Exploring Quantum Phases of Matter on Quantum Processors

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DFG TRR 80

MCQST

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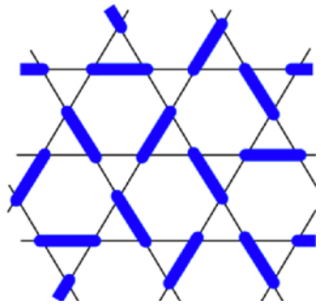
EQUAHUMO Efficient Quantum Algorithms for the Hubbard Model

Munich Quantum Valley

Novel Quantum States in Condensed Matter 2022
Kyoto Nov. 21, 2022

Complexity of Quantum Many-Body Systems

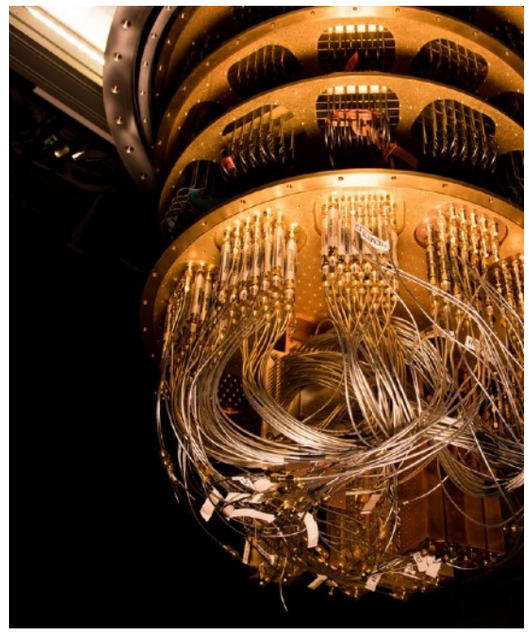
Computational Complexity grows **exponentially** with system size!



$$H|\psi\rangle = E|\psi\rangle$$

~40 qubits

~50 qubits

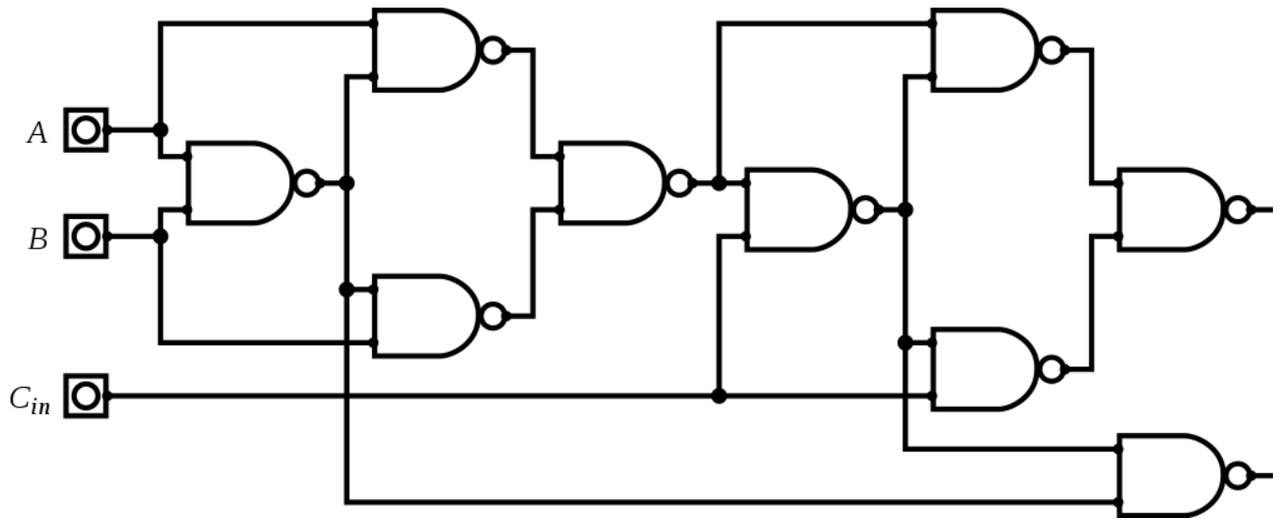


Noise Intermediate Scale Quantum (NISQ)

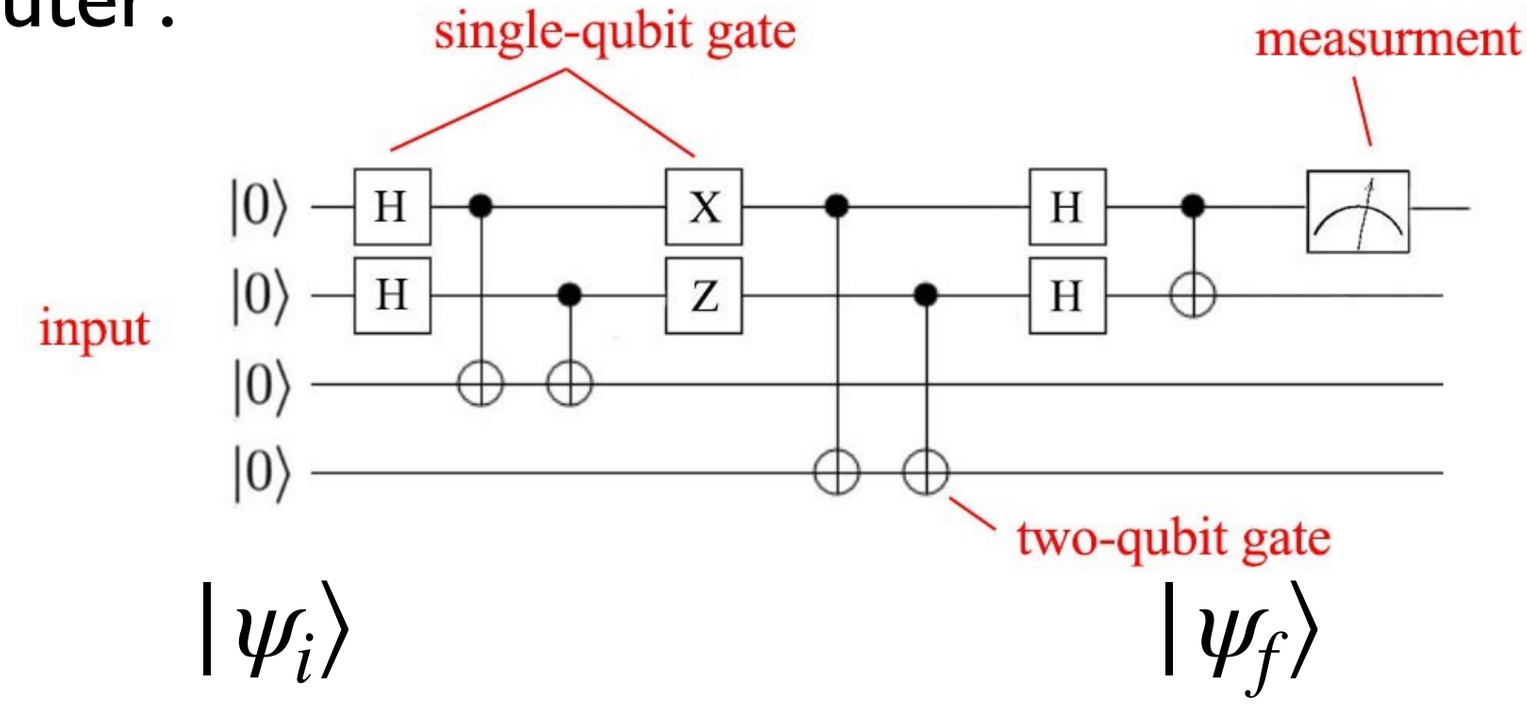
Identify problems that are hard on classical computers but doable on near term NISQ devices!

Classical versus Quantum Computer

Classical Computer:



Quantum Computer:



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Crossing a Symmetry Protected Topological (SPT) phase transition on quantum processor

[Smith, Jobst, Green, FP, PRR **4**, L022020 (2022)]

