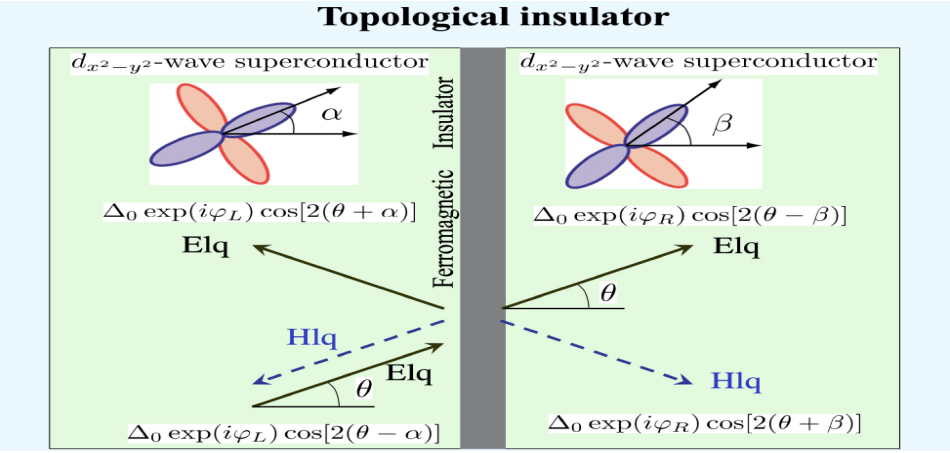
Theory of giant diode effect of d-wave superconductor junction on the surface of topological insulator



Yukio Tanaka

Nagoya University

NQS2022, Yukawa Institute: November 17, 2022

Y. Tanaka, Bo Lu and N. Nagaosa

arXiv:2205.13177

Contents of this talk

Background of this talk

1 d-wave superconductor,
topological surface Andreev bound states

2 Josephson effect in d-wave superconductor junctions without TI

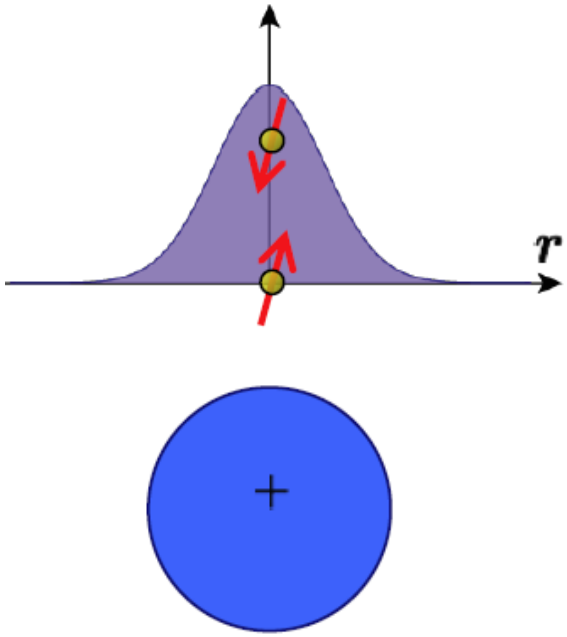
3 Superconducting junctions on TI

4 Diode effect by d-wave superconductor junctions on TI

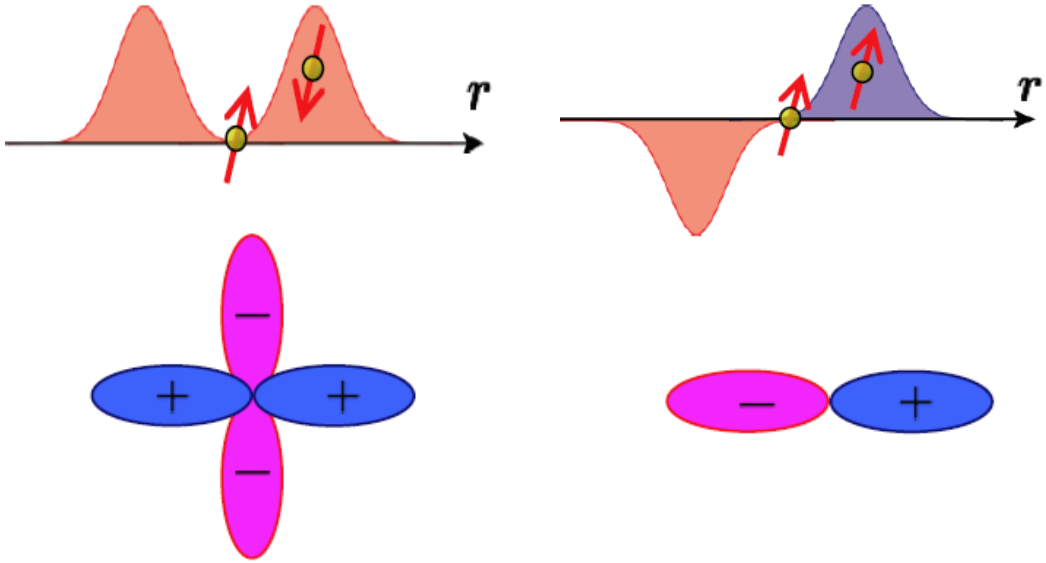
Unconventional superconductor

In strongly correlated systems, pair potential changes sign on the Fermi surface.

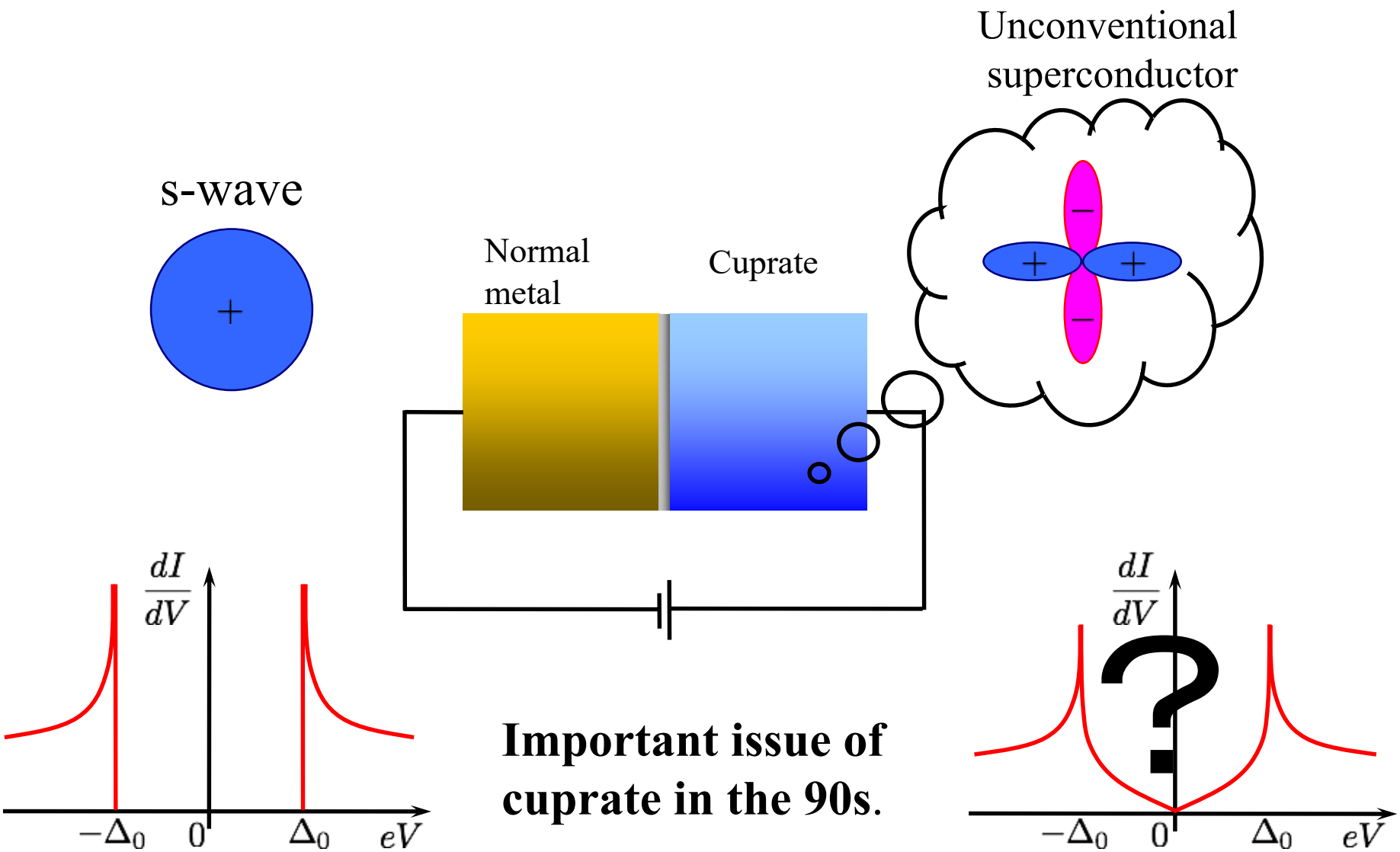
Isotropic pairing



Anisotropic pairing



Tunneling effect in unconventional superconductors



Important issue of cuprate in the 90s.

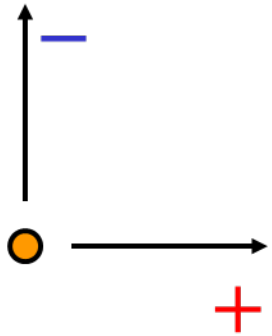
Properties of unconventional superconductors

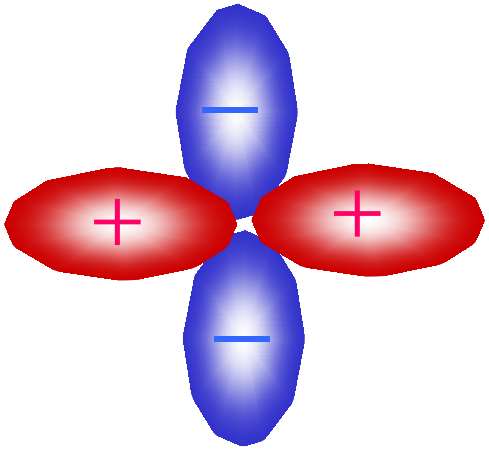
Pair potential becomes zero on the Fermi surface.

We call it nodal points or nodes.

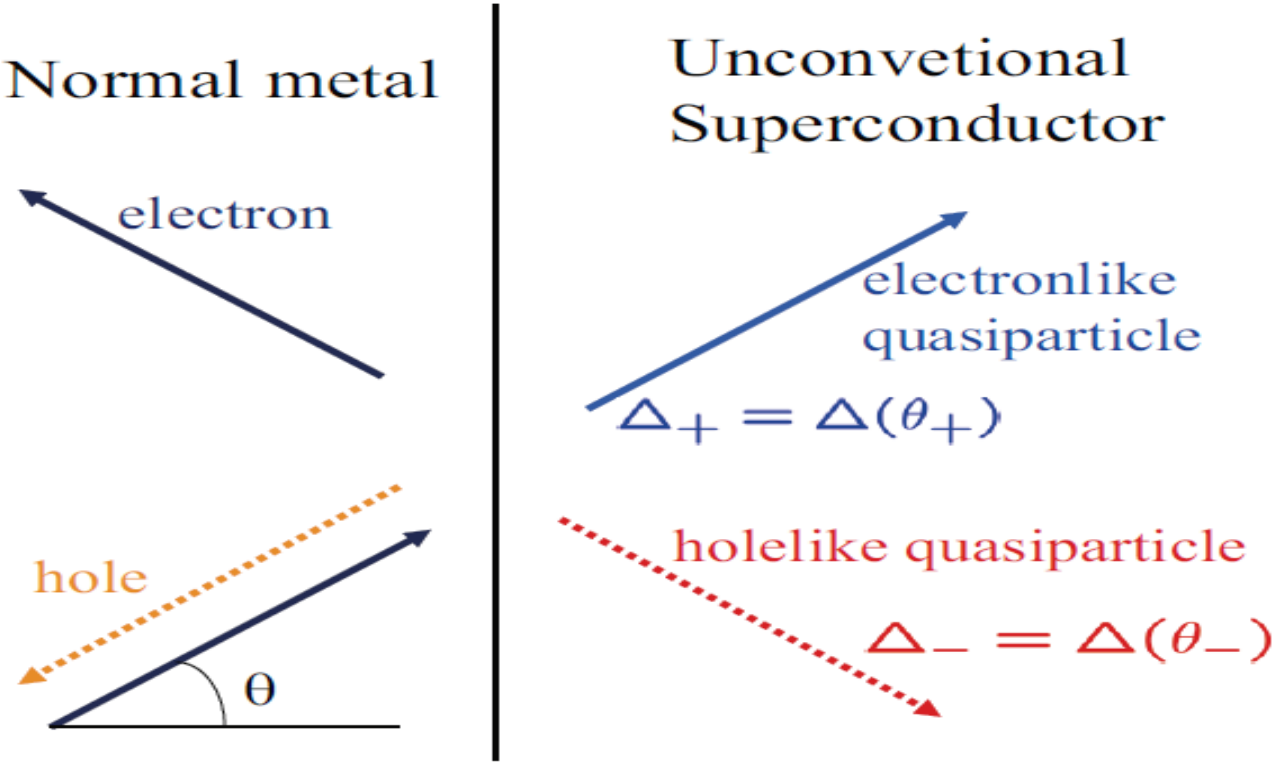
Quasiparticle（Bogoliubov particle）feels different sign (phase) of pair potential depending on their directions of motions.

d-wave case





Wave function in N/S junction (2)



Spin-singlet s-wave superconductor

$$\Delta(\theta_+) = \Delta(\theta_-) = \Delta_0$$

$$\Psi(\theta, x) = \begin{pmatrix} u_+(\theta, x) \\ v_+(\theta, x) \end{pmatrix} \exp(ik_F \cos\theta x) + \begin{pmatrix} u_-(\theta, x) \\ v_-(\theta, x) \end{pmatrix} \exp(-ik_F \cos\theta x)$$

Formula of differential conductance 1

$$\sigma_T(E) = \frac{\int_{-\pi/2}^{\pi/2} d\theta \sigma_N(\theta)\sigma_R(E,\theta)\cos\theta}{\int_{-\pi/2}^{\pi/2} d\theta \sigma_N(\theta)\cos\theta}$$

$$\sigma_R(E,\theta) = \frac{1+\sigma_N(\theta)\mid\Gamma_+\mid^2 + [\sigma_N(\theta)-1]\mid\Gamma_+\Gamma_-\mid^2}{\mid 1+[\sigma_N(\theta)-1]\Gamma_+\Gamma_-\mid^2},$$

transparency

$$\sigma_N(\theta) = \frac{\cos^2\theta}{\cos^2\theta + Z^2}$$

$$E = eV$$

$$\Gamma_+ = v_+/u_+ = \frac{\Delta_+^*}{E+\Omega_+} \qquad \Gamma_- = v_-/u_- = \frac{\Delta_-}{E+\Omega_-}$$

(Tanaka and Kashiwaya PRL 74 3451)

$$\sigma_N(\theta)\sigma_R(E,\theta) = 1 + \mid a(E,\theta)\mid^2 - \mid b(E,\theta)\mid^2 = \sigma_S$$

Conductance formula & Surface Andreev bound state (SABS)

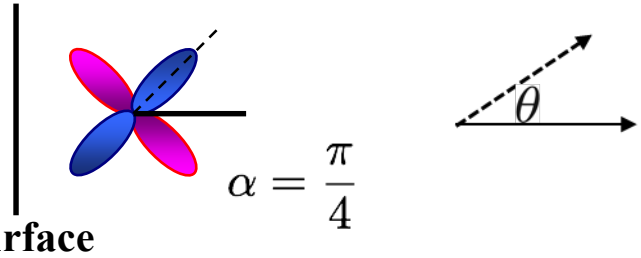
Condition for SABS

$$1 = \Gamma_+ \Gamma_-$$

d_{xy}-wave

$$\Delta_+ = -\Delta_- = \Delta_0 \sin(2\theta) = \Delta_0 \cos[2(\theta - \alpha)]$$

$$1 = \frac{-E + \sqrt{E^2 - \Delta_0^2 \sin^2 2\theta}}{E + \sqrt{E^2 - \Delta_0^2 \sin^2 2\theta}}$$



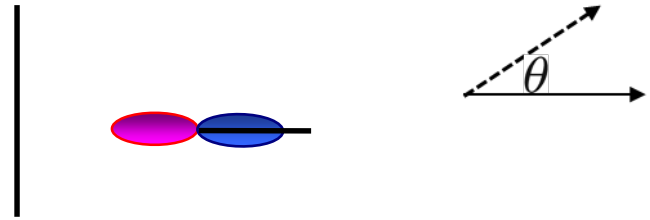
Flat zero energy band $E = 0$

C.R. Hu : Phys. Rev. Lett. 72 (1994) 1526.

p_x-wave

$$\Delta_+ = -\Delta_- = \Delta_0 \cos\theta$$

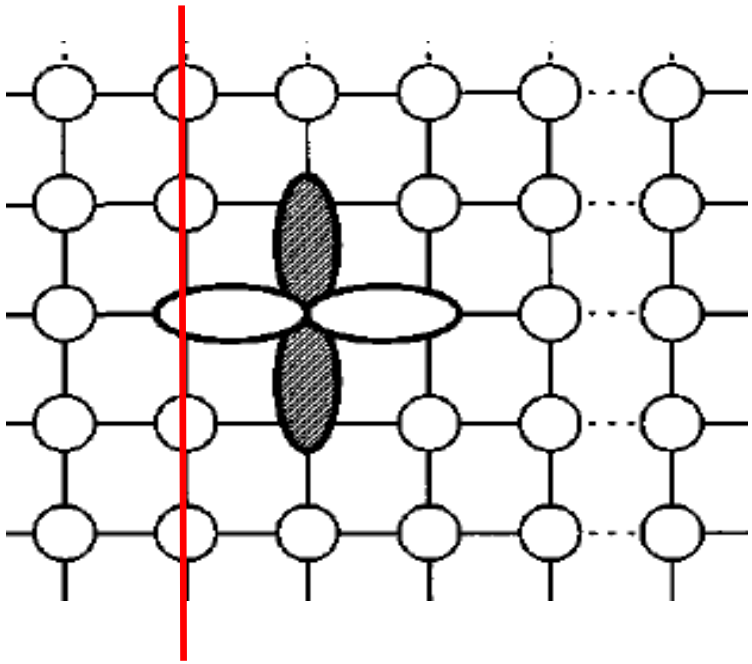
$$1 = \frac{-E + \sqrt{E^2 - \Delta_0^2 \cos^2 \theta}}{E + \sqrt{E^2 - \Delta_0^2 \cos^2 \theta}}$$

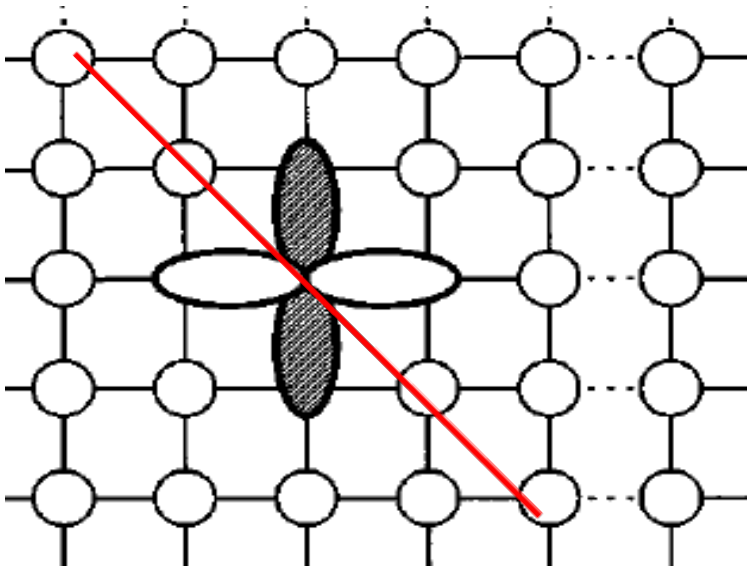


Flat zero energy band $E = 0$

J. Hara, K. Nagai : Prog.Theor. Phys. 74 (1986) 1237.

d-wave pairing in cuprate





100 surface

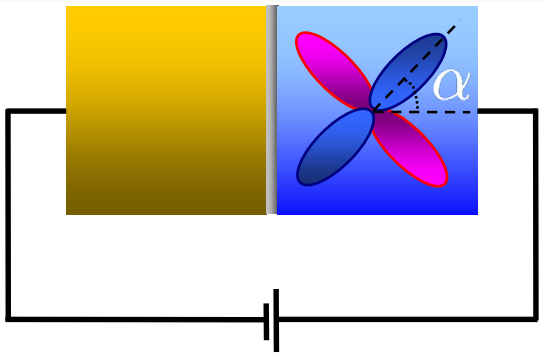
110 surface

$$\Delta_+ = \Delta_- = \Delta_0 \cos 2\theta$$

$$\Delta_+ = -\Delta_- = \Delta_0 \sin 2\theta$$

$$\Delta_+ = -\Delta_- = \Delta_0 \sin(2\theta) = \Delta_0 \cos[2(\theta - \alpha)]$$

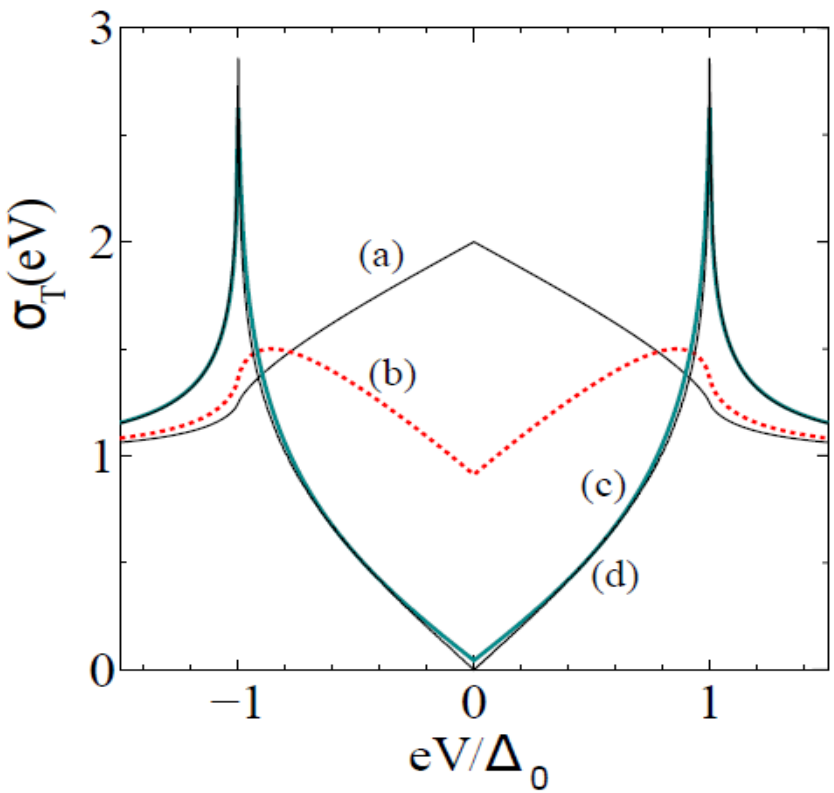
Conductance in d-wave superconductor junctions 1



Y. Tanaka & S. Kashiwaya: Phys. Rev. Lett. **74** (1995) 3451.

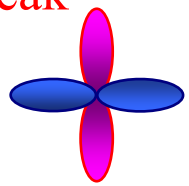
$$\Delta_{\pm} = \Delta_0 \cos[2(\theta \mp \alpha)]$$

