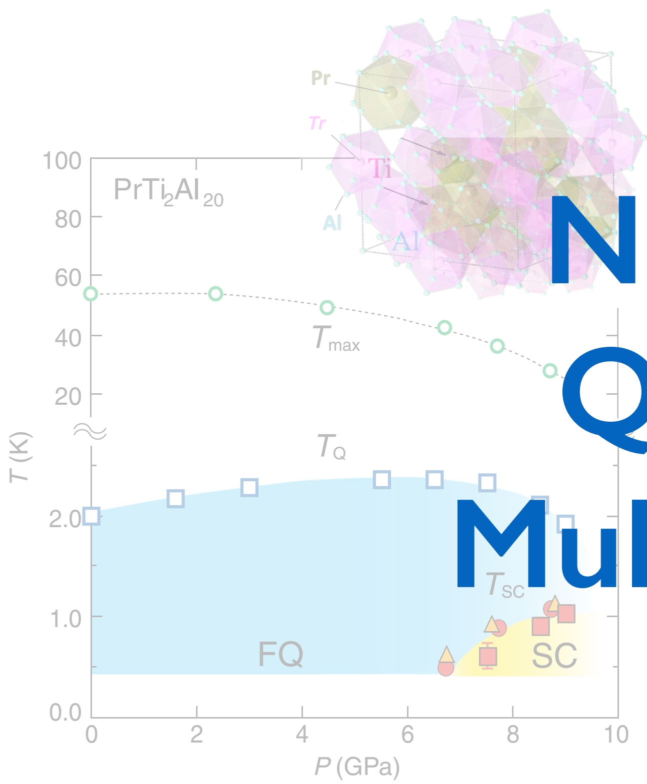
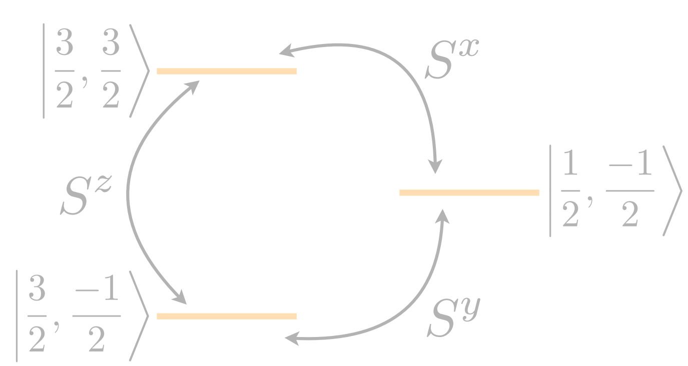
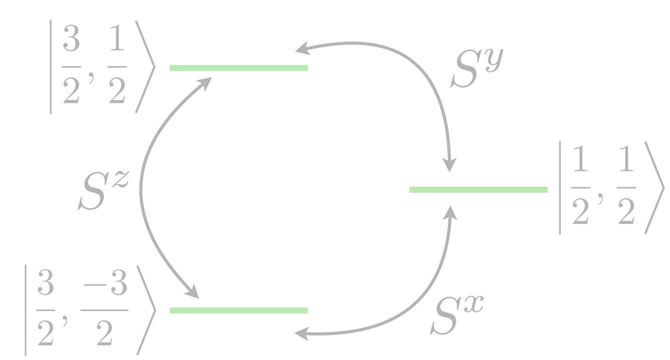


Non-Fermi Liquids and Quantum Criticality in Multipolar Kondo Systems

Yong Baek Kim
University of Toronto

Yukawa Institute
November 21, 2022





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Outline

1. Non-Fermi liquids in multipolar Kondo systems

Single multipolar local moment coupled to conduction electrons

(Focus on cubic Pr^{3+} systems)

RG and Conformal Field Theory

2. Competition with RKKY and quantum criticality

Bose-Fermi Kondo model and quantum criticality

3. Two-stage Kondo destruction and Fermi surface reconstruction: cubic Ce^{3+} systems

Collaborators

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=> MIT

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U of Toronto

A. Patri, YBK, PRX (2020), arXiv:2005.08973

A. Patri, I. Khait, YBK, PR Research (2020), arXiv:1904.02717

Kondo problem

S. Han, D. Schultz, YBK, arXiv:2206.02808 (2022)

S. Han, D. Schultz, YBK, arXiv:2207.07661 (2022)

Kondo + RKKY

Multipolar local moments

Many f-electron localized ions carry multipolar moments

Example:

Pr^{3+}

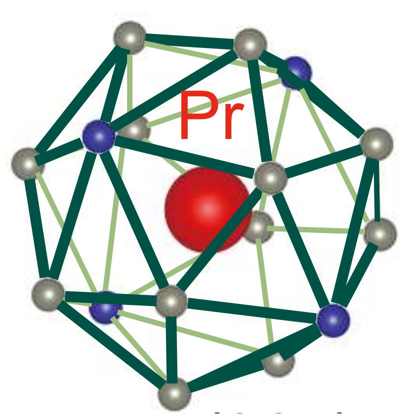
$4f^2$

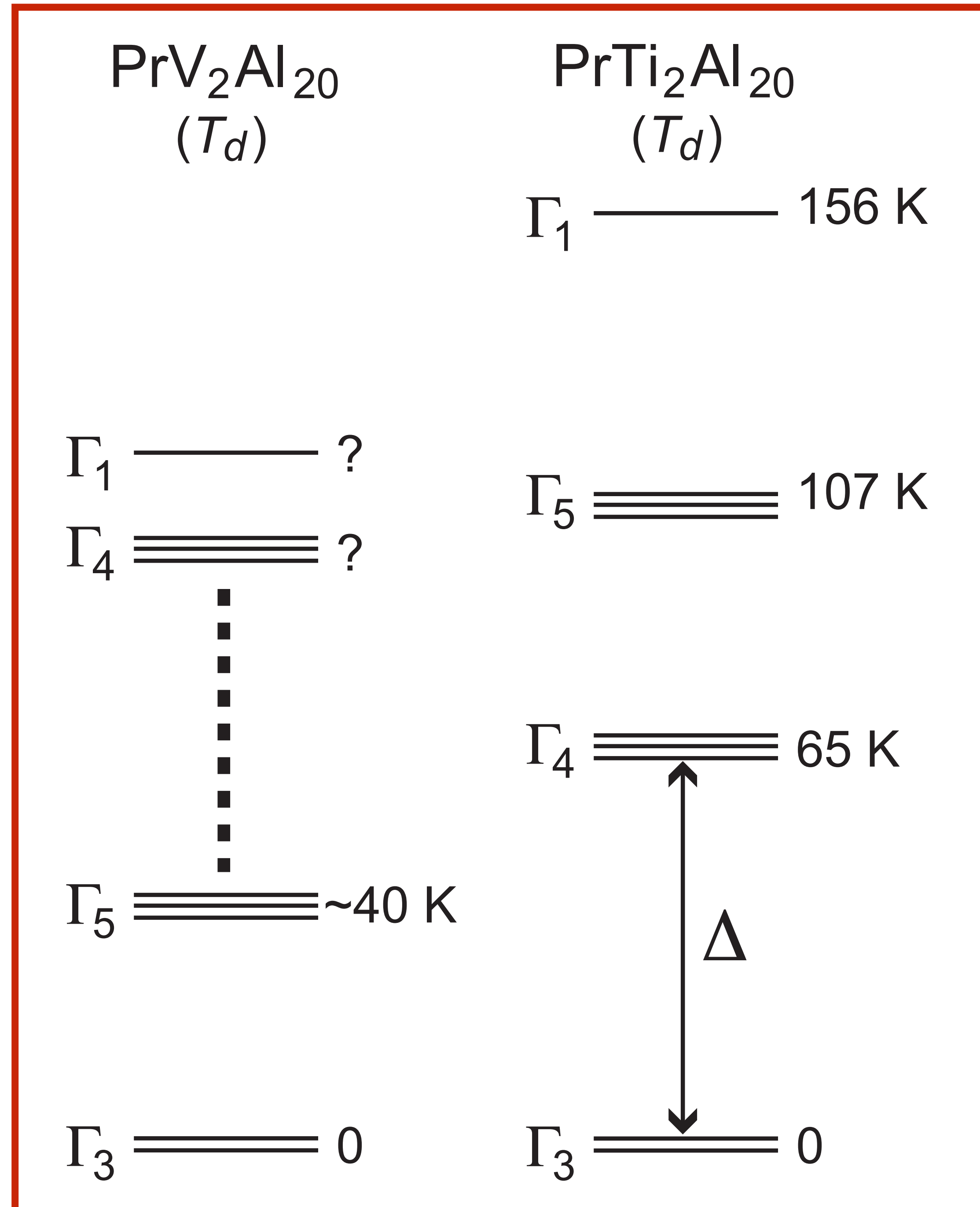
J=4

Crystal Electric Field Splitting

Point Group Symmetry

T_d





Multipolar local moments

Many f-electron localized ions carry multipolar moments

Example:

Pr^{3+}

$4f^2$

J=4

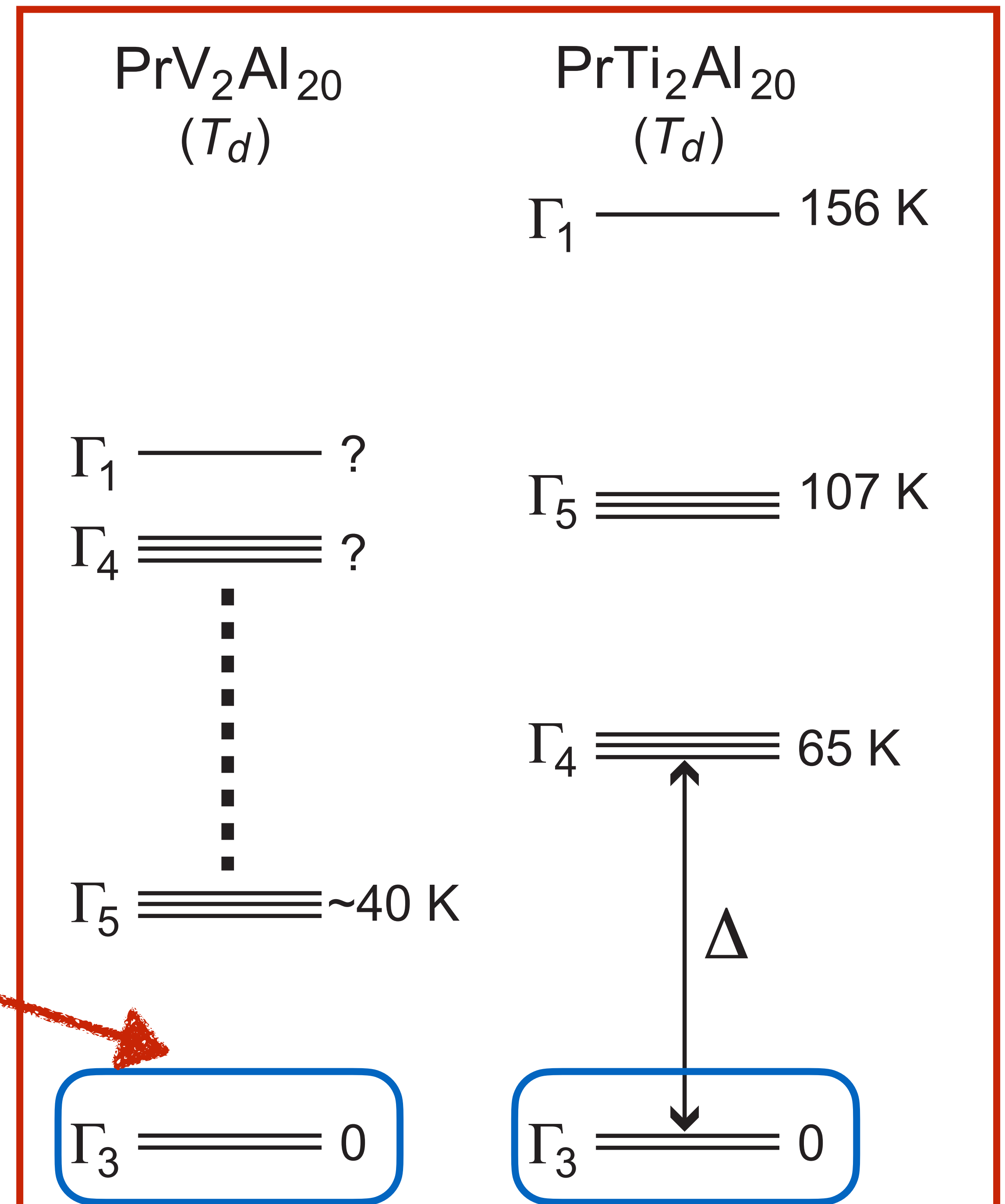
Crystal Electric Field Splitting

Point Group Symmetry

T_d

Non-Kramers Doublet

$\Gamma_3^{(1)}$	$\frac{1}{2}\sqrt{\frac{7}{6}}\|4\rangle - \frac{1}{2}\sqrt{\frac{5}{3}}\|0\rangle + \frac{1}{2}\sqrt{\frac{7}{6}}\|-4\rangle$
$\Gamma_3^{(2)}$	$\sqrt{\frac{1}{2}}\|2\rangle + \sqrt{\frac{1}{2}}\|-2\rangle$



Non-Kramers Doublet: Pr^{3+} $|+\rangle \equiv \Gamma_3^{(1)}$ $|-\rangle \equiv \Gamma_3^{(2)}$

$\langle \pm | J_\alpha | \pm \rangle = 0$ No dipole moment !