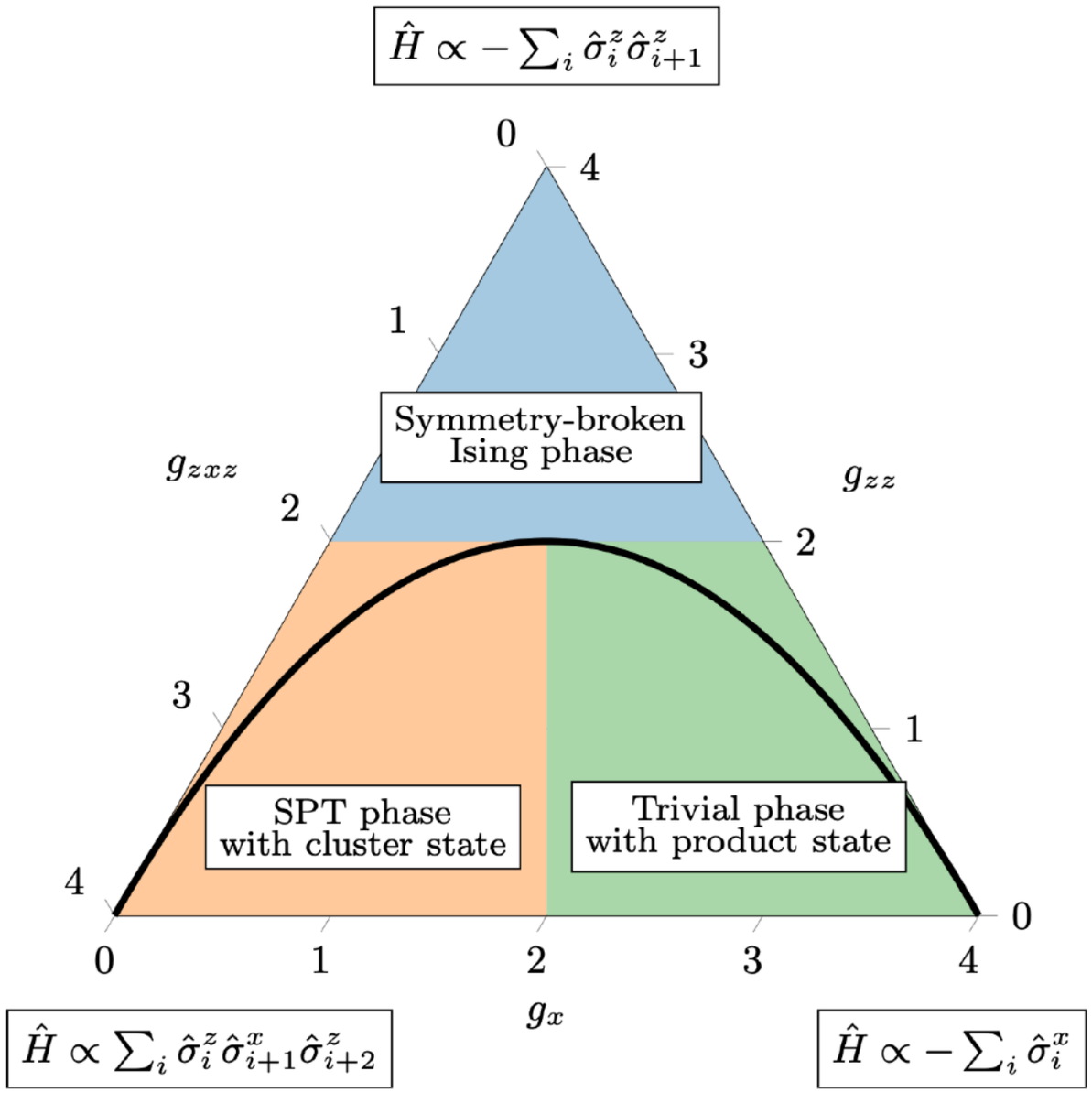
[Liu, Smith, Knap, FP (in preparation)]

Realizing and characterizing Topologically Ordered States on a quantum processor

[K. J. Satzinger, Y. Liu, A. Smith, C. Knapp et al., Science **374**, 6572 (2021)]

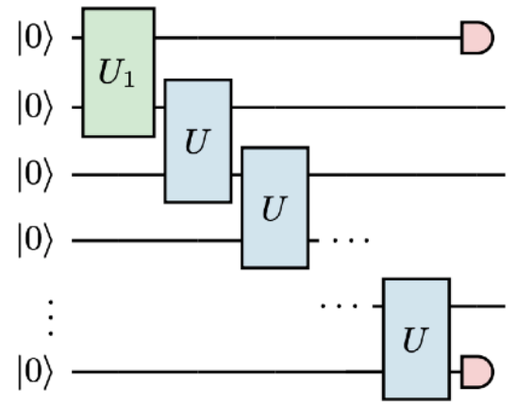
Exact quantum circuit crossing a phase transition

$$\hat{H} = \sum_i \left[-g_{zz}\, \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z - g_x\, \hat{\sigma}_i^x + g_{zxz}\, \hat{\sigma}_i^z \hat{\sigma}_{i+1}^x \hat{\sigma}_{i+2}^z \right]$$



Exact quantum circuit connecting the SPT and the trivial phase

$|\psi_0\rangle =$

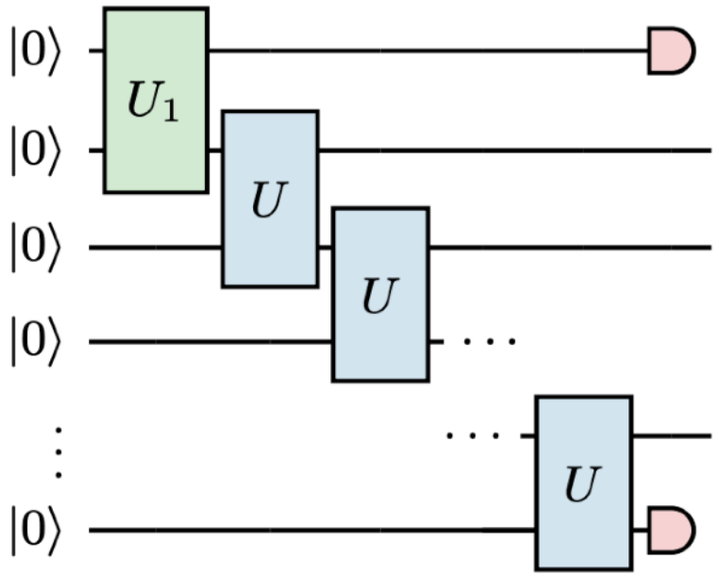


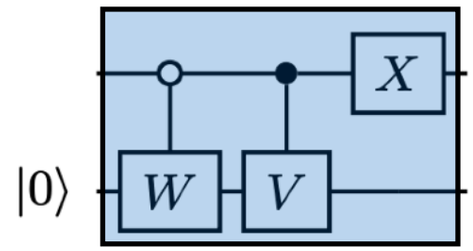
[Smith, Jobst, Green, FP, PRR **4**, L022020 (2022)]

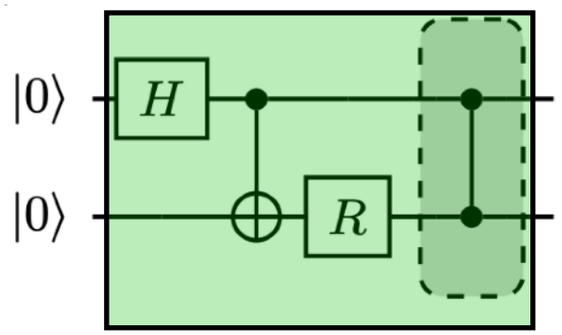
Circuit Construction

Exact quantum circuit

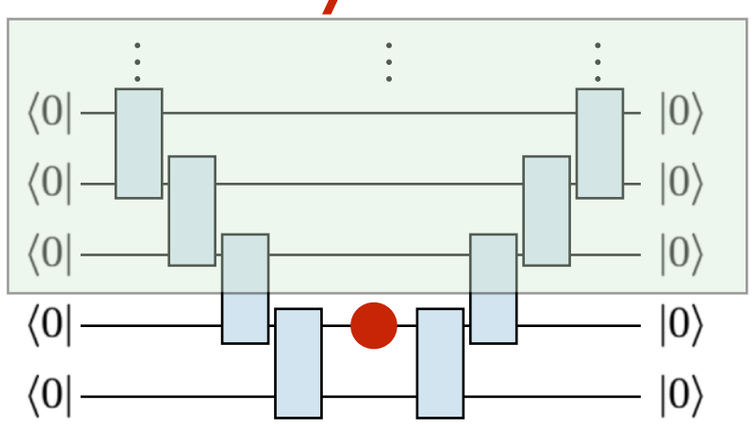
$|\psi_0\rangle =$

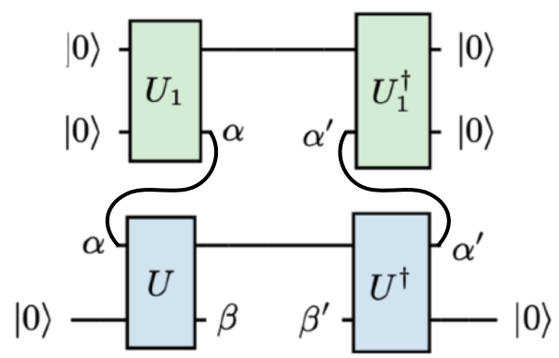






Thermodynamic limit: Dominant eigenvector as circuit:





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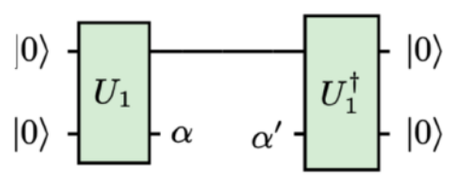


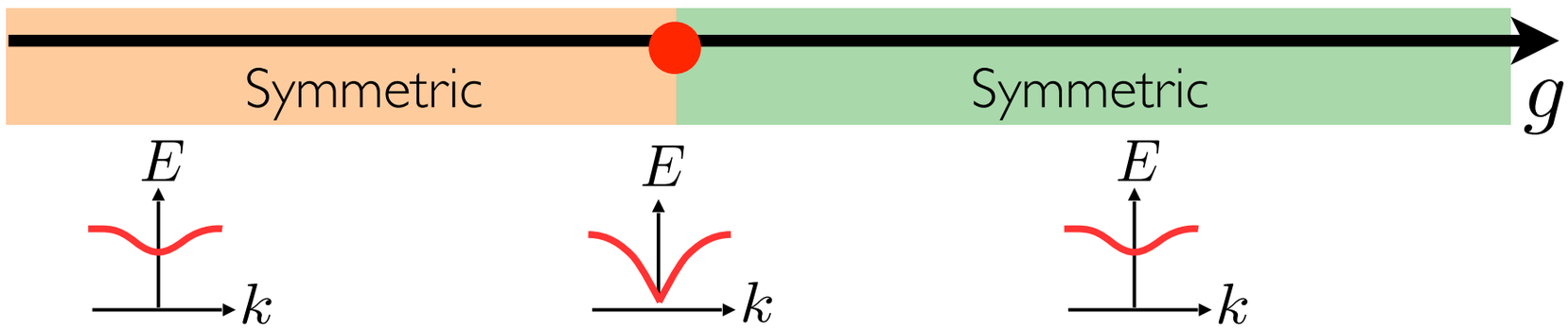
[Smith, Jobst, Green, FP, PRR **4**, L022020 (2022)]

