Spin Parity Effect in monoaxial chiral ferromagnetic chain*

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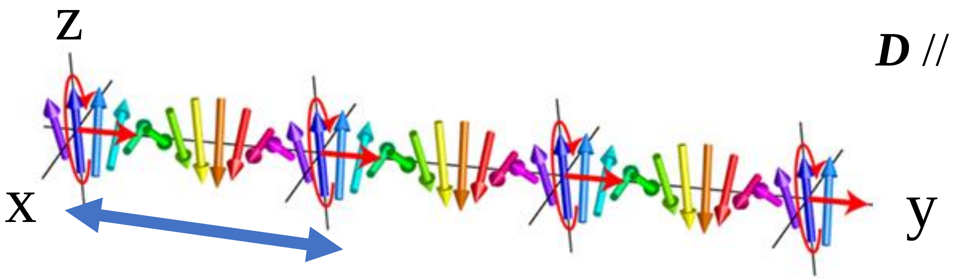
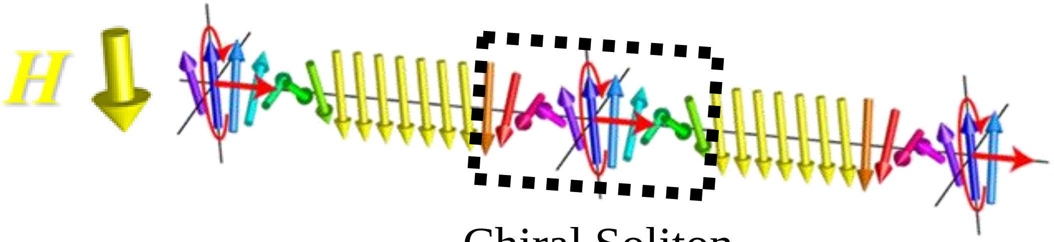
Background: monoaxial chiral magnets

$$\hat{H}_{\rm chiral} = \sum_i \left[-J\hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_{i+1} - D\left(\hat{\boldsymbol{S}}_i \times \hat{\boldsymbol{S}}_{i+1}\right)_y - H\hat{S}_i^z + K\left(\hat{S}_i^y\right)^2 \right]$$

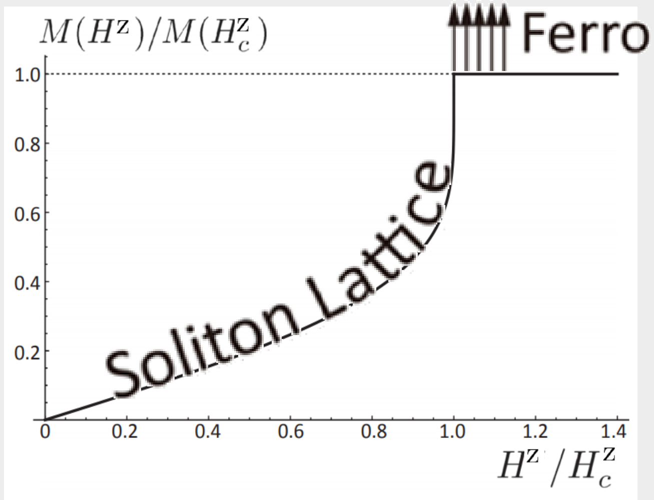
$\boldsymbol{D}$ // $\mathbf{e}_y$ “Monoaxial”



Period of
Chiral Soliton Lattice $\dfrac{2\pi}{\arctan(D/J)}$ $\sim J/D$ for $J \gg D$



Chiral Soliton



$$\frac{M(H^z)}{M(H_c^z)} \simeq 1 + \frac{1}{\log(H_c^z - H^z)}$$

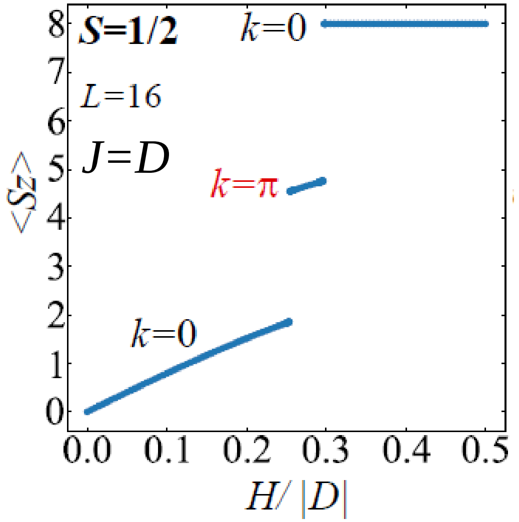
Review: Togawa *et al.*, J. Phys. Soc. Jpn. **85**, 112001 (2016)
Kishine-Ovchinnikov, Solid State Physics(2015)

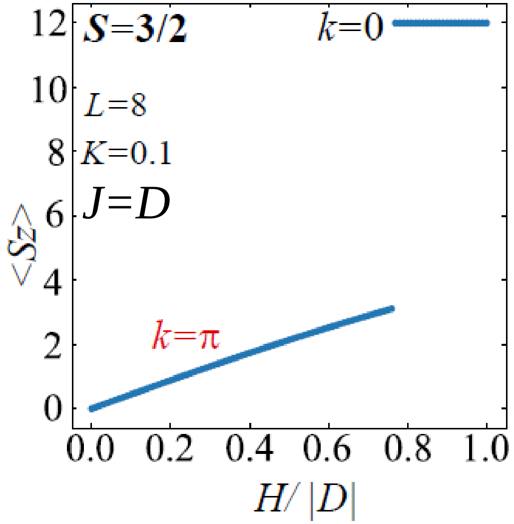
Dzyaloshinskii (1965)
Chiral Soliton Lattice.

