NQS2022, Yukawa Institute, Kyoto Uniersity, Oct. 31-Dec. 2 (Nov. 24)

新学術領域研究 令和元年度～５年度
量子液晶の物性科学
Quantum Liquid Crystals

科研費
KAKENHI

Nonreciprocal responses in superconductors: Finite-q pairing, parity mixing, and quantum geometry

Akito Daido
(Kyoto)

Hikaru Watanabe
(Univ. Tokyo)

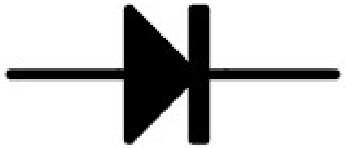
Yuhei Ikeda
(Kyoto)

Hiroto Tanaka
(Kyoto)

Kyoto University
Youichi Yanase

Why nonreciprocal responses?

Nonreciprocal response: left-mover ≠ right-mover



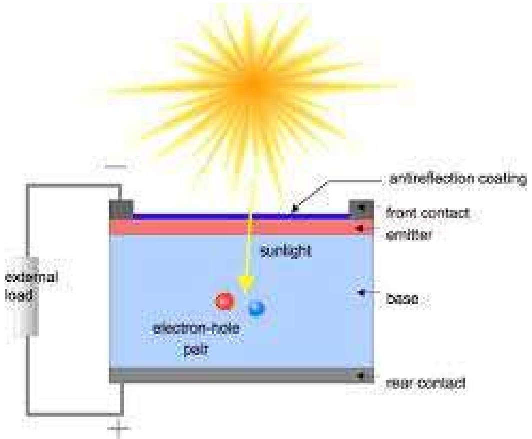
Diode: Building block of modern technology

LED = light-emitting diode

Photocurrent

Solar cell

Light sensor



Second harmonic generation

Green laser

Why nonreciprocal responses?

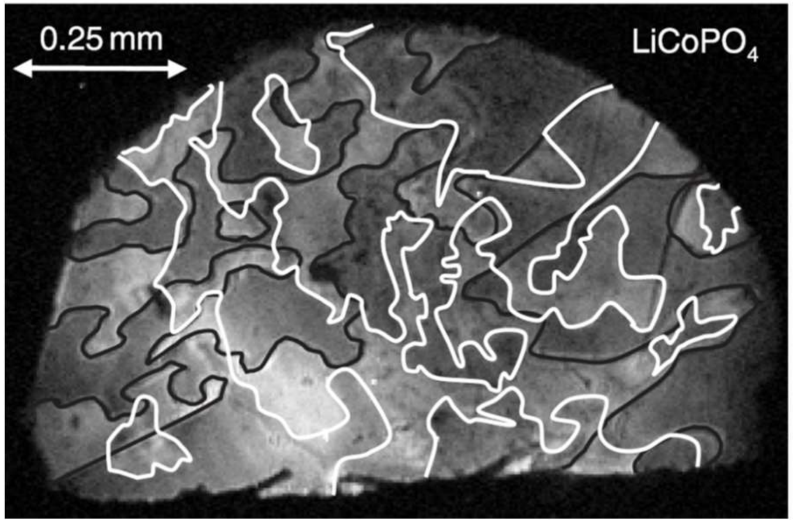
Sensitive to Inversion symmetry breaking, since it is needed.

Probing the quantum materials

Visualization of ferro toroidal domains by SHG

N. A. Spaldin, M. Fiebig, and M. Mostovoy, J. Phys. Condens. Matter **20**, 434203 (2008).

Application to many parity-breaking magnet



Question: probing parity-breaking superconducting phases possible?

Order parameter, symmetry, topology, criticality ......

This talk: finite-q pairing, parity mixing in Cooper pairs, quantum geometry

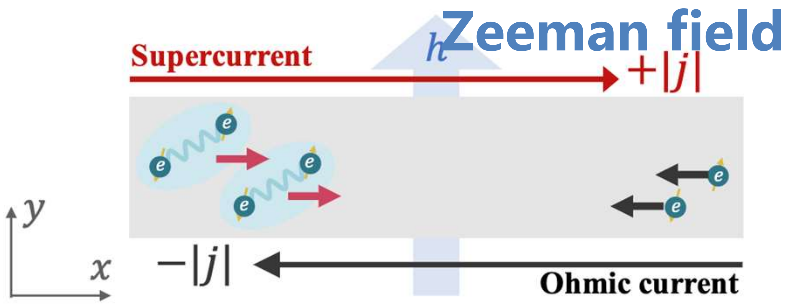
Studied for a long time, but clarification has been awaited.

Contents

1. Superconducting diode effect (SDE)

Finite-q pairing in the SC state

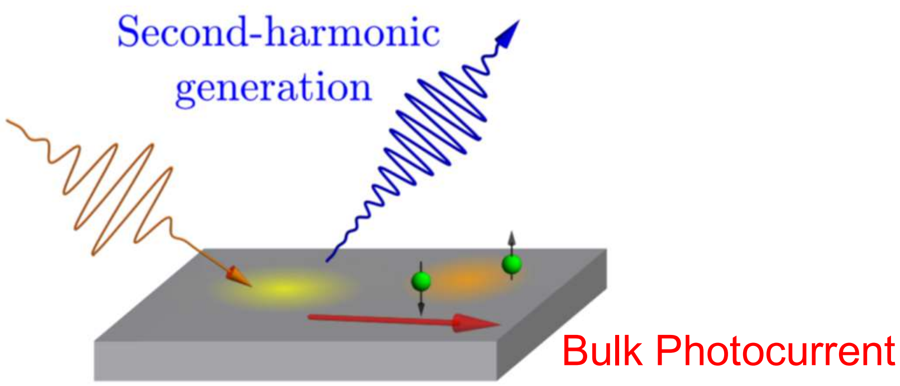
(FFLO, helical, anapole)



2. Nonreciprocal optics

Quantum geometry of Bloch electrons

Parity mixing in Cooper pairs



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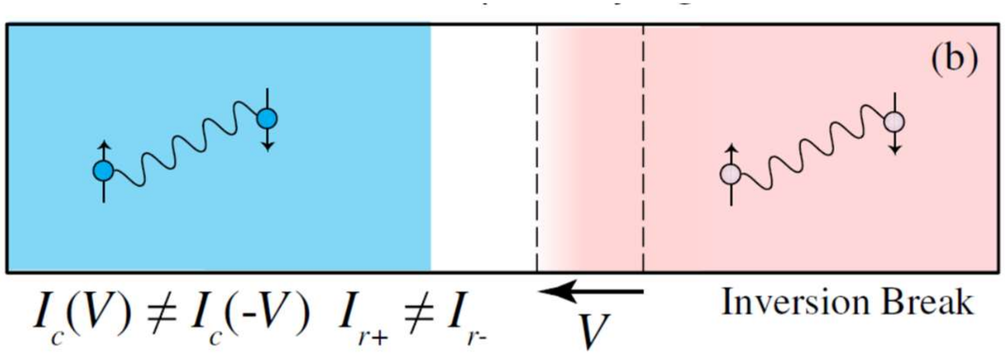
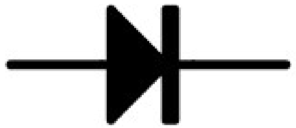
Josephson diode





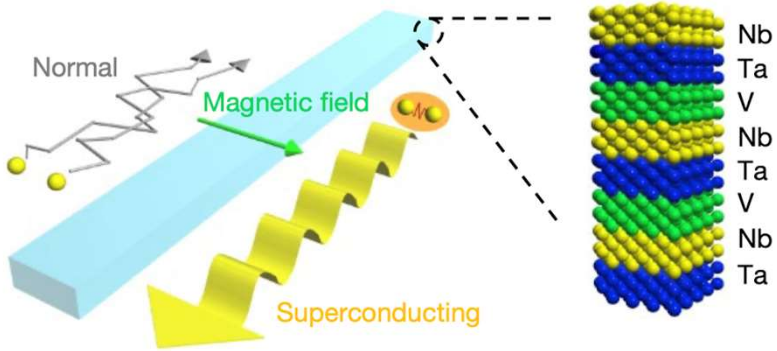
Y. Zhang et al. Phys. Rev. X 12, 041013 (2022)

$$J_{\mathrm{T}} = J_0 \sin\theta + J_1 \cos 2\theta + \ldots..$$

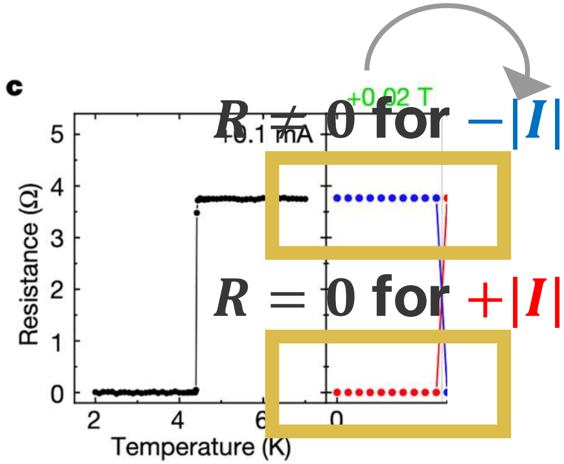
- J. Hu, C. Wu, and X. Dai, Phys. Rev. Lett. 99, 067004 (2007).
- A. A. Reynoso, G. Usaj, C. A. Balseiro, D. Feinberg, and M. Avignon, Phys. Rev. Lett. 101, 107001 (2008).
- A. Zazunov, R. Egger, T. Jonckheere, and T. Martin, Phys. Rev. Lett. 103, 147004 (2009).
- I. Margaris, V. Paltoglou, and N. Flytzanis, J. Phys. Condens. Matter 22, 445701 (2010).
- T. Yokoyama, M. Eto, and Y. V. Nazarov, Phys. Rev. B 89, 195407 (2014).
- M. A. Silaev, A. Y. Aladyshkin, M. V. Silaeva, and A. S. Aladyshkina, J. Phys. Condens. Matter 26, 095702 (2014).
- G. Campagnano, P. Lucignano, D. Giuliano, and A. Tagliacozzo, J. Phys. Condens. Matter 27, 205301 (2015).
- F. Dolcini, M. Houzet, and J. S. Meyer, Phys. Rev. B 92, 035428 (2015).
- C.-Z. Chen, J. J. He, M. N. Ali, G.-H. Lee, K. C. Fong, and K. T. Law, Phys. Rev. B 98, 075430 (2018).
- M. Minutillo, D. Giuliano, P. Lucignano, A. Tagliacozzo, and G. Campagnano, Phys. Rev. B 98, 144510 (2018).
- S. Pal and C. Benjamin, EPL 126, 57002 (2019).