Detecting a topological phase transition

Cluster state model:

$$H = \sum_j \sigma_j^z \sigma_{j+1}^x \sigma_{j+2}^z - g \sum_j \sigma_j^x$$

$$\lim_{|i-j|\to\infty} \langle \prod_{i<k<j} \sigma_k^x \rangle = 0 \qquad \lim_{|i-j|\to\infty} \langle \prod_{i<k<j} \sigma_k^x \rangle \neq 0$$

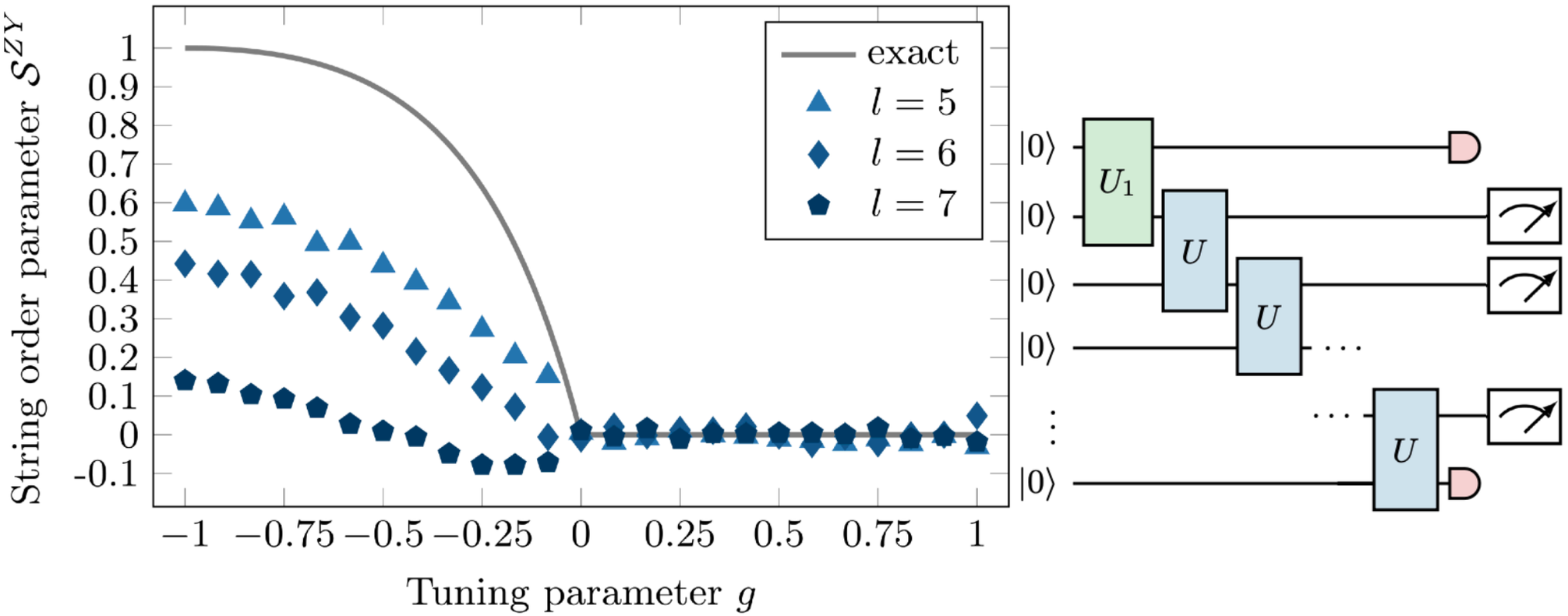


Non-local "string" order parameter [den Nijs and Rommelse '89, FP and Turner '12]

[Smith, Jobst, Green, FP, PRR **4**, L022020 (2022)]

Quantum circuit crossing an SPT phase transition

Results on a **20 qubit IBM-Q device:** $\mathcal{S}^O(g) = \langle\psi|\hat{O}_i\left(\prod_{j=i+2}^{k-2}\hat{\sigma}_j^x\right)\hat{O}'_k|\psi\rangle$



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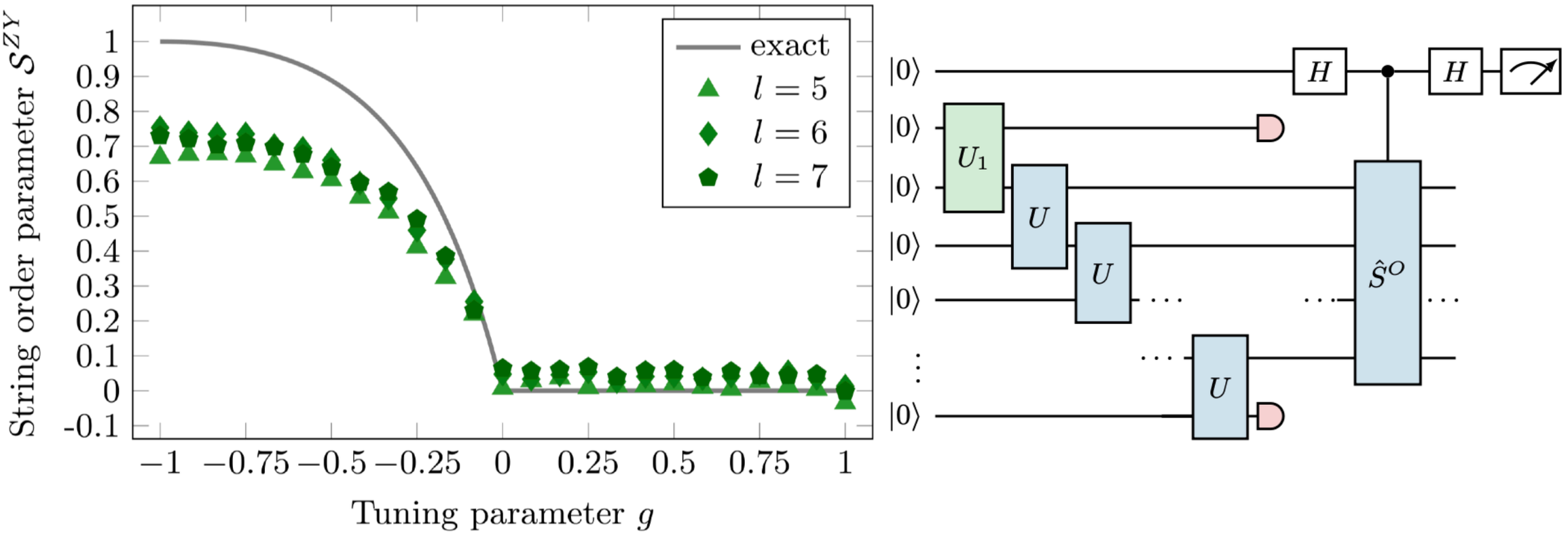


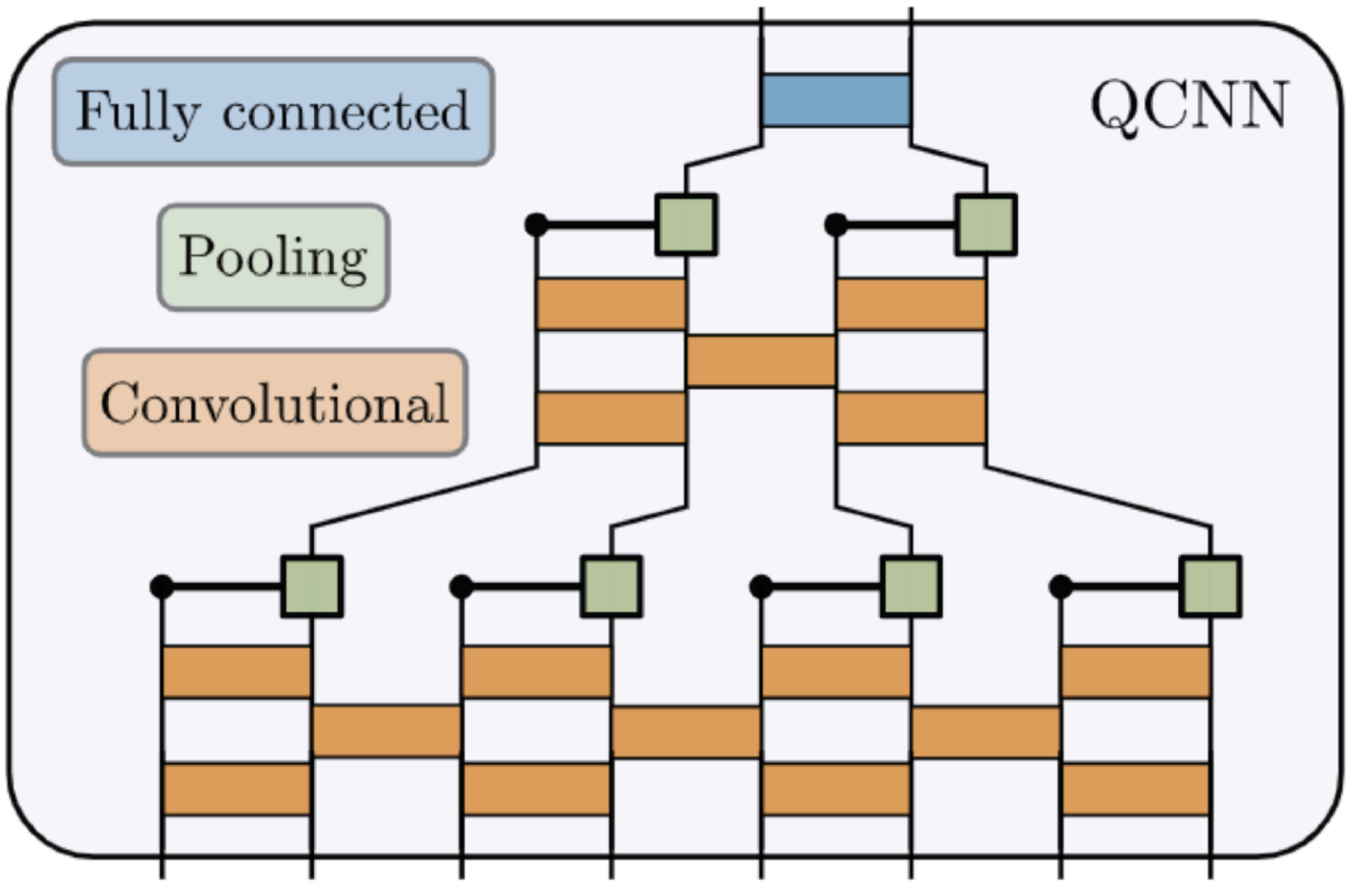
- Accurate representation on real machine!
- Generalization to the entire one-dimensional BDI class