α Angle between the crystal axis and normal to the interface



Exceptional case without zero bias conductance peak

$\alpha = 0$



transparency $\sigma_N(\theta) = \frac{\cos^2\theta}{\cos^2\theta + Z^2}$

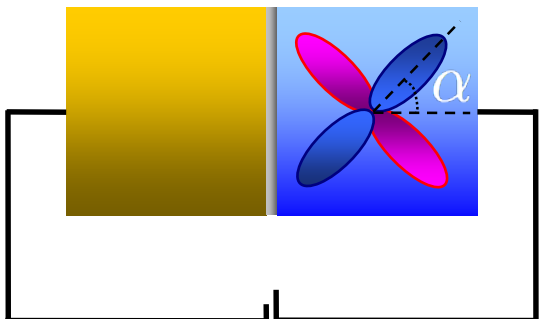
$(a) Z = 0, (b) Z = 0.5$

$(c) Z = 3$

(d) LDOS in bulk

S. Kashiwaya and Y. Tanaka, Rep. Prog. Phys. 63 (2000) 1641.

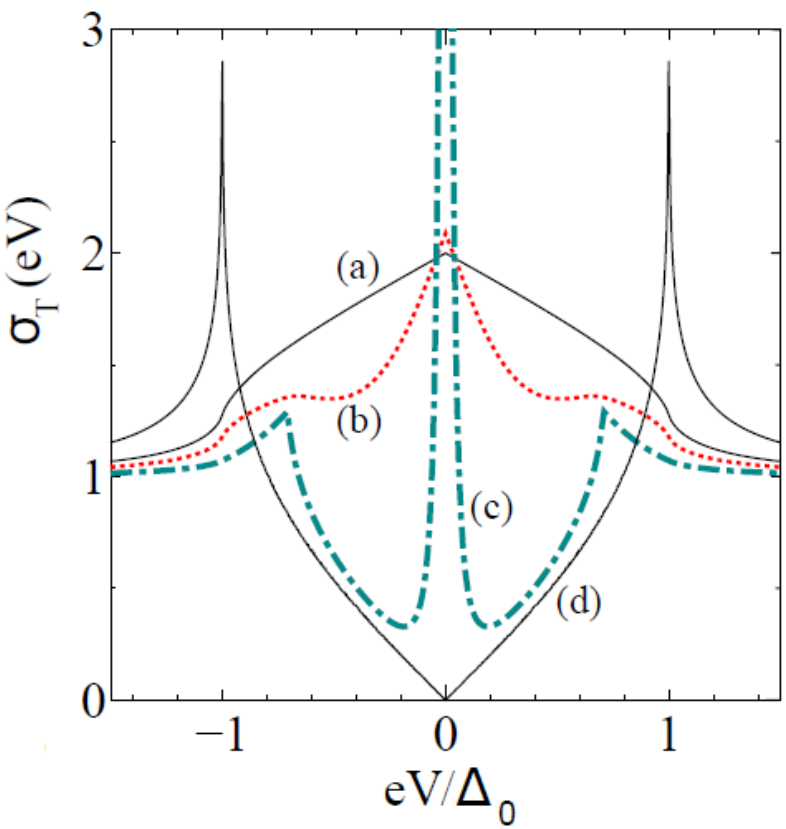
Conductance in d-wave superconductor junctions 2



Y. Tanaka & S. Kashiwaya: Phys. Rev. Lett. **74** (1995) 3451.

$$\Delta_{\pm} = \Delta_0 \cos[2(\theta \mp \alpha)]$$

α Angle between the crystal axis and normal to the interface

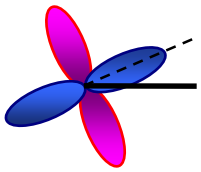


$(a) Z = 0, (b) Z = 0.5$

$(c) Z = 3$

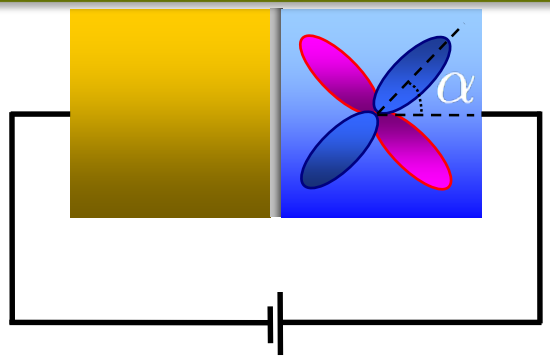
(d) LDOS in bulk

$$\alpha = \frac{\pi}{8}$$



S. Kashiwaya and Y. Tanaka, Rep. Prog. Phys. 63 (2000) 1641.

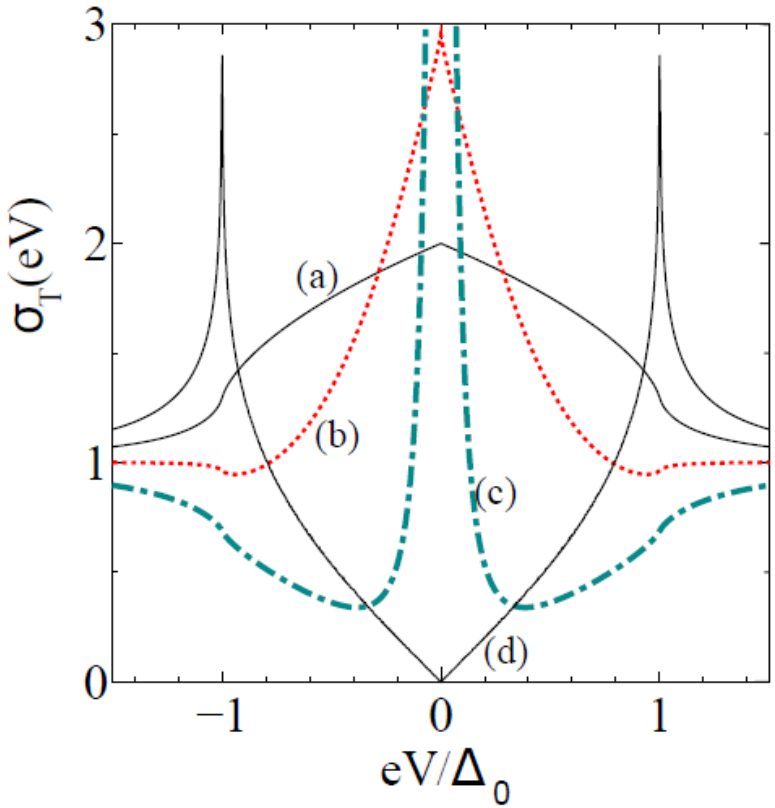
Conductance in d-wave superconductor junctions 3



Y. Tanaka & S. Kashiwaya: Phys. Rev. Lett. **74** (1995) 3451.

$$\Delta_{\pm} = \Delta_0 \cos[2(\theta \mp \alpha)]$$

α Angle between the crystal axis and normal to the interface

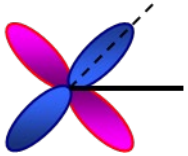


$$(a) Z = 0, (b) Z = 0.5$$

$$(c) Z = 3$$

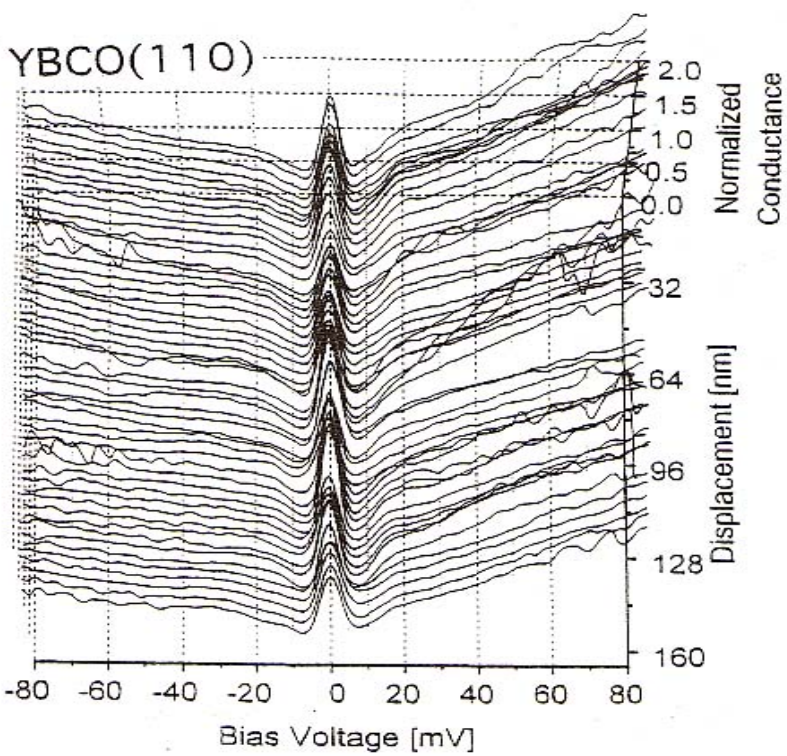
(d) LDOS in bulk

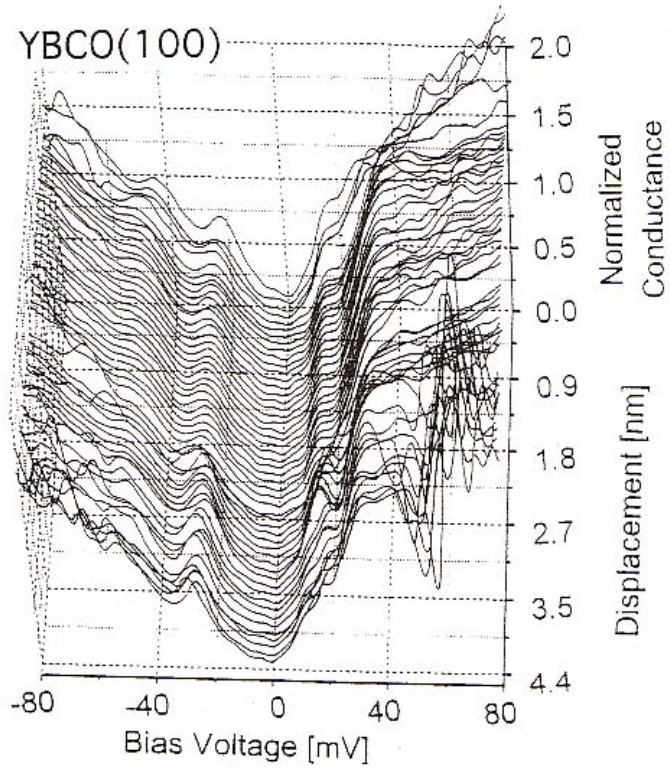
$$\alpha = \frac{\pi}{4}$$



S. Kashiwaya and Y. Tanaka, Rep. Prog. Phys. 63 (2000) 1641.

STS measurement in high Tc cuprate





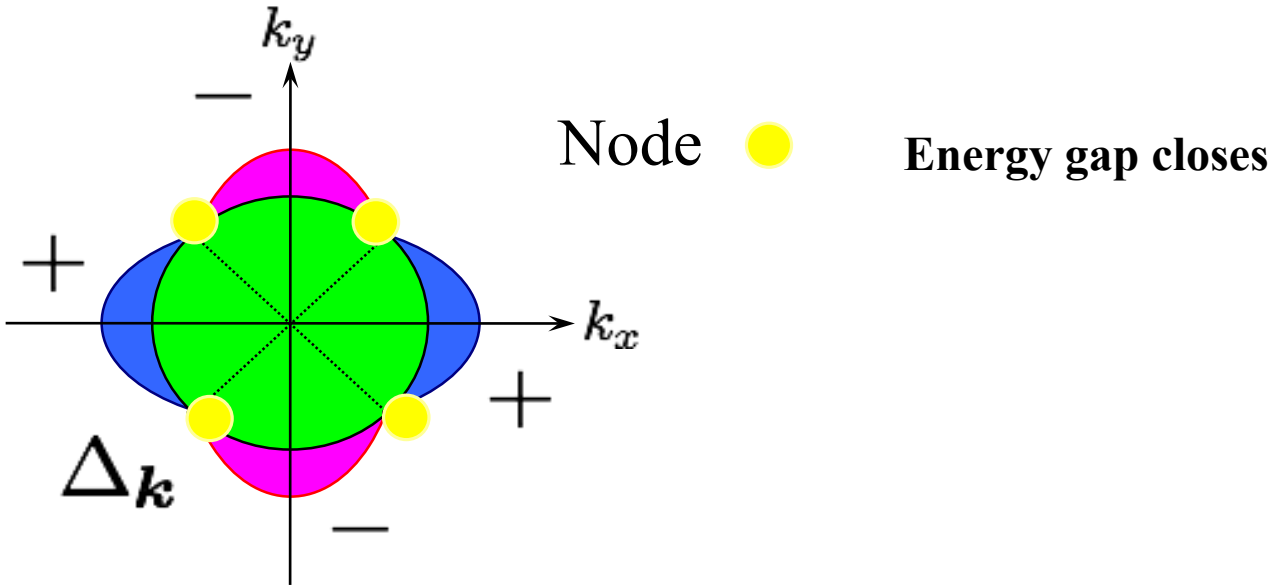
Strong evidence supporting
d-wave pairing symmetry in cuprate

S. Kashiwaya & Y.Tanaka : Rep. Prog. Phys. ***63*** *(2000) 1641.*

Physical origin of zero bias conductance peak?

Why zero energy surface Andreev bound state appears?

It appears not accidentally.



Topological invariant defined in momentum space.

Bulk-edge correspondence

M. Sato, Y. Tanaka, K. Yada, T.Yokoyama、Phys. Rev. B 83, 224511 (2011)

Topological invariant

BdG Hamiltonian 2×2 matrix

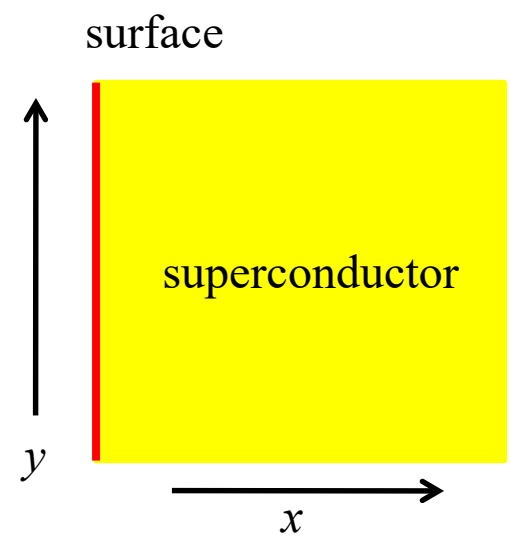
$$\mathcal{H} = \sum_{\boldsymbol{k}} \left(c^\dagger_{\boldsymbol{k}\uparrow}, c_{-\boldsymbol{k}\downarrow} \right) \mathcal{H}(\boldsymbol{k}) \begin{pmatrix} c_{\boldsymbol{k}\uparrow} \\ c^\dagger_{-\boldsymbol{k}\downarrow} \end{pmatrix} \qquad \mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} \xi(\boldsymbol{k}) & \Delta(\boldsymbol{k}) \\ \Delta(\boldsymbol{k}) & -\xi(\boldsymbol{k}) \end{pmatrix},$$

$$\Delta(\boldsymbol{k}) = \begin{cases} \psi(\boldsymbol{k}) = \psi(\boldsymbol{k}) & \text{for spin-singlet} \\ d_z(\boldsymbol{k}) = -d_z(-\boldsymbol{k}) & \text{for spin-triplet} \end{cases}.$$

Winding number

$$w_{1\mathrm{d}} = \frac{1}{2\pi} \int_{C_1} dk \partial_{k_x} \theta(\boldsymbol{k}).$$

$$\cos\theta(\boldsymbol{k}) = \frac{\xi(\boldsymbol{k})}{\sqrt{\xi(\boldsymbol{k})^2 + \Delta(\boldsymbol{k})^2}}, \quad \sin\theta(\boldsymbol{k}) = \frac{\Delta(\boldsymbol{k})}{\sqrt{\xi(\boldsymbol{k})^2 + \Delta(\boldsymbol{k})^2}},$$



Sato, Tanaka, Yada , Yokoyama, PRB 83 224511 (2011)

Topological invariant

$$w_{1\mathrm{d}}(k_y) = \frac{1}{2\pi}\int_{-\pi}^{\pi} dk_x \partial_{k_x}\theta(\boldsymbol{k})$$

$$\cos\theta(\boldsymbol{k}) = \frac{\xi(\boldsymbol{k})}{\sqrt{\xi(\boldsymbol{k})^2+\Delta(\boldsymbol{k})^2}}$$

$$\sin\theta(\boldsymbol{k}) = \frac{\Delta(\boldsymbol{k})}{\sqrt{\xi(\boldsymbol{k})^2+\Delta(\boldsymbol{k})^2}}$$

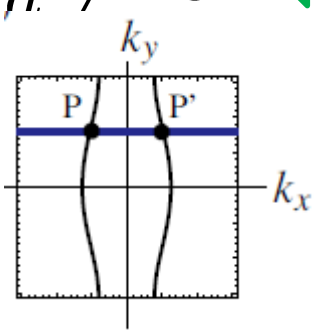
$$w_{1\mathrm{d}}(k_y) = -\frac{1}{2}\sum_{k_x;\xi(\boldsymbol{k})=0} \mathrm{sgn}[\Delta(\boldsymbol{k})]\cdot\mathrm{sgn}[\partial_{k_x}\xi(\boldsymbol{k})],$$

Topological invariant is defined for effective 1d Hamiltonian for fixed *k*y.

Single Fermi surface

$$w_{1\mathrm{d}} = -\frac{1}{2}\mathrm{sgn}[\partial_{k_x}\xi(-k_x^0,k_y)]\left[\mathrm{sgn}[\Delta(-k_x^0,k_y)] - \mathrm{sgn}[\Delta(k_x^0,k_y)]\right].$$

$$w_{1d} \neq 0 \Longleftrightarrow \Delta(-k_x^0,k_y)\Delta(k_x^0,k_y) < 0$$



P: $(-k_x^0, k_y)$

P': (k_x^0, k_y)

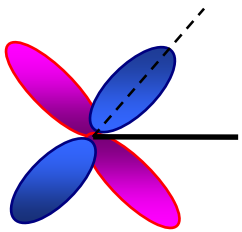
Sign change of the pair potential at the reflection

Sato, Tanaka, et al, PRB 83 224511 (2011)

Winding number

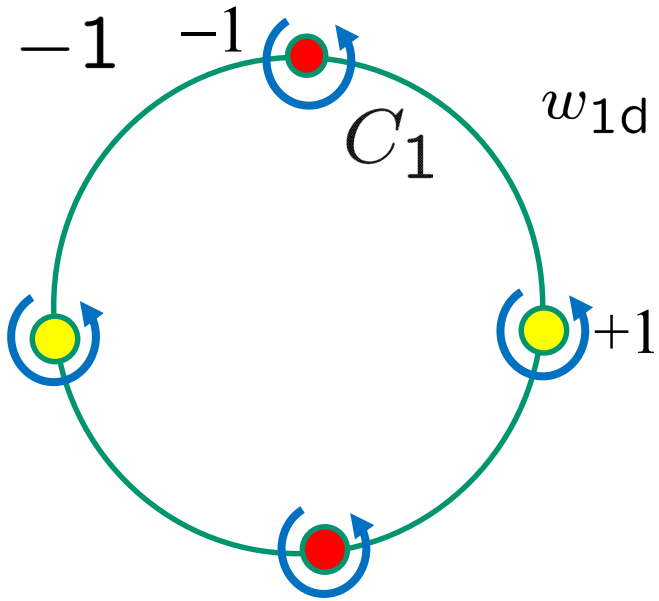
Winding number defined around nodes

d-wave superconductor



$$w_{1d} = -1$$

$$w_{1d} = 1$$



$$w_{1d} = 1$$

$$w_{1d} = -1$$

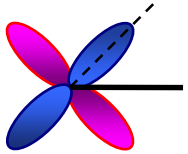
$$w_{1\mathrm{d}} = \frac{1}{2\pi}\int_{C_1} dk \partial_{k_x}\theta(\boldsymbol{k}).$$

$$\cos\theta(\boldsymbol{k}) = \frac{\xi(\boldsymbol{k})}{\sqrt{\xi(\boldsymbol{k})^2 + \Delta(\boldsymbol{k})^2}}, \quad \sin\theta(\boldsymbol{k}) = \frac{\Delta(\boldsymbol{k})}{\sqrt{\xi(\boldsymbol{k})^2 + \Delta(\boldsymbol{k})^2}},$$

Nodal structure and winding number

d-wave superconductor

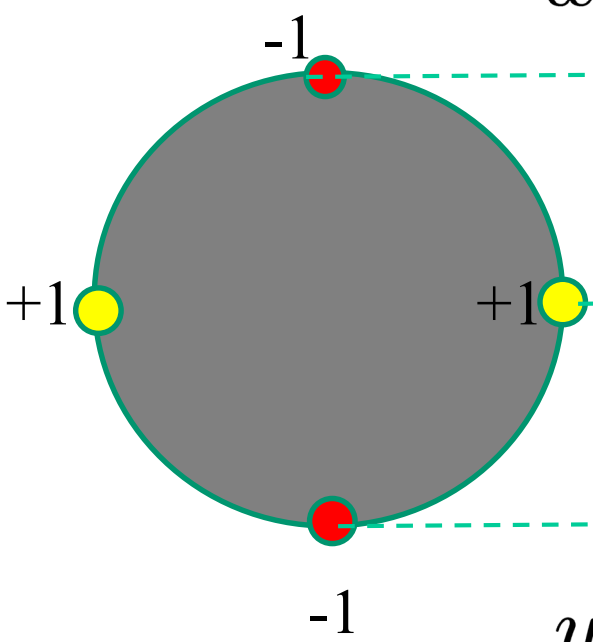
(110) oriented surface



d_{xy} wave superconductor

$\alpha = \pi/4$

$w_{1d}(k_y) = 0$



$w_{1d}(k_y) = -1$

$w_{1d}(k_y) = 1$

$w_{1d}(k_y) = 0$

ZESABS

k_y

Winding number & Index theorem

From the <u>bulk-edge correspondence</u>, there exists the gapless states on the edge only when integer w_{1d} is nonzero.

BdG Hamiltonian has a symmetry (chiral symmetry)

$$\{\mathcal{H}(\boldsymbol{k}), \sigma_y\} = 0$$

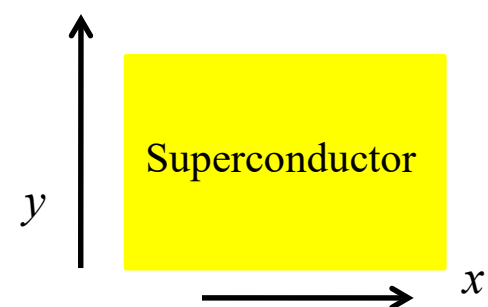
Zero energy ABS is an eigenstate of σ_y.

