

Symmetry & Strategy to extract Kitaev interaction in spin-S Kitaev Materials

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Collaborators

Theory

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Experiment

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Review article on “Beyond Kitaev physics”, N. Perkins, I. Rousochatzakis, Q. Luo, HYK, in preparation



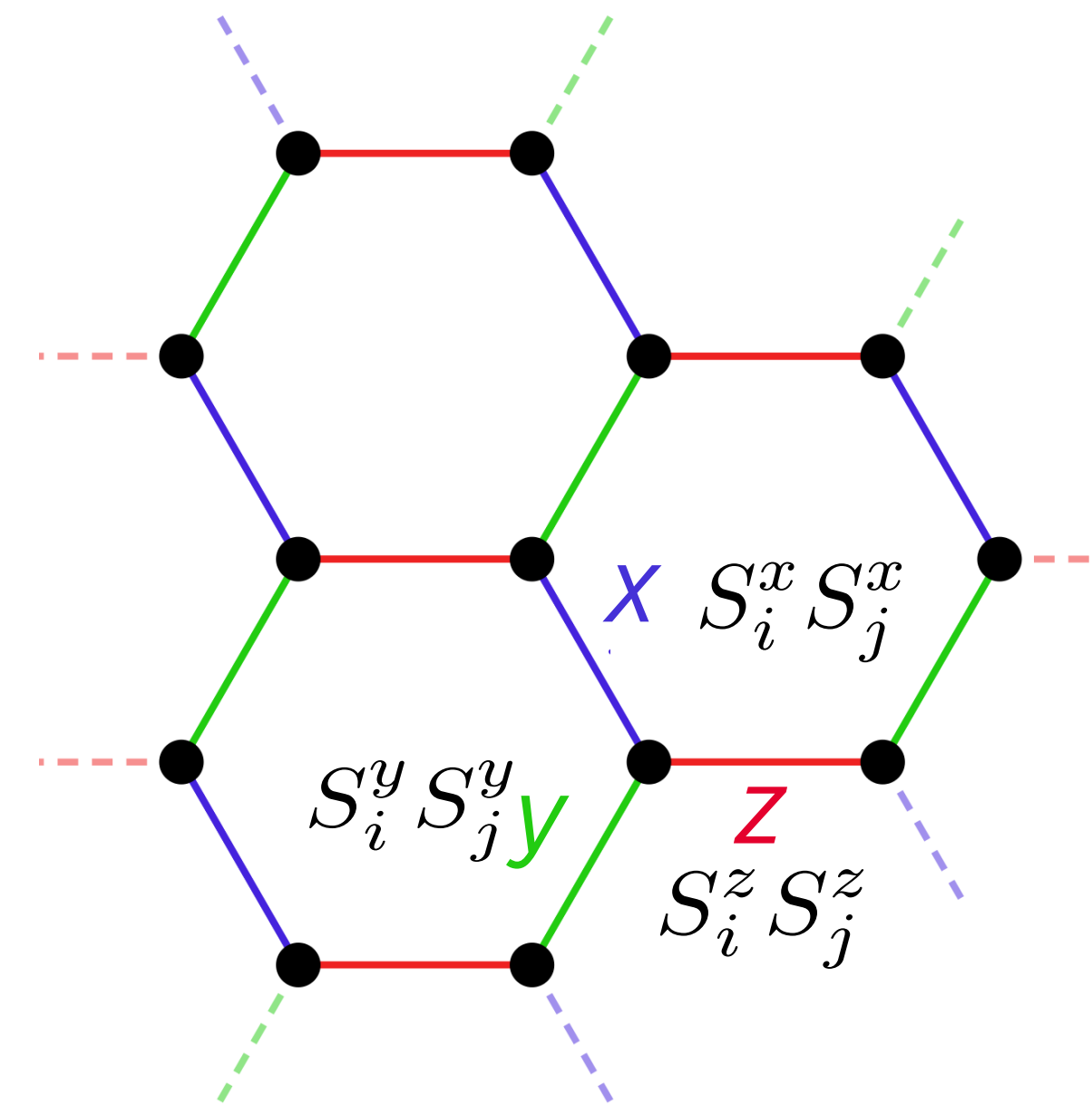
S= 1/2 Kitaev interaction & Kitaev spin liquid

Kitaev Exchange

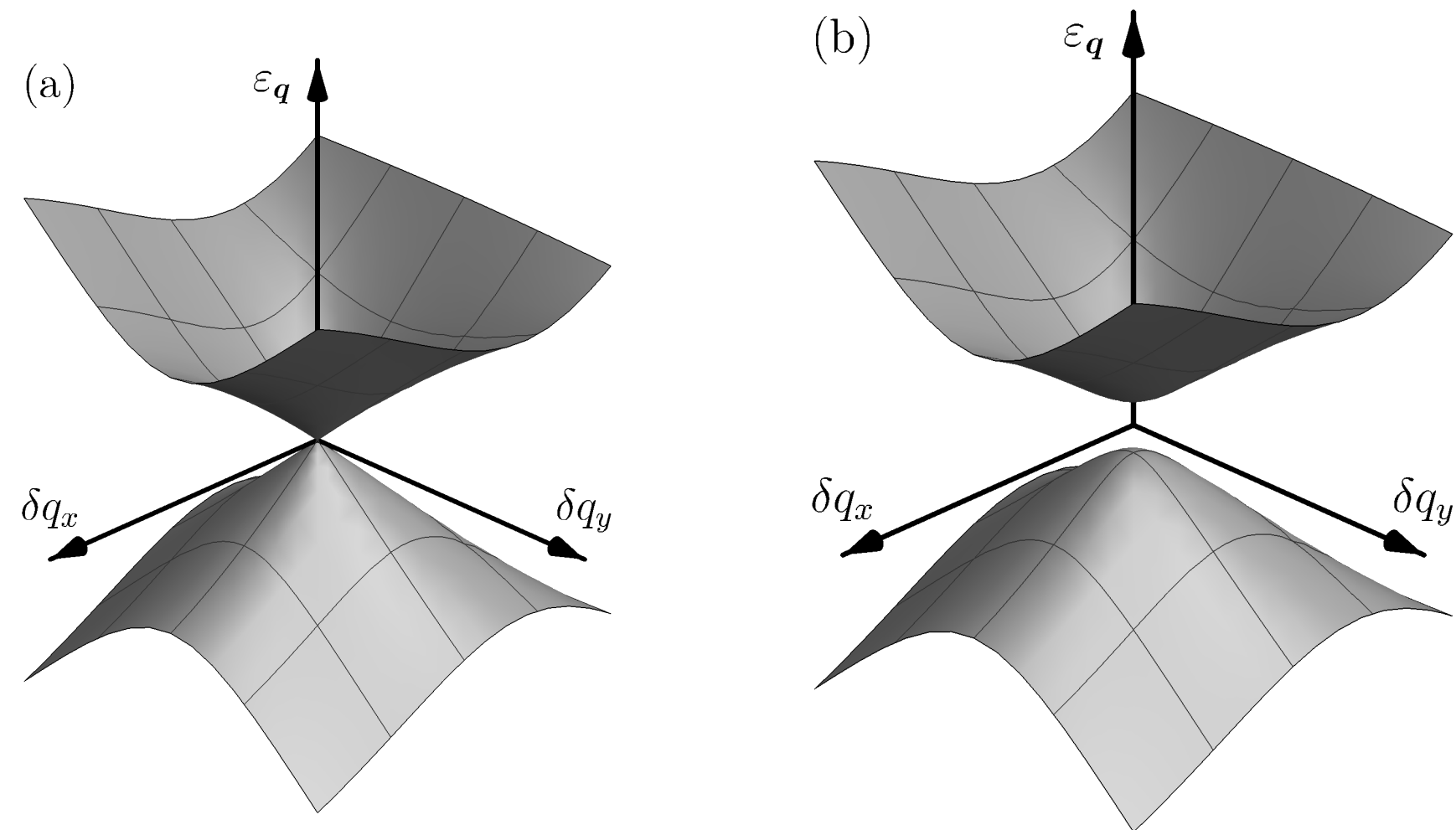
$$H = K \sum_{\langle ij \rangle \in \gamma} S_i^\gamma S_j^\gamma$$

where $\gamma = x, y, z$

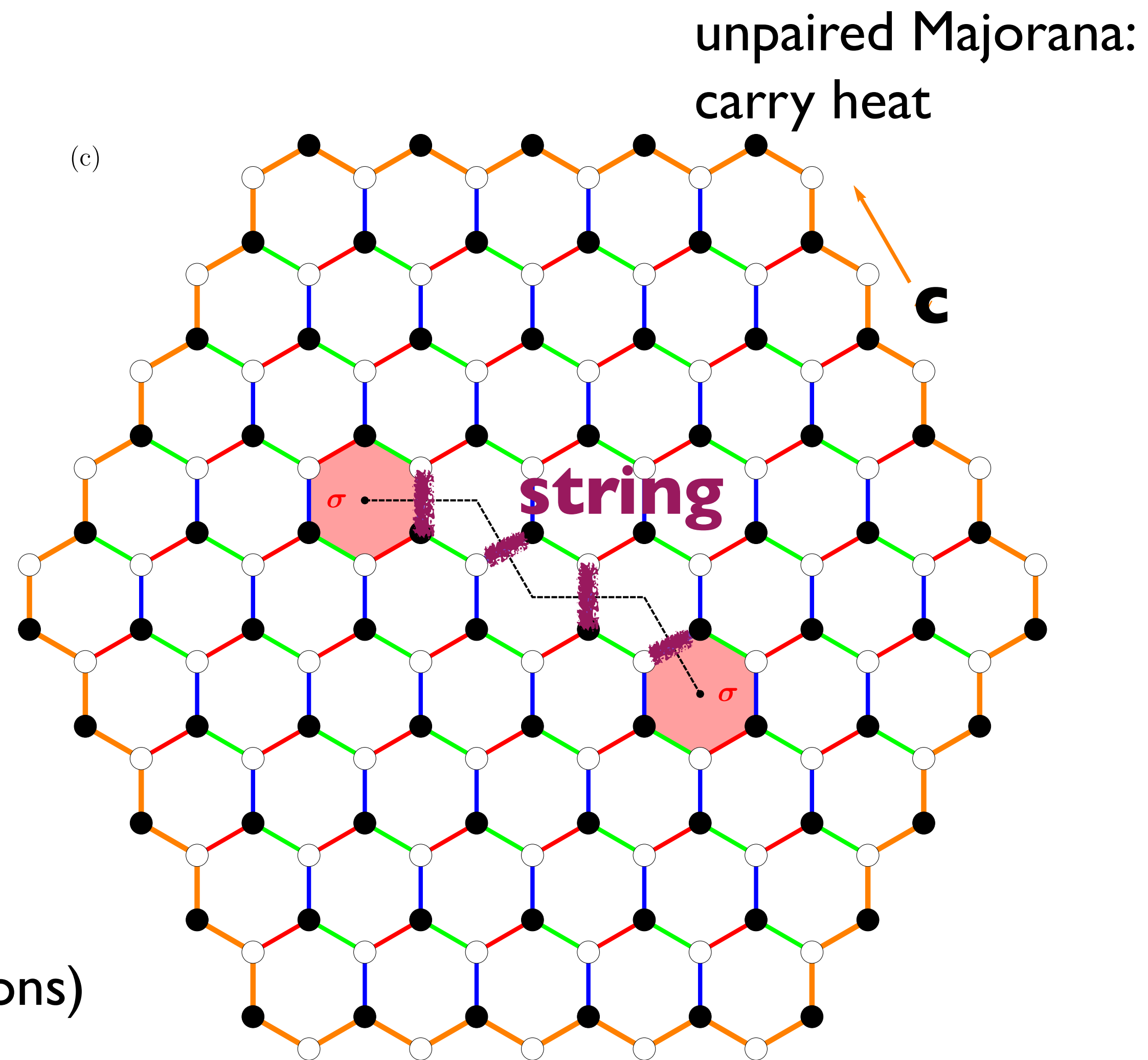
bond-dependent **Ising** interaction



Kitaev spin liquid: emergent particles - Majorana fermion and vortices



under magnetic field



Example: Ising Topological Order (\mathbb{Z}_2 vortex & Fermions)

$$\psi \times \psi = 1 \quad \psi \times \sigma = \sigma \quad \boxed{\sigma} \times \sigma = 1 + \psi$$

vortex carrying unpaired MF

Spin Interaction: low energy model of $t + U$

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \quad \text{SU(2): U and single-orbital}$$

When orbital & spin are not conserved via SOC

$$H_{ij} = \begin{pmatrix} S_i^x & S_i^y & S_i^z \end{pmatrix} \begin{pmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{yx} & J_{yy} & J_{yz} \\ J_{zx} & J_{zy} & J_{zz} \end{pmatrix} \begin{pmatrix} S_j^x \\ S_j^y \\ S_j^z \end{pmatrix}$$

: symmetry constrains matrix elements

insufficient to understand why one term is larger over other

Kitaev Materials

: Kitaev interaction is the largest interaction in full H

Necessary (not sufficient) Requirements

- honeycomb Mott insulator : strong e-e interaction
transition metals
- bond-dependent spin interaction **for $S=1/2$**
multi-orbital systems with Hund's coupling
spin-orbit coupling

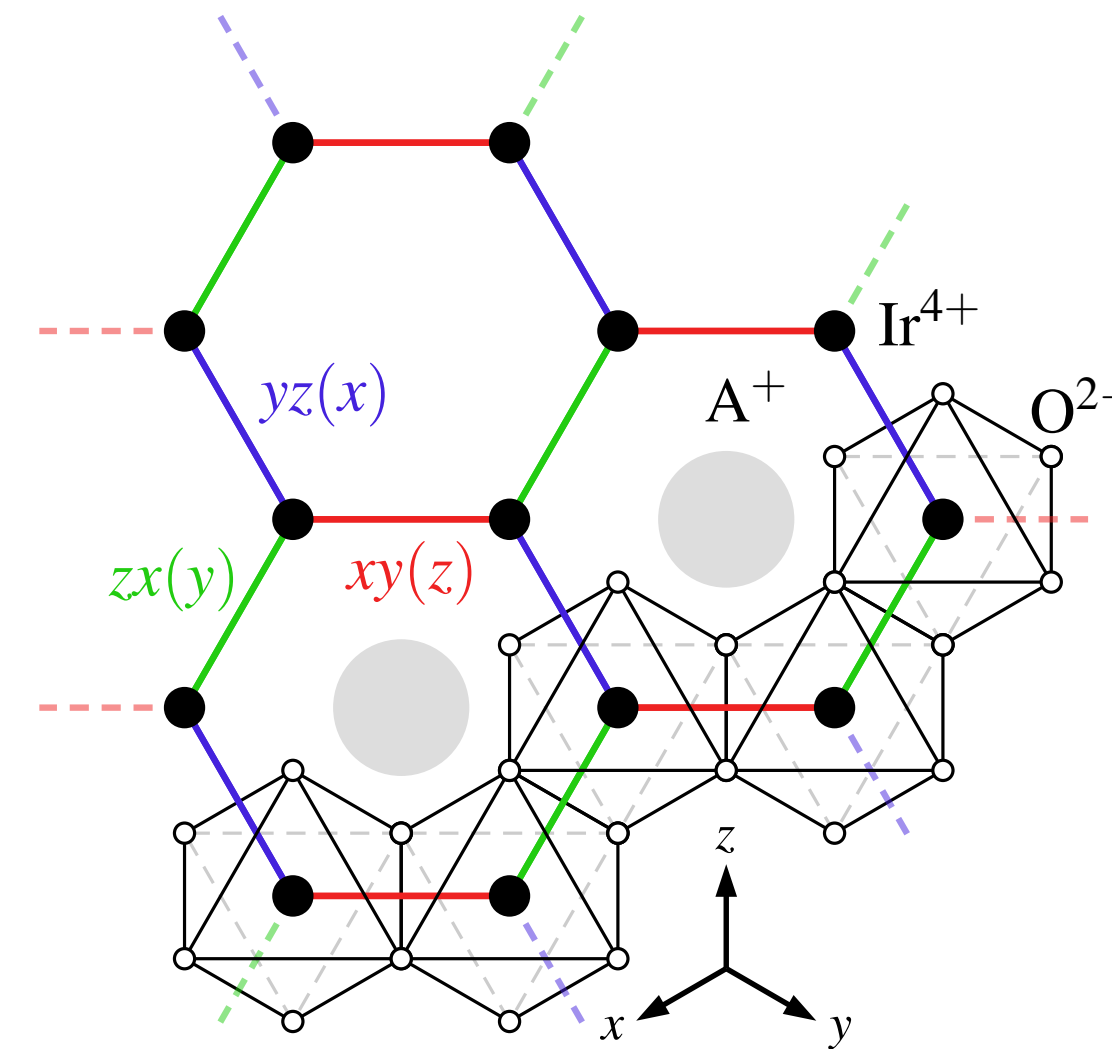
G. Khaliullin on triangular lattice (2005);

G. Jackeli, G. Khaliullin, Phys. Rev. Lett. 102, 017205 (2009)

Generic Spin Model in 2D honeycomb

nearest neighbour:
ideal honeycomb

$$H = \sum_{\gamma \in x, y, z} H^\gamma,$$



another bond-dep. interaction

$$H^z = \sum_{\langle ij \rangle \in z\text{-bond}} [K_z S_i^z S_j^z + \Gamma_z (S_i^x S_j^y + S_i^y S_j^x)] + J \mathbf{S}_i \cdot \mathbf{S}_j$$

+ other interactions allowed by the symmetry

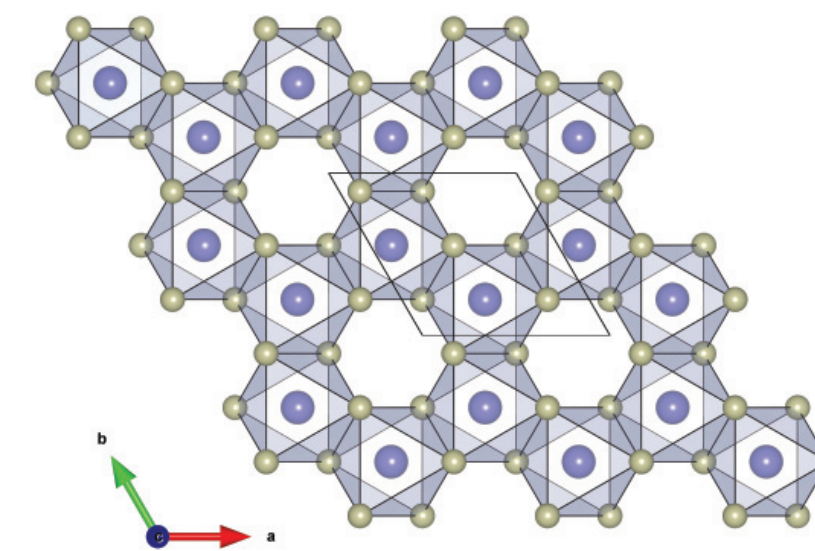
$$H^x = H^z (x \rightarrow y \rightarrow z \rightarrow x)$$

J. Rau, E. Lee, HYK, PRL 112, 077204 (2014)

other ref. - V. Katukuri, et al, New J. Phys. 16, 013056 (2014); Y. Yamaji, et al, PRL 113, 107201 (2014)

Kitaev interaction in Candidates with $J_{\text{eff}}=1/2$ (doublet)

Ti	V	Cr	Mn	Fe	Co	Ni
Zr	Nb	Mo	Tc	Ru	Rh	Pd
Hf	Ta	W	Re	Os	Ir	Pt

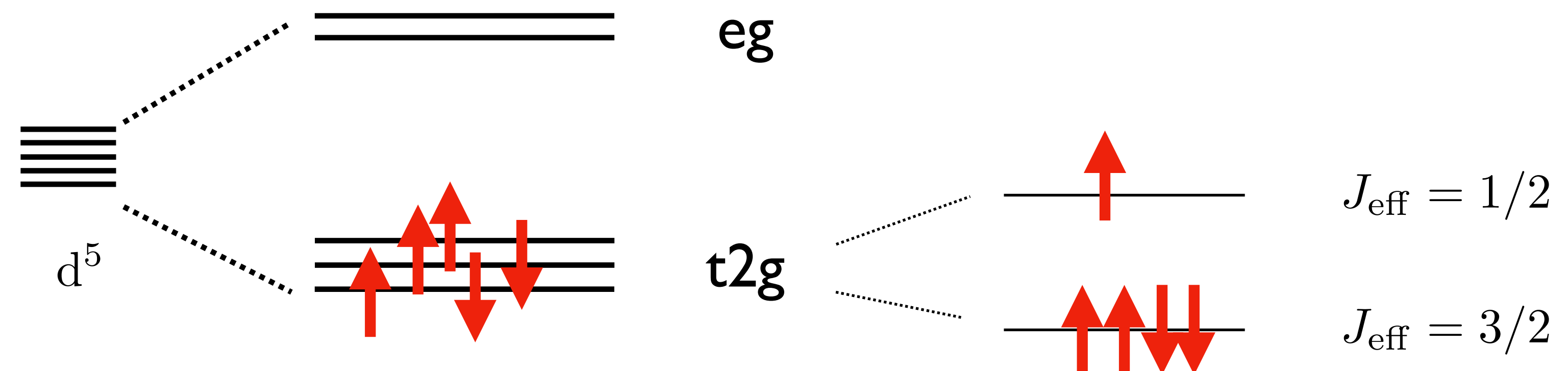


alpha-RuCl3

$A_2\text{IrO}_3$ ($A = \text{Na, Li}$): Y. Singh, et al, PRB 82, 064412 (2010);...

alpha-RuCl3 : K. Plumb, et al, PRB 90 041112(R) (2014); ...

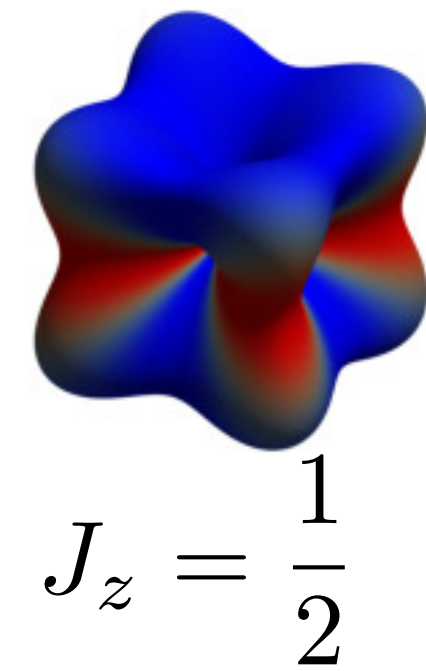
$A\text{IrO}_3$ ($A = \text{Mg, Zn}$): Y. Haraguchi, et al, PRM 2, 054411 (2018),....



