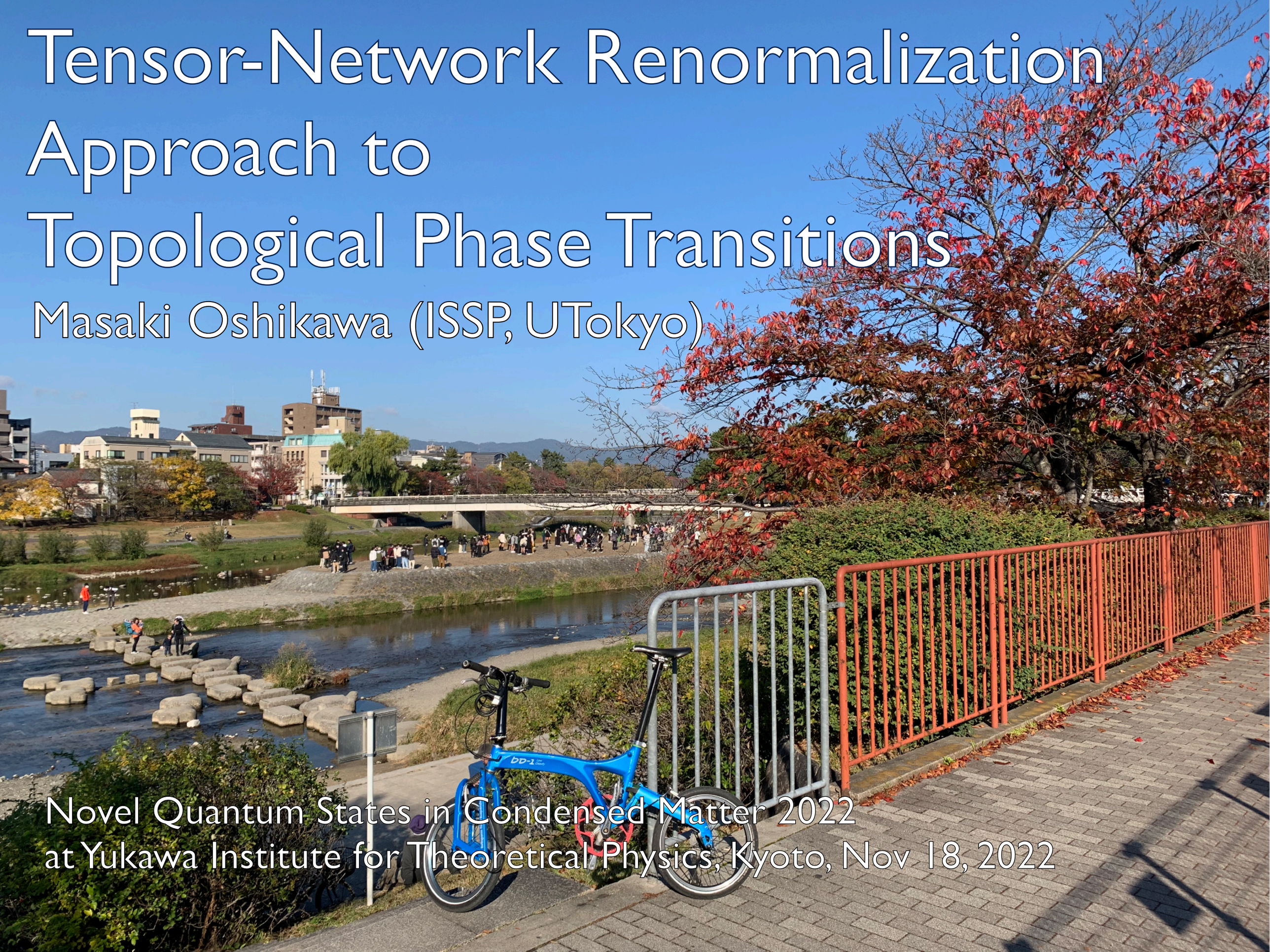
Tensor-Network Renormalization Approach to Topological Phase Transitions

Masaki Oshikawa (ISSP, UTokyo)



Novel Quantum States in Condensed Matter 2022
at Yukawa Institute for Theoretical Physics, Kyoto, Nov 18, 2022

PHYSICAL REVIEW E
covering statistical, nonlinear, biological, and soft matter physics

Tensor network renormalization study on the crossover in classical Heisenberg and RP^2 models in two dimensions

Atsushi Ueda and Masaki Oshikawa
Phys. Rev. E **106**, 014104 – Published 7 July 2022

Atsushi Ueda

authors implement the level-spectroscopy method, originally developed for quantum systems. They utilize the modern tensor-network renormalization scheme. This allows for an extremely accurate determination of the critical point as well as for a visualization of the celebrated Kosterlitz renormalization-group flow.

Atsushi Ueda and Masaki Oshikawa
Phys. Rev. B **104**, 165132 (2021)

Tensor network renormalization study on the crossover in classical Heisenberg and RP^2 models in two dimensions

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(Received 10 March 2022; accepted 17 June 2022; published 7 July 2022)

We study the classical two-dimensional RP^2 and Heisenberg models, using the tensor-network renormalization (TNR) method. The determination of the phase diagram of these models has been challenging and controversial due to the very large correlation lengths at low temperatures. The finite-size spectrum of the transfer matrix obtained by TNR is useful in identifying the conformal field theory describing a possible critical point. Our results indicate that the ultraviolet fixed point for the Heisenberg model and the ferromagnetic RP^2 model in the zero-temperature limit corresponds to a conformal field theory with central charge $c = 2$, in agreement with two independent would-be Nambu-Goldstone modes. On the other hand, the ultraviolet fixed point in the zero-temperature limit for the antiferromagnetic Lebwohl-Lasher model, which is a variant of the RP^2 model, seems to have a larger central charge. This is consistent with $c = 4$ expected from the effective SO(5) symmetry. At $T > 0$, the convergence of the spectrum is not good in both the Heisenberg and ferromagnetic RP^2 models. Moreover, there seems to be no appropriate candidate of conformal field theory matching the spectrum, which shows the effective central charge $c \sim 1.9$. These suggest that both models have a single disordered phase at finite temperatures, although the ferromagnetic RP^2 model exhibits a strong crossover at the temperature where the dissociation of $\mathbb{Z}_2$ vortices has been reported.

The Nobel Prize in Physics 2016

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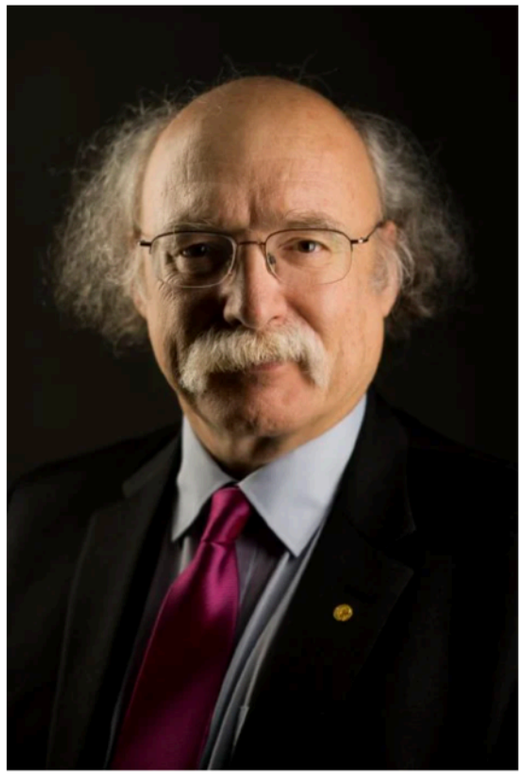
David J. Thouless

Prize share: 1/2



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F. Duncan M. Haldane

Prize share: 1/4

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J. Michael Kosterlitz

Prize share: 1/4

The Nobel Prize in Physics 2016 was awarded with one half to David J. Thouless, and the other half to F. Duncan M. Haldane and J. Michael Kosterlitz "for theoretical discoveries of topological phase transitions and topological phases of matter."

“Topological Phase Transitions”

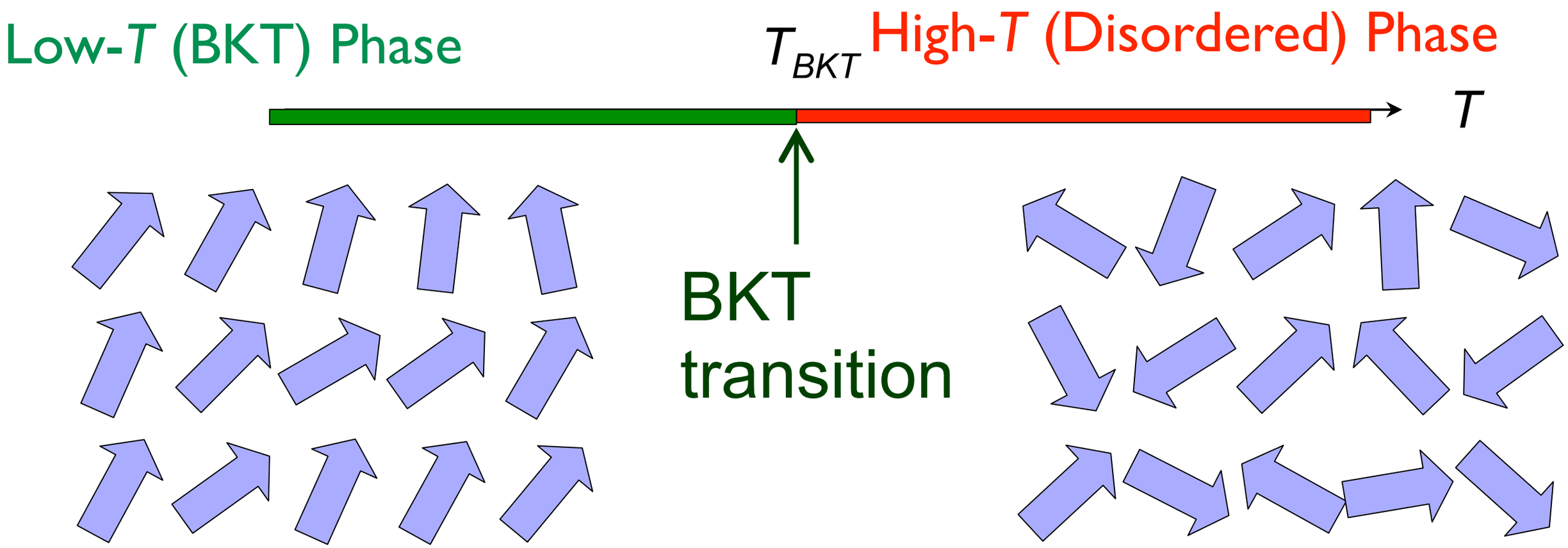
Prototype: Berezinskii-Kosterlitz-Thouless Transition
which is also closely related to “Haldane Gap”

Canonical model for the BKT transition:
2D classical XY model

$$H_{XY} = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

BKT Transition

$$H_{XY} = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$ Classical 2D XY model



$$\vec{s}_j = (\cos\theta_j, \sin\theta_j)$$

$$\langle \vec{s}_i \cdot \vec{s}_j \rangle \propto \left(\frac{1}{r}\right)^{\eta}$$

power-law decay

$$\langle \vec{s}_i \cdot \vec{s}_j \rangle \propto \exp\left(-\frac{r}{\xi}\right)$$

exponential decay

2D XY Model

$$H_{XY} = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

- Classical spin model with positive Boltzmann weight
  $\Rightarrow$ no sign problem
- Just 2 dimensions
- Efficient cluster algorithms available

Easily studied with Monte Carlo, right?