$n_0^{(+)}$ **<u>Number of ZES where the eigenvalue of σ_y is 1</u>**

$n_0^{(-)}$ **<u>Number of ZES where the eigenvalue of σ_y is –1</u>**

Index Theorem

$$w_{1d} = (n_0^{(+)} - n_0^{(-)})$$

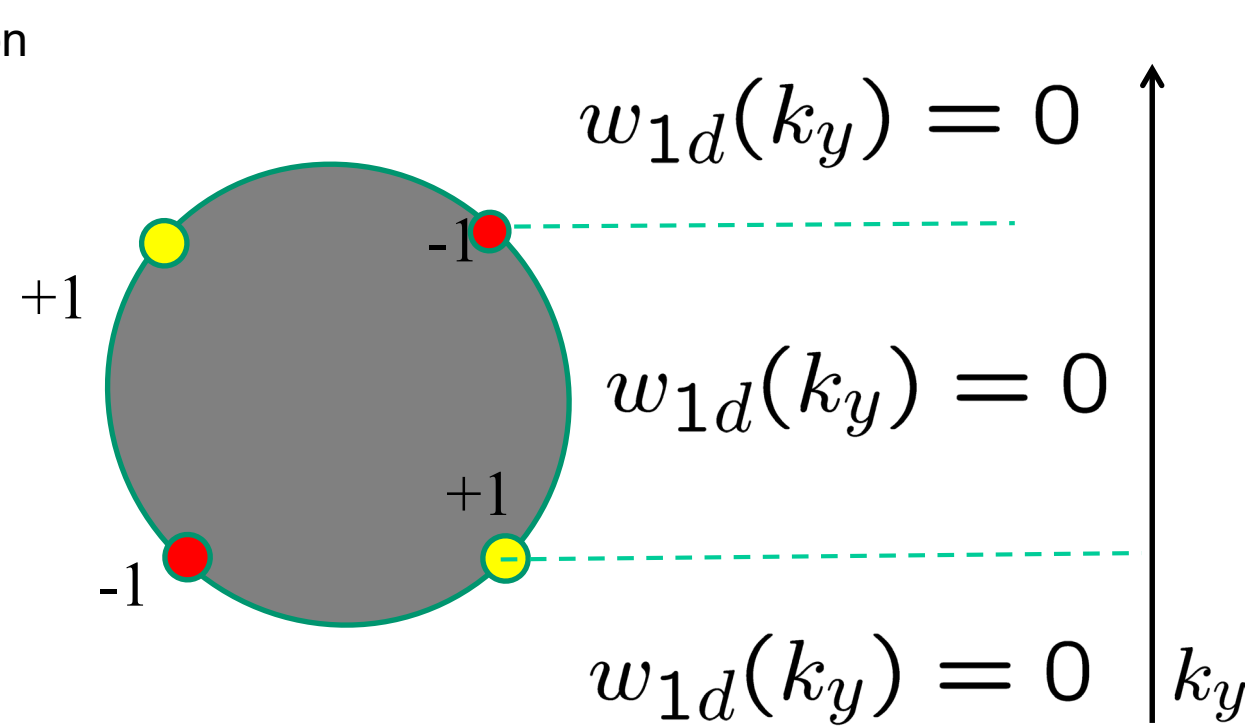


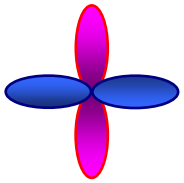
Sato, Tanaka, Yada , Yokoyama, PRB 83 224511 (2011)

Nodal structure and winding number

d-wave superconductor **Misorientation Angle** $\alpha = 0$

(100) direction





ZESABS vanishes due to the topological cancellation

Contents of this talk

Background of this talk

1 d-wave superconductor,
topological surface Andreev bound states

2 Josephson effect in d-wave superconductor junctions without TI

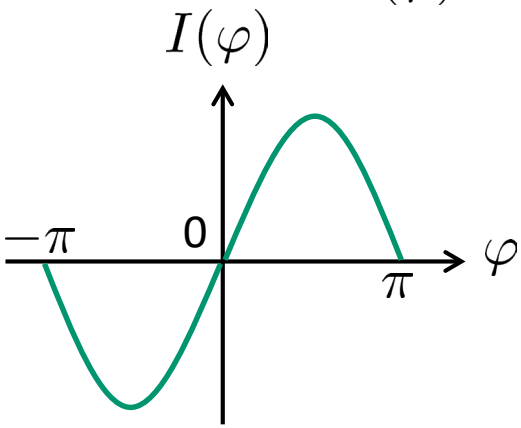
3 Superconducting junctions on TI

4 Diode effect by d-wave superconductor junctions on TI

Various types of Josephson junctions

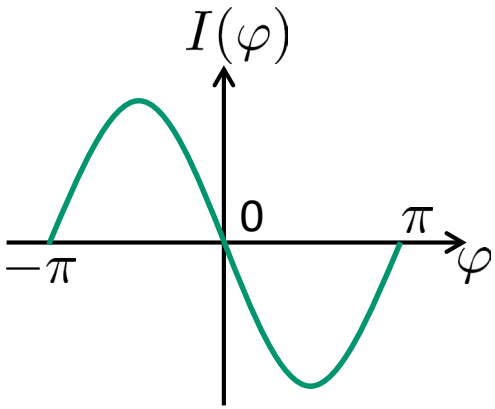
Current phase relations

$$I(\varphi) \sim I_1 \sin\varphi$$



$I_1 > 0$

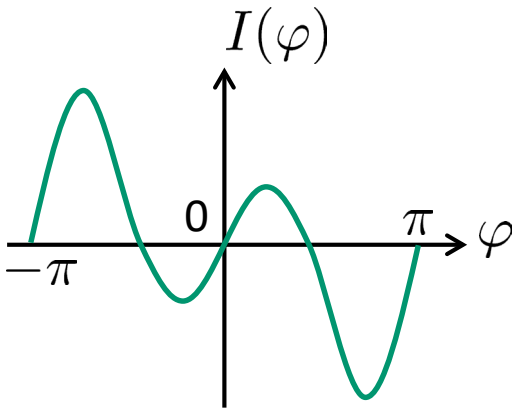
0-junction



$I_1 < 0$

π-junction

$$I(\varphi) \sim I_1 \sin\varphi + I_2 \sin 2\varphi$$



$I_1 I_2 < 0$

φ -junction

L. N. Bulaevskii (1977)　強磁性
D. J. van Harlingen (1995)　d 波
V. V. Ryazanov (2001)　強磁性

Y. Tanaka and S. Kashiwaya (1997)

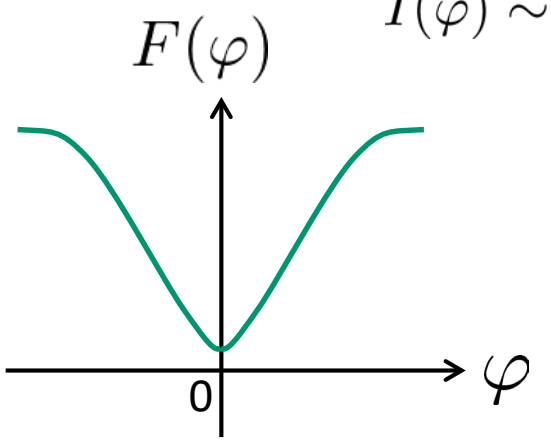
Various types of Josephson junctions

$$I(\varphi) = \frac{2e}{\hbar}\left(\frac{\partial F(\varphi)}{\partial \varphi}\right)$$

Free energy of the junctions

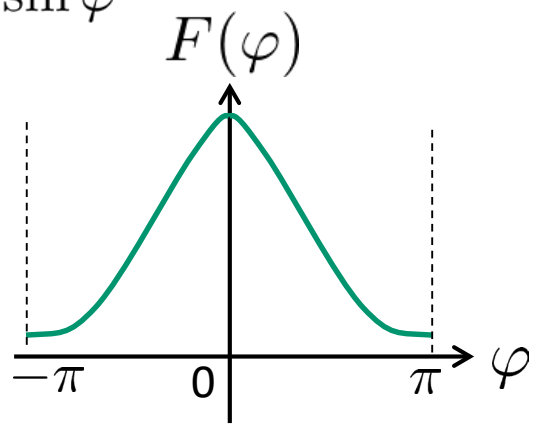
$$I(\varphi) \sim I_1 \sin\varphi$$

$$I(\varphi) \sim I_1 \sin\varphi + I_2 \sin 2\varphi$$



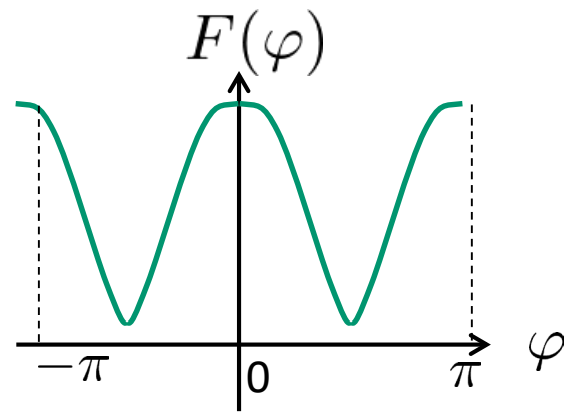
$I_1 > 0$

0-junction



$I_1 < 0$

π-junction



$I_1 I_2 < 0$

φ -junction

L. N. Bulaevskii (1977) F/S
D. J. van Harlingen (1995) d -wave
V. V. Ryazanov (2001) F/S

Y. Tanaka and S. Kashiwaya (1997)
d-wave

Josephson current formula available for unconventional superconductors

Phys. Rev. B ***53*** *(1996) 11957.*

$$a_1(\theta,\phi,E) \to a_1(\theta,\phi,\mathrm{i}\omega_n),\ a_2(\theta,\phi,E) \to a_2(\theta,\phi,\mathrm{i}\omega_n),$$

$$I(\phi) = \frac{2e}{\hbar}\int_{\pi/2}^{\pi/2} d\theta \sum_{\omega_n} \frac{|\Delta_{L,+}|}{\Omega_{n,+}}[a_1(\phi,\mathrm{i}\omega_n) - a_2(\phi,\mathrm{i}\omega_n)]\cos\theta,$$

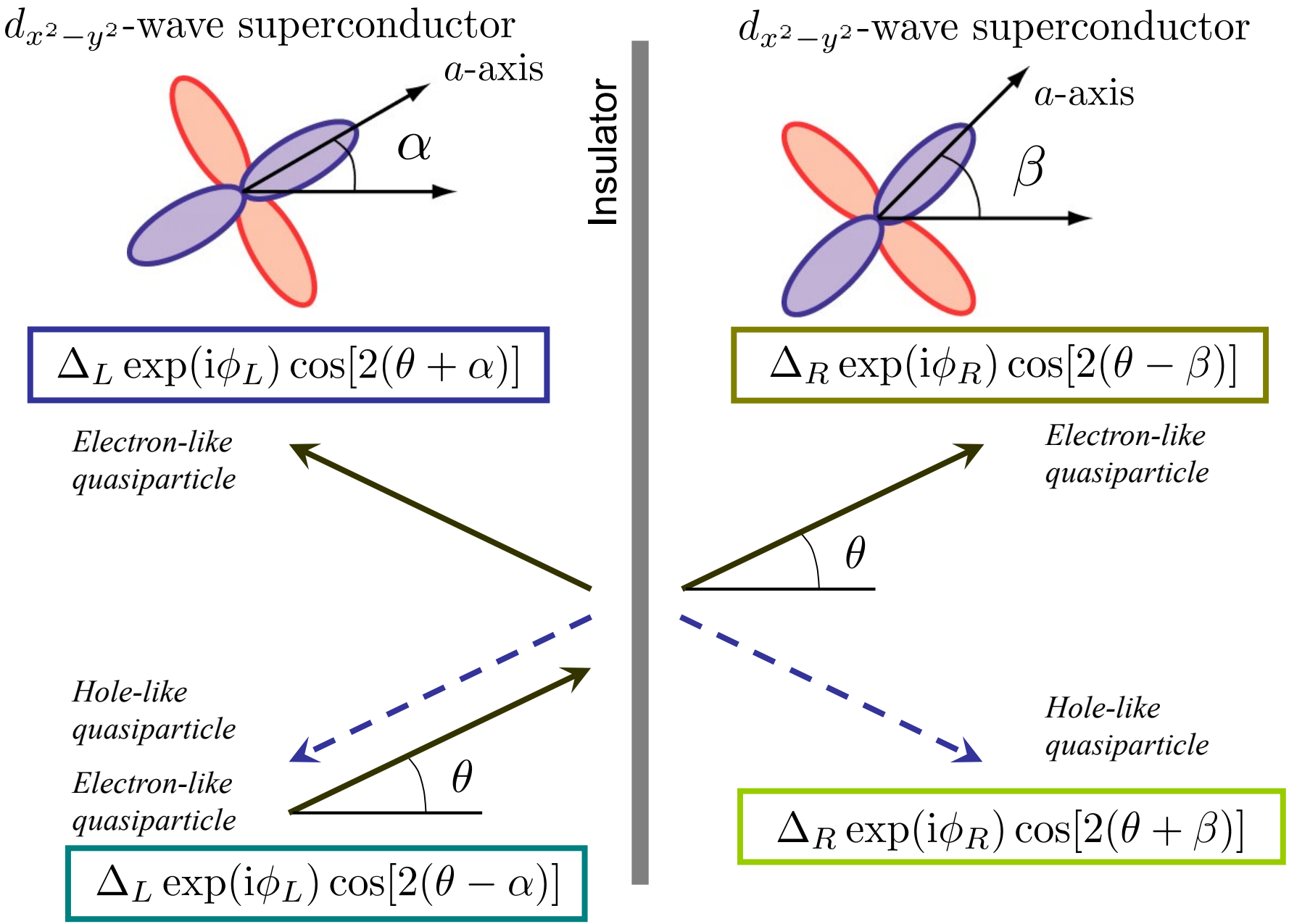
(1)SABS is taken into account.

(2)Coexistence of 0 and π junctions

(3)Realization of ϕ-junction

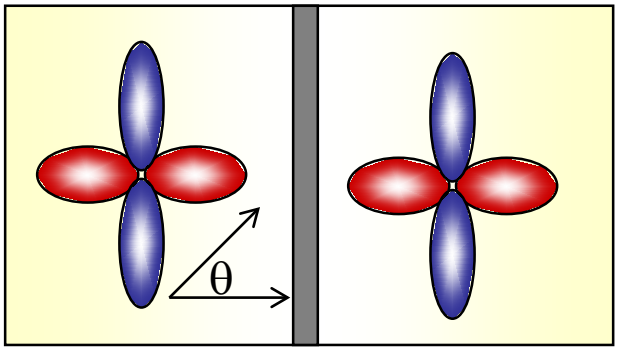
Including spin-triplet case; Y. Asano PRB 2001

Scattering Process



Andreev bound states in d-wave junctions

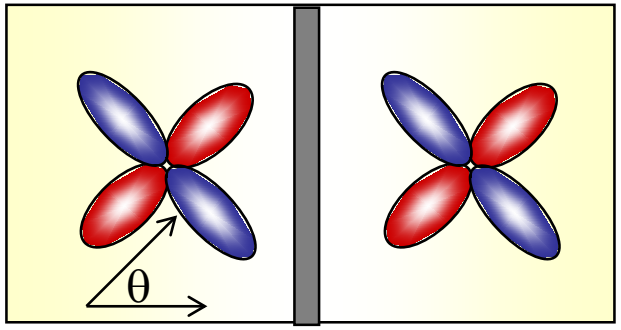
φ Phase difference σ_N transmissivity θ injection angle



$$\Delta_{eff} = \Delta \cos 2\theta$$

$$E_b = \Delta_{eff}\sqrt{\cos^2(\varphi/2) + (1-\sigma_N)\sin^2(\varphi/2)}$$

$\sigma_N \to 0$ $E_b = \Delta_{eff}$



$$\Delta_{eff} = \Delta \sin 2\theta$$

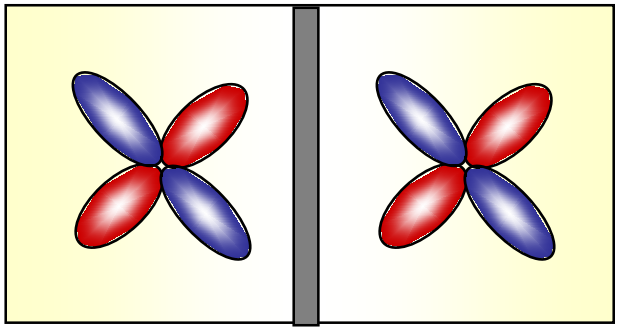
$$E_b = \Delta_{eff}\cos(\varphi/2)\sqrt{\sigma_N}$$

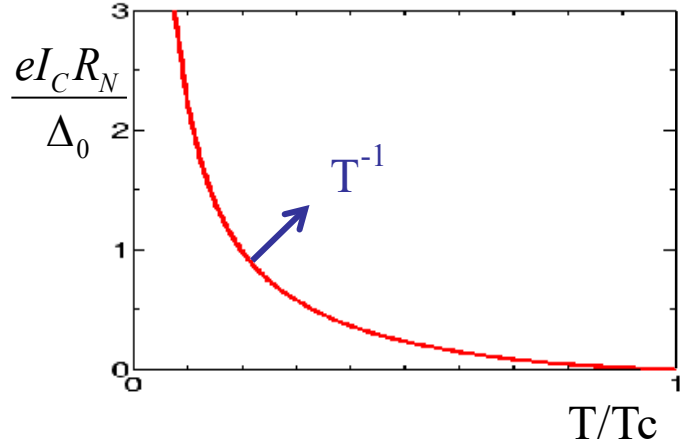
4π periodicity

$\sigma_N \to 0$ $E_b = 0$

PRB 53 9371 (1997)

Enhanced d.c. Josephson current by ABS





$$R_N I(\varphi) = \frac{\pi \bar{R}_N}{e} \int_{-\pi/2}^{\pi/2} \Delta_{eff} \sqrt{\sigma_N} \cos\theta \sin(\varphi/2) \tanh[\frac{\Delta_{eff} \cos(\varphi/2) \sqrt{\sigma_N}}{2k_B T}] d\theta$$

D.C. Josephson current does not have 4π periodicity (conventional 2π periodicity)

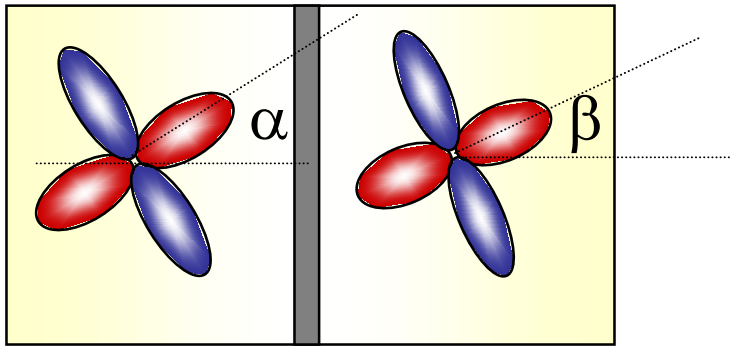
I_c maximum Josephson current I_C is proportional to $(R_N)^{-1/2}$

Barash (1996) **Tanaka, Kashiwaya(1996)**

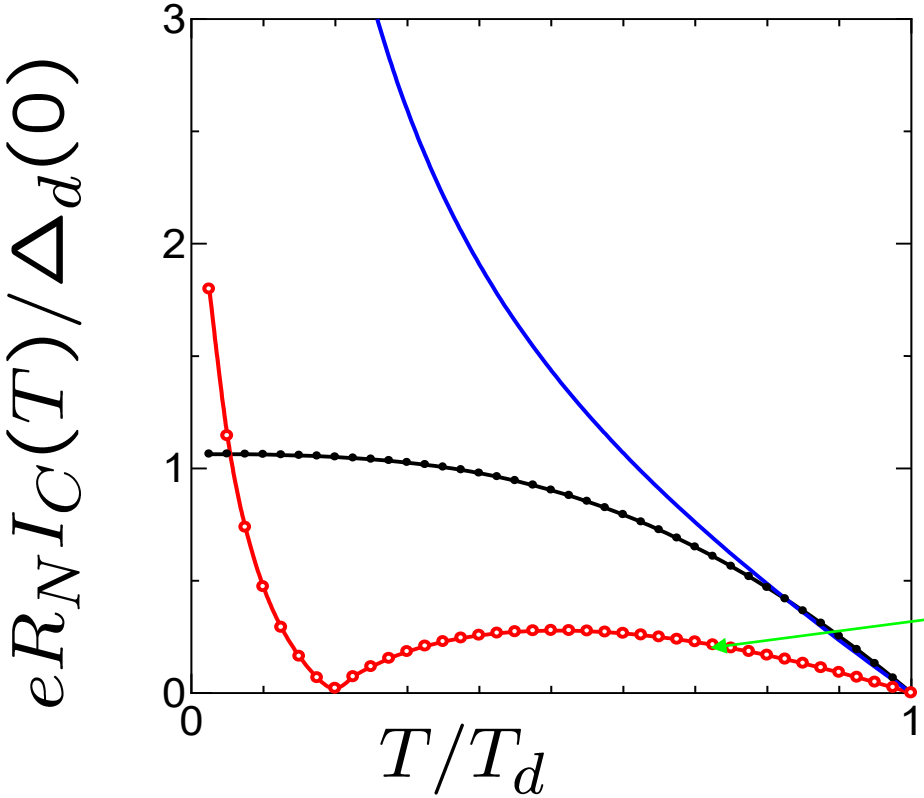
A.C. Josephson current has 4π periodicity

Yakovenko et al (2004)

Josephson current in mirror type junctions



$\alpha = -\beta$



(a) $\alpha=0$

(b) $\alpha=0.1\pi$

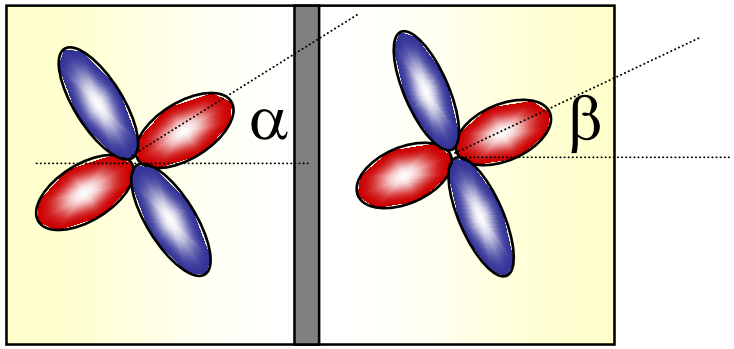
(c) $\alpha=0.25\pi$

Non-monotonic temperature dependence

Phys. Rev. B 53 11957(1996)
Phys. Rev. B 56 892(1997)

Barash, Burkhardt, Rainer, Phys. Rev. Lett.77 4070 (1996)

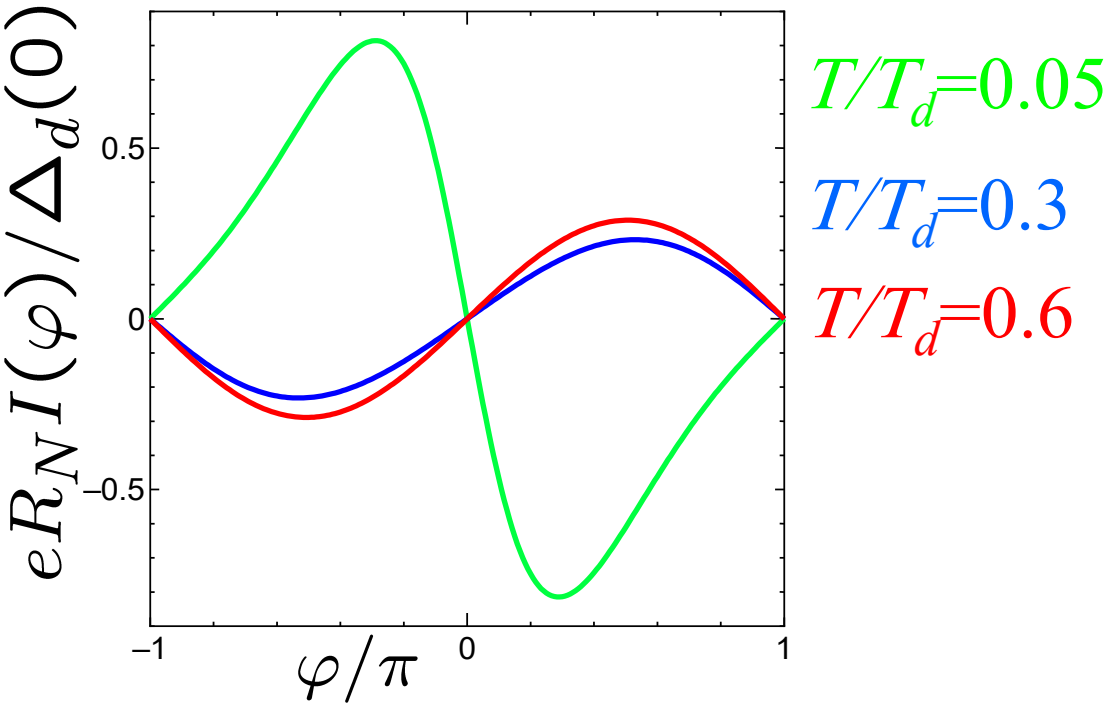
Josephson current in mirror type junctions

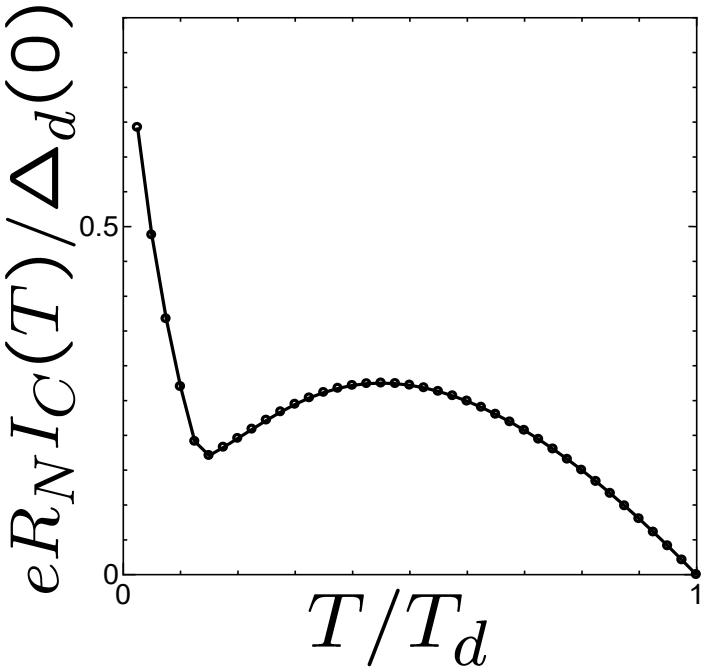


$\alpha = -\beta$

$$\varphi = \phi_L - \phi_R$$

Transition from 0 junction to π junction

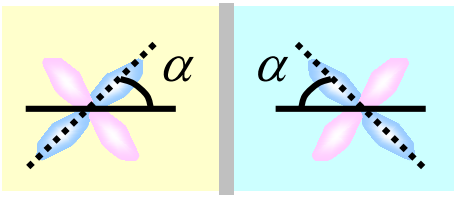




Physical Review B 56 892(1997)
Physical Review B 53 11957(1996)

Mirror type junctions

$$R_N I(\varphi) = \frac{\pi \bar{R} k_B T}{e} \{ \sum_{\omega_n} \int_{-\pi/2}^{\pi/2} F(\theta, i\omega_n, \varphi) \sin\varphi \sigma_N \cos\theta d\theta$$



$$F(\theta, i\omega_n, \varphi) = \frac{2\Delta(\theta_+)\Delta(\theta_-)}{\Omega_{n,+}\Omega_{n,-} + \omega_n^2 + (1 - 2\sigma_N \sin^2\varphi/2)\Delta(\theta_+)\Delta(\theta_-)}$$

$$\Delta(\theta_\pm) = \Delta_0 \cos[2(\theta \mp \alpha)]$$

φ phase difference $\varphi = \phi_L - \phi_R$

$$\Omega_{n,\pm} = \mathrm{sgn}(\omega_n)\sqrt{\Delta^2(\theta) + \omega_n^2} \qquad \bar{R}_N^{-1} = \int_{-\pi/2}^{\pi/2} d\theta \sigma_N \cos\theta$$

$$\Delta(\theta_+)\Delta(\theta_-) < 0, \quad \pi/4 - |\alpha| < |\theta| < \pi/4 + |\alpha|$$
$$\Delta(\theta_+)\Delta(\theta_-) > 0, \quad otherwise$$

$\Delta(\theta_+)\Delta(\theta_-)$	Sign of $F(\theta, i\omega_n, \varphi)$ $0 < \varphi < \pi$	Zero energy states
+	+	No
−	−	Yes

Change of the position of the free energy minima

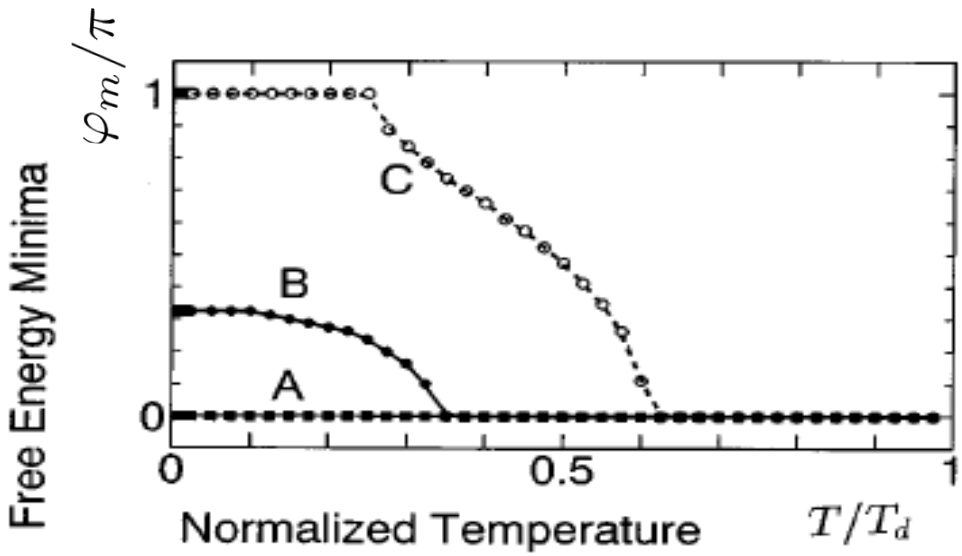


FIG. 5. Position of the free-energy minima φ_0 plotted as a function of temperature. A: $\alpha=\beta=0$, $\lambda_0 d_i=1$, and $\kappa=0.5$, B: $\alpha=-\beta=0.1\pi$, $\lambda_0 d_i=0$, and $\kappa=0.5$, C: $\alpha=-\beta=0.12\pi$, $\lambda_0 d_i=1$, and $\kappa=0.5$.