Issue:
Understanding of Different behaviour in magnetization process between half-Integer and Integer Spins (a "Spin Parity Effect")

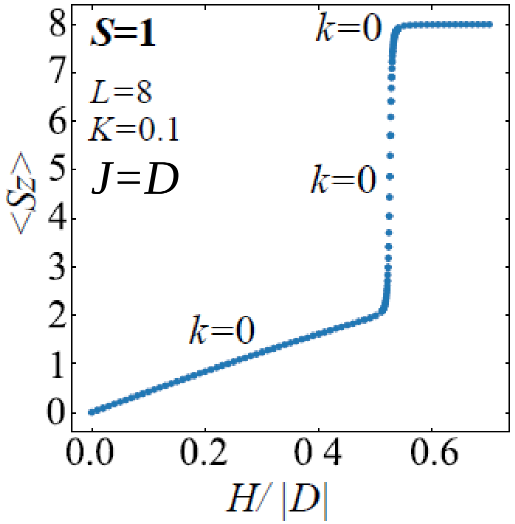
Numerical Diagonalization

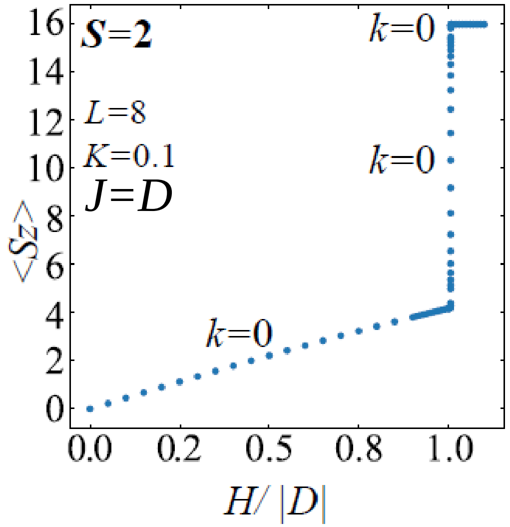
Under the periodic boundary condition





Discontinuous Magnetization =Level-Crossing





Numerical Diagonalization

Under the periodic boundary condition

Kodama et al arXiv:2209.04227

Continuous Magnetization =Level-Repulsion

Earlier Studies

Semiclassical Approach (Spin coherent state and Berry phase argument) for

- Nonchiral nanomagnets (Braun-Loss PRB1996) :
  solitons (domain wall) are generated by Ising anisotropy

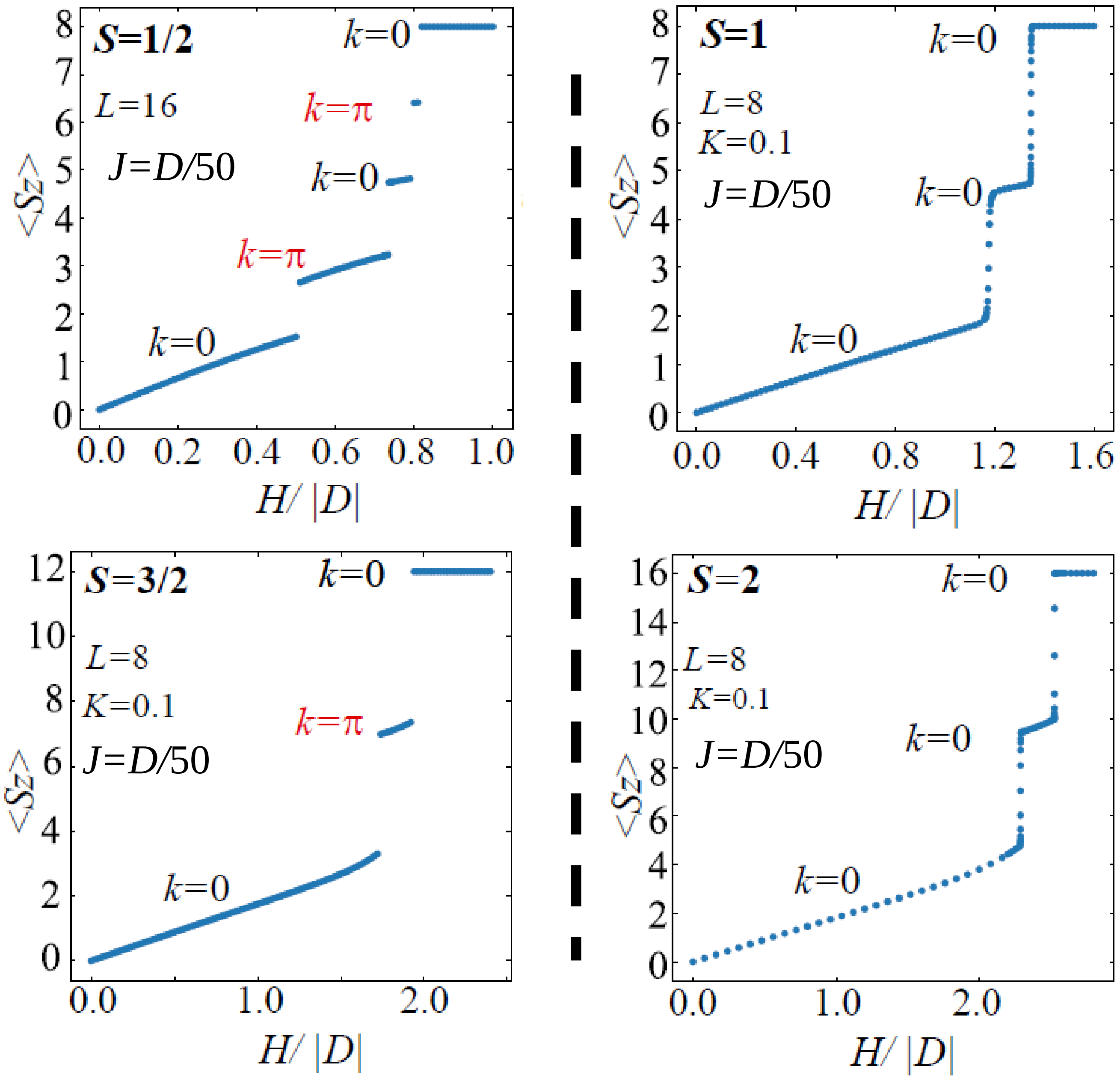
- 2D chiral magnets (Takashima-Ishizuka-Balents 2016 PRB):
  Quantum skyrmion

Those are valid for **large S**, ⇔ We seek for a theory valid for **small S**.