But, finite matrix elements for

$O_{22} = \frac{\sqrt{3}}{2}(J_x^2 - J_y^2)$ $O_{20} = \frac{1}{2}(3J_z^2 - J^2)$ Quadrupolar

$T_{xyz} = \frac{\sqrt{15}}{6}\overline{J_x J_y J_z}$ Octupolar

Pseudo-spin basis

$|\uparrow\rangle \equiv \frac{1}{\sqrt{2}}(|\Gamma_3^{(1)}\rangle + i\,|\Gamma_3^{(2)}\rangle)$

$|\downarrow\rangle \equiv \frac{1}{\sqrt{2}}(i\,|\Gamma_3^{(1)}\rangle + |\Gamma_3^{(2)}\rangle)$

$$S^x = -\frac{1}{4}\mathcal{O}_{22}$$

$$S^y = -\frac{1}{4}\mathcal{O}_{20}$$

$$S^z = \frac{1}{3\sqrt{5}}\mathcal{T}_{xyz}$$

$[S^\alpha, S^\beta] = i\epsilon_{\alpha\beta\gamma}S^\gamma$

(S^x, S^y) Quadrupolar — time-reversal even

S^z Octupolar — time-reversal odd

Multipolar local moments with non-Kramers doublet

$PrTi_2Al_{20}$ PrV_2Al_{20}

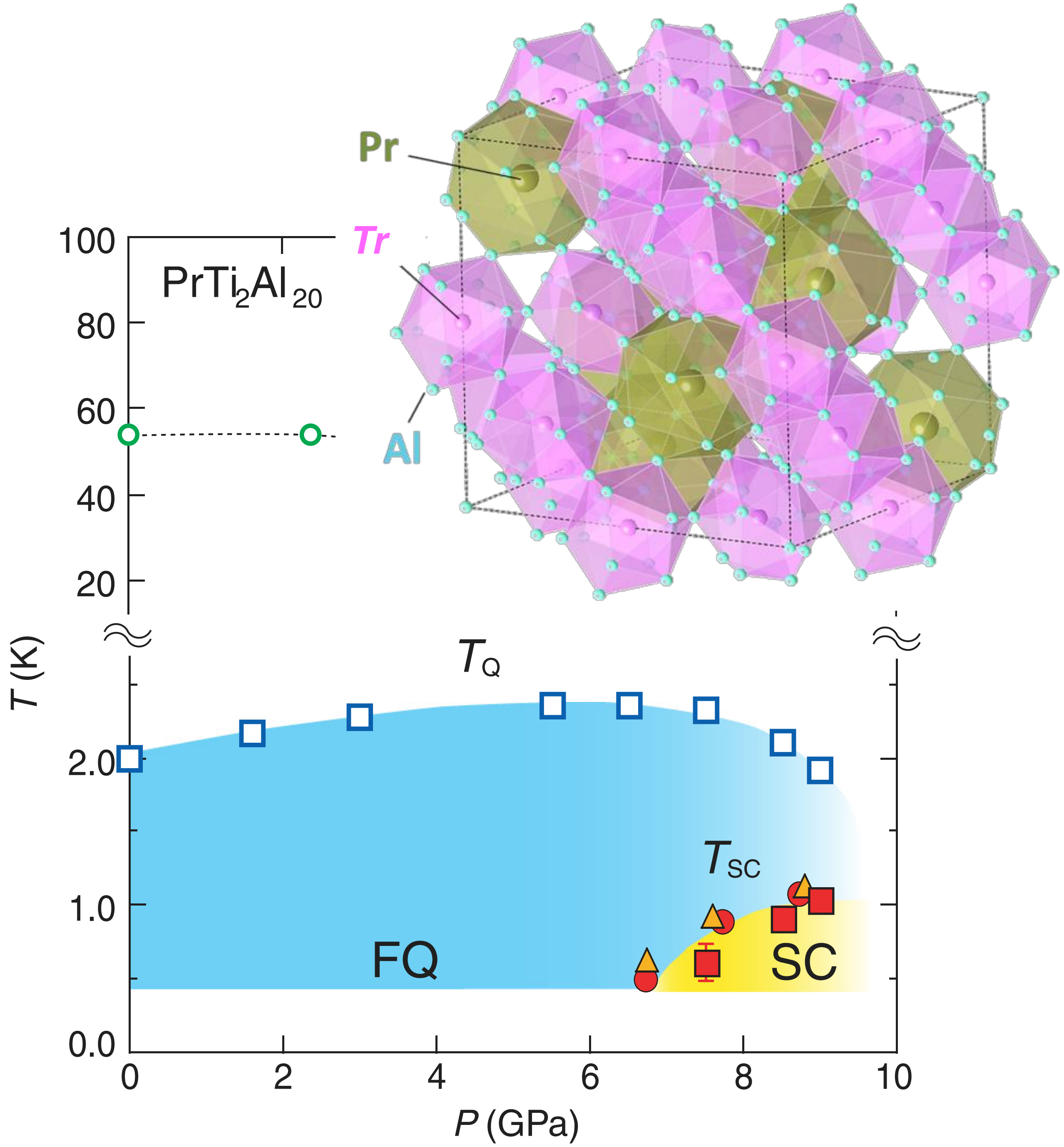
$PrIr_2Zn_{20}$ $PrRh_2Zn_{20}$

$PrNi_2Cd_{20}$ $PrPd_2Cd_{20}$

Pr^{3+} $4f^2$

S. Nakatsuji, A. Sakai,
P. Gegenwart,
T. Onimaru,
B. Maple, …

Pr: Diamond Lattice



Non-Kramers Doublet: Pr^{3+} $|+\rangle \equiv \Gamma_3^{(1)}$ $|-\rangle \equiv \Gamma_3^{(2)}$

Unusual forms of the Kondo and RKKY interaction

First, look at the single impurity Kondo problem !

Pseudo-spin basis

$$|\uparrow\rangle \equiv \frac{1}{\sqrt{2}}(|\Gamma_3^{(1)}\rangle + i\,|\Gamma_3^{(2)}\rangle)$$

$$|\downarrow\rangle \equiv \frac{1}{\sqrt{2}}(i\,|\Gamma_3^{(1)}\rangle + |\Gamma_3^{(2)}\rangle)$$

$$S^x = -\frac{1}{4}\mathcal{O}_{22}$$

$$S^y = -\frac{1}{4}\mathcal{O}_{20}$$

$$S^z = \frac{1}{3\sqrt{5}}\mathcal{T}_{xyz}$$

$$[S^\alpha, S^\beta] = i\epsilon_{\alpha\beta\gamma}S^\gamma$$

(S^x, S^y) Quadrupolar

time-reversal even

S^z Octupolar

time-reversal odd

$PrIr_2Zn_{20}$ Quadrupolar ordering → Dilution → $Y_{1-x}Pr_xIr_2Zn_{20}$ Non-Fermi liquid

PHYSICAL REVIEW LETTERS **121,** 077206 (2018)

Single-Site Non-Fermi-Liquid Behaviors in a Diluted $4f^2$ System $Y_{1-x}Pr_xIr_2Zn_{20}$

Y. Yamane,[1] T. Onimaru,[1,*] K. Wakiya,[1,†] K. T. Matsumoto,[1,‡] K. Umeo,[2] and T. Takabatake[1]
[1]*Graduate School of Advanced Sciences of Matter, Hiroshima University, Higashi-Hiroshima 739-8530, Japan*
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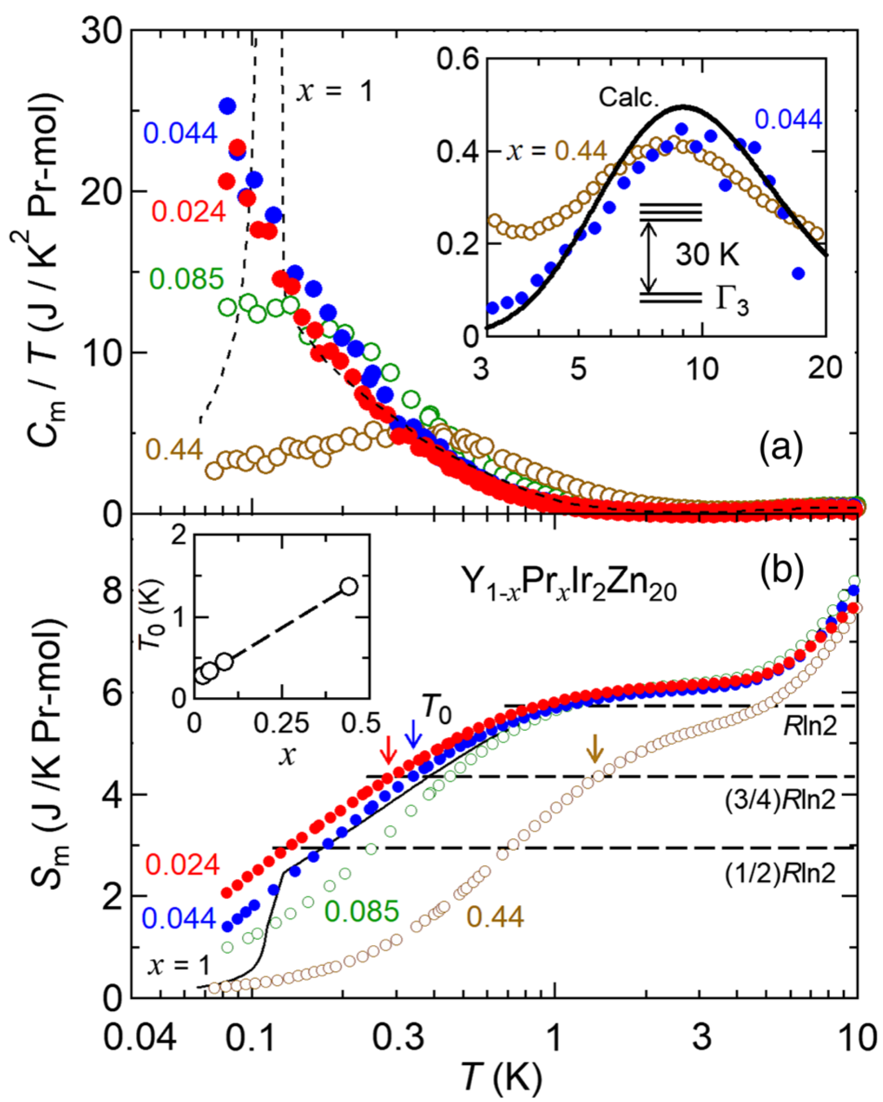