φ-junction

The position of the free energy minima, φ_m can locate neither 0 nor π.

⇩

ground state is doubly degenerate.

Y. Tanaka and S. Kashiwaya, Phys. Rev. B 53 11957(1996)

Yip J. Low Temp. Phys. 1997

Experiments of dc Josephson current non-monotonic temperature dependence

Testa *et. al*, Phys. Rev. B 71, 134520 (2005)

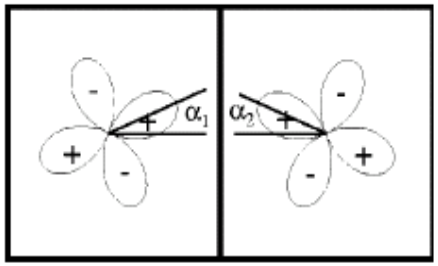


FIG. 1. Schematic geometry of the grain-boundary interface. α_1 and α_2 are the angles between the normal to the interface and the crystallographic axes on the left and right sides, respectively.

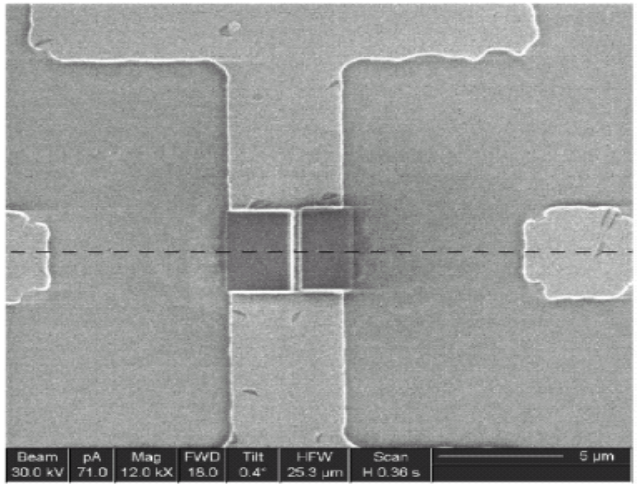


FIG. 5. FIB picture of a 350-nm-wide GBJ (the dashed line indicates the bicrystal line, not visible by FIB).

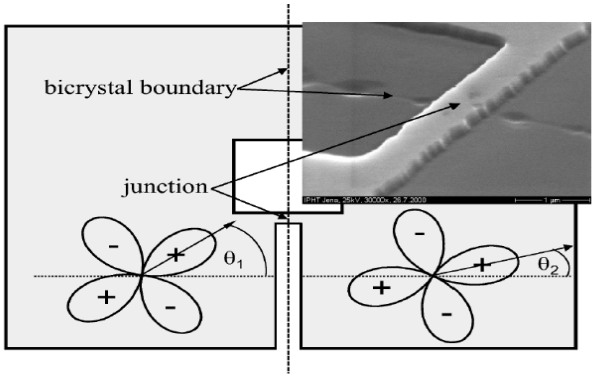


FIG. 1. Schematic picture of the rf SQUID. The YBCO thin film occupies the gray area. The inset shows an electron microscope image of the narrow grain boundary Josephson junction.

Ilichev, PRL 86 5369 (2001)

Experiments of dc Josephson current non-monotonic temperature dependence

Testa *et. al*,
Phys. Rev. B 71, 134520 (2005)

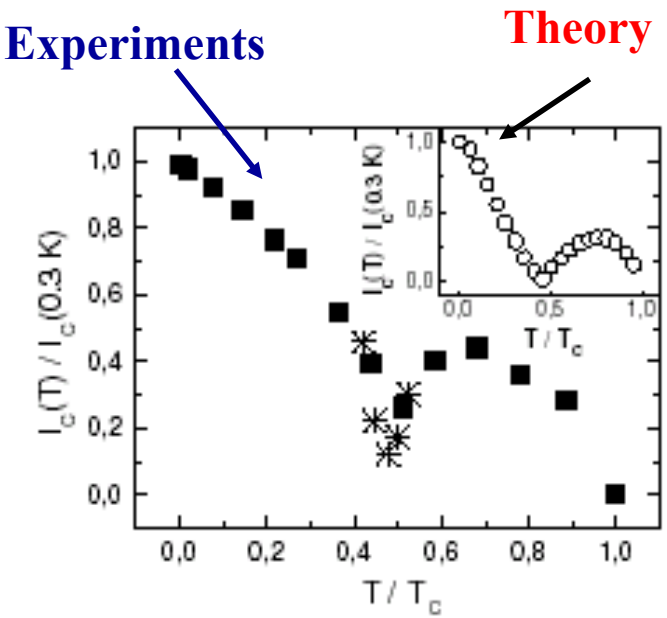


FIG. 3: Normalized temperature dependence of the Josephson current for the dc SQUID (squares, 2nd set of measurements and stars, 3rd set of measurements). In the inset we show the theoretical curve obtained by using the TK formula with $\Delta_d(0) = 0.018$, $\kappa = 0.5$, $\lambda_D d_i = 3$ and $\gamma = 0.2\Delta(0)$.

Ilichev, PRL 86 5369 (2001)

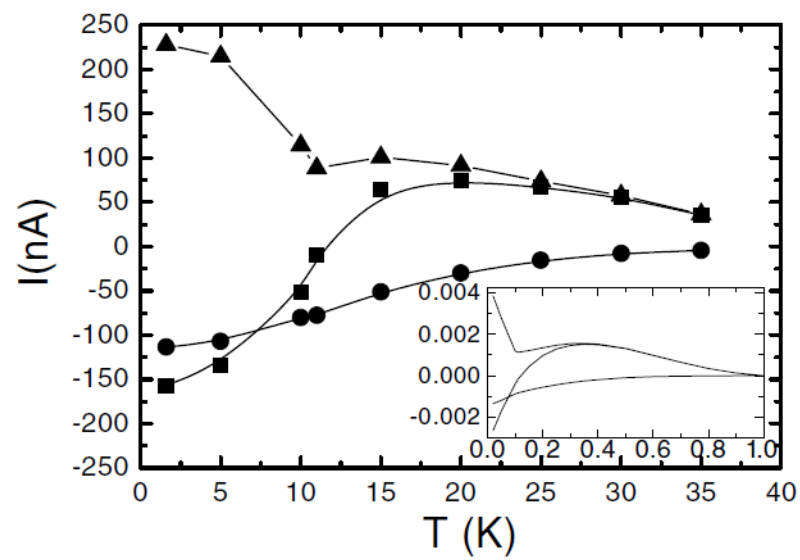


FIG. 4. The critical current I_c (triangles) and the harmonic components I_1 (squares) and I_2 (circles) of the Josephson current as a function of temperature for sample No. 2. The figure is obtained by the Fourier analysis of $I(\varphi)$ shown in Fig. 3b. Inset: Theoretical prediction for the temperature dependence of j_c, j_1, and j_2 for a junction with $\mathcal{D} = 0.3$ and $\rho = 0.3$. The current densities are plotted in units of the Landau critical current density; the temperature is in units of T_c.

Contents of this talk

Background of this talk

1 d-wave superconductor,
topological surface Andreev bound states

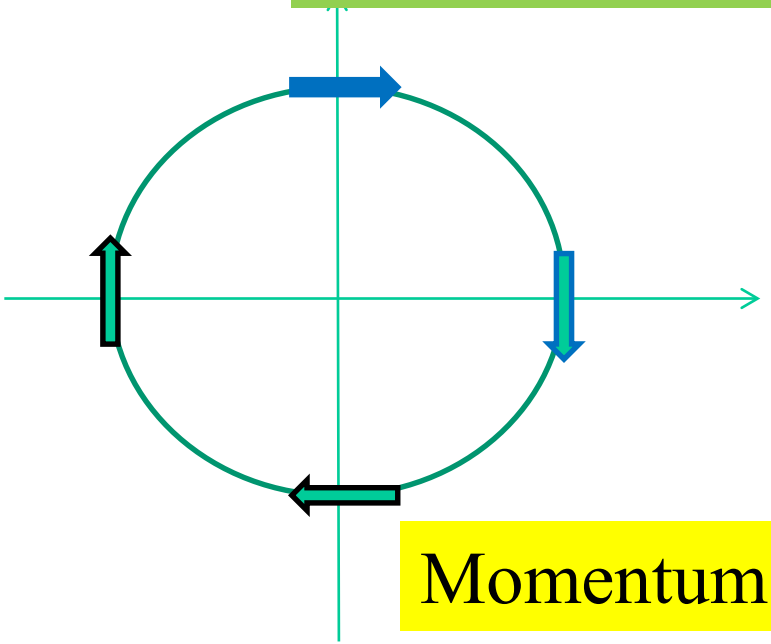
2 Josephson effect in d-wave superconductor
junctions without TI

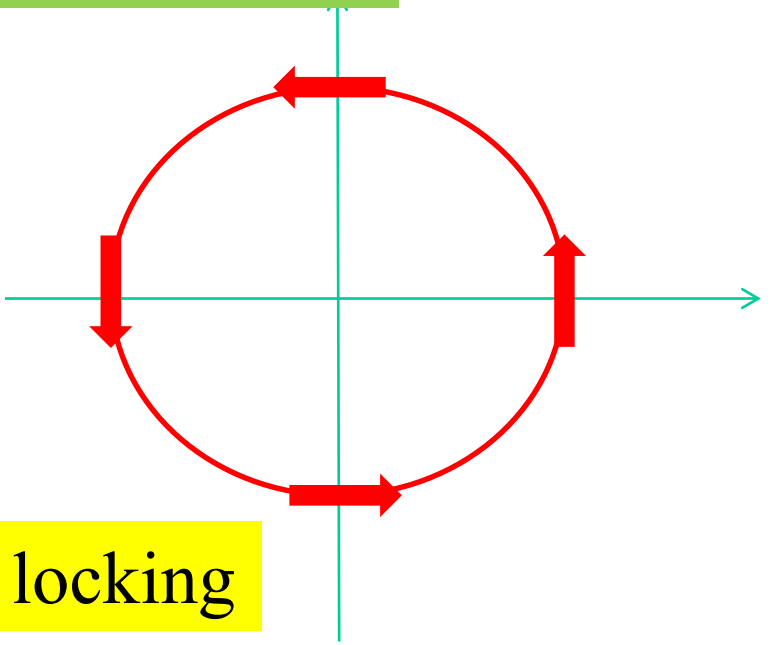
3 Superconducting junctions on TI

4 Diode effect by d-wave superconductor
junctions on TI

Surface state of topological insulator

Massless Dirac cone (Weyl cone)

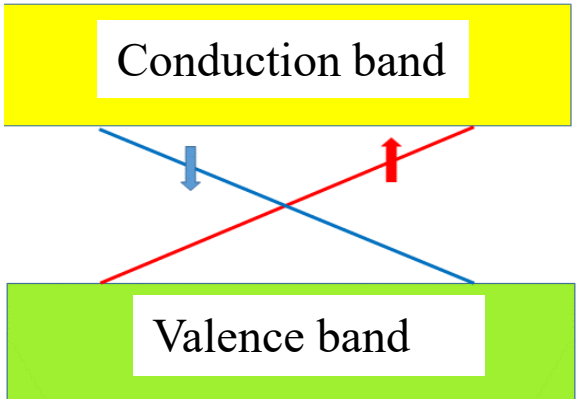




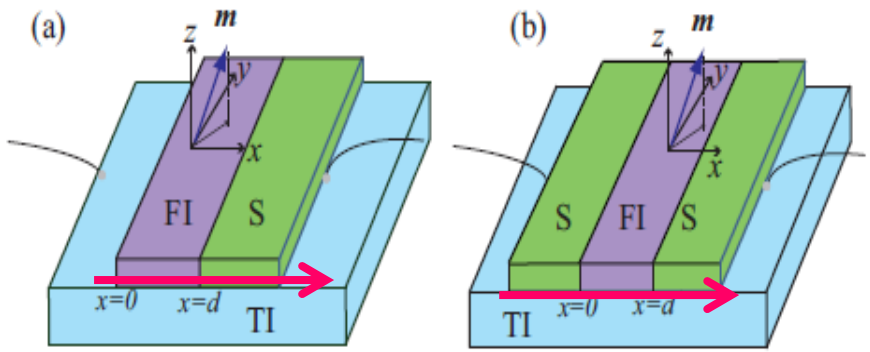
Momentum spin locking

Fermi level: below Dirac point

Fermi level: above Dirac point



Hamiltonian of the surface state of Topological insulator



S: s-wave pair potential

$$\check{H}_S = \begin{pmatrix} \hat{H}(\boldsymbol{k}) + \hat{M} & \hat{\Delta} \\ -\hat{\Delta}^* & -\hat{H}^*(-\boldsymbol{k}) - \hat{M}^* \end{pmatrix}$$

$$\hat{H}(\boldsymbol{k}) = v_F(\hat{\sigma}_x k_x + \hat{\sigma}_y k_y) - \mu[\Theta(-x) + \Theta(x-d)]$$

$$\hat{M} = \boldsymbol{m} \cdot \hat{\boldsymbol{\sigma}} \Theta(d-x)\Theta(x)$$

N/TI/S $\quad \hat{\Delta} = i\hat{\sigma}_y \Delta \Theta(x-d)$

S/TI/S $\quad \hat{\Delta} = i\hat{\sigma}_y[\Delta\Theta(x-d) + \Delta\Theta(-x)\exp(i\varphi)]$

Tanaka, Yokoyama, Nagaosa, PRL 103, 107002 (2009)

SABS and differential conductance

$$\sigma_S(\theta) = \frac{\sigma_N[1+\sigma_N \mid \Gamma \mid^2 -(1-\sigma_N) \mid \Gamma \mid^4]}{\mid 1+(1-\sigma_N)\exp(i\gamma)\Gamma^2 \mid^2}$$

SABS is obtained when denominator becomes zero for $\sigma_N \to 0$

$$E_b = -\frac{\Delta\mu \sin\theta \mathrm{sgn}(m_z)}{\sqrt{\mu^2 \sin^2\theta + m_z^2 \cos^2\theta}}.$$

$$m_z = \mu \qquad E_b = -\Delta \sin\theta \mathrm{sgn}(m_z)$$

Conductance formula obtained in unconventional superconductors

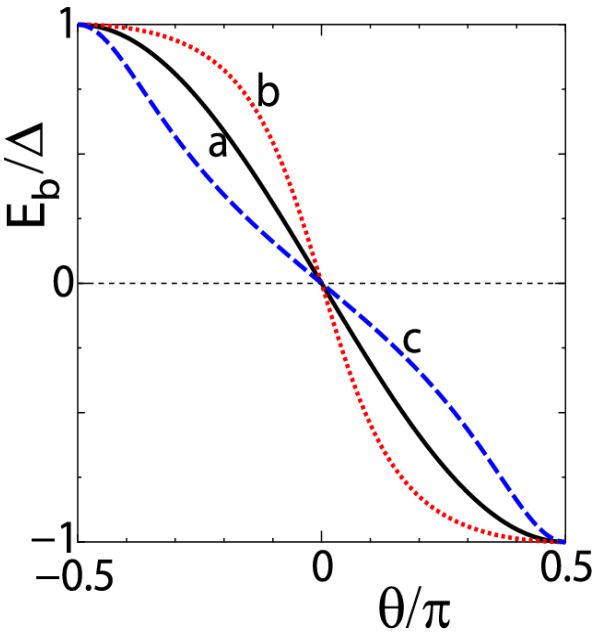
$$\sigma_S(\theta) = \frac{\sigma_N(\theta)[1+\sigma_N(\theta) \mid \Gamma_+ \mid^2 +[\sigma_N(\theta)-1] \mid \Gamma_+\Gamma_- \mid^2]}{\mid 1+[\sigma_N(\theta)-1]\Gamma_+\Gamma_- \mid^2}, \qquad \sigma_N(\theta) = \sigma_N$$

We assume m_y=0. The component m_x does not influence the charge conductance.

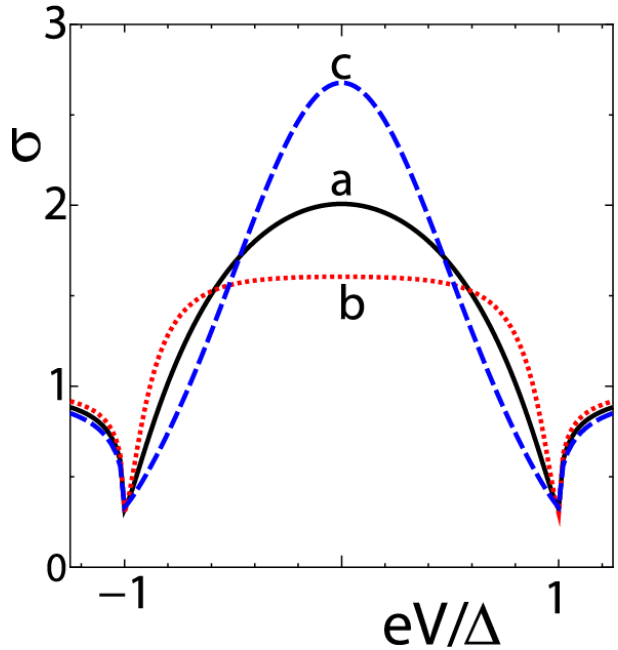
Tanaka, Yokoyama, Nagaosa, PRL 103, 107002 (2009)

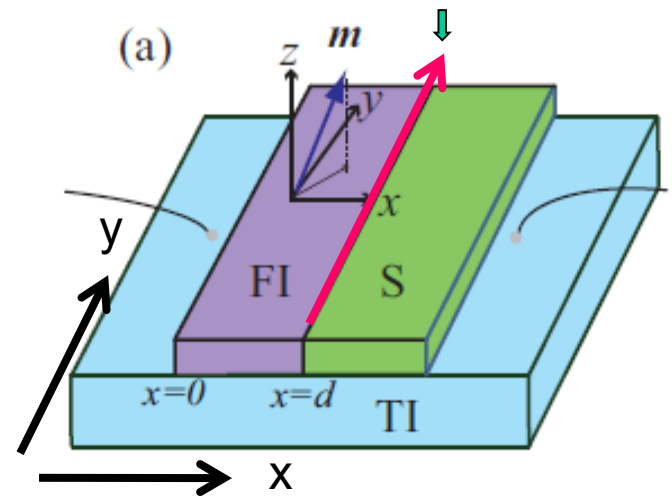
Chiral Majorana mode (CMM) on topological insulator

ABS



Conductance



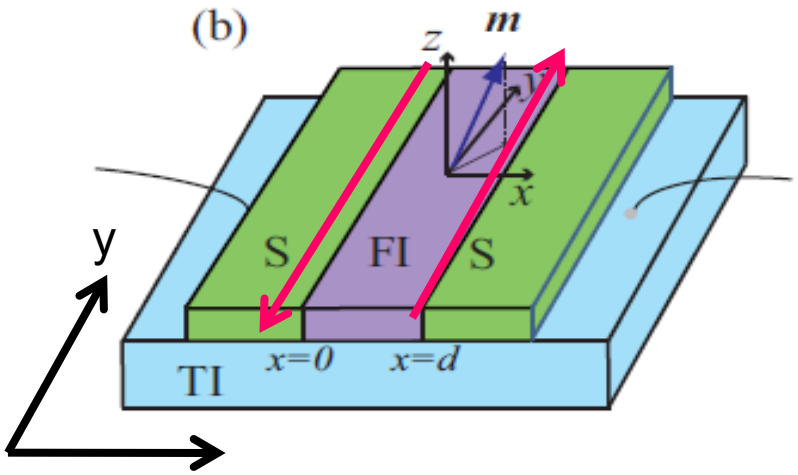


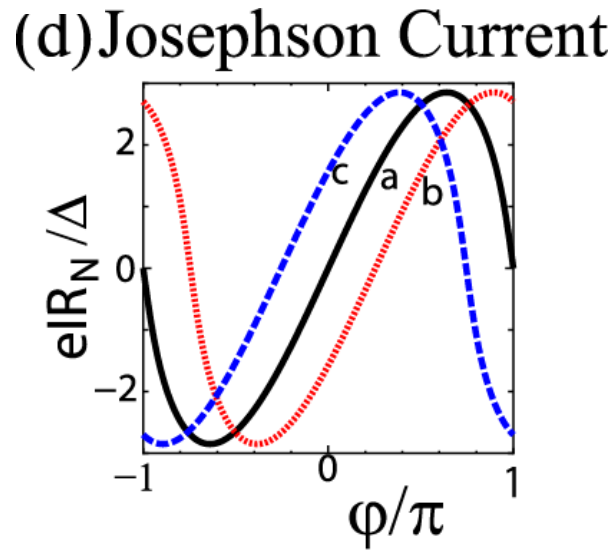
$m_z d/v_F = 1$ $m_y/m_z = 0$ a $\mu/m_z = 1$ b $\mu/m_z = 0.5$ c $\mu/m_z = 2$

ABS as a chiral Majorana mode (CMM)

Tanaka, Yokoyama, Nagaosa, PRL 103, 107002 (2009)