Journal of the Physical Society of Japan **81** (2012) 113001 **LETTERS**
http://dx.doi.org/10.1143/JPSJ.81.113001

Large-Scale Monte Carlo Simulation of Two-Dimensional Classical XY Model Using Multiple GPUs

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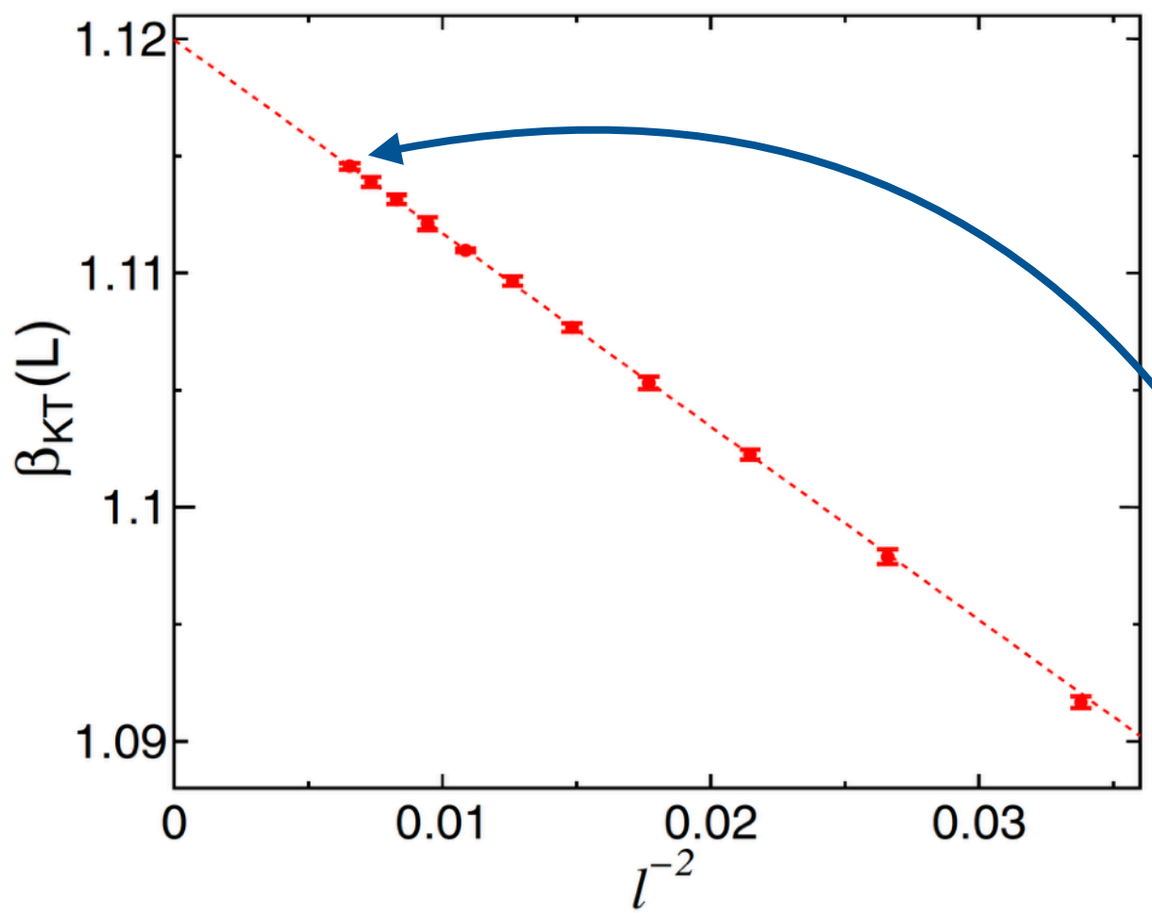
(Received August 27, 2012; accepted September 24, 2012; published online October 12, 2012)



finite-size scaling of T_c

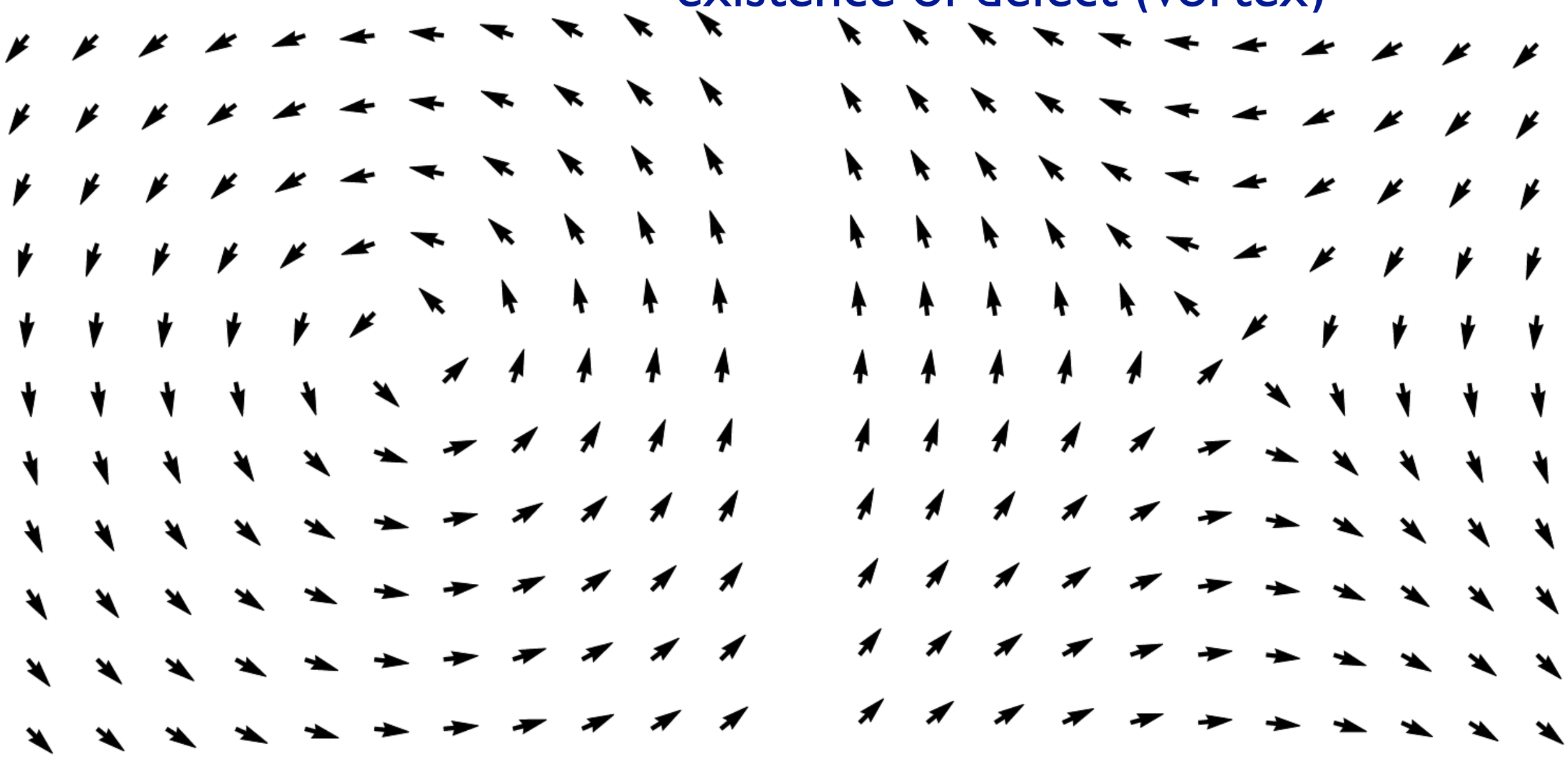
$$l = \ln bL$$

largest system:
L=65536
calculation using 256GPUs

Vortex in the 2D XY model

XY spin goes back to itself by 2π-rotation ⇒

existence of defect (vortex)



attraction

vortex

antivortex

BKT Transition

Low-*T* phase：vortex and antivortex form a pair

cf.) formation of atoms by nuclei and electrons

vortices are effectively absent at lengthscales larger than the pair size

$$\langle \vec{s}_i \cdot \vec{s}_j \rangle \propto \left(\frac{1}{r} \right)^{\eta}$$

High-*T* phase：vortices/antivortices dissociate from pairs and move freely

cf.) plasma state formed by dissociation of nuclei/electrons

$$\langle \vec{s}_i \cdot \vec{s}_j \rangle \propto \exp\left(-\frac{r}{\xi} \right)$$

Sine-Gordon Field Theory for BKT

$$\mathcal{L} = \frac{1}{2\pi K}(\partial_\mu \phi)^2 - y_\mathcal{K}(\partial_\mu \phi)^2 + y_V V$$

θ angle of the XY spin $\longleftrightarrow$ ϕ

dual (mutually non-local)

$\theta \sim \theta + 2\pi$

$\phi \sim \phi + \pi$

$$V = \cos 2\phi = \frac{1}{2}\left(e^{2i\phi} + e^{-2i\phi}\right)$$

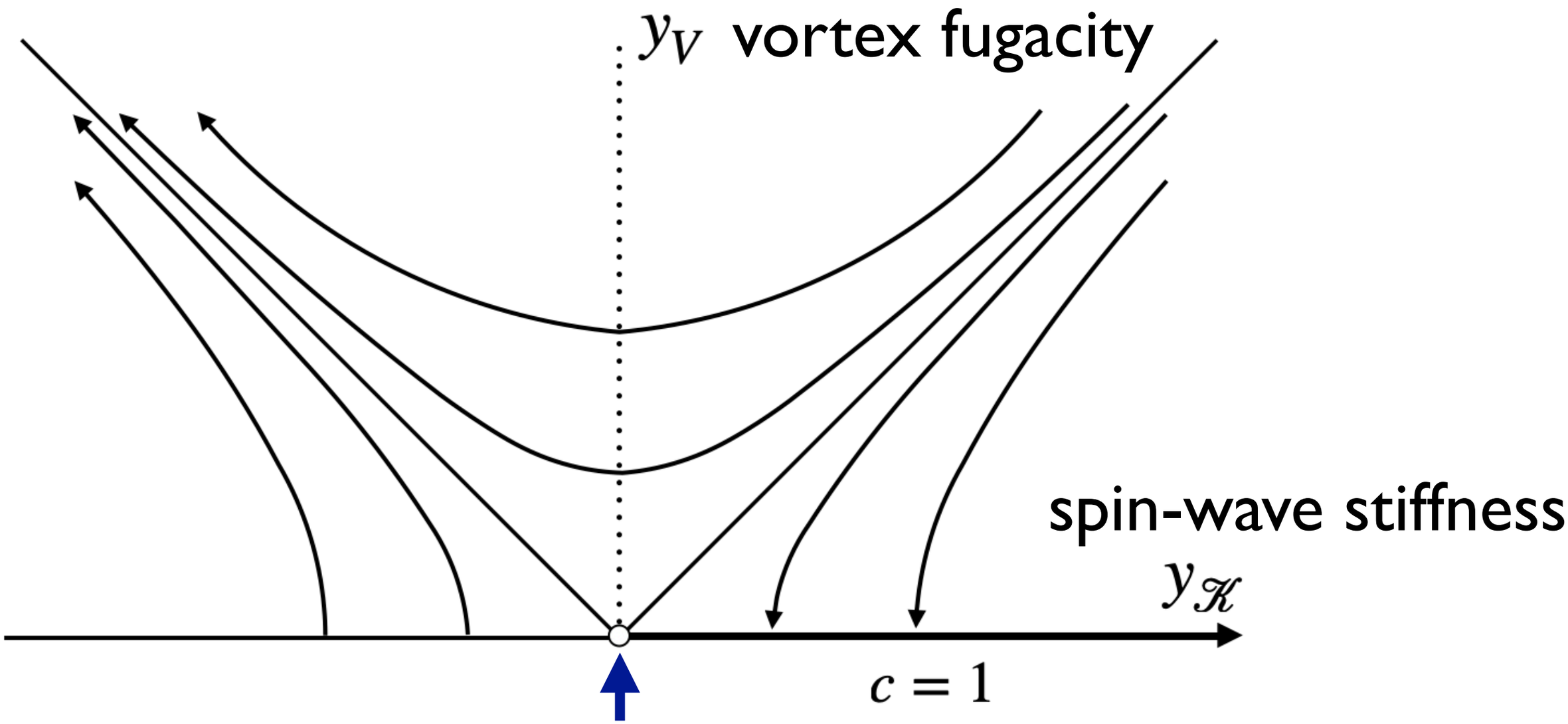
scaling dimension 2/*K*
marginal at *K*=2

Single vortex creation / annihilation operator

y_V vortex fugacity

$y_\mathcal{K}$ renormalization of Luttinger parameter *K*

Kosterlitz RG Flow



Luttinger parameter *K*=2 (vortex operator marginal)

BKT transition: $y_V = y_{\mathcal{K}} = g$

$$\frac{dg}{dl} = -g^2 \qquad g \sim \frac{1}{l} \sim \frac{1}{\ln L}$$

slow decay

↓

log-corrections

BKT Transition in S=1/2 XXZ Chain

$$\mathcal{H} = \sum_j \left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z \right)$$