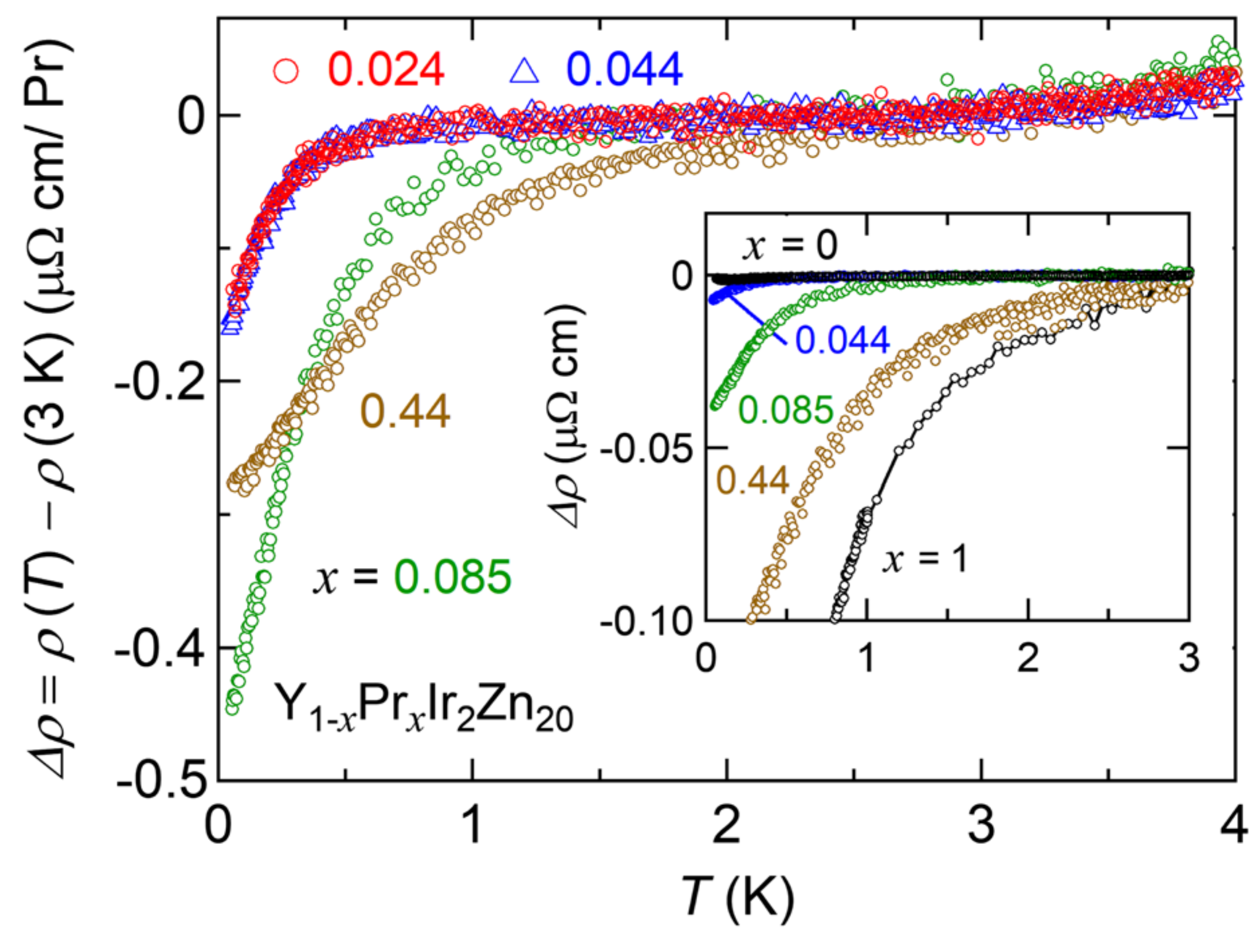

How to model conduction electrons ?

Local symmetry around Pr ion is T_d

Fermi pockets around zone center
(quantum oscillations, S. Nagashima et al, 2014)

Conduction electron orbitals can be classified in terms of irreducible representation of T_d

A_1	xyz, $x^4+y^4+z^4$, $x^2y^2z^2$
A_2	$x^4(y^2-z^2)+y^4(z^2-x^2)+z^4(x^2-y^2)$
E	$\{x^2-y^2,\ 2z^2-x^2-y^2\}$
T_1	$\{x(z^2-y^2),\ y(z^2-x^2),\ z(x^2-y^2)\}$
T_2	$\{x,\ y,\ z\}$, $\{xy,\ xz,\ yz\}$,

For concrete models, we consider

T_2 (p- or t_{2g} d-orbitals) - "molecular" orbitals

The "Kondo" model

S^x, S^y, S^z couple to fermion spin-orbital 'currents' that transform exactly in the same way

For example,

$$H_3 = K_3 S^z \Big[\sigma^x_{\alpha\beta}(c^\dagger_{p_y,\alpha} c_{p_z,\beta} + h.c.) + \sigma^y_{\alpha\beta}(c^\dagger_{p_z,\alpha} c_{p_x,\beta} + h.c.) + \sigma^z_{\alpha\beta}(c^\dagger_{p_x,\alpha} c_{p_y,\beta} + h.c.)\Big]$$

$\alpha, \beta = \uparrow, \downarrow$

K_1, K_2, K_3 three independent couplings

Entangled fluctuations of both orbital and spin !

Perform RG computations up to 3rd order:
two stable fixed points

A. Patri, I. Khait, YBK, PR Research 2, 013257 (2020)

The "Kondo" model

S^x, S^y, S^z couple to fermion spin-orbital 'currents' that transform exactly in the same way

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$\alpha, \beta = \uparrow, \downarrow$

K_1, K_2, K_3 three independent couplings

L=1, S=1/2 → Spin-orbital entangled basis

$|j, j_z\rangle = |j = 3/2, j_z\rangle, \ |j = 1/2, j_z\rangle$

$\lvert p_x, \uparrow\rangle$	$\lvert p_x, \downarrow\rangle$
$\lvert p_y, \uparrow\rangle$	$\lvert p_y, \downarrow\rangle$
$\lvert p_z, \uparrow\rangle$	$\lvert p_z, \downarrow\rangle$

$\lvert 3/2, 3/2\rangle$	$\lvert 3/2, 1/2\rangle$
$\lvert 3/2, -1/2\rangle$	$\lvert 3/2, -3/2\rangle$
$\lvert 1/2, 1/2\rangle$	$\lvert 1/2, -1/2\rangle$

Fixed point I Two channel Kondo model !

Only j=3/2 electrons are involved

$$H_{\text{tot}} = S^x\left(\tau_A^x + \tau_B^x\right) - S^y\left(\tau_A^z + \tau_B^z\right) + S^z\left(\tau_A^y + \tau_B^y\right)$$

$$\left|\frac{3}{2},\frac{3}{2}\right\rangle \quad \left|\frac{3}{2},\frac{-1}{2}\right\rangle$$

$$(\uparrow_A, \quad \downarrow_A)$$

conduction electron pseudospin for channel A $\vec{\tau}_A$

$$\left|\frac{3}{2},\frac{1}{2}\right\rangle \quad \left|\frac{3}{2},\frac{-3}{2}\right\rangle$$

$$(\uparrow_B, \quad \downarrow_B)$$

conduction electron pseudospin for channel B $\vec{\tau}_B$

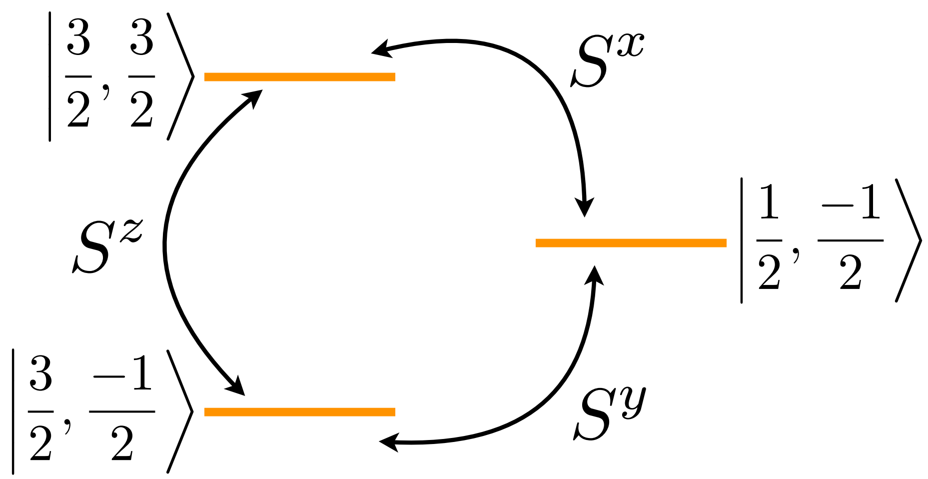
$$C_v \propto T\,\ln T$$

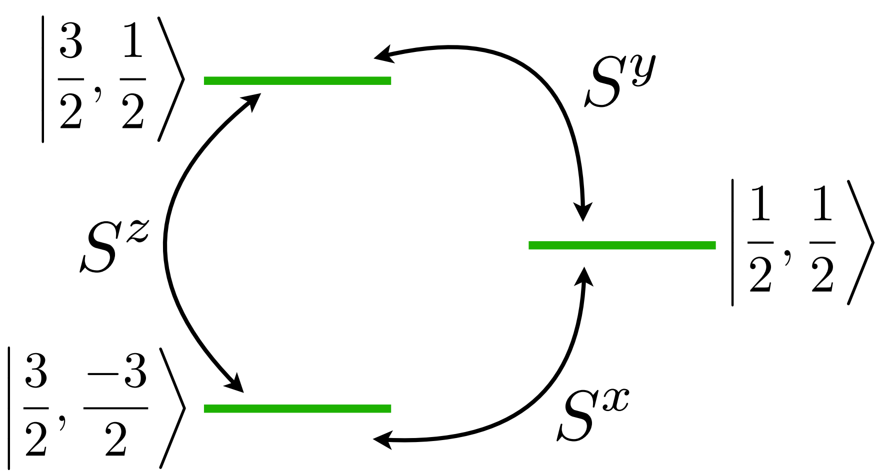
$$\rho \propto \sqrt{T}$$

A. Patri, YBK, PRX, arXiv:2005.08973 (2020)

Fixed point II

All of j=3/2 and j=1/2 electrons are involved





Two channels

But each channel involves three flavors of conduction electrons - SU(3) ‘current’

This is not the usual Kondo problem

A. Patri, YBK, PRX, arXiv:2005.08973 (2020)

Fixed point II

From the perturbative RG

Approaching the fixed point by fixing the ratios of the couplings

$$K_1 = -\frac{1}{2\sqrt{6}} g_k, \; K_2 = -\frac{1}{12\sqrt{2}} g_k, \; K_3 = -\frac{1}{4\sqrt{3}} g_k$$

$$\frac{dg_k}{dl} = \frac{g_k^2}{4} - \frac{g_k^3}{4} \approx -\frac{1}{4}(g_k - g_k^*)$$

$g_k^* = 1$ at the fixed point

$1 + \Delta$ dimension of the leading irrelevant operator

$\Delta = 1/4$

$$C_v \propto T^{2\Delta} = T^{1/2} \qquad \rho \propto T^{\Delta} = T^{1/4}$$

Fixed point II

True nature of this new fixed point ?

Rewriting the Kondo coupling

$$H_k = g_k \sum_{m=1,2} \vec{\psi}^\dagger_m(0) \left[\frac{S^x}{2}\frac{\lambda^4}{2} + \frac{S^y}{2}\frac{\lambda^6}{2} + \frac{S^z}{2}\frac{\lambda^2}{2} \right] \vec{\psi}_m(0)$$

$\lambda^a \qquad a = 1, ..., 8$ **3x3 SU(3) Gell-Mann matrices**

$$\vec{\psi}^\dagger_{m=1} = \left(-c^\dagger_{\frac{3}{2},\frac{3}{2}}, -c^\dagger_{\frac{3}{2},\frac{-1}{2}}, c^\dagger_{\frac{1}{2},\frac{-1}{2}} \right)$$

$$\vec{\psi}^\dagger_{m=2} = \left(c^\dagger_{\frac{3}{2},-\frac{3}{2}}, c^\dagger_{\frac{3}{2},\frac{1}{2}}, c^\dagger_{\frac{1}{2},\frac{1}{2}} \right)$$

$g_k^* = 1$

at the perturbative fixed point

This is not an SU(3) Kondo problem …

Current Algebra and Conformal Field Theory (Results)

Useful to generalize this to *k* channels   Here $k = 2$

$$g_k^* = \frac{2}{k+3} \qquad \Delta = \frac{2}{4k+2}$$

$1 + \Delta$ dimension of the leading irrelevant operator

$$C_v \propto T^{2\Delta} \qquad \rho \propto T^{\Delta}$$

Perturbative regime $k \gg 1$

$$g_k^* \to 2/k \qquad \Delta \to 1/2k$$

$$k = 2 \qquad g_k^* \to 1 \qquad \Delta \to 1/4$$

recover the perturbative results !