Superconducting diode effect

F. Ando, Y. Miyasaka, T. Li, J. Ishizuka, T. Arakawa, Y. Shiota, T. Moriyama, Y. Yanase, and T. Ono Nature **584**, 373 (2020)



B: flipped

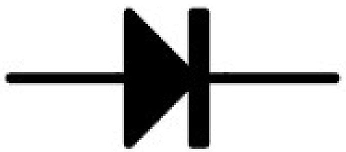


$R \neq 0$ for $-|I|$

$R = 0$ for $+|I|$

Superconducting diode effect $R(-I) > R(I) = 0$

- ✓ 100% rectification
- ✓ Ideal diode



= **nonreciprocity in *critical current*.**

$$\Delta j_c = j_{c+} - |j_{c-}| \neq 0$$

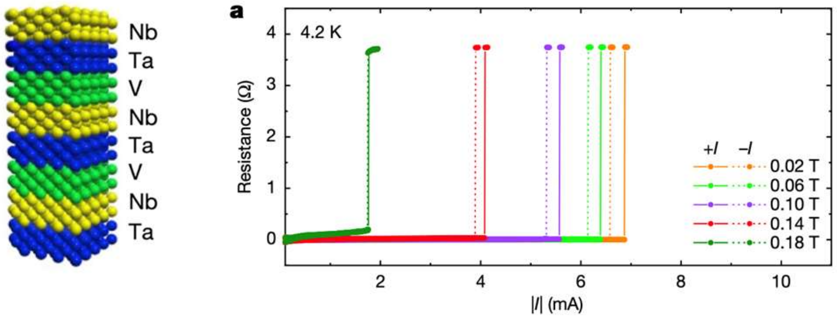
Rikken *et al.*, PRL (2001), (2005);
Magnetochiral anisotropy

cf. $R(I) = R_0(1 + \gamma B I)$

Experiments

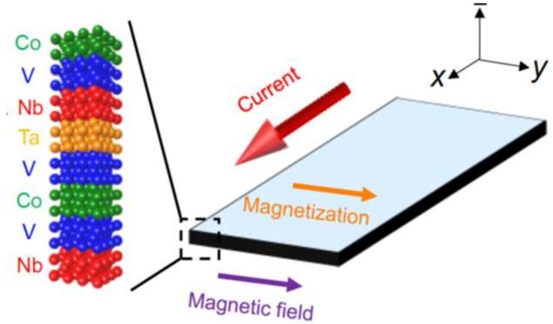
Rashba system + in-plane field

F. Ando *et al.*, Nature **584**, 373 (2020)



Rashba system/ferromagnet hybrid

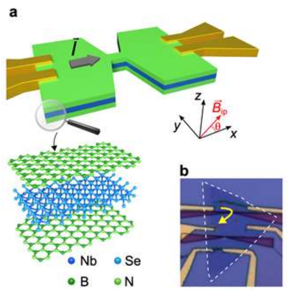
H. Narita *et al.*, Nature Nanotech. **17**, 823 (2022)

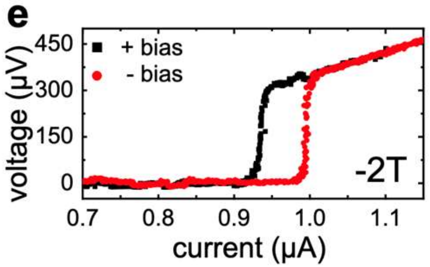


Ising (or Zeeman)-type SOC + out-of-plane field

NbSe$_2$ + mag. field

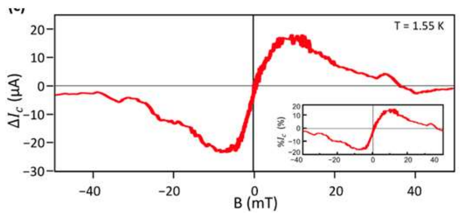
L. Bauriedl *et al.*, Nat. Commun. **13**, 4266 (2022)





NbSe$_2$/AFM + mag. field

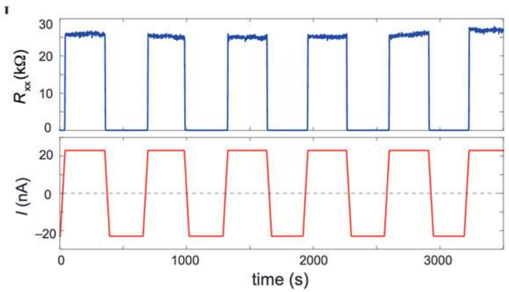
J. Shin *et al.*, arXiv:2111.05627



Zero field (triggered by SSB)

Twisted trilayer graphene/WSe$_2$

J.-X. Lin *et al.*, Nature Phys. **12**, 1221 (2022)



+ Remarkable development in Josephson diode effect H. Wu *et al.*, Nature **604**, 653 (2022)

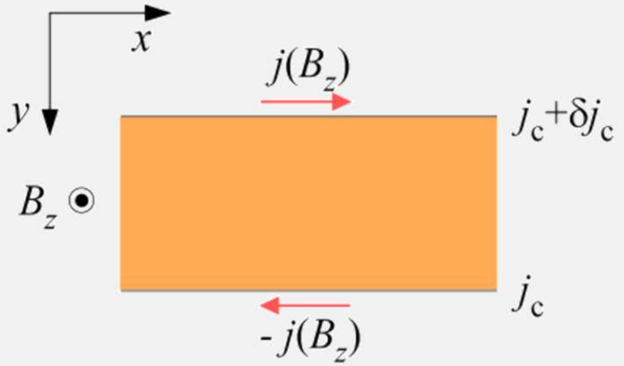
Extrinsic SC diode effect

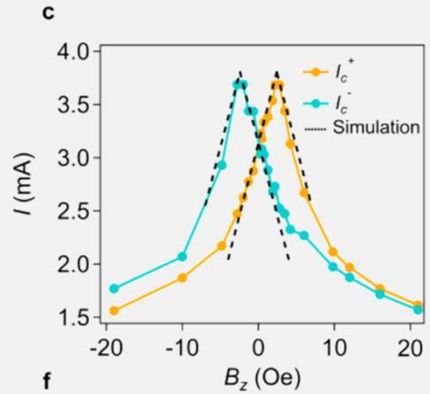
How can SC diode effect occur?

Some experiments might be understood by extrinsic mechanisms

Asymmetric
Meissner screening current

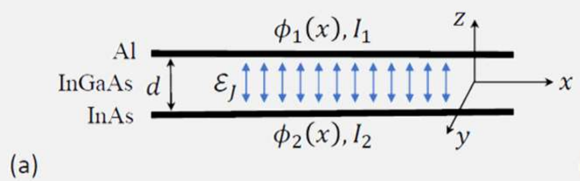
Y. Hou *et al.*, arXiv:2205.09276
M. K. Hope, *et al.*,
PRB **104**, 184512 (2021).

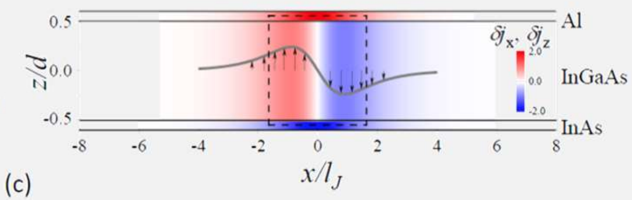




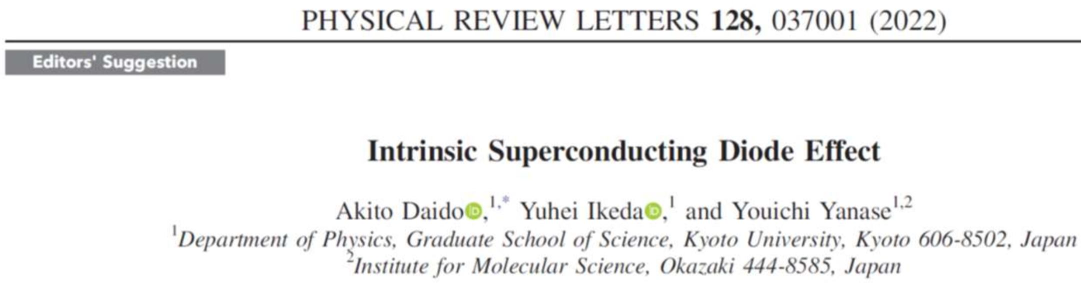
Vortex formation
in asymmetric multilayer SC

A. Sundaresh *et al.*, arXiv:2207.03633





SC diode effect as an *intrinsic material property*?


PHYSICAL REVIEW LETTERS **128**, 037001 (2022)
Editors' Suggestion
Intrinsic Superconducting Diode Effect
Akito Daido,[1,*] Yuhei Ikeda,[1] and Youichi Yanase[1,2]
[1]*Department of Physics, Graduate School of Science, Kyoto University, Kyoto 606-8502, Japan*
[2]*Institute for Molecular Science, Okazaki 444-8585, Japan*

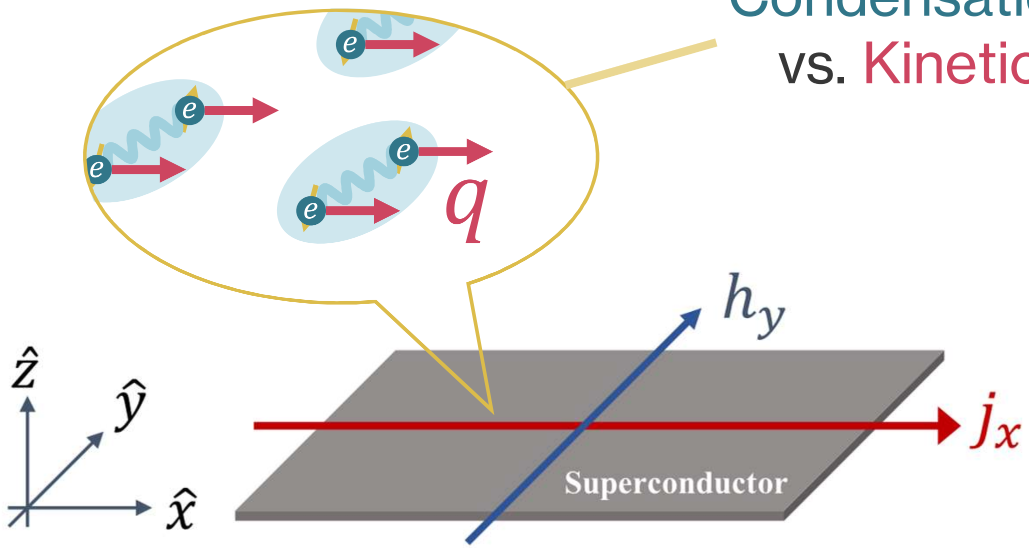
Our work

<u>The mechanism of SC diode effect is not clarified.</u>

We propose an intrinsic mechanism.

Target: ***depairing critical current***

Condensation energy
vs. Kinetic energy



Depairing critical current in FeSe

Y. Sun *et al.*, Phys. Rev. B 101, 134516 (2020).

Methods

How to evaluate Δj_c?

No diode effect in the simplest GL theory

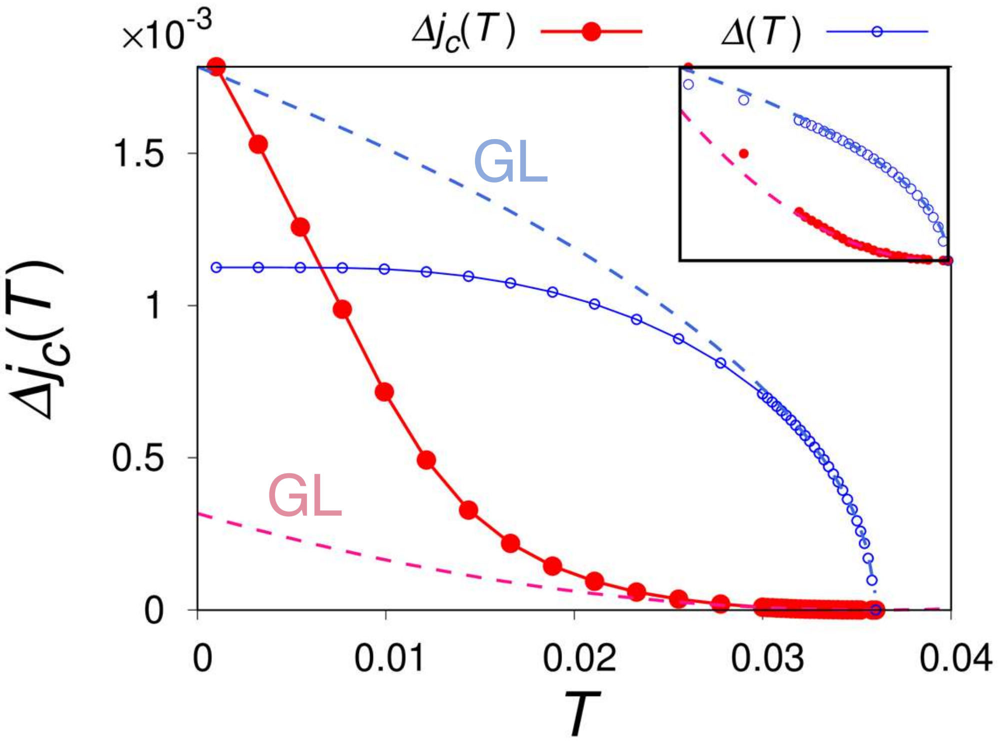
Smidman *et al.*, Rep. Prog. Phys. **80**, 036501 (2017).