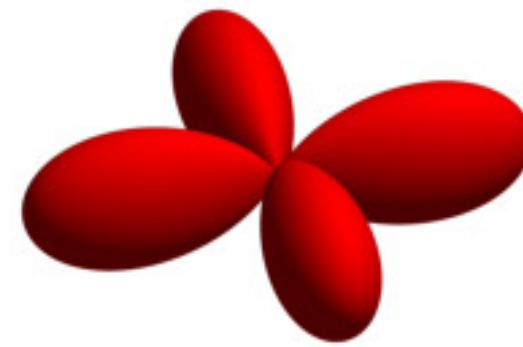
**Due to strong spin-orbit coupling:
mixture of different orbitals and different spins**

$J_{eff} = 1/2$ basis

spin down  spin up

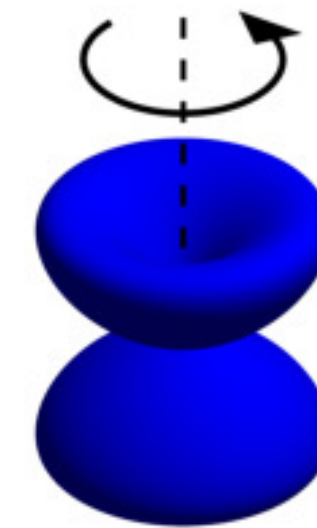


$$= +\frac{1}{\sqrt{3}}$$

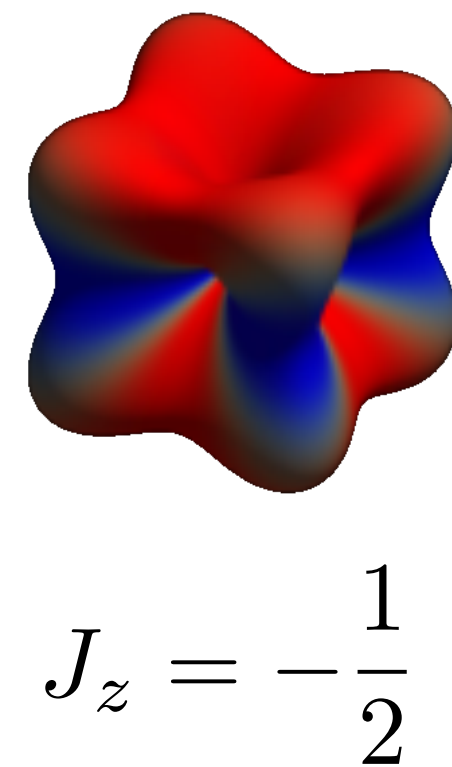


dxy

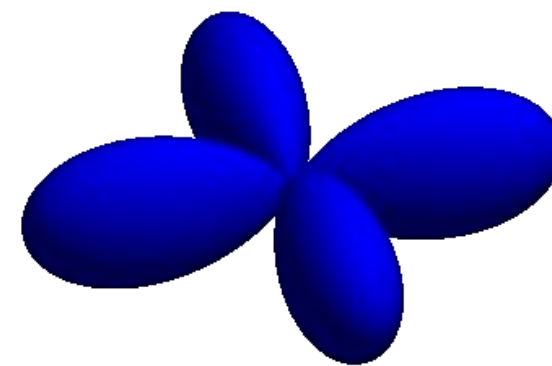
$$-\sqrt{\frac{2}{3}}$$



dxz + i dyz

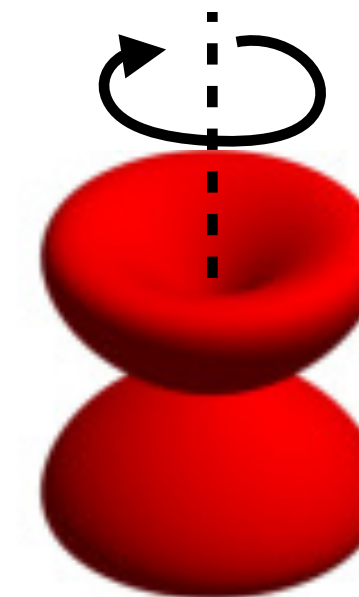


$$= \frac{1}{\sqrt{3}}$$



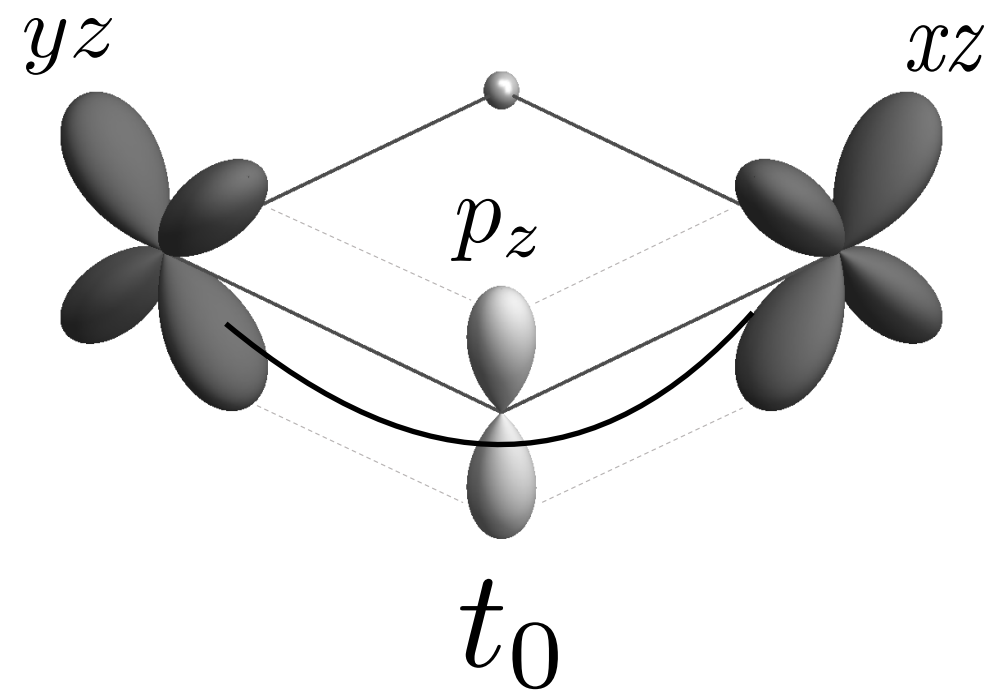
$L_z = 0$

$$-\sqrt{\frac{2}{3}}$$



$L_z = -1$

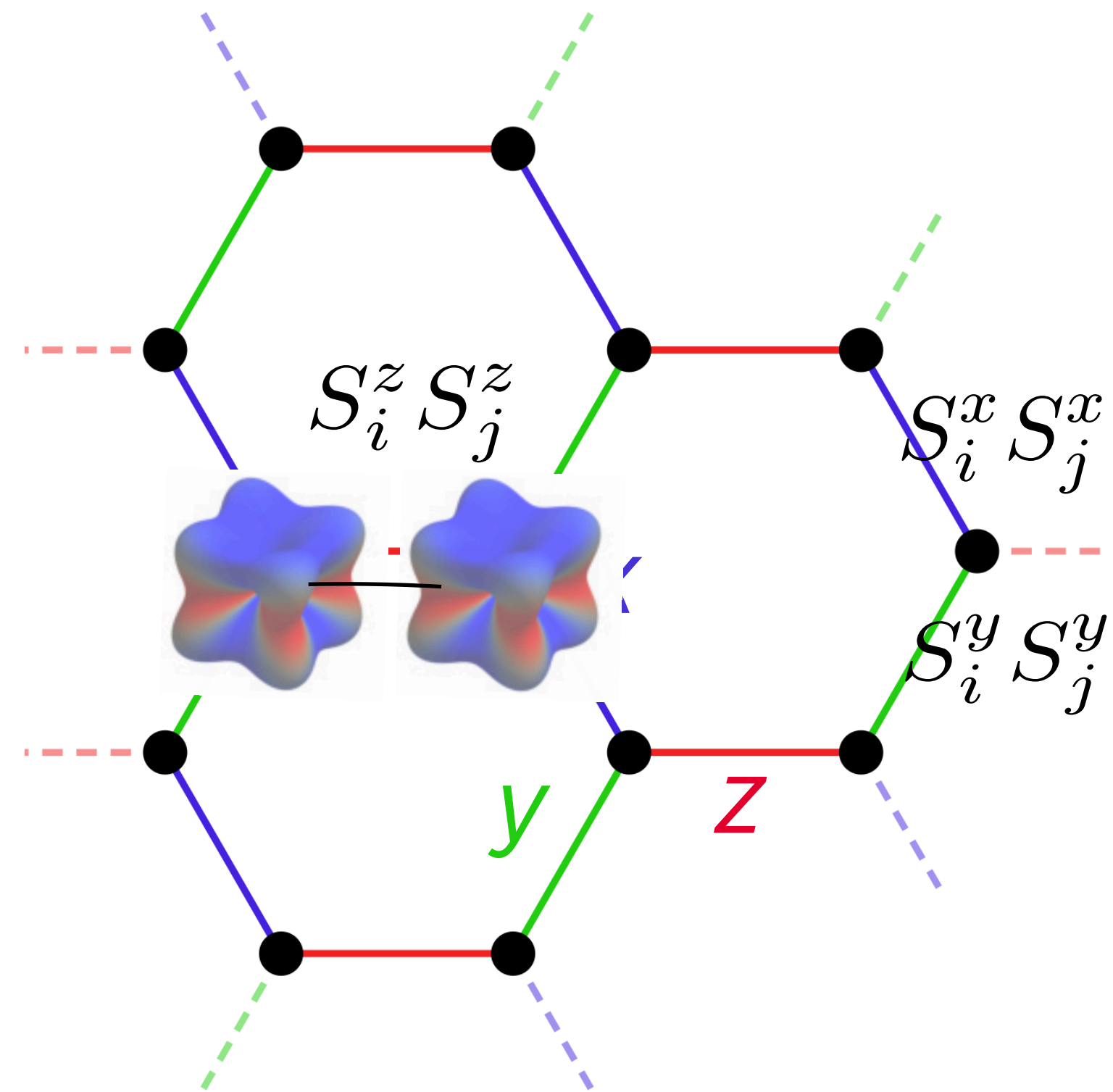
$$\frac{1}{\sqrt{3}} (|xy\rangle|\uparrow\rangle + |yz\rangle|\downarrow\rangle + i|xz\rangle|\downarrow\rangle)$$



$$\delta L^z = \pm 2 \quad \delta J_{\text{eff}}^z = \pm 2$$

orbital non-conserving hopping

~~Heisenberg term~~



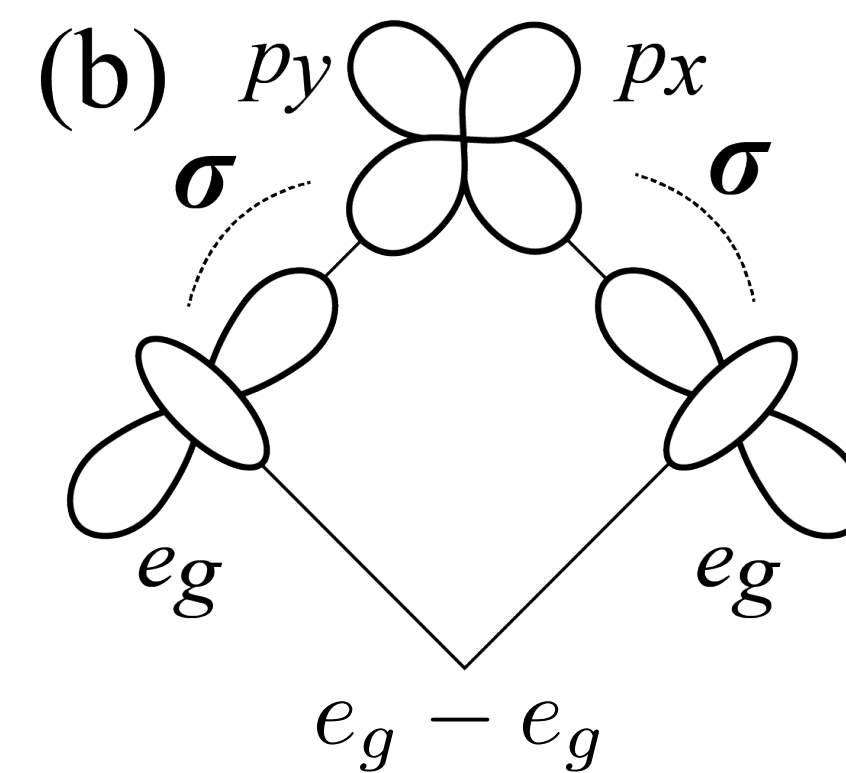
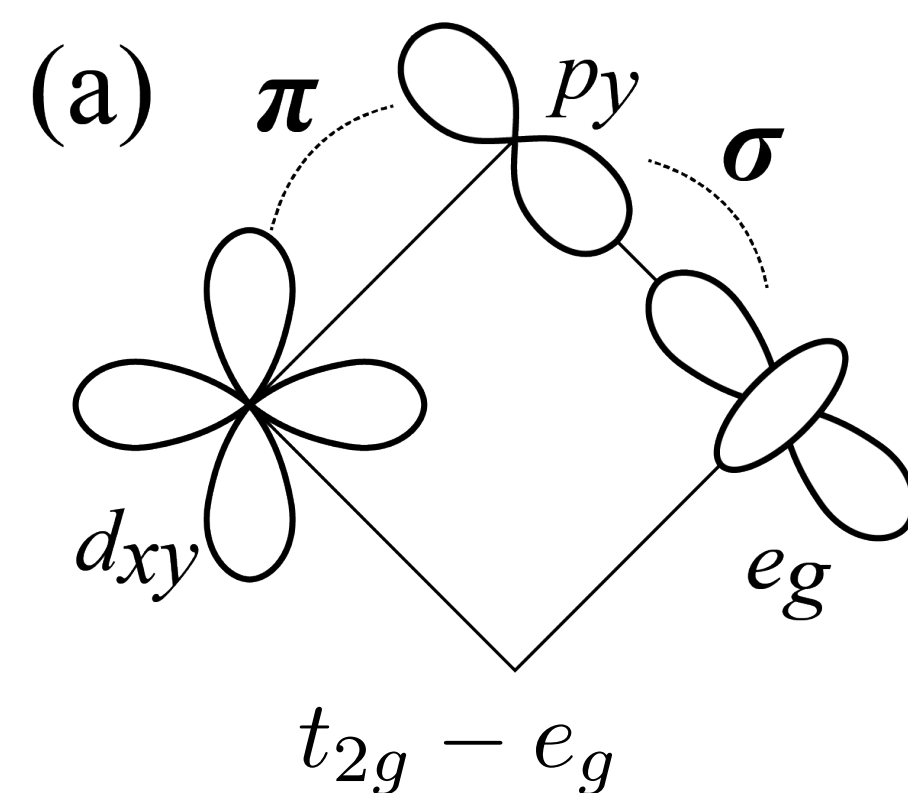
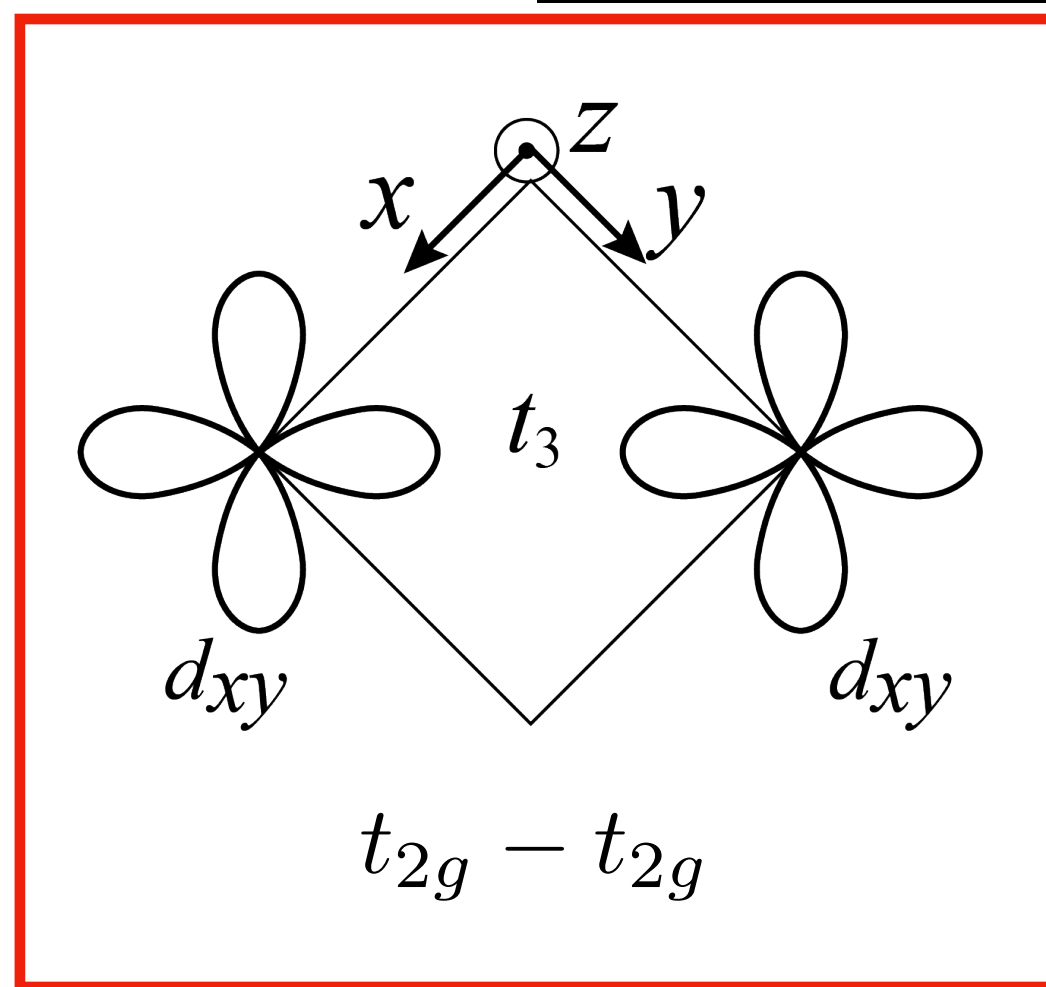
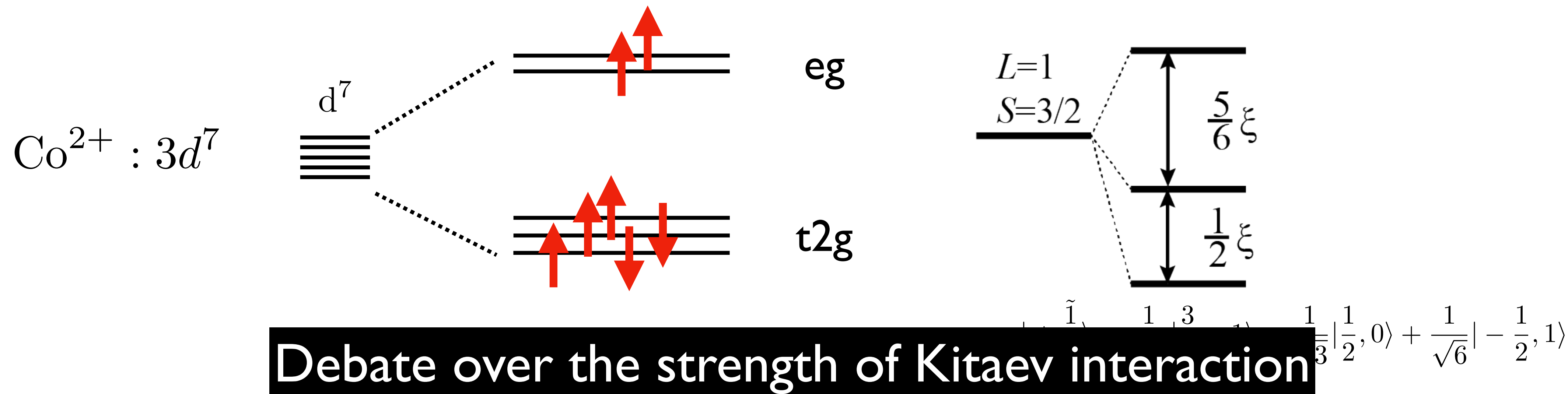
$$H_K = K S_i^z S_j^z$$

G. Jackeli, G. Khaliullin, Phys. Rev. Lett. 102, 017205 (2009)

orbital conserving hopping: Heisenberg + ...