Single vortex creation/annihilation operator

$$\cos 2\phi \sim (-1)^j \vec{S}_j \cdot \vec{S}_{j+1}$$

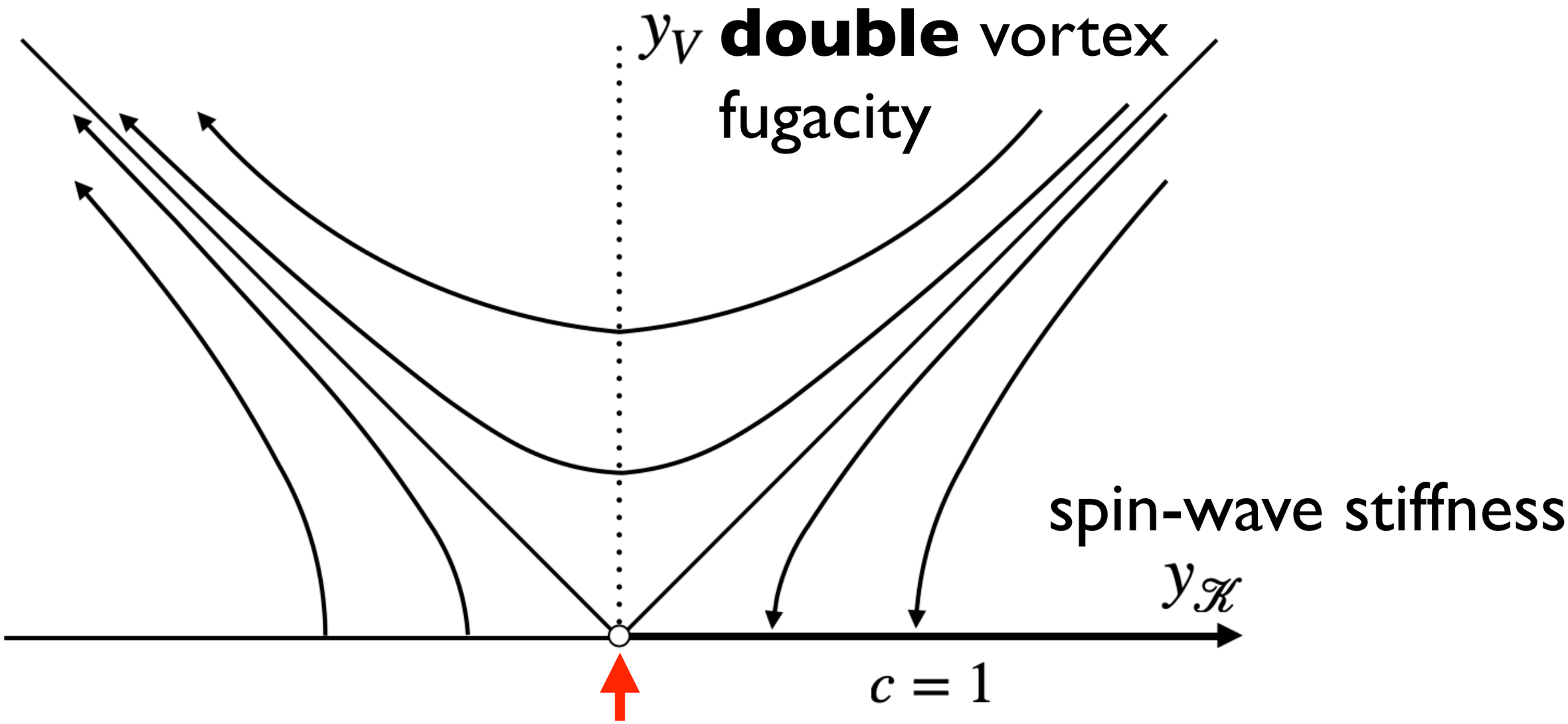
$$\sin 2\phi \sim (-1)^j \dot{S}_j^z$$

Forbidden in Hamiltonian by the translation symmetry!
(**Haldane 1980** → "Haldane conjecture"
related to Lieb-Schultz-Mattis theorem etc.)

The leading (most relevant) perturbation is thus
double vortex creation/annihilation op. $\cos 4\phi$

BTK Transition in *S*=1/2 XXZ Chain

$$\mathcal{L} = \frac{1}{2\pi K}(\partial_\mu \phi)^2 - y_{\mathcal{K}}(\partial_\mu \phi)^2 + y_V V \qquad V = \cos 4\phi$$



Luttinger parameter ***K*=1/2**
(**double** vortex operator marginal)

BKT Transition in S=1/2 XXZ Chain

$$\mathcal{H} = \sum_j \left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z \right)$$

BKT transition at Δ=1 (SU(2) symmetric point)

IR fixed point at the BKT transition:

Free boson (Tomonaga-Luttinger Liquid) at *K*=1/2

equivalent to Level 1 SU(2) Wess-Zumino-Witten

Effective theory in the vicinity of the BKT transition

$$\mathcal{L} = \mathcal{L}_{k=1}^{\mathrm{WZW}} + g \mathbb{J}^L \cdot \mathbb{J}^R + t \left(-\frac{1}{2} J_+^L J_+^R - \frac{1}{2} J_-^L J_+^R + J_z^L J_z^R \right)$$

$$y_V = g + t, y_{\mathcal{K}} = g - t$$

BKT transition ⇔ *t*=0 ⇔ SU(2) symmetry

Level Spectroscopy

Determination of the critical point from the finite-size spectrum [Okamoto-Nomura 1994, …]

"Double vortex" BKT transition at K=1/2 can be identified by SU(2) symmetry of the finite-size spectrum!

State-operator correspondence and Finite-Size Scaling in CFT [Cardy 1984, 1986]

$$E_n - E_0 = \frac{2\pi}{L}\left(x_n + \sum_m c_{nnm} y_m L^{2-x_m} + \dots\right)$$

BKT transition ⇔

Energy levels form SU(2) singlet, triplet, …

1D S=1/2 XXZ vs 2D Classical XY

$$\mathcal{L} = \frac{1}{2\pi K}(\partial_\mu \phi)^2 - y_\mathcal{K}(\partial_\mu \phi)^2 + y_V V$$

S=1/2 XXZ

K=1/2 (SU(2)$_1$ WZW)

$V \sim \cos 4\phi$

double vortex op.

SU(2) triplet (degenerate at BKT)

$n^x \sim \cos\theta,\ n^y \sim \sin\theta,\ n^z \sim \sin 2\phi$

$2\phi \to \phi$

Classical XY

K=2 $\quad \phi \sim \phi + \pi$

$V \sim \cos 2\phi$

single vortex op.

$\cos 2\theta, \sin 2\theta, \sin\phi$

half-vortex op.
(eigenstate under antiperiodic b.c.)

Nomura-Kitazawa 1998

Level Spectroscopy for 2D Stat Mech

Level spectroscopy has been developed for quantum 1D,
but why not for 2D stat mech models (such as XY)??

1D quantum Hamiltonian ⇔

Transfer matrix for 2D stat mech

Continuous spin: series expansion of Boltzmann weight

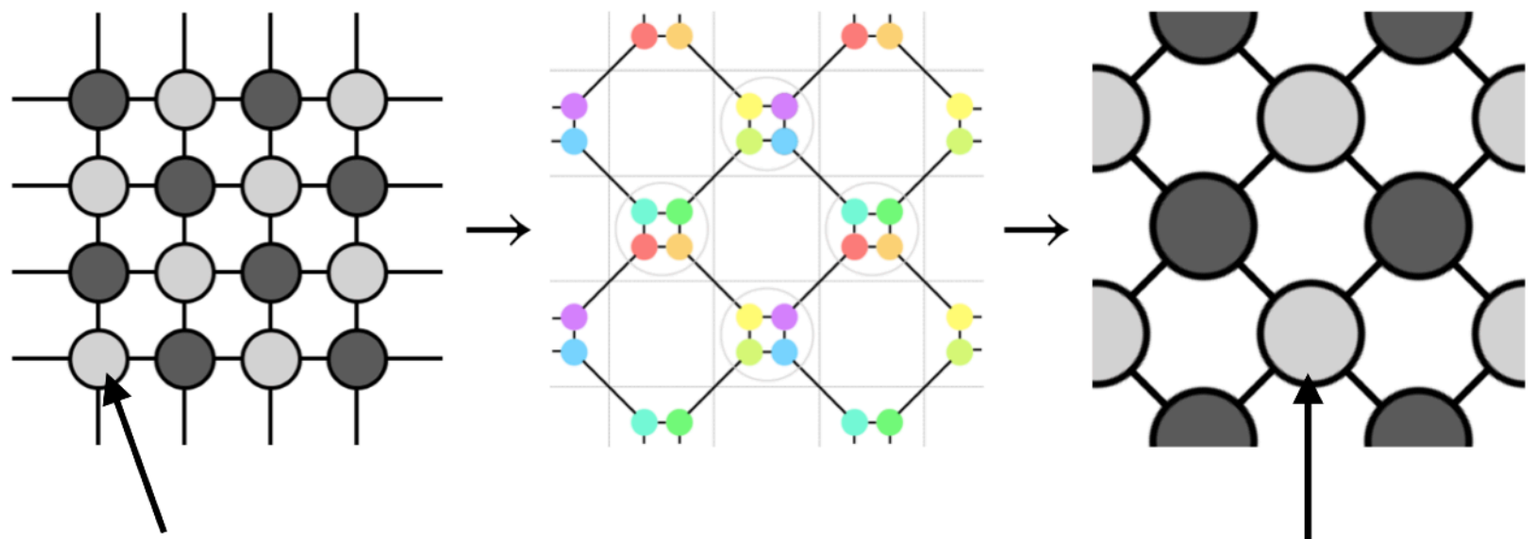
$$e^{\beta \cos(\theta_i - \theta_j)} = \sum_{n=-\infty}^{\infty} e^{in(\theta_i - \theta_j)} I_n(\beta),$$

the series may be
truncated to $-15 \leqq n \leqq 15$

Transfer matrix still "too large" to be diagonalized
⇒ we utilize Tensor Network Renormalization

(We used Loop-TNR)

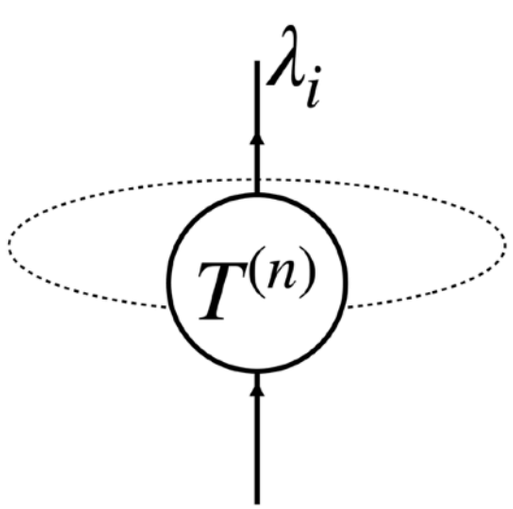
TNR Construction of Transfer Matrix



Boltzmann weight at $L = \sqrt{2}^n$

Boltzmann weight at $L = \sqrt{2}^{n+1}$

after *n* steps, a single tensor represents
a square block of linear size $L = \sqrt{2}^n$



