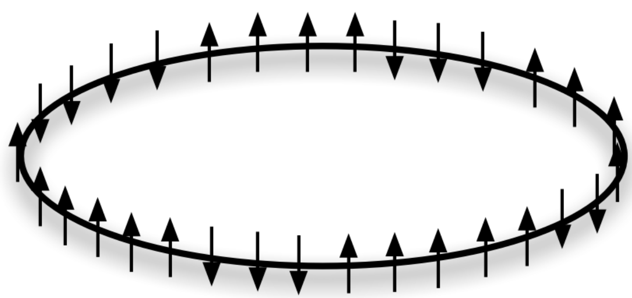

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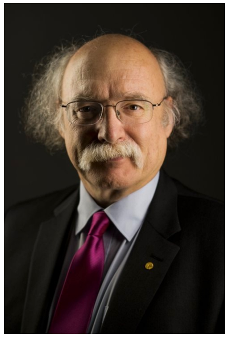
$$H^{s=1}_{\mathrm{HAF}} = \sum_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$

unique gap

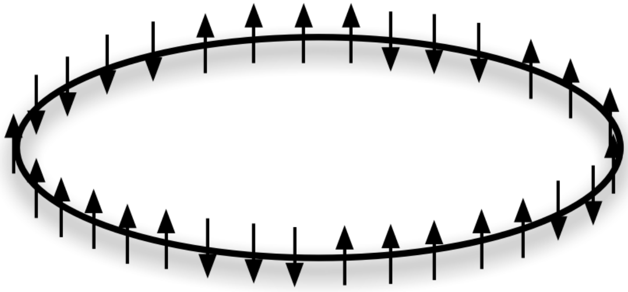
$$H^{s=1/2}_{\mathrm{HAF}} = \sum_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$

gapless



"Haldane's conjecture"

[Haldane ('83)]



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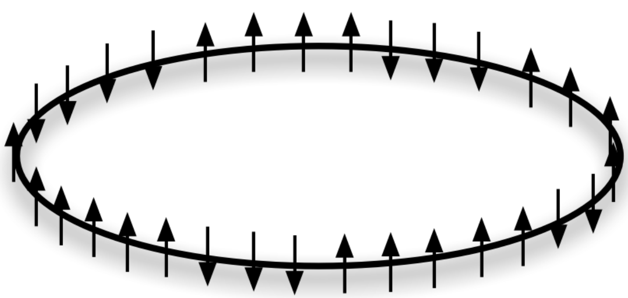
$$H_{\mathrm{MG}}^{s=1/2} = \sum_i \left(\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + \frac{1}{2} \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+2} + \frac{3}{8} \right)$$

dimerized

(gapped w/ 2-fold deg.)



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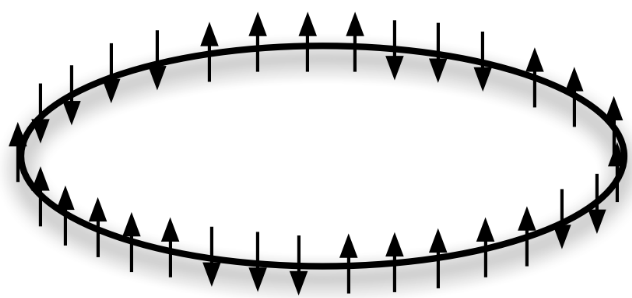
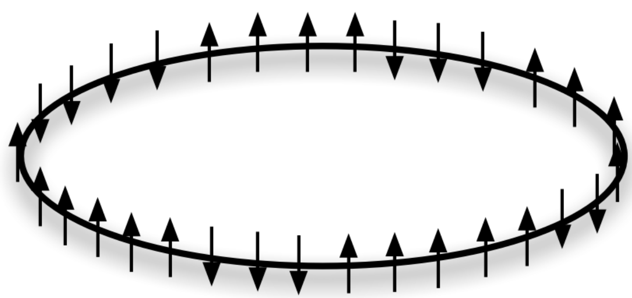


1d spin chains w/ spin-rotation and translation symm

In fact, *any* spin-half-integer chain is *always* non-featureless, indicated by the **Lieb-Schultz-Mattis (LSM)** theorem [Lieb-Schultz-Mattis ('61)]:

> *A 1d antiferromagnetic spin chain* ***cannot*** *have a unique gapped GS if the spin per unit cell is half-integral and if the lattice-transl and spin-rotation symm are strictly imposed.*



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Various versions of LSM theorem

Various versions of LSM thm for 1d quantum magnets w/ diff symm $G_{int}\times G_{space}$ were known: [Watanabe et al. (15); Po et al. (17); Ogata et al. (18, 20); Else-Thorngren (19); Yao-Oshikawa (21)]

- $G_{int} = SO(3)$ or $\mathbb{Z}_2\times\mathbb{Z}_2$ or $\mathbb{Z}_2^T$, $G_{space} = \mathbb{Z}^{Tr}$
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($Tr =$ transl, $T =$ time reversal, $I_s =$ site-centered inversion)

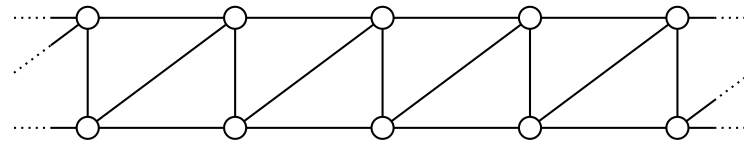
However, there are non-featureless systems not respecting the above symm, e.g.

$$\mathcal{H} = J \sum_i (-1)^i \vec{S}_i \cdot (\vec{S}_{i+1} \times \vec{S}_{i+2})$$

Chiral ladder w/ 3-spin interactions

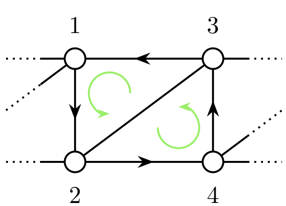
[Schmoll et al. ('18)]

$$H = \sum_i J_i \, \mathbf{S}_i \cdot (\mathbf{S}_{i+1} \times \mathbf{S}_{i+2})$$



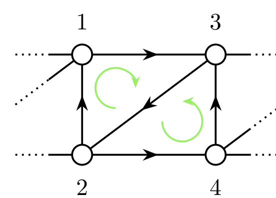
- $J_i = \pm 1$: $SO(3) \times \mathbb{Z}^{Tr}$ (cases H_3, H_4)
- $J_i = \pm(-1)^i$: $SO(3) \times \mathbb{Z}^{Tr \cdot T}$ (cases H_1, H_2)