3d7: $J_{eff} = 1/2$ honeycomb Cobaltates:

R. Sano et al, PRB 97, 014408 (2018), H. Liu, G. Khaliulin, PRB 97, 014407 (2018),

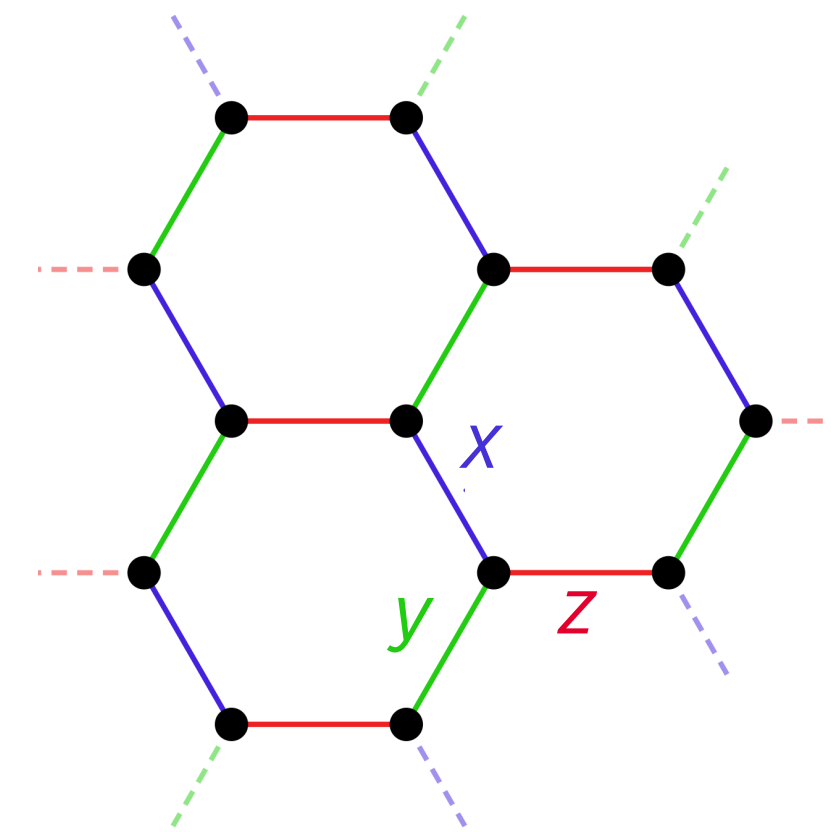


When dominated: Non-Kitaev, e.g., $BaCo_2(XO_4)_2$, $X=As, P$: X. Liu, HYK, arXiv:2211.03737

Spin-S Kitaev

Kitaev Exchange

$$K \sum_{\langle ij \rangle \in \gamma} S_i^\gamma S_j^\gamma \quad \text{where } \gamma = x, y, z$$



For arbitrary S, $W_p = e^{i\pi(S_1^y + S_2^z + S_3^x + S_4^y + S_5^z + S_6^x)}$

ultra-short range correlations

G. Baskaran, D. Sen, R. Shankar, PRB 78, 115116 (2008)

Quantum spin liquid?

Majorana excitations?

half-integer vs. integer-S Kitaev?

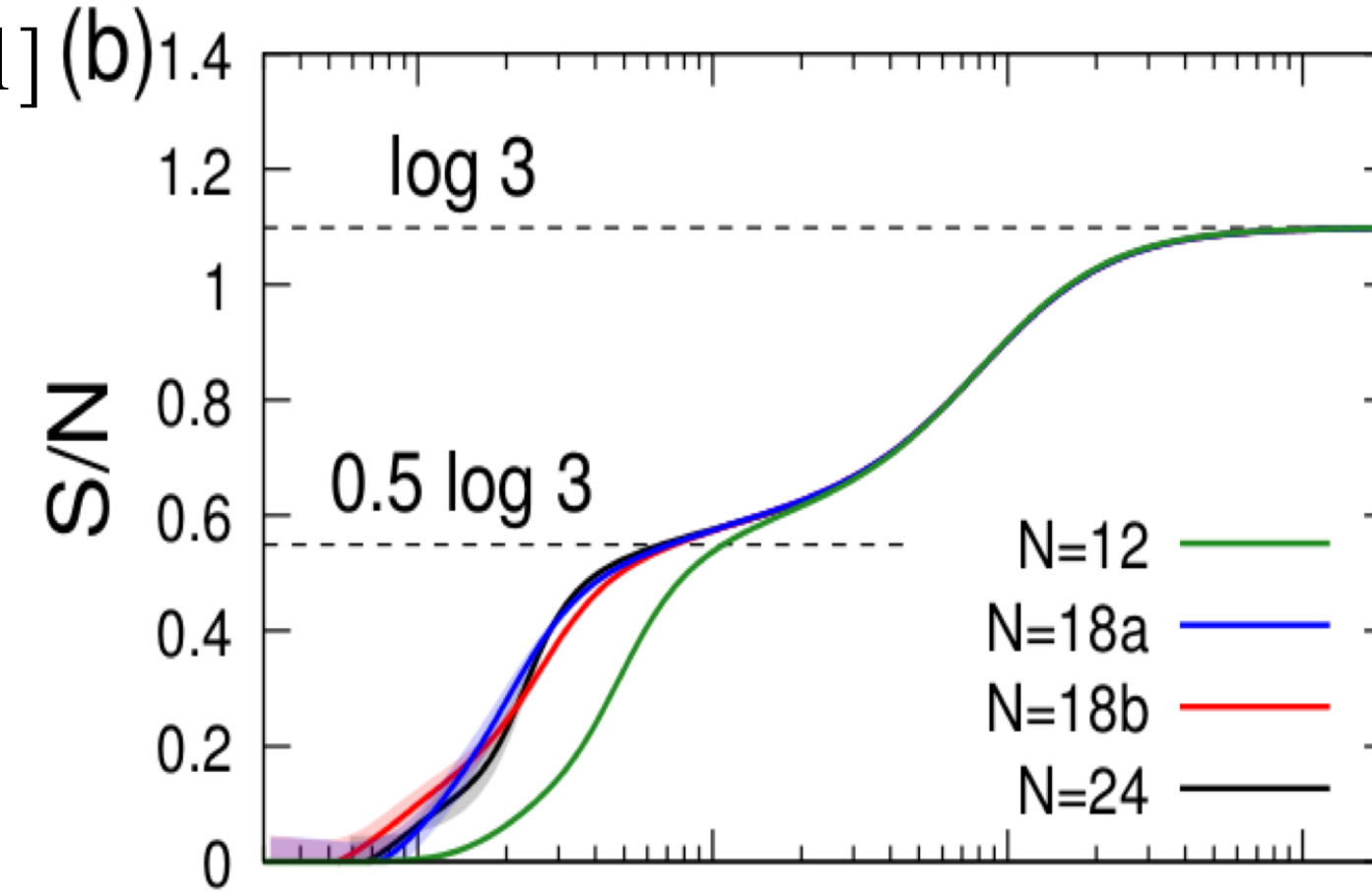
Spin $S=1$ Kitaev model in the literature.

$S=1$ Kitaev model:

Plaquette operators that commute with the Hamiltonian [1]

May be a gapless spin liquid [2]

Has incipient entropy plateau [2,3]



Q: How do we get $S=1$ Kitaev interaction?

10

[1] G. Baskaran, D. Sen, and R. Shankar, *Phys. Rev. B* **78**, 115116 (2008).

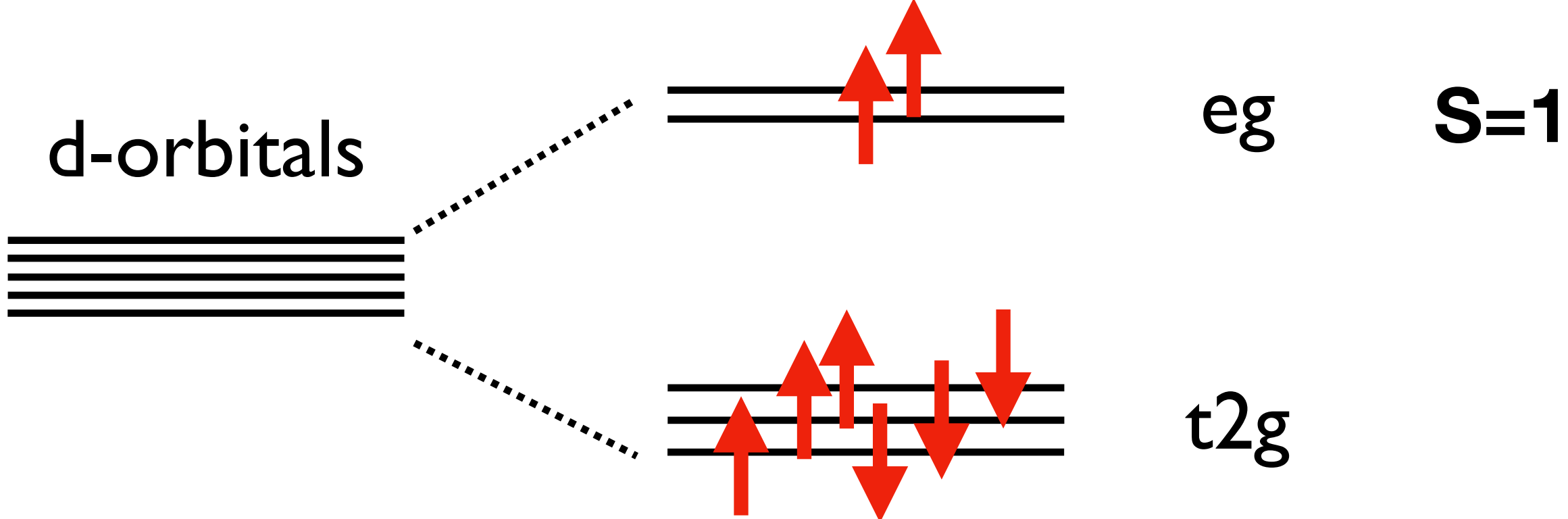
[2] A. Koga, H. Tomishige, and J. Nasu, *Journal of the Physical Society of Japan* **87**, 063703 (2018).

[3] J. Oitmaa, A. Koga, and R. R. P. Singh, *Phys. Rev. B* **98**, 214404 (2018).

Kitaev interaction in honeycomb insulators with general S

Ti	V	Cr	Mn	Fe	Co	Ni
Zr	Nb	Mo	Tc	Ru	Rh	Pd
Hf	Ta	W	Re	Os	Ir	Pt

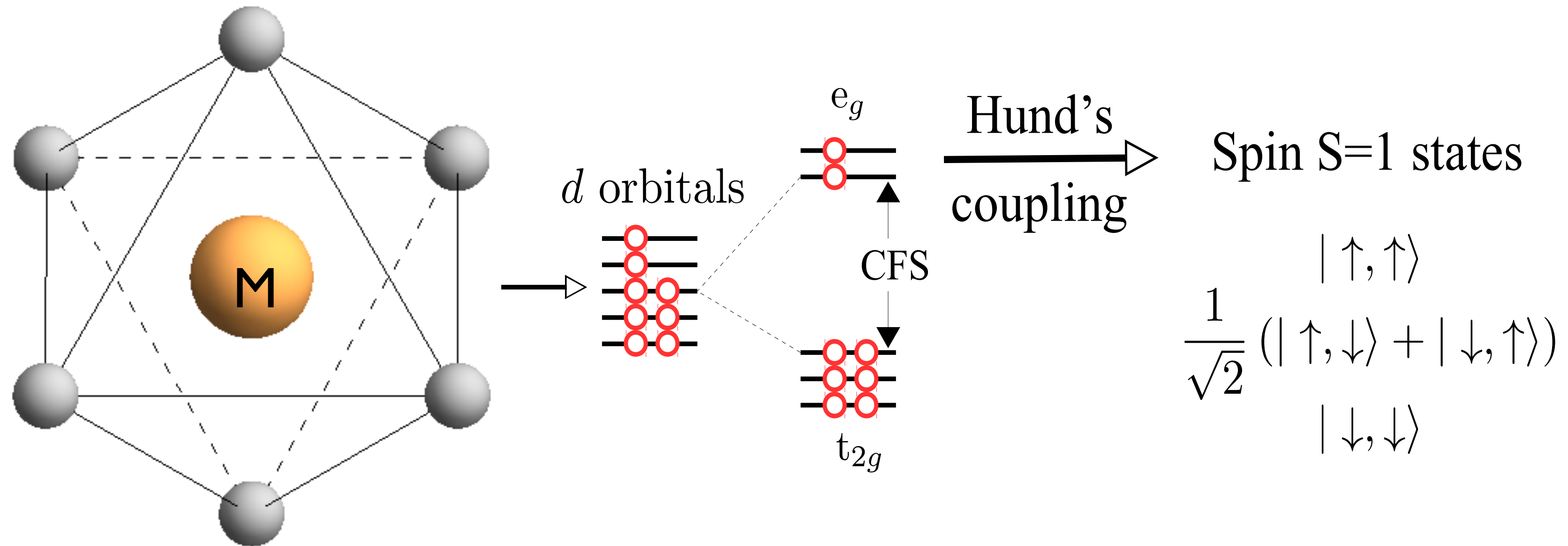
$\text{Ni}^{2+} : 3d^8$



Higher-spin Kitaev interaction

P. Peter Stavropoulos, D. Pereira, HYK PRL 123, 037203 (2019)

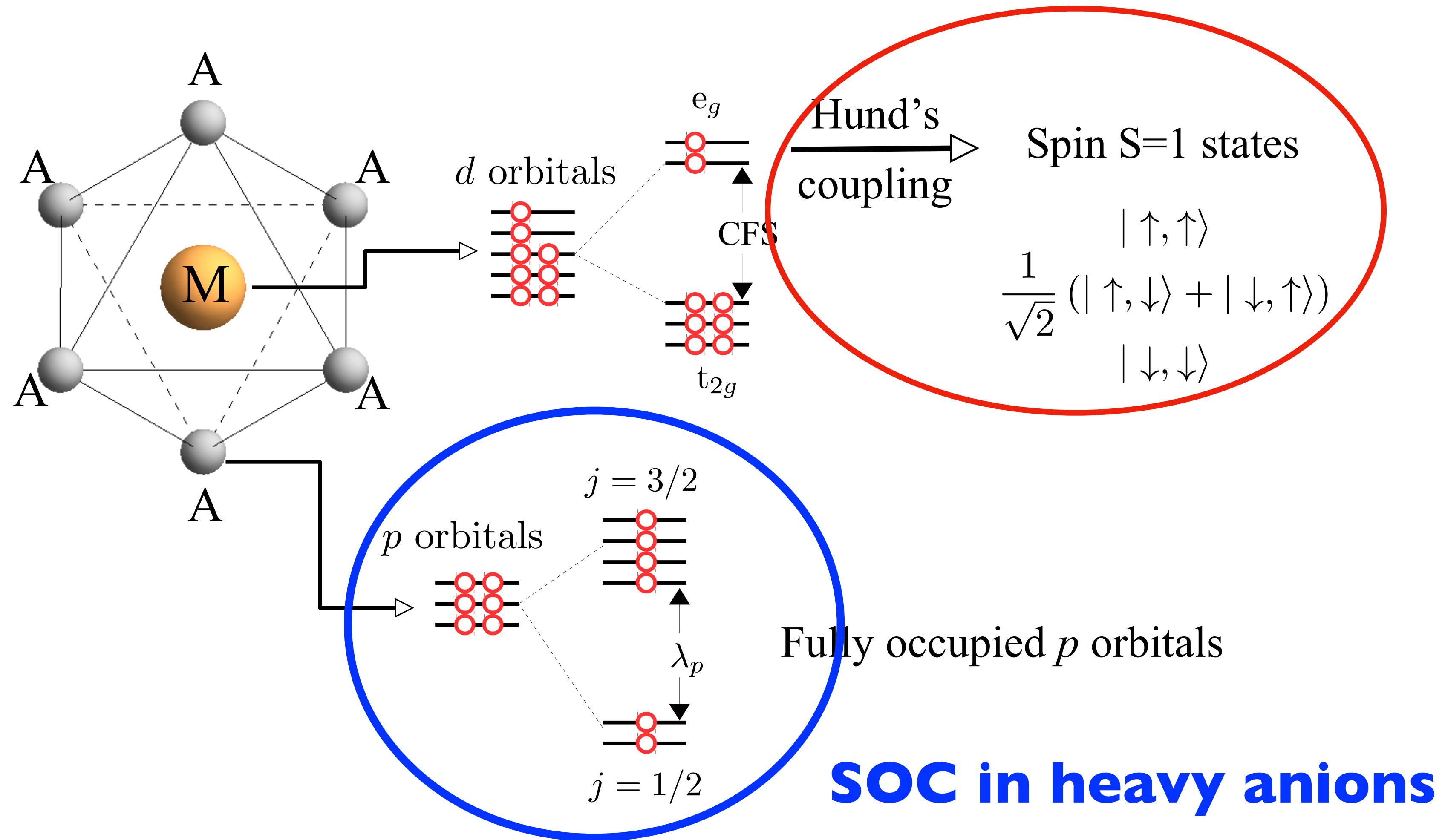
ex: spin one (d8)



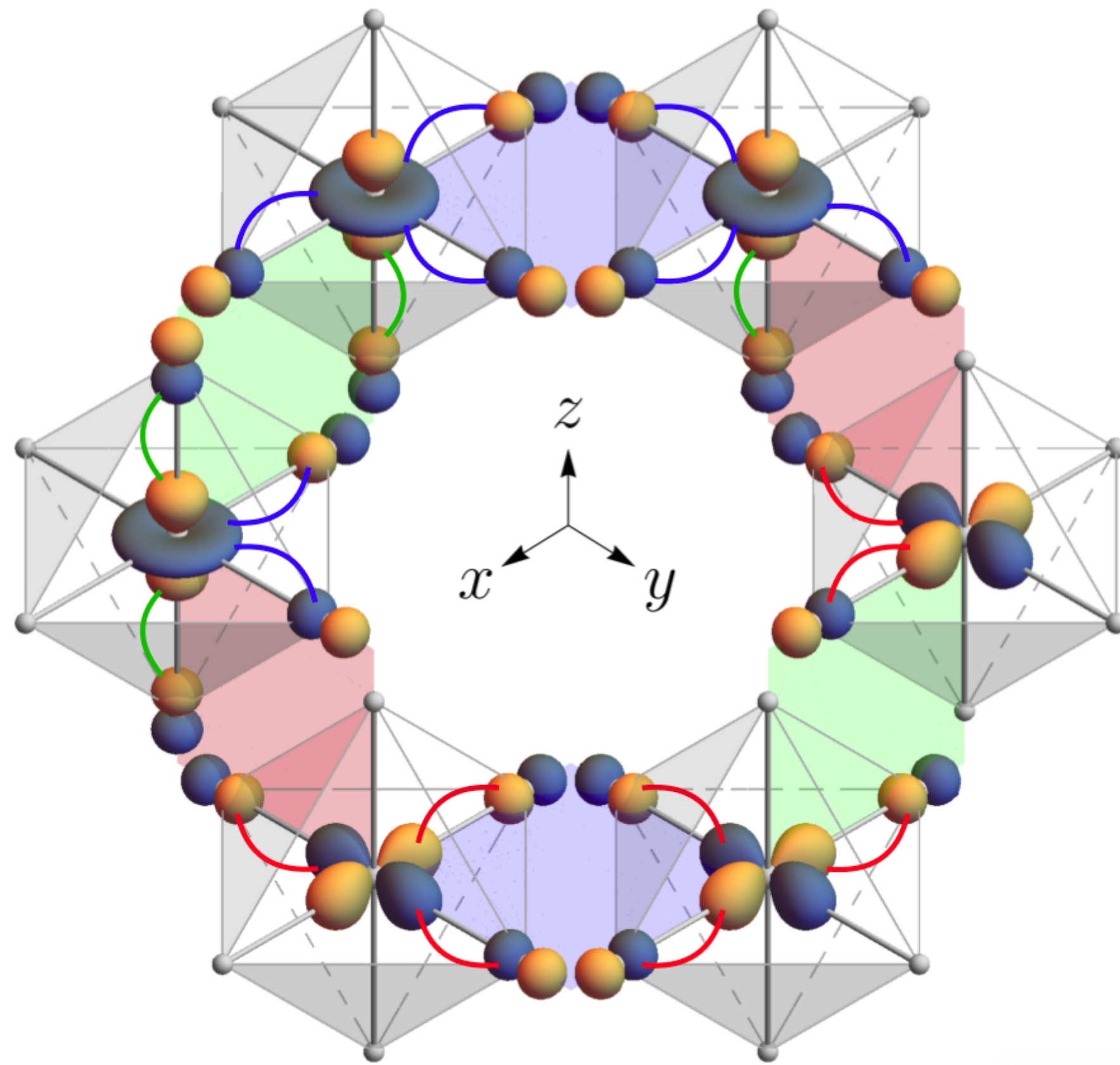
Crystal field splitting > Hund's coupling >> spin-orbit coupling (SOC)

No mixture of different orbitals & spins - No Kitaev?

on-site $H_0 = \text{Kanamori (U, U', Hund's)} + \text{SOC}$



perturbation theory: hopping between two M sites via heavy A sites



$$t_1 \text{ --- red line ---}$$

$$t_2 \text{ --- blue line ---}$$

$$t_3 \text{ --- green line ---}$$

Hopping integrals
M to A sites:

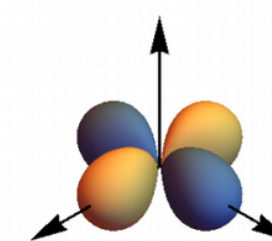
Cubic symmetry:

$$t_1 = \frac{\sqrt{3}}{2} t_{pd\sigma},$$

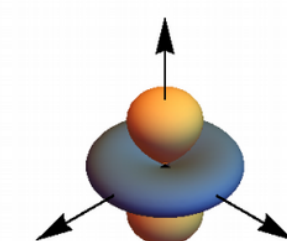
$$t_2 = \frac{1}{2} t_{pd\sigma},$$

$$t_3 = t_{pd\sigma}$$

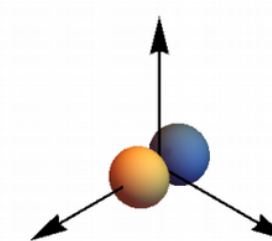
$$d_{x^2-y^2}$$



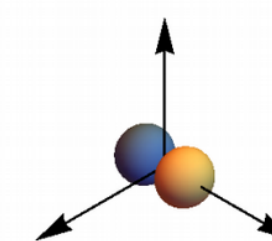
$$d_{3r^2-z^2}$$



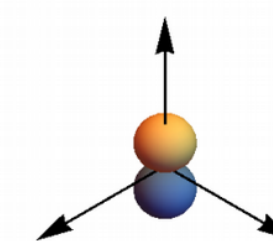
$$p_x$$



$$p_y$$



$$p_z$$



Indirect

$$K = -2J_{ind}$$

Direct

$$J_d = 4t^2/U$$

$$\Gamma = 0$$

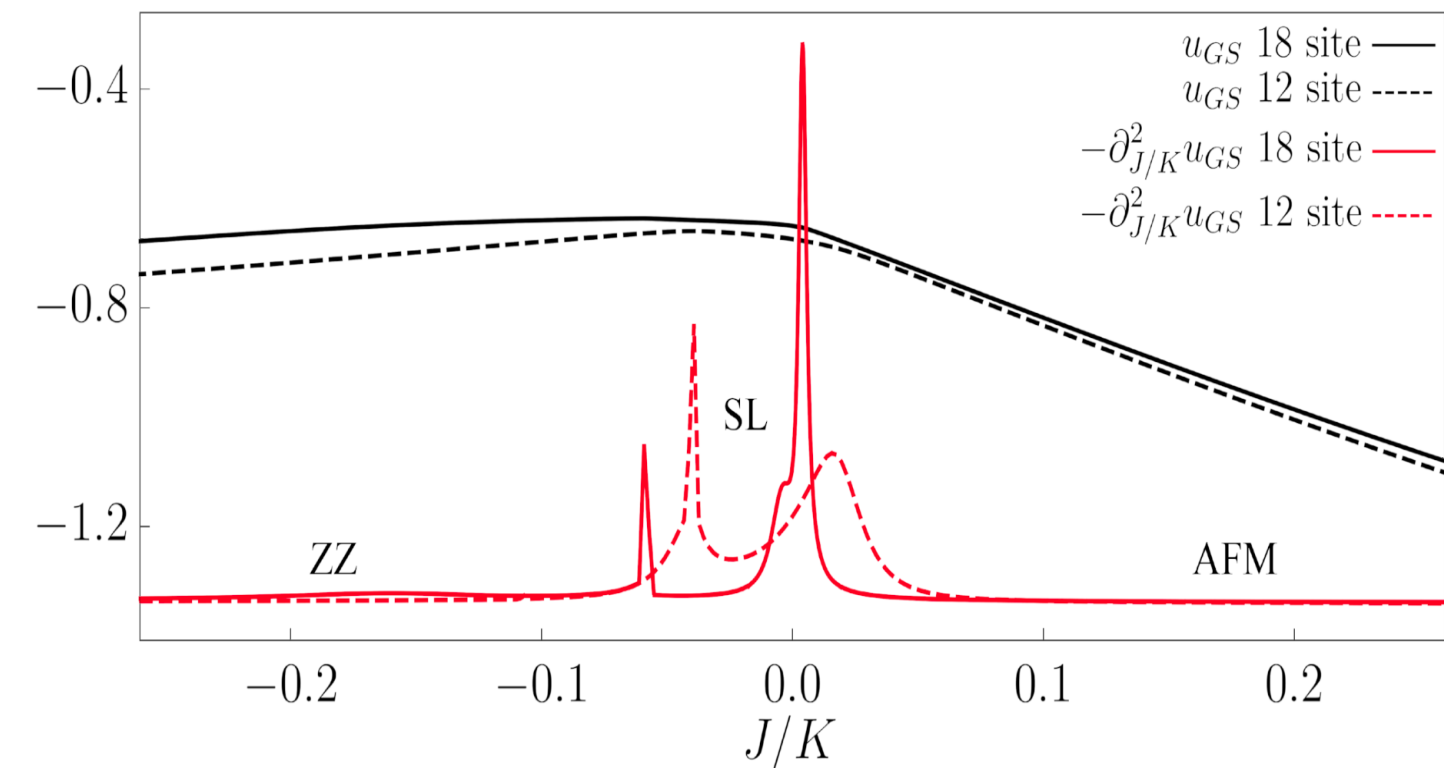
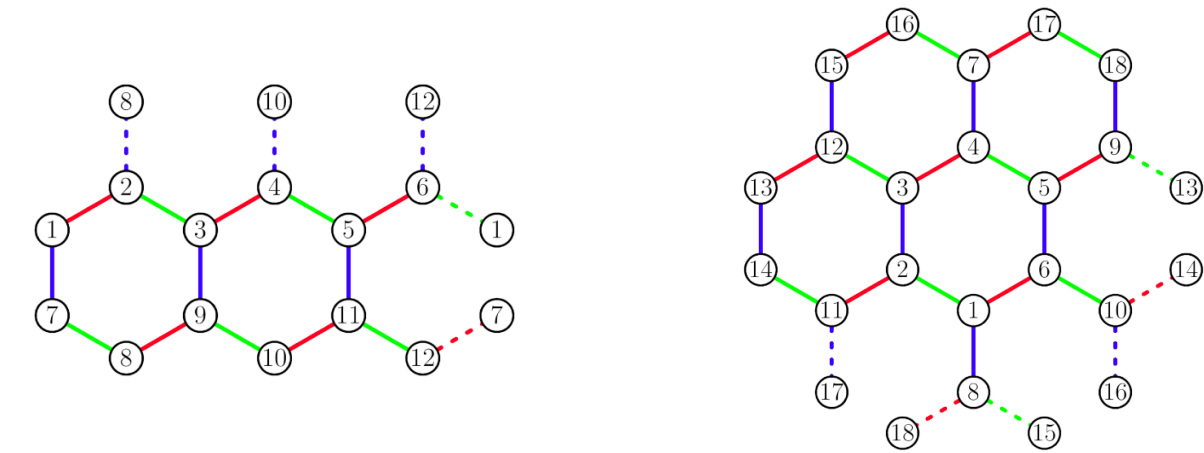
: up to 4th order

$$H_{ij}^\gamma = K S_k^\gamma S_j^\gamma + JS_i \cdot S_j$$

AF Kitaev

$$J = -|J_{ind}| + J_d$$

ED calculation: S=1 KJ model
12 & 18 sites

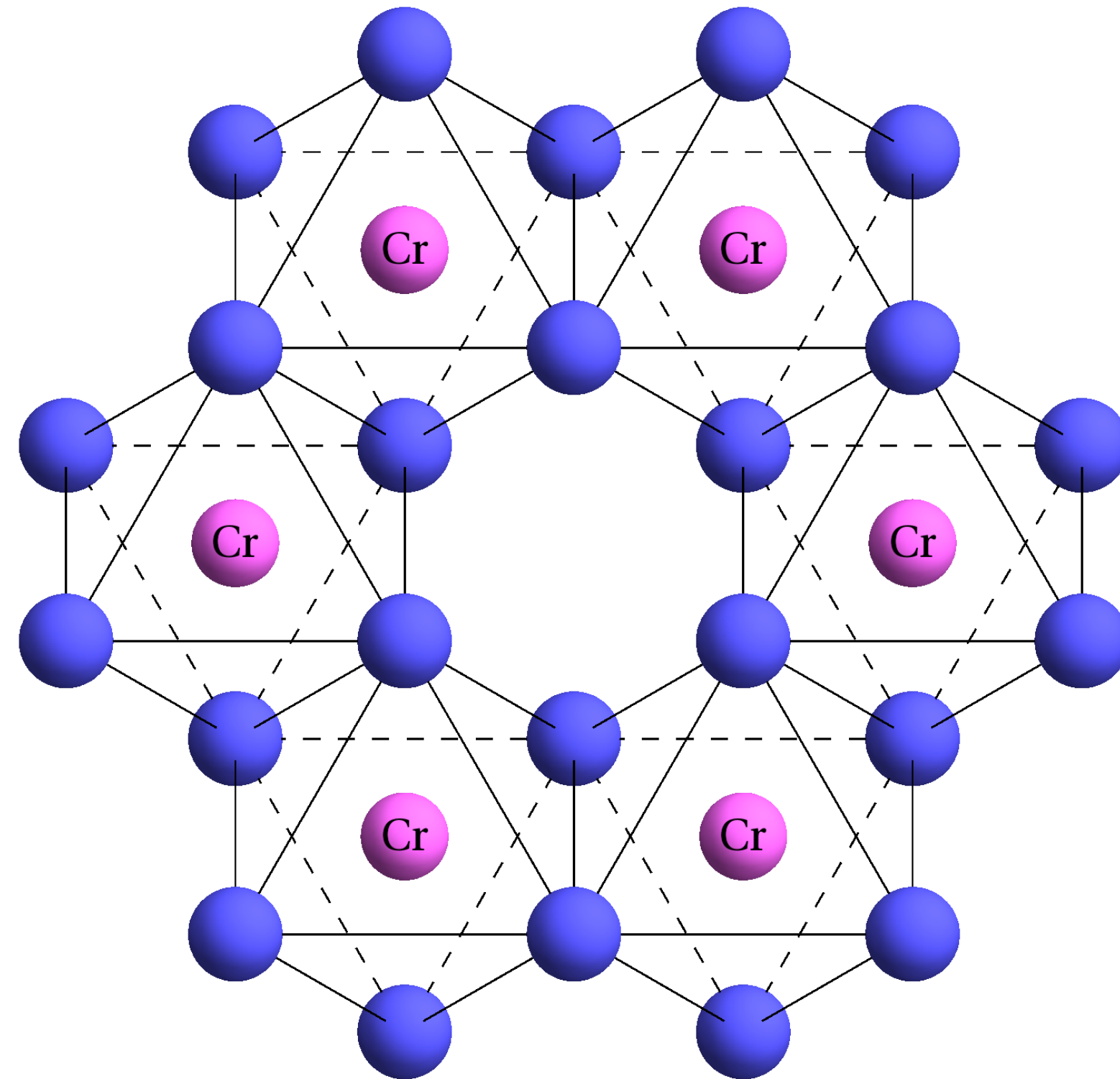


P. Peter Stavropoulos, D. Pereira, HYK, PRL 123, 037203 (2019)

Kitaev from heavy anion from ab-intio: C. Xu et al, npj Comput. Mater. 4, 57 (2018)

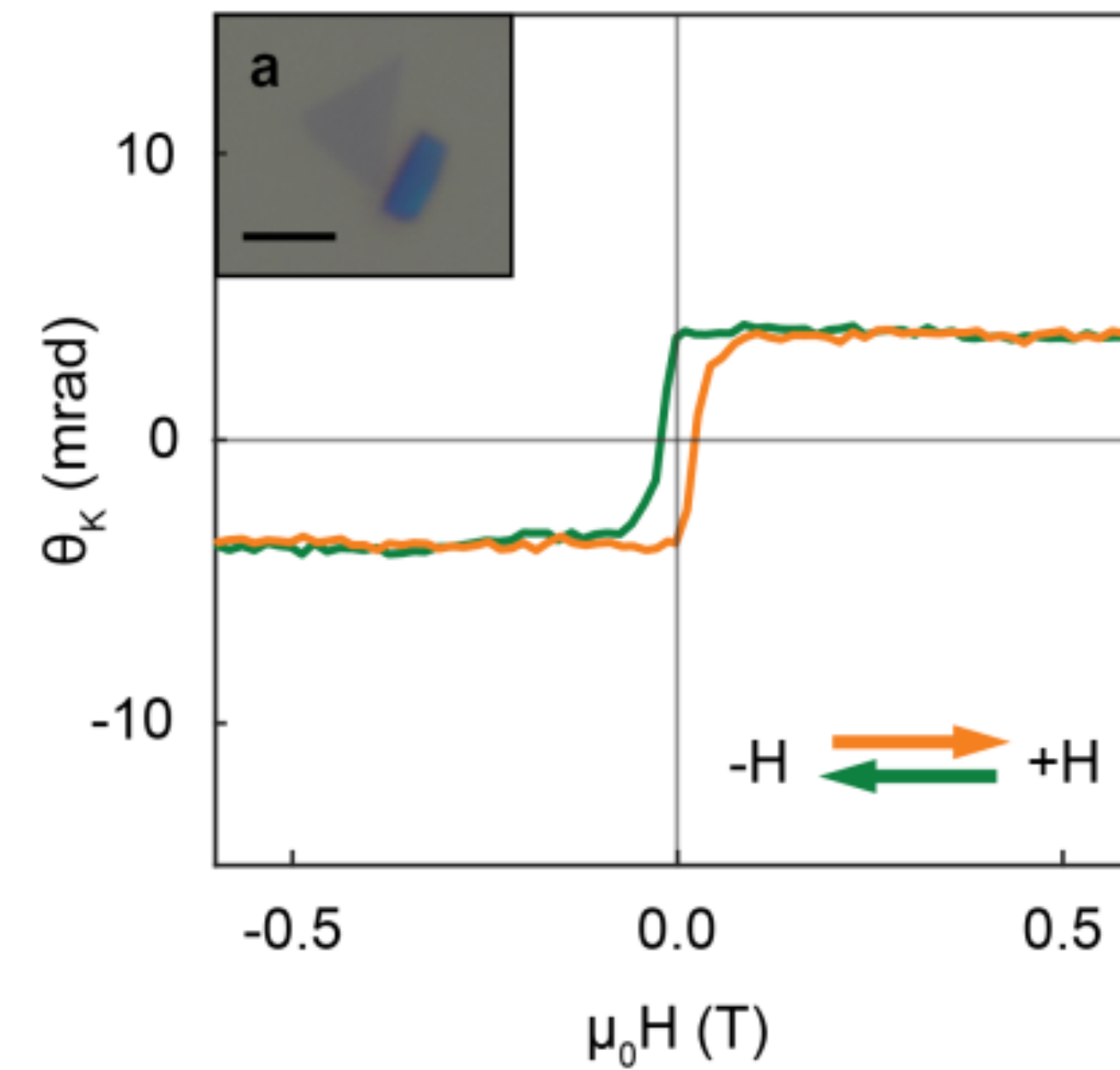
Example

Single layer of CrI₃ ($S=3/2$ honeycomb):
Ferromagnetic insulator



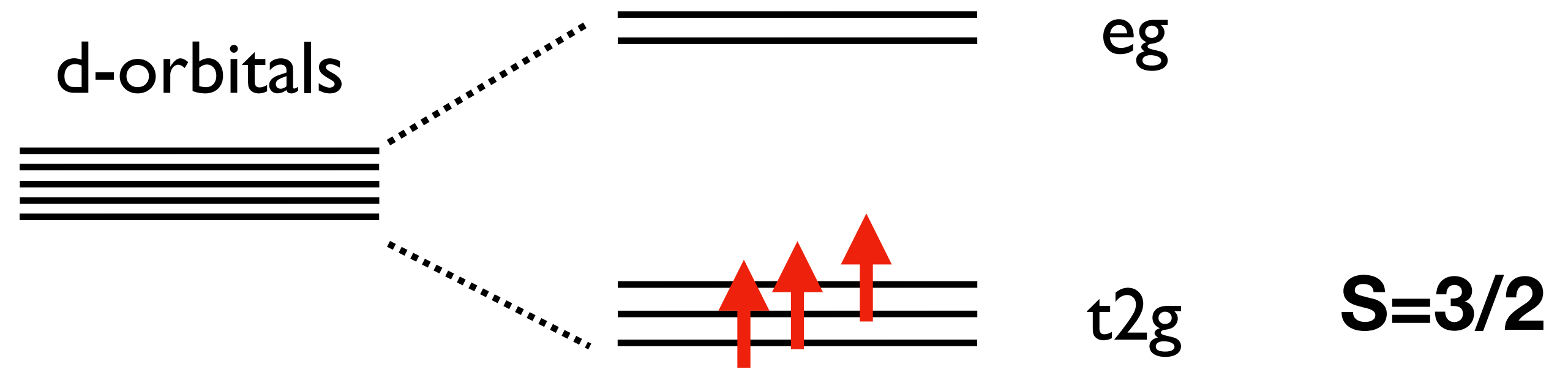
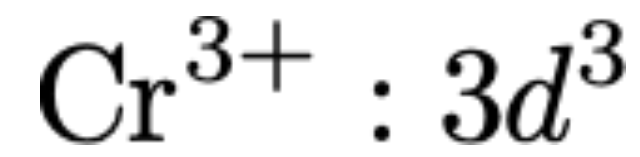
Kitaev interaction?

Kerr rotation



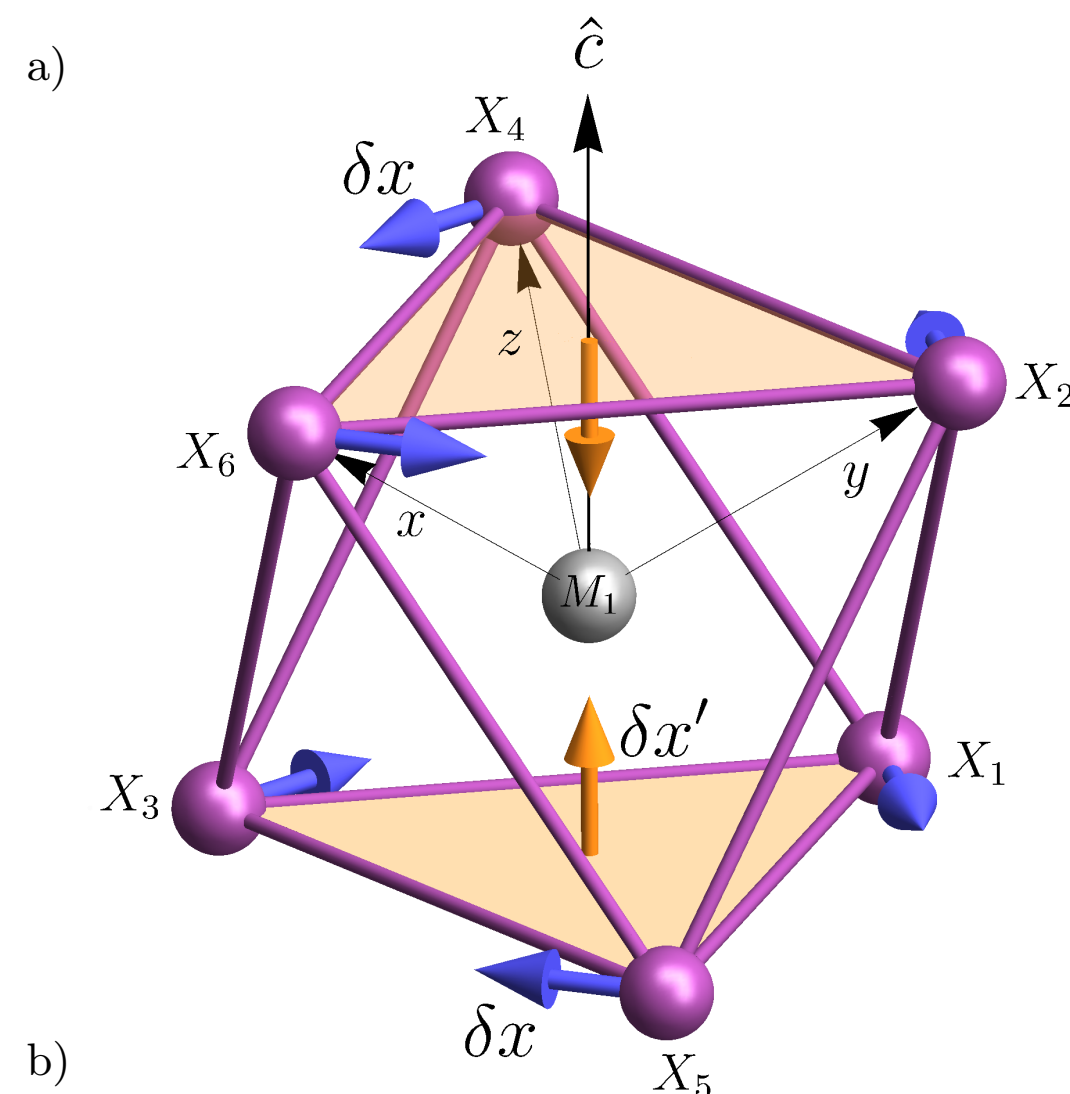
Single layer (2D) Ferromagnet

B. Huang et al, Nature 546, 7657 (2017)



n.n. model

$$H = \sum_{\langle ij \rangle \in \alpha\beta(\gamma)} \left[J \mathbf{S}_i \cdot \mathbf{S}_j + K S_i^\gamma S_j^\gamma + \Gamma (S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) \right. \\ \left. + \Gamma' (S_i^\alpha S_j^\gamma + S_i^\beta S_j^\gamma + S_i^\gamma S_j^\alpha + S_i^\gamma S_j^\beta) \right] \\ + \sum_i A_c (\mathbf{S}_i \cdot \hat{\mathbf{c}})^2$$