[Jones et al., PRR 3, 033265 (2021)]

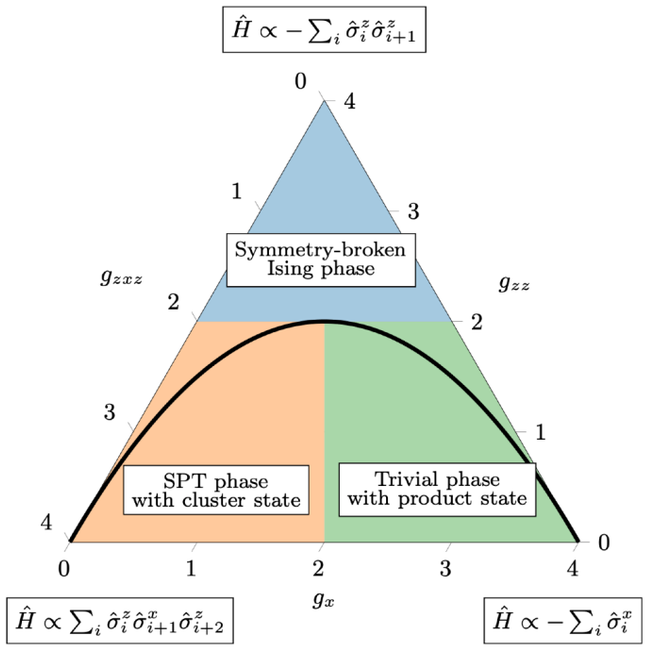
[Smith, Jobst, Green, FP, PRR **4**, L022020 (2022)]

Quantum Convolutional Networks (QCNN)

QCNN instead of string order parameter [Cong et al. 2018]



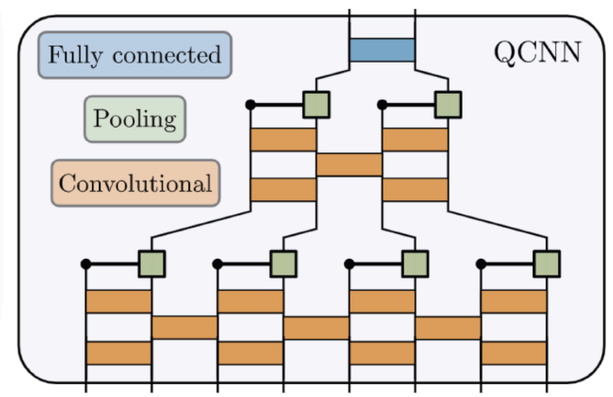
$|00\rangle \rightarrow SB$
$|01\rangle \rightarrow PM$
$|10\rangle \rightarrow SPT$
$|11\rangle \rightarrow Fail$

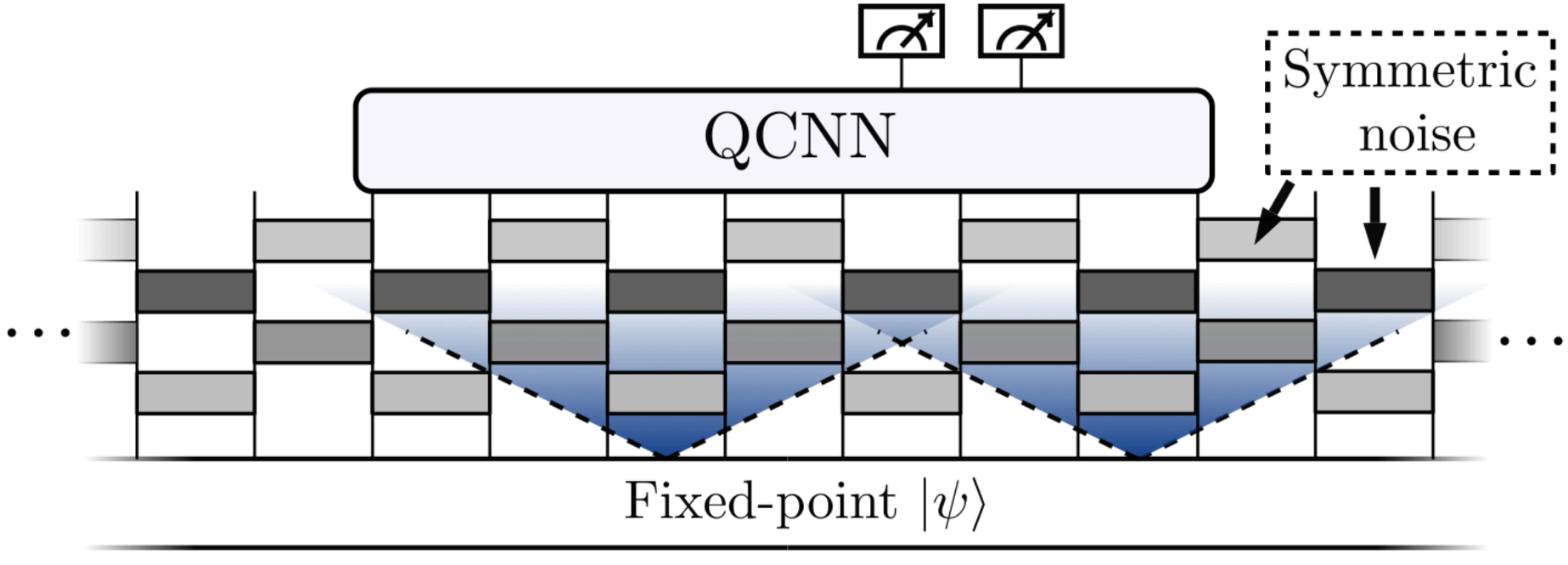


[Smith, Jobst, Green, FP, PRR **4**, L022020 (2022)]

Model independent learning

Mask short range correlations with symmetric noise

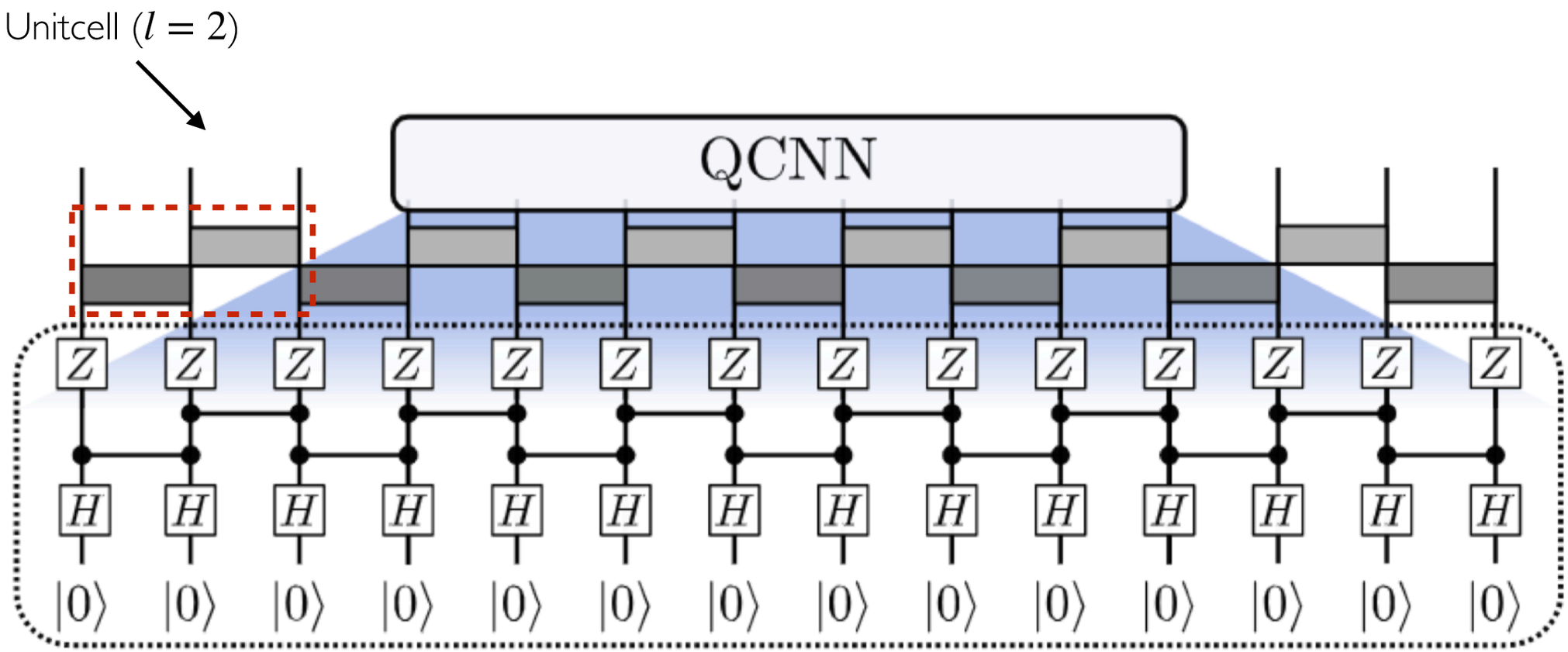




[Liu, Smith, Knap, FP (in preparation)]