S/FI/S junction on top of TI 1





a $m_x/m_z = 0$

b $m_x/m_z = 0.4$

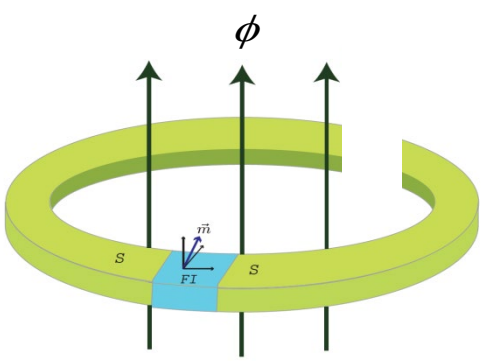
c $m_x/m_z = -0.4$

$\mu/m_z = 1$

$m_z d/v_F = 1$

Anomalous current phase relation by m_x

$$I \propto \sin(\varphi - 2\delta) \qquad \delta = m_x d/v_F$$

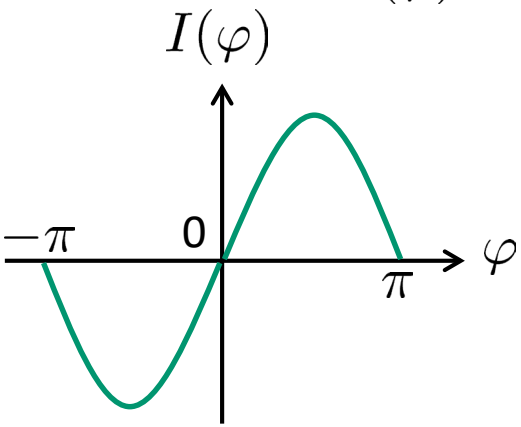


Anomalous current phase relation can be detected by interferometer

Tanaka, Yokoyama, Nagaosa, PRL 103, 107002 (2009)

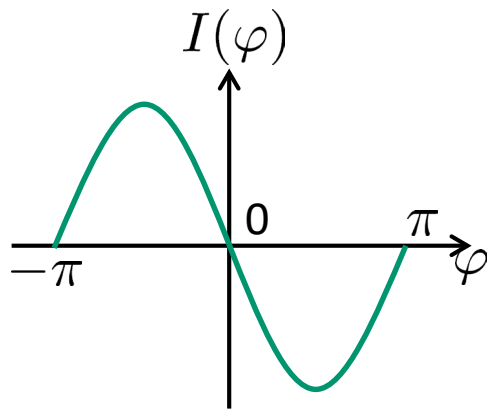
φ_0 junctions

$$I(\varphi) \sim I_1 \sin\varphi$$



$I_1 > 0$

0- junction

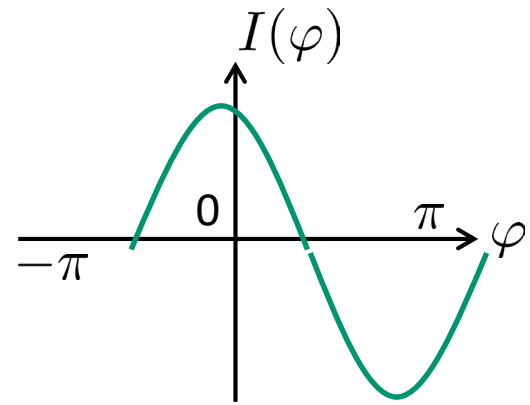


$I_1 < 0$

π- junction

L. N. Bulaevskii (1977) FS junction
D. J. van Harlingen (1995) d -wave
V. V. Ryazanov (2001) FS junction

$$I(\varphi) \sim I_1 \sin(\varphi - \varphi_0)$$

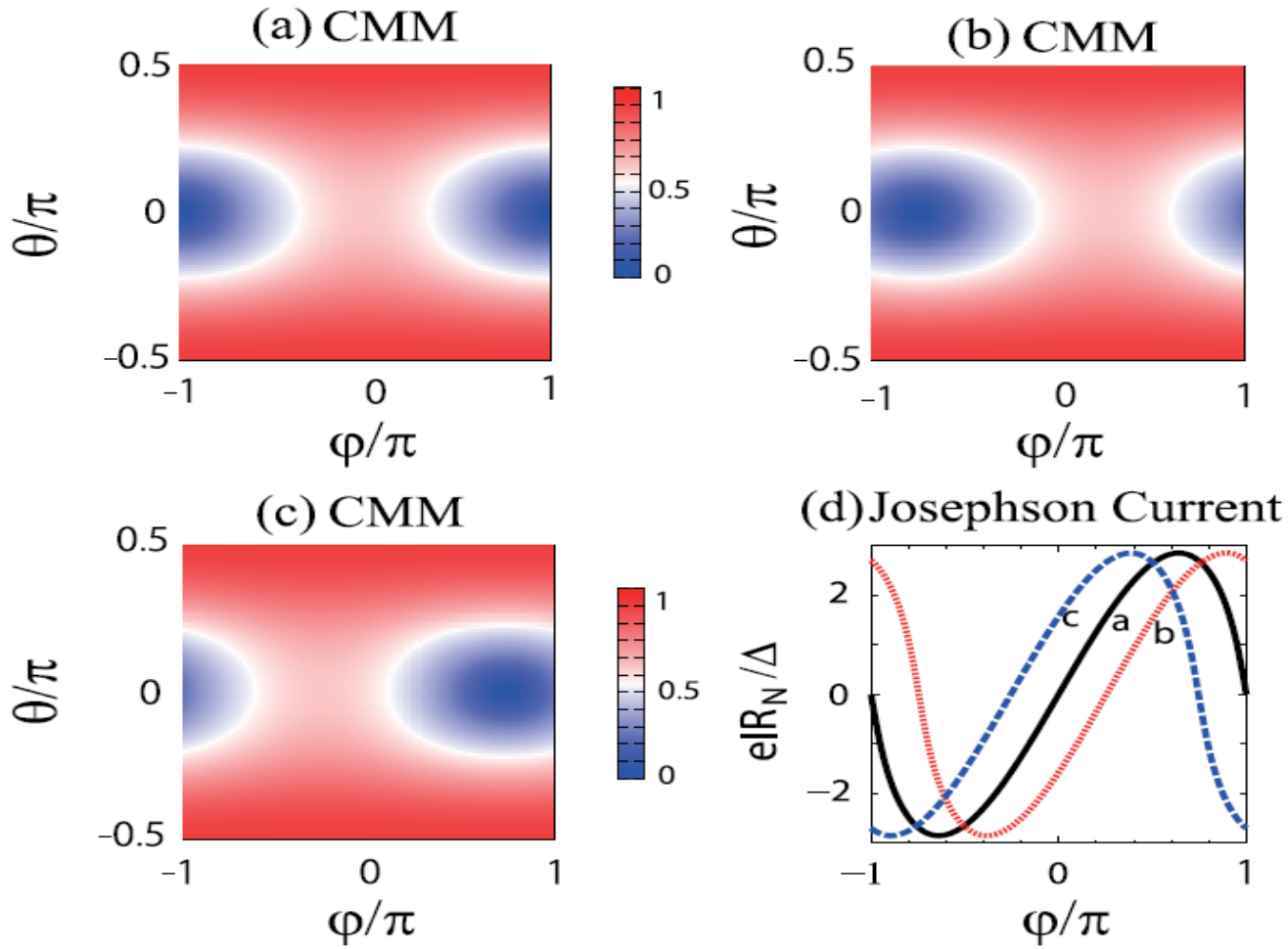


φ_0 -junction

Y. Tanaka et al (2009)
R. Grein et al., (2009);
M. Eschrig et al., (2007);
Y. Asano et a(2007)
A. Buzdin, et al (2008)

Both time reversal and spatial inversion symmetry should be broken!!

S/FI/S junction on top of TI 2



(a) $m_x = 0$

(b) $m_x = 0.4 m_z$

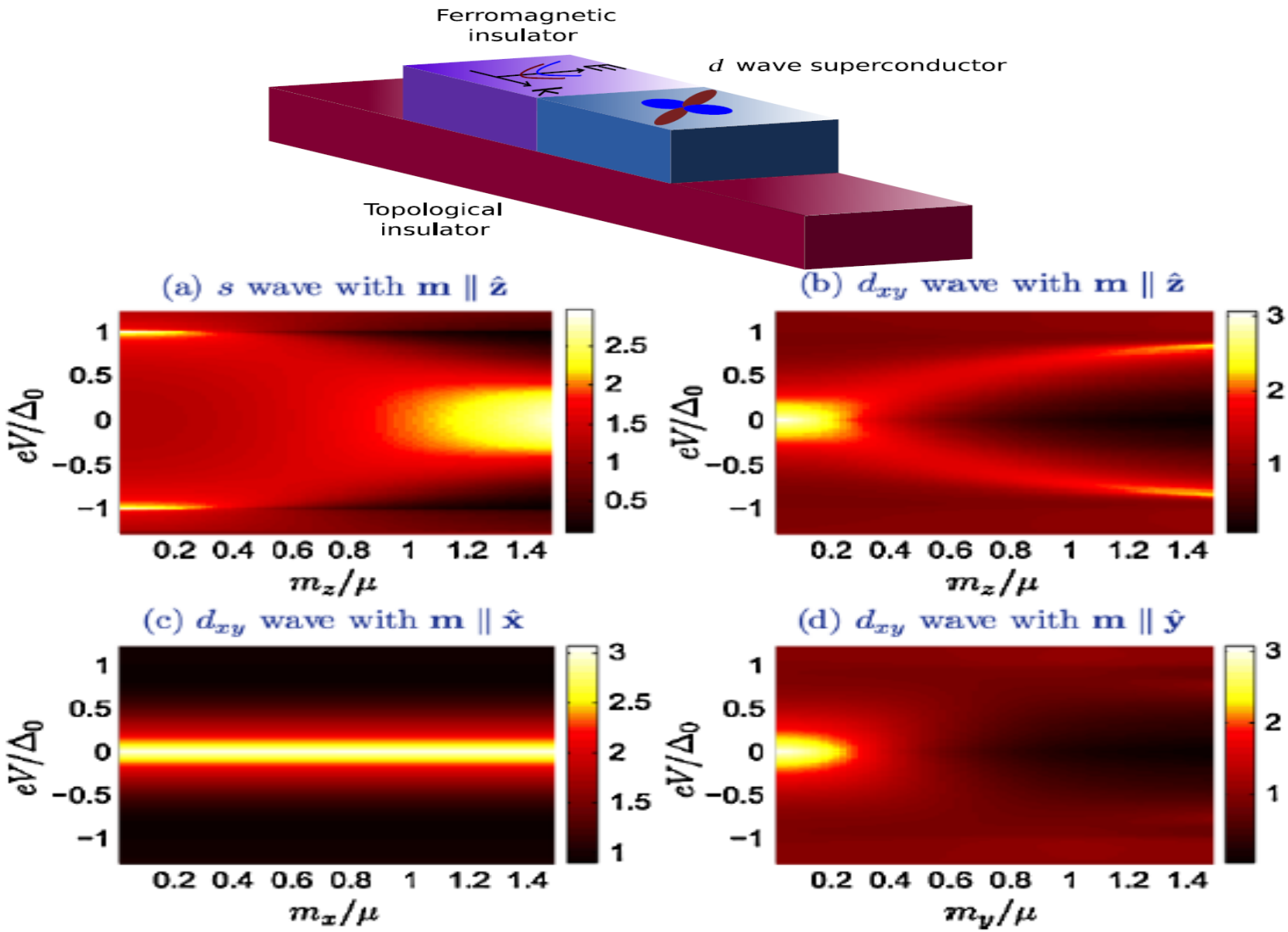
(c) $m_x = -0.4 m_z$

$$E_J = \sqrt{\sigma_N \cos^2(\varphi/2 - \delta) + (1 - \sigma_N)(E_b/\Delta)^2}\,\Delta$$

$$\delta = m_x d / v_F$$

Tanaka, Yokoyama, Nagaosa, PRL 103, 107002 (2009)

d-wave superconductor junctions on TI



Linde Tanaka et al, PRL(2010)

Diode effect in Josephson junctions

Background of this talk

1 d-wave superconductor,
topological surface Andreev bound states

2 Josephson effect in d-wave superconductor junctions without TI

3 Superconducting junctions on TI

4 Diode effect by d-wave superconductor junctions on TI

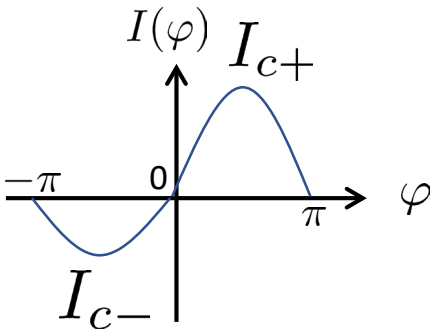
arXiv:2205.13177

Diode effect in Josephson junctions

I_c^+ **Positive maximum Josephson current**

I_c^- **Negative maximum Josephson current**

$$Q = \frac{I_c^+ - |I_c^-|}{I_c^+ + |I_c^-|}$$



If the value of Q is nonzero, we can expect a diode effect.

Superconducting diode effect

Observation of diode effect F. Ando et al, Nature 584 373 (2020)

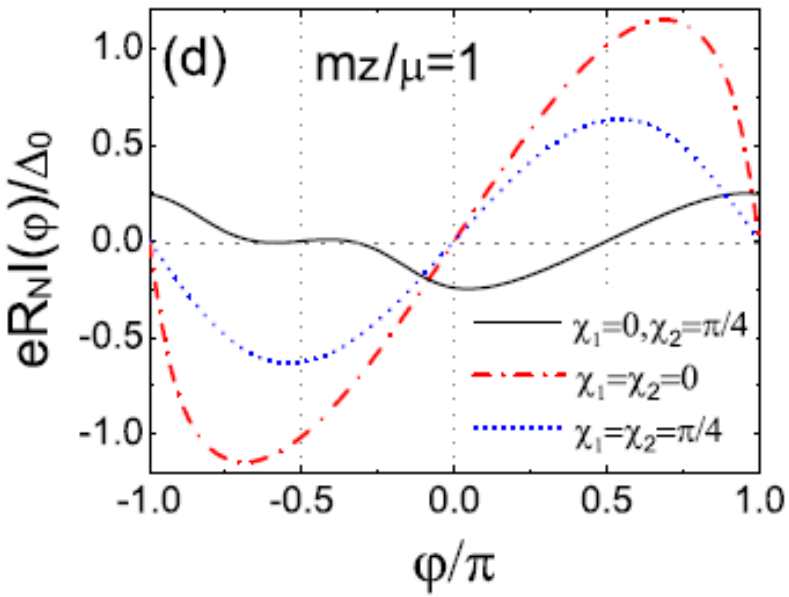
Recent experiments

- B. Pal, et al, Nature Physics 18, 1228 (2022).
- H. Narita, et al, Nature Nanotechnology 17, 823 (2022).
- K.-R. Jeon, et al, Nature Materials 21, 1008 (2022).
- L. Bauriedl, et al, Nature Communications 13, 4266 (2022),……

Theoretical works

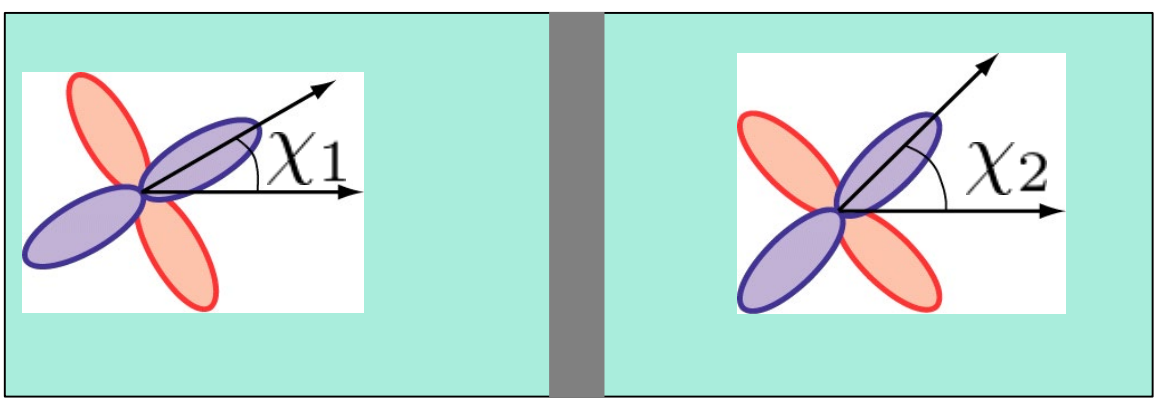
- K. Misaki and N. Nagaosa, Phys. Rev. B 103, 245302 (2021).
- J. J. He, Y. Tanaka and N. Nagaosa, New J. Phys. 24, 053014 (2022).
- A. Daido, Y. Ikeda, and Y. Yanase, Phys. Rev. Lett. 128, 037001 (2022).
- N. F. Q. Yuan and L. Fu, Proc. Nat. Acad. of Sci. 119, e2119548119 (2022).
- S. Ili´c and F. S. Bergeret, Phys. Rev. Lett. **128**, 177001 (2022).
- T. Karabassov, et al, arXiv:2203.15608 (2022).
- R. S. Souto, et al, arXiv:2205.04469 (2022).
- J. Jiang, et al, Rev. Applied **18**, 034064 (2022).
- A. Daido and Y. Yanase, arXiv:2209.03515.
- T. Kokkeler, et al, arXiv:2209.13987

d-wave superconductor junctions on TI



We are aware of
anomalous current phase relation

$$\chi_1 = 0, \chi_2 = \pi/4$$



Bo Lu, Yada, Golubov Tanaka (PRB 2015)

Merit of d-wave / ferromagnet junctions on TI

- High transition temperature compared to conventional BCS superconductor.
- Wide variety of temperature dependence
- Enhancement of Q factor at low temperature
- Simultaneous breaking of both time reversal and spatial inversion symmetry

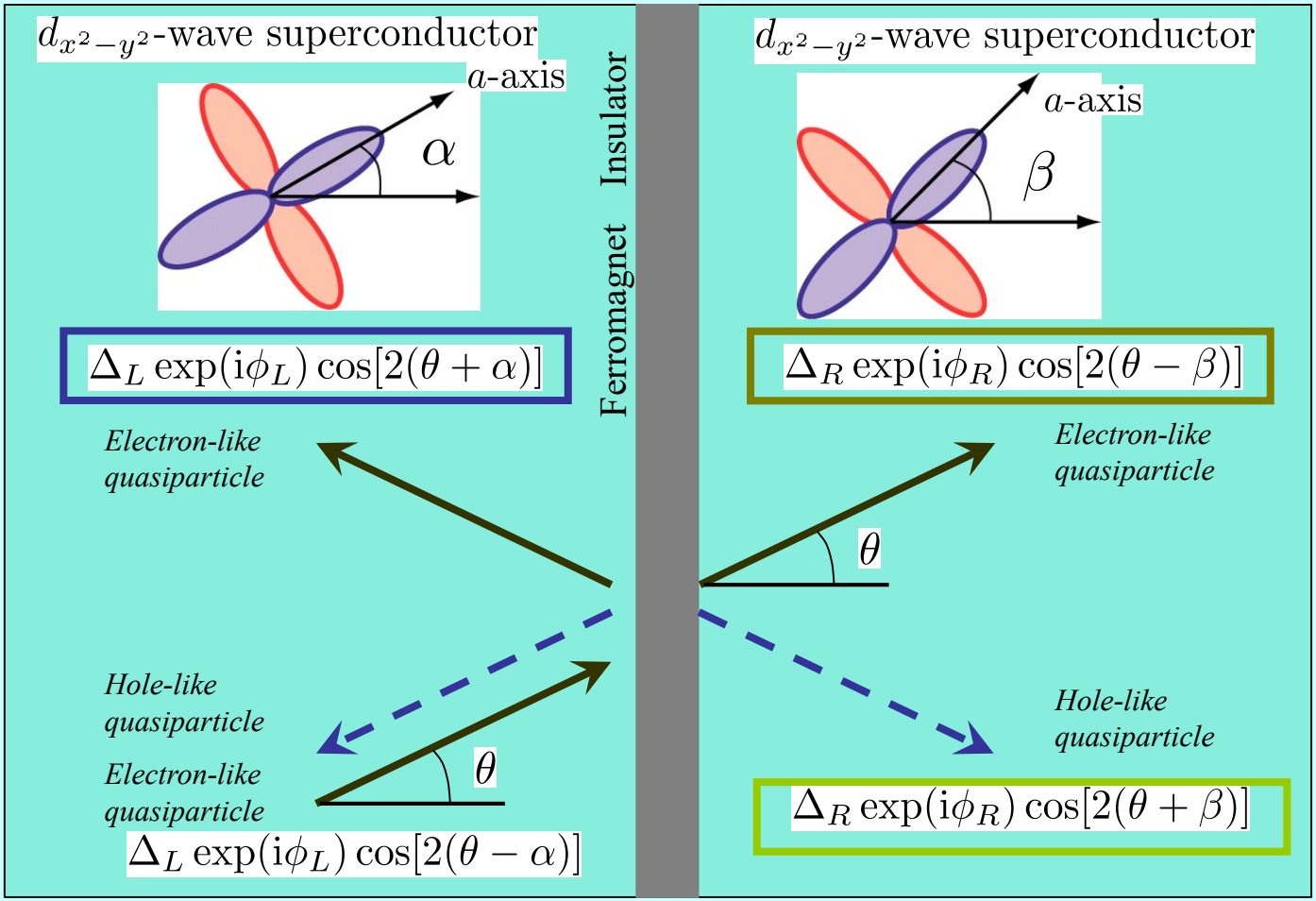
$$I(\varphi) \sim I_1 \sin\varphi + I_2 \sin 2\varphi + J_1 \cos\varphi$$

We can expect the same order of these three terms!!

arXiv:2205.13177

Model

Topological insulator



Model

$$\mathcal{H} = \begin{bmatrix} \hat{h}\left(k_x, k_y\right) + \hat{M} & i\hat{\sigma}_y \Delta\left(\theta, x\right) \\ -i\hat{\sigma}_y \Delta^*\left(\theta, x\right) & -\hat{h}^*\left(-k_x, -k_y\right) - \hat{M}^* \end{bmatrix}$$

Hamiltonian on the surface state of TI

$$\hat{h}\left(k_x, k_y\right) = v\left(k_x\hat{\sigma}_x + k_y\hat{\sigma}_y\right) - \mu\left[\Theta\left(-x\right) + \Theta\left(x - d\right)\right], k_x = \frac{\partial}{i\partial x}, \quad k_y = \frac{\partial}{i\partial y}$$

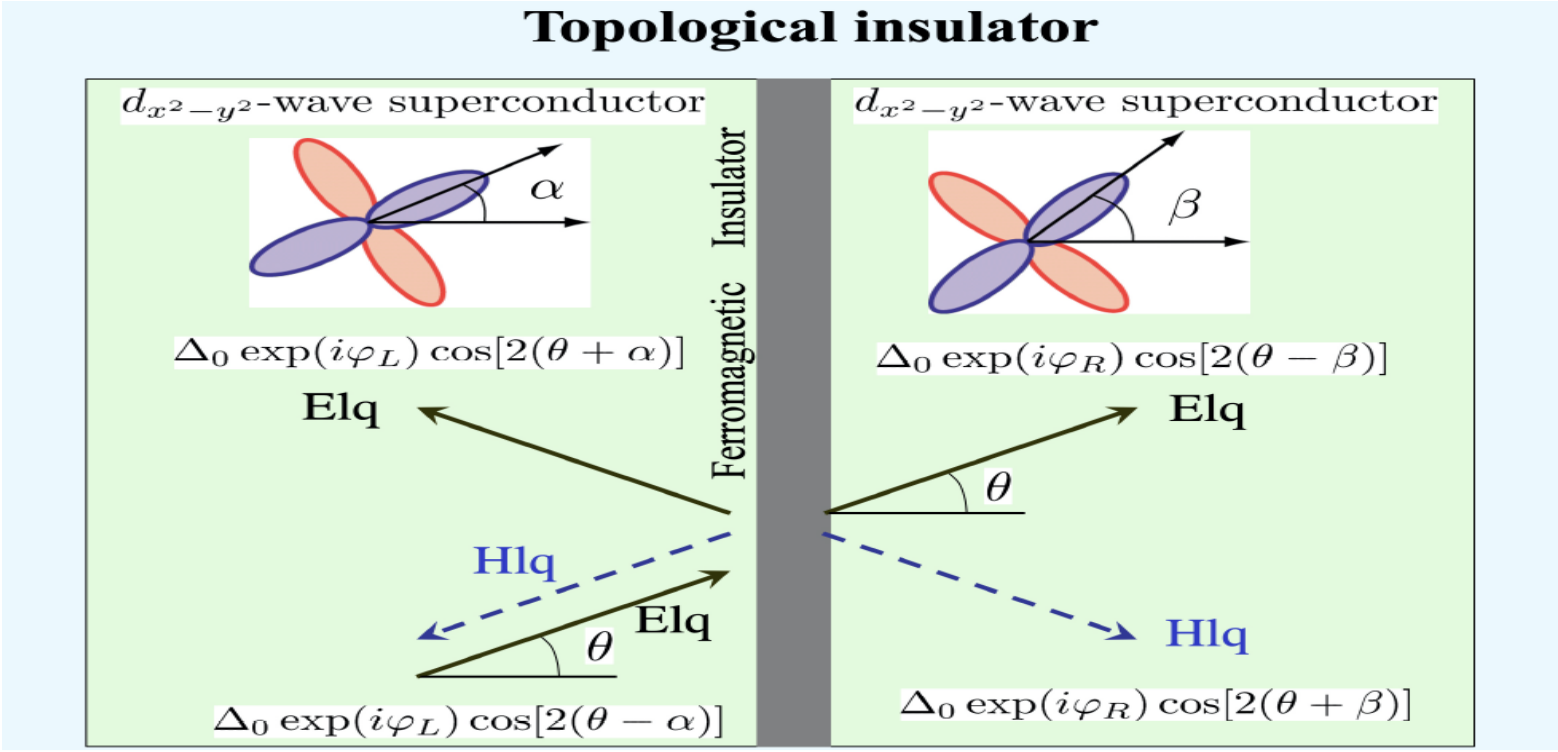
Magnetization

$$\hat{M} = m_z\hat{\sigma}_z\Theta\left(x\right)\Theta\left(d - x\right)$$

Pair potential

$$\Delta(\theta, x) = \begin{cases} \Delta_{L\pm}\left(\theta\right) = \Delta_0 \cos\left[2\left(\theta \mp \alpha\right)\right] \exp(i\varphi), & x < 0 \\ \Delta_{R\pm}\left(\theta\right) = \Delta_0 \cos\left[2\left(\theta \mp \beta\right)\right], & x > d. \end{cases}$$

Model (d/FI/d junction on TI)



d-wave pair potential

$$\Delta(\theta, x) = \begin{cases} \Delta_{L\pm}(\theta) = \Delta_0 \cos[2(\theta \mp \alpha)] \exp(i\varphi), & x < 0 \\ \Delta_{R\pm}(\theta) = \Delta_0 \cos[2(\theta \mp \beta)], & x > d. \end{cases}$$

arXiv:2205.13177

Model

$$\mathcal{H} = \begin{bmatrix} \hat{h}\left(k_x, k_y\right) + \hat{M} & i\hat{\sigma}_y \Delta\left(\theta, x\right) \\ -i\hat{\sigma}_y \Delta^*\left(\theta, x\right) & -\hat{h}^*\left(-k_x, -k_y\right) - \hat{M}^* \end{bmatrix}$$

Hamiltonian on the surface state of TI

$$\hat{h}\left(k_x, k_y\right) = v\left(k_x\hat{\sigma}_x + k_y\hat{\sigma}_y\right) - \mu\left[\Theta\left(-x\right) + \Theta\left(x - d\right)\right], k_x = \frac{\partial}{i\partial x}, \quad k_y = \frac{\partial}{i\partial y}$$

Magnetization

$$\hat{M} = m_z\hat{\sigma}_z\Theta\left(x\right)\Theta\left(d - x\right)$$

Pair potential

$$\Delta(\theta, x) = \begin{cases} \Delta_{L\pm}\left(\theta\right) = \Delta_0 \cos\left[2\left(\theta \mp \alpha\right)\right] \exp(i\varphi), & x < 0 \\ \Delta_{R\pm}\left(\theta\right) = \Delta_0 \cos\left[2\left(\theta \mp \beta\right)\right], & x > d. \end{cases}$$

Outline of calculation 1

Josephson current formula expressed by Andreev reflection coefficient

A. Furusaki and M. Tsukada, Solid State Commun. Vol. 78, 299 (1991).

(1)Retarded Green's function is calculated by using the scattering state of the wave function.