H_1



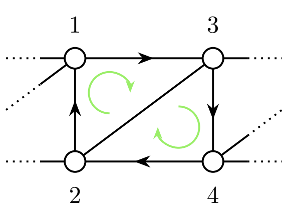
$H_1 = +\vec{S}_1 \cdot \left(\vec{S}_2 \times \vec{S}_3\right) - \vec{S}_2 \cdot \left(\vec{S}_3 \times \vec{S}_4\right)$

H_3



$H_3 = -\vec{S}_1 \cdot \left(\vec{S}_2 \times \vec{S}_3\right) - \vec{S}_2 \cdot \left(\vec{S}_3 \times \vec{S}_4\right)$

H_2



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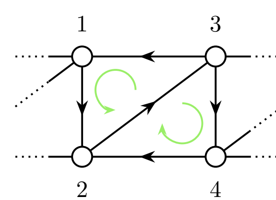
H_4



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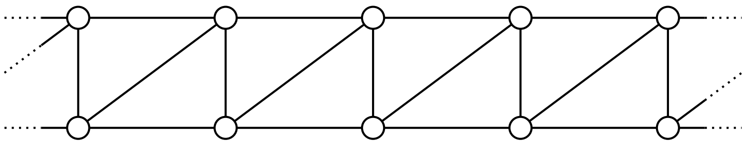

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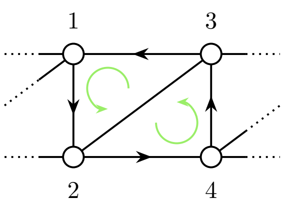
H_1



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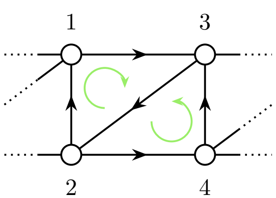

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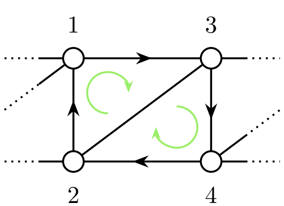

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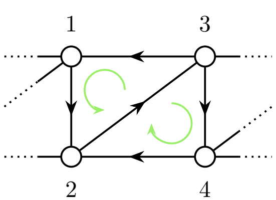

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Chiral ladder w/ 3-spin interactions

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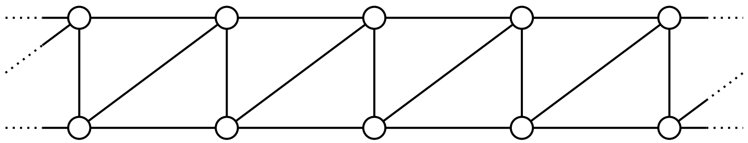

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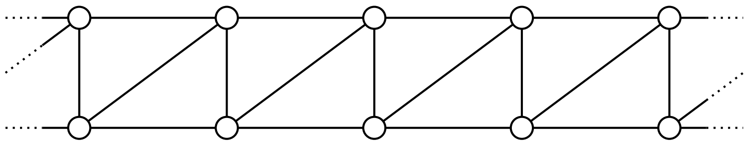

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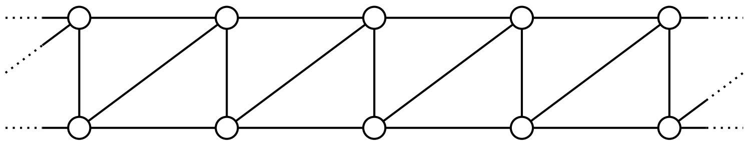

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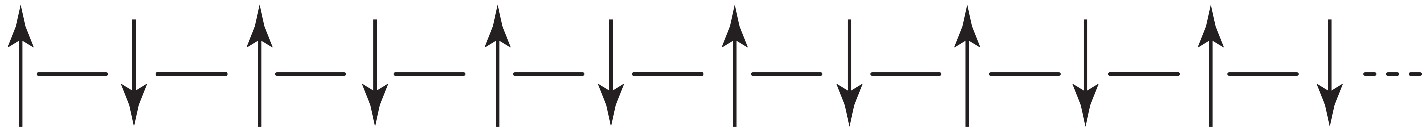
Is such a non-featureless H dictated by spin-rot and time-rev-transl symm?

LSM thm for spin-rot and time-rev-transl symm

Indeed, 1d quantum magnets w/ spin-rot = $SO(3)$ or $\mathbb{Z}_2 \times \mathbb{Z}_2$ and time-rev-transl = $\mathbb{Z}^{(Tr \cdot T)}$ symm must be **non-featureless**



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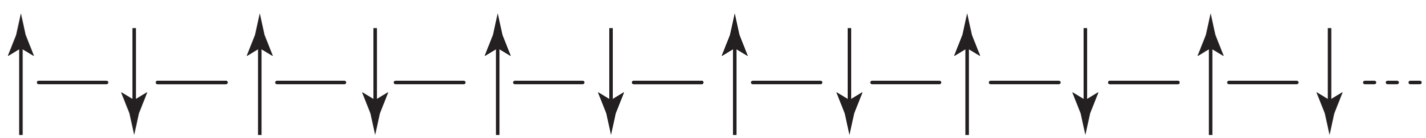
- Assume H has $(\mathbb{Z}_2 \times \mathbb{Z}_2) \times \mathbb{Z}^{(Tr\cdot T)}$ symm:

$$\mathbb{Z}_2 \times \mathbb{Z}_2 : \quad R_x^\pi = \exp(i\pi \sum_r S_r^x); \; R_z^\pi = \exp(i\pi \sum_r S_r^z)$$

$$\mathbb{Z}^{(Tr\cdot T)} : \quad A \equiv Tr \cdot T, \quad A\vec{S}_r A^{-1} = -\vec{S}_{r+1}; \; AiA^{-1} = -i$$



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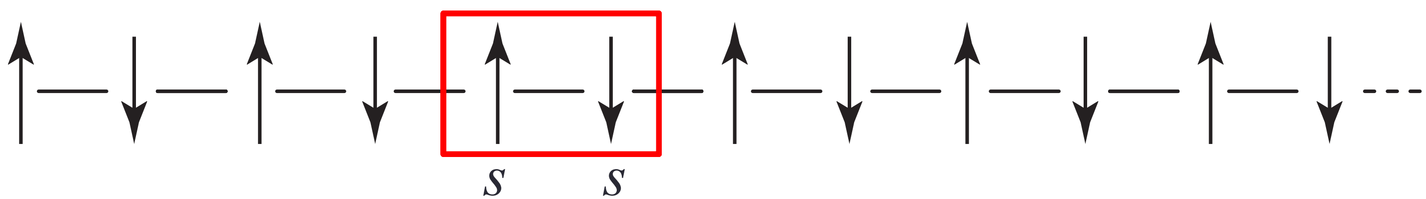
- # of spins per unit cell: $s + s = 2s \in \mathbb{Z}$
  - ➢ Spectrum is not constrained by conventional LSM thm