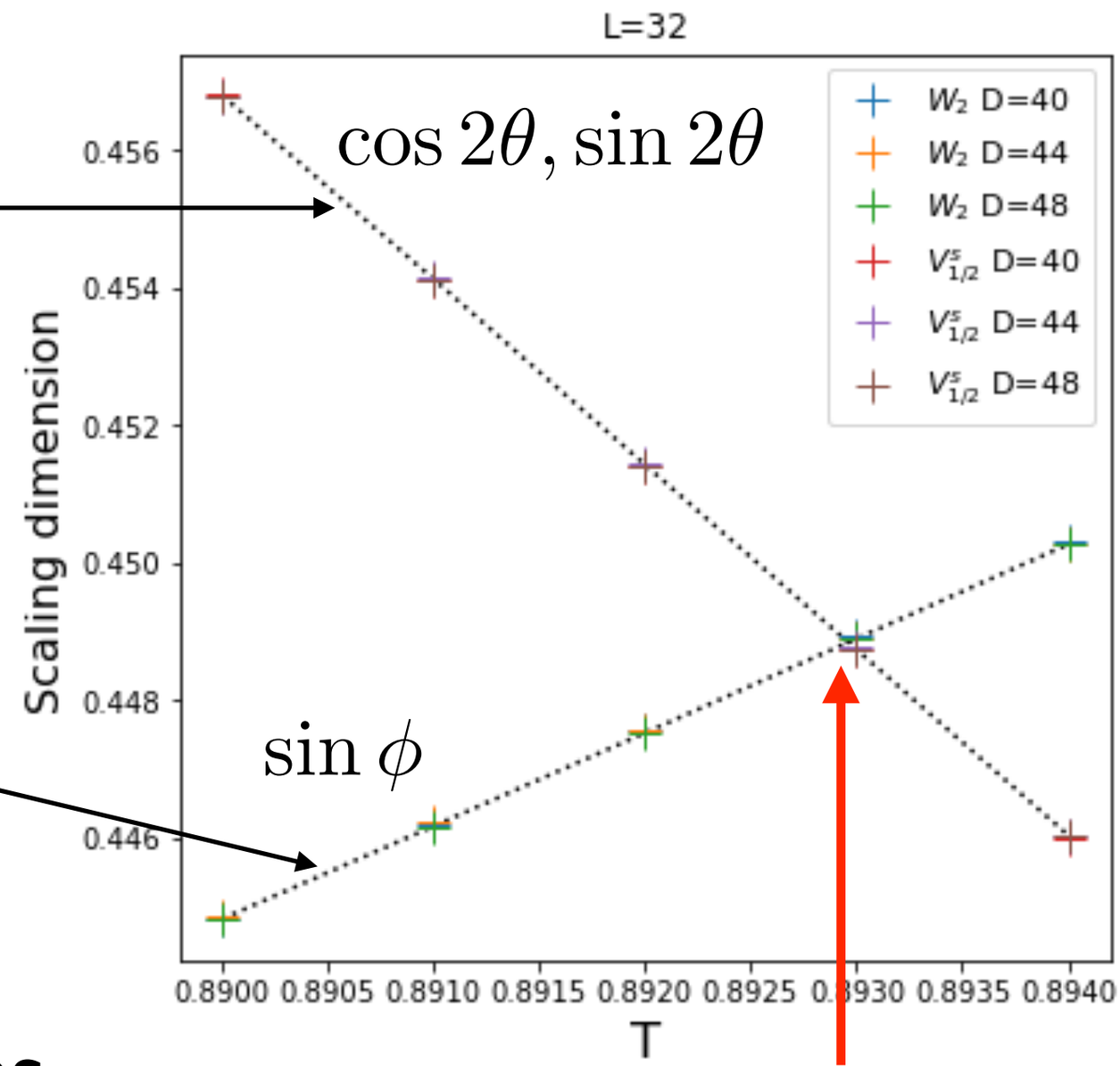
contract horizontal indices
$\Rightarrow$ transfer matrix in vertical direction

$$\lambda_i = e^{-LE_i(L)}$$

Identifying T_c with Level Crossing

"spin-wave" excited states under periodic b.c.

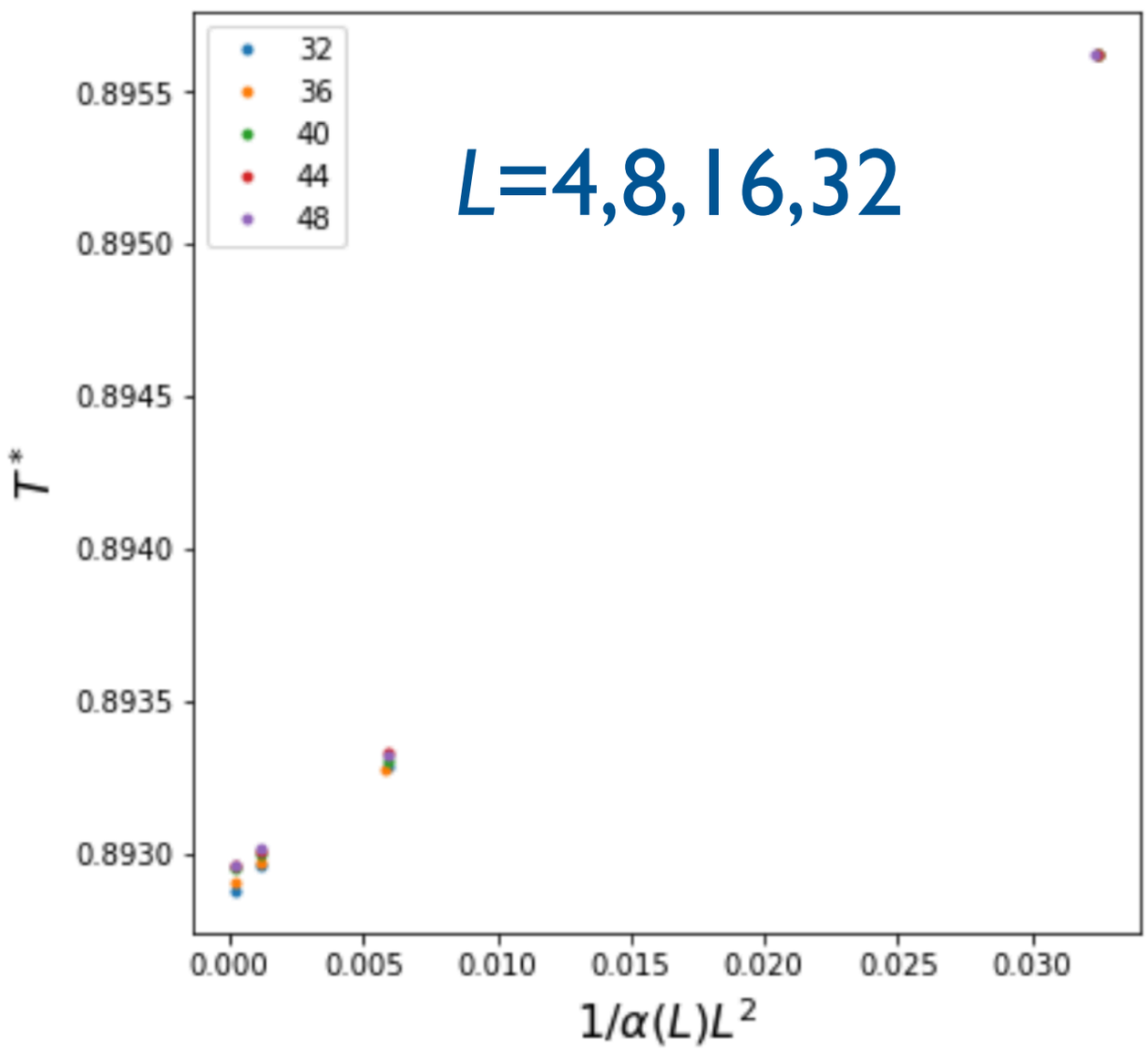
ground state under antiperiodic b.c.



This procedure eliminates logarithmic corrections to all orders in g

extra degeneracy forming SU(2) triplet $\sim$ BKT transition

Remaining Finite-Size Effect



Level crossing point weakly depends on the system size *L*

Effect of irrelevant perturbations

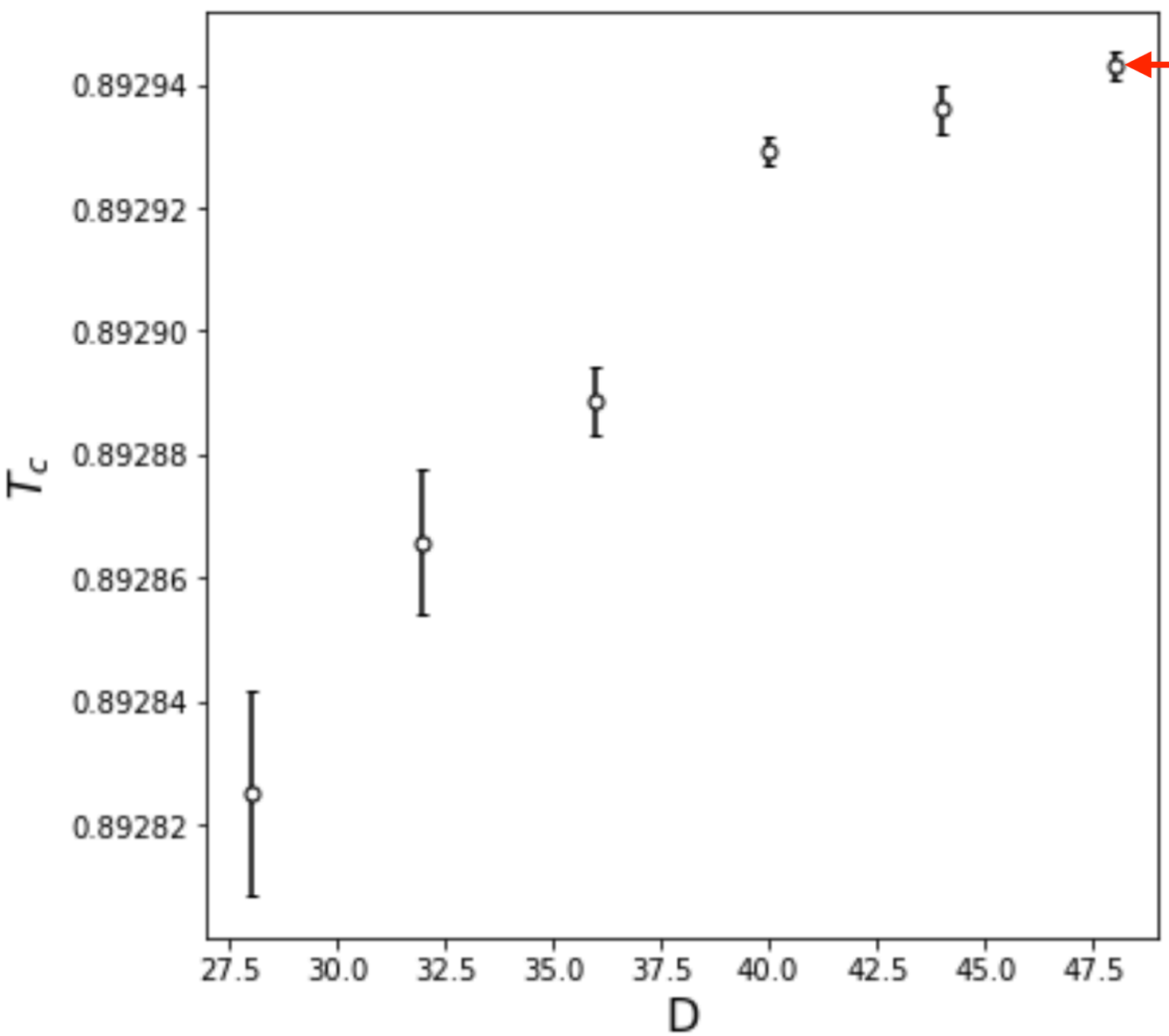
$$T^2, \bar{T}^2, T\bar{T}, \ldots$$

T: holomorphic part of the energy-momentum tensor

$$T^* \sim T_c + \text{const.}\frac{1}{L^2}$$

Extrapolate to *L*=∞

Dependence on Bond Dimension *D*



Our final estimate
T_c=0.892943(2)