Exact $k = 2$

$$g_k^* = 2/5$$

$$\Delta = 1/5$$

$$C_v/T \propto T^{-3/5}$$

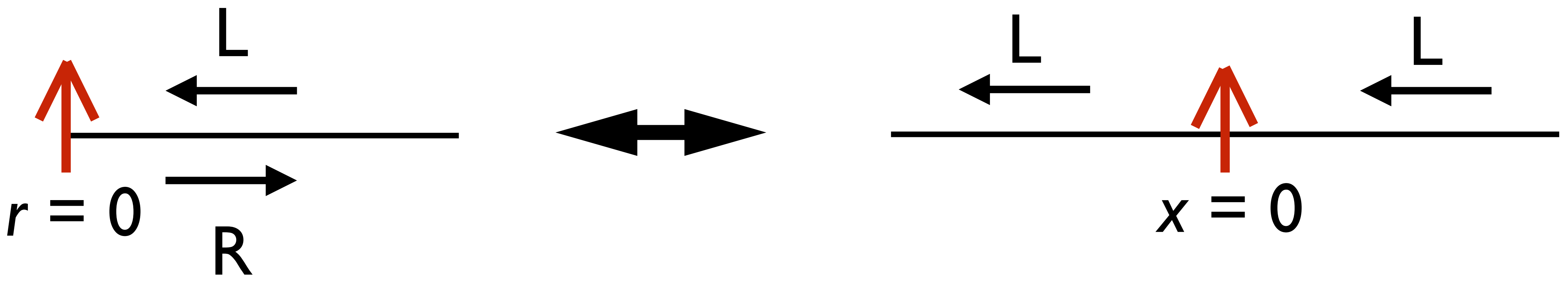
$$\rho \propto T^{1/5}$$

A. Patri, YBK, PRX, arXiv:2005.08973 (2020)

Current Algebra and Conformal Field Theory Solution (Schematic)

Consider δ-function interaction and s-wave scattering

Mapping to 1+1 dimensional (chiral) model



$$\psi_L(x,\tau) = \psi_L(z = \tau + ix)$$

$$\psi_R(x,\tau) \equiv \psi_L(-x,\tau)$$

$$\psi_R(x,\tau) = \psi_R(z^* = \tau - ix)$$

$$\psi_L(0,\tau) = \psi_R(0,\tau)$$

Current Algebra and Conformal Field Theory Solution (Schematic)

Bulk free theory (before coupling to impurity)

Current algebra and conformal embedding

$$\mathrm{U}(1)\times\mathrm{SU}(3)_2\times\mathrm{SU}(2)_3$$

Charge Spin-Orbital Flavor(channel)

3 = three spin-orbital bands in each channel

2 = two channels

$$H_0 = \frac{1}{12} : JJ : (z) + \frac{1}{5} : J^a J^a : (z) + \frac{1}{5} : J^A J^A : (z)$$

Charge Spin-Orbital Flavor(channel)

$a = 1, ..., 8$ $A = 1, ..., 3$

A. Patri, YBK, arXiv:2005.08973 (2020)

Current Algebra and Conformal Field Theory Solution (Schematic)

Bulk free theory (before coupling to impurity)

Current algebra and conformal embedding

$$\mathrm{U}(1)\times\mathrm{SU}(3)_2\times\mathrm{SU}(2)_3$$

Charge Spin-Orbital Flavor(channel)

But only three of eight generators of SU(3) are coupled to the impurity

$$H_K = g_k\left(J^4(z)\frac{S^x}{2} + J^6(z)\frac{S^y}{2} + J^2(z)\frac{S^z}{2}\right)$$ Kondo coupling

We need a different conformal embedding …

A. Patri, YBK, arXiv:2005.08973 (2020)

Current Algebra and Conformal Field Theory Solution (Schematic)

$\mathrm{U}(1)\times\mathrm{SU}(3)_2\times\mathrm{SU}(2)_3$ Bulk free theory

Charge Spin-Orbital Flavor(channel) Conformal embedding

Coset construction

$$\mathrm{SU}(3)_2 = [\text{3-state Potts model}] \times \widetilde{\mathrm{SU}}(2)_8$$

$$J^2, J^4, J^6 \in \widetilde{\mathrm{SU}}(2)_8$$

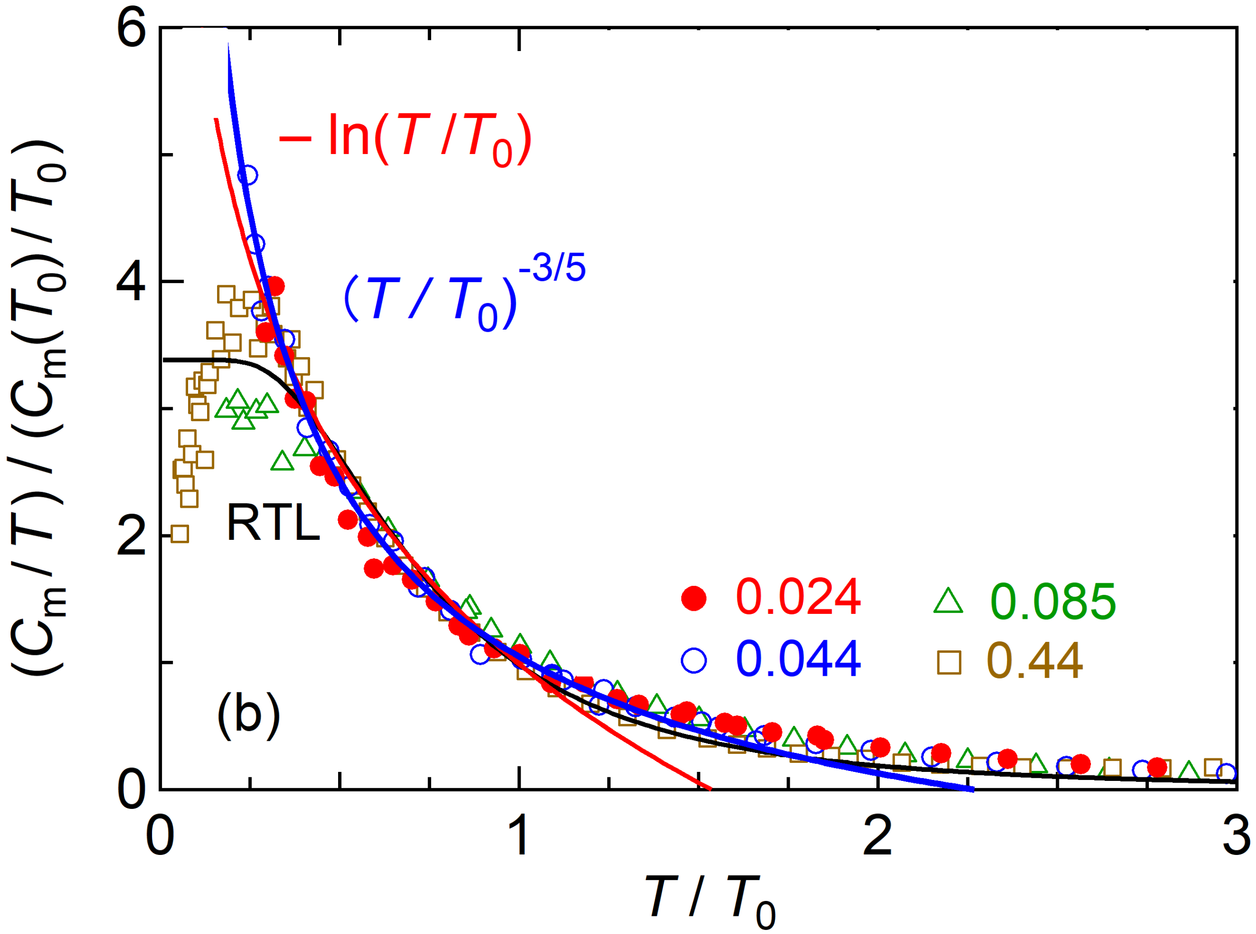
Kondo coupling imposes (conformally invariant) boundary conditions on the currents

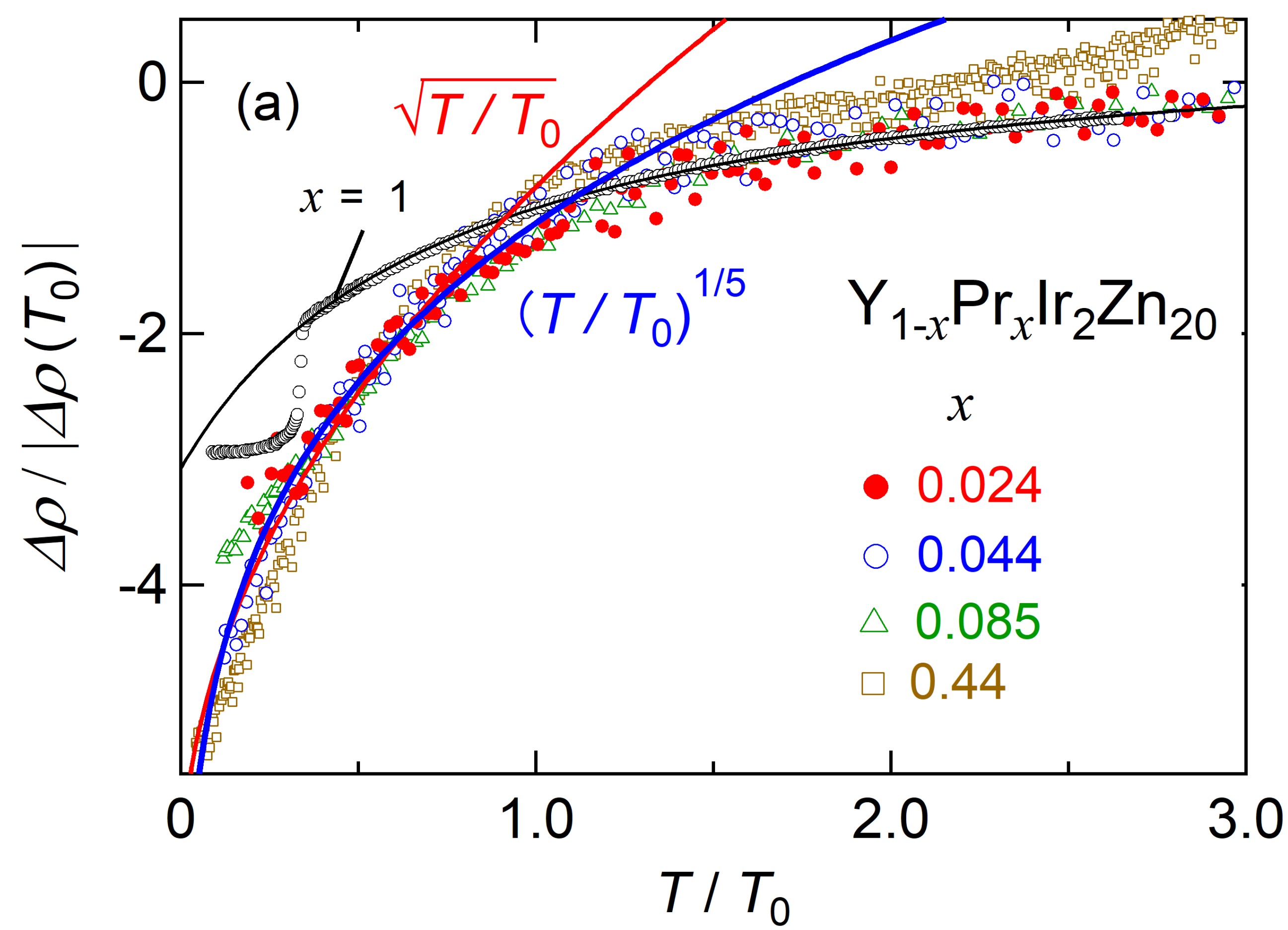
↓ Boundary CFT (John Cardy)

Affleck + Ludwig (1991)

Rearrangement of the conformal towers

Experiments ?