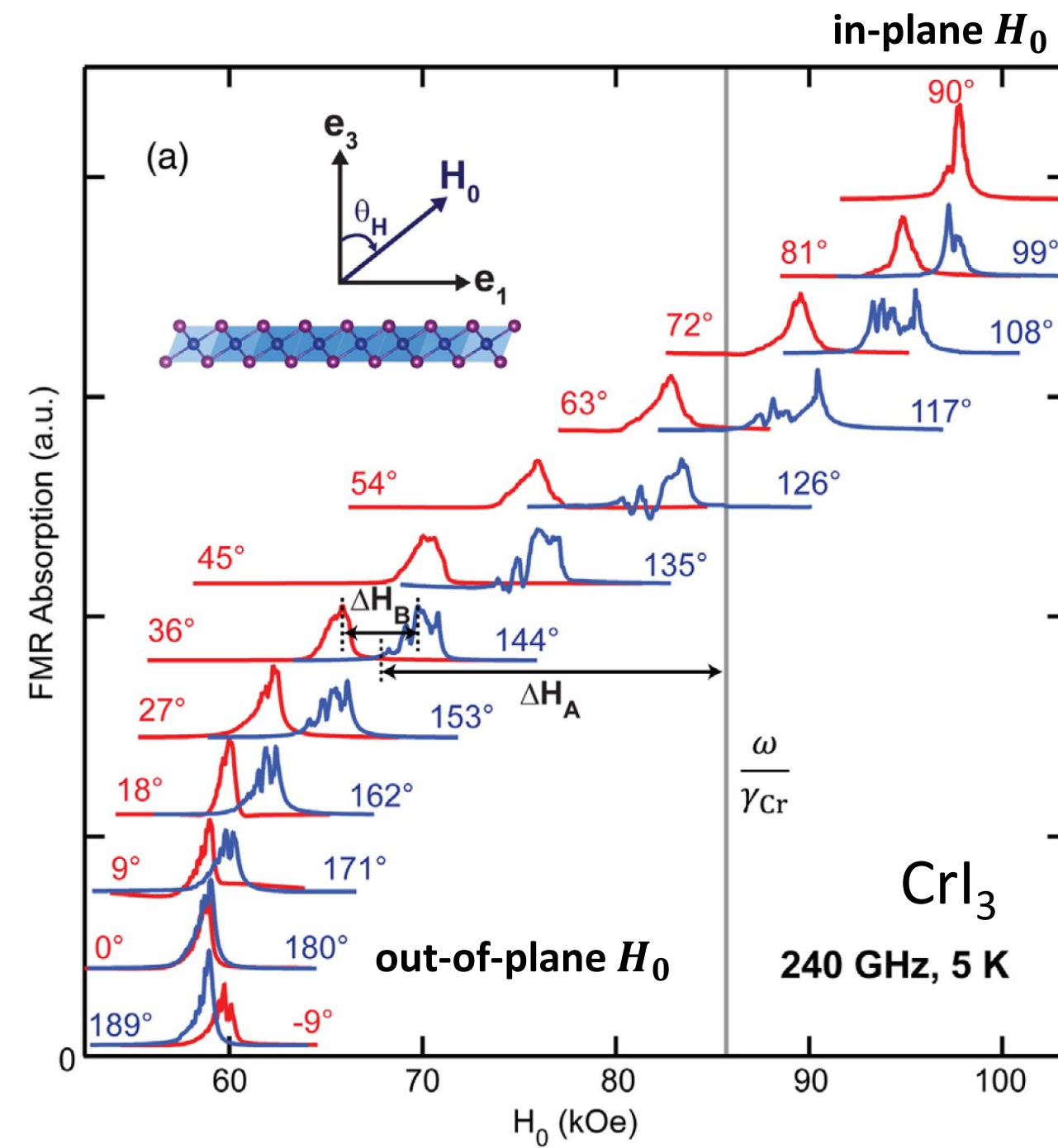
trigonal distortion

single-ion anisotropy, Γ, Γ'
from SOC + distortion

CrI₃ : debate

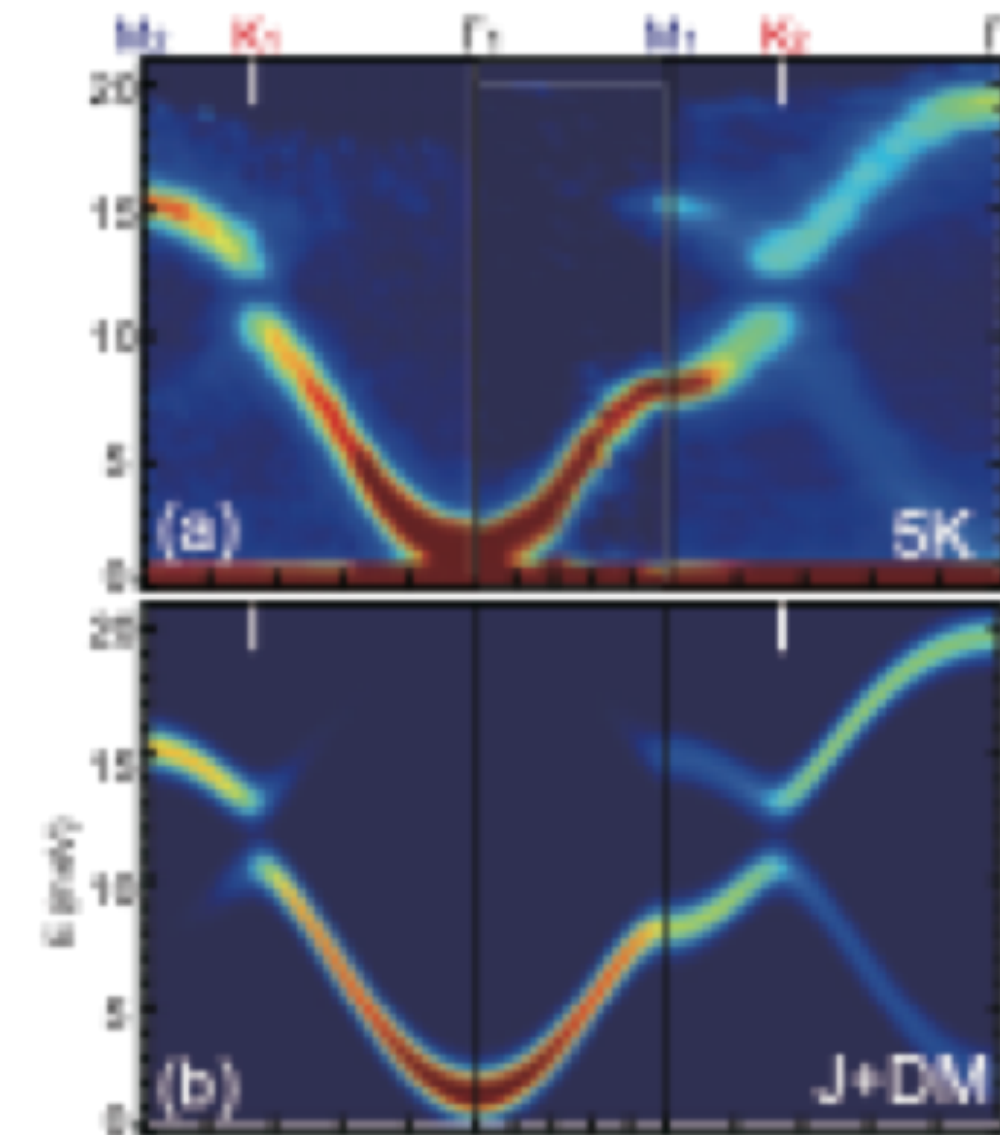
Angle dependent Ferromagnetic Resonance (FMR): CrI₃

Inelastic neutron scattering



Kitaev dominant

Inhee Lee, et al, Phys. Rev. Lett. 124, 017201 (2020)



J + DM interaction

L. Chen et al, PRX 11, 031047 (2021)

Kitaev Materials

: Kitaev interaction is the largest interaction in full H

Necessary (not sufficient) Requirements

- honeycomb Mott insulator : strong e-e interaction

~~transition metals~~

Debate over the strength of Kitaev interaction

- bond-dependent spin interaction for $S=1/2$

multi-orbital systems with Hund's coupling

spin-orbit coupling

$S > 1/2$: spin-orbit coupling in heavy ligand + Hund's coupling in transition metal

Goal: how to determine the Kitaev strength?

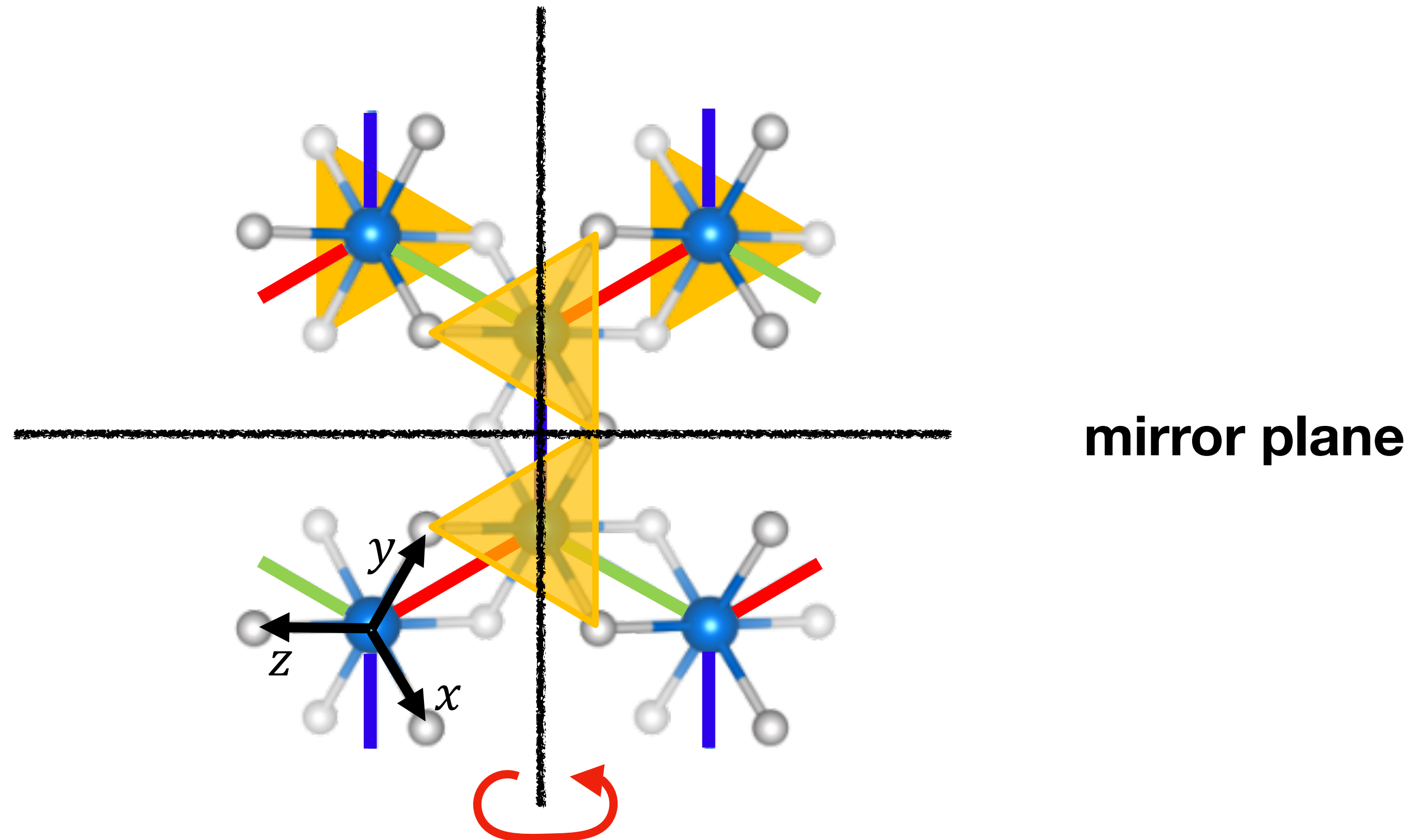
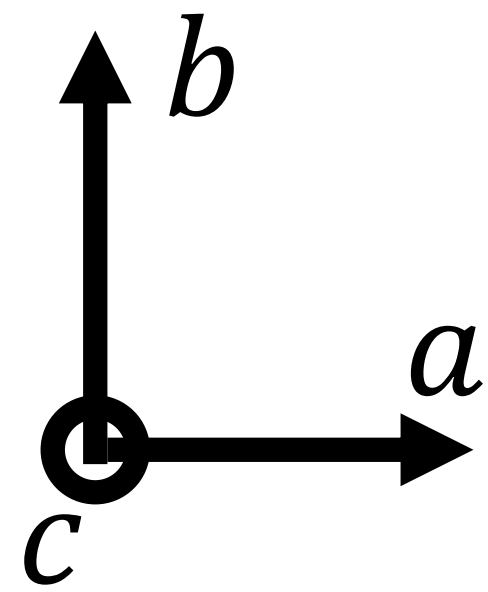
Use Symmetry of full Hamiltonian

Jiefu Cen, HYK, Commun. Phys. 5, 119 (2022);

Jiefu Cen, HYK, arXiv:2208.13807 (2022)

Generic n.n. **ideal** octahedra model

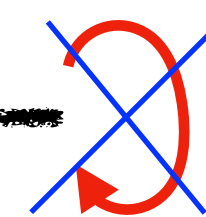
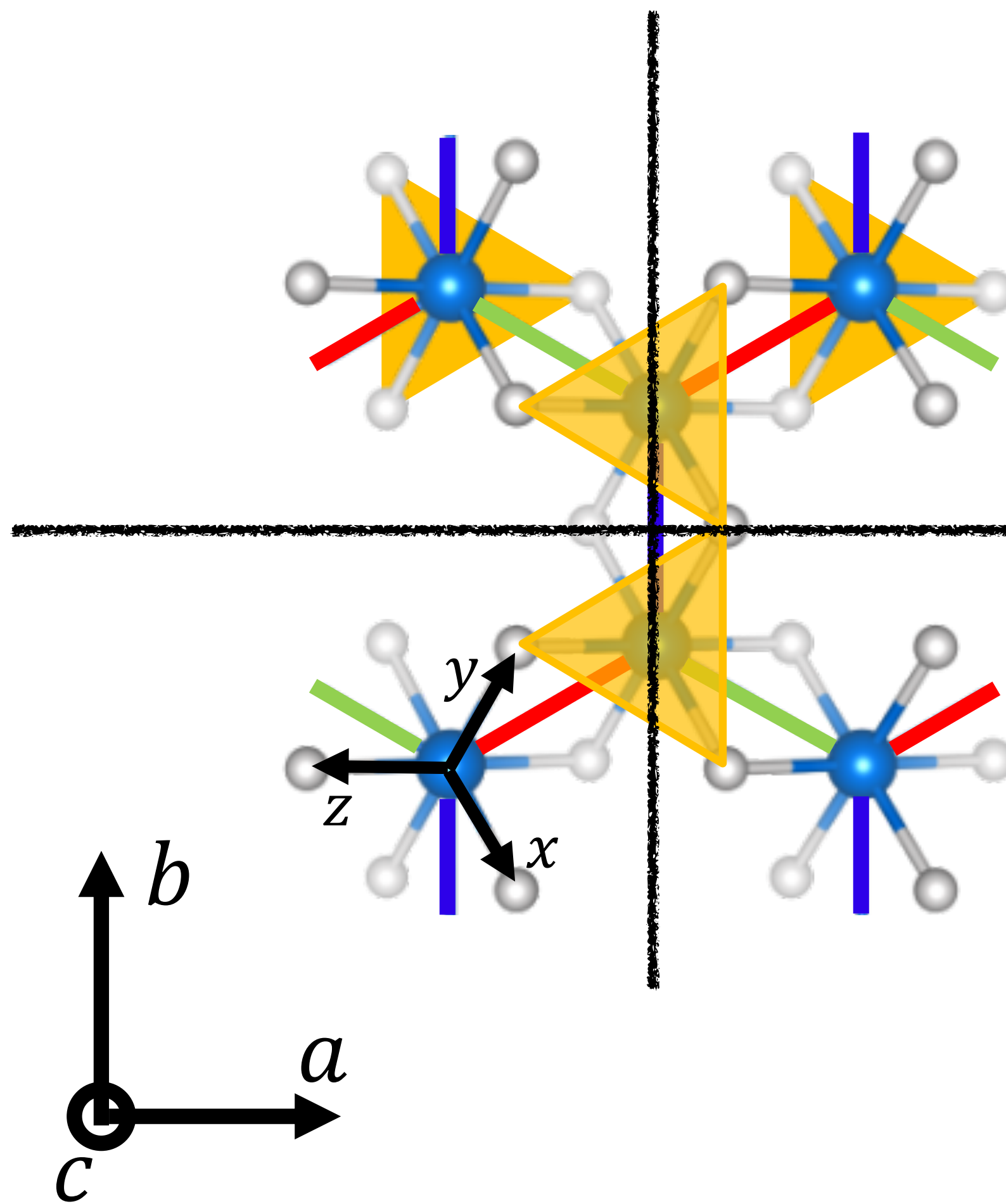
$$\mathcal{H} = \sum_{\langle ij \rangle \in \alpha\beta(\gamma)} \left[JS_i \cdot S_j + K S_i^\gamma S_j^\gamma + \Gamma (S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) \right],$$



C_{2b} = 180 rotation about b-axis

b is parallel to the bond; a/c-axis are perp. to the bond/plane

NO mirror plane (bc plane)



C2a C2a= 180 rotation about a-axis

$$\begin{pmatrix} \hat{e}_a \\ \hat{e}_b \\ \hat{e}_c \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 1 & -2 \\ -\sqrt{3} & \sqrt{3} & 0 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \end{pmatrix} \begin{pmatrix} \hat{e}_x \\ \hat{e}_y \\ \hat{e}_z \end{pmatrix} .$$

b is parallel to the bond; a/c-axis are perp. to the bond/plane

H in abc coordinate

$$\begin{aligned} \mathcal{H} = \sum_{\langle i,j \rangle} & \left[J_{XY} \left(S_i^a S_j^a + S_i^b S_j^b \right) + J_Z S_i^c S_j^c \right. \\ & + J_{ab} \left[\cos \phi_\gamma \left(S_i^a S_j^a - S_i^b S_j^b \right) - \sin \phi_\gamma \left(S_i^a S_j^b + S_i^b S_j^a \right) \right] \\ & \left. - \sqrt{2} J_{ac} \left[\cos \phi_\gamma \left(S_i^a S_j^c + S_i^c S_j^a \right) + \sin \phi_\gamma \left(S_i^b S_j^c + S_i^c S_j^b \right) \right] \right], \end{aligned}$$

$\phi_\gamma = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$ for $\gamma = z, x, y$ - bond

$$J_{XY} = J + J_{ac}, \quad J_Z = J + J_{ab},$$

$$J_{ab} = \frac{1}{3}K + \frac{2}{3}\Gamma, \quad J_{ac} = \frac{1}{3}K - \frac{1}{3}\Gamma.$$

Note: $J_Z - J_{XY} = J_{ab} - J_{ac} = \Gamma$

in-plane vs. out-of-plane anisotropy due to Gamma

J. Chaloupka, G. Khaliullin, PRB 92, 24413 (2015);

Pyrochlore lattice, S. Onoda, J. Phys.:Conf. Ser. 320, 012065 (2011); K. Ross, et al, PRX 1, 021002 (2011)

$$C_{2a} : (S^a, S^b, S^c) \rightarrow (S^a, -S^b, -S^c) \text{ and } \phi_x \leftrightarrow \phi_y$$

Consequence of broken C_{2a}

$$\mathcal{H} = \sum_{\langle i,j \rangle} \left[J_X (S_i^a S_j^a + S_i^b S_j^b) + J_Z S_i^c S_j^c \right. \\ \left. + J_{ab} [\cos \phi_\gamma (S_i^a S_j^a - S_i^b S_j^b) - \sin \phi_\gamma (S_i^a S_j^b + S_i^b S_j^a)] \right. \\ \left. - \sqrt{2} J_{ac} [\cos \phi_\gamma (S_i^a S_j^c + S_i^c S_j^a) + \sin \phi_\gamma (S_i^b S_j^c + S_i^c S_j^b)] \right]$$

+ single-ion anisotropy, DM on 2nd n.n. with d//c, further n.n. Heisenberg + ...

All interactions are invariant under C_{2a} *Except*

$$J_{ac} \rightarrow -J_{ac}$$

$$J_{ac} = \frac{1}{3}K - \frac{1}{3}\Gamma.$$

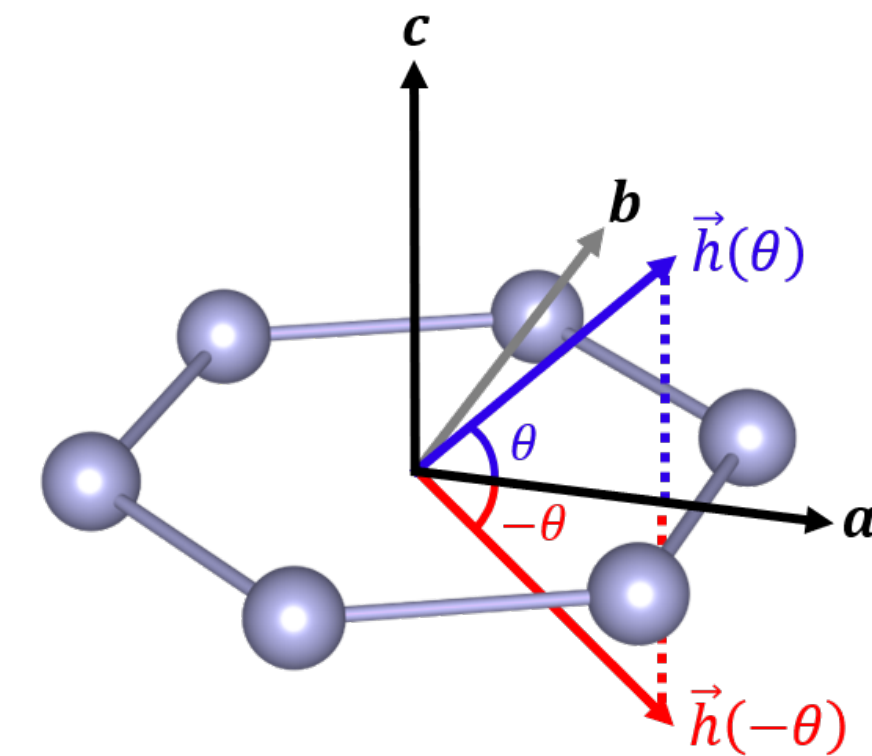
- A magnetic field can detect the broken C_{2a} symmetry and be used to isolate the interaction that breaks C_{2a} .

$$H_{total} = \mathcal{H} - g\mu_B \vec{h} \cdot \vec{S}$$

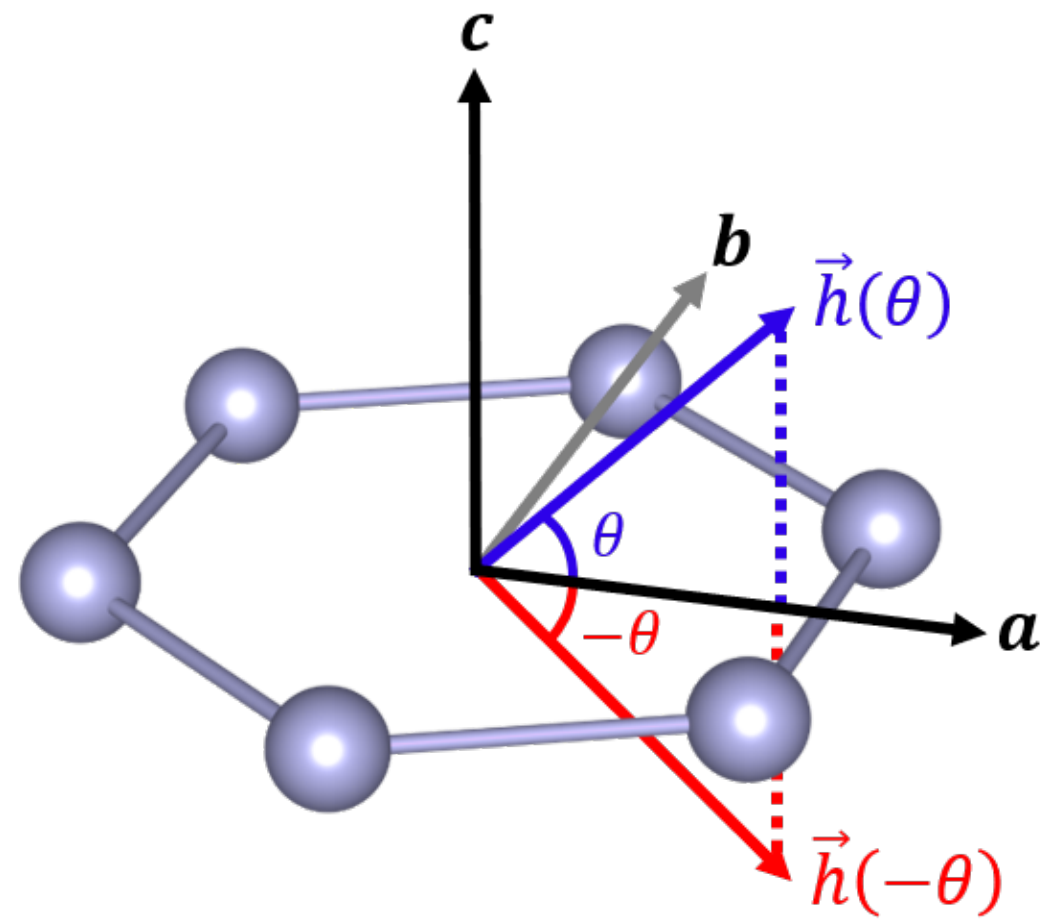
Two different directions of field related by C_{2a} :
different excitation energies **only if Jac finite**

$$E_n(\theta) - E_n(-\theta) \propto J_{ac}$$

field is in the ac-plane



$$J_{ac} = \frac{1}{3}K - \frac{1}{3}\Gamma. \quad \text{needs to isolate both Gamma and K}$$