1. **GL with higher-order spatial derivatives**

2. **Microscopic analysis: Relation to finite-q helical SC**

   [s-wave/d-wave Rashba-Zeeman model, Mean-field theory]

Δj_c : Temperature dependence



- **Temperature scaling**

$q^3\Delta^2$ $q\Delta^4$

$$\text{GL: } \Delta j_c = \left(\frac{16}{27\tilde{\beta}_0\tilde{\alpha}_2}\alpha_3 - \frac{8}{9\tilde{\beta}_0^2}\beta_1 \right)\tilde{\alpha}_0^2,$$

$\tilde{\alpha}_0 \propto T_c - T$

$$\Delta j_c(T) \propto (T_c - T)^2$$

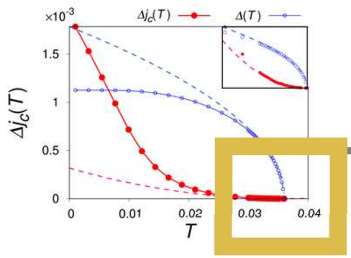
$$\Delta(T) \propto \sqrt{T_c - T}$$

cf.) N. Yuan and L. Fu, PNAS 119, e2119548119 (2022).
J. J. He, Y. Tanaka, and N. Nagaosa, New J. Phys. 24, 053014 (2022).

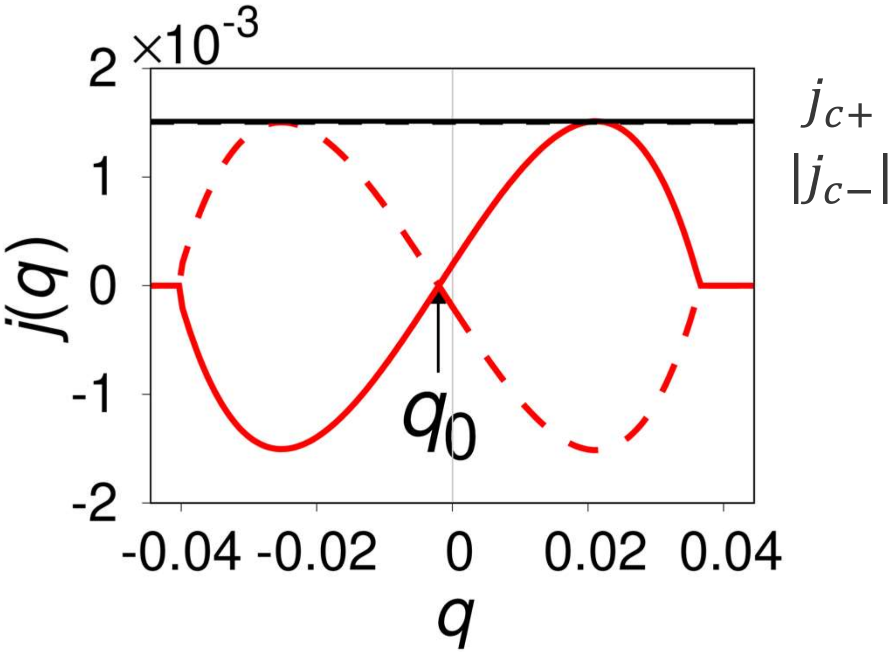
consistent with microscopic calc.

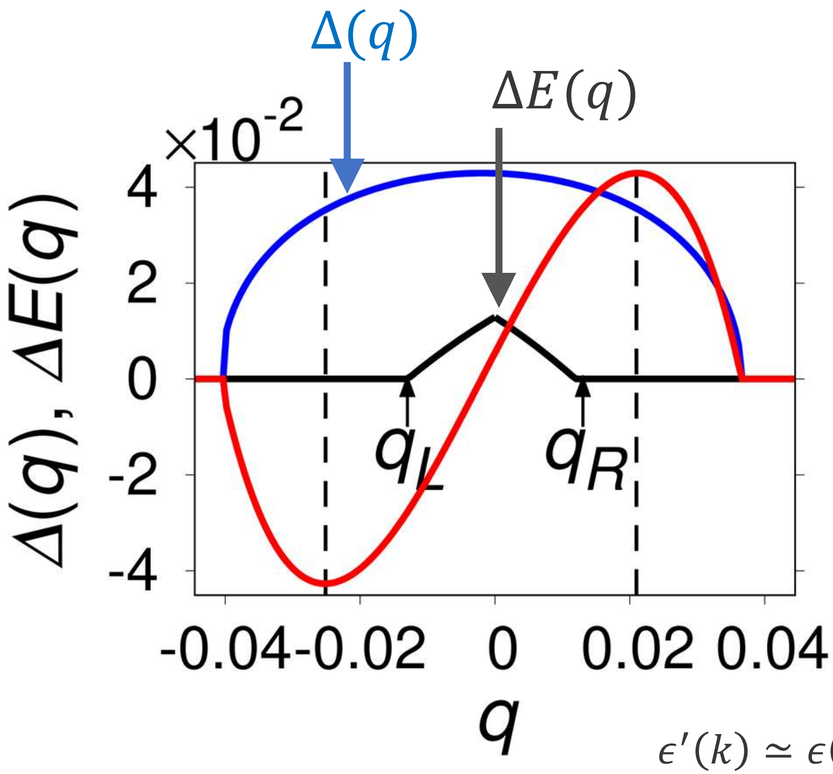
- **Enhancement at low temperature**

Why?

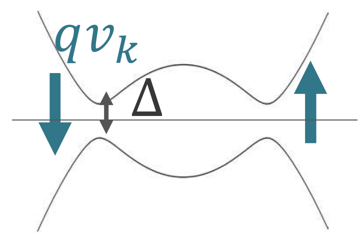


Diode effect at $T \sim T_c$

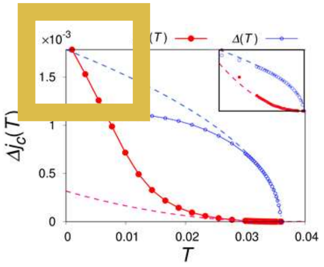




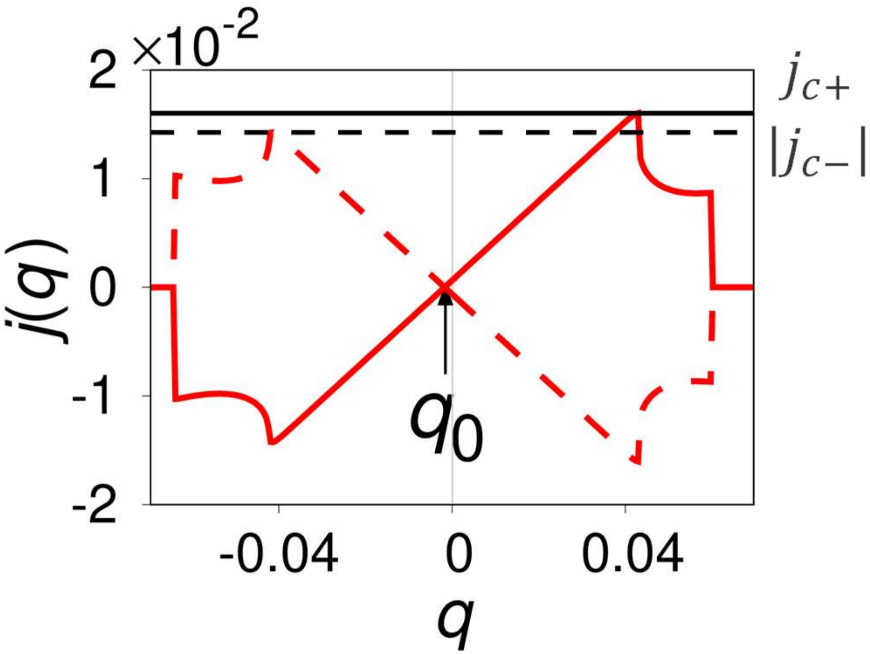
$\epsilon'(k) \simeq \epsilon(k) + q \cdot v_k$

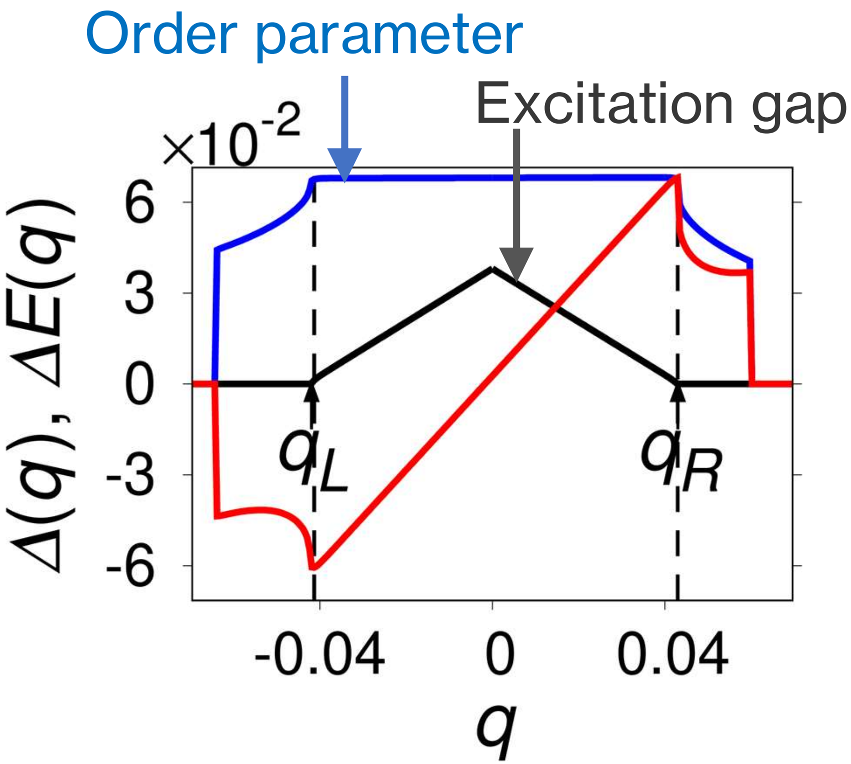


- Helical superconductivity $q_0 \neq 0$
- Tiny diode effect by the asymmetry of $j(q)$ (i.e. $f(q)$)
- j_{c+}& j_{c-} achieved in the gapless region



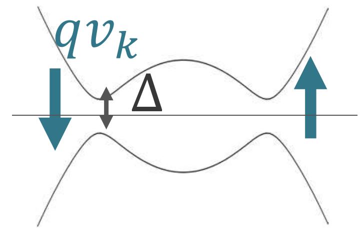
Diode effect at $T \sim 0$





- **j_{c+} & j_{c-} achieved at Landau critical momenta**
- $j(q_R) = n^s(q_R - q_0)/2 \quad j(q_L) = n^s(q_L - q_0)/2$