Cf：Haldane Gap problem

O(3)Nonlinear Sigma model (large S + Berry phase) ⇔ AKLT model (S=1)

Clue: $J/D \Rightarrow 0$ limit



Discontinuous Magnetization =Level-Crossing

Continuous Magnetization =Level-Repulsion

Model in the J/D ⇒ 0 limit is a **canonical model** to understand for spin parity effect in chiral magnet

$$\mathcal{H}_{DH} = \sum_i \left[D\left(\hat{\boldsymbol{S}}_i \times \hat{\boldsymbol{S}}_{i+1}\right)^y - H\hat{S}_i^z \right]$$

- Difference between half-odd-Integer S and Integer S is also observed in *M*-*H* curves in the limit J/D ⇒ 0 (i.e. **Spin Parity Effect exists:**) cf Previous page

- Number of Solitons becomes a **conserved quantity**. (Next page)

$$\hat{N} = \sum_{i=1}^{L} \left(\frac{1}{4} - \hat{S}_i^z \hat{S}_{i+1}^z \right)$$

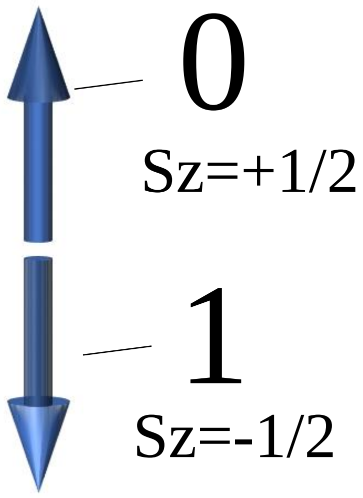
Conserved Quantity=Soliton Number Operator

$$\hat{N} = \sum_{i=1}^{L} \left(\frac{1}{4} - \hat{S}_i^z \hat{S}_{i+1}^z \right)$$

$$\hat{N}|0001111100\rangle = |0001111100\rangle,$$

$$\hat{N}|0011100110\rangle = 2|0011100110\rangle.$$

Conventional Basis

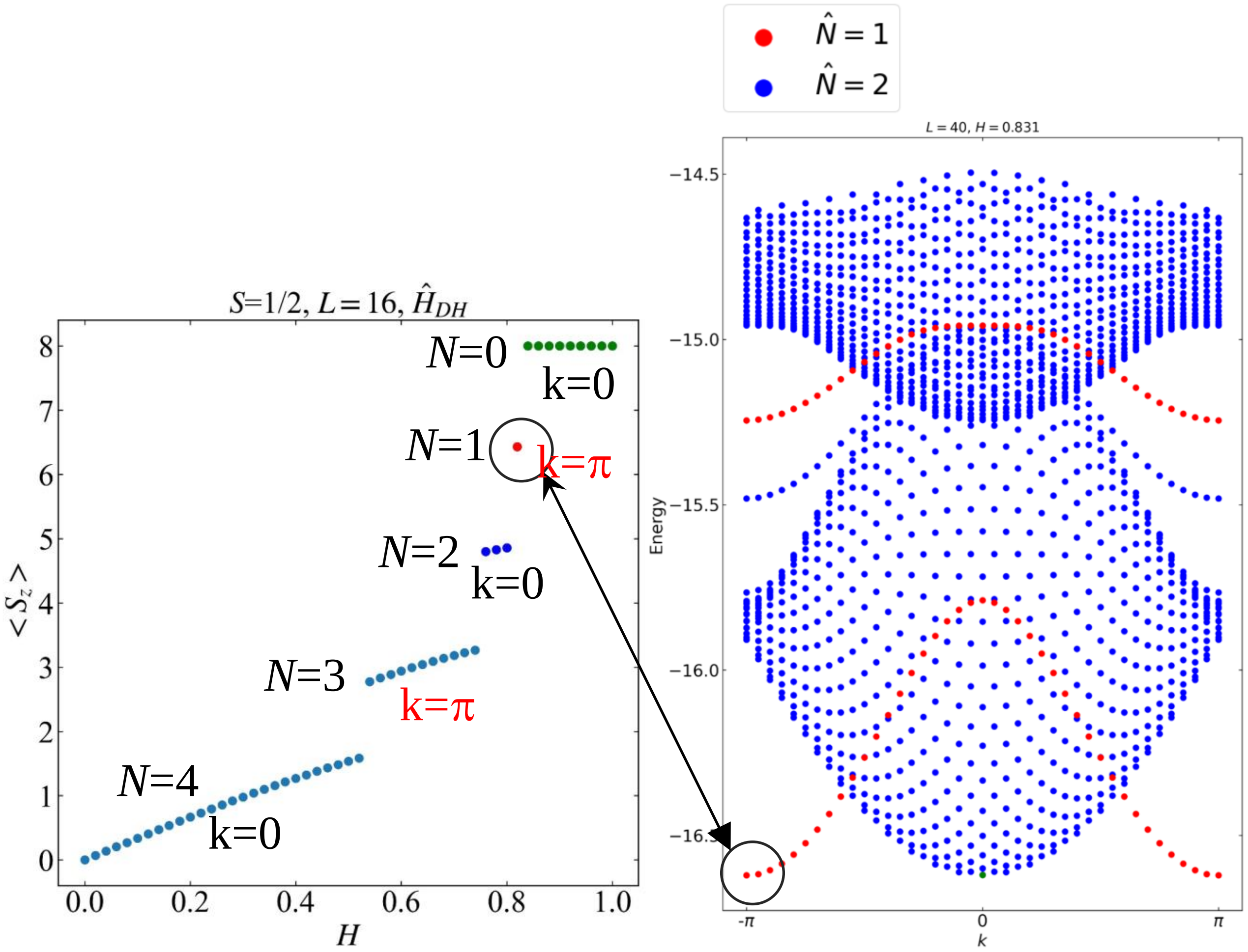


$$|n_1, n_2, \cdots, n_L\rangle =: |\boldsymbol{n}\rangle$$

$$n_i = 0, 1, \cdots 2S \ (\ i = 1, \cdots, L)$$

$$\hat{S}_{i,z}|\boldsymbol{n}\rangle = (S - n_i)|\boldsymbol{n}\rangle$$

Numerical Results for H_{DH} model (under the PBC)



- **Magnetization/deMagnetization Processes consisting of successive escape/penetration of Solitons,**

- **One-Soliton states (N=1) have minimum energy at k=π**
  Two-Soliton states (N=2) have minimum energy at k=0,
  which can be proven for the H_{DH} model.

Three ingredients for proof

- There exists a **signed basis** in which Off-diagonal Matrix Elements of the Hamiltonian (H_{DH}) are non-positive.