Recipe to isolate Kitaev

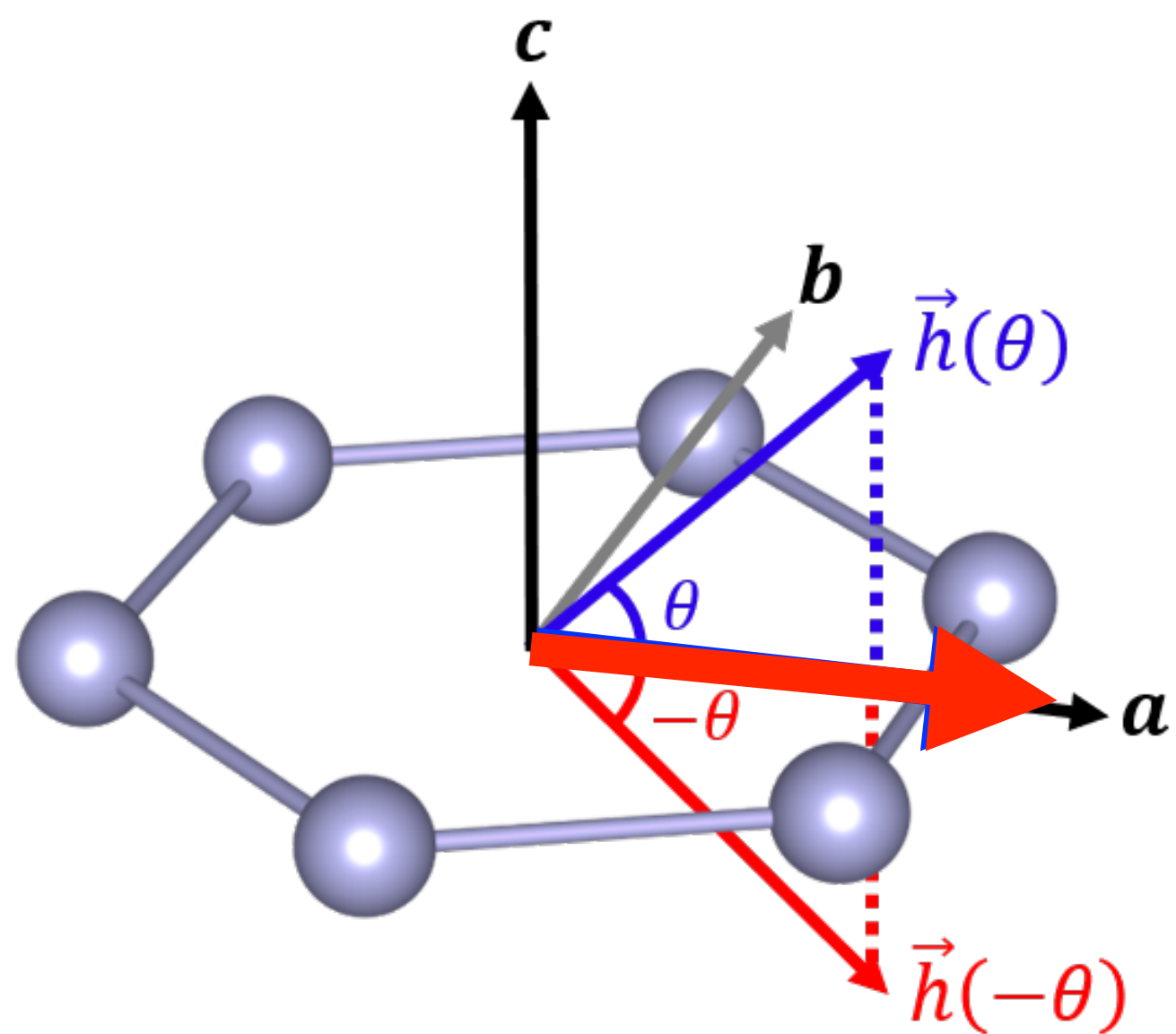
1. apply B in the **ac-plane** with θ
2. measure the spin excitations
3. rotate B with angle of $-\theta$
4. measure the spin excitations
5. energy difference between 2 & 4:

$$\delta\omega_K = \omega(\theta) - \omega(-\theta) \propto K - \Gamma$$

6. In-plane and out-of-plane energy excitation = Gamma

$$\delta\omega_A = \omega(\theta = 0) - \omega(\theta = 90^\circ) \quad \text{for } S=1/2$$

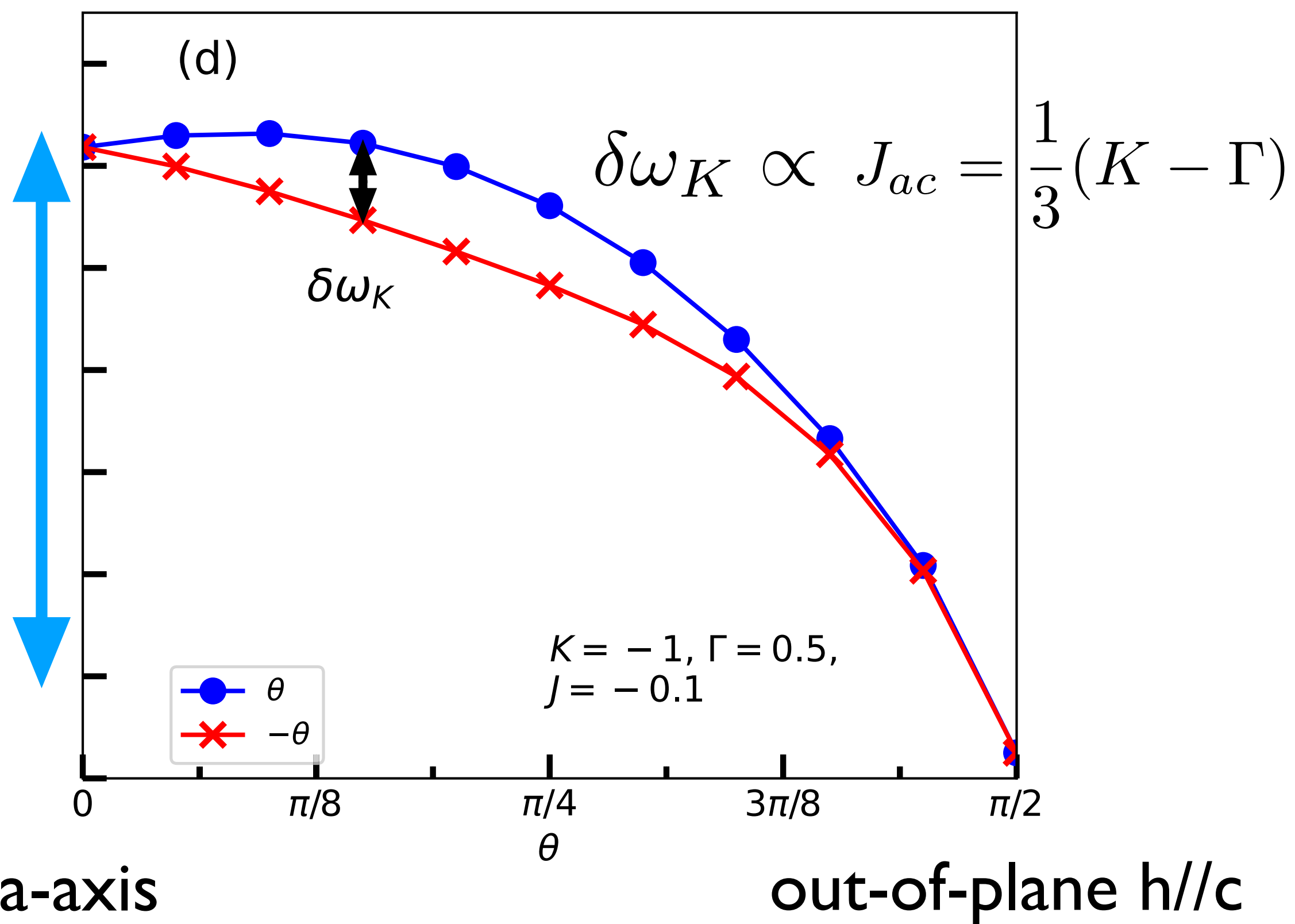
7. subtract Gamma from 5, then single out the Kitaev interaction!



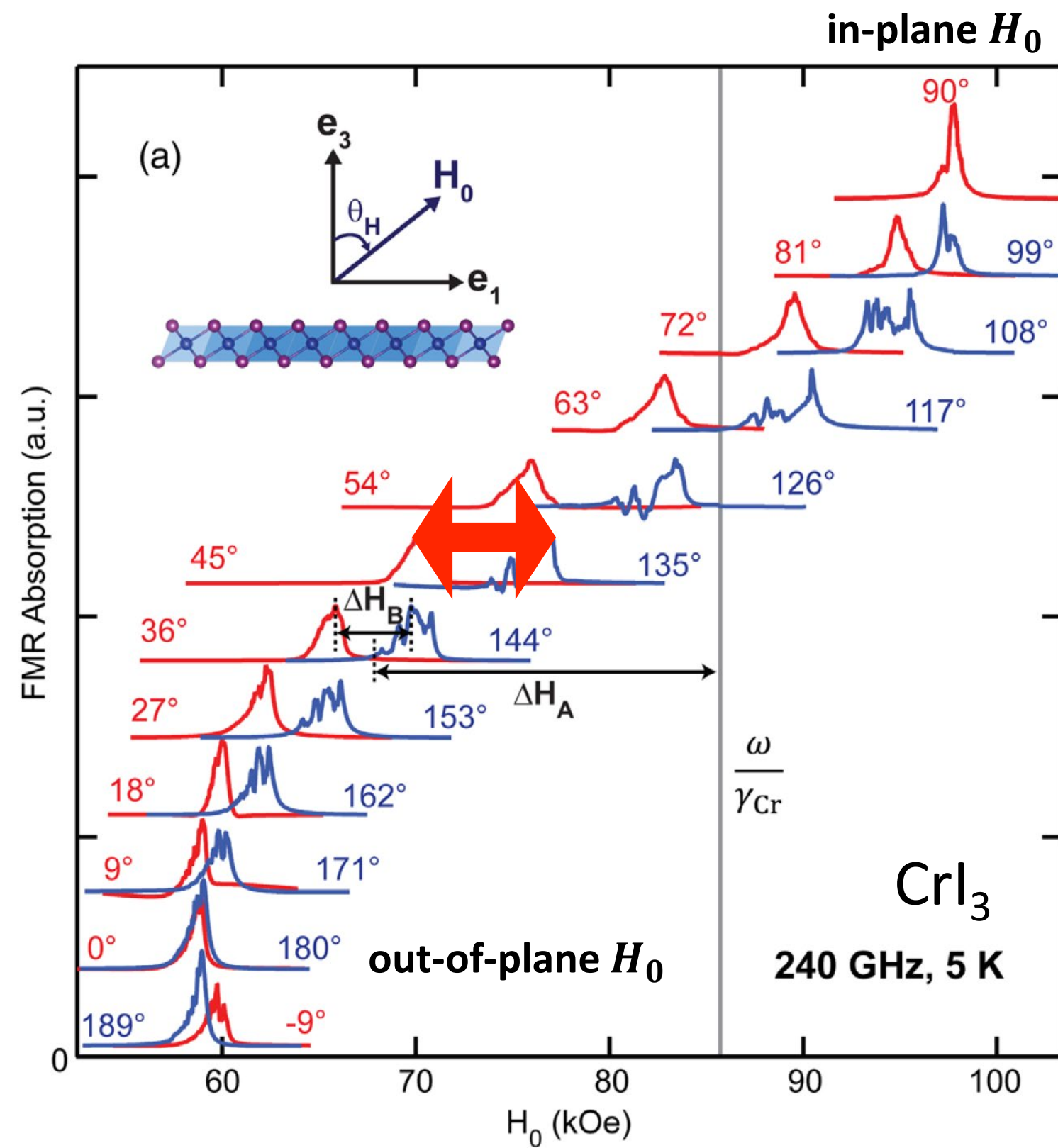
For $S=1/2$ $J_{ac} = \frac{1}{3}K - \frac{1}{3}\Gamma.$

Angle-dependent Ferromagnetic Resonance,
NMR, Optical spectroscopy

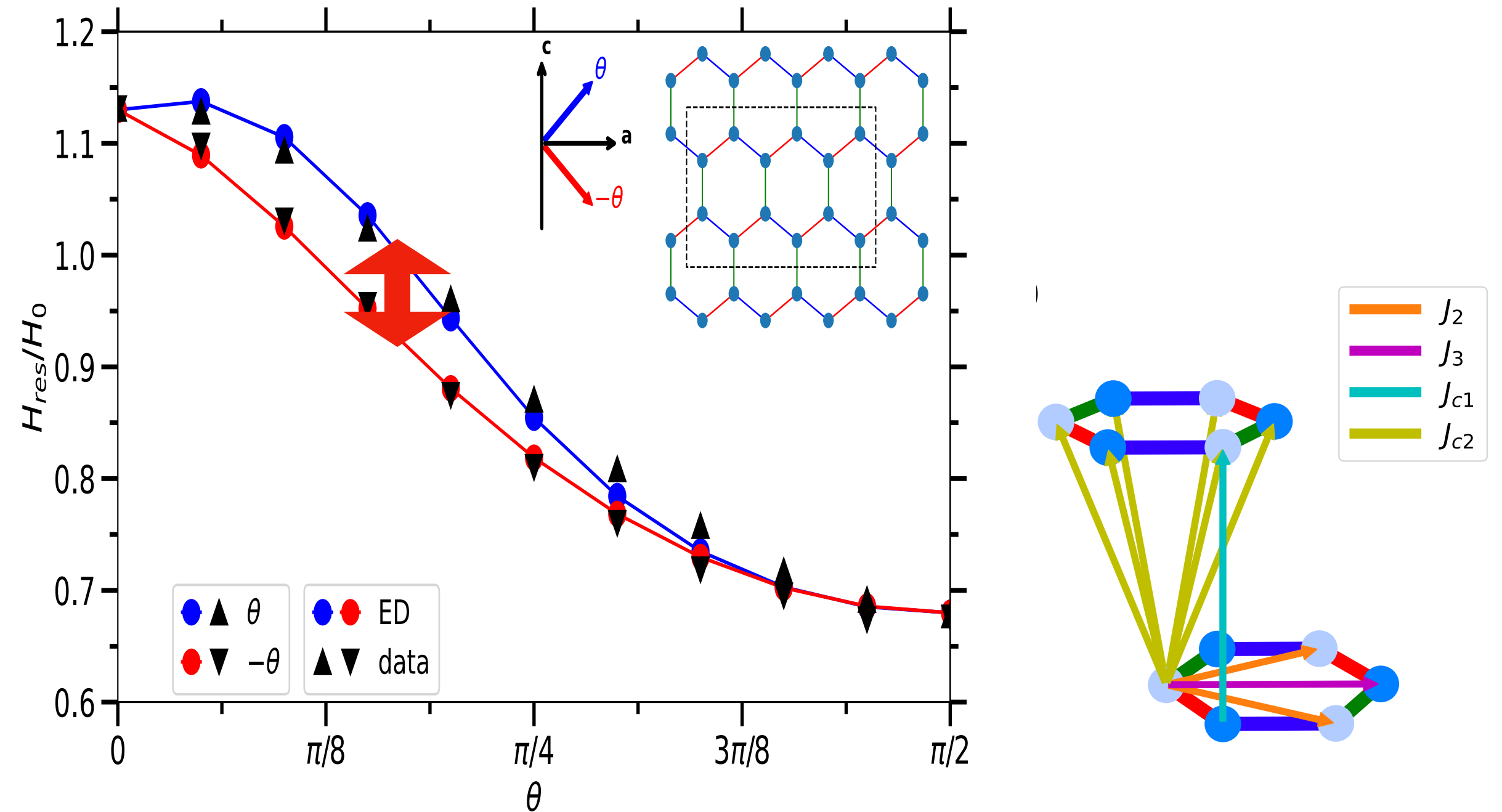
$\delta\omega_A \equiv \omega(\theta = 0) - \omega(\theta = 90^\circ) \propto J_{XY} - J_Z = \Gamma$



Application to CrI3



I. Lee et al, PRL 124, 017201 (2020)

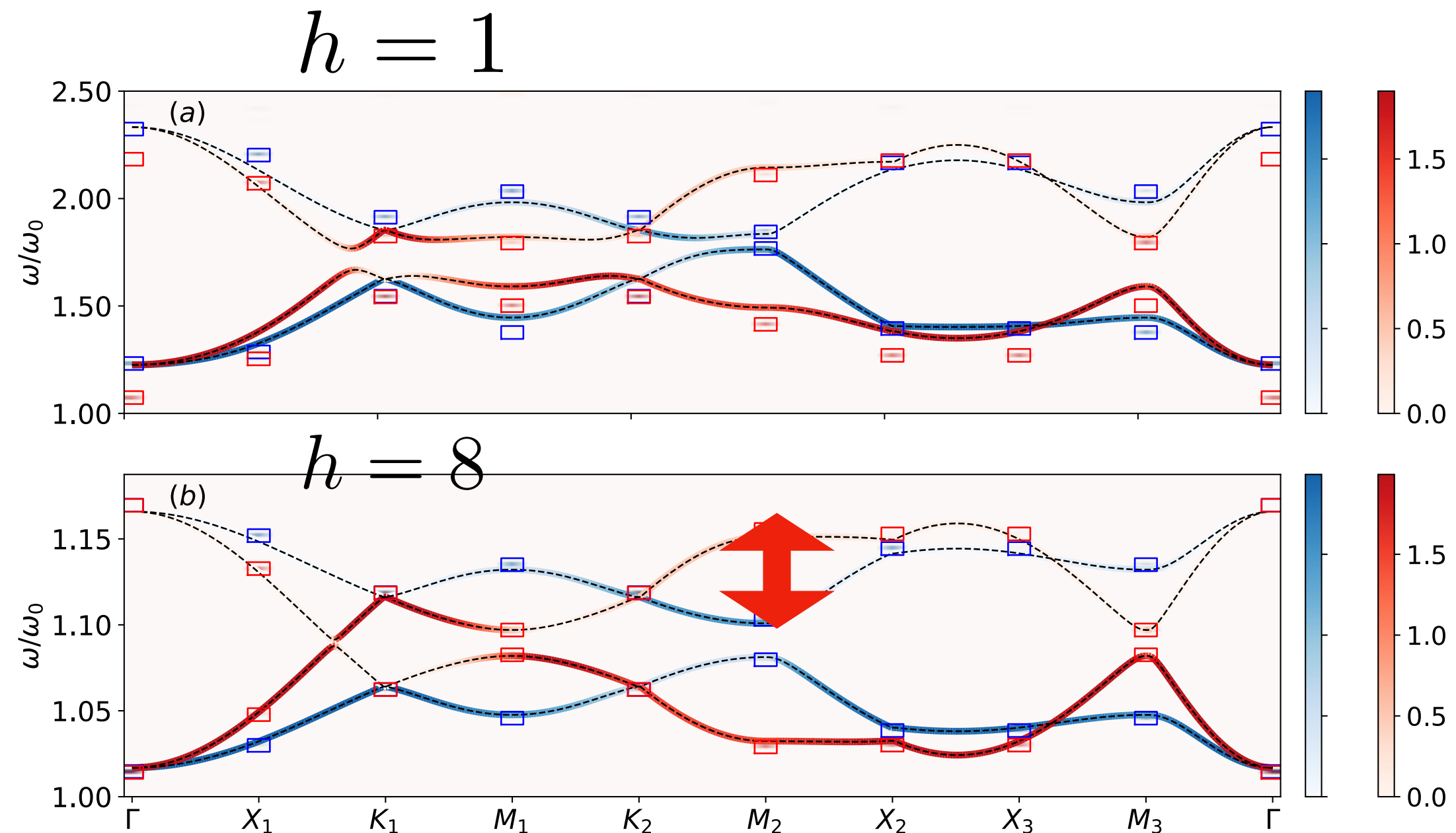
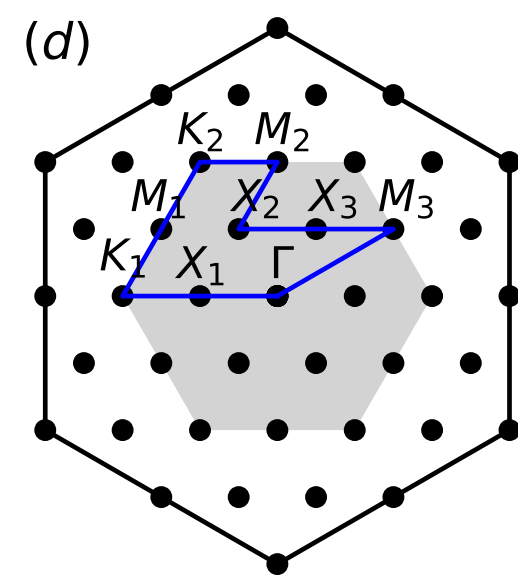
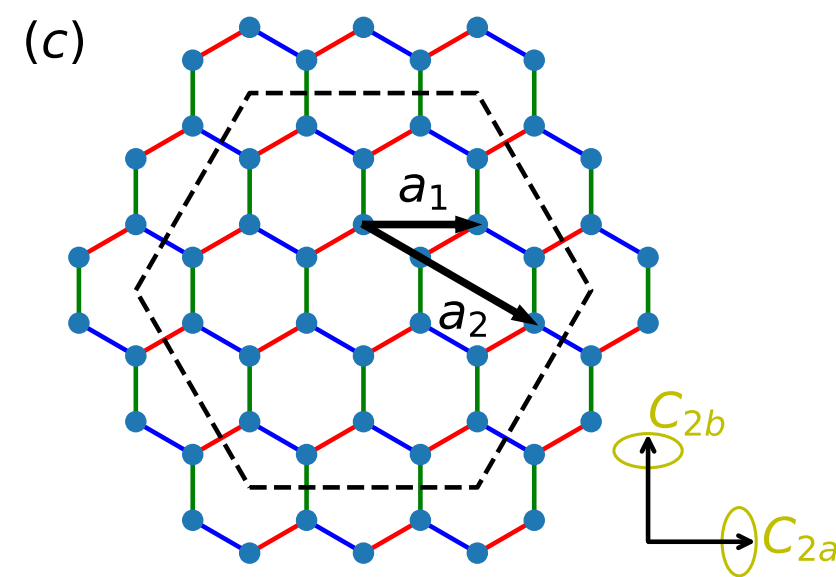
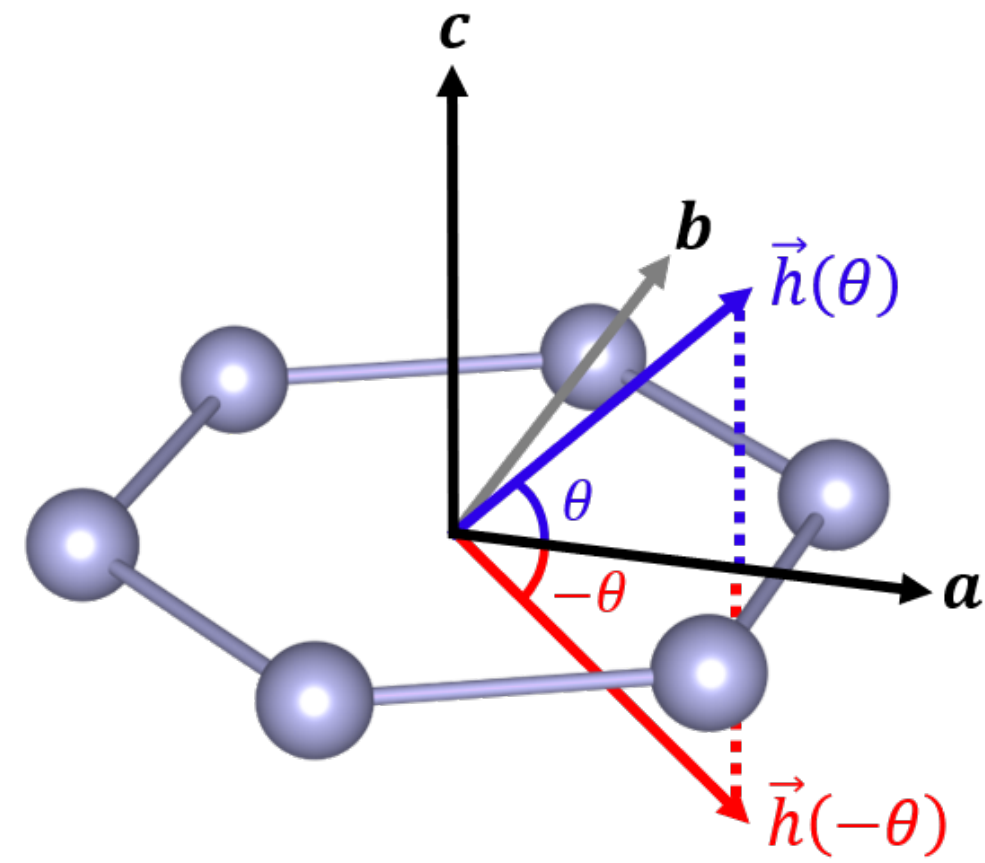