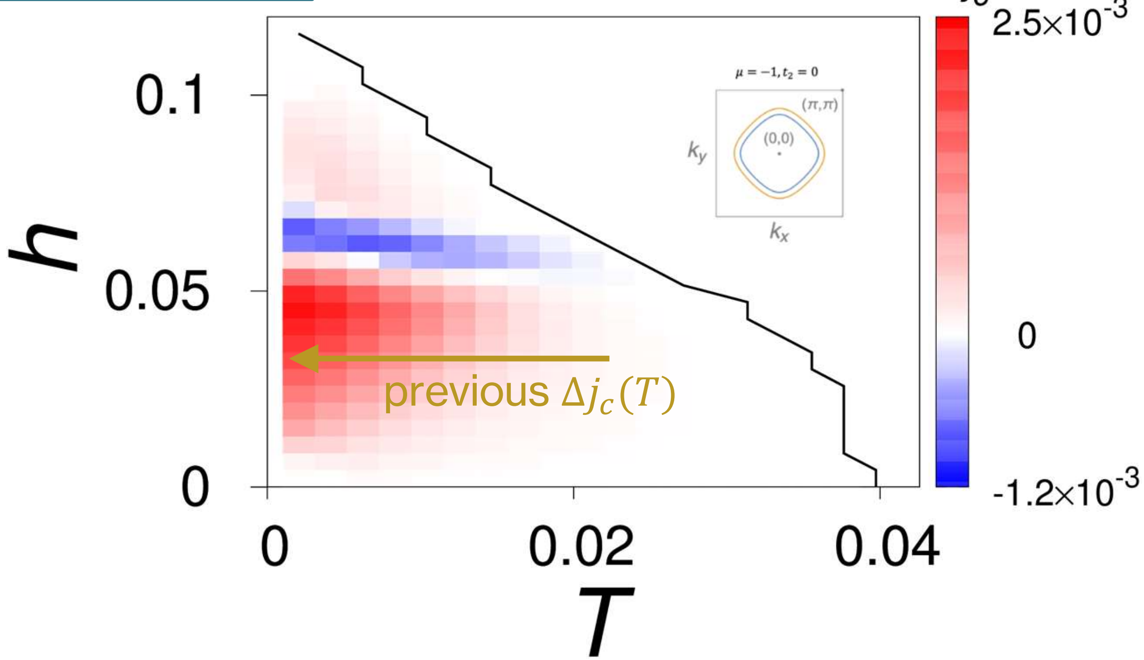
$$\Delta j_c = n^s(q_R + q_L - 2q_0)/2$$



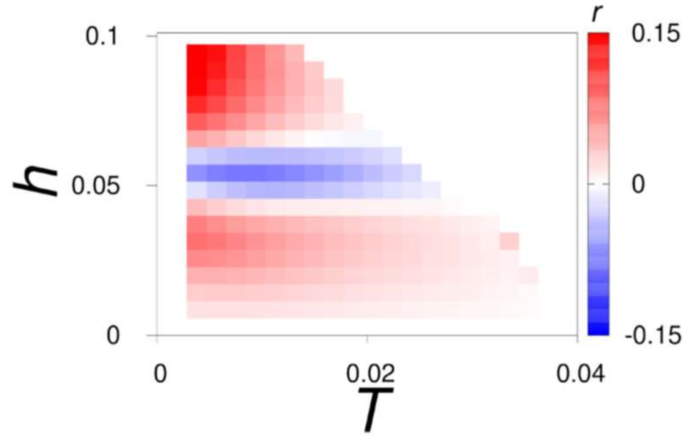
Phase diagram for $\Delta j_c(T,h)$

S-wave SC

$\Delta j_c(T,h)$



D-wave SC

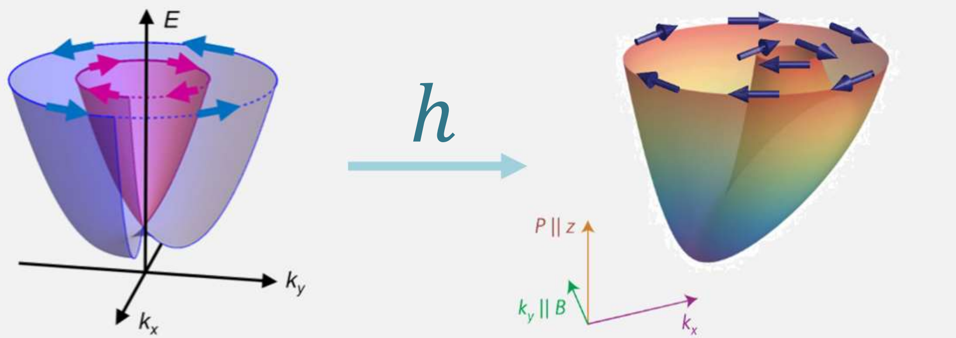


- **Large Δj_c at low T and sign reversals by $h\uparrow$**
- **Typical behavior of noncentrosymmetric SCs**

Why?

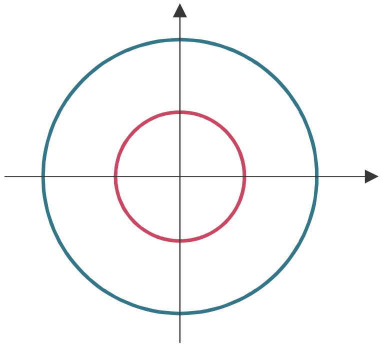
Nature of helical SC

Key: **Fermi-surface shift**



Shcherbakov *et al.*, Sci. Adv. (2021)

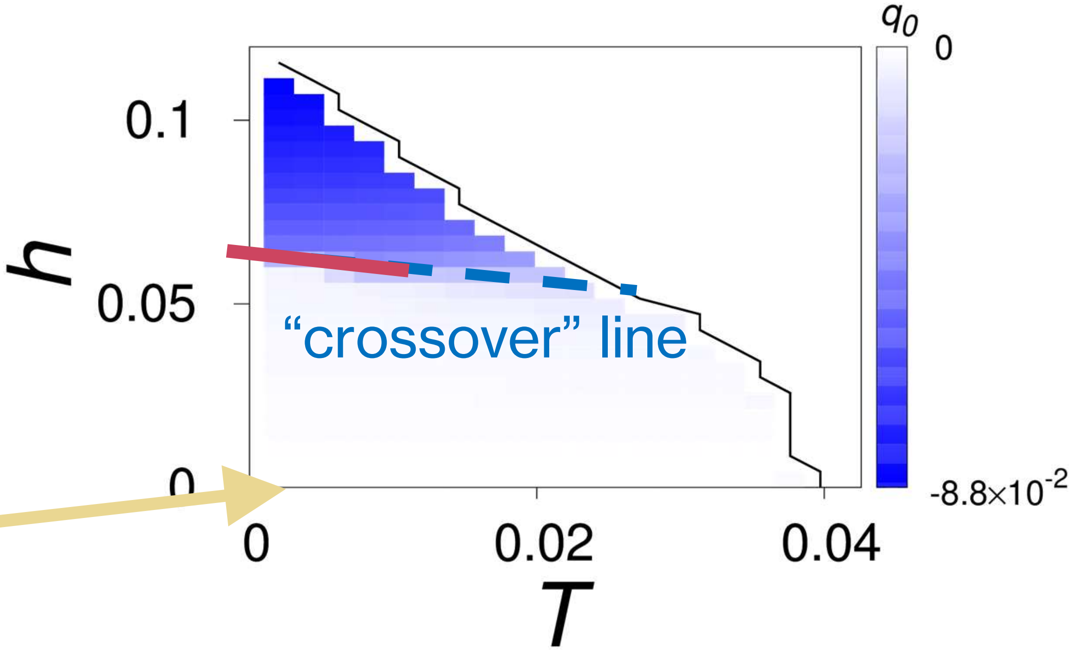
Ideue *et al.*, Nat. Phys. (2017)



$\underline{h = 0:}$

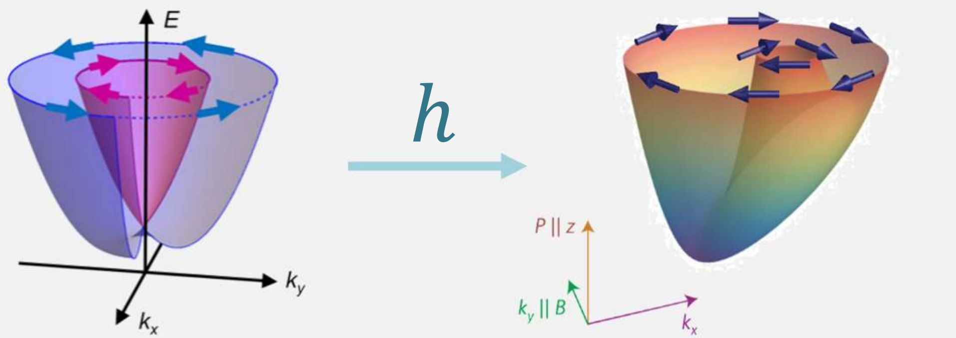
Both FS are happy with $q_0 = 0$.

Free-energy minimum at $q_0\,(h_y, T)$



Nature of helical SC

Key: **Fermi-surface shift**



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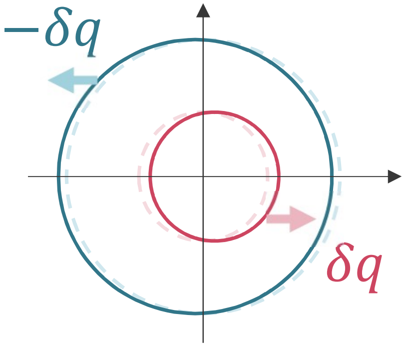
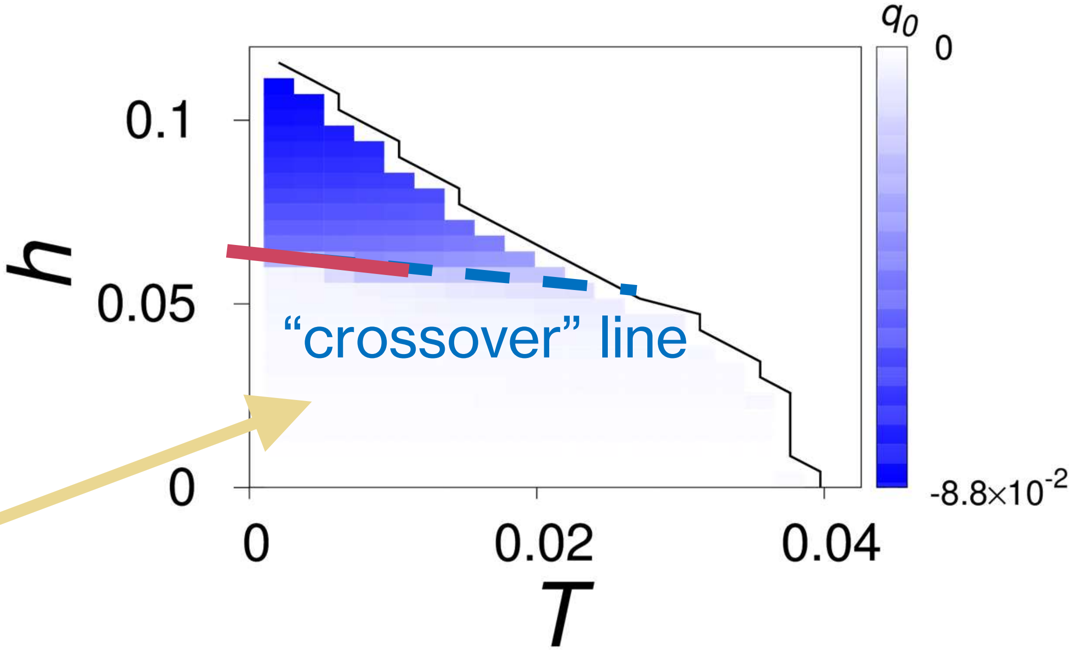


$\underline{\delta q \sim h \ll 1}$

Both FS are *almost happy* with $q_0 \sim 0$.