- The relation among **sign for each basis, translation, and Soliton numbers**

- All expansion coefficients for the ground state wavefunction in the signed basis have a definite sign (**Perron-Frobenius Theorem**)

Signed Basis $(-1)^{\delta(\boldsymbol{n})}|\boldsymbol{n}\rangle$

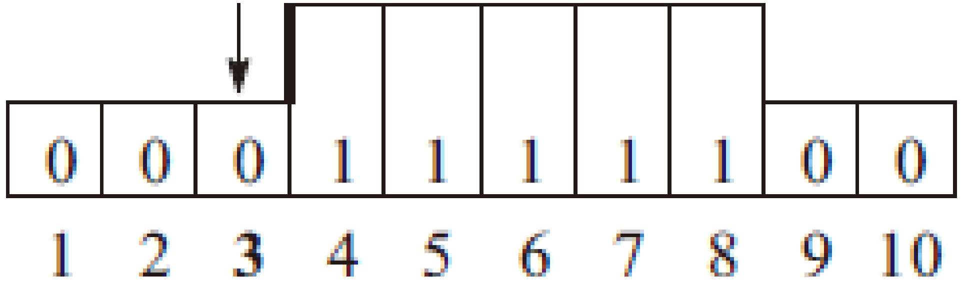
$$\delta(\boldsymbol{n}) = \sum_{i=1}^{L} i\left(n_{i+1} - n_i + |n_{i+1} - n_i|\right)/2.$$

δ(**n**): sum of the coordinates of the left edge of each soliton

Ex.

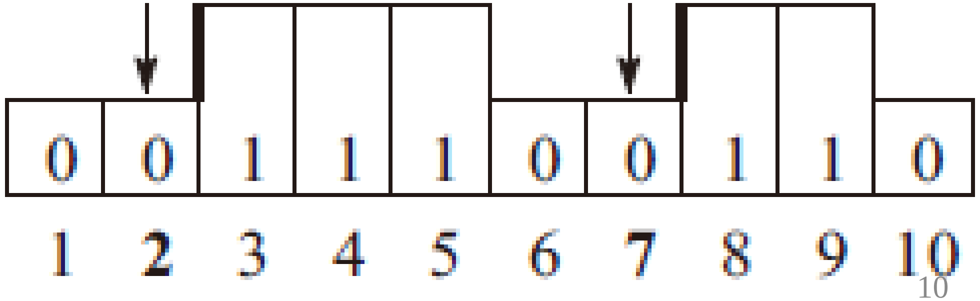
$$(-1)^3|0001111100\rangle$$

$(-1)^3$



$$(-1)^{2+7}|0011100110\rangle$$

$(-1)^{2+7}$



The relation among **sign for each basis, translation, and Soliton numbers**

$$(-1)^{\delta(\boldsymbol{n})} = (-1)^{\delta(T(\boldsymbol{n}))}(-1)^{N}$$

Translation Operator $\hat{T}|\boldsymbol{n}\rangle = |T(\boldsymbol{n})\rangle$ with $T(\boldsymbol{n}) := (n_2, n_3, \cdots, n_L, n_1)$

N=1,3,5 (N=2,4,6) Soliton states change (does not change) the sign under the one-site translation.

Off-diagonal Matrix Elements in *conventional basis*

$$\hat{\mathcal{H}}_{DH} = \hat{\mathcal{H}}_{\mathrm{DM}} + \hat{\mathcal{H}}_{\mathrm{Z}}$$

$$\hat{\mathcal{H}}_{\mathrm{DM}} = D \sum_j \left(\hat{\boldsymbol{S}}_j \times \hat{\boldsymbol{S}}_{j+1} \right)^y$$

$$-\frac{D}{2} \sum_{i=1}^{L} \hat{h}_i$$

$$\hat{\mathcal{H}}_{\mathrm{Z}} = -H \sum_j \hat{S}_j^z$$

diagonal

$$\hat{h}_i = \left(\hat{S}_{i+1,z} - \hat{S}_{i-1,z} \right) \left(\hat{S}_{i,+} + \hat{S}_{i,-} \right)$$

local Dzyaloshinskii-Moriya interaction
= a three-site term

Ex.

$$\hat{h}_i | \cdots 0\overset{i}{0}1 \cdots \rangle = -| \cdots 0\overset{i}{1}1 \cdots \rangle$$

$$\hat{h}_i | \cdots 0\overset{i}{1}1 \cdots \rangle = -| \cdots 0\overset{i}{0}1 \cdots \rangle$$

$$\hat{h}_i | \cdots 1\overset{i}{0}0 \cdots \rangle = +| \cdots 1\overset{i}{1}0 \cdots \rangle$$

$$\hat{h}_i | \cdots 1\overset{i}{1}0 \cdots \rangle = +| \cdots 1\overset{i}{0}0 \cdots \rangle$$

Off-diagonal Matrix Elements in *conventional basis*

$$\hat{h}_i|\boldsymbol{n}\rangle = (n_{i-1} - n_{i+1})|\bar{\boldsymbol{n}}\rangle \quad \text{where } \bar{\boldsymbol{n}} = (\bar{n}_1\bar{n}_2\cdots\bar{n}_L) \text{ with } \quad \bar{n}_j = \begin{cases} n_j, & j \neq i \\ 1-n_i, & j = i \end{cases}$$

$$\hat{h}_i|\cdots 0\overset{i}{0}1\cdots\rangle = -|\cdots 0\overset{i}{1}1\cdots\rangle$$

$$\hat{h}_i|\cdots 0\overset{i}{1}1\cdots\rangle = -|\cdots 0\overset{i}{0}1\cdots\rangle$$

$$\hat{h}_i|\cdots 1\overset{i}{0}0\cdots\rangle = +|\cdots 1\overset{i}{1}0\cdots\rangle$$

$$\hat{h}_i|\cdots 1\overset{i}{1}0\cdots\rangle = +|\cdots 1\overset{i}{0}0\cdots\rangle.$$

Left edge is shifted under the action of h_i (first two lines)

Off-diagonal Matrix Elements in the *signed basis*

$$\hat{h}_i(-1)^{\delta(\boldsymbol{n})}|\boldsymbol{n}\rangle = |n_{i-1} - n_{i+1}|(-1)^{\delta(\bar{\boldsymbol{n}})}|\bar{\boldsymbol{n}}\rangle$$

non-positive off-diagonal matrix elements

$(-1)^{\delta(\mathbf{n})}$: absorbs the minus sign in the matrix elements

(Off-diagonal matrix element with definite sign)
+Irreducibility. i.e.

$$(-1)^{\delta(\boldsymbol{n})+\delta(\boldsymbol{n}')}\langle \boldsymbol{n}|(-\hat{\mathcal{H}}_{DH})^{l}|\boldsymbol{n}'\rangle > 0$$

Theorem (Perron-Frobenius)

$\Rightarrow$

Lowest energy eigenstate in eigenspace of *N* soliton states

$$|E_{\min,N}\rangle = \sum_{\boldsymbol{n}\in V_N} a(\boldsymbol{n})(-1)^{\delta(\boldsymbol{n})}|\boldsymbol{n}\rangle \quad \text{with } a(\boldsymbol{n}) > 0$$