Model independent learning

Example: Custer state with two layers of noise

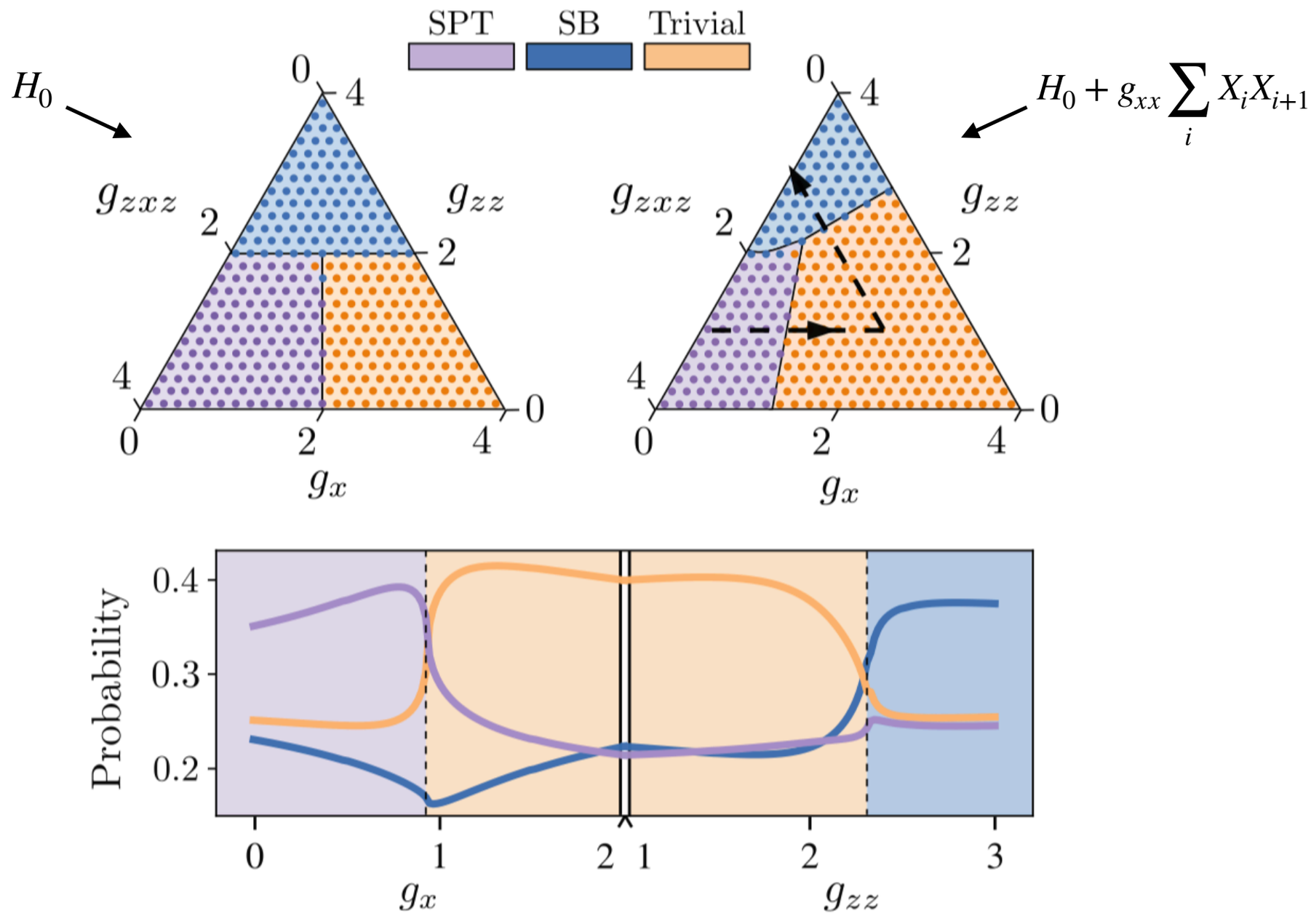


$$U = \exp(i \sum_k \theta_k P_k), P_k \in \{Z_1 Y_2, Y_1 Z_2, X_1 Z_2, Z_1 X_2, Z_1, Z_2, \}$$

[Liu, Smith, Knap, FP (in preparation)]

Model independent learning

Benchmark on physical models



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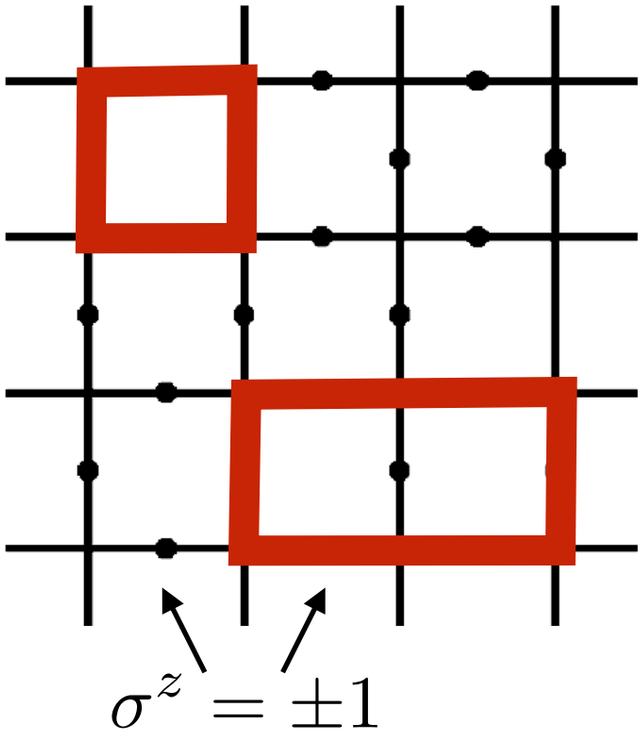
[K. J. Satzinger, Y. Liu, A. Smith, C. Knapp et al., Science **374**, 6572 (2021)]

Intrinsic topological order and anyons

Toric code model

[Kitaev '03]