Y. Yamane, T. Onimaru (2018, 2020)

The fits are encouraging ! We need sharper predictions …

Need more systematic understanding as to

Quadrupolar ordering (non-diluted) ⟶ "Kondo" regime (diluted)

Competition with RKKY: Bose-Fermi Kondo model

$$H = H_{\text{Fermi Kondo}} + H_{\text{Bose Kondo}}$$

$$H_{\text{RKKY}} = \sum_{ij} \left[J_{ij}^Q (S_i^x S_j^x + S_i^y S_j^y) + J_{ij}^O S_i^z S_j^z \right]$$

$$\rightarrow \quad H_{\text{Bose Kondo}} = g_Q (S^x \phi_0^x + S^y \phi_0^y) + g_O S^z \phi_0^z$$
$$+ \sum_{\mathbf{k}} \left[\Omega_{Q\mathbf{k}} (\phi_{\mathbf{k}}^{x\dagger} \phi_{\mathbf{k}}^x + \phi_{\mathbf{k}}^{y\dagger} \phi_{\mathbf{k}}^y) + \Omega_{O\mathbf{k}} \phi_{\mathbf{k}}^{z\dagger} \phi_{\mathbf{k}}^z \right]$$

Boson density of states

$$D_i(\omega) = C_i |\omega|^{1-\epsilon_i} \text{sgn}(\omega) \qquad i = Q, O$$

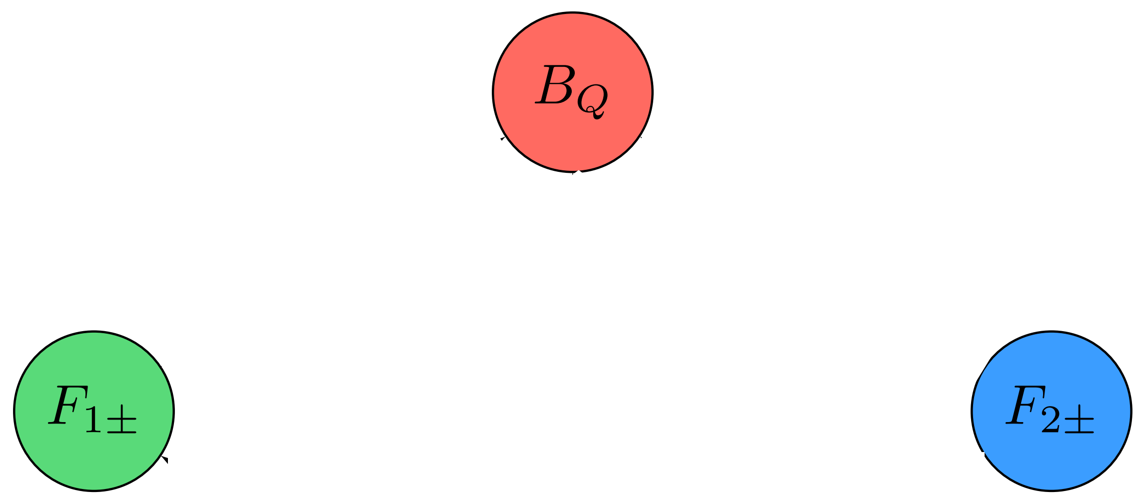
This is a local approximation of the RKKY interaction in the spirit of DMFT

J. L. Smith, Q. Si (96,97), A. M. Sengupta (97)

S. Han, D. Schultz, YBK, arXiv:2206.02808 (2022)

Renormalization Group Analysis

RKKY Quadrupolar Ordering (Small FS)

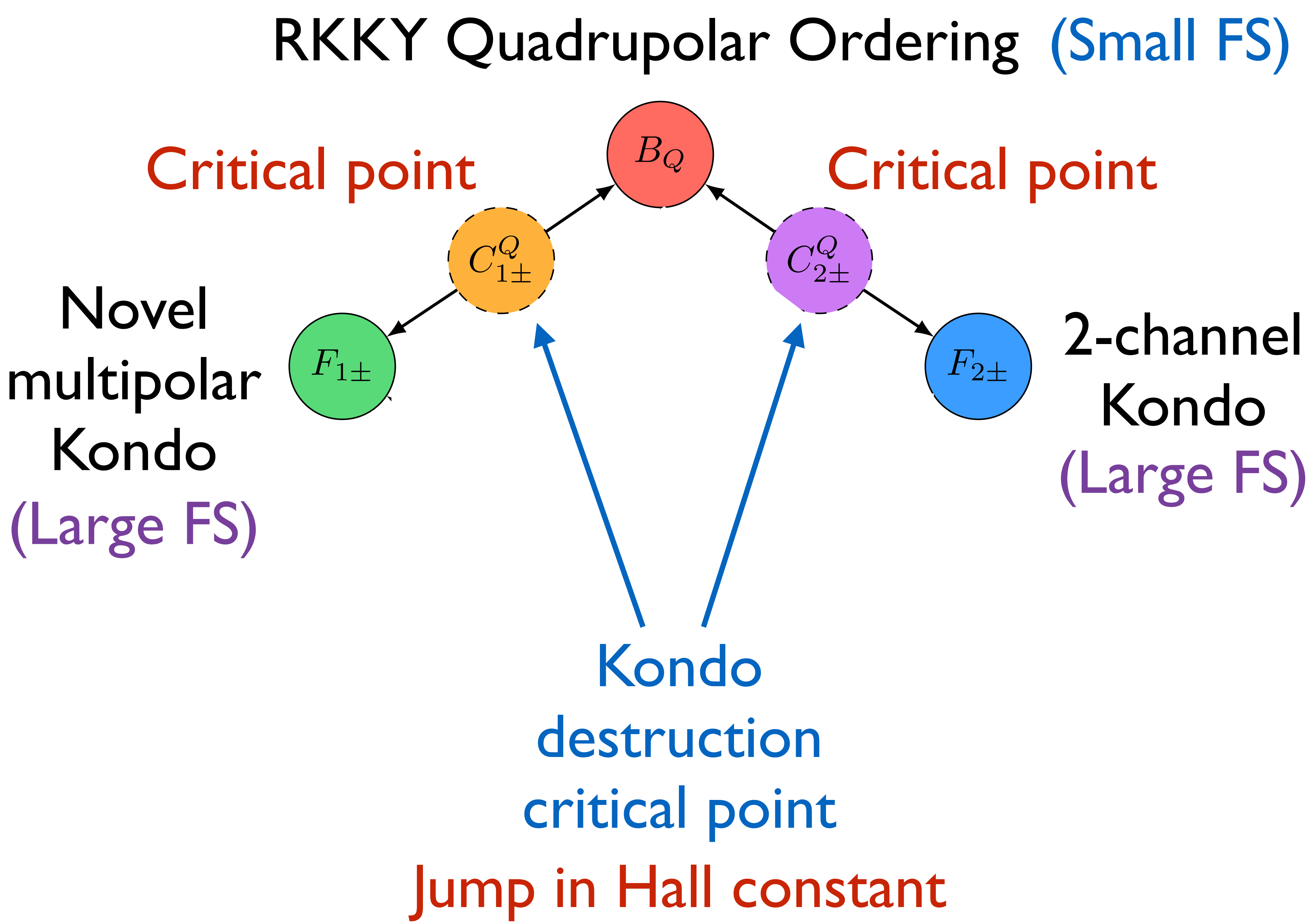


Novel multipolar Kondo (Large FS)

2-channel Kondo (Large FS)

S. Han, D. Schultz, YBK, arXiv:2206.02808 (2022)

Renormalization Group Analysis



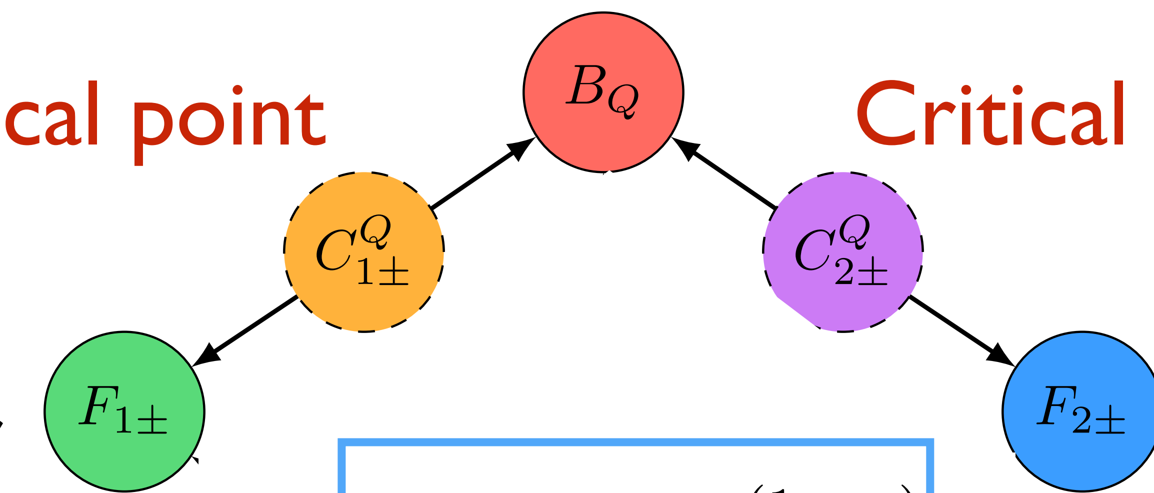
S. Han, D. Schultz, YBK, arXiv:2206.02808 (2022)

Renormalization Group Analysis

RKKY Quadrupolar Ordering (Small FS)

Critical point

Critical point



Novel multipolar Kondo (Large FS)

2-channel Kondo (Large FS)

$$\chi_i(T) \propto T^{-(1-\gamma_i)}$$

$$i = Q, O$$

Quadrupolar susceptibility $\chi^Q(\tau) = \langle T_\tau S^{x,y}(\tau) S^{x,y}(0)\rangle \propto \left(\frac{\tau_0}{|\tau|}\right)^{\gamma_Q}$

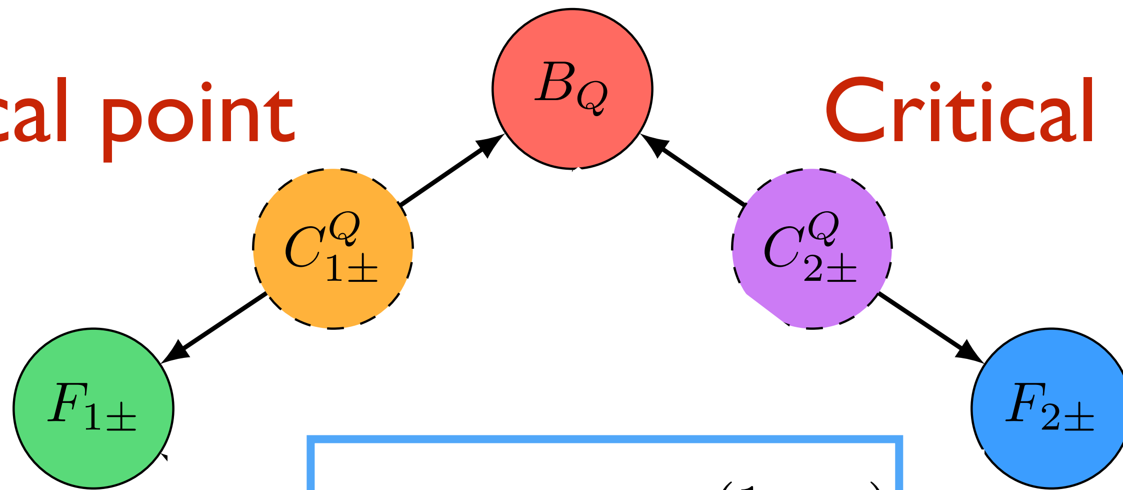
Octupolar susceptibility $\chi^O(\tau) = \langle T_\tau S^{z}(\tau) S^{z}(0)\rangle \propto \left(\frac{\tau_0}{|\tau|}\right)^{\gamma_O}$

Renormalization Group Analysis

RKKY Quadrupolar Ordering (Small FS)

Critical point

Critical point



Novel multipolar Kondo (Large FS)