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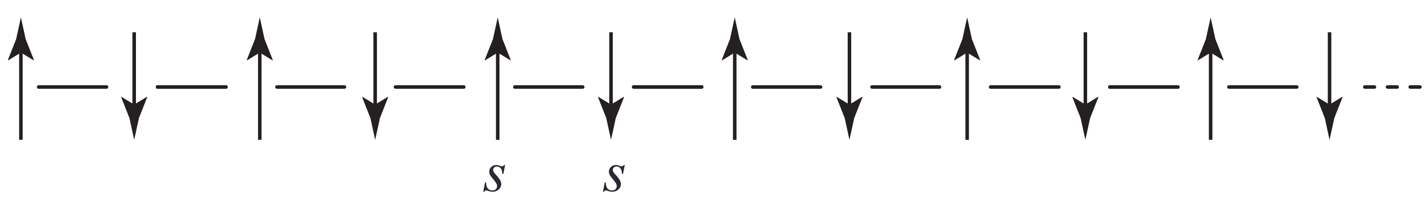
LSM thm for spin-rot and time-rev-transl symm

Claim: *H,* w/ **half-integer *s*** per **site**, has a doubly degenerate spectrum under the following twisted BC:

$$\vec{S}_{r+L} = R_x^{\pi} \vec{S}_r (R_x^{\pi})^{-1} \equiv \tilde{\vec{S}}_r, \quad L \in 2\mathbb{Z}$$



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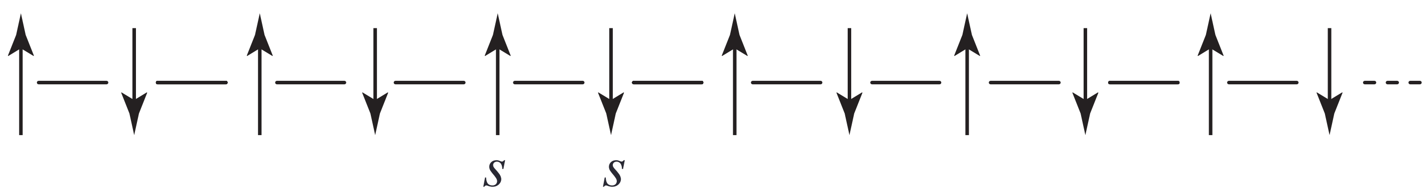
Ex: $$\mathcal{H}_{\mathrm{PBC}} = J \sum_{r=0}^{L-1} (-1)^r \vec{S}_r \cdot (\vec{S}_{r+1} \times \vec{S}_{r+2})$$

$$\mathcal{H}_{\mathrm{TBC}} = J \sum_{r=0}^{L-3} (-1)^r \vec{S}_r \cdot (\vec{S}_{r+1} \times \vec{S}_{r+2}) + \vec{S}_{L-2} \cdot (\vec{S}_{L-1} \times \tilde{\vec{S}}_0) - \vec{S}_{L-1} \cdot (\tilde{\vec{S}}_0 \times \tilde{\vec{S}}_1)$$



LSM thm for spin-rot and time-rev-transl symm

Claim: *H*, w/ **half-integer *s*** per **site**, has a doubly degenerate spectrum under the following twisted BC:

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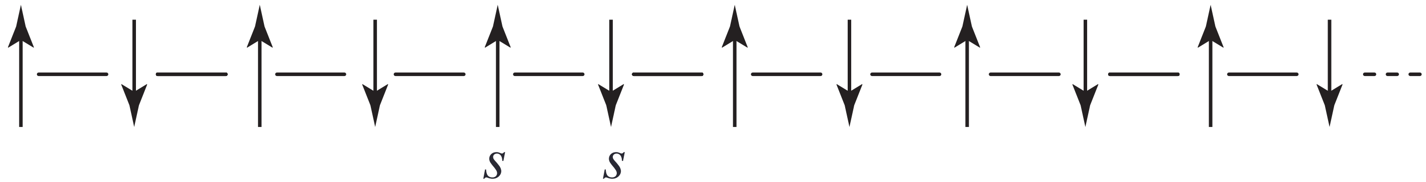

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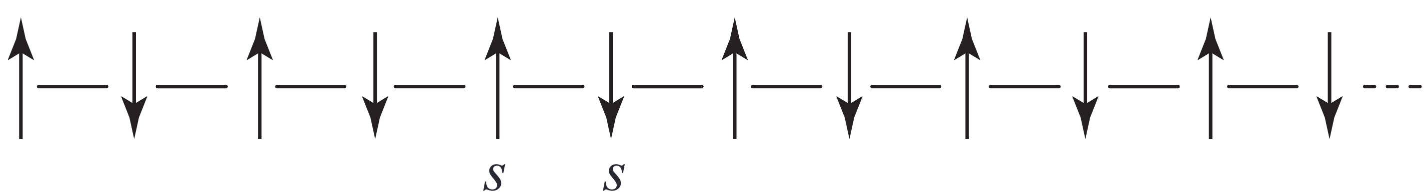

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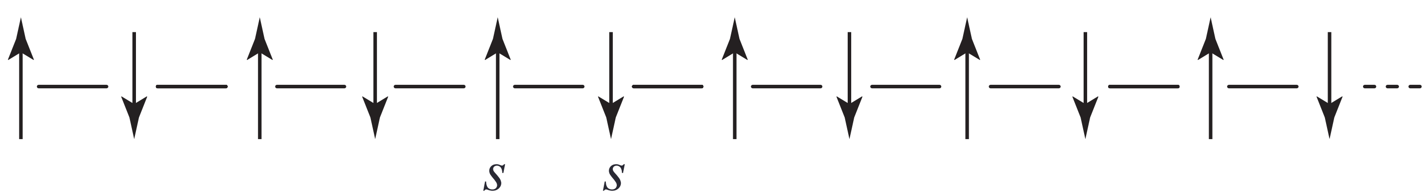