(2)Matsubara Green's function by analytical continuation.

(3) Josephson current by Matsubara Green's function.

(4)Josephson current given by Andreev reflection coefficient which is expressed by Matsubara frequency.

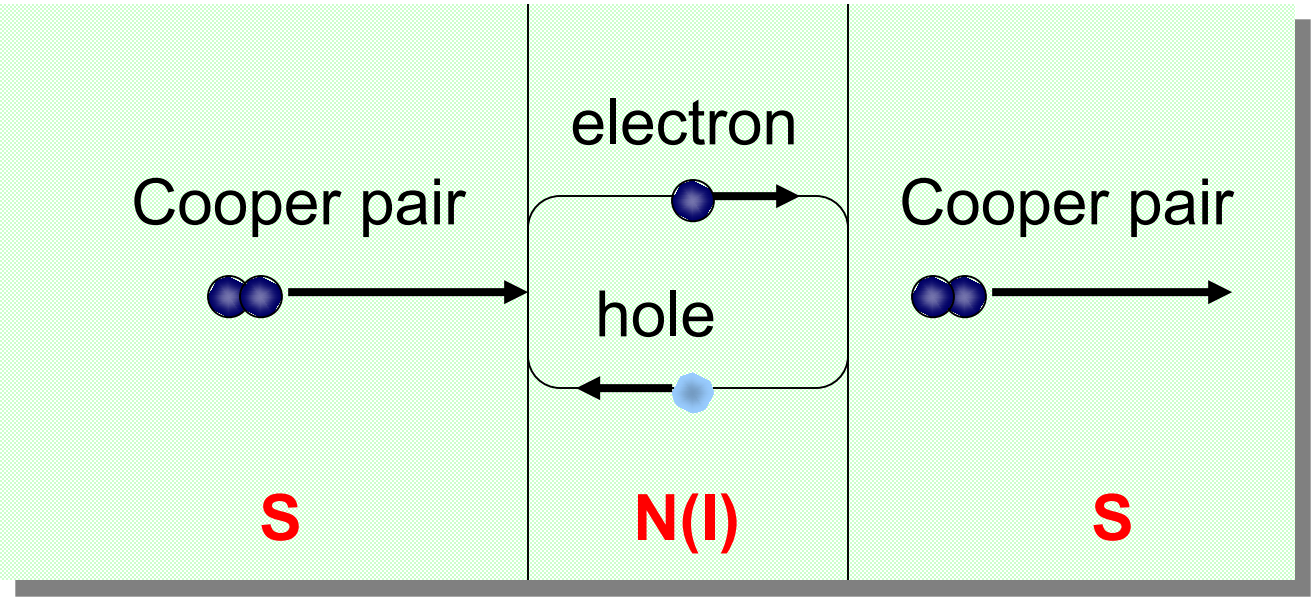
d-wave superconductor

Y. Tanaka and S. Kashiwaya, PRB 53 11957 (1996),
PRB 56 892 (1997)

Josephson junctions on TI

Lu Bo and T. Yukio, Phil. Trans. R. Soc. A. 376, 20150246 (2018).

Andreev reflection and Josephson current



Josephson current I(φ) by Andreev reflection coefficient

$$R_N I(\varphi) = \frac{\pi \bar{R}_N k_B T}{e} \left\{ \sum_{\omega_n} \int_{-\pi/2}^{\pi/2} \left[\frac{a_{en}(\theta,\varphi)}{\Omega_{nL+}} \Delta_{L+}(\theta) - \frac{a_{hn}(\theta,\varphi)}{\Omega_{nL-}} \Delta_{L-}(\theta) \right] \cos\theta d\theta \right\}$$

$a_{en}(\theta,\varphi)$ Andreev reflection coefficient by an electron-like quasiparticle injection

$a_{hn}(\theta,\varphi)$ Andreev reflection coefficient by an hole-like quasiparticle injection

Josephson current

$$R_N I(\varphi) = \frac{\pi \bar{R}_N k_B T}{e} \sum_n \int_{-\pi/2}^{\pi/2} d\theta \frac{2\cos\theta \sigma_N}{|\Lambda_{dn}(\theta)|^2} \left[A(\theta)\sin\varphi + B(\theta)\sin 2\varphi + C(\theta)\cos\varphi\right]$$

$$I(\varphi) = \sum_n \left[I_n \sin n\varphi + J_n \cos n\varphi\right].$$

- The simultaneous existence of I_1, I_2 J_1 at the same time in the same order is the source of the giant diode effect.

Josephson current 1

$$R_N I(\varphi) = \frac{\pi \bar{R}_N k_B T}{e} \sum_n \int_{-\pi/2}^{\pi/2} d\theta \frac{2\cos\theta \sigma_N}{|\Lambda_{dn}(\theta)|^2} \left[A(\theta)\sin\varphi + B(\theta)\sin 2\varphi + C(\theta)\cos\varphi\right]$$

$$A(\theta) = \left(\Gamma_{nL+}\Gamma_{nR+} + \Gamma_{nL-}\Gamma_{nR-}\right)\left[(1-\sigma_N)\Lambda_{ne} + \sigma_N\left(1 + \Gamma_{nL+}\Gamma_{nL-}\Gamma_{nR+}\Gamma_{nR-}\right)\right],$$

$$B(\theta) = 2\sigma_N \Gamma_{nL+}\Gamma_{nL-}\Gamma_{nR+}\Gamma_{nR-}, \quad C(\theta) = (1-\sigma_N)\left(\Gamma_{nL+}\Gamma_{nR+} - \Gamma_{nL-}\Gamma_{nR-}\right)\Lambda_{no}$$

σ_N transmissivity at the interface

$$\Lambda_{dn}(\theta) = [1-\sigma_N]\left[1 - \exp(-i\eta)\Gamma_{nR+}\Gamma_{nR-}\right]\left[1 - \exp(i\eta)\Gamma_{nL+}\Gamma_{nL-}\right]$$
$$+ \sigma_N\left[1 + \exp(-i\varphi)\Gamma_{nL-}\Gamma_{nR-}\right]\left[1 + \exp(i\varphi)\Gamma_{nL+}\Gamma_{nR+}\right]$$

$$\Lambda_{ne} = 1 + \Gamma_{nL+}\Gamma_{nL-}\Gamma_{nR+}\Gamma_{nR-} - \cos\eta\left(\Gamma_{nL+}\Gamma_{nL-} + \Gamma_{nR+}\Gamma_{nR-}\right)$$

$$\Lambda_{no} = \left(\Gamma_{nL+}\Gamma_{nL-} - \Gamma_{nR+}\Gamma_{nR-}\right)\sin\eta$$

$$\Gamma_{nL\pm} = \frac{\Delta_{L\pm}(\theta)}{\omega_n + \Omega_{nL\pm}}, \quad \Gamma_{nR\pm} = \frac{\Delta_{R\pm}(\theta)}{\omega_n + \Omega_{nR\pm}} \qquad \Omega_{nL\pm} = \mathrm{sgn}(\omega_n)\sqrt{\Delta_L^2(\theta_\pm) + \omega_n^2}$$

Josephson current 2

$$R_N I(\varphi) = \frac{\pi \bar{R}_N k_B T}{e} \sum_n \int_{-\pi/2}^{\pi/2} d\theta \frac{2\cos\theta\sigma_N}{|\Lambda_{dn}(\theta)|^2} \left[A(\theta)\sin\varphi + B(\theta)\sin 2\varphi + C(\theta)\cos\varphi\right]$$

$$A(\theta) = \left(\Gamma_{nL+}\Gamma_{nR+} + \Gamma_{nL-}\Gamma_{nR-}\right)\left[(1-\sigma_N)\Lambda_{ne} + \sigma_N\left(1 + \Gamma_{nL+}\Gamma_{nL-}\Gamma_{nR+}\Gamma_{nR-}\right)\right],$$

$$B(\theta) = 2\sigma_N \Gamma_{nL+}\Gamma_{nL-}\Gamma_{nR+}\Gamma_{nR-}, \quad C(\theta) = (1-\sigma_N)\left(\Gamma_{nL+}\Gamma_{nR+} - \Gamma_{nL-}\Gamma_{nR-}\right)\Lambda_{no}$$

$$\Lambda_{dn}(\theta) = [1-\sigma_N]\left[1 - \exp(-i\eta)\Gamma_{nR+}\Gamma_{nR-}\right]\left[1 - \exp(i\eta)\Gamma_{nL+}\Gamma_{nL-}\right]$$
$$+ \sigma_N\left[1 + \exp(-i\varphi)\Gamma_{nL-}\Gamma_{nR-}\right]\left[1 + \exp(i\varphi)\Gamma_{nL+}\Gamma_{nR+}\right]$$

$$\Lambda_{ne} = 1 + \Gamma_{nL+}\Gamma_{nL-}\Gamma_{nR+}\Gamma_{nR-} - \cos\eta\left(\Gamma_{nL+}\Gamma_{nL-} + \Gamma_{nR+}\Gamma_{nR-}\right)$$

$$\Lambda_{no} = \left(\Gamma_{nL+}\Gamma_{nL-} - \Gamma_{nR+}\Gamma_{nR-}\right)\sin\eta$$

$$\cos\eta = \frac{m_z^2\cos^2\theta - \mu^2\sin^2\theta}{m_z^2\cos^2\theta + \mu^2\sin^2\theta}, \quad \sin\eta = \frac{-2m_z\mu\cos\theta\sin\theta}{m_z^2\cos^2\theta + \mu^2\sin^2\theta}$$

Josephson current 3

$$R_N I(\varphi) = \frac{\pi \bar{R}_N k_B T}{e} \sum_n \int_{-\pi/2}^{\pi/2} d\theta \frac{2\cos\theta\sigma_N}{|\Lambda_{dn}(\theta)|^2} \left[A(\theta)\sin\varphi + B(\theta)\sin 2\varphi + C(\theta)\cos\varphi\right]$$

$$A(\theta) = \left(\Gamma_{nL+}\Gamma_{nR+} + \Gamma_{nL-}\Gamma_{nR-}\right)\left[(1-\sigma_N)\Lambda_{ne} + \sigma_N\left(1 + \Gamma_{nL+}\Gamma_{nL-}\Gamma_{nR+}\Gamma_{nR-}\right)\right],$$

$$B(\theta) = 2\sigma_N \Gamma_{nL+}\Gamma_{nL-}\Gamma_{nR+}\Gamma_{nR-}, \quad C(\theta) = (1-\sigma_N)\left(\Gamma_{nL+}\Gamma_{nR+} - \Gamma_{nL-}\Gamma_{nR-}\right)\Lambda_{no}$$

Condition for nonzero cos φ term

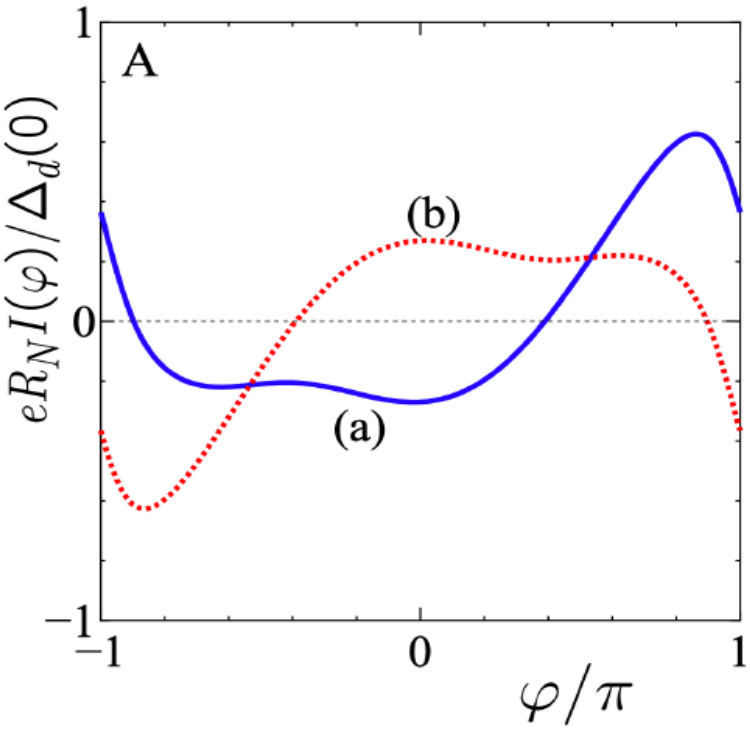
$$\Lambda_{no} \neq 0 \qquad \Gamma_{nL+}\Gamma_{nR+} \neq \Gamma_{nL-}\Gamma_{nR-}$$

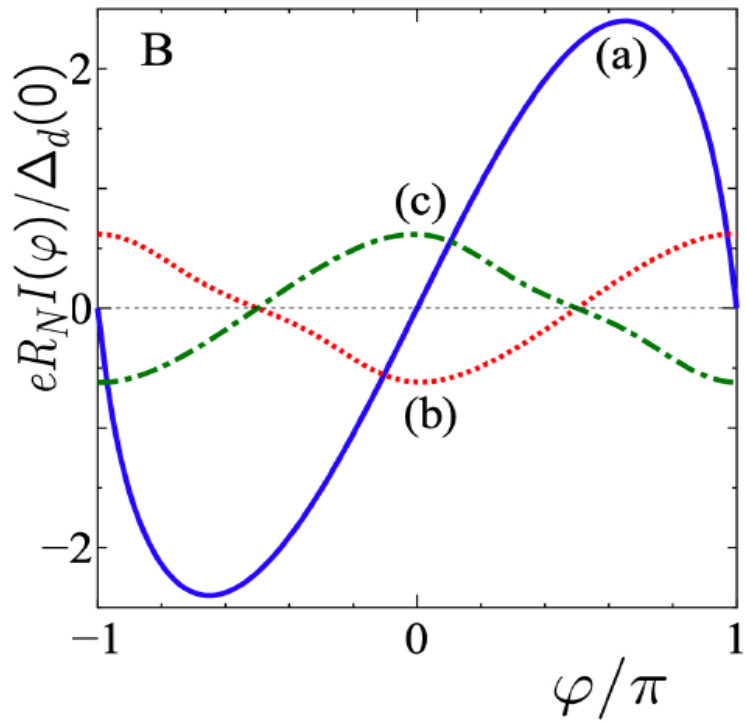
$$\Downarrow$$

$$m_z \neq 0$$

Condition for diode effect

Current phase relation





$\alpha = -0.2\pi, \beta = 0.09\pi$

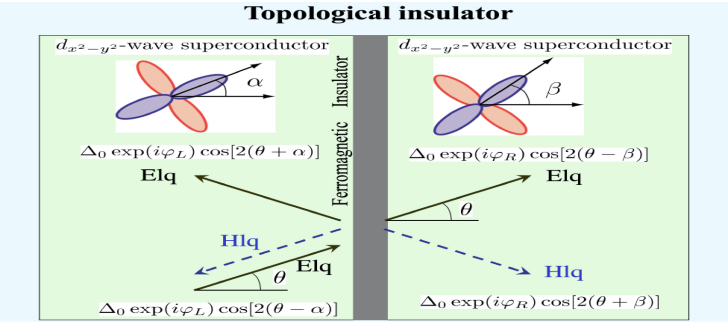
$(a) m_z = 0.5\mu$

$(b) m_z = -0.5\mu$

$(a) \alpha = 0, \beta = 0, m_z = 0.5\mu$

$(b) \alpha = 0, \beta = 0.25\pi, m_z = 0.5\mu$

$(c) \alpha = 0, \beta = 0.25\pi, m_z = -0.5\mu$



Josephson current and sign inversion of m_z

$$R_N I(\varphi) = \frac{\pi \bar{R}_N k_B T}{e} \sum_n \int_{-\pi/2}^{\pi/2} d\theta \frac{2\cos\theta\sigma_N}{|\Lambda_{dn}(\theta)|^2} \left[A(\theta)\sin\varphi + B(\theta)\sin 2\varphi + C(\theta)\cos\varphi\right]$$

$$A(\theta) = \left(\Gamma_{nL+}\Gamma_{nR+} + \Gamma_{nL-}\Gamma_{nR-}\right)\left[\left(1-\sigma_N\right)\Lambda_{ne} + \sigma_N\left(1 + \Gamma_{nL+}\Gamma_{nL-}\Gamma_{nR+}\Gamma_{nR-}\right)\right],$$

$$B(\theta) = 2\sigma_N \Gamma_{nL+}\Gamma_{nL-}\Gamma_{nR+}\Gamma_{nR-}, \quad C(\theta) = \left(1-\sigma_N\right)\left(\Gamma_{nL+}\Gamma_{nR+} - \Gamma_{nL-}\Gamma_{nR-}\right)\Lambda_{no}$$

$$\Lambda_{dn}(\theta) = \Lambda_{dn}(\theta, m_z, \varphi)$$

$$\Lambda_{dn}(\theta, -m_z, \varphi) = \Lambda_{dn}(-\theta, m_z, -\varphi), \quad \Lambda_{dn}(-\theta, m_z, \varphi) = \Lambda_{dn}^*(\theta, m_z, \varphi)$$

Josephson current 2

$$R_N I(\varphi) = \frac{\pi \bar{R}_N k_B T}{e} \sum_n \int_{-\pi/2}^{\pi/2} d\theta \frac{2\cos\theta\sigma_N}{|\Lambda_{dn}(\theta)|^2} \left[A(\theta)\sin\varphi + B(\theta)\sin 2\varphi + C(\theta)\cos\varphi\right]$$

$$A(\theta) = (\Gamma_{nL+}\Gamma_{nR+} + \Gamma_{nL-}\Gamma_{nR-})\left[(1-\sigma_N)\Lambda_{ne} + \sigma_N(1 + \Gamma_{nL+}\Gamma_{nL-}\Gamma_{nR+}\Gamma_{nR-})\right],$$

$$B(\theta) = 2\sigma_N\Gamma_{nL+}\Gamma_{nL-}\Gamma_{nR+}\Gamma_{nR-}, \quad C(\theta) = (1-\sigma_N)(\Gamma_{nL+}\Gamma_{nR+} - \Gamma_{nL-}\Gamma_{nR-})\Lambda_{no}$$

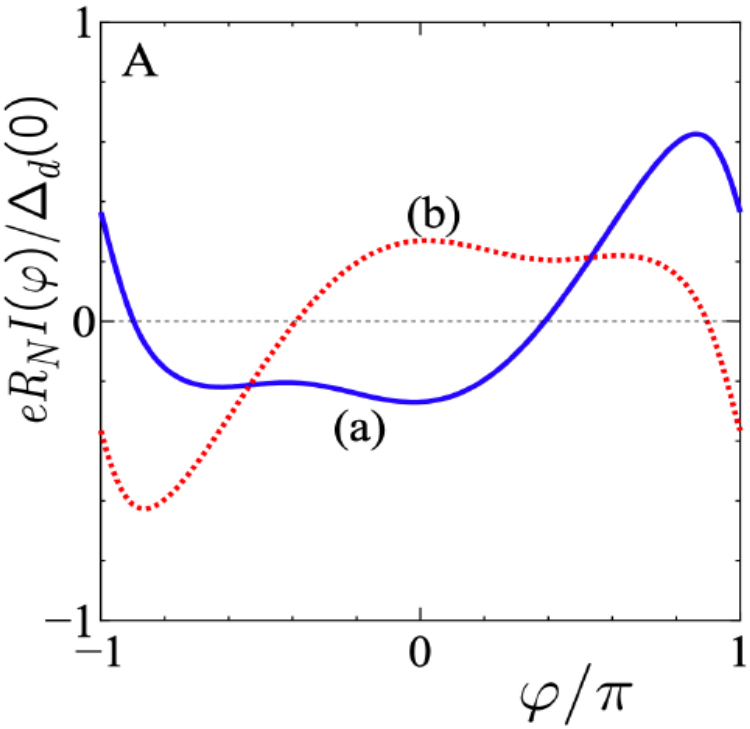
$$|\Lambda_{dn}(\theta, -m_z, \varphi)|^2 = |\Lambda_{dn}(\theta, m_z, -\varphi)|^2$$