Free-energy minimum at $q_0\,(h_y, T)$



Nature of helical SC

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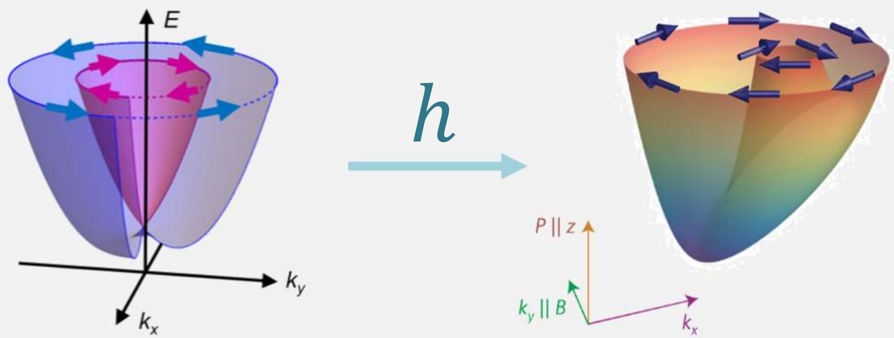
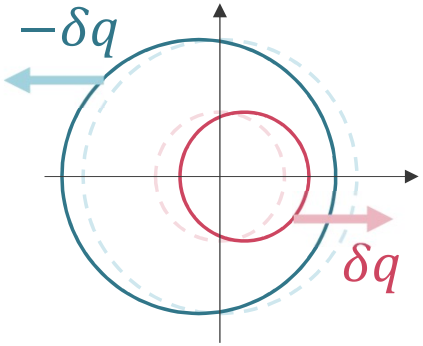


Shcherbakov *et al.*, Sci. Adv. (2021)

Ideue *et al.*, Nat. Phys. (2017)

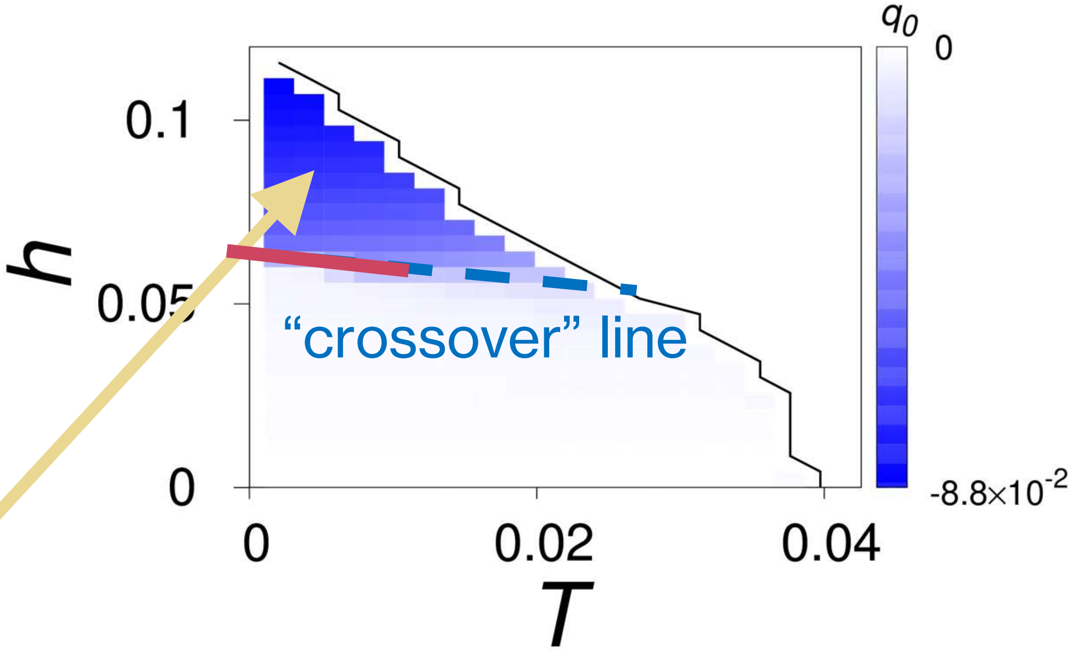


$\underline{\delta q \gg 1}$

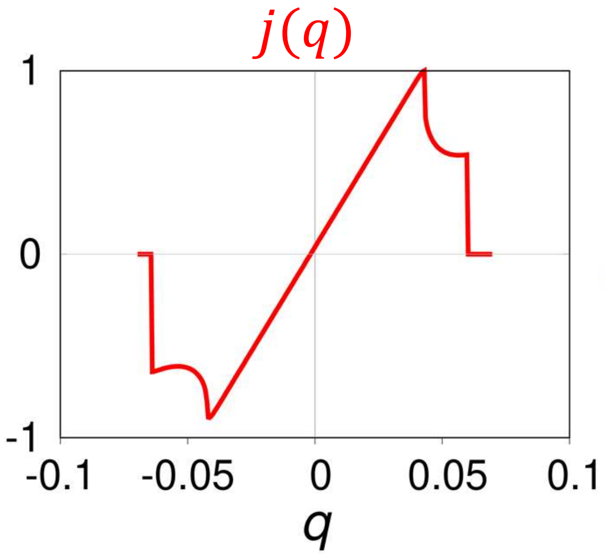
Both FS are *unhappy* with $q_0 \sim 0$.

Pairing on one FS is better: $q_0 \sim -\delta q$.

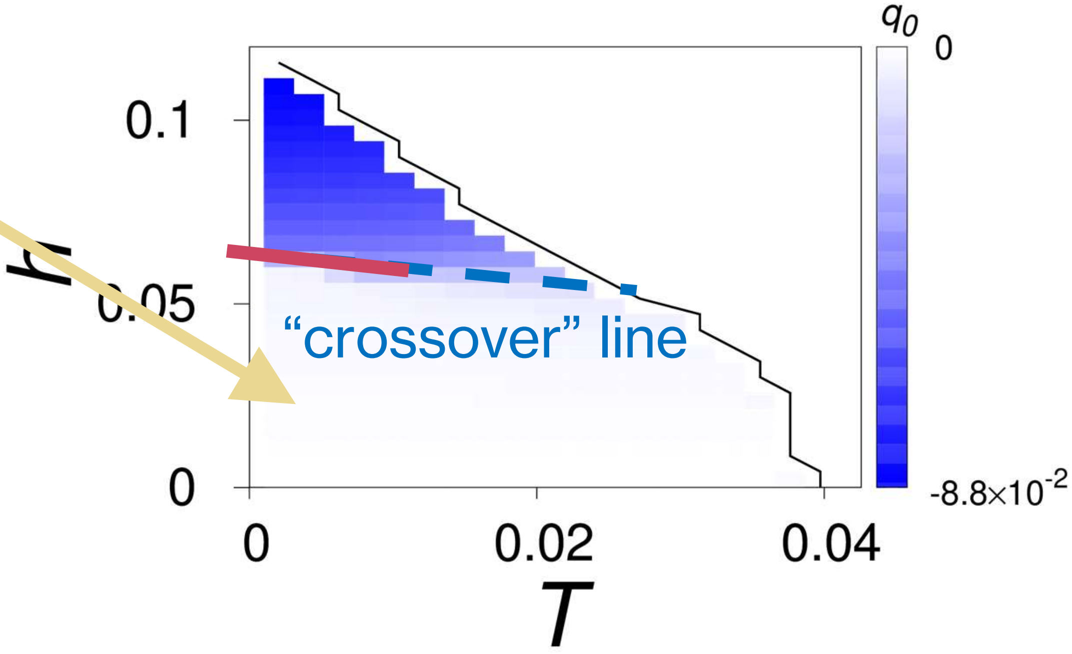
Free-energy minimum at $q_0\ (h_y, T)$



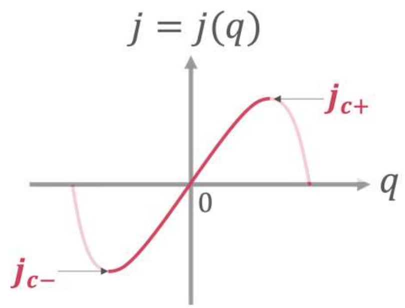
Current density $j(q)$



Equilibrium solution at
$q_0\ (T, h)$

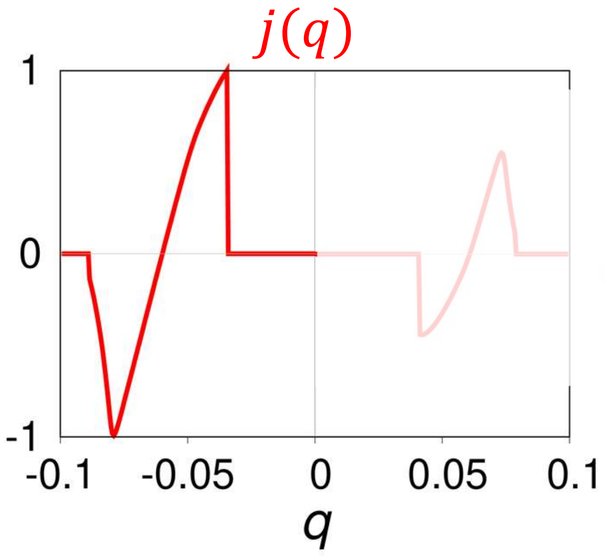


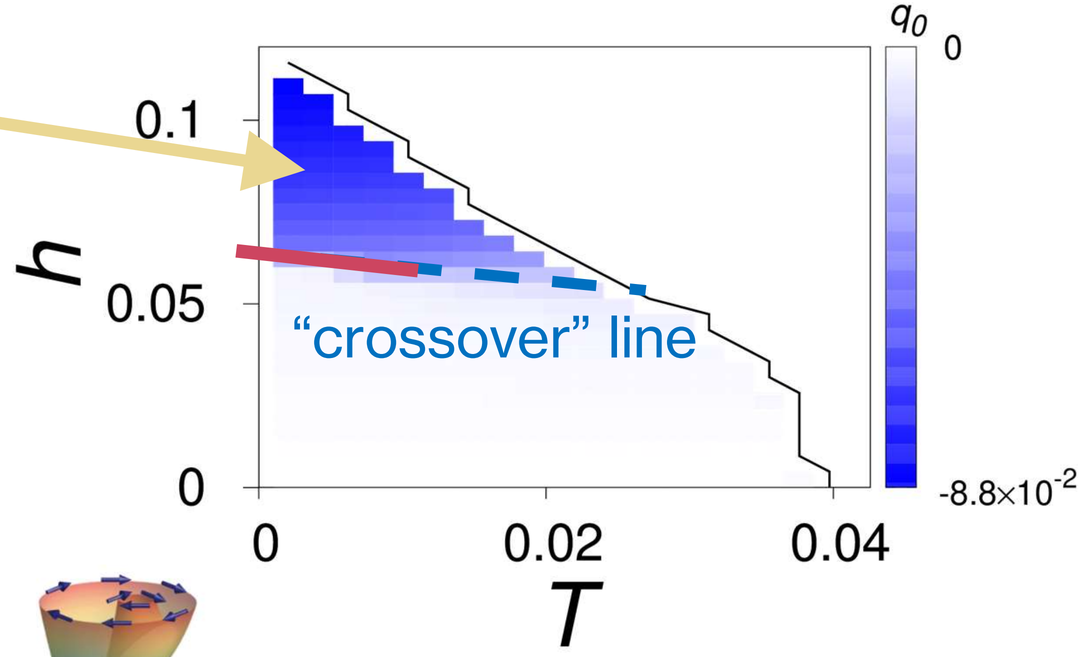
$\simeq$



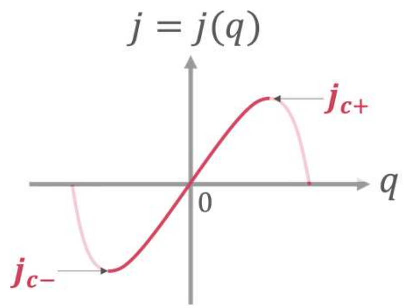
$\times 1$

Current density $j(q)$

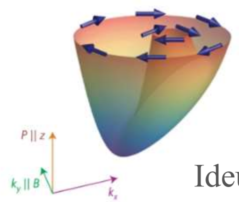




$\simeq$

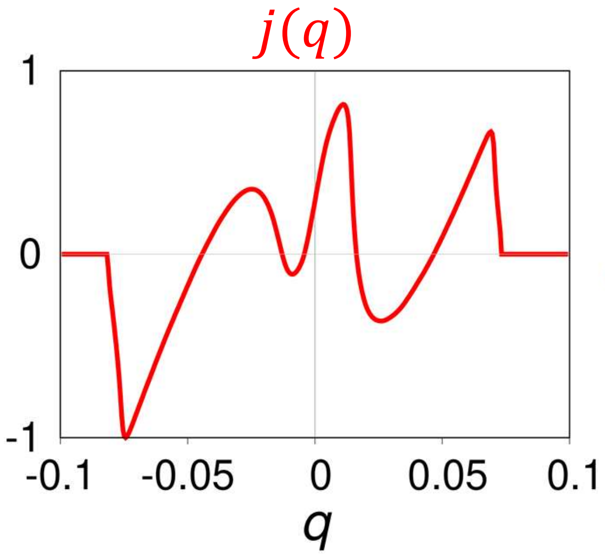


$\times\, 2$

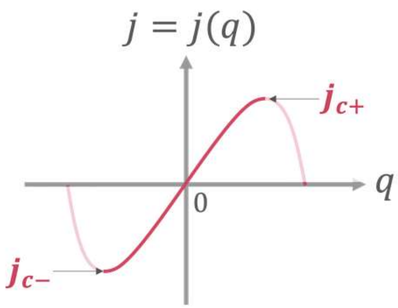


Ideue *et al.*, Nat. Phys. (2017)