Interaction	value (meV)	Interaction	value (meV)
J	-2.5	J_2	-0.09
K	1.1	J_3	0.13
Γ	~ 0	J_{c1}	0.048
Γ'	~ 0	J_{c2}	-0.071
A_c	-0.23	D_c	0.17

Inelastic Neutron Scattering

$$K = -1, \Gamma = 0.5, J = -0.1$$

$$\theta = \pm 30^\circ$$



Can we use a “fixed” magnetic field direction?

— Two different momenta under a certain magnetic field

Symmetry of H

	\mathcal{T}	C_{2b}	$\mathcal{T}C_{2b}$	C_{2a}	$\mathcal{T}C_{2a}$
$\mathbf{h} = 0$	○	○	○	×/○	×/○
\mathbf{h} in $a - c$ plane	×	×	○	×	×
\mathbf{h} in $b - c$ plane	×	×	×	×	×/○

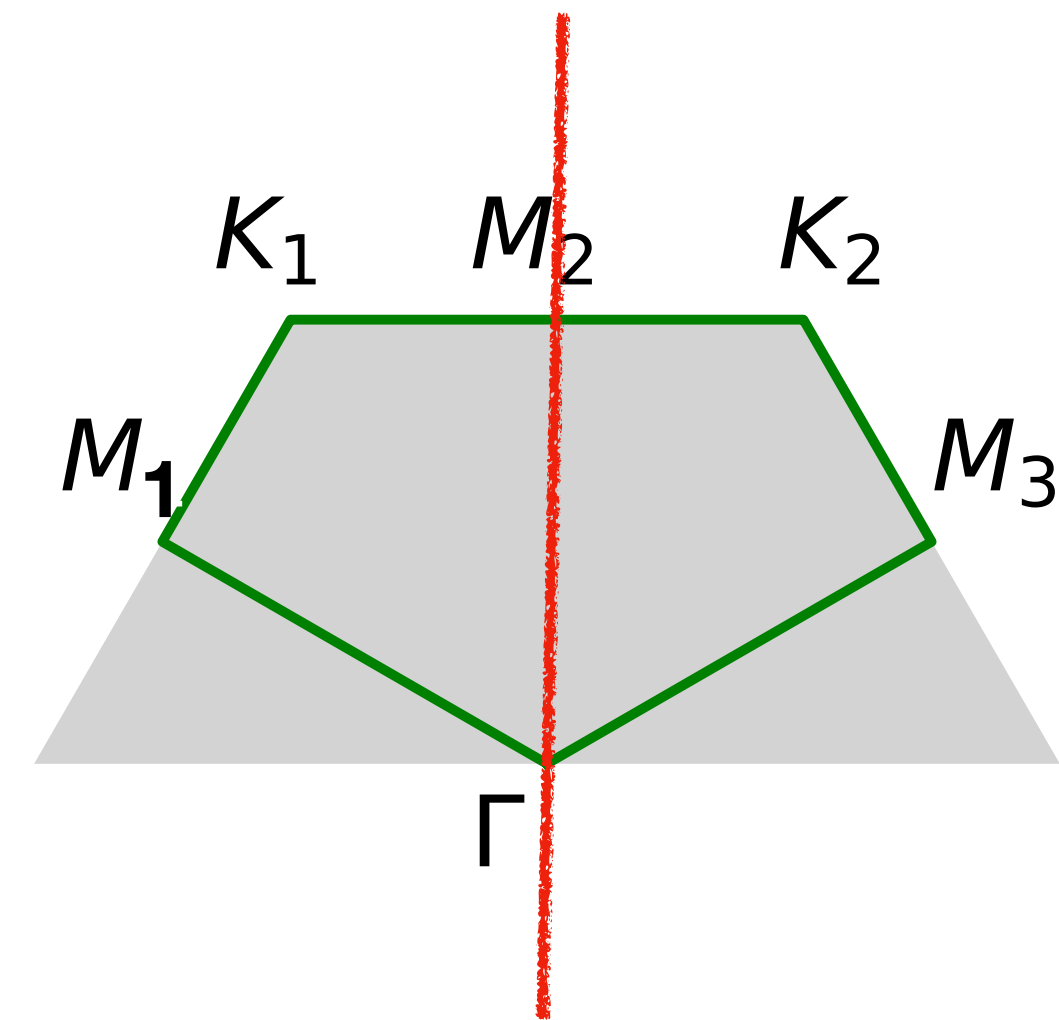
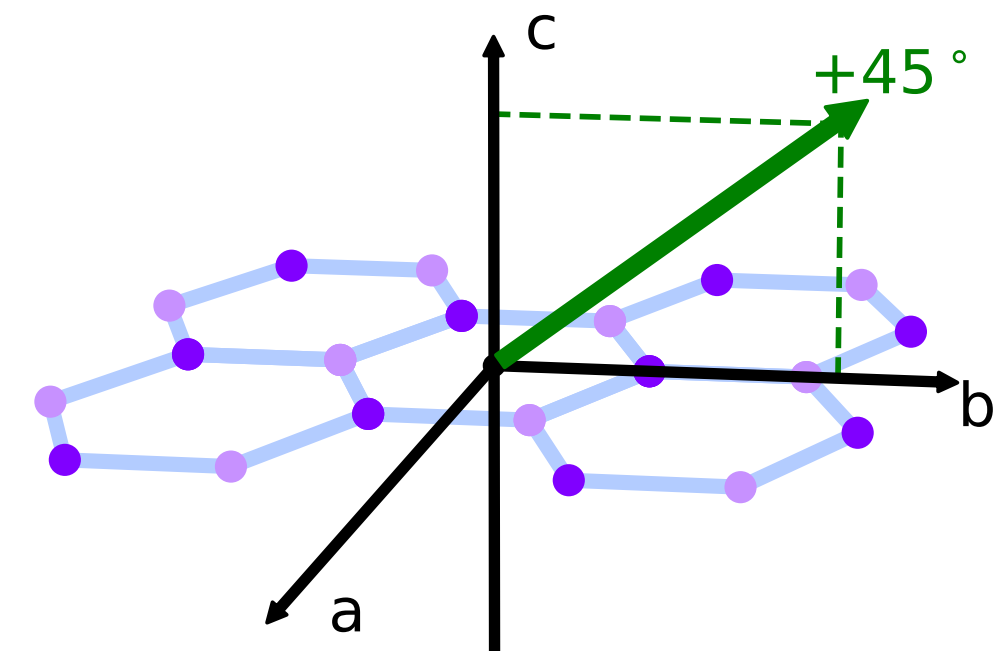
$J_{ac} \neq 0$

$J_{ac} = 0$

2nd Recipe to isolate Jac

$$J_{ac} \propto (K - \Gamma)$$

1. apply B in the **bc-plane** with angle theta
2. measure the spin excitations at M_1 and M_3
3. energy difference between $M_1 - M_3 \propto (K - \Gamma)$

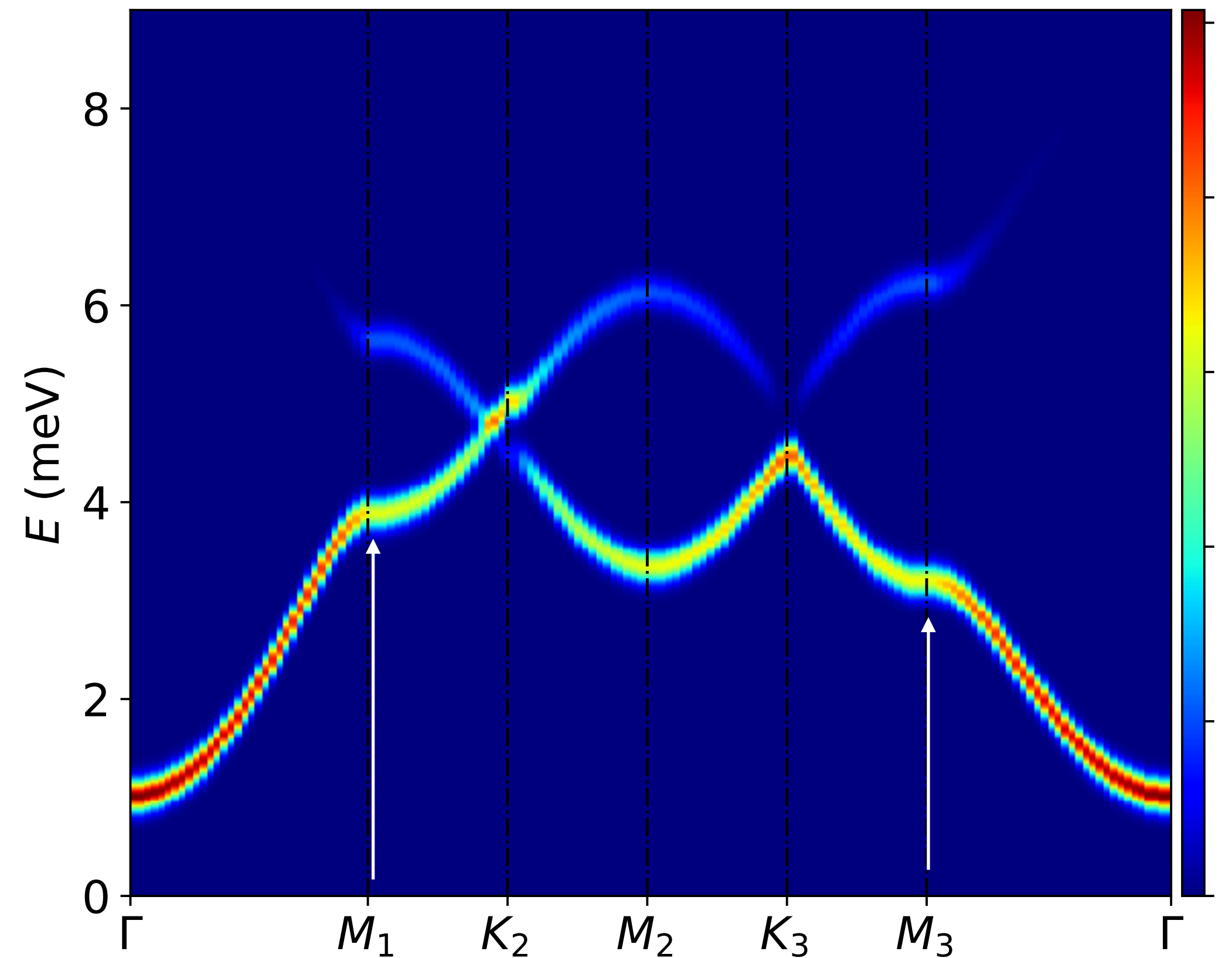
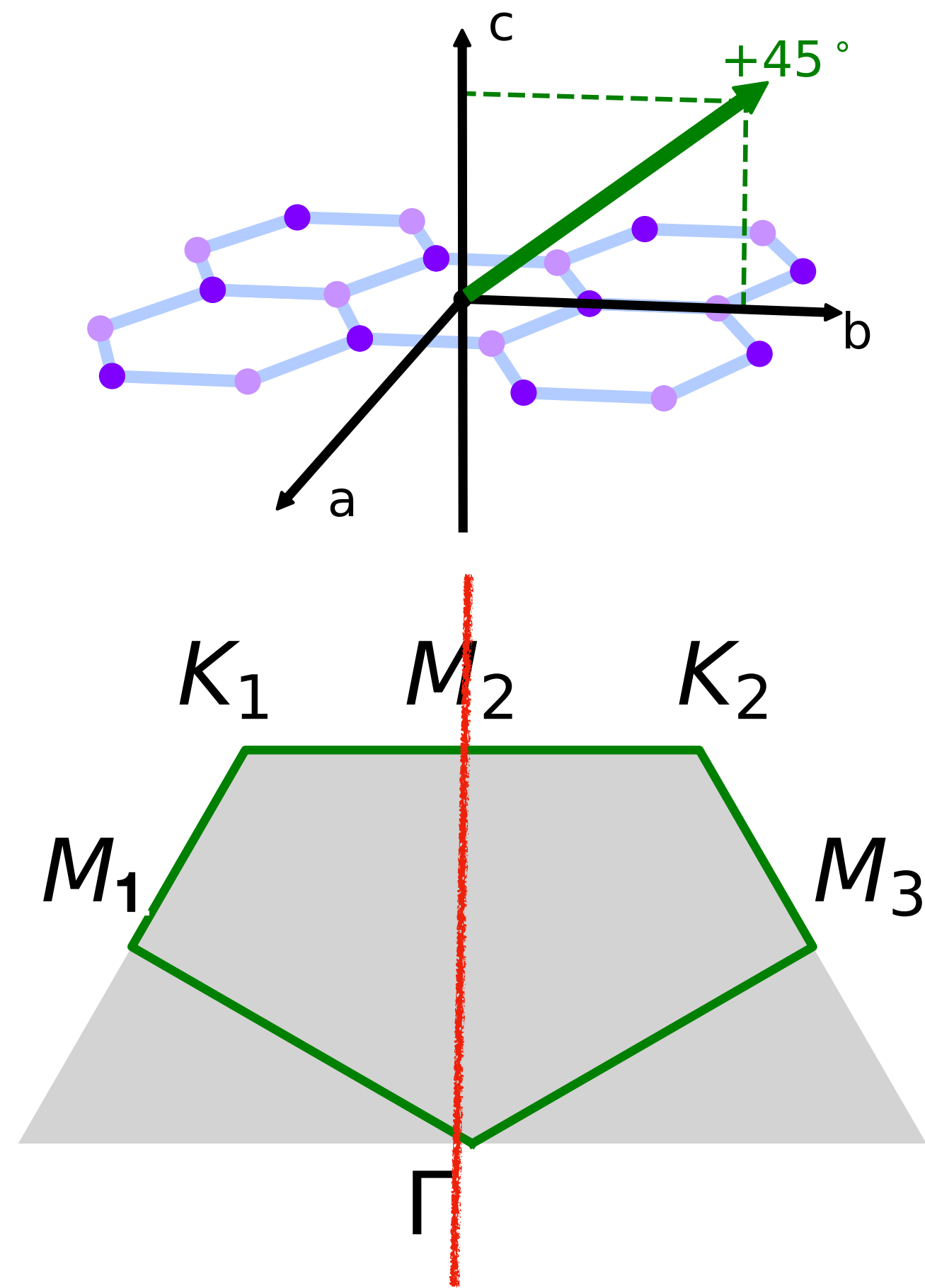


bc-plane: no mirror symmetry

In higher-S, $\Gamma \sim 0 \rightarrow J_{ac} \sim K$

Example

$J = -1, K=0.5, h = 1$



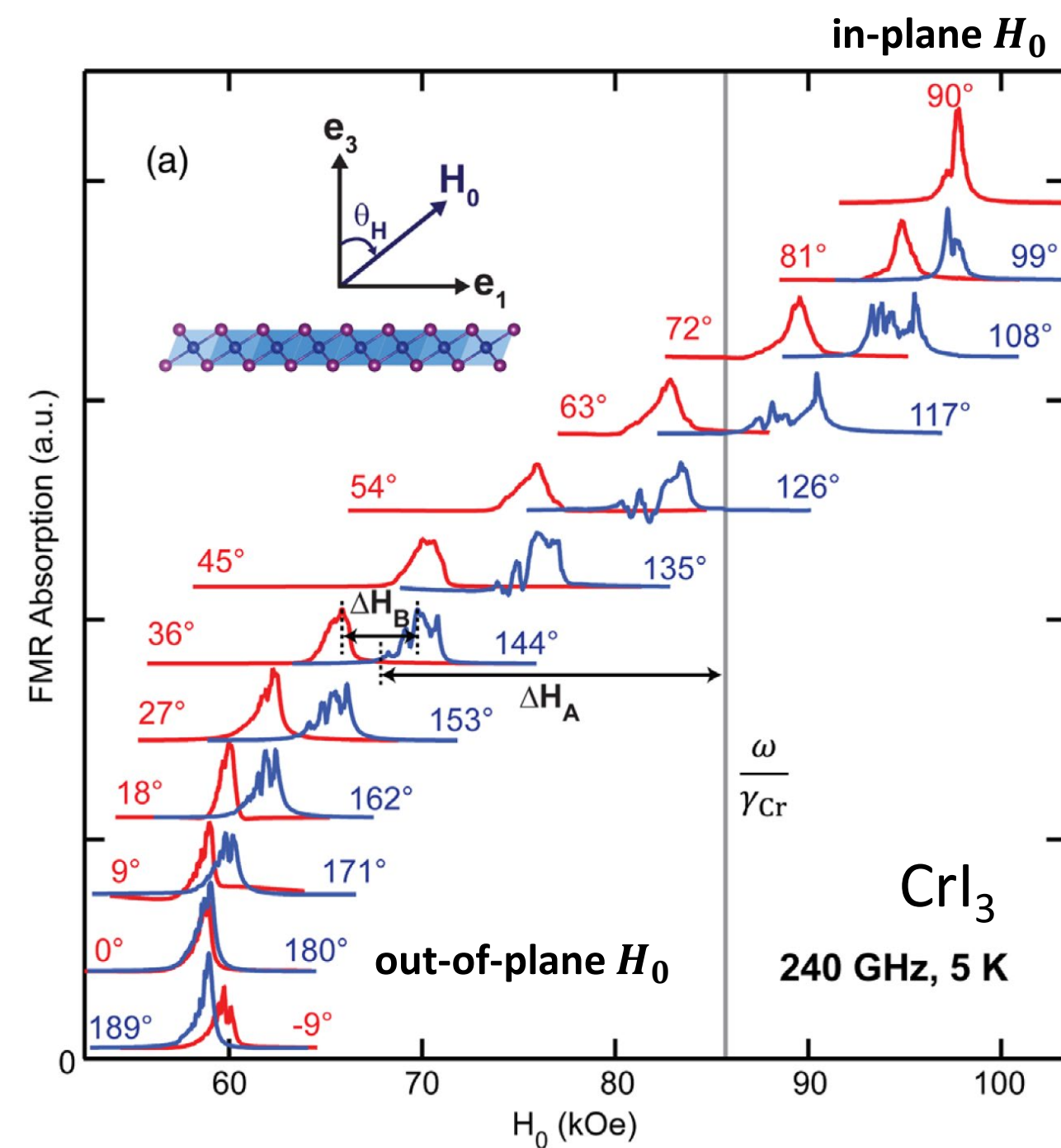
bc-plane: no mirror symmetry \rightarrow finite (Kitaev - Γ) interaction