V_N: set of basis for N-soliton states

Ex. *N*=1

$$\begin{aligned}|E_{\min,1}\rangle =& a_1(-1)^1|010000\rangle + a_2(-1)^2|001000\rangle + \cdots \\ &+ b_1(-1)^1|011000\rangle + b_2(-1)^2|001100\rangle + \cdots\end{aligned} \quad \text{with } a_i, b_i \cdots > 0$$

$$|E_{\min,N}\rangle = \sum_{\boldsymbol{n}\in V_N} a(\boldsymbol{n})(-1)^{\delta(\boldsymbol{n})}|\boldsymbol{n}\rangle \quad \text{with } a(\boldsymbol{n}) > 0$$ (Perron-Frobenius)

From Translation Symmetry $|a(\boldsymbol{n})| = |a(T(\boldsymbol{n}))|$, $\Rightarrow$ $a(\boldsymbol{n}) = a(T(\boldsymbol{n}))$

$$\begin{aligned}
&|E_{\min,N}\rangle \\
=&a(\boldsymbol{n})\left((-1)^{\delta(\boldsymbol{n})}|\boldsymbol{n}\rangle + (-1)^{\delta(T(\boldsymbol{n}))}|T(\boldsymbol{n})\rangle + \cdots\right) \\
&+\cdots \\
=&a(\boldsymbol{n})(-1)^{\delta(\boldsymbol{n})}\left(|\boldsymbol{n}\rangle + (-1)^{N}|T(\boldsymbol{n})\rangle + \cdots\right) \\
&+\cdots
\end{aligned}$$

$\Rightarrow$

$$\hat{T}|E_{\min,N}\rangle = (-1)^N|E_{\min,N}\rangle$$

for $N \in [1, L/2-1]$

- **One-Soliton states (N=1) have minimum energy at k=π**
- **Two-Soliton states (N=2) have minimum energy at k=0,**

The above constitutes proof for the H_{DH} model for S=1/2.

Clues for understanding **higher S** cases: J/D ⇒0 limit

Soliton Number is not conserved in the *DH* model (H_{DH}) for higher S .

However:

- there exists a **model (H_p**:projected *DH* model**) , which approximates H_{DH}** well .

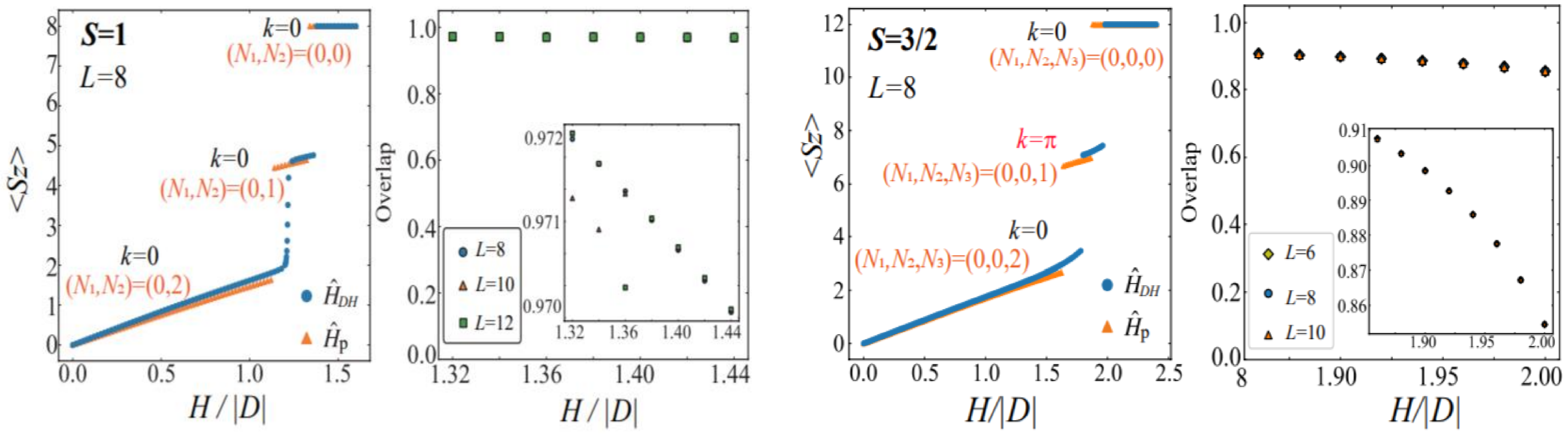
$$\hat{\mathcal{H}}_{\rm p} = \sum_{N_1,\cdots,N_{2S}} \hat{P}(\{N_f\})\hat{\mathcal{H}}_{DH}\hat{P}(\{N_f\})$$

$\hat{P}(\{N_f\})$ projection operator into the eigenspace with $N_1,\cdots,N_{2S}$

- **Soliton numbers** N_f **(f=1,2….2S) with various heights** f are conserved in the H_p model.

- There exists a **signed basis** in which **Off-diagonal Matrix Elements of the Hamiltonian (H_p) are non-positive.**

How well H_p model approximates H_{DH} model?



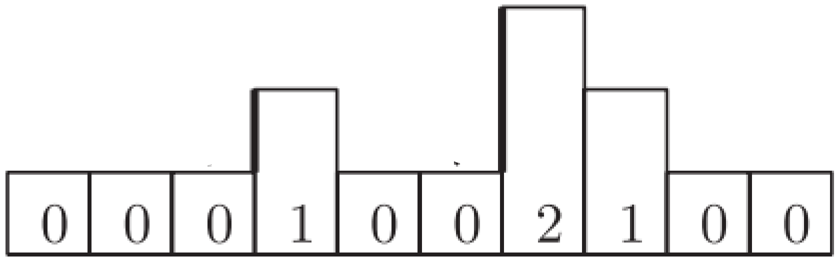
- ➢Approximate model H_p accounts for overall features of magnetization process in H_{DH}.
- ➢Only solitons with maximum height (amplitude) contribute to the ground state