Effect of Finite Bond-Dimension

Finite bond dimension *D* ⇔ finite "correlation length"

$$\xi_D \sim 0.3 D^{\kappa}$$

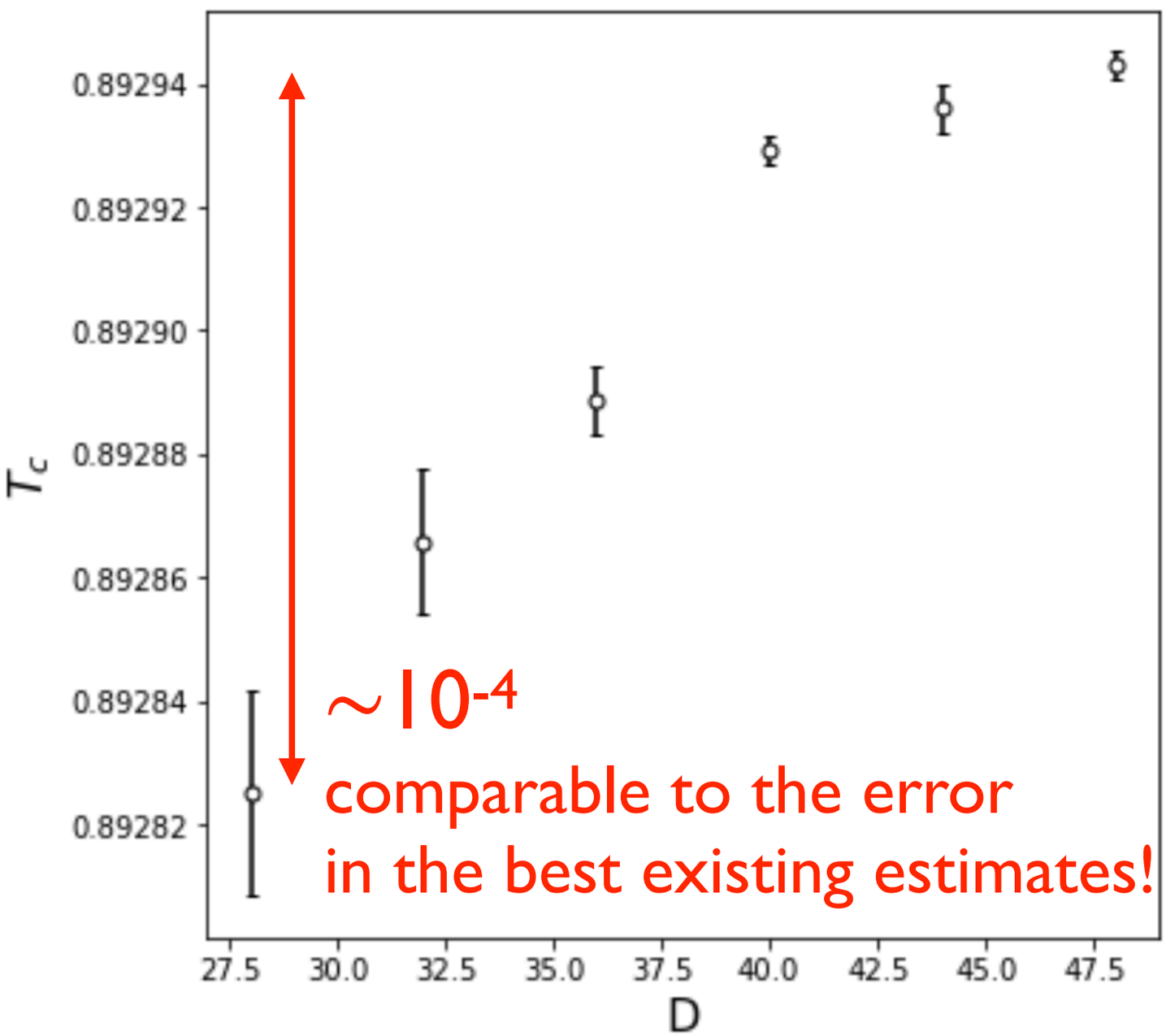
$$\kappa = \frac{6}{c\left(\sqrt{\frac{12}{c}}+1\right)}$$

[Pollmann et al. 2008]

$\xi_D > L$ low-energy finite-size spectrum almost exact!

$\xi_D < L$ low-energy spectrum still reasonably accurate, but some error due to the finite *D*

T_c dependence on D



D=48 gives $\xi \sim 54$

enough for up to L=32

D=28 gives $\xi \sim 26$

too small for L=32
BUT….

Error in (Loop-)TNR

(Loop-)TNR is often used to construct the "fixed point" tensor, which would describe the large scale behaviors, by iterating TNR many times

This approach has given accurate results for large systems, but small errors due to the finite bond dimension remain

[A. Ueda & M.O., in preparation]

In our approach, we study the spectrum of finite-size systems with TNR. TNR is almost exact when the system size is less than the effective correlation length.

TNR calculation of the finite-size spectrum
+ Level Spectroscopy → better accuracy

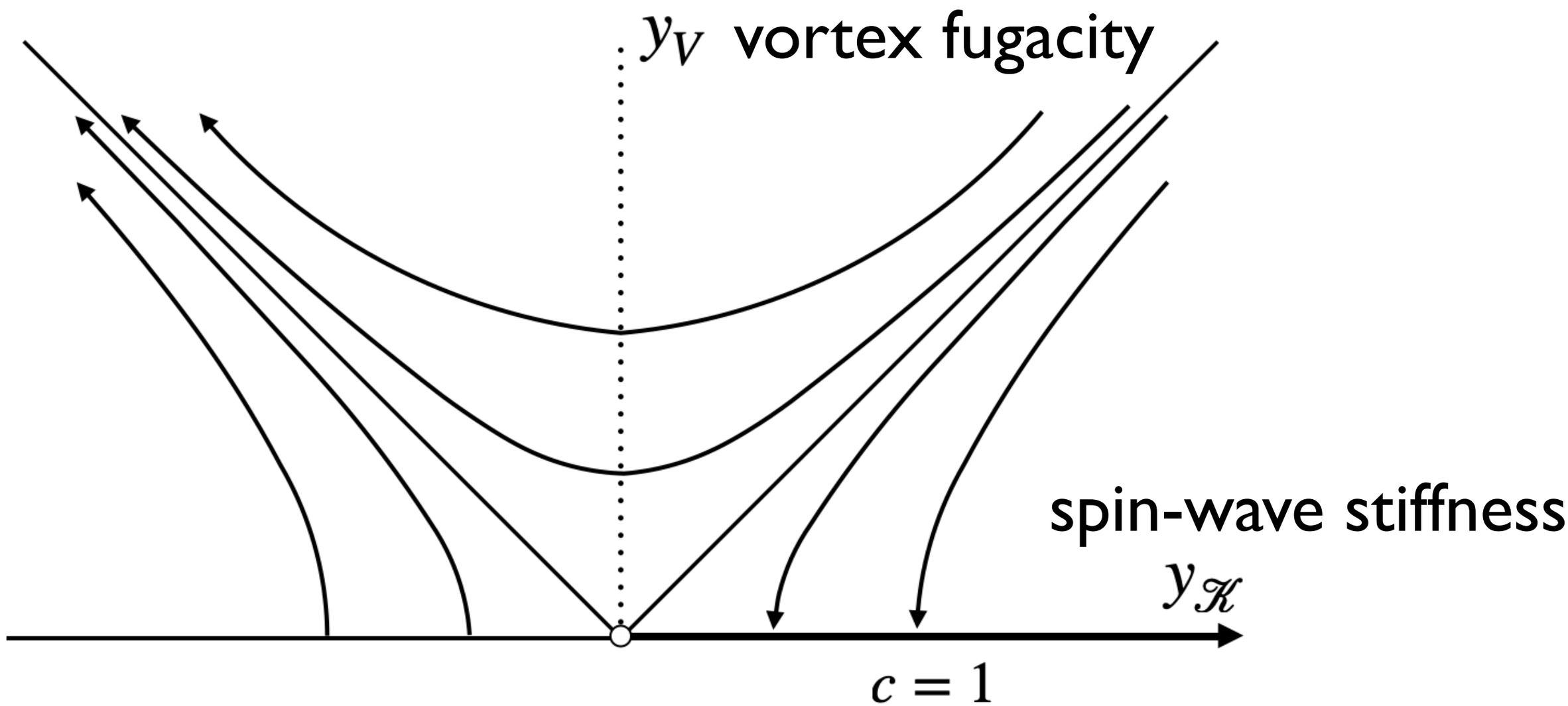
Estimates of T_c

Monte Carlo(1979)[35]	0.89
Monte Carlo(2005)[36]	0.8929
Monte Carlo(2012)[37]	0.89289
Monte Carlo(2013)[38]	0.8935
Series expansion(2009)[39]	0.89286
HOTRG(2014)[40]	0.8921
VUMPS(2019)[41]	0.8930
HOTRG(2020)[42]	0.89290(5)
present work	0.892943(2)

TABLE I. Comparison of the estimated critical temperature of the 2D classical XY model.

Kosterlitz RG Flow

You must have seen this diagram many times….



*but have you **really** seen the RG flow?*

ELSEVIER

Nuclear Physics B 522 [FS] (1998) 533–549

NUCLEAR PHYSICS B

Low energy effective Hamiltonian for the XXZ spin chain

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L.D. Landau Institute for Theoretical Physics, Kosygina 2, Moscow, Russia

Received 19 February 1998; accepted 18 March 1998

"vacuum energy" under the twisted boundary condition with the twist angle θ [notation clash…]

$$\delta^{\mathrm{RG}} = -\frac{1}{12}\left\{1 + \frac{3}{8} g_{\parallel} g_{\perp}^2\right\} + \frac{s^2}{2}\left\{1 - \frac{g_{\parallel}}{2} + \frac{1}{4} g_{\perp}^2 - \frac{7}{32} g_{\parallel} g_{\perp}^2\right\}$$
$$+ \frac{|s|}{16}\left\{2 g_{\perp}^2 - g_{\parallel} g_{\perp}^2\right\} + \frac{\theta^2}{2}\left\{1 + \frac{g_{\parallel}}{2} + \frac{g_{\parallel}^2}{4} - \frac{g_{\perp}^2}{4} + \frac{g_{\parallel}^3}{8} - \frac{g_{\parallel} g_{\perp}^2}{32}\right\} + O(g^4), \tag{4.6}$$

Extrapolating Lukyanov's Result

In our notations, the energy levels would be given by

$$E_n - E_0 = \frac{2\pi}{L} x_n$$

$$e^{\pm i2\theta} \qquad x_{2,0} = \frac{1}{2} - \frac{y_\mathcal{K}}{4} + (\frac{1}{4} - \frac{11}{64} y_\mathcal{K}) y_V^2,$$

$$e^{\pm i\phi} \qquad x_{0,1/2} = \frac{1}{2} + \frac{y_\mathcal{K}}{4} + \frac{1}{8}(y_\mathcal{K}^2 - y_V^2) + \frac{y_\mathcal{K}^3}{16} - \frac{1}{64} y_\mathcal{K} y_V^2.$$

They form two doublets, and no triplet is formed even on the BKT transition line $y_\mathcal{K} = y_V$?!

The two states corresponding to $e^{\pm i\phi}$ are mixed by the vortex perturbation $\cos 2\phi$ and split into two levels corresponding to $V_{1/2}^s = \sin\phi, V_{1/2}^c = \cos\phi$

Energy Levels up to 2nd Order

- split between $x_{V^s_{1/2}}, x_{V^c_{1/2}}$ should be odd in y_V
- SU(2) triplet should be formed on the BKT transition line $y_\mathcal{K} = y_V$

$\Rightarrow$ uniquely determines the energy levels up to $O(y^2)$

$$x_{W_{\pm 2}} = \frac{1}{2} - \frac{y_\mathcal{K}}{4} + \frac{1}{4} y_V^2,$$

$$x_{V^s_{1/2}} = \frac{1}{2} + \frac{y_\mathcal{K}}{4} - \frac{y_V}{2} + \frac{1}{8}(y_\mathcal{K}^2 + 2 y_\mathcal{K} y_V - y_V^2),$$

$$x_{V^c_{1/2}} = \frac{1}{2} + \frac{y_\mathcal{K}}{4} + \frac{y_V}{2} + \frac{1}{8}(y_\mathcal{K}^2 - 2 y_\mathcal{K} y_V - y_V^2),$$

Obtaining Running Coupling Constants

$$x_{W_{\pm 2}} = \frac{1}{2} - \frac{y_\mathcal{K}}{4} + \frac{1}{4} y_V^2,$$
$$x_{V^s_{1/2}} = \frac{1}{2} + \frac{y_\mathcal{K}}{4} - \frac{y_V}{2} + \frac{1}{8}(y_\mathcal{K}^2 + 2 y_\mathcal{K} y_V - y_V^2),$$
$$x_{V^c_{1/2}} = \frac{1}{2} + \frac{y_\mathcal{K}}{4} + \frac{y_V}{2} + \frac{1}{8}(y_\mathcal{K}^2 - 2 y_\mathcal{K} y_V - y_V^2),$$