Application to CrI3

Current debate

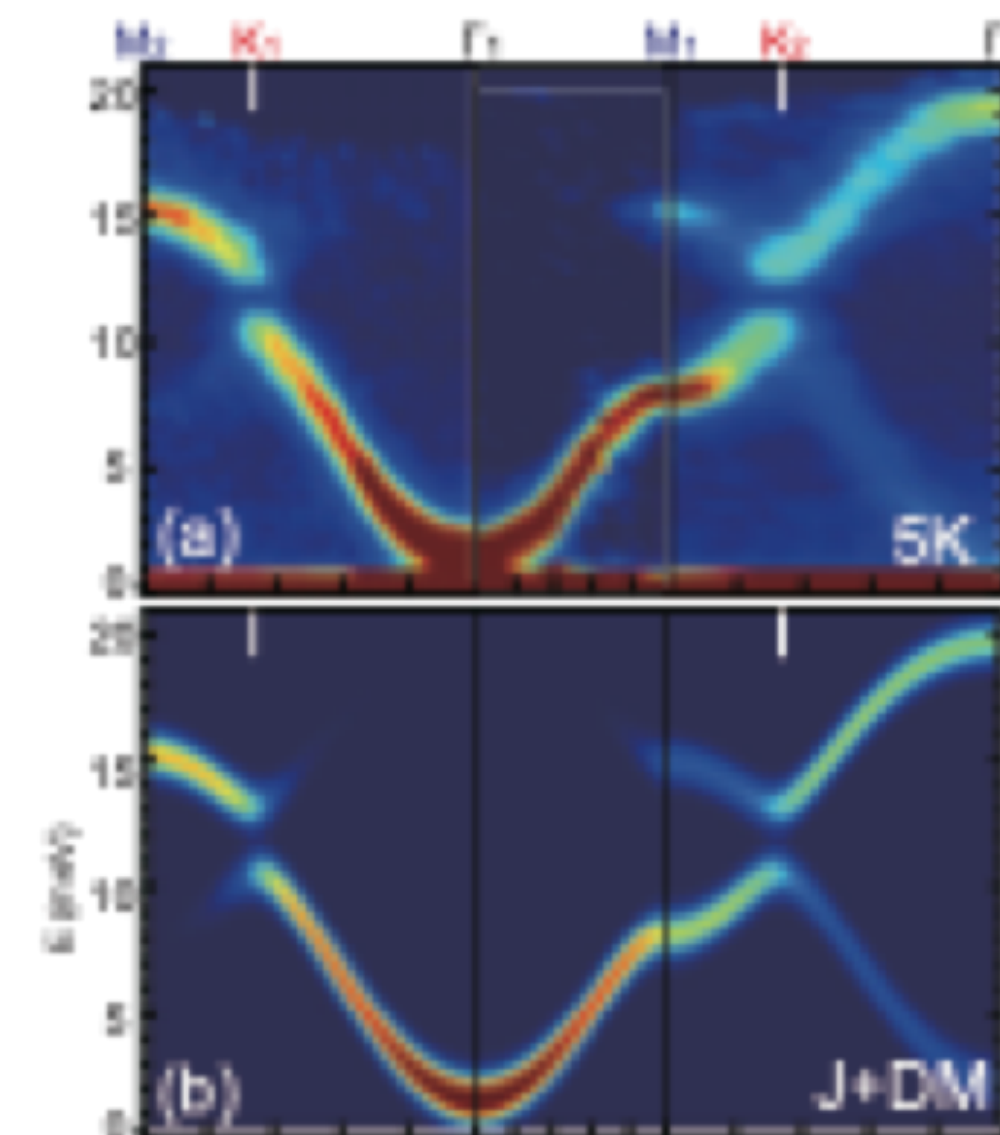
Angle dependent Ferromagnetic Resonance (FMR): CrI₃

Inelastic neutron scattering



Kitaev dominant

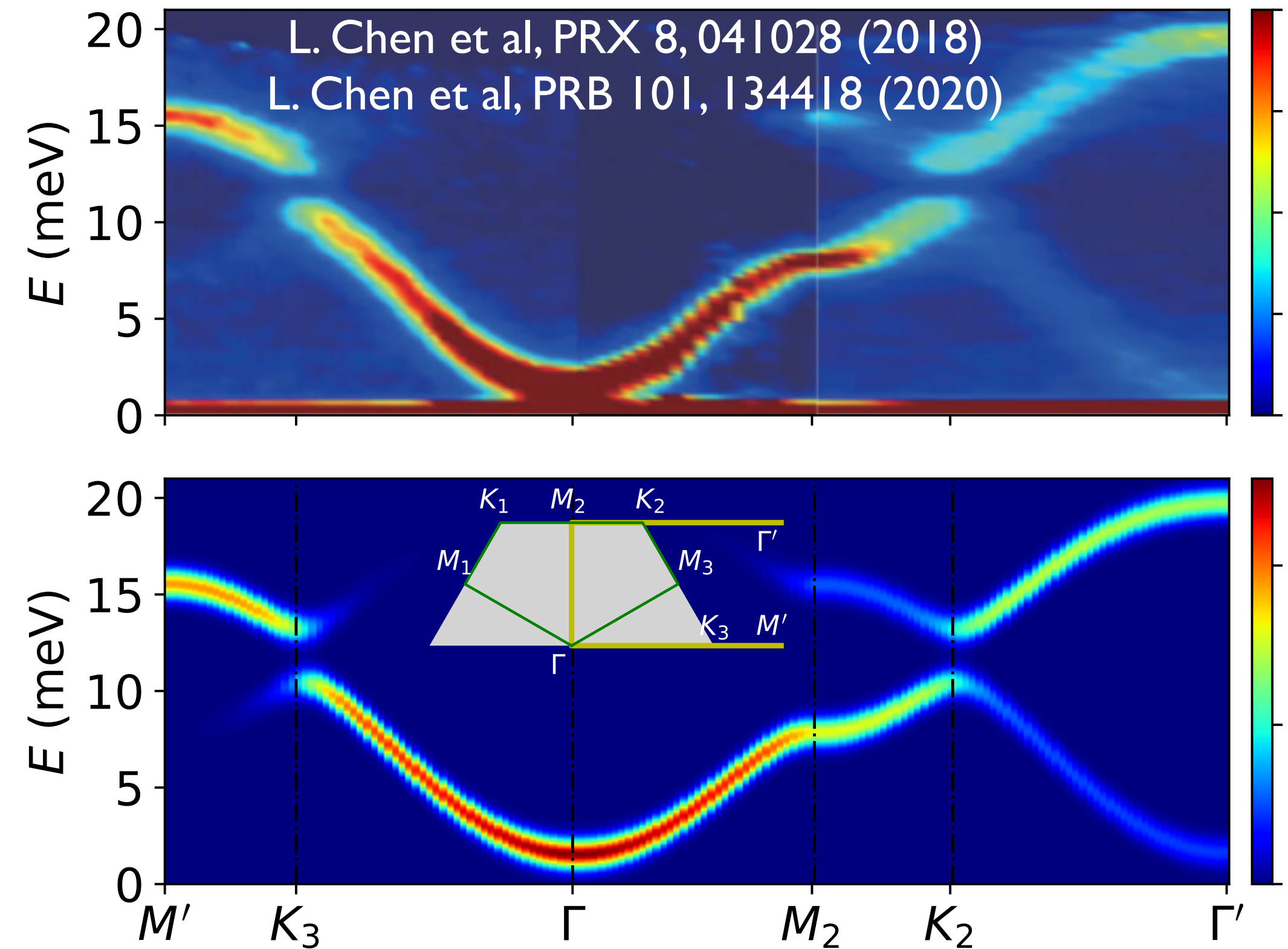
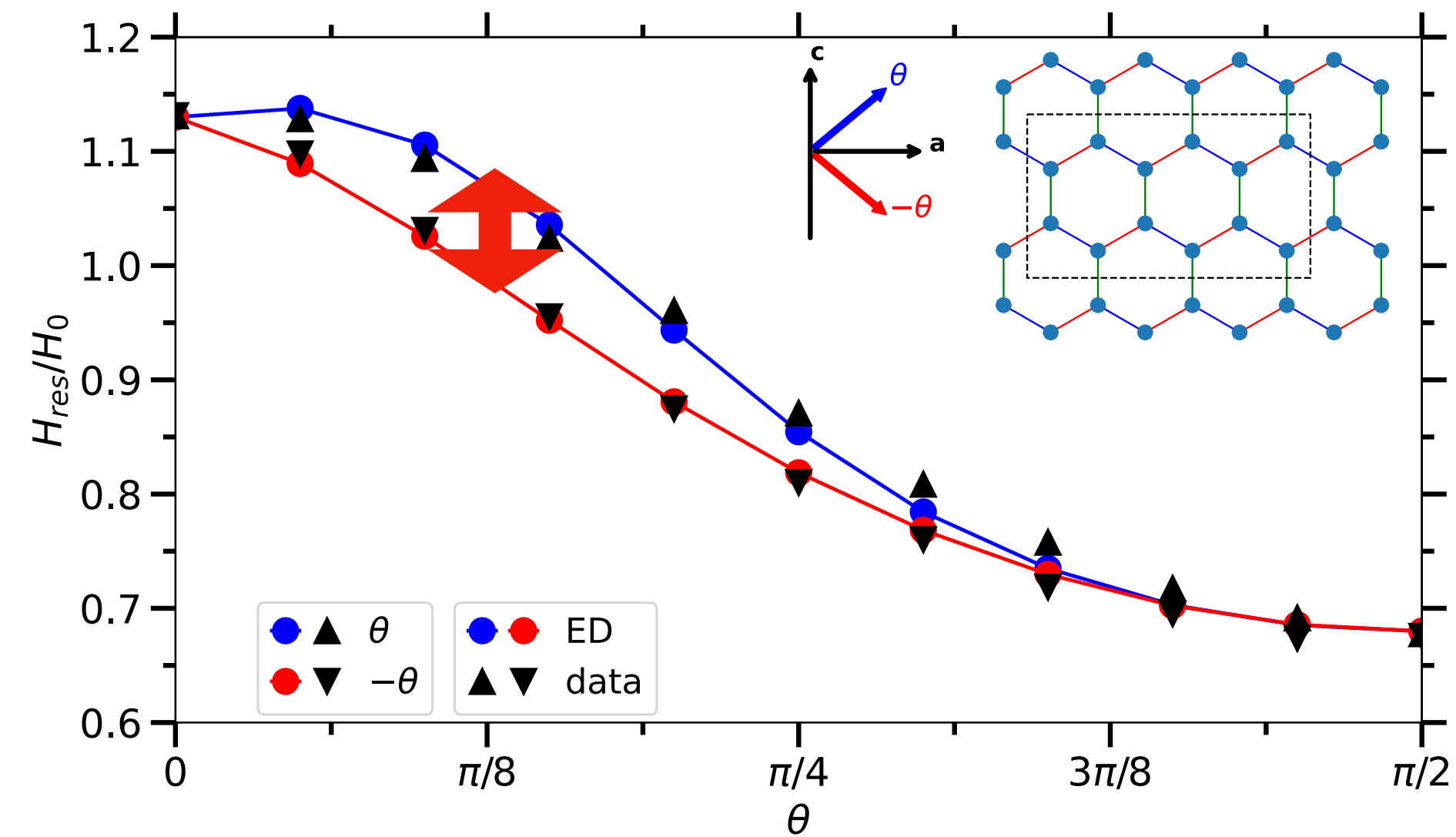
Inhee Lee, et al, Phys. Rev. Lett. 124, 017201 (2020)



J + DM interaction

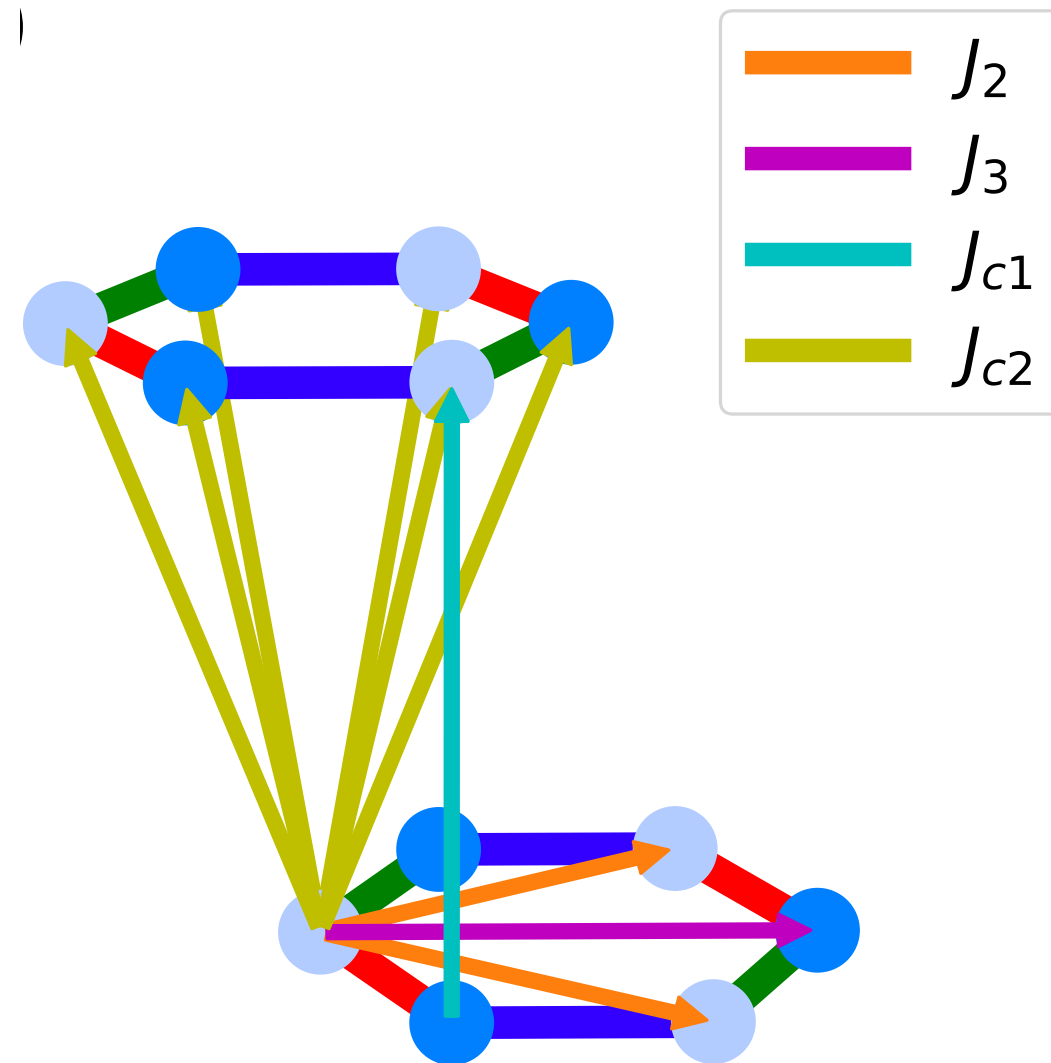
L. Chen et al, PRX 11, 031047 (2021)

Applying our theory



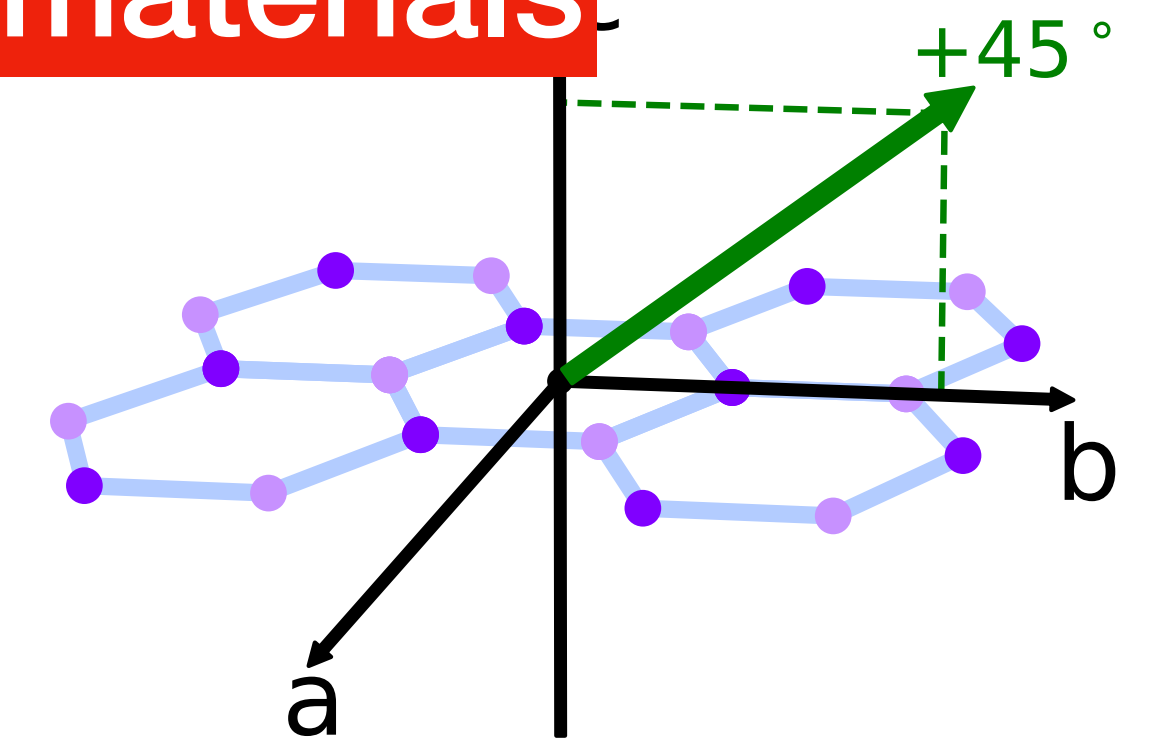
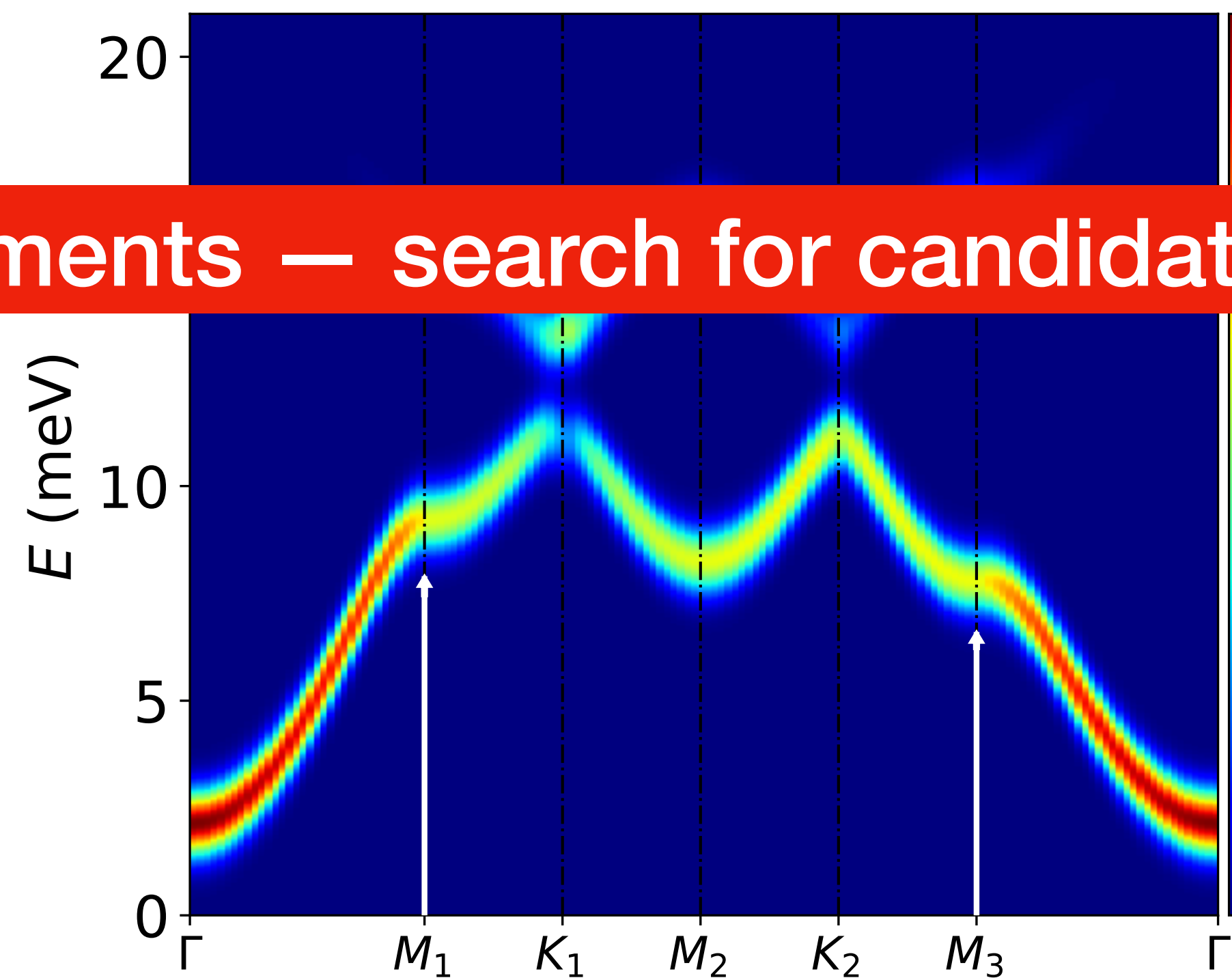
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A_c	-0.23	D_c	0.17

Prediction using our 2nd recipe: CrI3



Interaction	value (meV)	Interaction	value (meV)
J	-2.5	J_2	-0.09
K	1.1	J_3	0.13
Γ	~ 0	J_{c1}	0.048
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A_c	-0.23	D_c	0.17

Future experiments — search for candidate materials



Summary

- Higher-spin S Kitaev interaction:
combination of Hund's at transition metal + SOC in ligand
- Proposal to estimate the Kitaev interaction for general S :
apply magnetic field in the bc -plane;
measure spin excitations at two momenta ($M1$ and $M3$) - broken mirror symmetry
difference is due to Kitaev interaction ($\Gamma, \Gamma' \sim 0$ for spin $> 1/2$)