Conserved Quantities

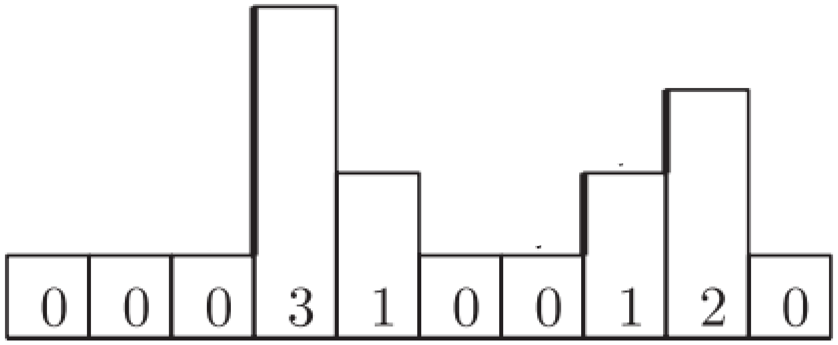
N_f number of soliton with height f = 1,2, ・・・2S

Ex. S=3/2,

$$|00100000\rangle, \quad (N_1, N_2, N_3) = (1, 0, 0)$$
$$|01211000\rangle, \quad (N_1, N_2, N_3) = (0, 1, 0)$$
$$|01123100\rangle, \quad (N_1, N_2, N_3) = (0, 0, 1)$$
$$|01310230\rangle, \quad (N_1, N_2, N_3) = (0, 0, 2)$$



(N_1, N_2, N_3)=(1,1,0)

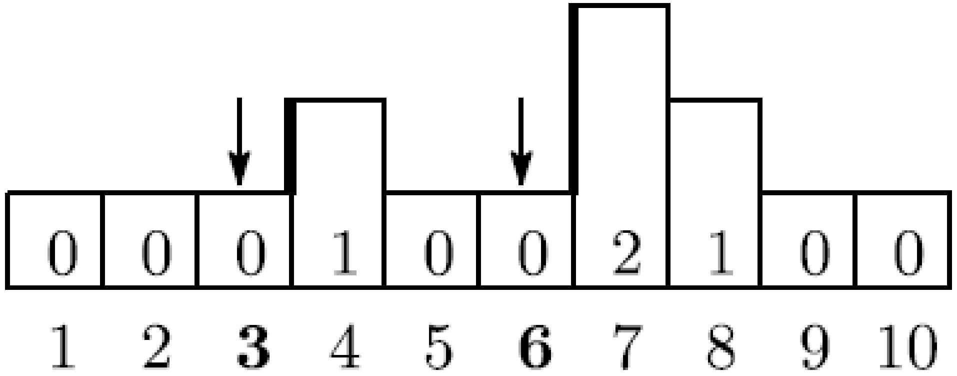


(N_1, N_2, N_3)=(0,1,1)

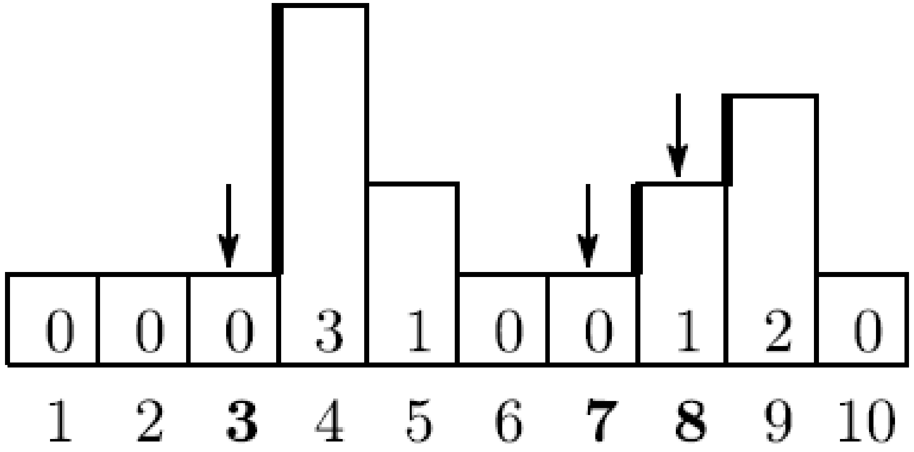
Signed Basis $(-1)^{\delta(\boldsymbol{n})}|\boldsymbol{n}\rangle$

$$\delta(\boldsymbol{n}) = \sum_{i=1}^{L} i\left(n_{i+1} - n_i + |n_{i+1} - n_i|\right)/2.$$

$(-1)^{\mathbf{3}+\mathbf{6}*2}$



$(-1)^{\mathbf{3}*3+\mathbf{7}*1+\mathbf{8}*1}$



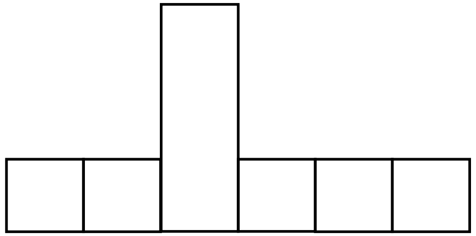
δ(**n**): depends on **height-difference** in **the left side of the maximum height of solitons**.

The relation among **sign** for each basis, **translation**, and **Soliton height and numbers**

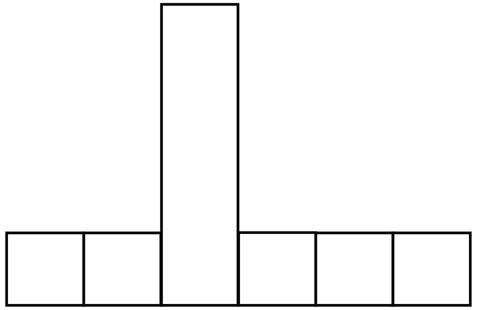
$$(-1)^{\delta(\boldsymbol{n})} = (-1)^{\delta(T(\boldsymbol{n}))}(-1)^{\sum_{f=1}^{2S} f N_f}$$

A soliton state with odd f (even f) changes (does not change) the sign under one-site translation.