2-channel Kondo (Large FS)

$$\chi_i(T) \propto T^{-(1-\gamma_i)}$$

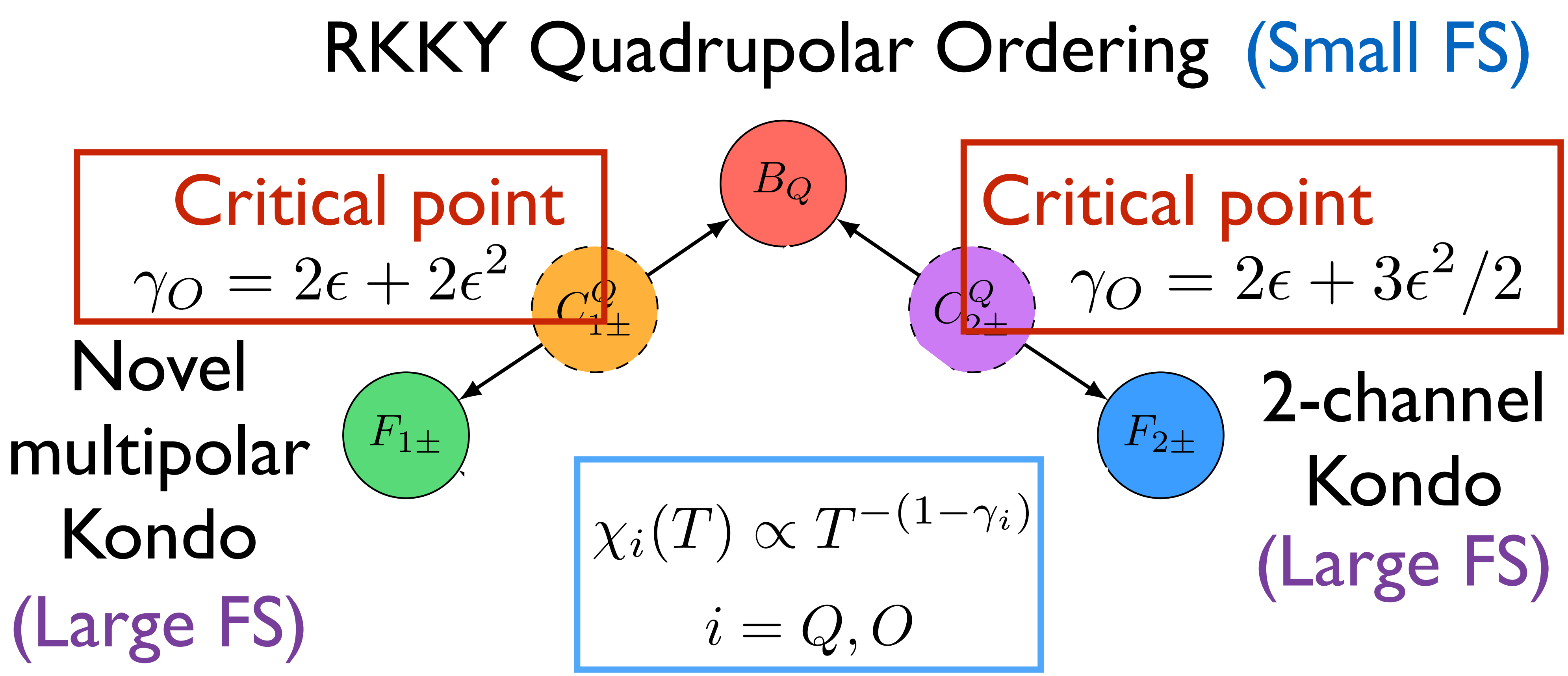
$$i = Q, O$$

Measurement via elastic constants (ultrasound)

$$(C_{11} - C_{12}) = (C^0_{11} - C^0_{12}) - (s^2_Q)\chi'_Q,$$

$$C_{44} = C^0_{44} - (s^2_O h^2)\chi'_O,$$

Renormalization Group Analysis



Measurement via elastic constants (ultrasound)

$$(C_{11} - C_{12}) = (C^0_{11} - C^0_{12}) - (s^2_Q)\chi'_Q,$$

$$C_{44} = C^0_{44} - (s^2_O h^2)\chi'_O,$$

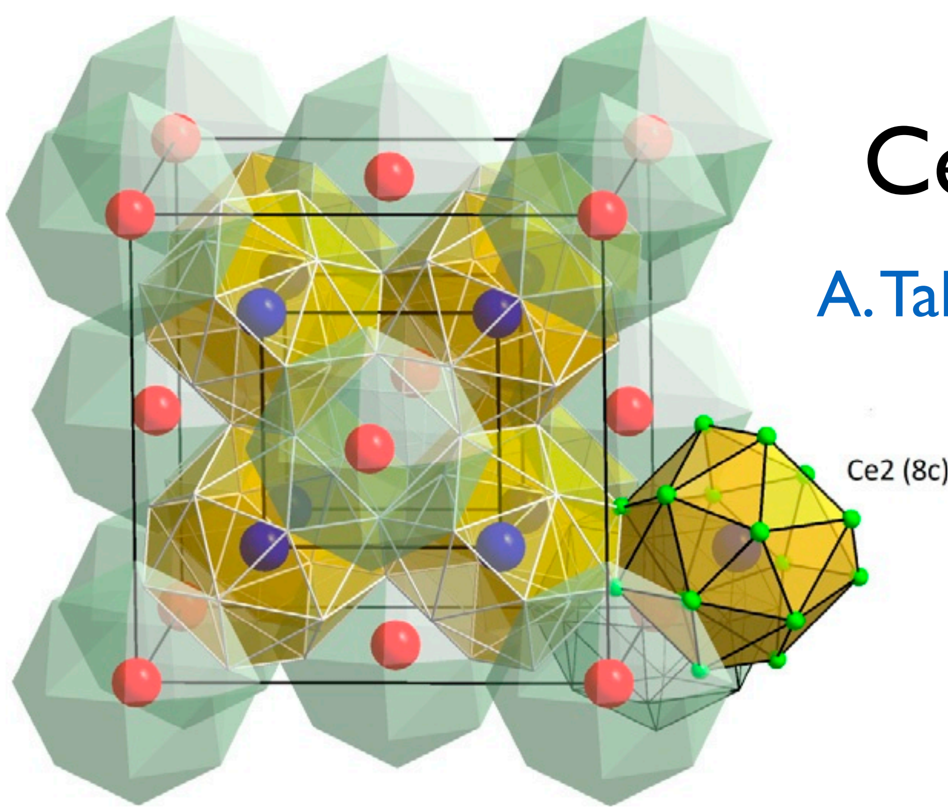
Multipolar local moments with two-degenerate Kramers doublets (Quartet)

Ce^{3+} [Xe]$4f^1$ J=5/2

$Ce_3Pd_{20}Si_6$ S. Paschen et al (2008)

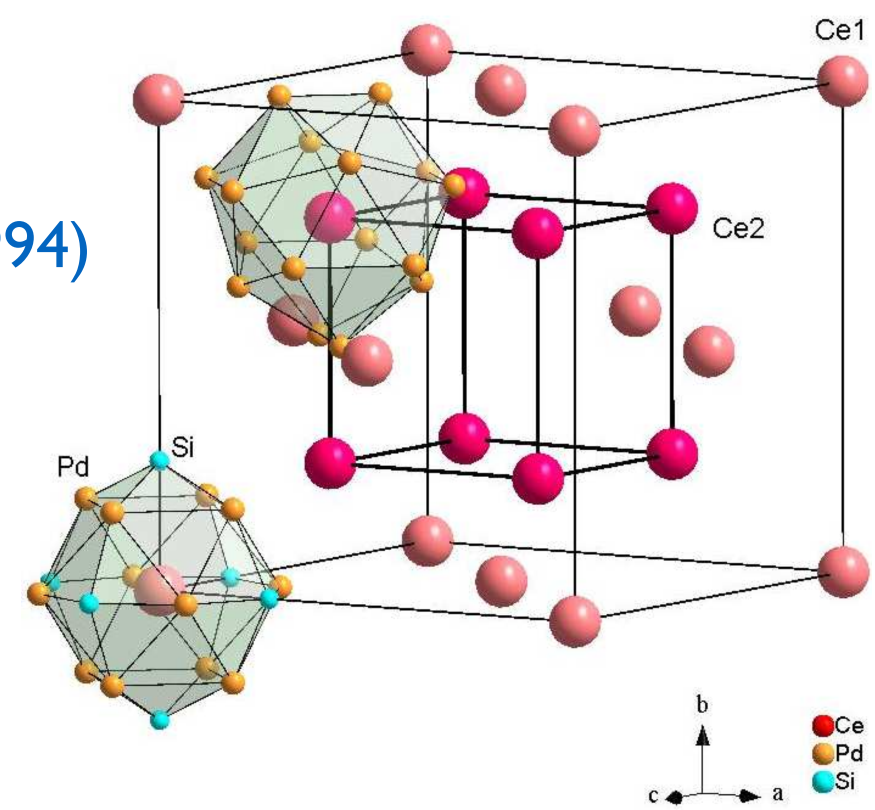
$Ce_3Pd_{20}Ge_6$ J.Kitagawa et al (1996)

In cubic systems, two lowest Kramers doublet are degenerate, which makes 4-fold degenerate Γ_8 CEF level



$Ce_3Pd_{20}Si_6$

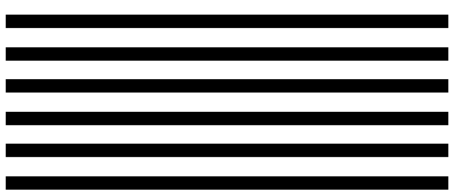
A. Takeda et al (1994)



Crystal field splitting

Ce^{+3}

$4f^1$

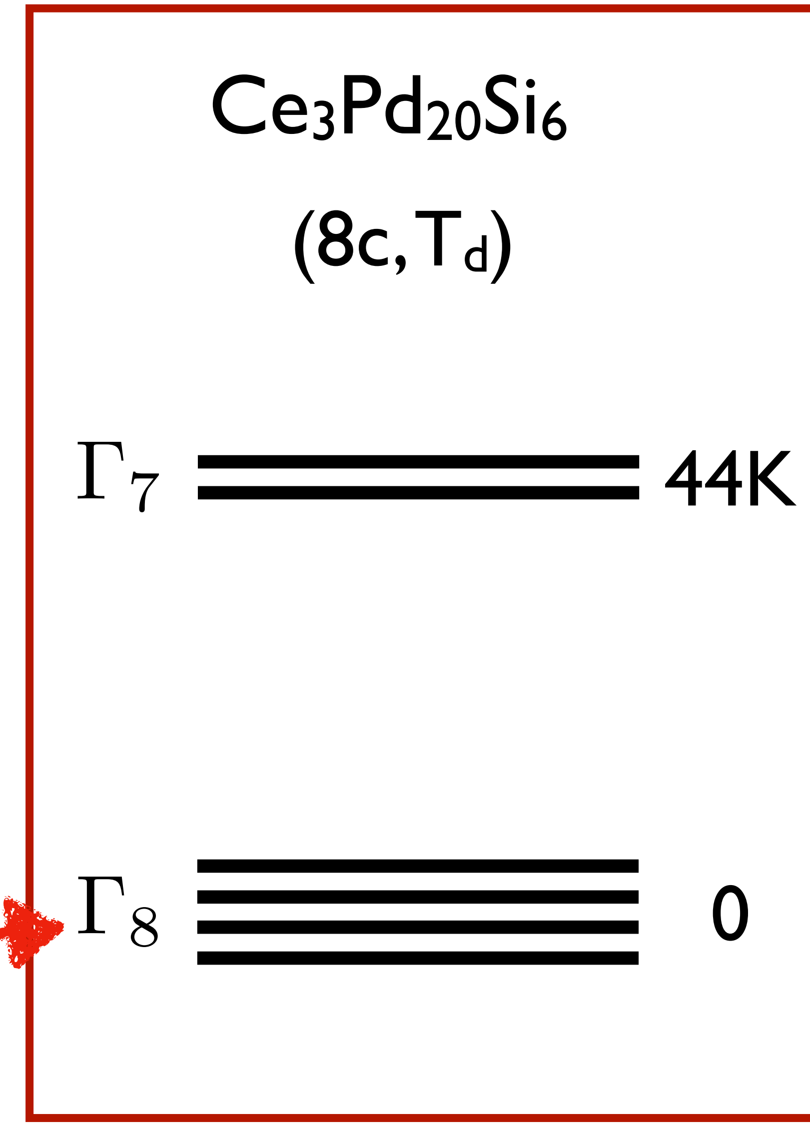


j=5/2

Crystal Electric Field Splitting

Point group symmetry (8c)

T_d



S. Paschen, et al (2008)

Γ_8 Quartet

$$\Gamma_8^{(1)} = \sqrt{\frac{5}{6}}|+\frac{5}{2}\rangle + \sqrt{\frac{1}{6}}|-\frac{3}{2}\rangle, \quad \Gamma_8^{(3)} = |+\frac{1}{2}\rangle,$$