Current density $j(q)$



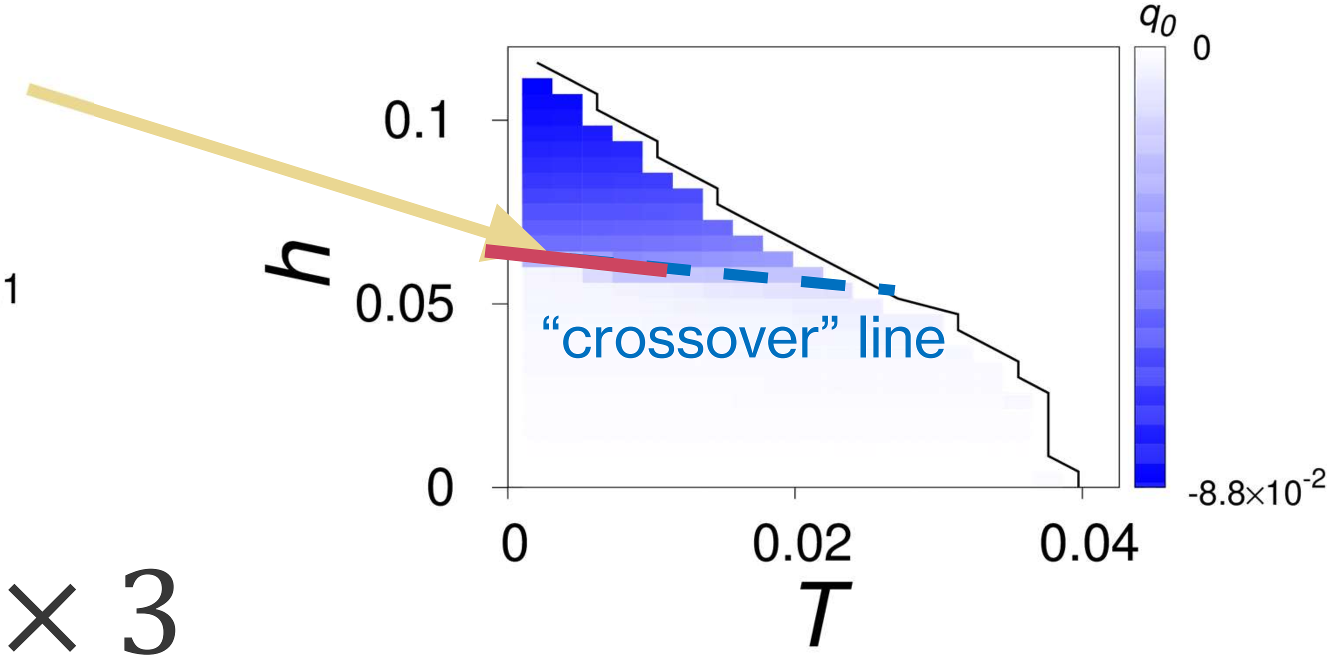
$\simeq$



$\times 3$

Equilibrium solution at

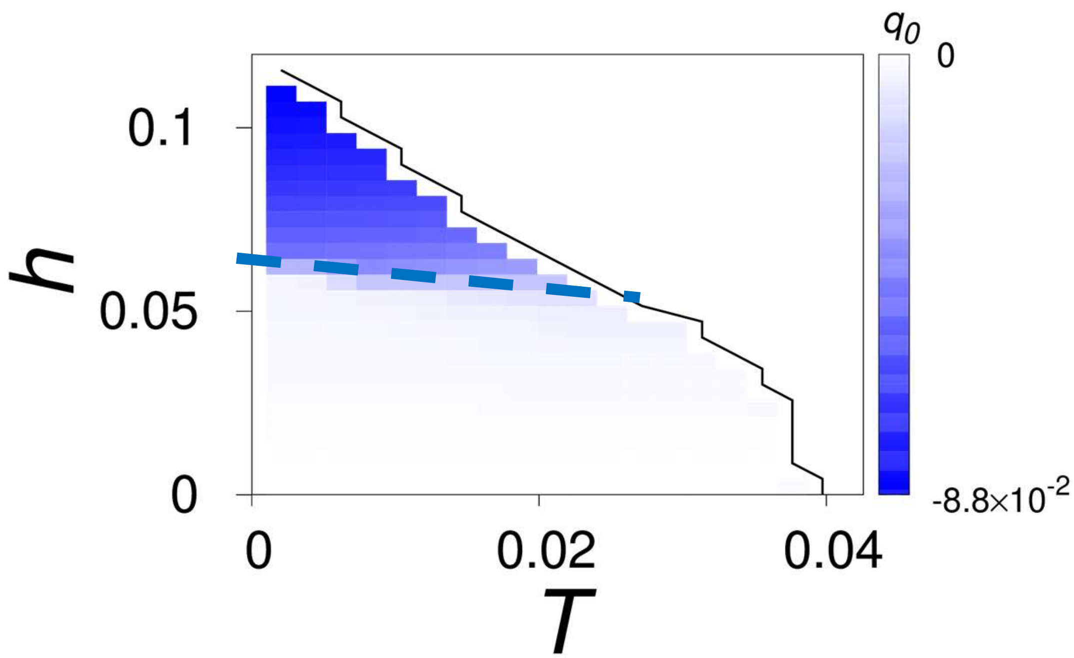
$$q_0\ (T, h)$$



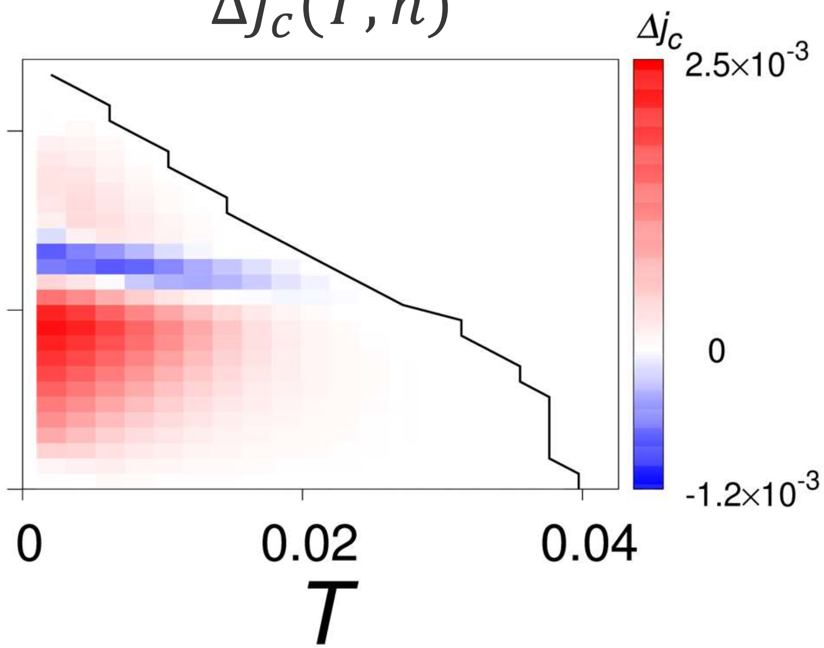
Phase diagram: origin of sign-reversals

A. Daido, Y. Ikeda, and YY, PRL 128, 037001 (2022)

$q_0\,(h_y, T)$



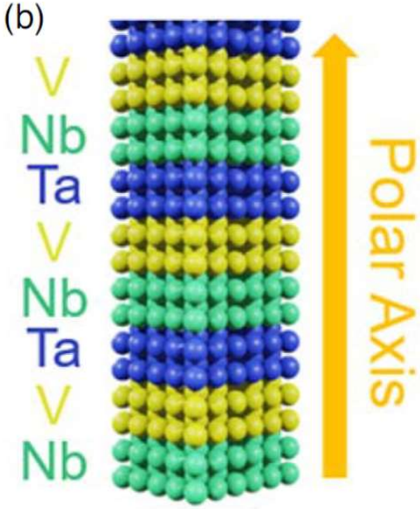
$\Delta j_c(T, h)$

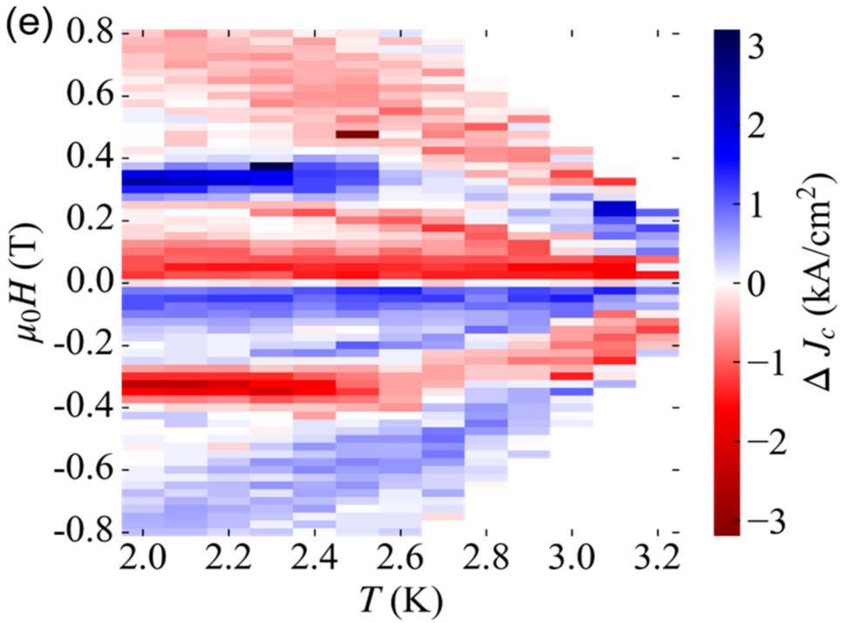


- **Helical superconductivity is realized**
- **Sign reversal of SDE ↔ Change in the nature of helical SC**
- **SC diode effect: promising probe of finite-q helical SC!**

Experiment: Polarity oscillation of SDE in H

R. Kawarazaki *et al*. Applied Physics Express 15, 113001 (2022)





- Sign reversal of SDE
- Magnetic field is smaller than theoretical prediction

Future issues

Theory:
- Layered structure
- Vortex state

Experiment:
- More 2D
- Exclusion of extrinsic mechanism (small device)

General theory of intrinsic SDE

A. Daido and YY, arXiv:2209.03515

General GL model

$$f(\boldsymbol{q},\psi) = \alpha(\boldsymbol{q})\psi^2 + \frac{\beta(\boldsymbol{q})}{2}\psi^4$$

$$\alpha(\boldsymbol{q}) = \alpha^{(0)} + \alpha_i^{(1)} q_i + \alpha_{ij}^{(2)} q_i q_j + \alpha_{ijk}^{(3)} q_i q_j q_k + \alpha_{ijkl}^{(4)} q_i q_j q_k q_l + O(q^5),$$

$$\beta(\boldsymbol{q}) = \beta^{(0)}(1 + \beta_i^{(1)} q_i + \beta_{ij}^{(2)} q_i q_j) + O(q^3).$$