$$H_{TC} = -J\sum_v A_v - J\sum_p B_p, \quad J > 0$$



$$A_v = \prod_{i\in v} \sigma_i^z, \; B_p = \prod_{i\in p} \sigma_i^x$$

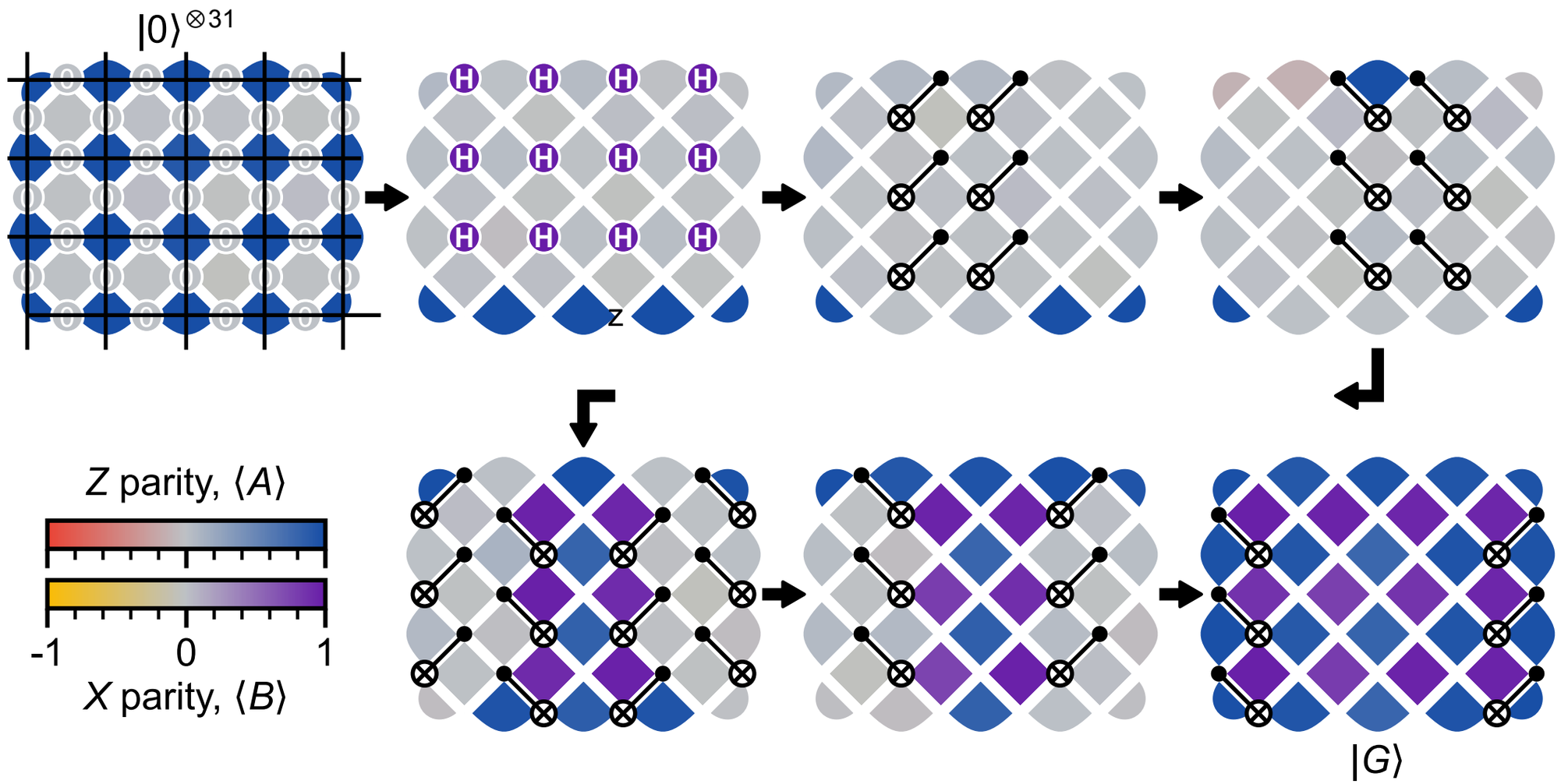
$[A_v, B_p] = 0$ ➡ Exactly solvable

$|\psi_0\rangle = |$  $\rangle + |$  $\rangle + \ldots$

- $\mathbb{Z}_2$ topological order

Realizing the toric code on a quantum processor

Toric code ground state $|G\rangle \propto \prod_p (1 + B_p)|0\rangle$



- Linear depth in system width

[K. J. Satzinger, Y. Liu, A. Smith, C. Knapp et al., Science **374**, 1237 (2021)]
[See also Semeghini et al., Science **374**, 1242 (2021)]

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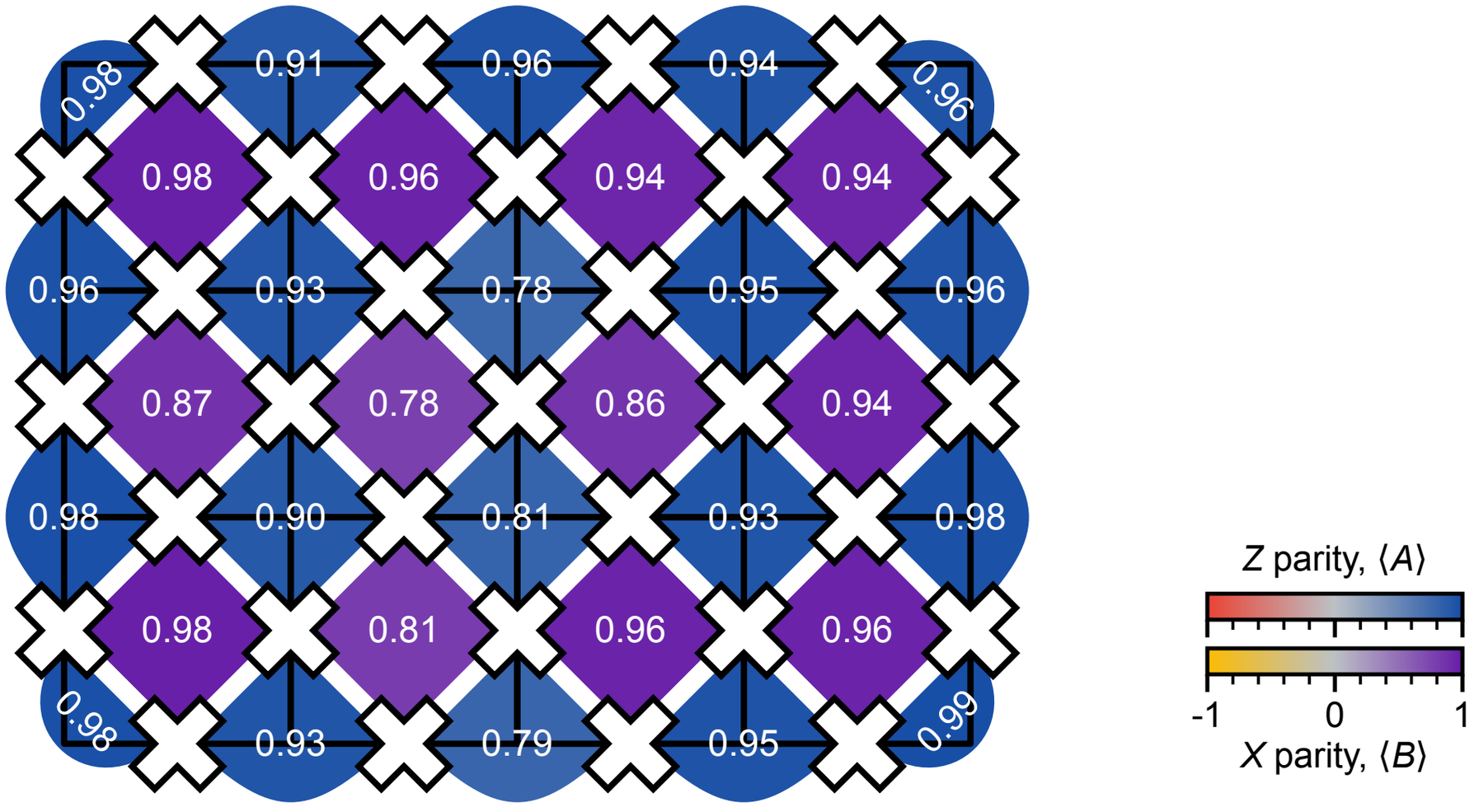


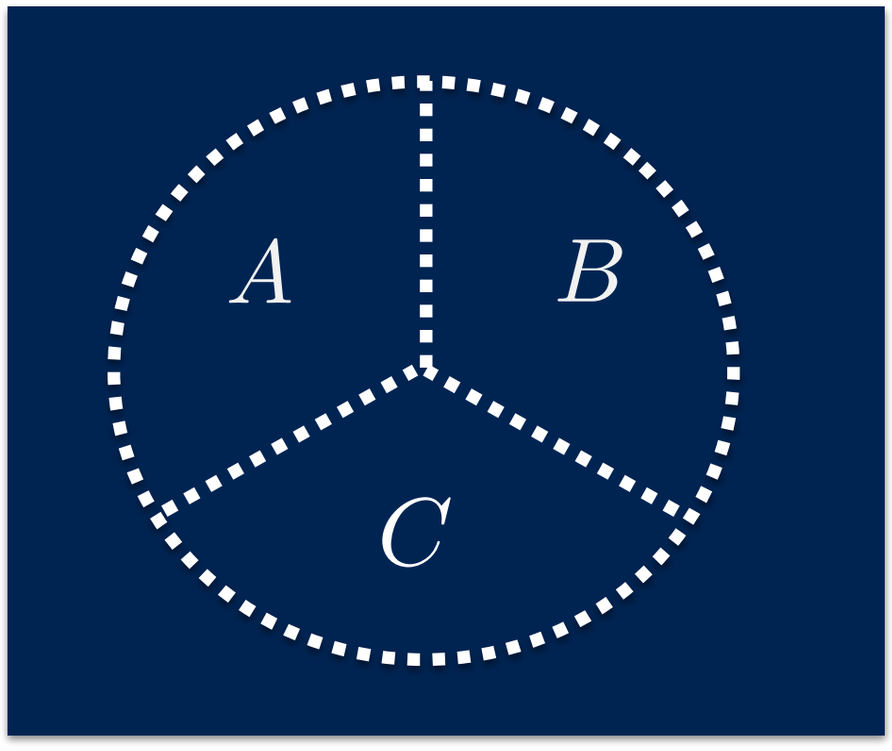
31 qubits, average stabilizer fidelity 0.92 ±0.06

[K. J. Satzinger, Y. Liu, A. Smith, C. Knapp et al., Science **374**, 1237 (2021)]

Probing topological entanglement

Topological entanglement entropy $S = \alpha L - \gamma$

[Kitaev and Preskill '06, Levin and Wen '06]