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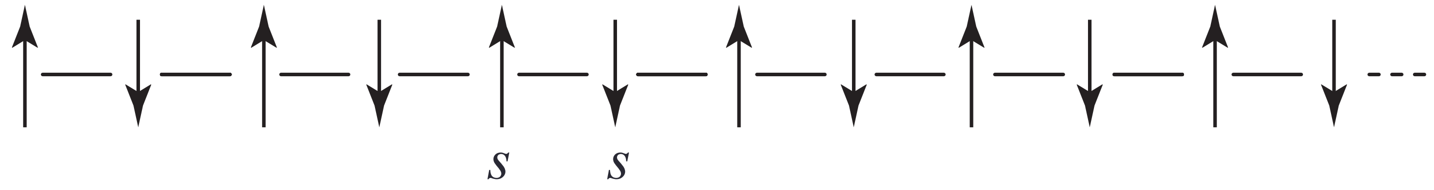

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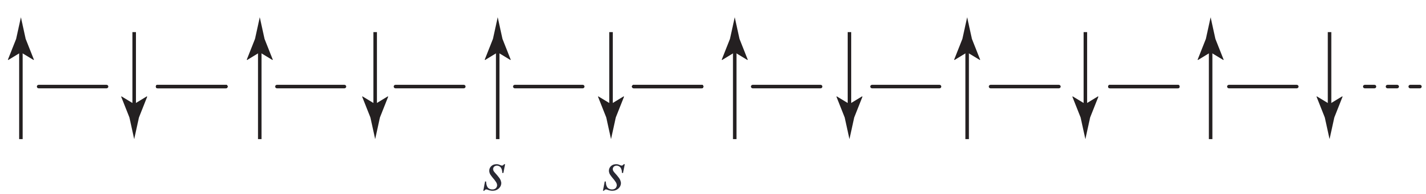
H_{PBC} cannot have a unique gapped GS as well!



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- Nevertheless, all these cases can be understood from lattice homotopy

  [Po et al. ('17); Else-Po-Watanabe ('18); Else-Thorngren ('19)]

  (and also from bulk-bdry corresp of 2d crystalline SPTs)

Outline

- **Introduction:** non-featureless spectrum and conventional LSM thm
- **LSM thm for magnetic space groups (MSGs)**
  - Symmetry-twisting method
  - Lattice homotopy
- **LSM constraints from MSGs in 1d**
- **Summary**

Projective rep and anomalous texture

A representation U of G satisfies

$$U(gh) = \omega(g,h)U(g)U(h), \quad g,h \in G \qquad\qquad \omega(g,h) = e^{i\phi(g,h)}$$

- If $\omega(g,h)$ is trivial $(= 1)$, U is called a *linear rep*
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Ex1: $G = SO(3)$, one can choose $U(\hat{n},\theta) = e^{i\theta\hat{n}\cdot\vec{S}}$

$$U(\hat{n},\theta)U(\hat{n},2\pi-\theta) = e^{2\pi i\hat{n}\cdot\vec{S}} = (-1)^{2s} = (-1)^{2s}U(\mathbb{1})$$

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$$U(R^{\pi}_{\hat{n}})U(R^{\pi}_{\hat{n}}) = e^{2i\pi S_n} = (-1)^{2s} = (-1)^{2s}U(\mathbb{1})$$

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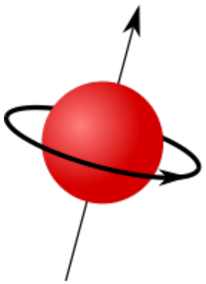

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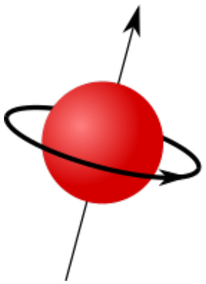

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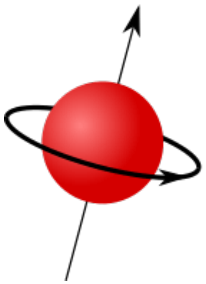

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Higher-dim system of spins?

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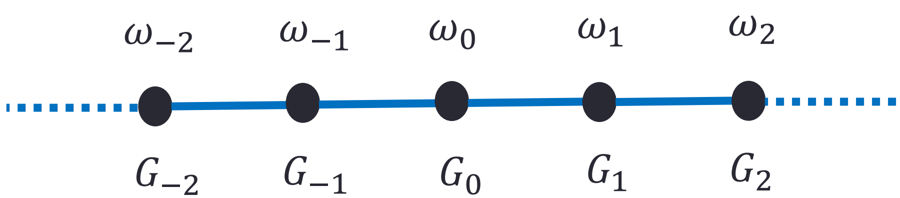
A "proj rep" of $G = G_{int} \times G_{space}$ acting on a d-dim system is defined as an *anomalous texture* which corresp to consistent assignments of proj rep of the isotropy group at each site r [Else-Thorngren ('19)]

- Isotropy group $G_r \subseteq G$ leaving r invariant
- Anomalous texture = $\{\omega_r \in H^2\big(G_r, U(1)\big)\}$

Ex1: $G_{int} = SO(3)$, $G_{space} = \mathbb{Z}^{Tr}$

$d = 1$



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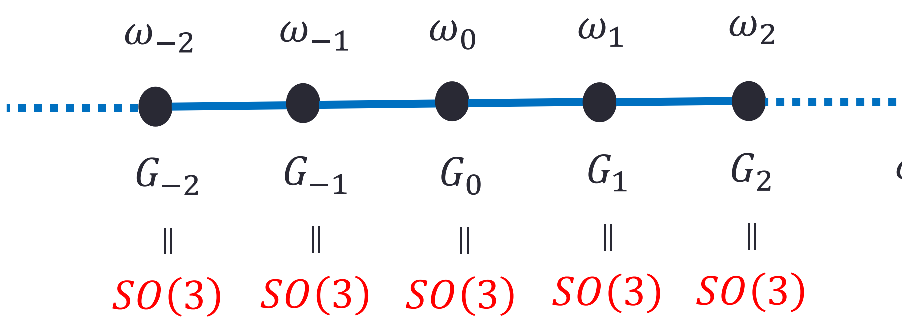

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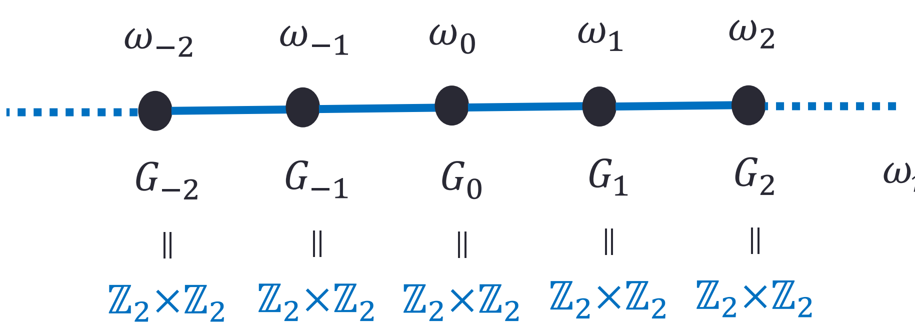