Effective SOC of Cooper pairs

$$\boldsymbol{g}_{\mathrm{eff}}(\boldsymbol{q}) \equiv \frac{2\boldsymbol{g}_3(\boldsymbol{q})}{\sum_i q_i^2/2m_i} - \boldsymbol{g}_1(\boldsymbol{q})$$

$$[\alpha_3]_{ijk}\delta q_i \delta q_j \delta q_k \equiv \boldsymbol{h}\cdot\boldsymbol{g}_3(\delta\boldsymbol{q}),$$
$$[\beta_1]_{ij}\delta q_i \equiv \boldsymbol{h}\cdot\boldsymbol{g}_1(\delta\boldsymbol{q}),$$

Intrinsic SDE in GL region

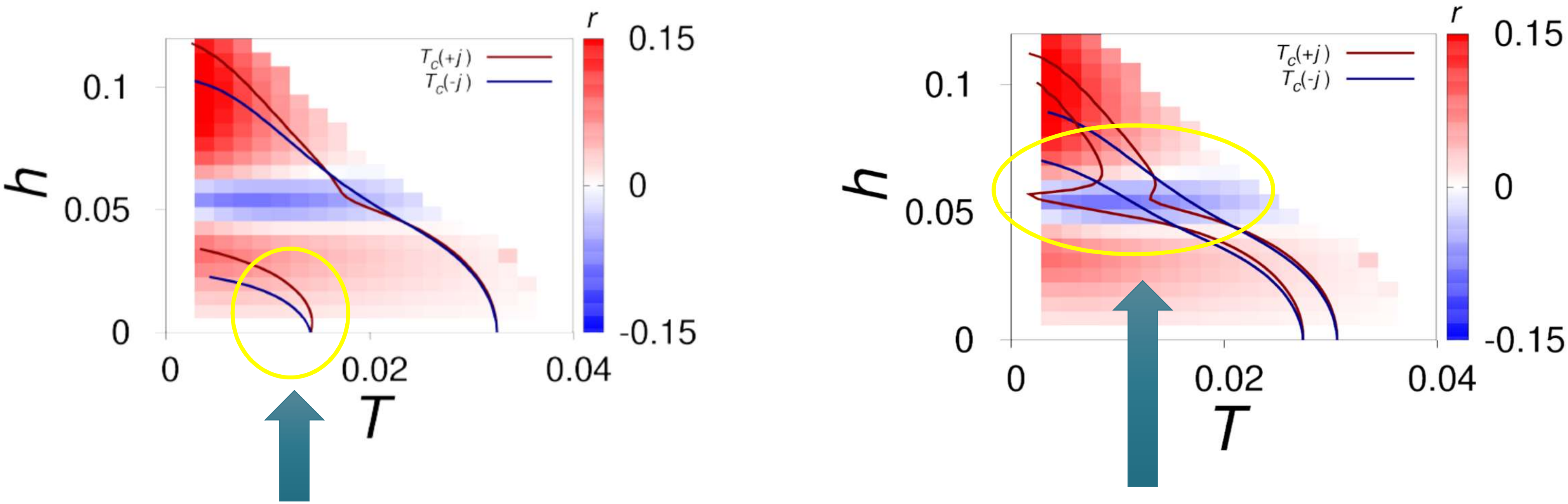
$$\Delta j_c(\hat{n}) = \frac{8a_0^2}{9\beta_0}(T_c - T)^2\, \boldsymbol{g}_{\mathrm{eff}}(\hat{n})\cdot\boldsymbol{h}$$

- All the 21 noncentrosymmetric point group allows the SDE. (even non-gyrotropic)
- Behaviors of SDE can be predicted by the point group.

Nonreciprocal phase transition lines

A. Daido and YY, arXiv:2209.03515

Solid lines = phase transition lines under supercurrent



Large current: field-enhanced SC

Moderate current:
nonreciprocal reentrant SC

Contents

1. Superconducting diode effect (SDE)

   Finite-q pairing in the SC state

   (FFLO, helical, anapole)

2. Nonreciprocal optics

   Quantum geometry of Bloch electrons

   Parity mixing in Cooper pairs

Hikaru Watanabe
(Univ. Tokyo)

Hiroto Tanaka
(Kyoto)

Formulation for nonlinear conductivity (normal state)

■ Response formula for nonlinear conductivity

$$J^{\mu}_{(2)}(\omega) = \int \frac{d\omega_1 d\omega_2}{(2\pi)^2} \tilde{\sigma}^{\mu;\nu\lambda}(\omega;\omega_1,\omega_2) E^{\nu}(\omega_1) E^{\lambda}(\omega_2)$$

(2^{nd}-) nonlinear current

- Dipole-approximation (no photo-magnetic field, not spatially-dispersive)
- Non-interacting electrons + phenomenological scattering

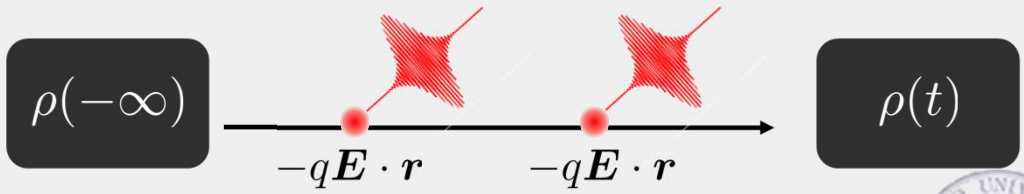
■ Perturbation from photo-electric field

J. E. Sipe & A. I. Shkrebtii, Phys. Rev. B 61, 5337 (2000);
G. B. Ventura et al., Phys. Rev. B 96, 035431 (2017)

$$i\partial_t \rho(t) = [\mathcal{H}(t), \rho(t)],$$

von Neumann equation

- $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_E(t),\ \mathcal{H}_E(t) = -q\boldsymbol{E}(t)\cdot\boldsymbol{r}$
- $\partial_t \rho \to \partial_t \rho + i\gamma(\rho - \rho_{\mathrm{eq}})$



Density mat.
in eq

Formulation for nonlinear conductivity (normal state)

- Second order term in electric field

Monochromatic light induced photocurrent

$$J(t) = \mathrm{Tr}[qv\,\rho(t)] \Rightarrow J^{\mu}_{\mathrm{dc}} = \sigma^{\mu;\nu\lambda}(\omega = 0; \omega_1 = -\Omega, \omega_2 = \Omega) E^{\nu}(-\Omega) E^{\lambda}(\Omega)$$

- Polarization analysis of photocurrent conductivity

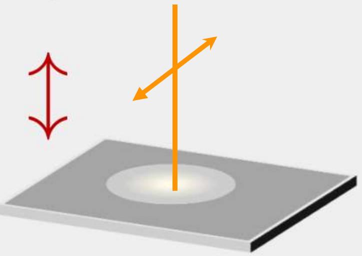
B. I. Sturman & V. M. Fridkin, (CRC Press,1992).
Photovoltaic and Photo-refractive Effects in Noncentrosymmetric Materials

$$J^{\mu}_{\mathrm{dc}} = \sigma^{\mu;\nu\lambda} E^{\nu}(-\Omega) E^{\lambda}(\Omega) = \sigma^{\mu;\nu\lambda}[E^{\nu}(\Omega)]^{*} E^{\lambda}(\Omega)$$

$$\sigma^{\mu;\nu\lambda}(E^{\nu})^{*}E^{\lambda} = \frac{1}{2}\left(\sigma^{\mu;\nu\lambda} + \sigma^{\mu;\lambda\nu}\right)\mathrm{Re}[(E^{\nu})^{*}E^{\lambda}] + \frac{i}{2}\left(\sigma^{\mu;\nu\lambda} - \sigma^{\mu;\lambda\nu}\right)\mathrm{Im}[(E^{\nu})^{*}E^{\lambda}],$$

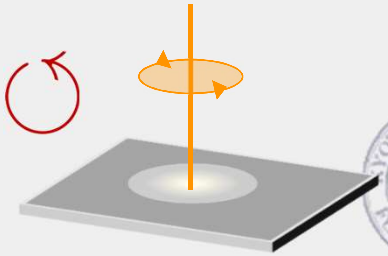
$\mathrm{Re}[(E^{\nu})^{*}E^{\lambda}]$, (ν, λ): symmetric

Linearly polarized light



$\mathrm{Im}[(E^{\nu})^{*}E^{\lambda}]$, (ν, λ): anti-sym.

Circularly polarized light



Classification of photocurrent [Watanabe and YY, Phys. Rev. X 11, 011001 (2021)]

	P × T ○ PT × **T-symmetric**	P × T × PT ○ **PT-symmetric**
Metal	**Berry curvature dipole** ↻ J. E. Moore & J. Orenstein, Phys. Rev. Lett. 105, 026805 (2010).	**Drude photocurrent** ↕ T. Holder et al., Phys. Rev. Research 2, 033100 (2020) .
	Intrinsic Fermi surface term ↻ F. de Juan et al., Phys. Rev. Research 2, 012017 (2020).	**Intrinsic Fermi surface term** ↕ **This work**
Metal and insulator	**Electric injection current** ↻ J. E. Sipe & A. I. Shkrebtii, Phys. Rev. B 61, 5337 (2000).	**Magnetic injection current** ↕ Y. Zhang et al., Nat. Commun. 10, 3783 (2019).
	Shift current ↕ R. von Baltz & W. Kraut, Phys. Rev. B 23, 5590 (1981); J. E. Sipe & A. I. Shkrebtii, Phys. Rev. B 61, 5337 (2000).	**Gyration current** ↻ **This work**

Electric/Magnetic multipole ⇔ T/PT symmetry ⇔ Linearly/Circularly polarized light

Classification of photocurrent based on quantum geometry

$$\sigma^{\mu;\nu\lambda} \sim \int \frac{d\boldsymbol{k}}{(2\pi)^d} \sum_{a\neq b} G^{\mu\nu\lambda}_{ab} \; [f(\epsilon_a) - f(\epsilon_b)]\,\delta(\hbar\Omega - \epsilon_b - \epsilon_a), \quad G^{\mu\nu\lambda} = X^{\mu} T^{\nu\lambda}$$

	Director X^{μ}	Transition amp. $T^{\nu\lambda}$
Electric injection current	Δ^{μ} Group velocity difference	$\Omega^{\nu\lambda}$
Shift current	R^{μ} Shift vector	$g^{\nu\lambda}$
Magnetic injection current	Δ^{μ}	$g^{\nu\lambda}$
Gyration current	$R^{\mu}_{\pm}$	$g^{\nu\lambda}, \Omega^{\nu\lambda}$

$\Omega^{\nu\lambda}$ (band-resolved) Berry curvature $\quad g^{\nu\lambda}$ (band-resolved) quantum metric

→ **Diverging photocurrent in AFM Dirac/Weyl semimetals**

Watanabe and YY, Phys. Rev. X 11, 011001 (2021), J. Ahn *et al.*, Phys. Rev. X 10, 041041 (2020).