$(-1)^{2*2}$

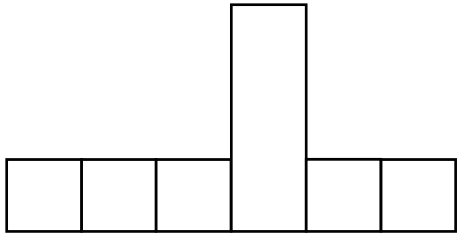


$(-1)^{2*3}$



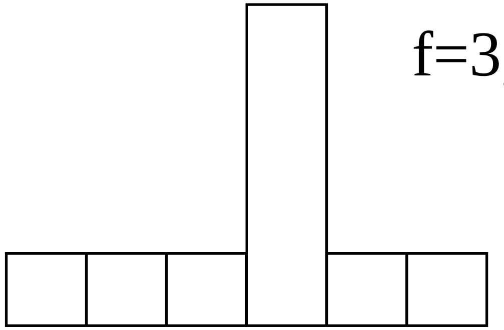
f=2, N_2=1

$(-1)^{3*2}$



f=3, N_3=1

$(-1)^{3*3}$



Off-diagonal Matrix Elements in the signed basis

$$(-1)^{\delta(\boldsymbol{n}')+\delta(\boldsymbol{n})}\langle \boldsymbol{n}'|\hat{\mathcal{H}}_{\rm p}|\boldsymbol{n}\rangle \leq 0,$$ non-positive off-diagonal matrix elements

⇒Lowest energy eigenspace for N_f solitons with height f is given by

$$|E_{\min,\{N_f\},\mu}\rangle = \sum_{\boldsymbol{n}\in V_\mu(\{N_f\})} a(\boldsymbol{n})(-1)^{\delta(\boldsymbol{n})}|\boldsymbol{n}\rangle$$ with $a(\boldsymbol{n}) > 0.$

From Translation Symmetry $|a(\boldsymbol{n})| = |a(T(\boldsymbol{n}))|,$ ⟹ $a(\boldsymbol{n}) = a(T(\boldsymbol{n}))$

$$|E_{\min,\{N_f\},\mu}\rangle = \sum_{\boldsymbol{n}\in V_\mu(\{N_f\})} a(\boldsymbol{n})(-1)^{\delta(\boldsymbol{n})}|\boldsymbol{n}\rangle$$

$$\Downarrow \quad a(\boldsymbol{n}) = a(T(\boldsymbol{n}))$$

$$\begin{aligned}
&|E_{\min,\{N_f\},\mu}\rangle \\
=&a(\boldsymbol{n})\left((-1)^{\delta(\boldsymbol{n})}|\boldsymbol{n}\rangle + (-1)^{\delta(T(\boldsymbol{n}))}|T(\boldsymbol{n})\rangle + \cdots\right) \\
&+\cdots \\
=&a(\boldsymbol{n})(-1)^{\delta(\boldsymbol{n})}\left(|\boldsymbol{n}\rangle + \underline{(-1)^{\sum_{f=1}^{2S} fN_f}}|T(\boldsymbol{n})\rangle + \cdots\right. \\
&+\cdots .
\end{aligned}$$

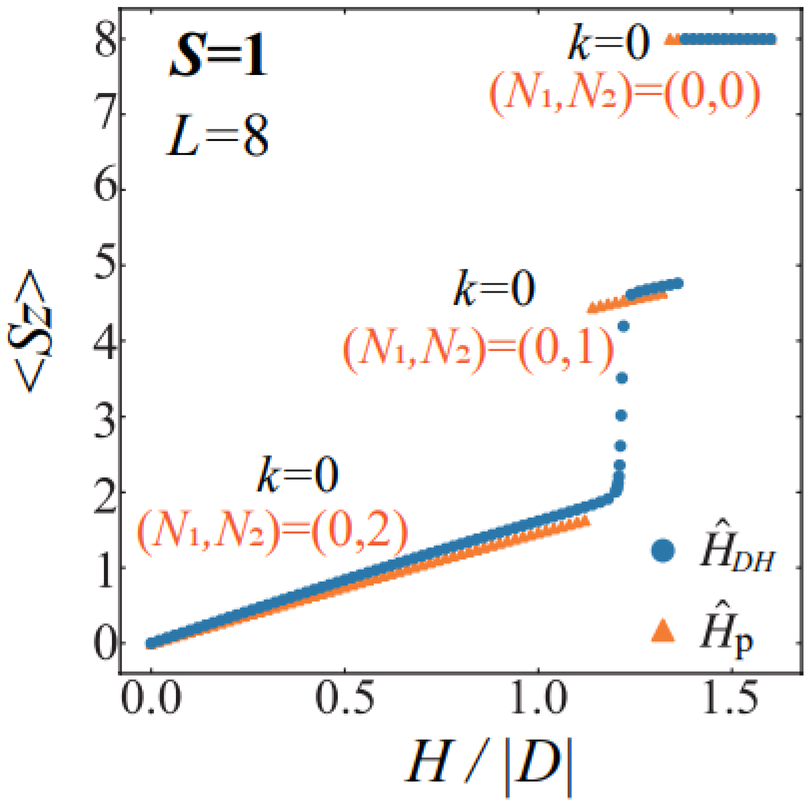
This implies that

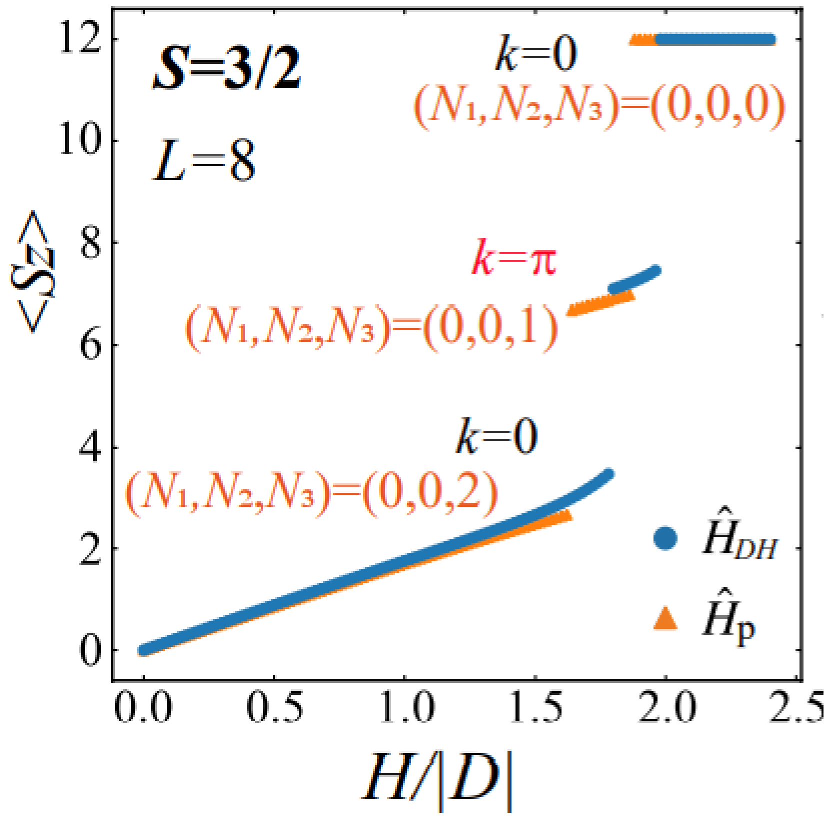
$$\hat{T}|E_{\min,\{N_f\},\mu}\rangle = \underline{(-1)^{\sum_{f=1}^{2S} fN_f}}|E_{\min,\{N_f\},\mu}\rangle.$$

