Quantum structural-spin glass in two-dimension at finite temperature

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Hotta, Ueda, Imada, arXiv:2207.07293

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Take home message

We found a finite *T* glass transition in 2D for the first time
in the triangular lattice transverse (quantum) AF Ising model

What is glass ?

→ Loss of ergodicity between numerous competing states

→ “Order parameter” is a replica overlap

It has something in common to other correlated disorders(QSL, MBL, et

How to make a glass.

It is very difficult to form a glass in 2D and 3D lattice models.

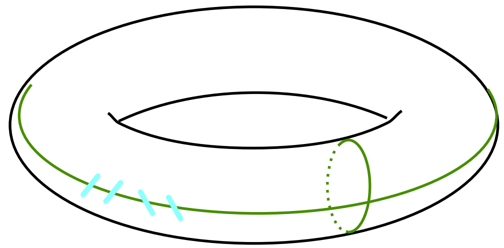
- two coupled degrees of frustrated freedom
- classical 3D or quantum 2D

Correlated disorder

- Not a simple(free) paramagnet.
- No local order parameter.
- Hilbert space divided into groups

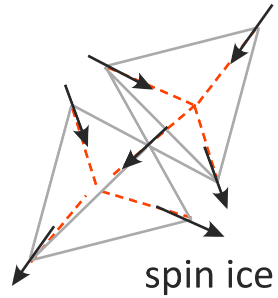
Quantum spin liquid(QSL) (topological order, QHE)

topologial degeneracy



- Hilbert space divided into topological sectors
- degenerate entanglement spectrum

Classical spin liquid(CSL)

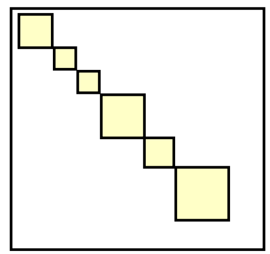


spin ice

- Only few low energy sectors (order-N degeneracy) join

mismatch of local symmetry and the (higher) average symmetry

Many-body localization (MBL)



- Hilbert space fragmentation
- Edwards-Anderson parameter

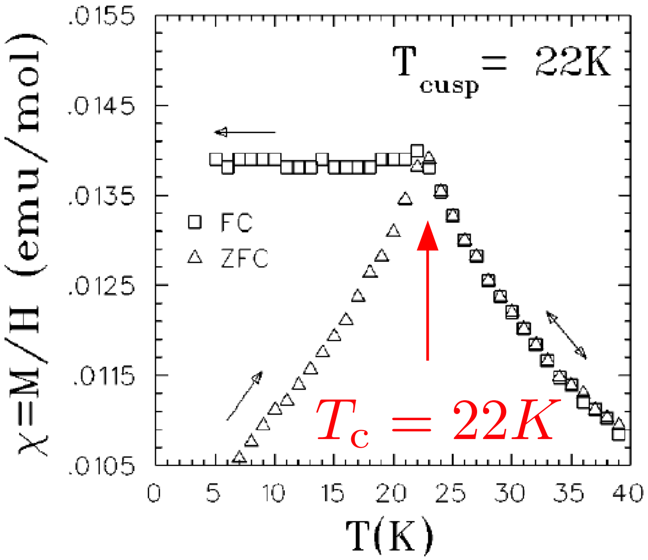
Glass

- Non-ergodicity (multi-valley energy landscape)
- Replica overlap

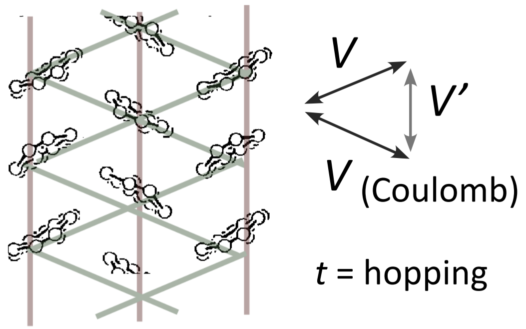
Glasses in crystalline solids

Spins glass in 3D pyrochlore

$Y_2Mo_2O_7$ Gingras, et al (1997)