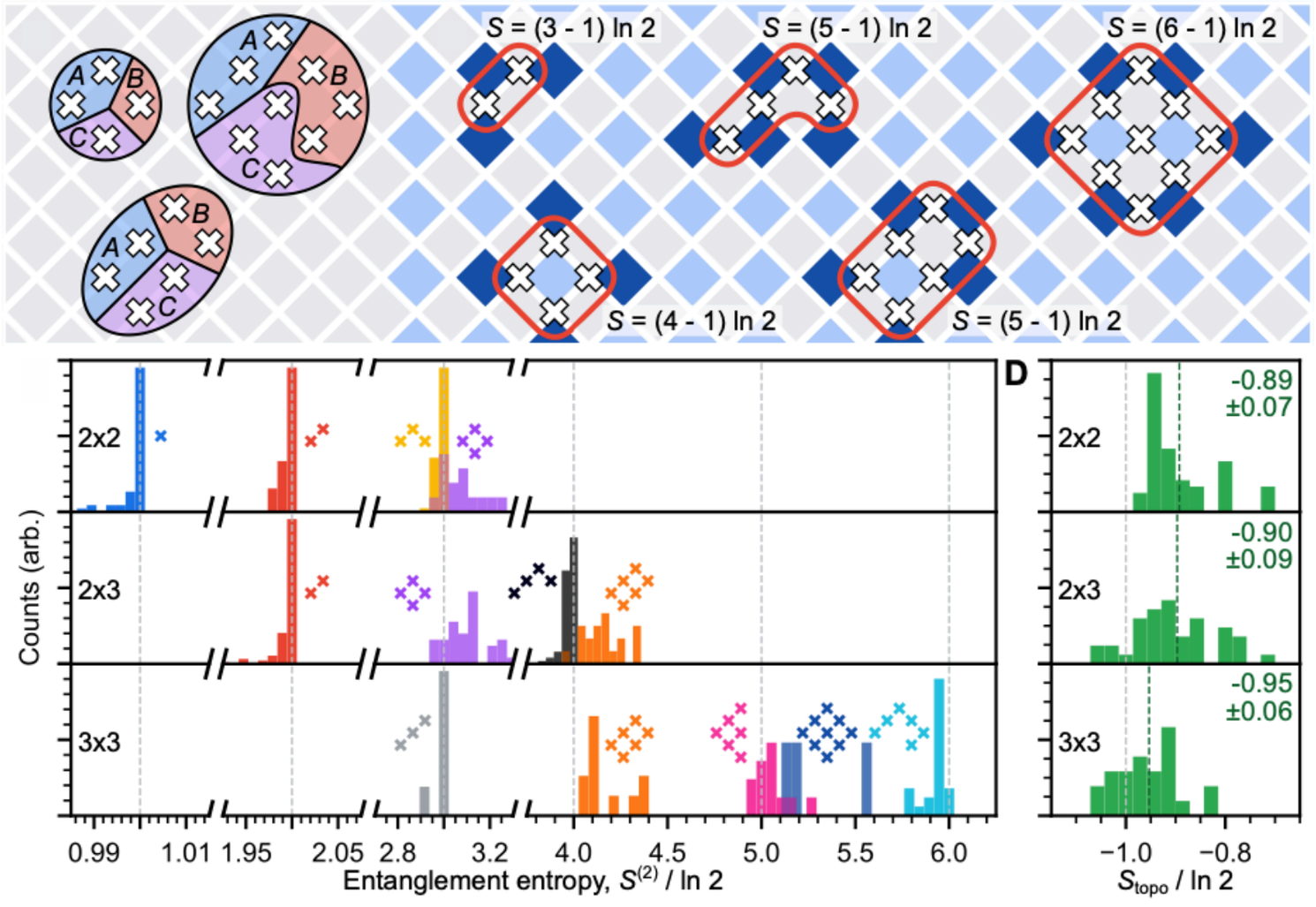
Subtraction scheme cancels area law boundary terms!

$$S_{\text{topo}} = -\gamma = S_A + S_B + S_C - S_{AB} - S_{AC} - S_{BC} + S_{ABC}$$

[K. J. Satzinger, Y. Liu, A. Smith, C. Knapp et al., Science **374**, 1237 (2021)]

Probing topological entanglement

Toric code: $S_{\text{topo}} = -\ln 2$



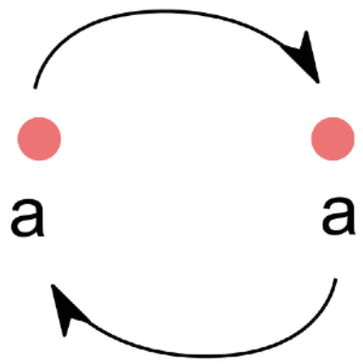
Full state tomography for 4 and 6 qubits

Randomized measurements for 9 qubits

Average over location and orientation

[K. J. Satzinger, Y. Liu, A. Smith, C. Knapp et al., Science **374**, 1237 (2021)]

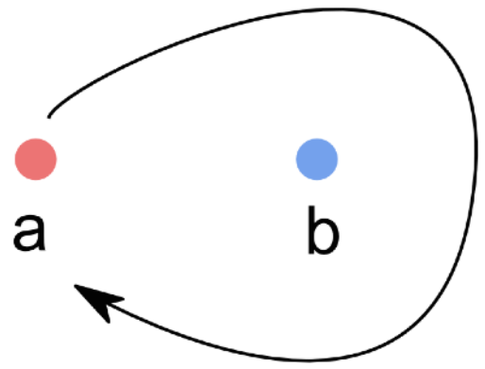
Anyonic braiding statistics



U_{aa}

Exchange: U-matrix

Can take rational phases other than ± 1

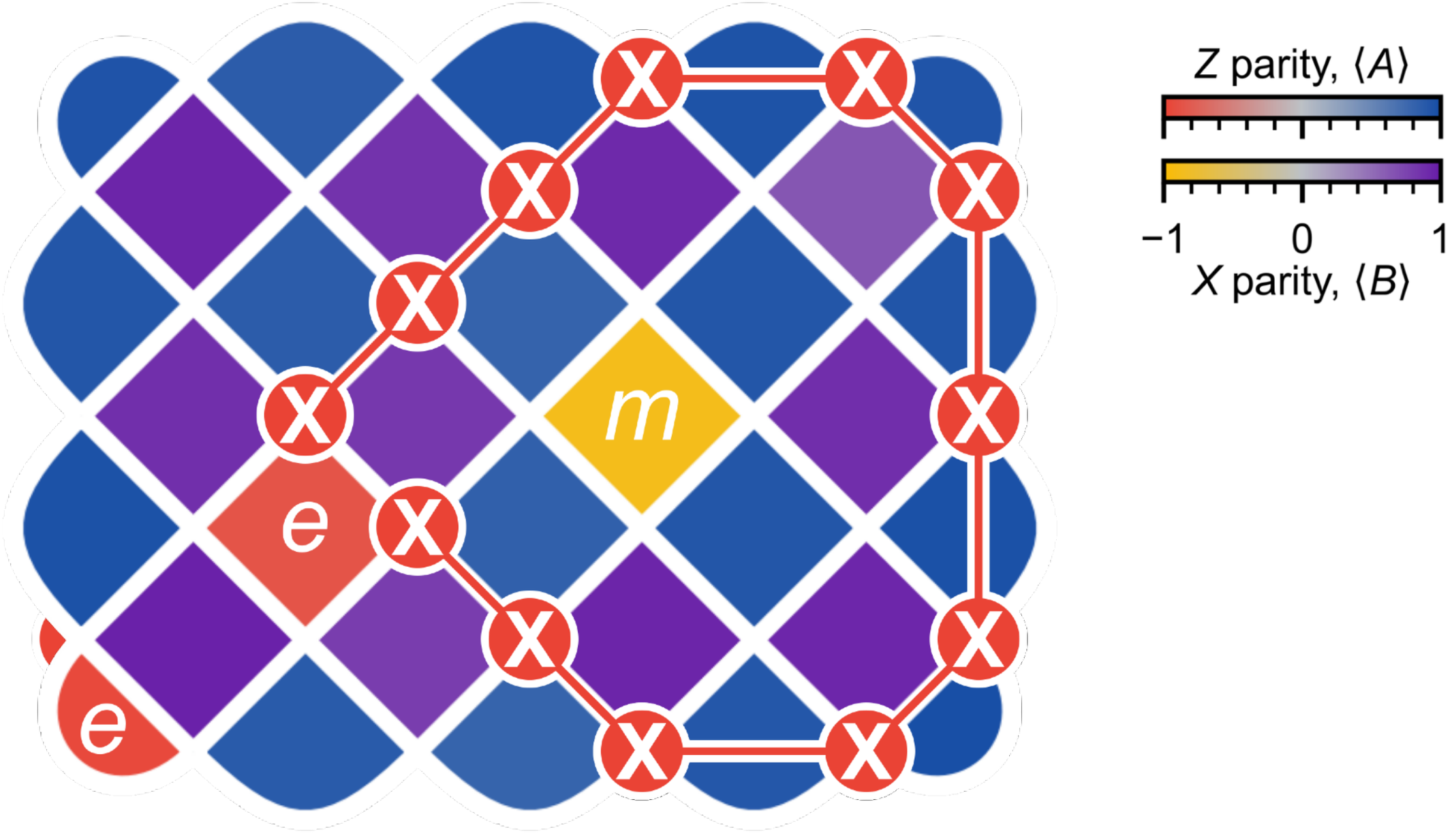


S_{ab}

Mutual: S-matrix

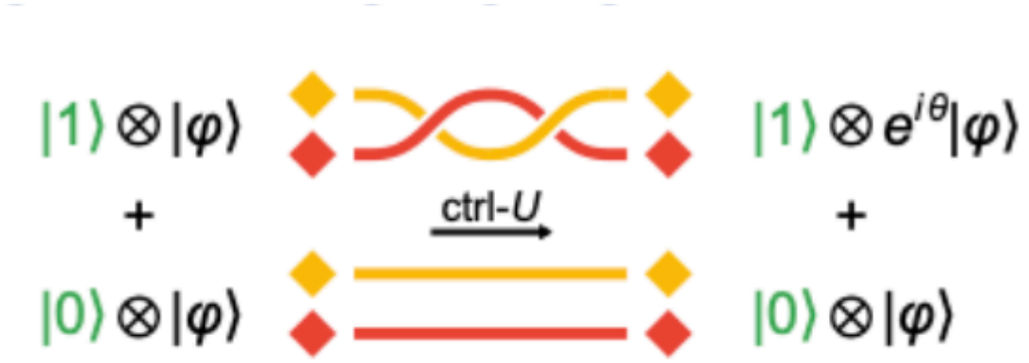
No analogue for fundamental fermions/ bosons in 3D