Superconductors

H. Watanabe, A. Daido, and YY, Phys. Rev. B 105, 024308 (2022)

H. Watanabe, A. Daido, and YY, Phys. Rev. B 105, L100504 (2022)

H. Tanaka, H. Watanabe, and YY, arXiv:2205.14445

Optical responses unique to superconductors

Variational parameter : $\boldsymbol{\lambda}$ (= vector potential A)

$$J^{\alpha}_{ab}(\boldsymbol{k}) = -\lim_{\boldsymbol{\lambda}\to\boldsymbol{0}} \left\langle a_{\boldsymbol{\lambda}} \middle| \frac{\partial H_{\boldsymbol{\lambda}}}{\partial \lambda_\alpha} \middle| b_{\boldsymbol{\lambda}} \right\rangle \qquad H_{\boldsymbol{\lambda}}(\boldsymbol{k}) = \begin{pmatrix} H_N(\boldsymbol{k}-\boldsymbol{\lambda}) & \Delta \\ \Delta^\dagger & -[H_N(-\boldsymbol{k}-\boldsymbol{\lambda})]^T \end{pmatrix}$$

λ -derivative and k -derivative are not equivalent. ⟺ **Opposite charges of electrons and holes**

Divergent response at THz or sub THz regime

$\sim O(\omega_1{}^{-1}\omega_2{}^{-1})$

Nonreciprocal superfluid weight

$$\sigma^{\alpha;\beta\gamma}_{\mathrm{NRSF}} = \frac{1}{2\omega_1\omega_2} f^{\alpha\beta\gamma}$$

$\sim O(\omega^{-1})$

Conductivity derivative

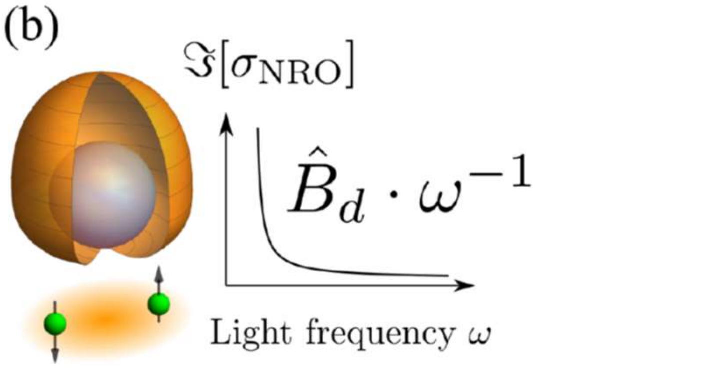
$$\sigma^{\alpha;\beta\gamma}_{\mathrm{sCD}} = -\frac{i}{4}\left(\frac{1}{\omega_2}\left(D_d^{\beta;\alpha\gamma} + \epsilon_{\alpha\gamma\delta} B_d^{\beta\delta}\right) + \frac{1}{\omega_1}\left(D_d^{\gamma;\alpha\beta} + \epsilon_{\alpha\beta\delta} B_d^{\gamma\delta}\right)\right)$$

Photocurrent in superconductors [Watanabe, Daido, YY, PRB 105, 024308 (2022)]

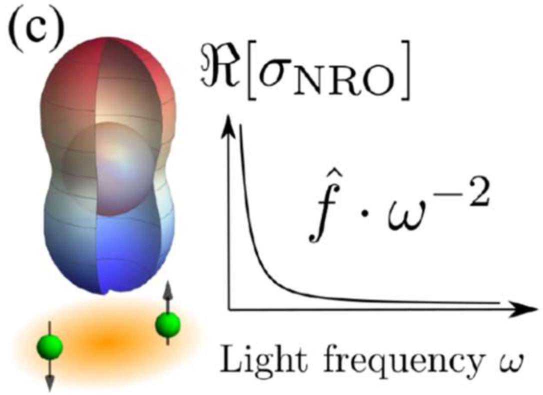
	T-symmetric		PT-symmetric	
Metal	**Berry curvature dipole**	↺	**Drude photocurrent**	↕
	Intrinsic Fermi surface term	↺	**Intrinsic Fermi surface term**	↕
Metal and insulator	**Electric injection current**	↺	**Magnetic injection current**	↕
	Shift current	↕	**Gyration current**	↺
Super-conductor	**Berry curvature derivative**	↺	**Nonreciprocal superfluid density**	↕
			Drude derivative	↺

Giant nonreciprocal optical responses in superconductors

(b)



(c)



$$\sigma^{\alpha;\beta\gamma}_{\rm sCD} = \lim_{\boldsymbol{\lambda}\to \mathbf{0}} \frac{i}{4\Omega}\epsilon_{\beta\gamma\delta}\partial_{\lambda_\alpha}\left(\sum_a \Omega_a^{\lambda_\delta} f_a\right) = \hat{B}_{\rm d}$$

Berry curvature derivative

$$\sigma^{\alpha;\beta\gamma}_{\rm NRSF} = \lim_{\boldsymbol{\lambda}\to \mathbf{0}} -\frac{1}{2\Omega^2}\partial_{\lambda_\alpha}\partial_{\lambda_\beta}\partial_{\lambda_\gamma}F_{\boldsymbol{\lambda}}, = \hat{f}$$

Nonreciprocal superfluid weight

Super-
conductor

Berry curvature derivative

Nonreciprocal superfluid density

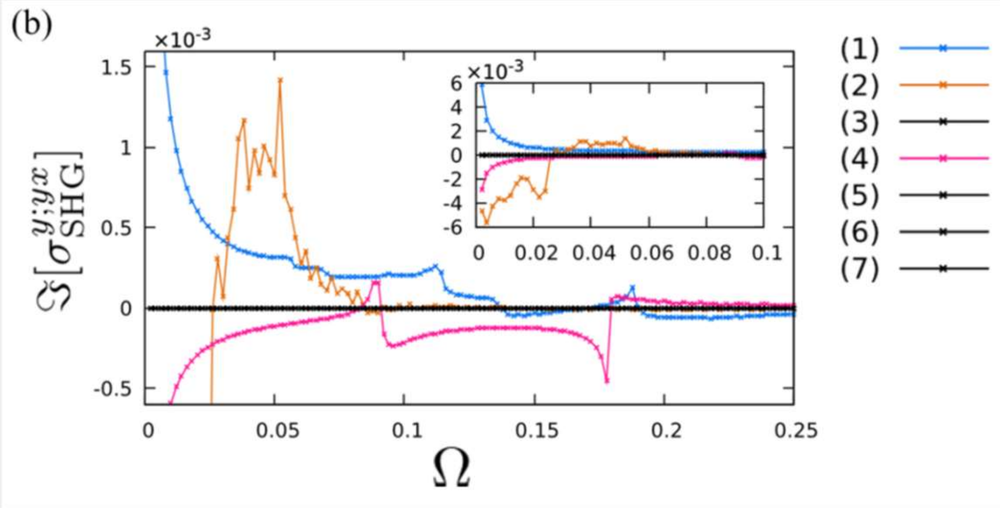
Drude derivative

Microscopic conditions [Tanaka, Watanabe, and YY, arXiv:2205.14445]

2D noncentrosymmetric superconductors

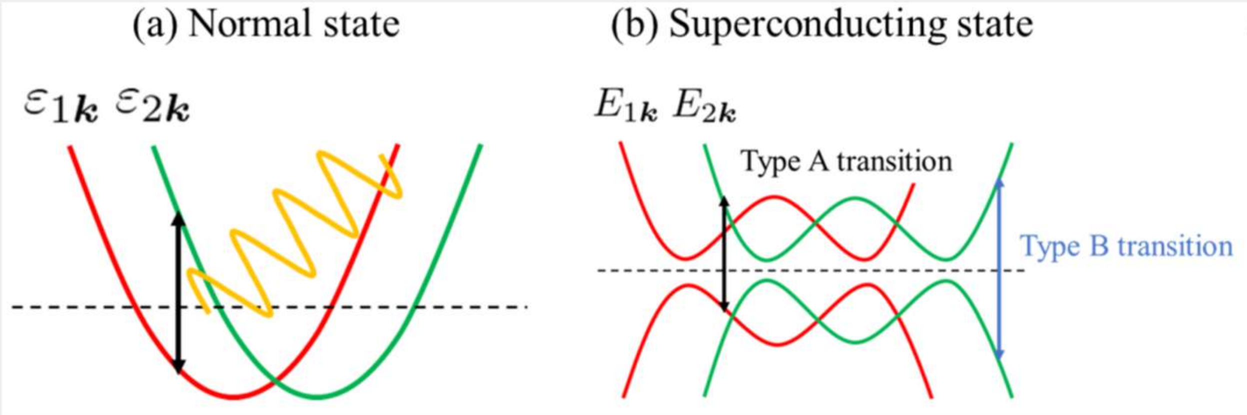
$$H_N(\boldsymbol{k}) = \xi(\boldsymbol{k}) + \underline{\alpha \boldsymbol{g}(\boldsymbol{k}) \cdot \boldsymbol{\sigma}}$$ ← **ASOC**

$$\Delta(\boldsymbol{k}) = [\psi(\boldsymbol{k}) + \boldsymbol{d}(\boldsymbol{k}) \cdot \boldsymbol{\sigma}]\, i\sigma_y$$ ← **Parity mixing**



No.	$(\alpha_1, \alpha_2, \alpha_3)$	ψ_0	(d_1, d_2, d_3)	$\boldsymbol{d_k} \cdot \boldsymbol{g_k}$	$\boldsymbol{d_k} \times \boldsymbol{g_k}$	$(\partial_x \boldsymbol{g_k} \times \partial_y \boldsymbol{g_k}) \cdot \hat{\boldsymbol{d}}_{\boldsymbol{k}}$	Δ_{intra}	Δ_{inter}	response
1	(0.3, 0.8, 0.1)	0.07	(0.04, 0.02, 0.05)	nonzero	nonzero	nonzero	nonzero	nonzero	nonzero
2	(0.3, 0.8, 0.1)	0	(0.04, 0.02, 0.05)	nonzero	nonzero	nonzero	nonzero	nonzero	nonzero
3	(0.2, 0.9, 0.1)	0.01	(0.06, 0.05, 0.03)	nonzero	nonzero	zero	nonzero	nonzero	zero
4	(0.7, 0.2, 0.3)	0.09	(0.03, 0, -0.07)	zero	nonzero	nonzero	nonzero	nonzero	nonzero
5	(0.7, 0.2, 0.3)	0	(0.03, 0, -0.07)	zero	nonzero	nonzero	zero	nonzero	zero
6	(0.4, 0.1, 0.6)	0.09	(0, 0, 0)	zero	zero	undefined	nonzero	zero	zero
7	(0.3, 0.2, 0.6)	0	(0, 0, 0)	zero	zero	undefined	zero	zero	zero

Microscopic conditions [Tanaka, Watanabe, YY, arXiv:2205.14445]



- Type B transition causes the nonlinear optical responses unique to superconductors
- Intraband Cooper pairs and interband Cooper pairs are necessary

1. Finite intraband pairing $\mathrm{Tr}\left[F_A(\boldsymbol{k})^\dagger F_A(\boldsymbol{k})\right] = 4\left[(\alpha \boldsymbol{g_k}\cdot \boldsymbol{d_k})^2 + \psi_{\boldsymbol{k}}^2 \alpha^2 g_{\boldsymbol{k}}^2\right] \neq 0$
2. Finite interband pairing $\mathrm{Tr}\left[F_C(\boldsymbol{k})^\dagger F_C(\boldsymbol{k})\right] = 8\alpha^2 |\boldsymbol{g_k}\times \boldsymbol{d_k}|^2 \neq 0$

Superconducting fitness [Ramires, 2018]

Conclusion:
Spin-triplet Cooper pairs are necessary for SC nonlinear optical responses.

Summary

Intrinsic SC diode effect

A. Daido, Y. Ikeda, and YY

Phys. Rev. Lett. 128, 037001 (2022)

A. Daido and YY, arXiv:2209.03515

- **Nonreciprocal depairing critical current**
- **Large Δj_c** at low T & moderate h_y
- **Sign reversals** as h_y ↑

→ **Promising probe of finite-q helical SC**

SC nonreciprocal optics

H. Watanabe, A. Daido, and YY,

Phys. Rev. B 105, 024308 (2022)

Phys. Rev. B 105, 024308 (2022)

Tanaka, Watanabe, and YY, arXiv:2205.14445

- **Nonlinear optical responses unique to SC**
- **Diverging photocurrent and SHG at sub THz**
- **Sensitive to quantum geometry**

→ **Promising probe of parity-mixed Cooper pairs**