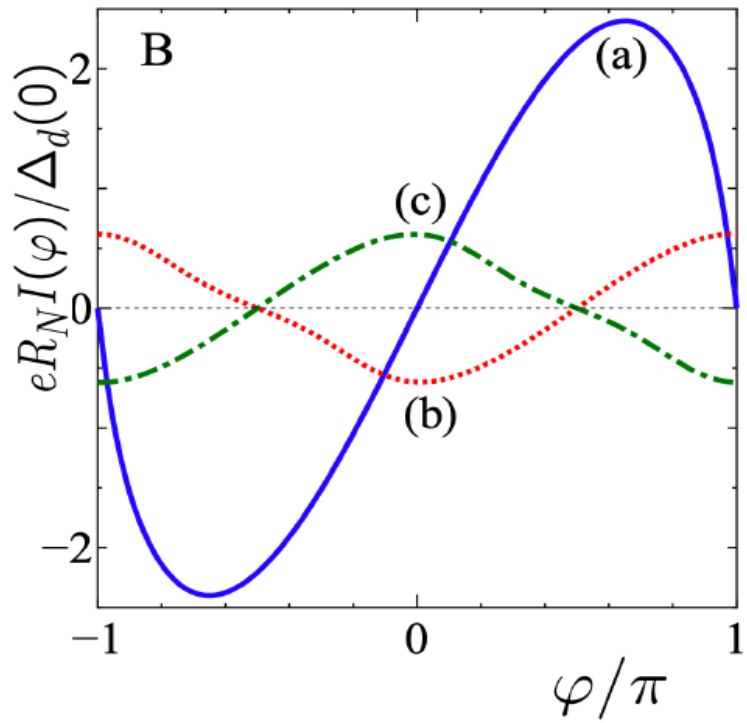
$$\Lambda_{dn}(\theta, -m_z, \varphi) = \Lambda_{dn}(-\theta, m_z, -\varphi), \ \Lambda_{dn}(-\theta, m_z, \varphi) = \Lambda_{dn}^*(\theta, m_z, \varphi)$$

$$A(\theta, m_z) = A(\theta, -m_z), \ C(\theta, m_z) = -C(\theta, -m_z)$$

$$\Rightarrow \quad I(\varphi, m_z) = -I(-\varphi, -m_z)$$

Current phase relation





$\alpha = -0.2\pi, \beta = 0.09\pi$

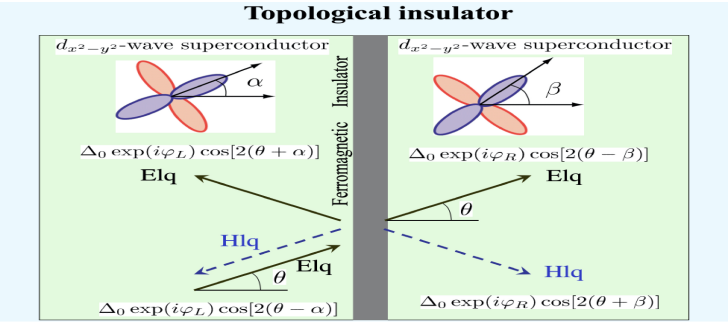
$(a) m_z = 0.5\mu$

$(b) m_z = -0.5\mu$

$(a) \alpha = 0, \beta = 0, m_z = 0.5\mu$

$(b) \alpha = 0, \beta = 0.25\pi, m_z = 0.5\mu$

$(c) \alpha = 0, \beta = 0.25\pi, m_z = -0.5\mu$

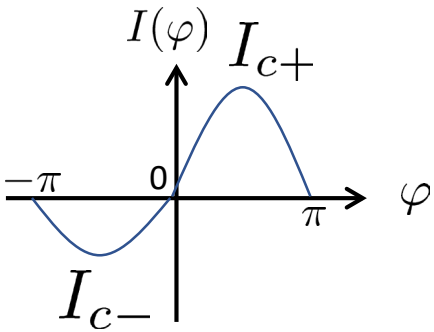


Quality factor 1

I_c^+ **Positive maximum Josephson current**

I_c^- **Negative maximum Josephson current**

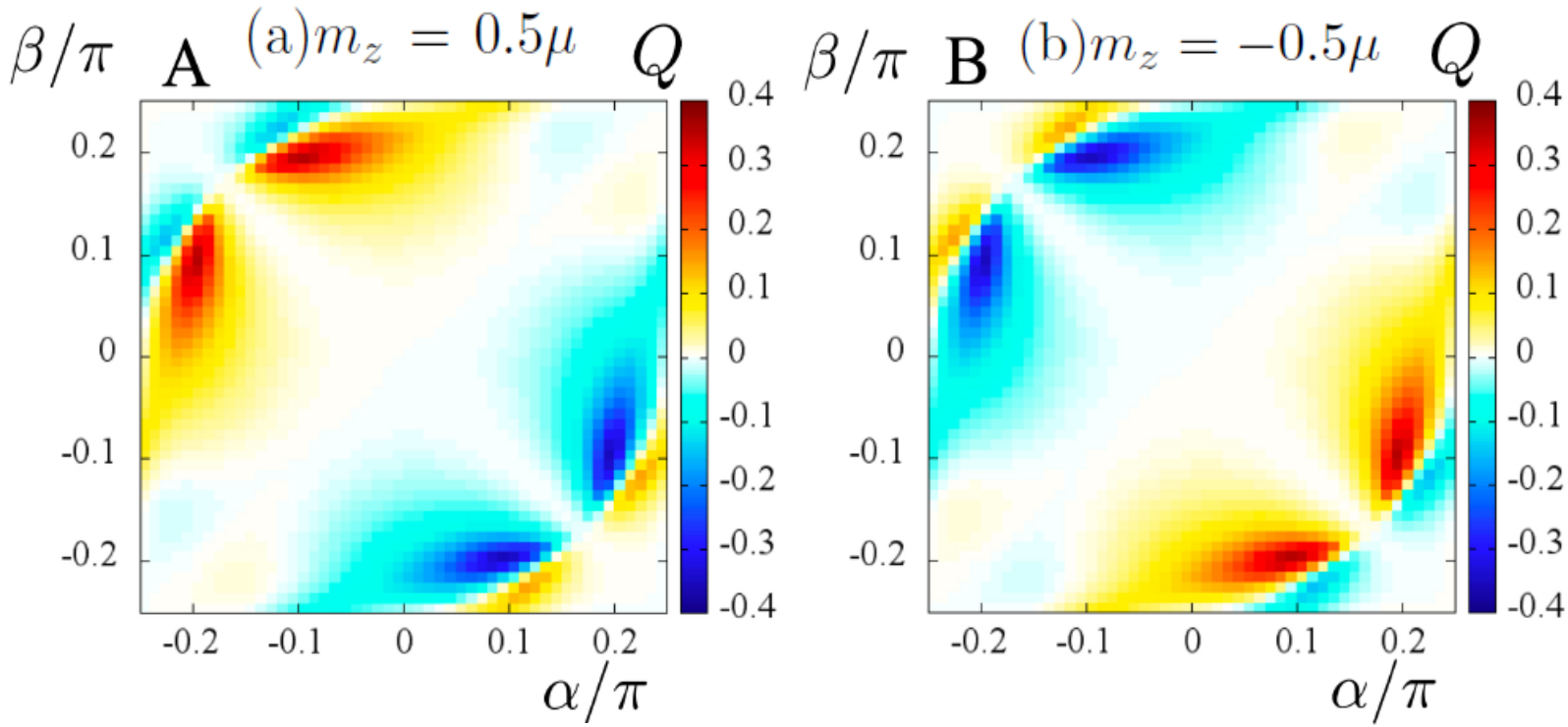
$$Q = \frac{I_c^+ - | I_c^- |}{I_c^+ + | I_c^- |}$$



We plot magnitude *of* Q in the following calculation.

$$| Q |$$

Quality factor 2

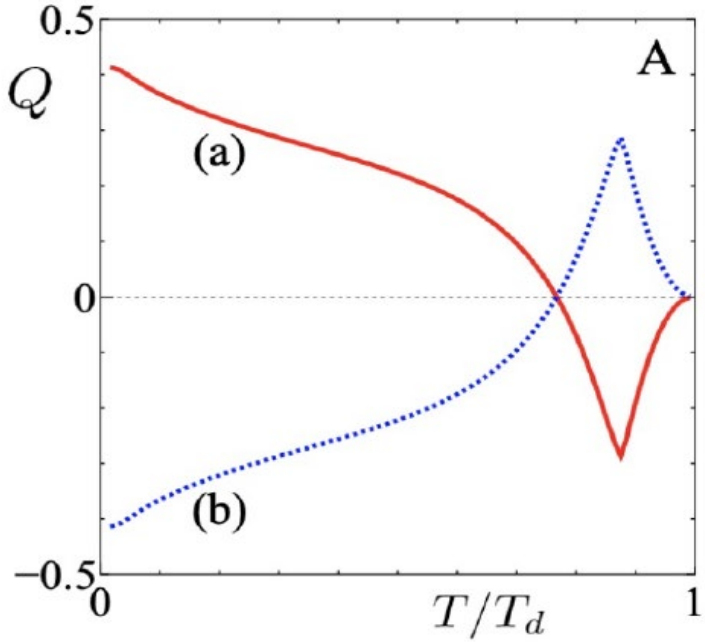


$$T = 0.05T_d \text{ and } d \mid m_z \mid /v = 1$$

$$Q(\alpha, \beta) = -Q(-\alpha, -\beta) \qquad I(\varphi, \alpha, \beta) = -I(-\varphi, \beta, \alpha)$$

arXiv:2205.13177

Quality factor 3



$$\alpha = -0.2\pi$$
$$\beta = 0.09\pi$$

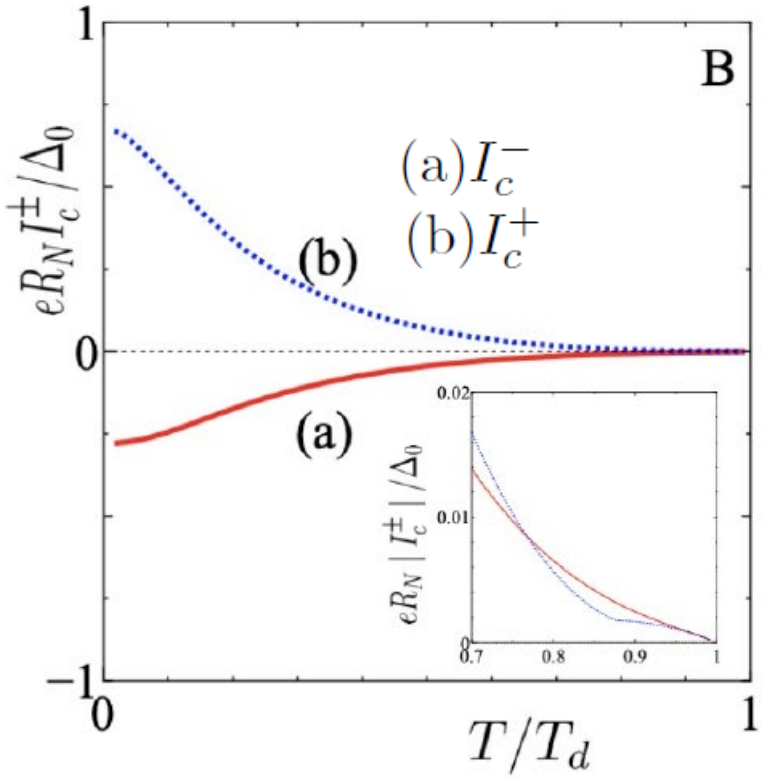
$$d\,|\,m_z\,|\,/v = 1$$

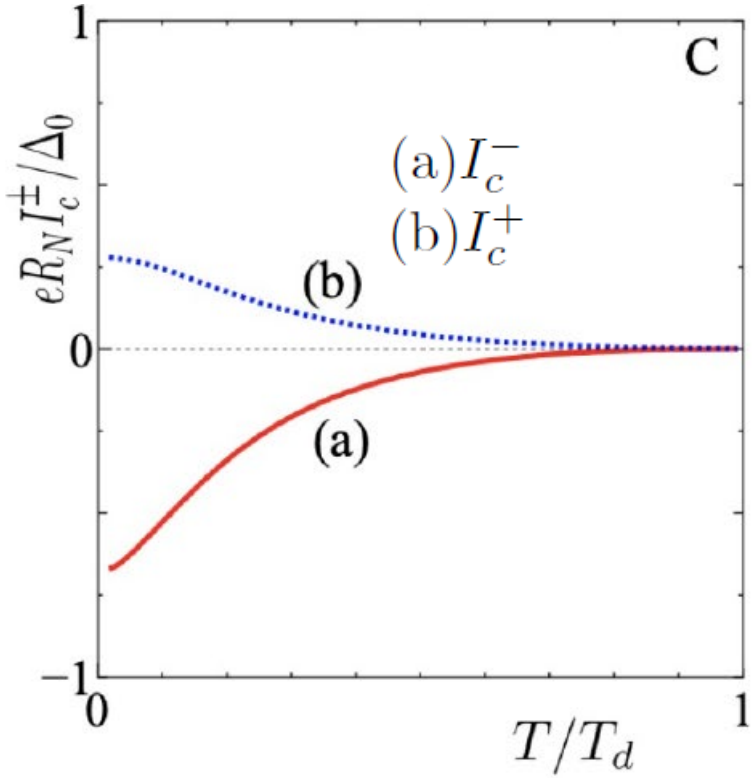
$$(\mathrm{a})m_z = 0.5\mu,\ (\mathrm{b})m_z = -0.5\mu.$$

Large magnitude of Q factor

Strong temperature dependence

Temperature dependence of maximum Josephson current





$m_z = 0.5\mu$

$m_z = -0.5\mu$

Inverse the direction of the magnetization

$$I_c^+ \leftrightarrow I_c^-$$

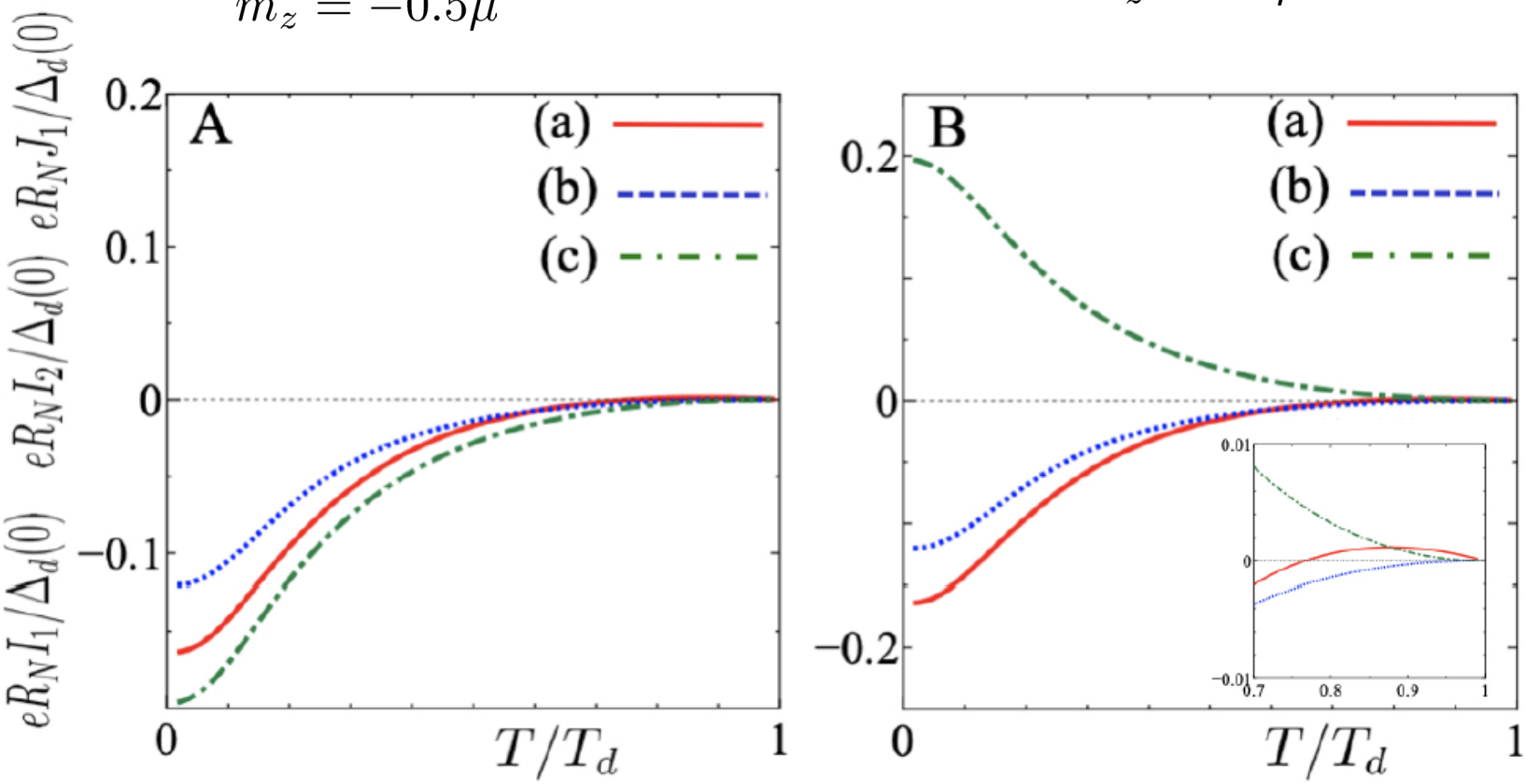
Temperature dependence of Fourier component 1

A: $(\alpha, \beta) = (-0.2\pi, 0.09\pi)$

$m_z = -0.5\mu$

B: $(\alpha, \beta) = (-0.2\pi, 0.09\pi)$

$m_z = 0.5\mu$

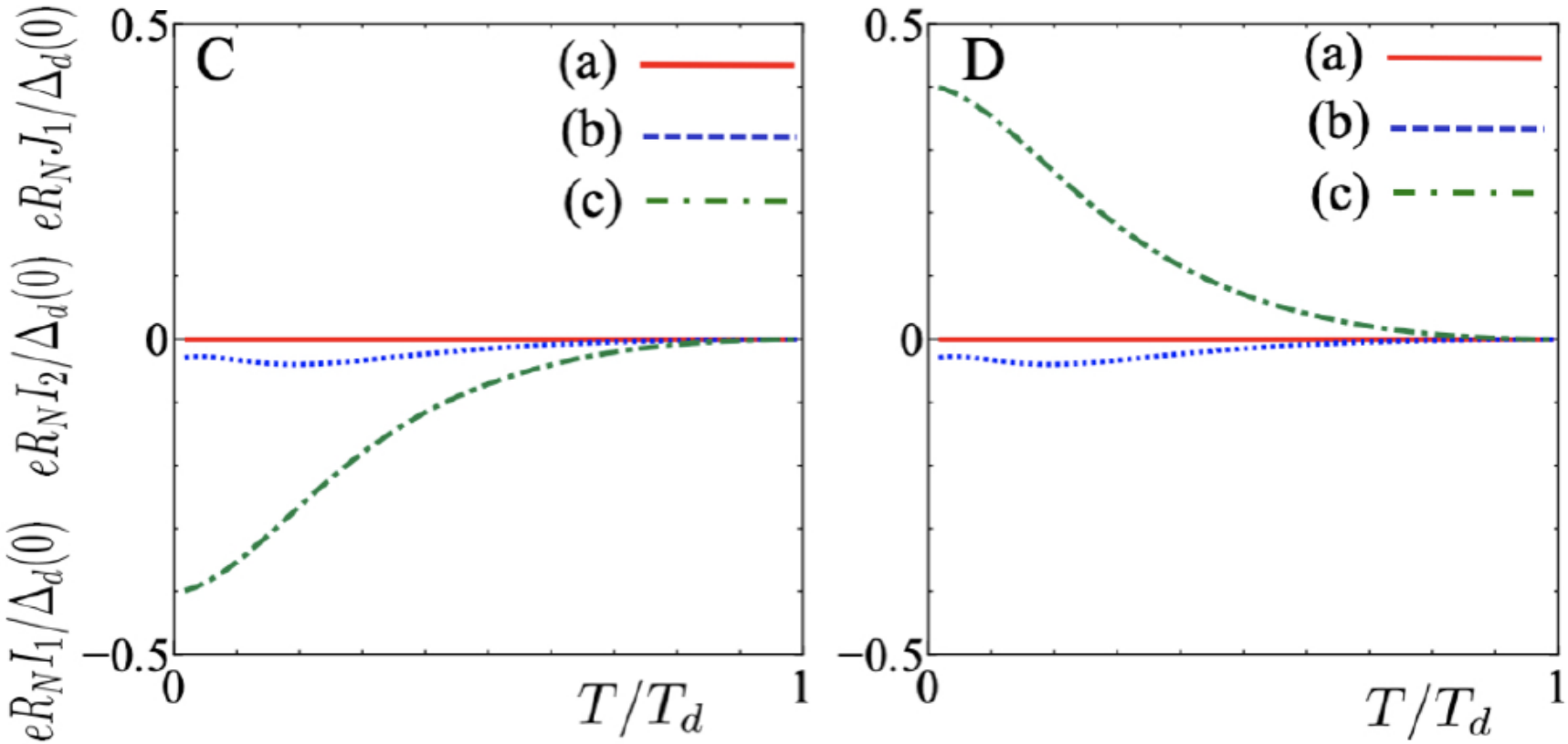


The sign of J_1 changes by the inversion of the sign of m_z.

$$I(\varphi) = \sum \left[I_n \sin n\varphi + J_n \cos n\varphi \right].$$

Temperature dependence of Fourier component 2

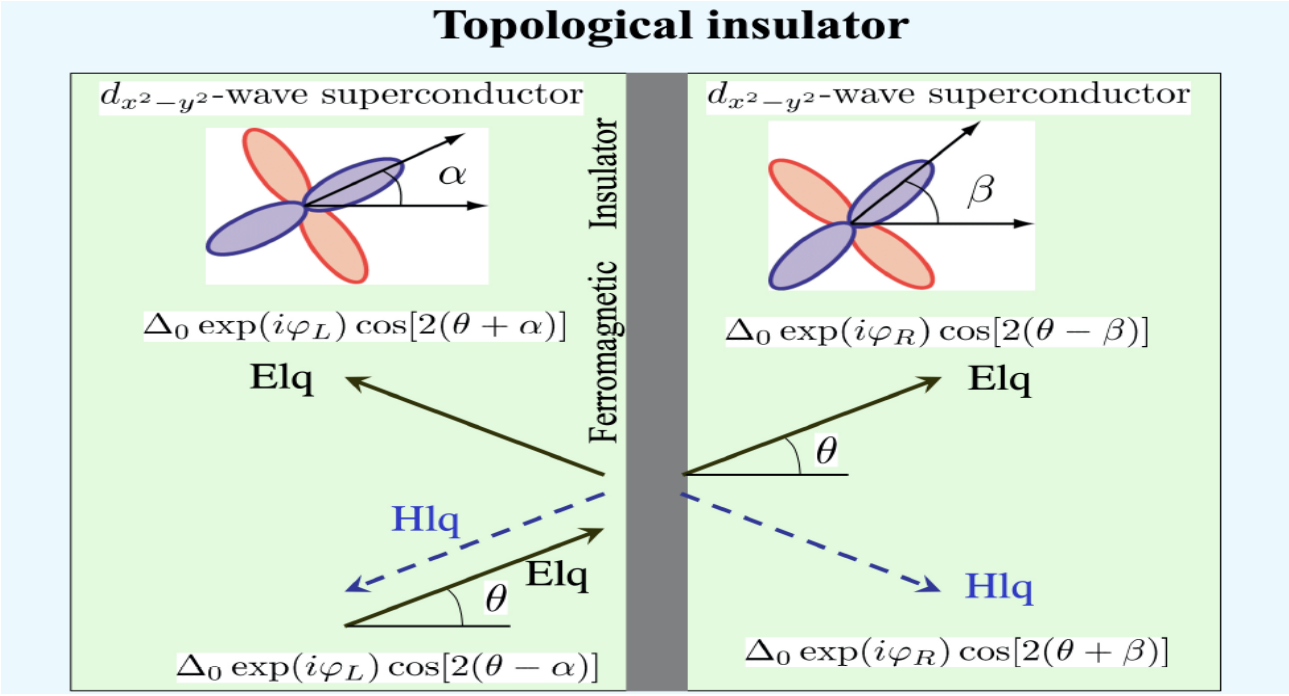
$(\alpha, \beta) = (0, 0.25\pi)\ m_z = -0.5\mu$ $(\alpha, \beta) = (0, 0.25\pi)\ m_z = 0.5\mu$



$$I(\varphi) = \sum_n [I_n \sin n\varphi + J_n \cos n\varphi] .$$

$$I(\varphi) = \sum [I_n \sin n\varphi + J_n \cos n\varphi] .$$

Andreev bound state at the interface 1

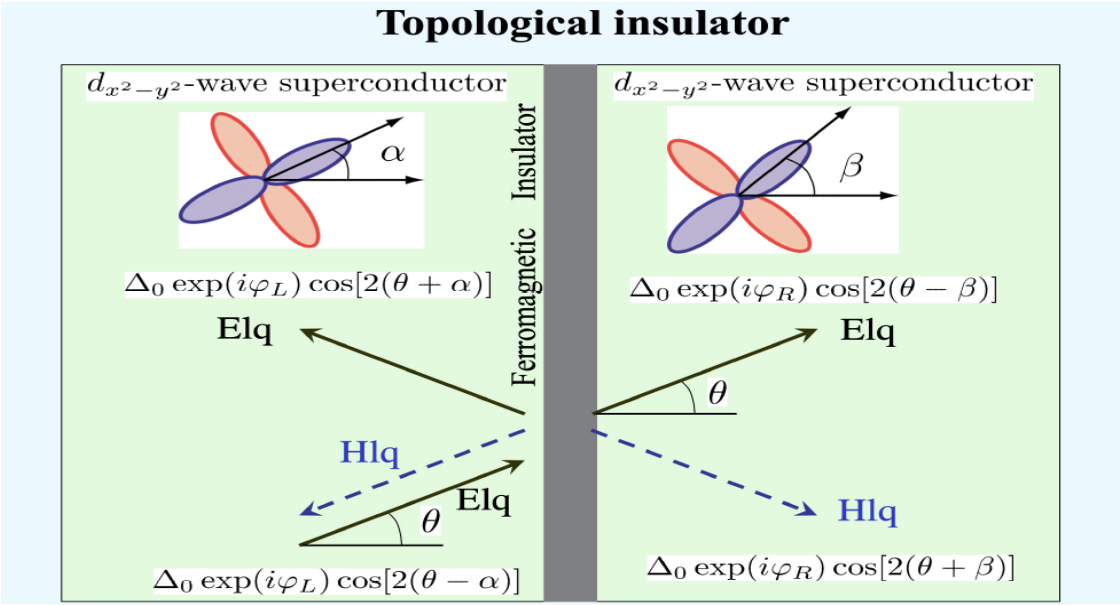


$$\varphi = \varphi_L - \varphi_R$$

Andreev bound states at the interface becomes Majorana fermions (chiral Majorana mode)

We can know the energy level by the zero of the denominator of the Andreev reflection coefficients.