Crystal momentum depends on height f (amplitude) of soliton as well as number N_f

"height parity effect"

Height Parity Effect ⇒Spin Parity effect



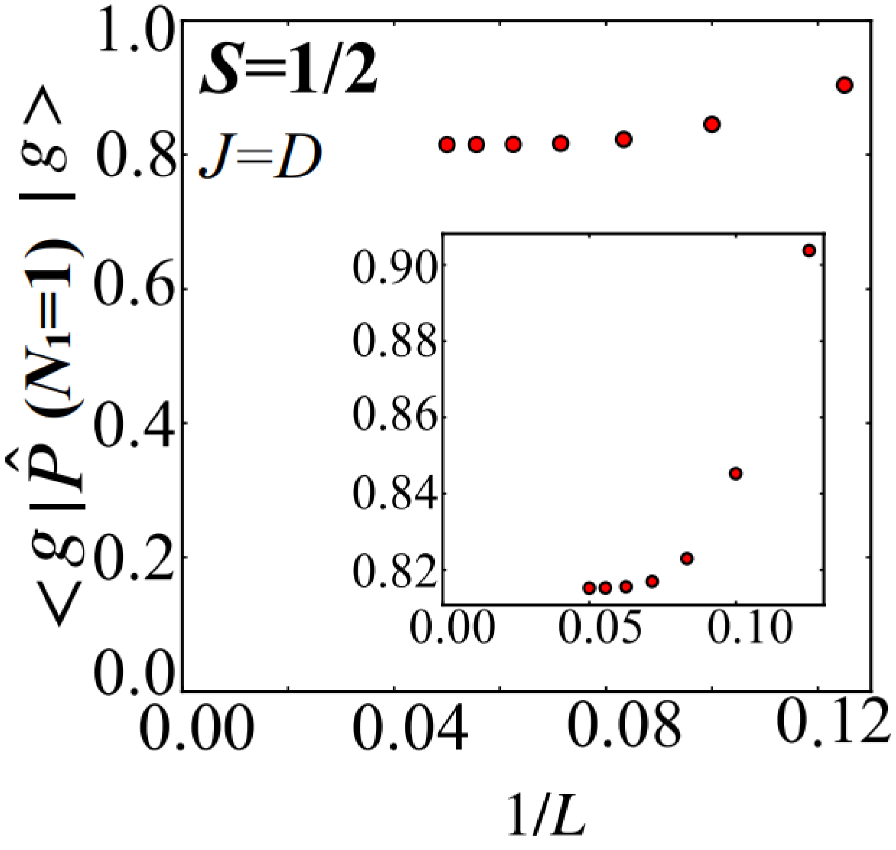


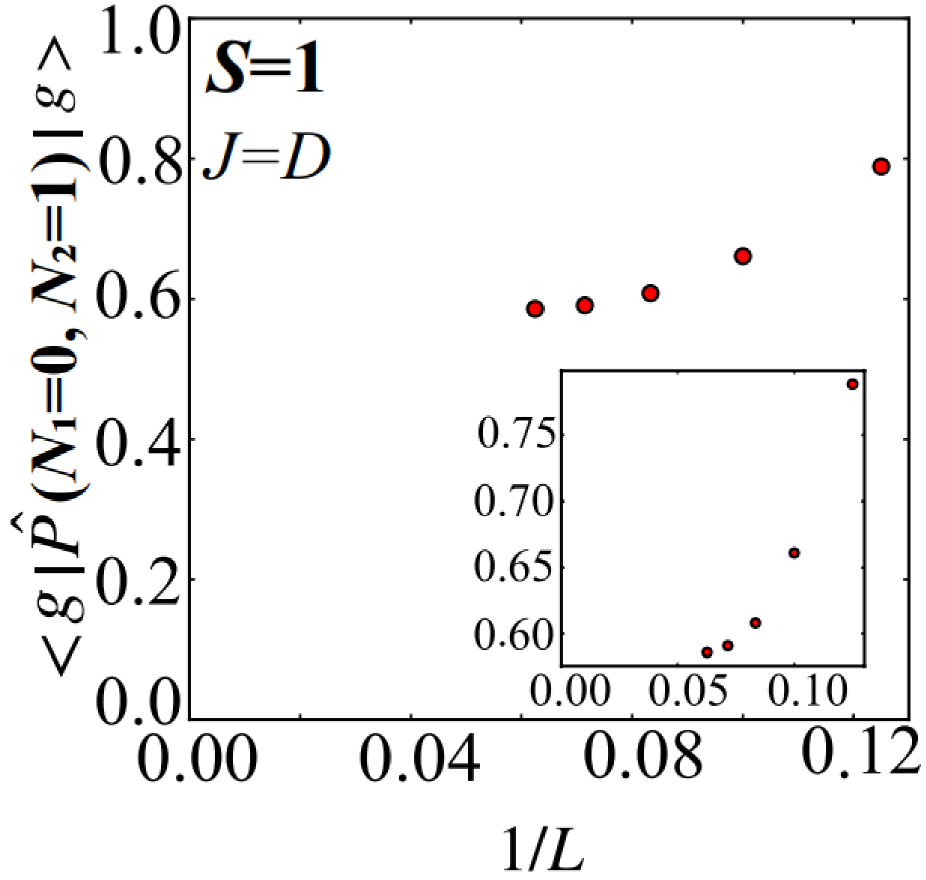
In the ground states, **only solitons with maximum height f=2S contribute**.

"Height parity effect" +Energetics leads to "Spin parity effect".

Effect of *J* (Exchange Interaction)

Size dependence of relative weight of One-soliton state in the ground state slightly below critical field *H*c





One soliton states account for 80% (58%) weight in the ground state for S=1/2 (S=1).

Summary:

- **Models in the limit D/J $\Rightarrow \infty$ are canonical models** to understand Spin parity effect in monoaxial chiral ferromagnetic chain.
- Essential is "height parity effect", a soliton with odd (even) height f has the $k=\pi$ ($k=0$) in the lowest energy state.
- In the low energy sector, only solitons with maximum height f=2S contribute. It results in the spin parity effect in the magnetization process.

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