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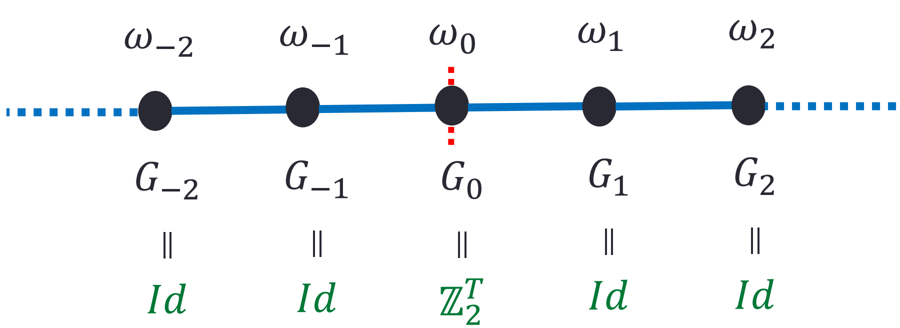

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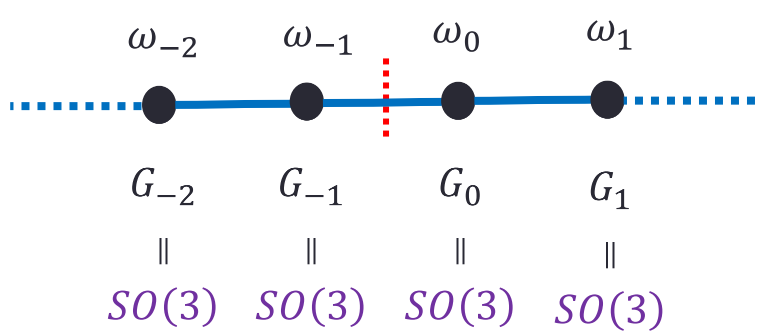

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Lattice homotopy

Equiv relation for anom texture:

- Symm deform: $h_t(g \cdot r) = g \cdot h_t(r),\ \ g \in G_{space}, t \in [0,1]$
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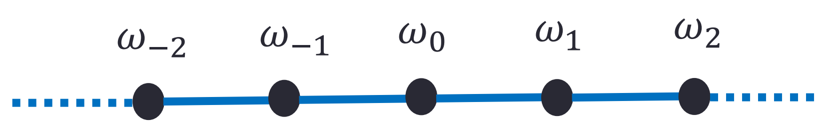

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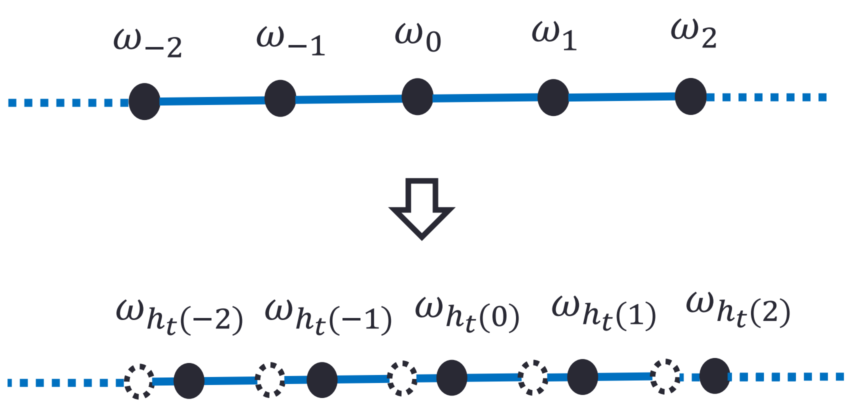

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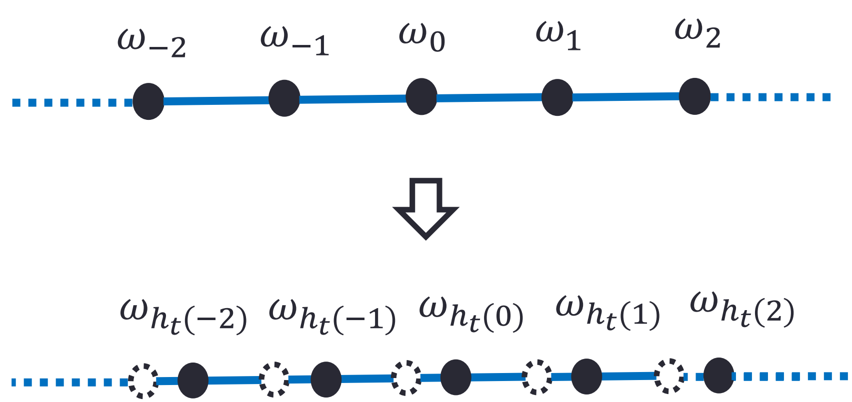

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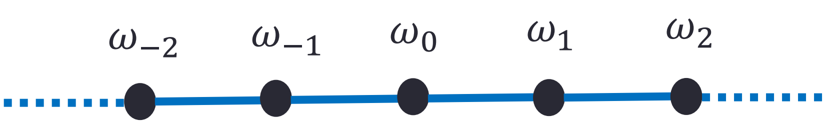

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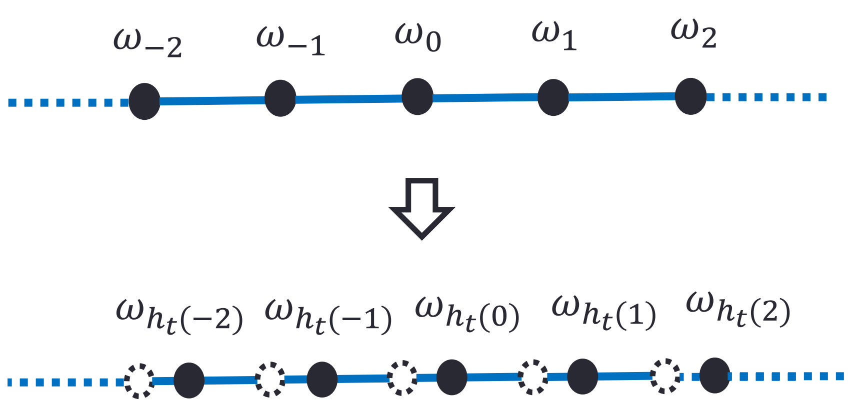