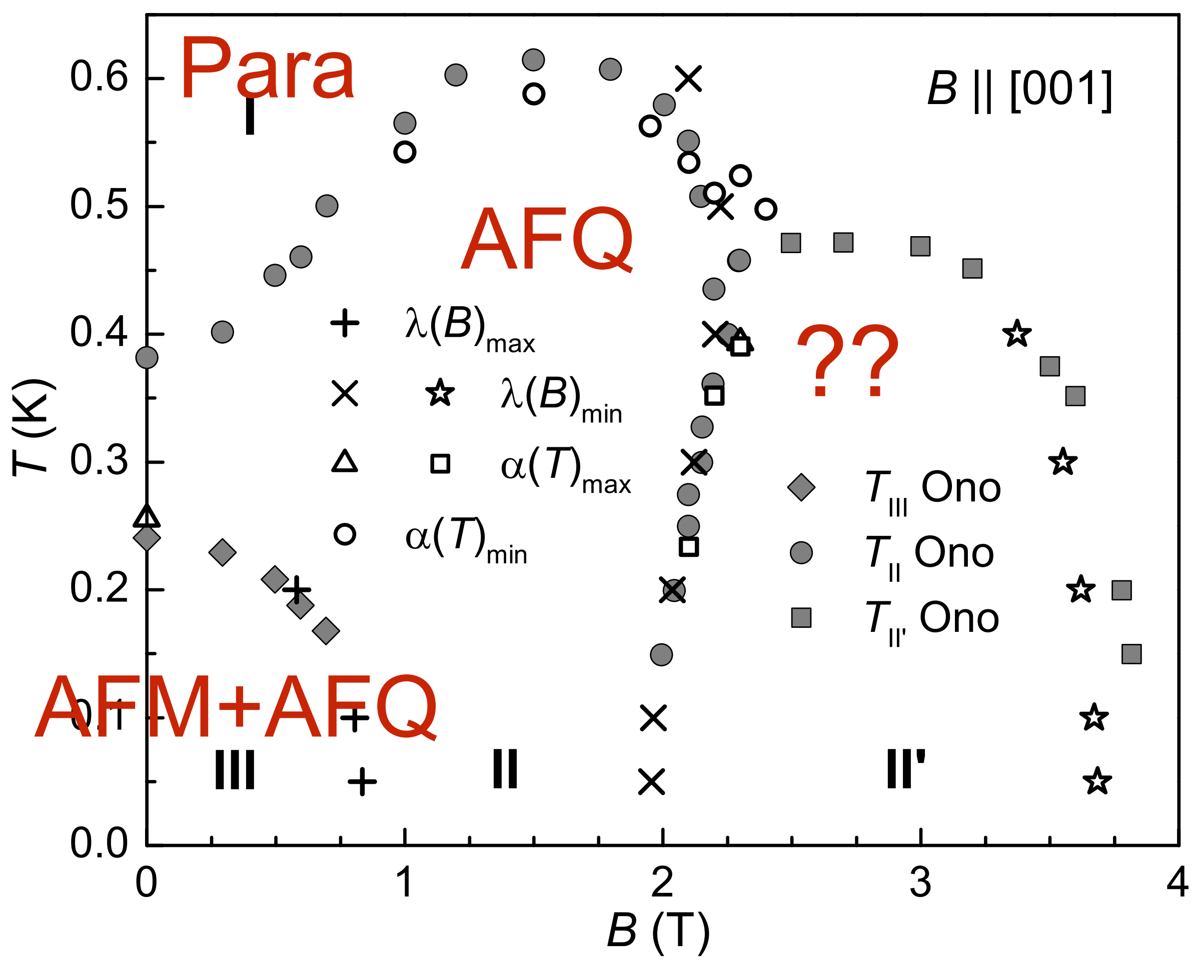
$$\Gamma_8^{(2)} = \sqrt{\frac{5}{6}}|-\frac{5}{2}\rangle + \sqrt{\frac{1}{6}}|+\frac{3}{2}\rangle, \quad \Gamma_8^{(4)} = |-\frac{1}{2}\rangle.$$

15 multipolar moments ⟷ 15 generators of SU(4)

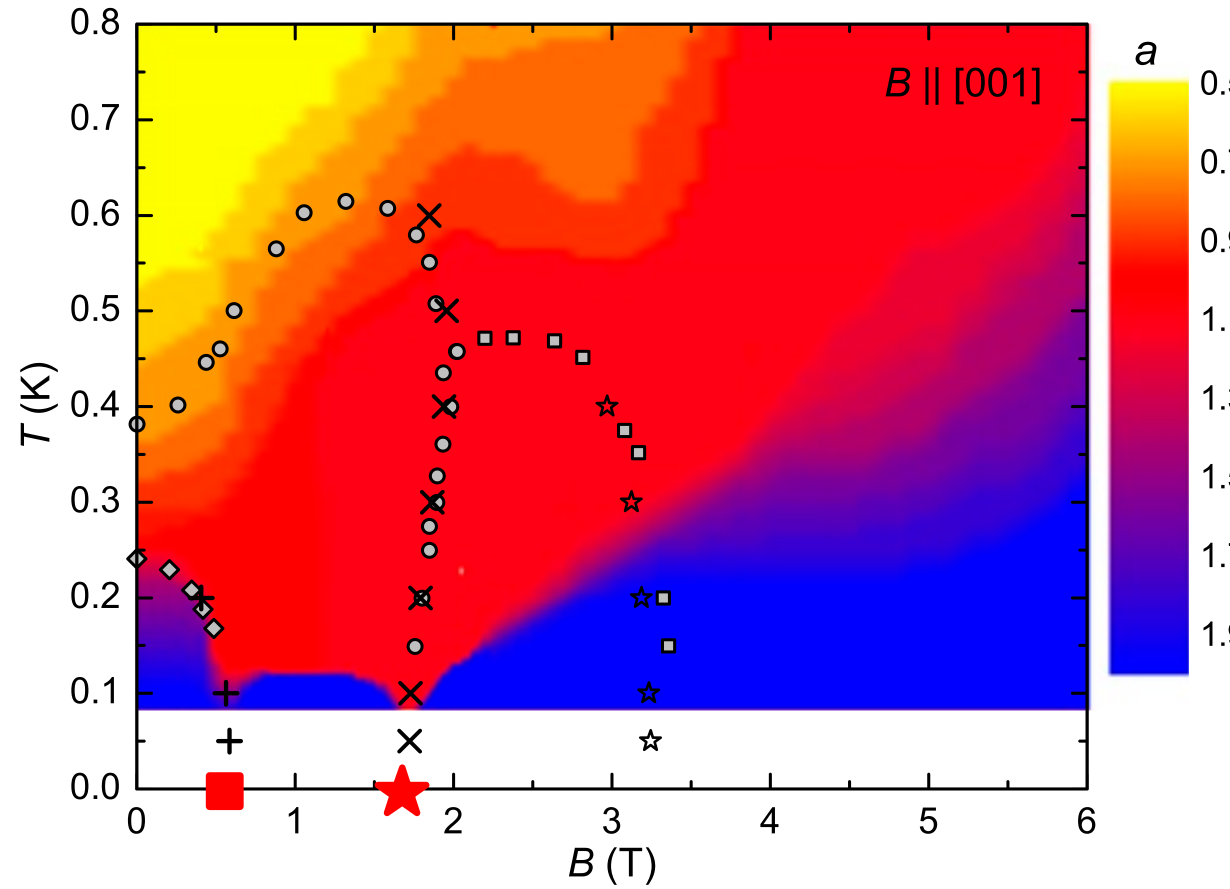
Moments	irreducible representations	Operators
Dipolar moments	T_1	J_x
		J_y
		J_z
Quadrupolar moments	E	$O_2^2 \equiv \frac{\sqrt{3}}{2}(J_x^2 - J_y^2)$
		$O_2^0 \equiv \frac{1}{2}(2J_z^2 - J_x^2 - J_y^2)$
	T_2	$O_{yz} \equiv \frac{\sqrt{3}}{2}\overline{J_y J_z}$
		$O_{zx} \equiv \frac{\sqrt{3}}{2}\overline{J_z J_x}$
		$O_{xy} \equiv \frac{\sqrt{3}}{2}\overline{J_x J_y}$
Octupolar moments	A_2	$T_{xyz} \equiv \frac{\sqrt{15}}{6}\overline{J_x J_y J_z}$
	T_1	$T_x^\alpha \equiv \frac{1}{2}(2J_x^3 - \overline{J_x J_y^2} - \overline{J_z^2 J_x})$
		$T_y^\alpha \equiv \frac{1}{2}(2J_y^3 - \overline{J_y J_z^2} - \overline{J_x^2 J_y})$
		$T_z^\alpha \equiv \frac{1}{2}(2J_z^3 - \overline{J_z J_x^2} - \overline{J_y^2 J_z})$
	T_2	$T_x^\beta \equiv \frac{\sqrt{15}}{6}(\overline{J_x J_y^2} - \overline{J_z^2 J_x})$
		$T_y^\beta \equiv \frac{\sqrt{15}}{6}(\overline{J_y J_z^2} - \overline{J_x^2 J_y})$
		$T_z^\beta \equiv \frac{\sqrt{15}}{6}(\overline{J_z J_x^2} - \overline{J_y^2 J_z})$

$Ce_3Pd_{20}Si_6$

Ono et al (2012,2013) Mutamura et al (2010) Goto et al (2009) Portnichenko et al (2016, 2018)



Jump in Hall constant at two consecutive phase transitions (not shown)



Resistivity exponent

Quantum critical behaviors

V.Martelli, … Q.Si, S. Paschen, PNAS (2019)

Previous theory: C.-C. Liu, S. Pachen, Q. Si (2021), arXiv:2101.01087

Bose-Fermi Kondo model (results)

15(Fermi)+6(Bose) coupling constants

Hybridization

$$\{J_x^2 - J_y^2, 3J_z^2 - \boldsymbol{J}^2\}$$
$$\{-21 J_{x,y,z} + 2\mathcal{T}^{\alpha}_{x,y,z}\}$$
$$\{J_x J_y J_z\}$$

F'_1

Ordering

$$\{J_x^2 - J_y^2, 3J_z^2 - \boldsymbol{J}^2\}$$
$$\{-21 J_{x,y,z} + 2\mathcal{T}^{\alpha}_{x,y,z}\}$$

B_M

Quadrupolar

$$\{J_x^2 - J_y^2, 3J_z^2 - \boldsymbol{J}^2\}$$

'Dipolar'