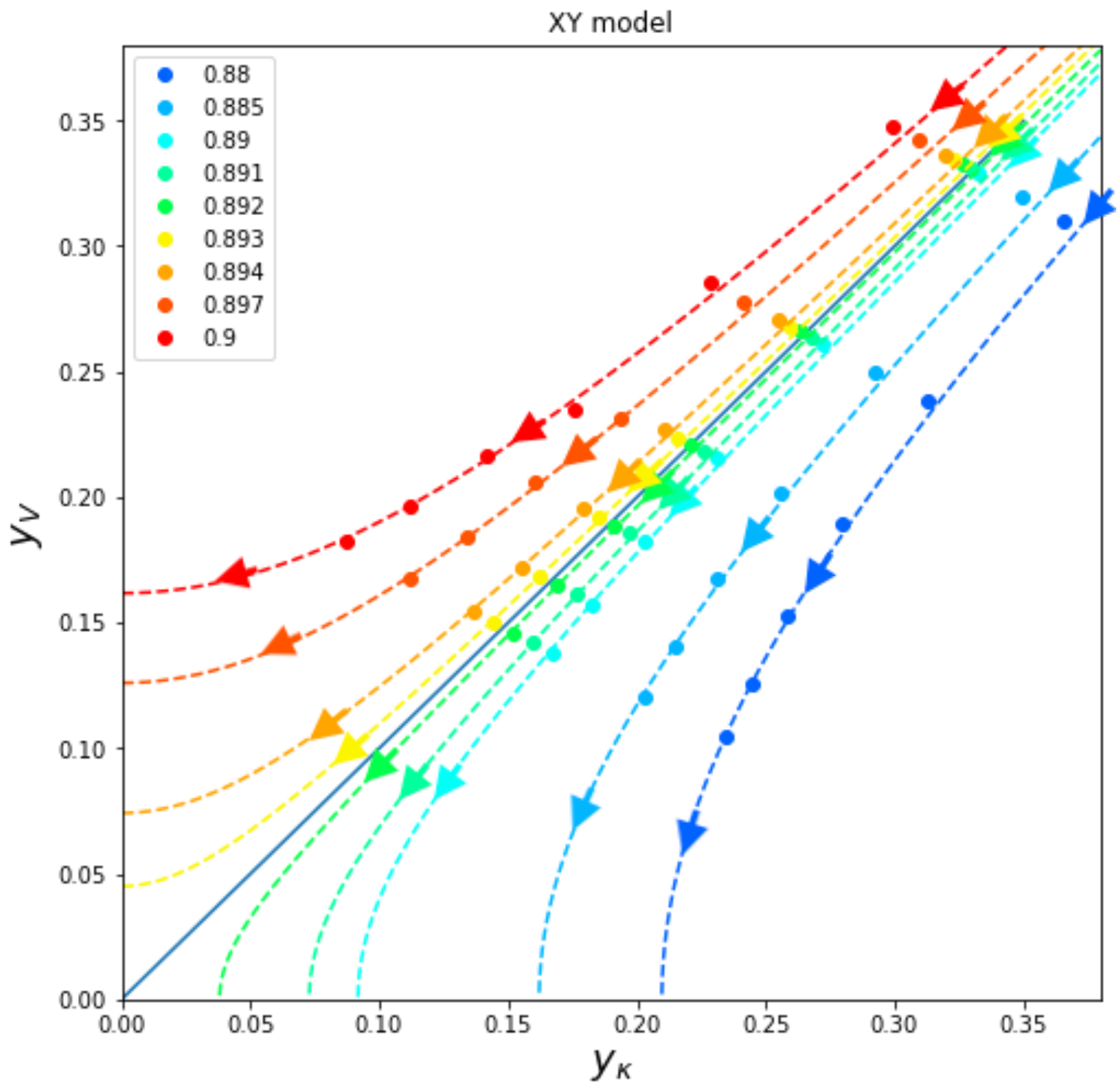
$$y_\mathcal{K} \sim 2 - 4 x_{W_{\pm 2}} + (x_{V^c_{1/2}} - x_{V^s_{1/2}})^2,$$

$$y_V \sim (x_{V^c_{1/2}} - x_{V^s_{1/2}}) / (1 - \frac{1}{2} y_\mathcal{K}),$$

We can estimate y_K & y_V from the finite-size energy levels

Less accuracy than T_c, but we can apply to larger systems (up to L=512)

Visualization of Kosterlitz RG Flow!



L=16
32
64
128
256
512

Conclusions (BKT)

TNR + Level Spectroscopy (finite size scaling of CFT) allows

- super accurate determination of BKT critical point
- visualization of Kosterlitz RG flow by extraction of running coupling constants from the spectrum

for continuous valued 2D classical spin system such as XY model

Future: extension/application to more nontrivial systems & unknown physics

Classical Heisenberg Model

$$H = -J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j ,$$

continuum limit

$$\mathcal{L} = \frac{1}{2g} (\partial_\mu \vec{n})^2 \qquad \vec{n}^2 = 1$$

O(3) Nonlinear Sigma Model

coupling *g* corresponds to temperature

Asymptotic freedom ⇒ disordered at any *T*>0

$$\frac{dg}{d \log L} = \frac{g^2}{2\pi} + \ldots$$

Supported also by factorizable *S*-matrix (Zamolodchikov[2])

BUT…

IOP Publishing — Journal of Physics A: Mathematical and Theoretical

J. Phys. A: Math. Theor. **40** (2007) 3741–3748 doi:10.1088/1751-8113/40/14/001

Quasi-long-range ordering in a finite-size 2D classical Heisenberg model

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Received 24 November 2006, in final form 13 February 2007
Published 20 March 2007
Online at stacks.iop.org/JPhysA/40/3741

Abstract
We analyse the low-temperature behaviour of the classical isotropic ferromagnetic Heisenberg model on a two-dimensional square lattice of finite size. Presence of a residual magnetization in a finite-size system enables us to use a low-temperature approximation, which is however more restricting than the usual spin-wave approximation known to give reliable results for the XY model at low temperatures T. For the system considered, we find that the spin–spin correlation function decays as $1/r^{\eta(T)}$ for large separations r bringing about the presence of a quasi-long-range ordering. We give analytic estimates for the exponent $\eta(T)$ in different regimes and support our findings by Monte Carlo simulations of the model on lattices of different sizes at different temperatures.

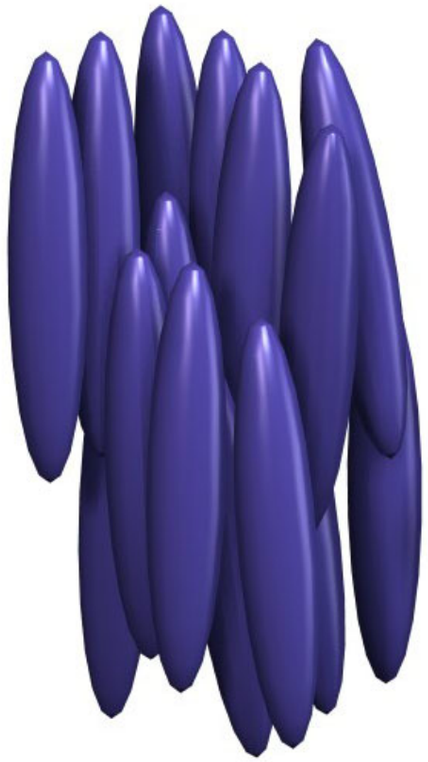
PACS numbers: 05.50.+q, 75.10

Really??

But not easy to prove/disprove

difficult to distinguish massive with very large correlation length from massless…

Z_2 vortex-driven transition?



Nematic liquid crystal:
symmetric rod-like molecules
(no distinction between head & tail)

$$H = -J \sum_{\langle i,j \rangle} (\vec{S}_i \cdot \vec{S}_j)^2,$$

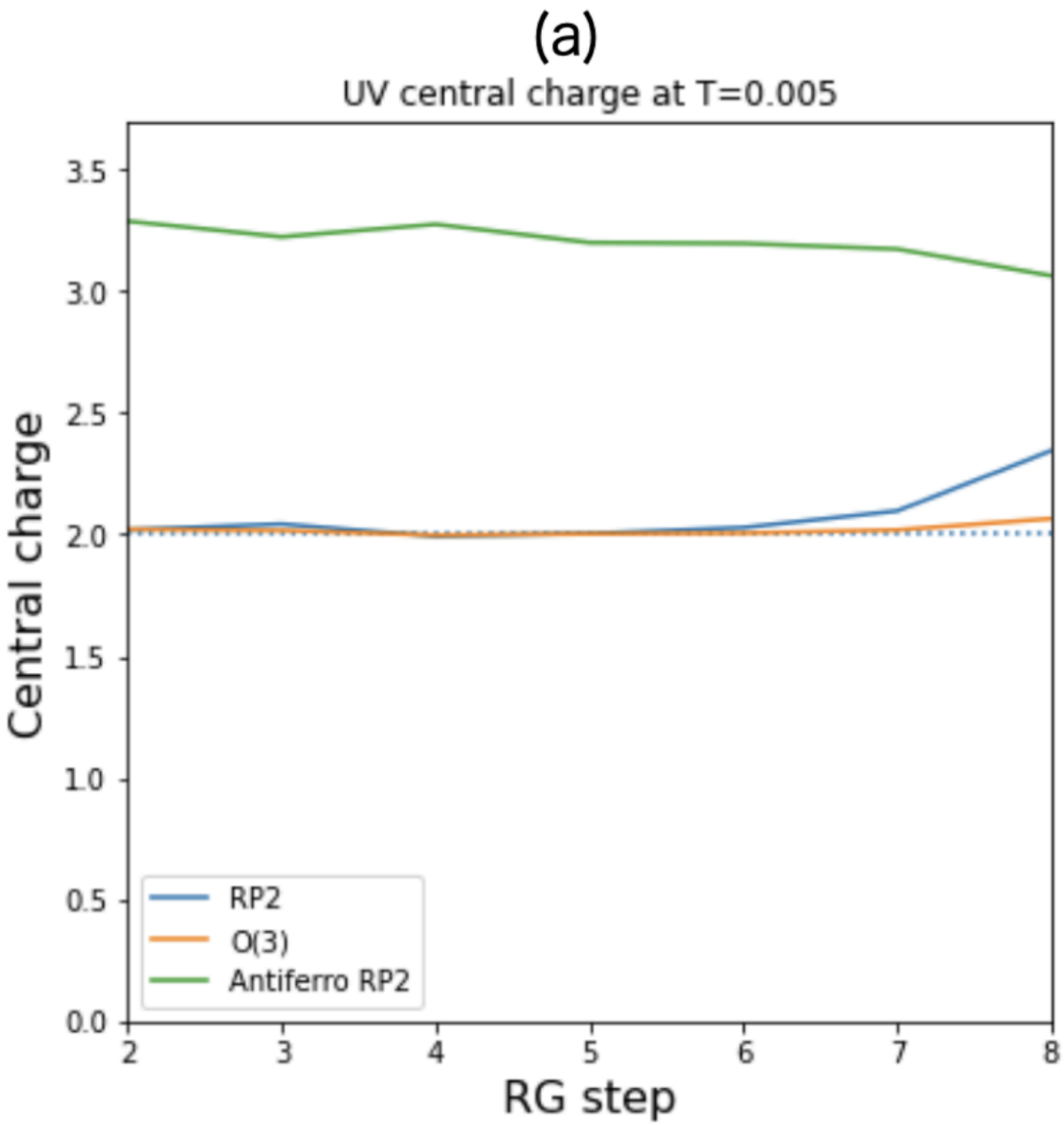
Lebwohl-Lascher 1972
Kawamura-Miyashita 1984

Target space = RP^2

$\pi_1(\mathrm{RP}^2) = \mathbb{Z}_2$ Z_2 vortex!

BKT-like transition driven by the Z_2 vortices?
But the RG equation also implies asymptotic freedom..