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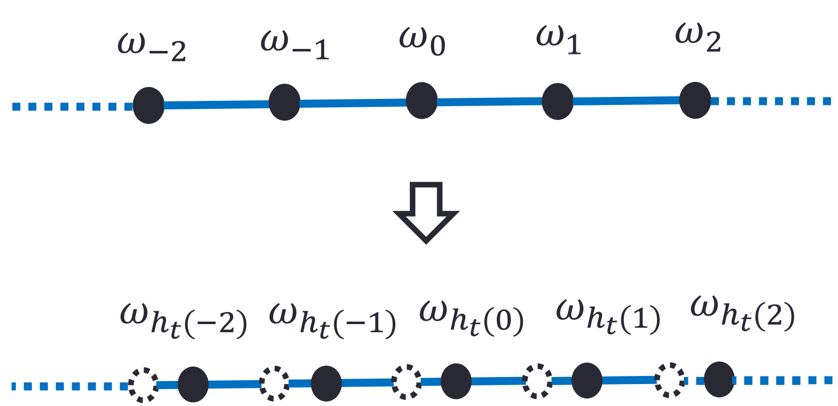

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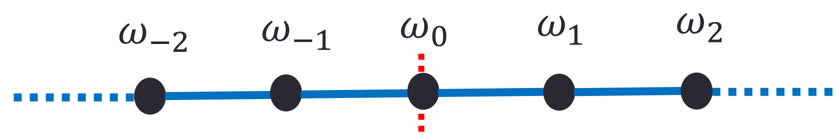

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Ex3: $G_{int} = Id,\ G_{space} = \mathbb{Z}_2^{I_s \cdot T}$

$$\omega_{-r} = \omega_r$$

$$\omega_{r \neq 0} \in H^2\big(Id, U(1)\big) = \mathbb{Z}_0$$
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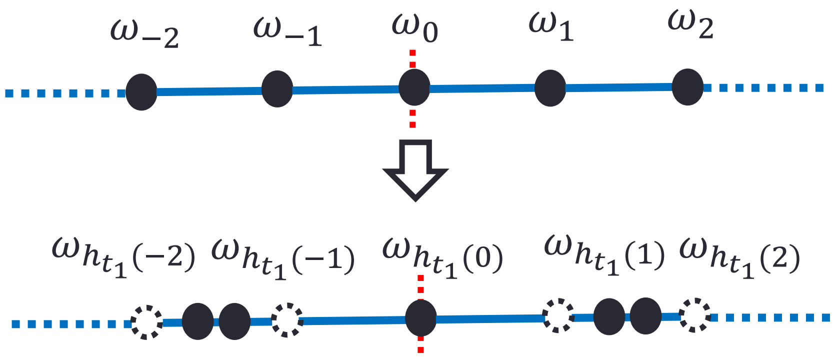

Lattice homotopy

Equiv relation for anom texture:

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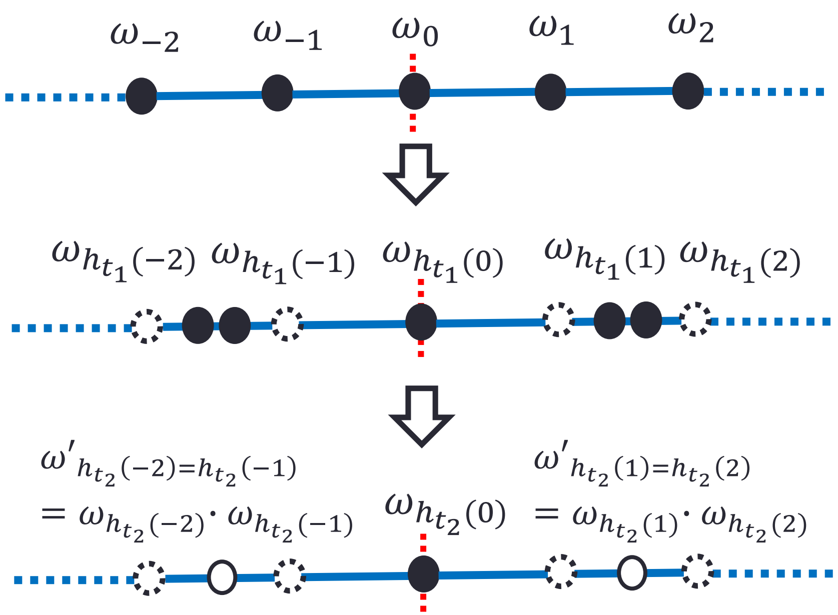

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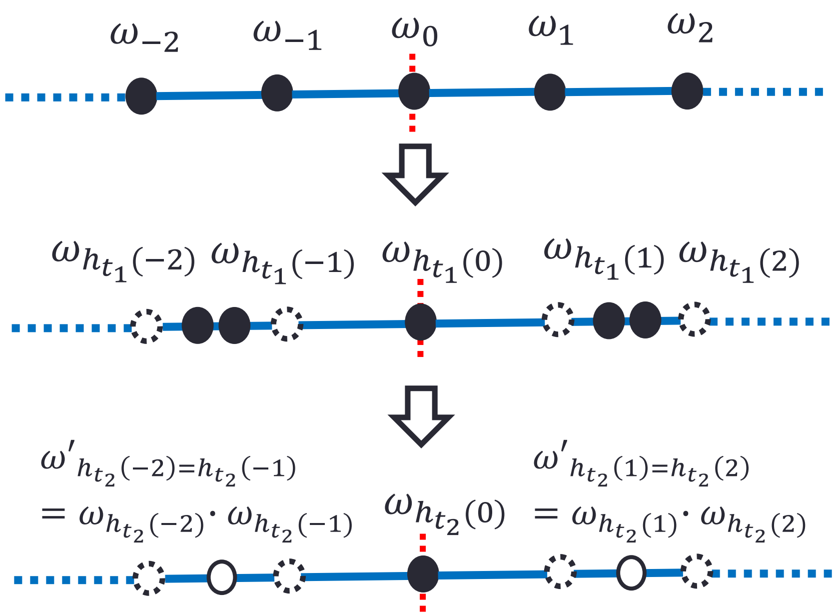

Half-integer *s* at inversion center $r = 0$:
$\{\omega_r\}$ is nontrivial => **non-featureless**