$$\{-21 J_{x,y,z} + 2\mathcal{T}^{\alpha}_{x,y,z}\}$$

Octupolar

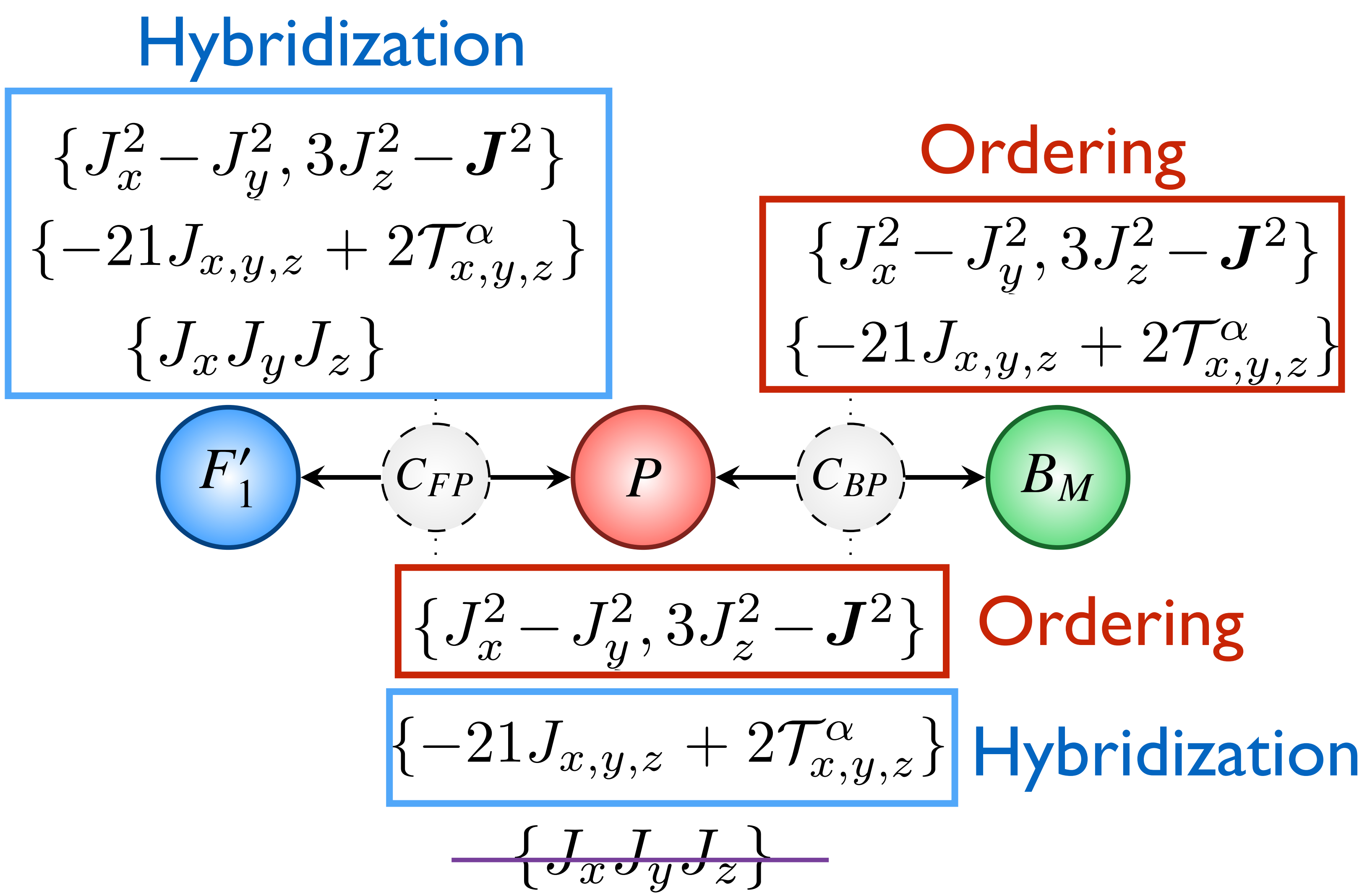
$$\{J_x J_y J_z\}$$

Kondo fixed point

RKKY fixed point

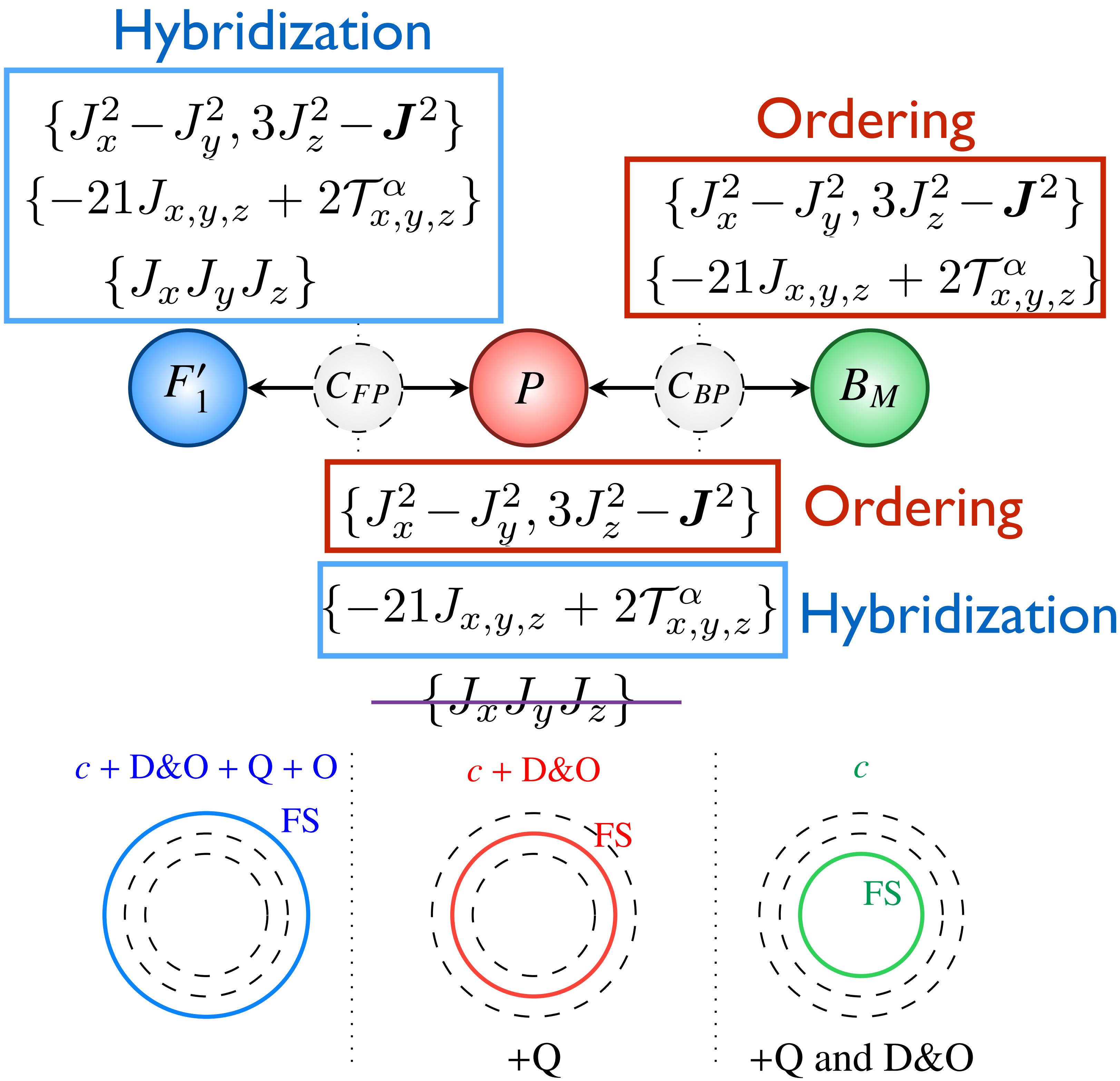
S. Han, D. Schultz, YBK, arXiv:2207.07661 (2022)

Bose-Fermi Kondo model (results)



S. Han, D. Schultz, YBK, arXiv:2207.07661 (2022)

Bose-Fermi Kondo model (results)



Two-stage Kondo destruction

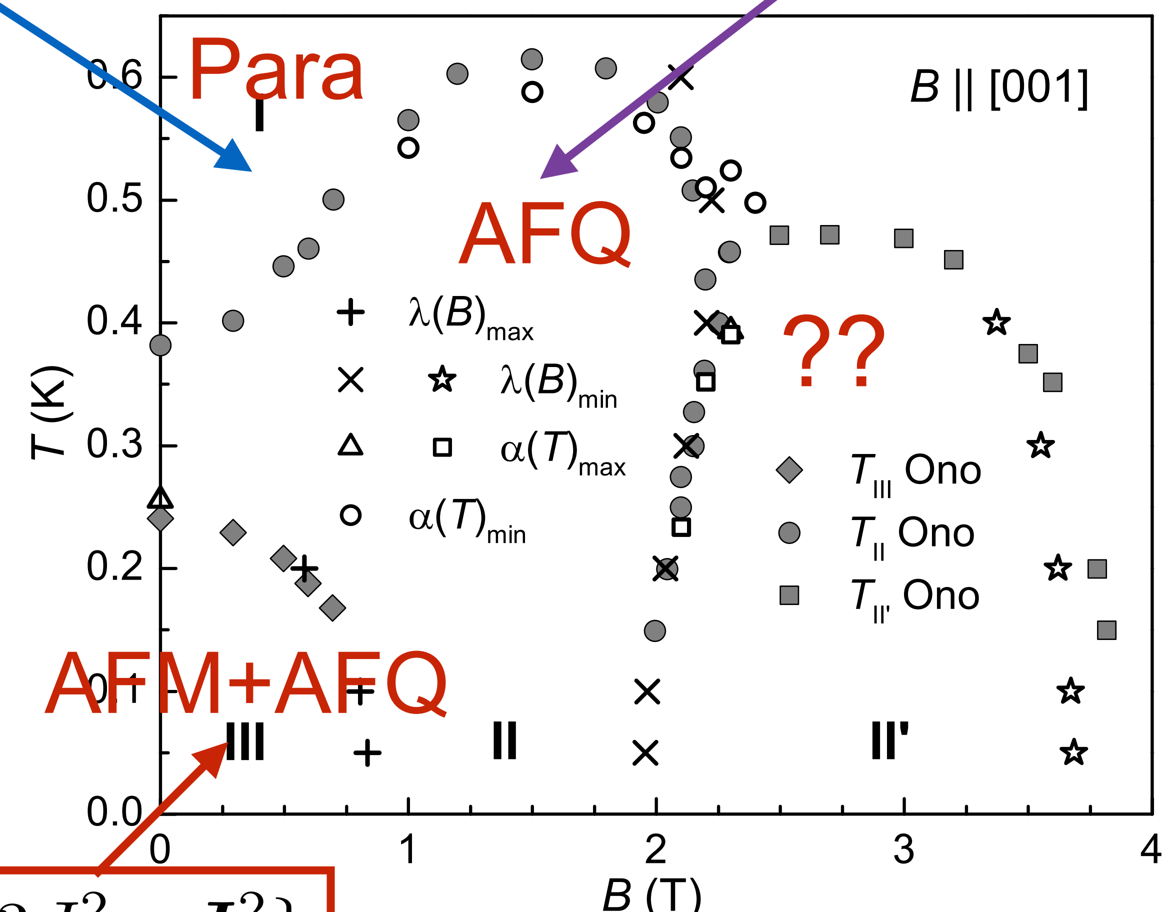
S. Han, D. Schultz, YBK, arXiv:2207.07661 (2022)

Hybridization

$$\{J_x^2 - J_y^2, 3J_z^2 - \boldsymbol{J}^2\}$$
$$\{-21 J_{x,y,z} + 2\mathcal{T}^{\alpha}_{x,y,z}\}$$
$$\{J_x J_y J_z\}$$

$$\{J_x^2 - J_y^2, 3J_z^2 - \boldsymbol{J}^2\}$$ Ordering

$$\{-21 J_{x,y,z} + 2\mathcal{T}^{\alpha}_{x,y,z}\}$$ Hybridization



Ordering

$$\{J_x^2 - J_y^2, 3J_z^2 - \boldsymbol{J}^2\}$$
$$\{-21 J_{x,y,z} + 2\mathcal{T}^{\alpha}_{x,y,z}\}$$

$Ce_3Pd_{20}Si_6$ Silke Paschen et al (2019)

Irrep.	Notation	In terms of Stevens	Moment
T_{1a}	S^1	$-\frac{1}{15}J_x + \frac{7}{90}\mathcal{T}_x^\alpha$	D&O
T_{1a}	S^2	$-\frac{1}{15}J_y + \frac{7}{90}\mathcal{T}_y^\alpha$	D&O
T_{1a}	S^3	$-\frac{1}{15}J_z + \frac{7}{90}\mathcal{T}_z^\alpha$	D&O
E	S^4	$\frac{1}{8}\mathcal{O}_{22}$	Q
E	S^5	$\frac{1}{8}\mathcal{O}_{20}$	Q
T_{2+}	S^6	$\frac{1}{2}\mathcal{O}_{yz}$	Q
T_{2+}	S^7	$\frac{1}{2}\mathcal{O}_{zx}$	Q
T_{2+}	S^8	$\frac{1}{2}\mathcal{O}_{xy}$	Q
A_2	S^9	$\frac{1}{9\sqrt{5}}\mathcal{T}_{xyz}$	O
T_{1b}	S^{10}	$-\frac{7}{15}J_x + \frac{2}{45}\mathcal{T}_x^\alpha$	D&O
T_{1b}	S^{11}	$-\frac{7}{15}J_y + \frac{2}{45}\mathcal{T}_y^\alpha$	D&O
T_{1b}	S^{12}	$-\frac{7}{15}J_z + \frac{2}{45}\mathcal{T}_z^\alpha$	D&O
T_{2-}	S^{13}	$\frac{1}{6\sqrt{5}}\mathcal{T}_x^\beta$	O
T_{2-}	S^{14}	$\frac{1}{6\sqrt{5}}\mathcal{T}_y^\beta$	O
T_{2-}	S^{15}	$\frac{1}{6\sqrt{5}}\mathcal{T}_z^\beta$	O

Irrep.	Moments
T_{1a}	S^1, S^2, S^3
E	S^4, S^5
T_{2+}	S^6, S^7, S^8
A_2	S^9
T_{1b}	S^{10}, S^{11}, S^{12}
T_{2-}	S^{13}, S^{14}, S^{15}

Multipolar susceptibility

Ultrasound

$$(C_{11} - C_{12}) = (C_{11}^0 - C_{12}^0) - (s_E^2)\chi_E' - 2(s_{2-}^2 h_z^2)\chi_{2-}', \tag{15}$$

$$C_{44} = C_{44}^0 - (s_{2+}^2)\chi_{2+}' - (s_A^2 h_z^2)\chi_A', \tag{16}$$

Summary and Outlook

Non-Fermi liquids in multipolar Kondo problems

Possibility of plethora of non-Fermi liquids

D.Schultz, A.Patri, YBK, PRR (2021), arXiv:2010.04731

D.Schultz, A.Patri, YBK, PRB (2021), arXiv:2104.11245

Kondo and RKKY: Bose-Fermi Kondo models

Multi-stage Fermi surface reconstruction and quantum critical behaviors (Ce-based system)

Kondo lattice problem ?

New opportunities for quantum criticality ?

New opportunities for unconventional superconductivity ?