UV Fixed Point



$T \to 0$

Spontaneous Symmetry Breaking of SO(3)

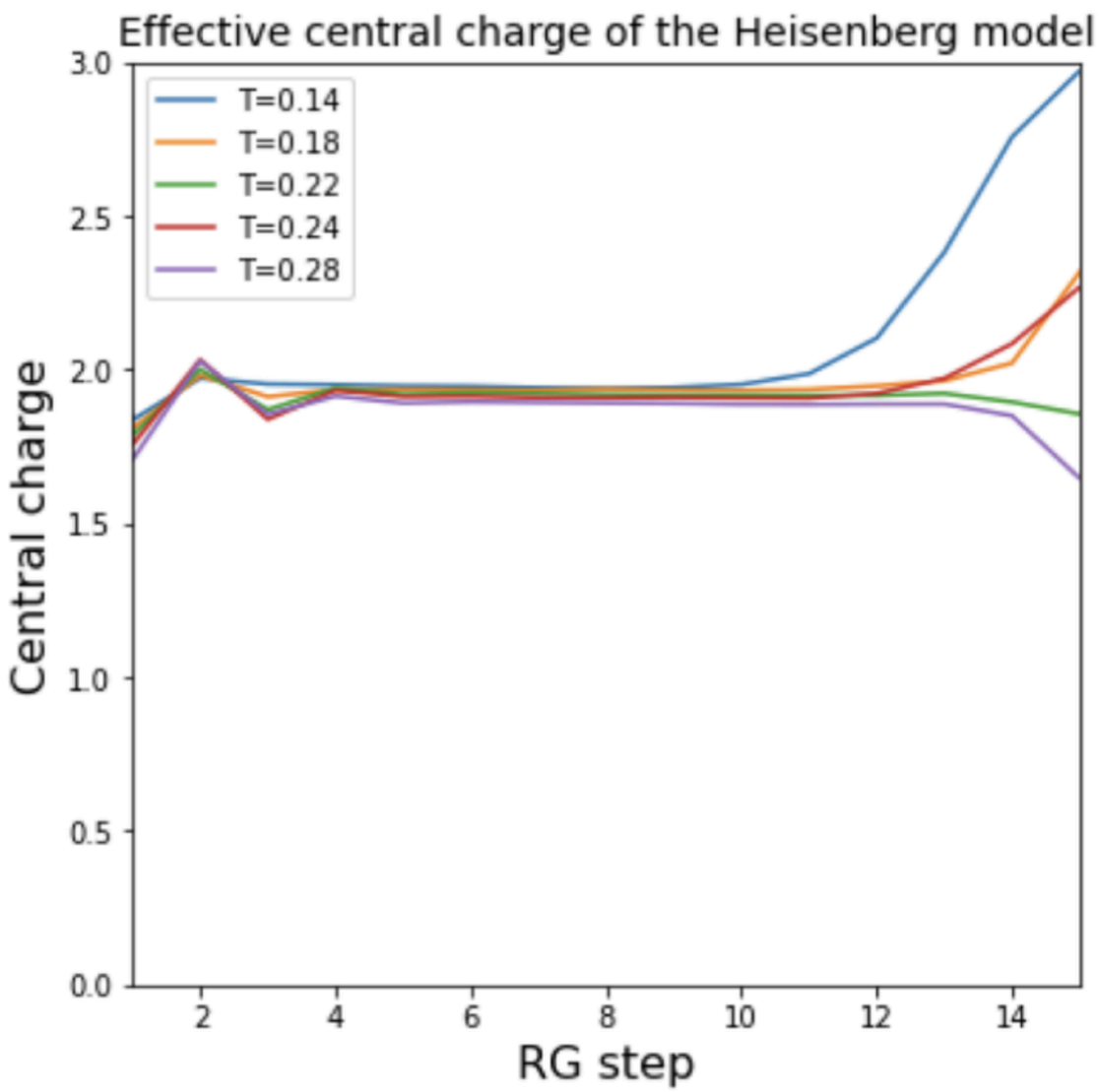
2 independent Nambu-Goldstone modes

$\Rightarrow c=2$

Confirmed by TNR

$$E_0(L) \sim \epsilon_0 L - \frac{\pi c}{6L}.$$

Heisenberg Model at Intermediate *T*

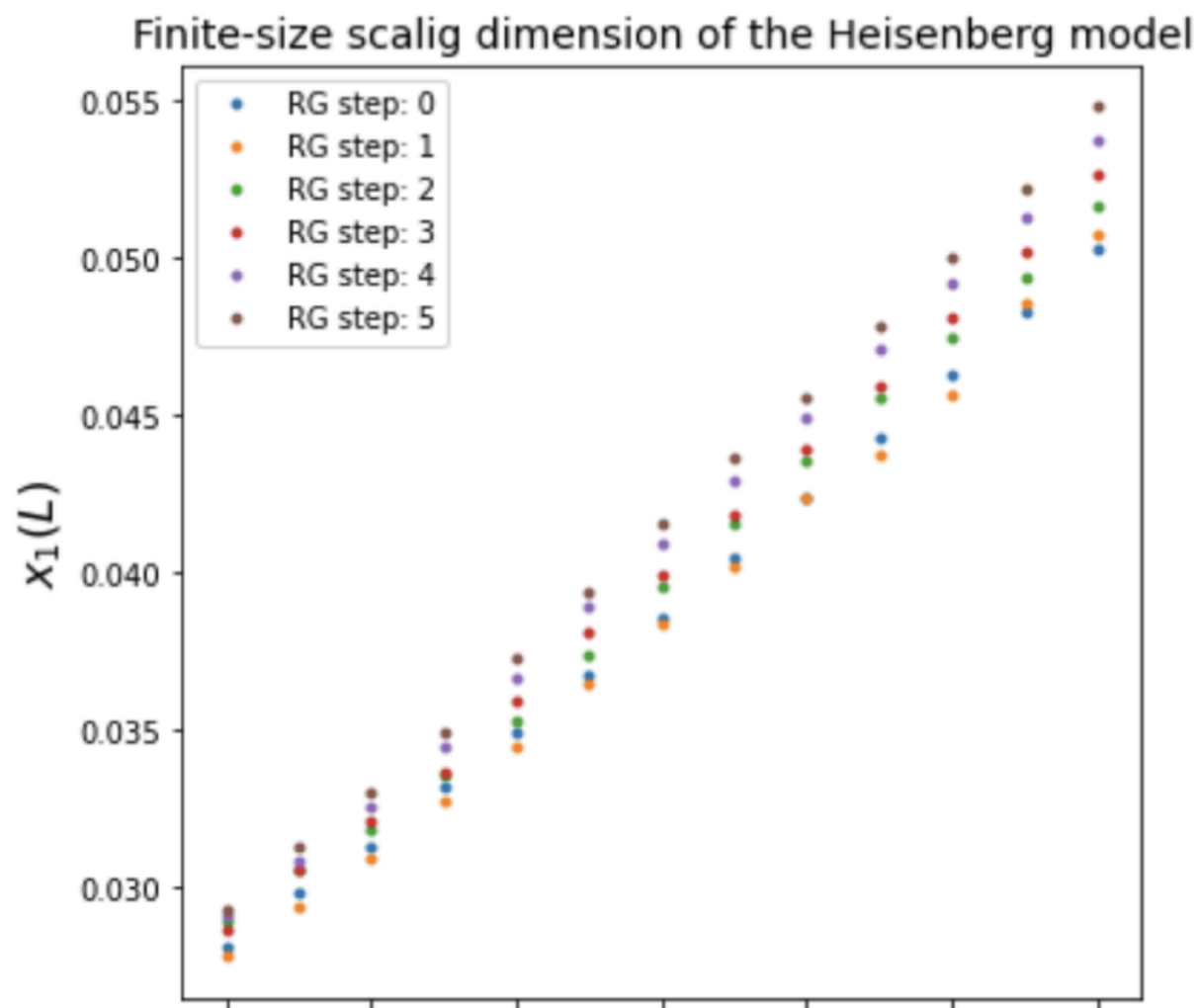


$$E_0(L) \sim \epsilon_0 L - \frac{\pi c}{6L}.$$

$c \sim 1.94 < 2$ no candidate CFT

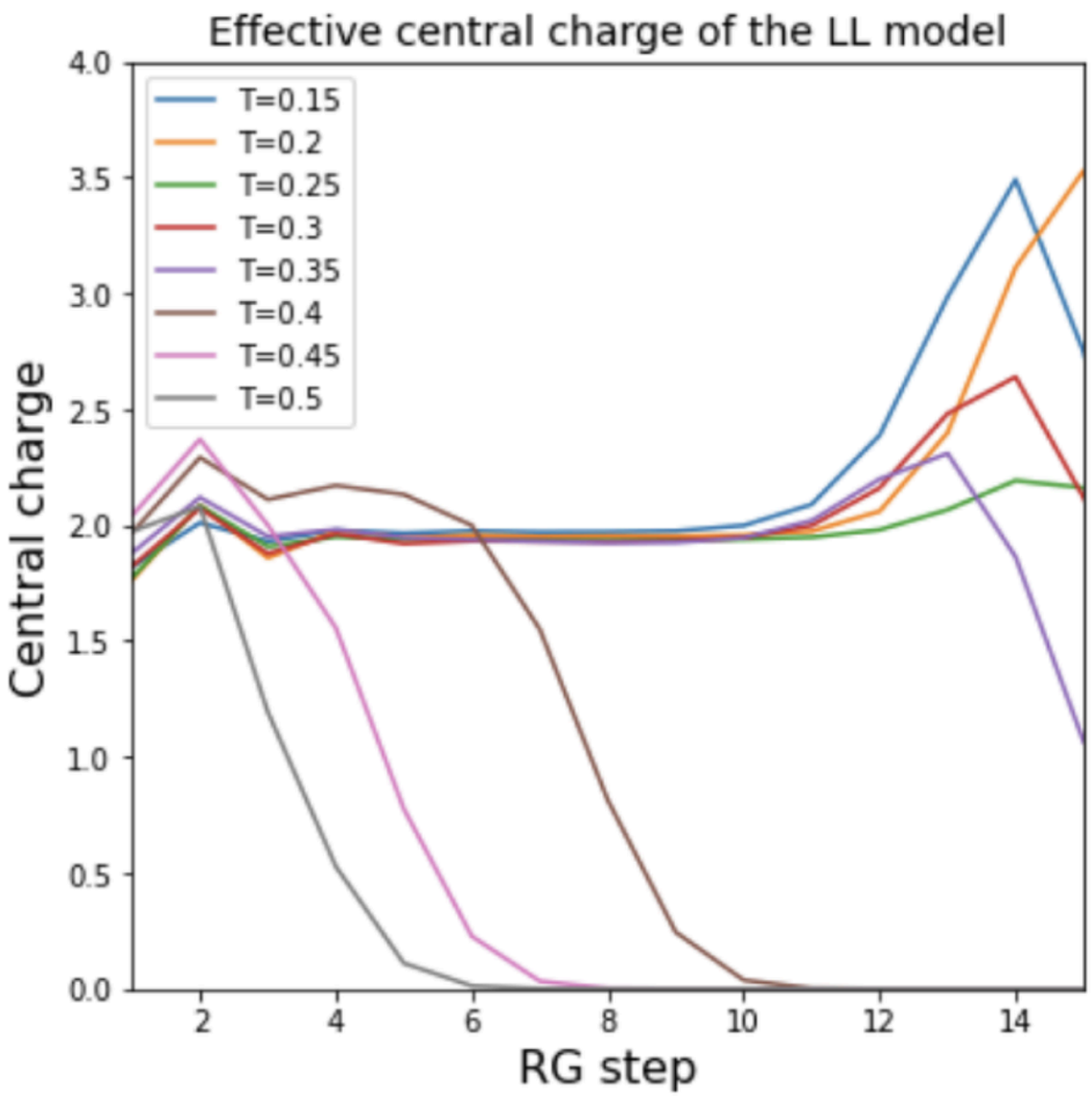
$$\frac{2\pi x_n}{L} = E_n(L) - E_0(L).$$