Lattice homotopy

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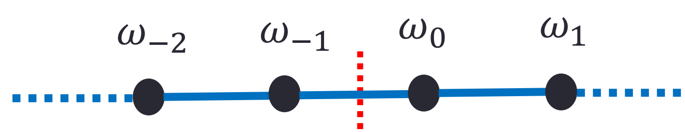
Ex4: $G_{int} = SO(3),\ G_{space} = \mathbb{Z}_2^{I_b}$



$$\omega_{-r-1} = \omega_r$$

$$\omega_r \in H^2\big(SO(3), U(1)\big) = \mathbb{Z}_2$$

Lattice homotopy

Equiv relation for anom texture:

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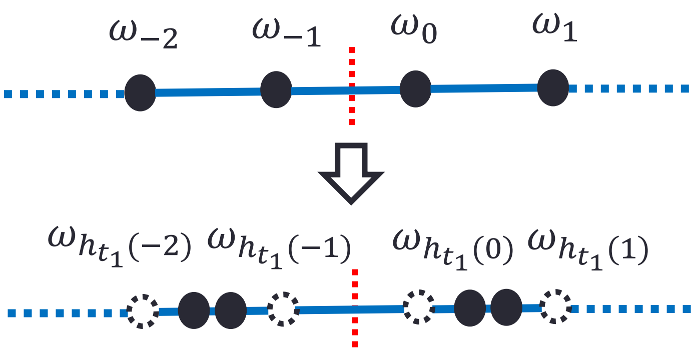

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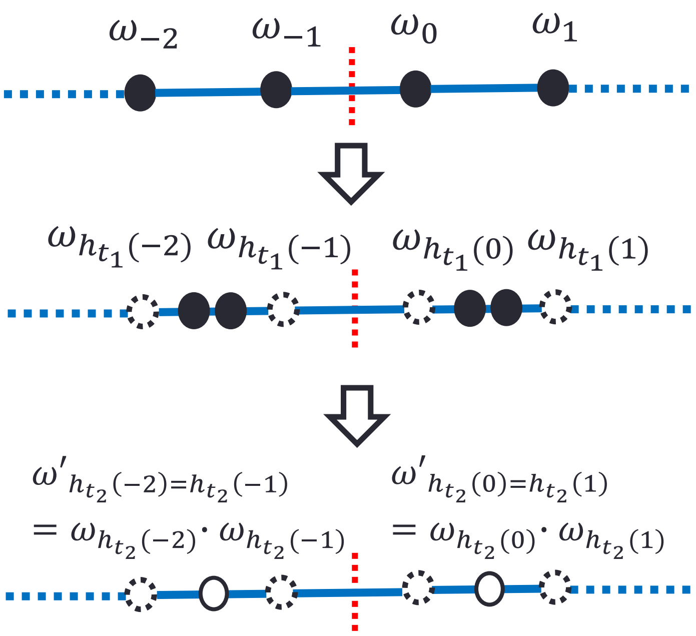

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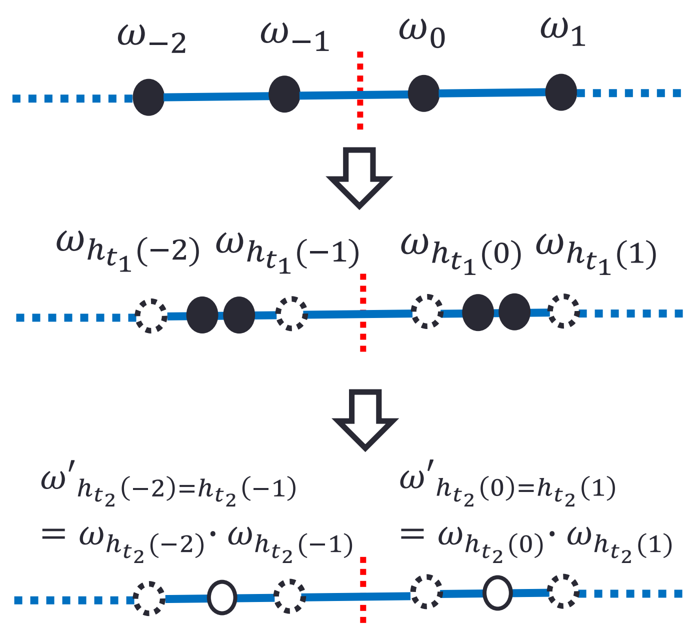


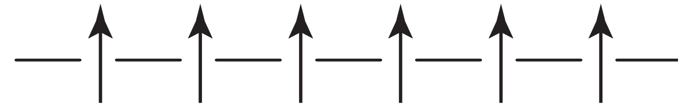
Trivial anom texture (**regardless of *s*** at each site)
=> (deformable to) unique symm gapped GS

Outline

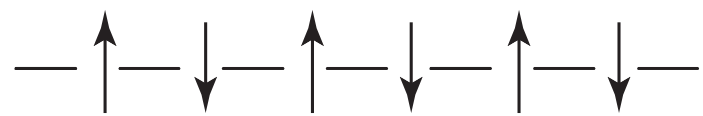
- **Introduction:** non-featureless spectrum and conventional LSM thm
- **LSM thm for magnetic space groups (MSGs)**
  - Symmetry-twisting method
  - Lattice Homotopy
- **LSM constraints from MSGs in 1d**
- **Summary**

(Minimal) MSGs giving rise to LSM constraints

- $G_{space} = \mathbb{Z}^{Tr}, G_{int} = \mathbb{Z}_2 \times \mathbb{Z}_2$ or $\mathbb{Z}_2^T$

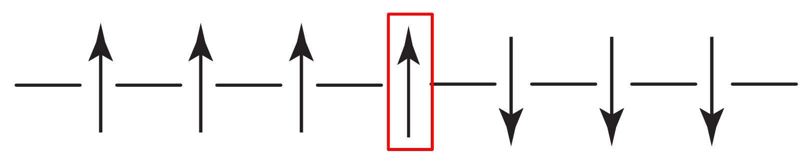


- $G_{space} = \mathbb{Z}^{Tr \cdot T}, G_{int} = \mathbb{Z}_2 \times \mathbb{Z}_2$

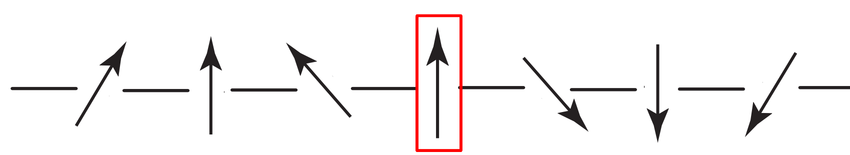


(π-rotation)×(site-centered inversion)