Simulating anyonic statistics

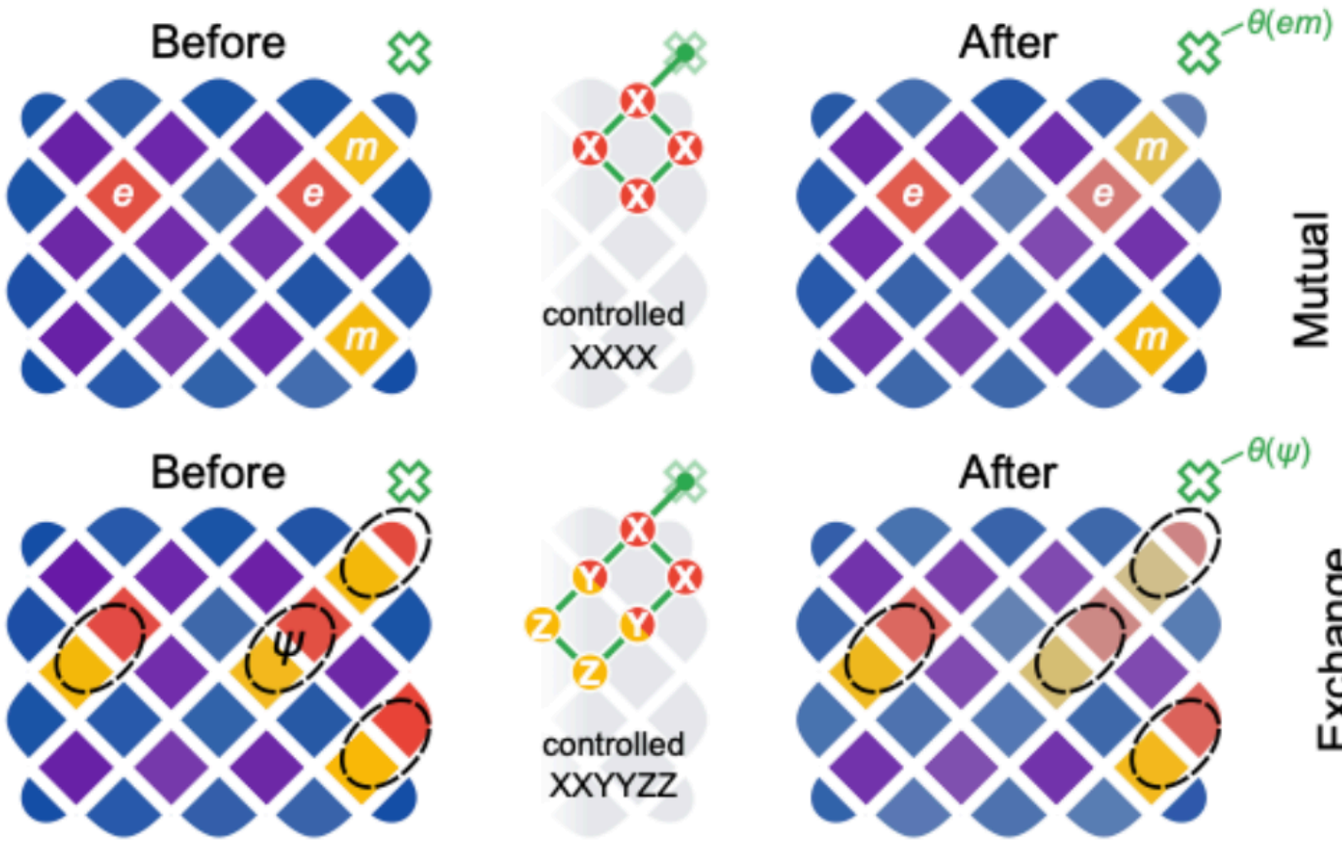


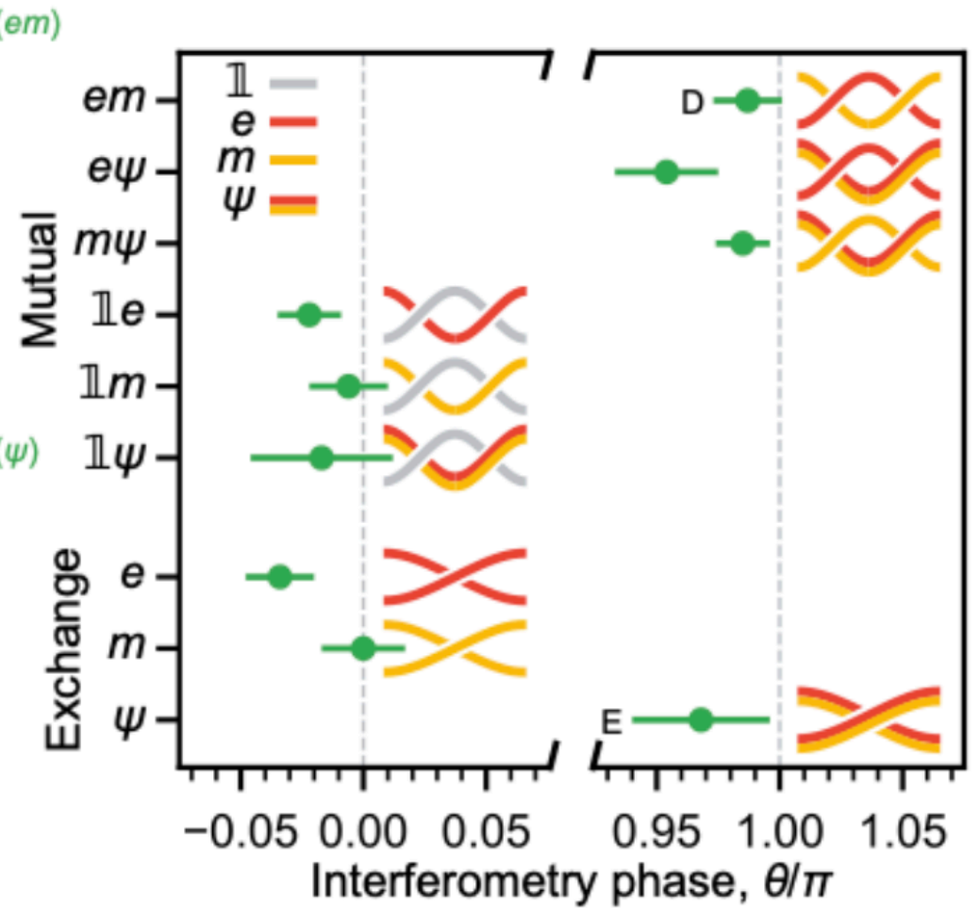
[K. J. Satzinger, Y. Liu, A. Smith, C. Knapp et al., Science **374**, 6572 (2021)]

Simulating anyonic statistics



[Jiang et al. '08]





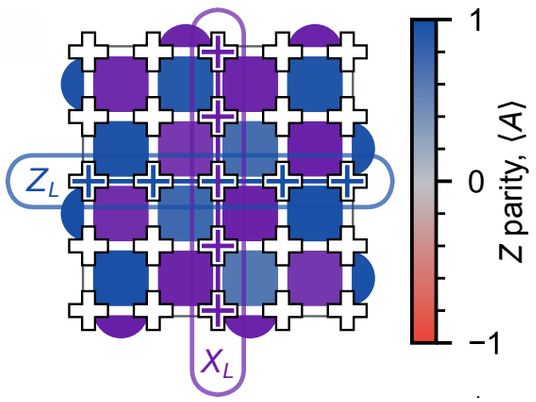
[K. J. Satzinger, Y. Liu, A. Smith, C. Knapp et al., Science **374**, 1237 (2021)]

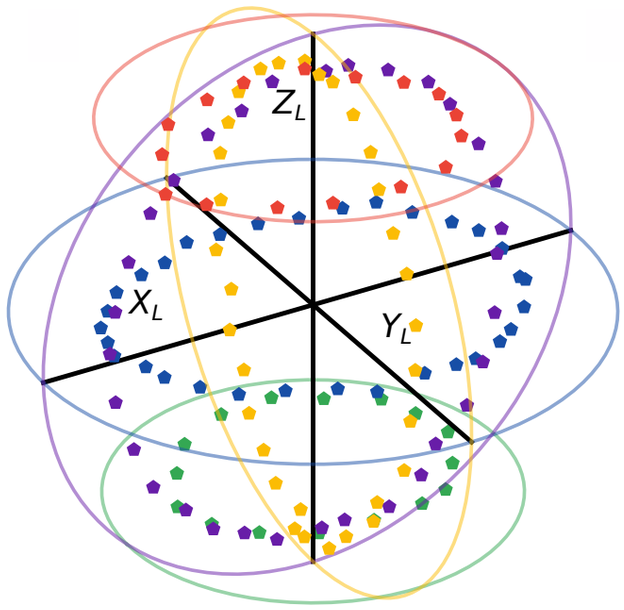
Surface code logical qubits

Towards logical qubits

Boundary conditions lead to ground state degeneracy

Perform state injection / readout over Bloch sphere





Linear quantum circuits for string-net models and quantum gates for braiding abelian and non-abelian anyons [Liu, Smith, Shtengel, FP, arXiv:2110.02020]

[K. J. Satzinger, Y. Liu, A. Smith, C. Knapp et al., Science **374**, 1237 (2021)]

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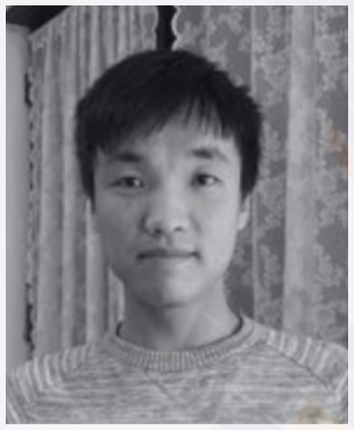
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