does not converge



likely to support "asymptotic freedom" RG flow c=2 → c=0

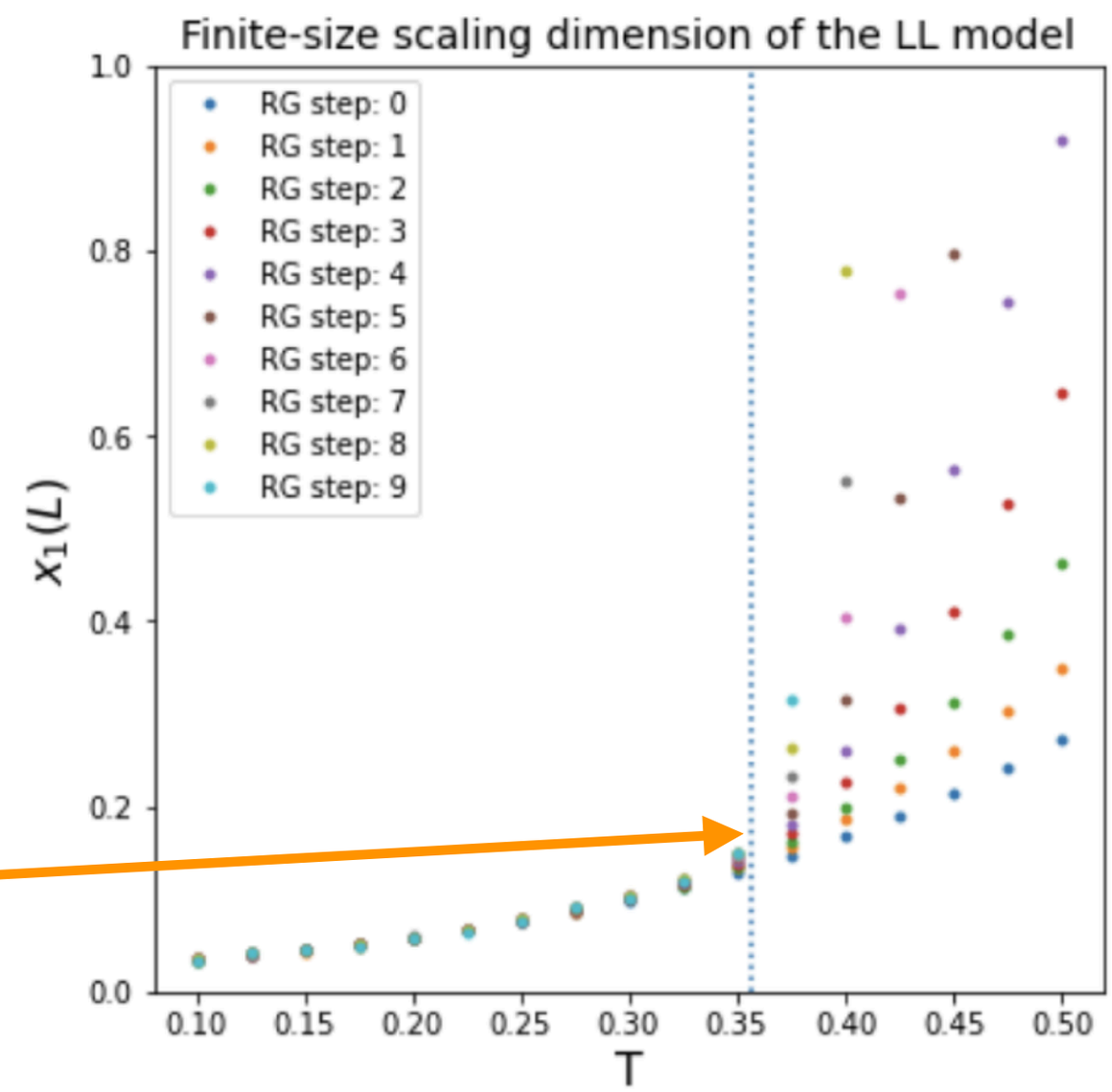
RP² Model at Intermediate *T*



$$E_0(L) \sim \epsilon_0 L - \frac{\pi c}{6L}.$$

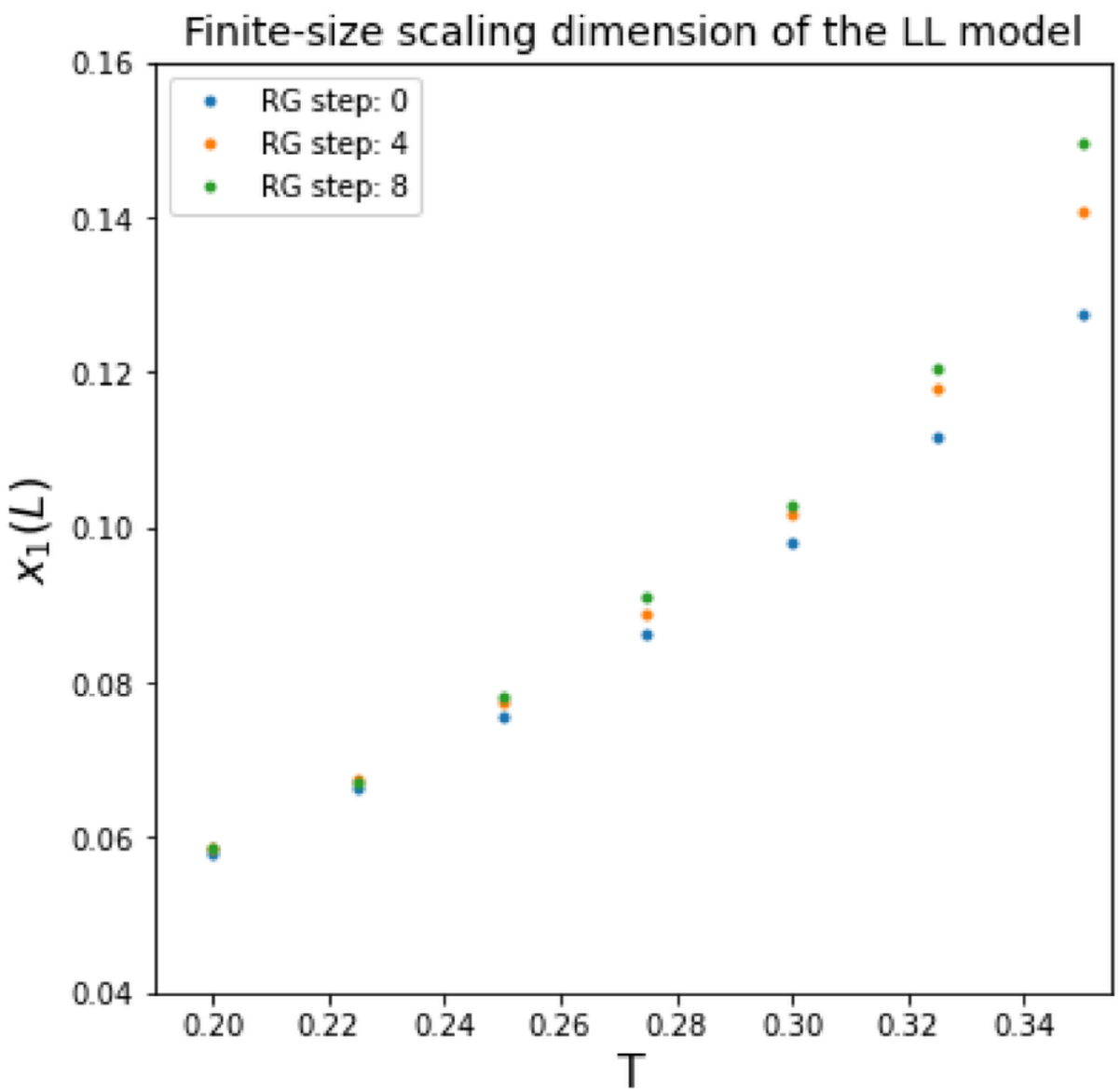
$c \sim 1.92 < 2$ no candidate CFT

$$\frac{2\pi x_n}{L} = E_n(L) - E_0(L).$$



Z2 vortex dissociation transition(?)

RP^2 Model at $T \leqq T_*$



Spectrum does not converge well even at $T \leqq T_*$

$c \sim 1.92 < 2$

SU(2) level 4

c=2 but smallest scaling dim

$$x_{j=1} = \frac{2}{3} = 0.666\ldots \gg 0.2$$

likely to support "asymptotic freedom"

RG flow c=2 → c=0 , but with a strong crossover at T_*

Conclusions (Heisenberg & RP^2)

Both Heisenberg & RP^2 Models
show c=2 at the UV fixed point for $T \to 0$
consistently with 2 (would-be) Nambu-Goldstone modes

At intermediate temperatures, the effective central charge
exhibit a "plateau" at $c \sim 1.9$

but there is no appropriate candidate CFT
and scaling dimensions do not converge

Our results support the "asymptotic freedom"
scenario with the single disordered phase in $T>0$

Discussion (Heisenberg & RP^2)

Difficulty in distinguishing very large but finite correlation length from criticality

Even when (system size) < (correlation length)
the Hamiltonian/transfer matrix spectrum
gives useful information!

We do not completely rule out the possibility of the critical point at $T \sim T_*$ or the critical phase in $T \lesssim T_*$ suggested in several papers. However, to pursue this viewpoint, one would need to explain the effective central charge and the spectra obtained in the present TNR study, which is perhaps more difficult than simply discussing whether the correlation length diverges or not.

Conclusions (Overall)

TNR + Level Spectroscopy (finite size scaling of CFT) allows

- super accurate determination of BKT critical point
- visualization of Kosterlitz RG flow by extraction of running coupling constants from the spectrum
- identify the UV fixed point for the Heisenberg & RP^2 models
- clarify(?) the crossover in the Heisenberg & RP^2 models

Future: extension/application to more nontrivial systems & unknown physics

Frank Verstraete
@fverstraete

A subjective list of 2021 highlights on new methods for Tensor Networks; please add your favorite ones.

1/ 2021 was the year for tensor networks for field theories:
@AntoineTilloy arxiv.org/abs/2102.07741
arxiv.org/abs/2104.10564 arxiv.org/abs/2105.00010

ツイートを翻訳

arxiv.org
Tensor network simulation of the (1+1)-dimensional $O(3)...
We perform a tensor network simulation of the (1+1)-dimensional $O(3)$ nonlinear $σ$-model with $θ=π$ ter...

午後7:50 · 2021年12月31日 · Twitter Web App

33 件のリツイート **4** 件の引用ツイート **112** 件のいい

Frank Verstraete
@fverstraete

4/ Backwards differentiation algorithms:
arxiv.org/abs/2101.03935
arxiv.org/abs/2107.03399

5/ Entanglement Scaling for TRG, TNR and PEPS:
arxiv.org/abs/2102.08136
arxiv.org/abs/2105.11460

ツイートを翻訳

arxiv.org
A scaling hypothesis for projected entangled-pair states

arXiv > cond-mat > arXiv:2202.07042

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Condensed Matter > Statistical Mechanics

[Submitted on 14 Feb 2022]

Tensor Network Renormalization Study on the Crossover in Classical Heisenberg and RP^2 Models in Two Dimensions

Atsushi Ueda, Masaki Oshikawa

We study the classical two-dimensional RP^2 and Heisenberg models, using the Tensor-Network Renormalization (TNR) method. The determination of the phase diagram of these models has been challenging and controversial, owing to the very large correlation lengths at low temperatures. The finite-size spectrum of the transfer matrix obtained by TNR is useful in identifying the conformal field theory describing a possible critical point. Our results indicate that the ultraviolet fixed point for the Heisenberg model and the ferromagnetic RP^2 model in the zero temperature limit corresponds to a conformal field theory with central charge $c = 2$, in agreement with two independent would-be Nambu-Goldstone modes. On the other hand, the ultraviolet fixed point in the zero temperature limit for the antiferromagnetic Lebwohl-Lasher model, which is a variant of the RP^2 model, seems to have a larger central charge. This is consistent with $c = 4$ expected from the effective SO(5) symmetry. At $T > 0$, the convergence of the spectrum is not good in both the Heisenberg and ferromagnetic RP^2 models. Moreover, there seems no appropriate candidate of conformal field theory matching the spectrum, which shows the effective central charge $c \sim 1.9$. These suggest that both models have a single disordered phase at finite temperatures, although the ferromagnetic RP^2 model exhibits a strong crossover at the temperature where the dissociation of $\mathbb{Z}_2$ vortices has been reported.

Subjects: **Statistical Mechanics (cond-mat.stat-mech)**
Cite as: arXiv:2202.07042 **[cond-mat.stat-mech]**
(or arXiv:2202.07042v1 **[cond-mat.stat-mech]** for this version)

Submission history

From: Atsushi Ueda [view email]
[v1] Mon, 14 Feb 2022 21:16:33 UTC (2,835 KB)

application to more controversial problems

arXiv > cond-mat > arXiv:2202.07597

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Condensed Matter > Statistical Mechanics

[Submitted on 15 Feb 2022]

Contrasting pseudo-criticality in the classical two-dimensional Heisenberg and RP^2 models: zero-temperature phase transition versus finite-temperature crossover

Lander Burgelman, Lukas Devos, Bram Vanhecke, Frank Verstraete, Laurens Vanderstraeten

Tensor-network methods are used to perform a comparative study of the two-dimensional classical Heisenberg and RP^2 models. We demonstrate that uniform matrix product states (MPS) with explicit $\mathrm{SO}(3)$ symmetry can probe correlation lengths up to $\mathcal{O}(10^3)$ sites accurately, and we study the scaling of entanglement entropy and universal features of MPS entanglement spectra. For the Heisenberg model, we find no signs of a finite-temperature phase transition, supporting the scenario of asymptotic freedom. For the RP^2 model we observe an abrupt onset of scaling behaviour, consistent with hints of a finite-temperature phase transition reported in previous studies. A careful analysis of the softening of the correlation length divergence, the scaling of the entanglement entropy and the MPS entanglement spectra shows that our results are inconsistent with true criticality, but are rather in agreement with the scenario of a crossover to a pseudo-critical region which exhibits strong signatures of nematic quasi-long-range order at length scales below the true correlation length. Our results reveal a fundamental difference in scaling behaviour between the Heisenberg and RP^2 models: Whereas the emergence of scaling in the former shifts to zero temperature if the bond dimension is increased, it occurs at a finite bond-dimension independent crossover temperature in the latter.

Subjects: **Statistical Mechanics (cond-mat.stat-mech)**; Quantum Physics (quant-ph)
Cite as: arXiv:2202.07597 **[cond-mat.stat-mech]**
(or arXiv:2202.07597v1 **[cond-mat.stat-mech]** for this version)

Submission history

From: Lander Burgelman [view email]
[v1] Tue, 15 Feb 2022 17:35:15 UTC (747 KB)


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