- $G_{space} = \mathbb{Z}_2^{\pi I_s}, G_{int} = \mathbb{Z}_2$



- $G_{space} = \mathbb{Z}_2^{I_s \cdot T}, G_{int} = Id$



Non-featureless spin interactions in 1d

	$(\mathbb{Z}_2 \times \mathbb{Z}_2) \times \mathbb{Z}^{Tr}$	$\mathbb{Z}_2^T \times \mathbb{Z}^{Tr}$	$(\mathbb{Z}_2 \times \mathbb{Z}_2) \times \mathbb{Z}^{Tr \cdot T}$	$\mathbb{Z}_2 \times \mathbb{Z}^{\pi I_s}$	$\mathbb{Z}_2^{I_s \cdot T}$
$\mathrm{XYZ}: \quad \sum_{\alpha,r} \mathcal{J}_\alpha S_r^\alpha S_{r+1}^\alpha$	✓	✓	✓	✓	✓
$\mathrm{uDM}: \quad \sum_r \vec{D} \cdot \left(\vec{S}_r \times \vec{S}_{r+1}\right)$	✗	✓	✗	✓	✗
$\mathrm{sDM}: \quad \sum_r (-1)^r \vec{D} \cdot \left(\vec{S}_r \times \vec{S}_{r+1}\right)$	✗	✗	✗	✗	✓
$\mathrm{uTP}: \quad \sum_r J\vec{S}_r \cdot (\vec{S}_{r+1} \times \vec{S}_{r+2})$	✓	✗	✗	✗	✓
$\mathrm{sTP}: \quad \sum_r (-1)^r J\vec{S}_r \cdot (\vec{S}_{r+1} \times \vec{S}_{r+2})$	✗	✗	✓	✗	✓

u: uniform s: staggered DM: Dzyaloshinskii-Moriya TP: (scalar-)triple-product

Non-featureless spin interactions in 1d

	$(\mathbb{Z}_2 \times \mathbb{Z}_2) \times \mathbb{Z}^{Tr}$	$\mathbb{Z}_2^T \times \mathbb{Z}^{Tr}$	$(\mathbb{Z}_2 \times \mathbb{Z}_2) \times \mathbb{Z}^{Tr \cdot T}$	$\mathbb{Z}_2 \times \mathbb{Z}^{\pi I_s}$	$\mathbb{Z}_2^{I_s \cdot T}$
XYZ : $\sum_{\alpha,r} \mathcal{J}_\alpha S_r^\alpha S_{r+1}^\alpha$	✓	✓	✓	✓	✓
uDM : $\sum_r \vec{D} \cdot \left(\vec{S}_r \times \vec{S}_{r+1}\right)$	✗	✓	✗	✓	✗
sDM : $\sum_r (-1)^r \vec{D} \cdot \left(\vec{S}_r \times \vec{S}_{r+1}\right)$	✗	✗	✗	✗	✓
uTP : $\sum_r J\vec{S}_r \cdot (\vec{S}_{r+1} \times \vec{S}_{r+2})$	✓	✗	✗	✗	✓
sTP : $\sum_r (-1)^r J\vec{S}_r \cdot (\vec{S}_{r+1} \times \vec{S}_{r+2})$	✗	✗	✓	✗	✓

u: uniform s: staggered DM: Dzyaloshinskii-Moriya TP: (scalar-)triple-product

Non-featureless combinations:

- XYZ + (any of the other 4 terms), e.g. HAF + sDM = XXZ [Oshikawa-Affleck ('97)]
- sDM + uTP + sTP

General phase structures

NF: non-featureless $\quad\quad$ $\overline{\text{NF}}$: not non-featureless

$$\underbrace{\overbrace{H_{\text{NF}}(\{J_\alpha\})}^{\text{FM/AFM/gapless}} + h\overbrace{H_{\overline{\text{NF}}}(\{J'_\beta\})}^{\text{PM/SPT}}}_{\text{QPT of Ginzburg-Landau, BKT, etc.}}$$

XYZ, DM, STP, etc.

external field (Z),
cluster model (XZX),
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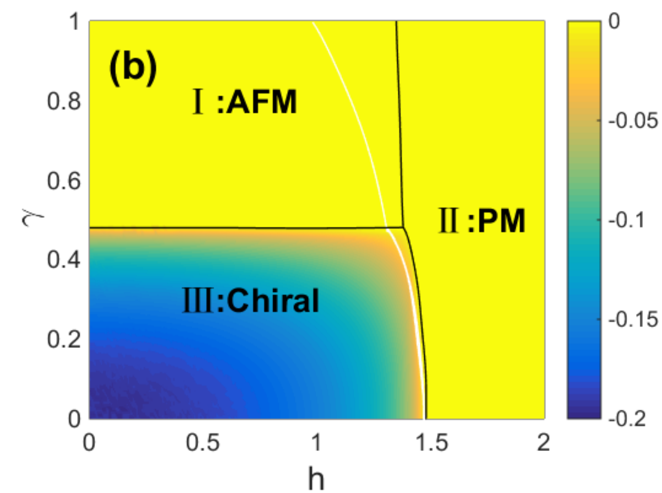
external field (Z),
cluster model (XZX),
etc.

E.g.

$$\mathcal{H} = J\sum_{j=1}^{N}\left(\frac{1+\gamma}{2}\,\sigma_j^x\sigma_{j+1}^x + \frac{1-\gamma}{2}\,\sigma_j^y\sigma_{j+1}^y + \Delta\sigma_j^z\sigma_{j+1}^z\right) + \sum_{j=1}^{N}\vec{D}\cdot(\vec{\sigma}_j\times\vec{\sigma}_{j+1}) - h\sum_{j=1}^{N}\sigma_j^z,, \qquad (1)$$

[Yi et al. (19')]



$D = 0.2$, $\Delta = 0.2$, and $J = 1$.

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- **Introduction:** non-featureless spectrum and conventional LSM thm
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Summary

- Conventional LSM thm: 1d spin chain w/ lattice-transl and spin-rot symm **cannot** have a unique gapped ground state if the spin per unit cell is **half-integral**.

- In this talk, we discuss various versions of 1d LSM thm for *magnetic space groups* (spin-rot + transl + time-rev + inv), basing on a symm-twisting method as well as lattice homotopy.

- The extended LSM constraints apply to systems with a **broader** class of spin interactions, such as Dzyaloshinskii-Moriya and scalar-triple-product 3-spin interactions.

Thank you for your attention!