Andreev bound state at the interface 2



$$\varphi = \varphi_L - \varphi_R$$

$$a_e = -\frac{\sigma_N \Lambda_{1e} + (1-\sigma_N)\Lambda_{2e}}{\Lambda_d(E,\theta)}, \quad a_h = -\frac{\sigma_N \Lambda_{1h} + (1-\sigma_N)\Lambda_{2h}}{\Lambda_d(E,\theta)}$$

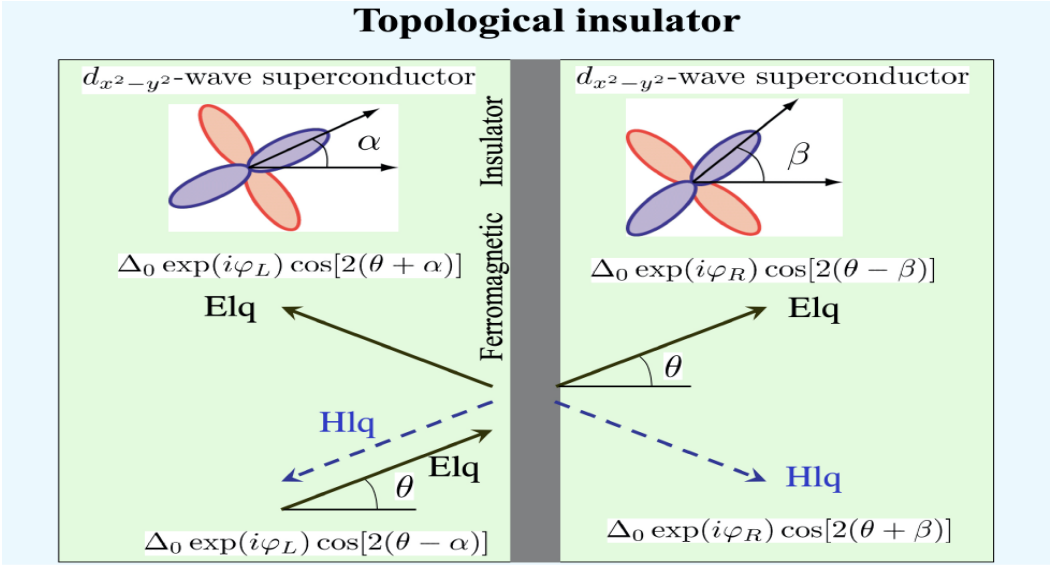
$$\Lambda_d(E,\theta) = [1-\sigma_N]\,[1+\exp(-i\eta)\,\Gamma_{R+}\Gamma_{R-}]\,[1+\exp(i\eta)\,\Gamma_{L+}\Gamma_{L-}]$$

$$+\,\sigma_N\,[1-\exp(-i\varphi)\,\Gamma_{L-}\Gamma_{R-}]\,[1-\exp(i\varphi)\,\Gamma_{L+}\Gamma_{R+}]$$

$$\Lambda(E,\theta) = 0$$

Andreev bound states condition

Andreev bound state at the interface 3



$$\varphi = \varphi_L - \varphi_R$$

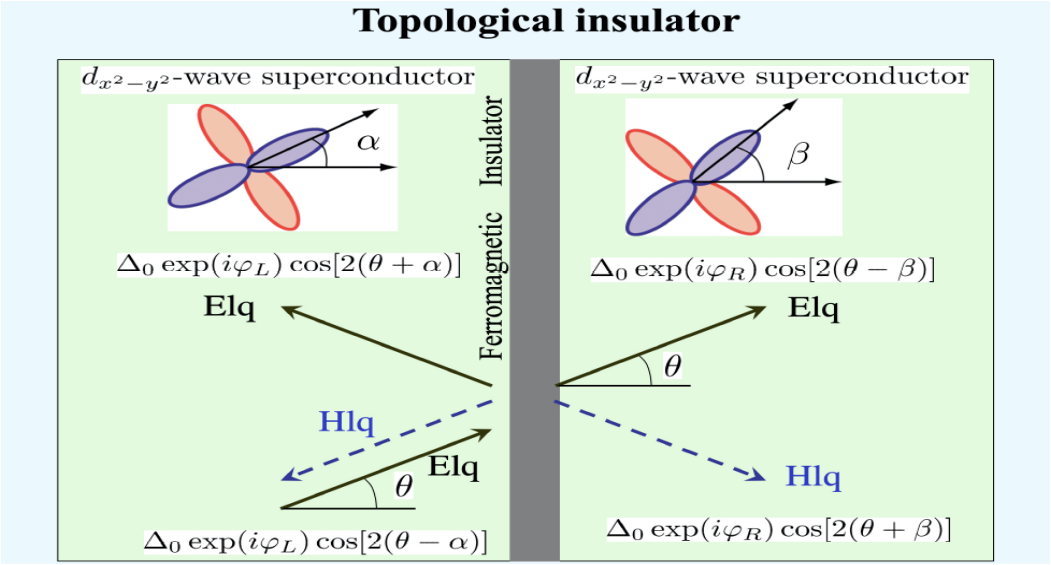
Limiting case where we can obtain analytical result

$$\alpha = \beta = 0 \qquad E_b = \pm\sqrt{\sigma_N \cos^2\frac{\varphi}{2} + (1-\sigma_N)\sin^2\frac{\eta}{2}}\,|\cos 2\theta|\,\Delta_0$$

$$\cos^2\frac{\eta}{2} = \frac{m_z^2\cos^2\theta}{m_z^2\cos^2\theta + \mu^2\sin^2\theta}, \qquad \sin^2\frac{\eta}{2} = \frac{\mu^2\sin^2\theta}{m_z^2\cos^2\theta + \mu^2\sin^2\theta}$$

$$E_b = 0, \quad \varphi = \pm\pi \quad \text{and} \quad \theta = 0$$

Andreev bound state at the interface 4



$$\varphi = \varphi_L - \varphi_R$$

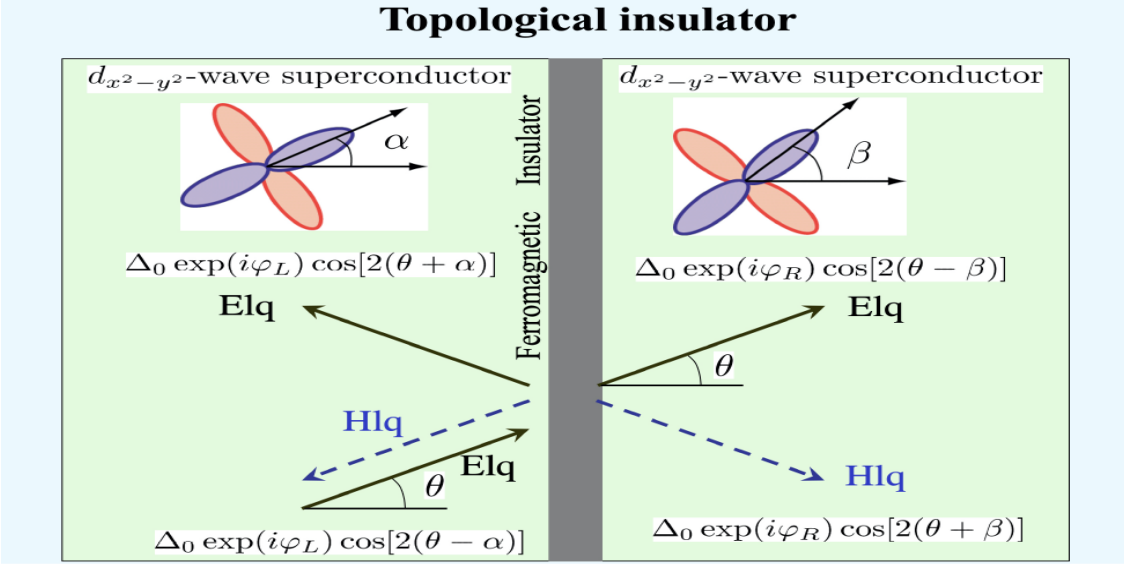
Limiting case where we can obtain analytical result

$$\alpha = \beta = \pi/4 \qquad E_b = \pm\sqrt{\sigma_N \cos^2\frac{\varphi}{2} + (1-\sigma_N)\cos^2\frac{\eta}{2}}\,|\sin 2\theta|\,\Delta_0$$

$$\cos^2\frac{\eta}{2} = \frac{m_z^2\cos^2\theta}{m_z^2\cos^2\theta + \mu^2\sin^2\theta}, \quad \sin^2\frac{\eta}{2} = \frac{\mu^2\sin^2\theta}{m_z^2\cos^2\theta + \mu^2\sin^2\theta}$$

$$E_b = 0, \quad \varphi = \pm\pi \quad \text{and} \quad \theta = \pm\pi/2$$

Andreev bound state at the interface 5

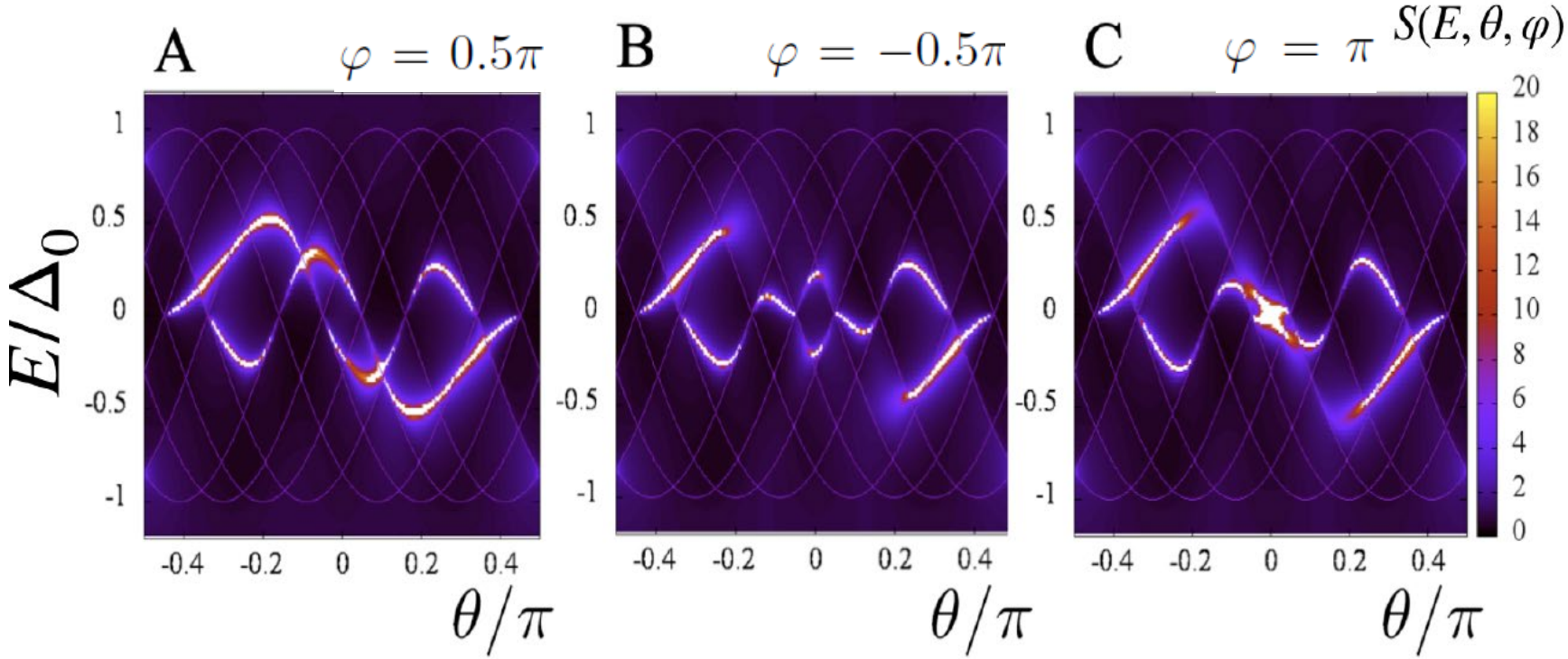


$$\varphi = \varphi_L - \varphi_R$$

In general, it is impossible to solve E_b analytically.
We plot inverse of $\Lambda(E,\theta) = \Lambda(E,\theta,\varphi)$

$$S(E,\theta,\varphi) = \frac{1}{|\ \Lambda_d\,(E,\theta,\varphi)\ |}$$

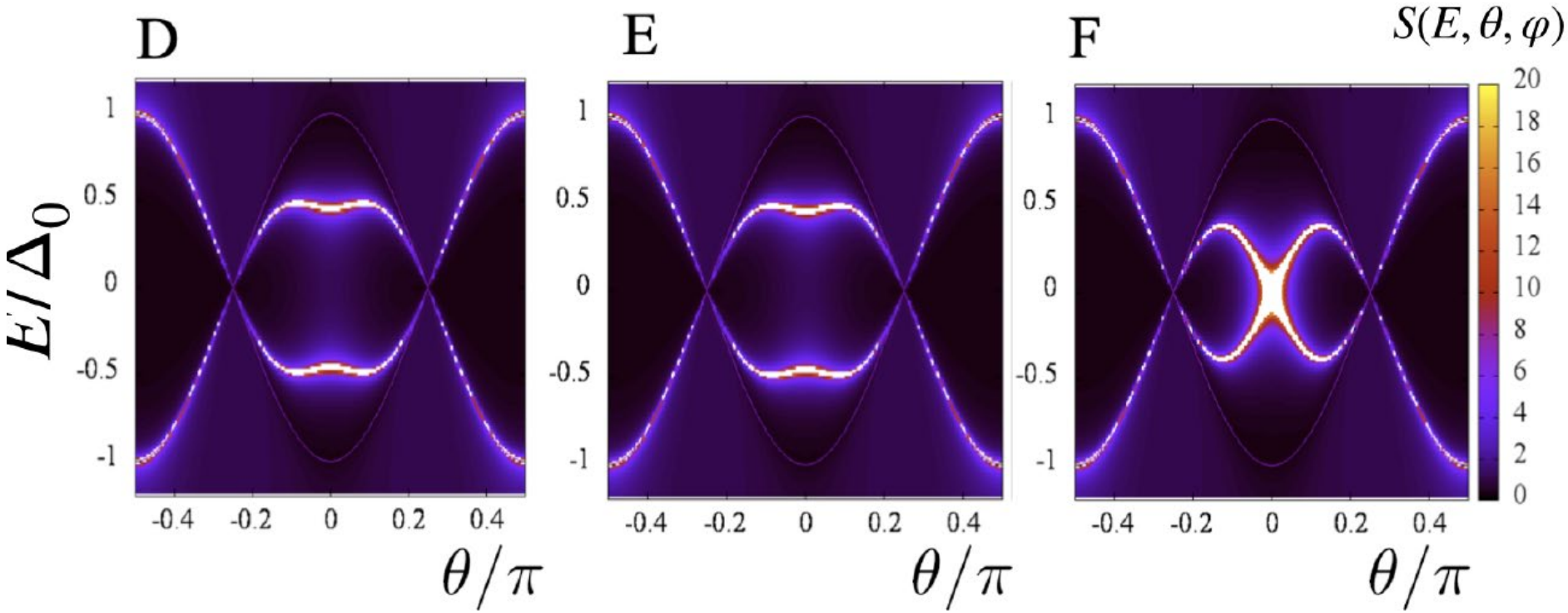
Andreev bound states(θ dependence)



$$\alpha = -0.2\pi, \beta = 0.09\pi$$

Q is nonzero (diode effect)

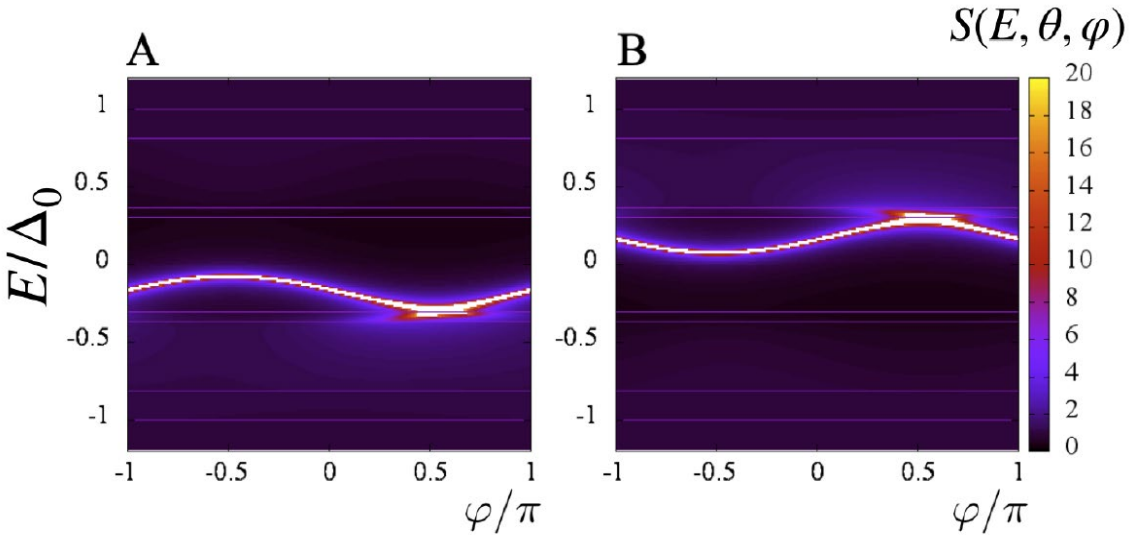
Andreev bound states(θ dependence)



No Diode effect case.

$$(\alpha, \beta) = (0, 0.25\pi)$$

Andreev bound states (Phase dependence)

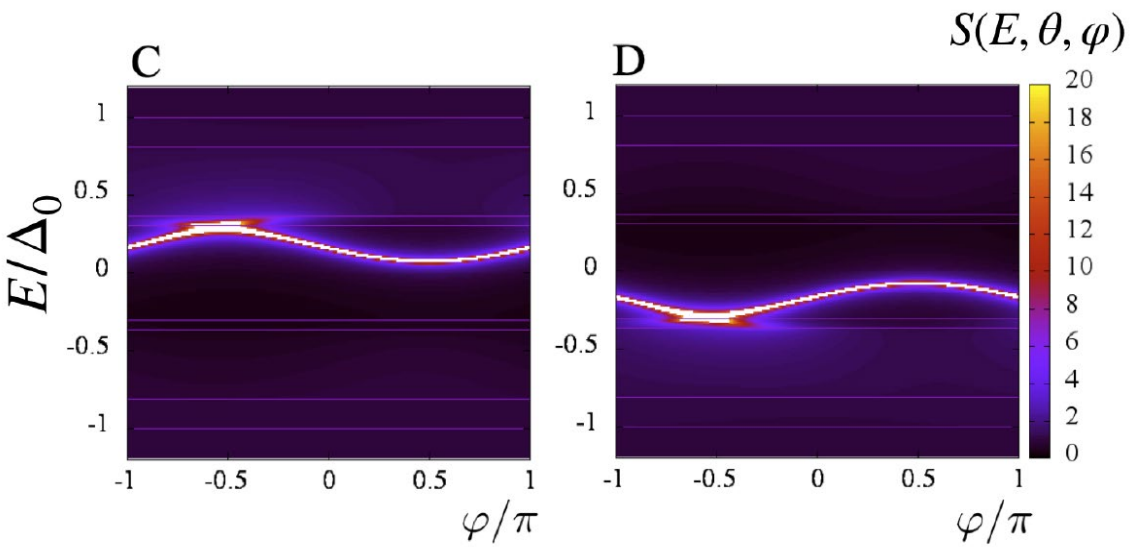


A: $\theta = 0.1\pi$ and $m_z = 0.5\mu$,

B: $\theta = -0.1\pi$ and $m_z = 0.5\mu$

C: $\theta = 0.1\pi$ and $m_z = -0.5\mu$

D $\theta = -0.1\pi$ and $m_z = 0.5\mu$

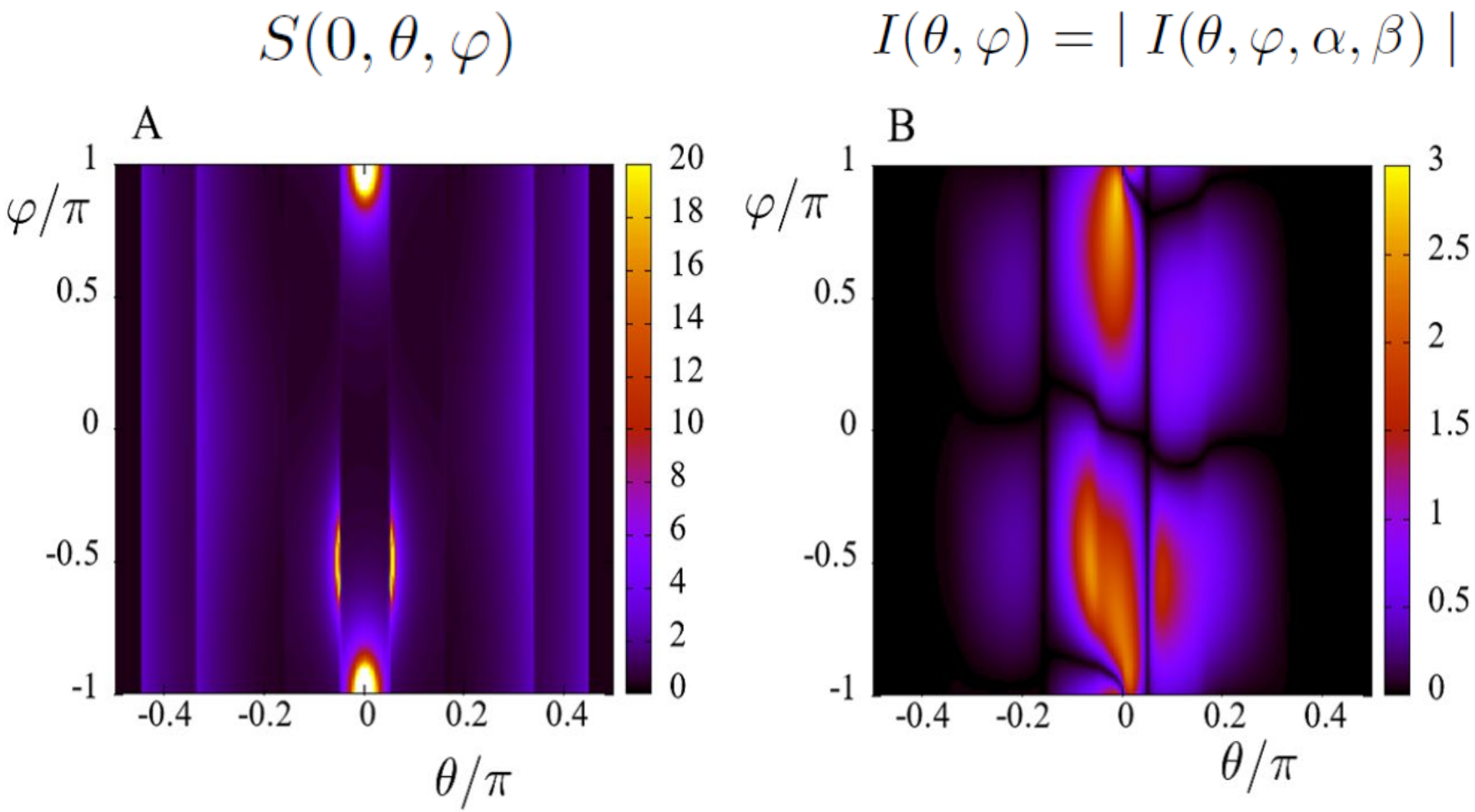


$$S(E,-\theta,-\varphi,-m_z) = S(E,\theta,\varphi,m_z)$$

$$\alpha = -0.2\pi, \beta = 0.09\pi$$

Q is nonzero (diode effect)

Zero energy Andreev bound states and Josephson current



Zero energy Andreev bound state (Majorana mode) and Josephson current are closely related to each other.

Summary and Conclusions

(1)The large magnitude of quality factor Q is realized by tuning the crystal axis of both left and right d-wave superconductors.

(2)The magnitude of Q becomes almost 0.4 at low temperatures and its sign is reversed by changing the direction of the magnetization in the FI.

(3)The large Q stems from the simultaneous existence of sin φ, cos φ and sin 2φ component.

(4)The strong temperature dependence of Q stems from the existence of the low energy Andreev bound state appearing as Majorana bound states.

arXiv:2205.13177