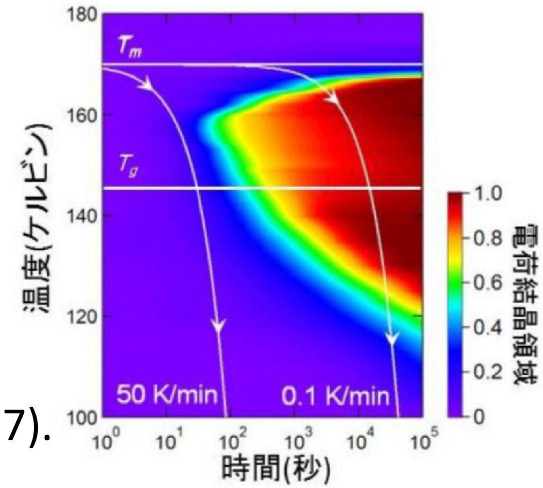


Charge glass in 2D triangular organics

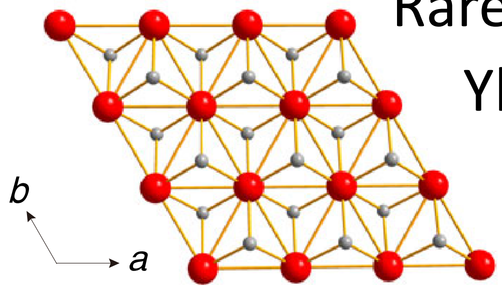


Kagawa, et.al. (2013),
Sasaki, Hashimoto. et.al.(2017).
Kanoda group.

X=TlZn(SCN)4



Glassy spin liquid?



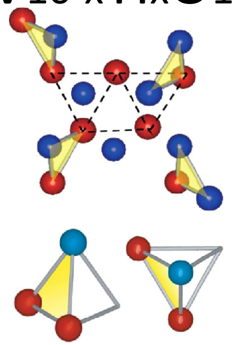
Rare-Earth material

$YbMgGaO_4$

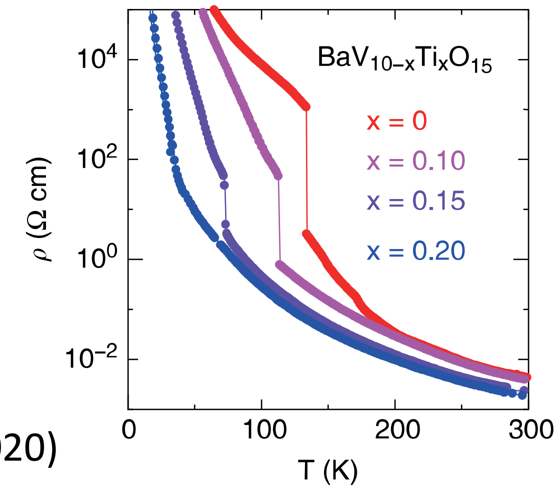
Paddison,*et.al.* (2017)
Li, *et.al* (2017)
Ma, *et.al.* (2018)

Supercooled orbital liquid

$BaV_{10-x}Tl_xO_{15}$

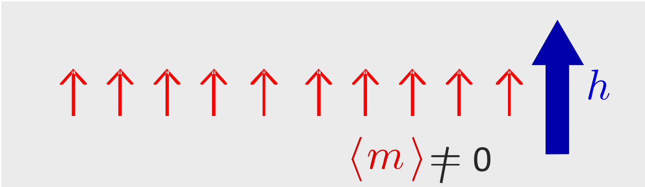


Katsufuji, et.al. (2020)



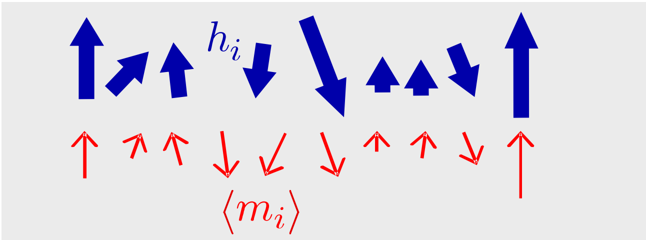
Glasses or not ?

$$\mathcal{H} = \mathcal{H}_0 - N\,m\,h$$



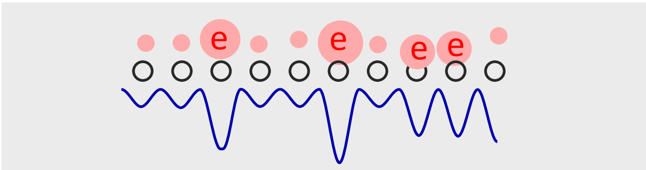
uniform field
spatially uniform magnetization

$$\mathcal{H} = \mathcal{H}_0 - \sum_i m_i\, h_i$$



site dependent random field
site-dependent random freezing of spins

$$\mathcal{H} = \mathcal{H}_0 - \sum_i n_i\, W_i$$

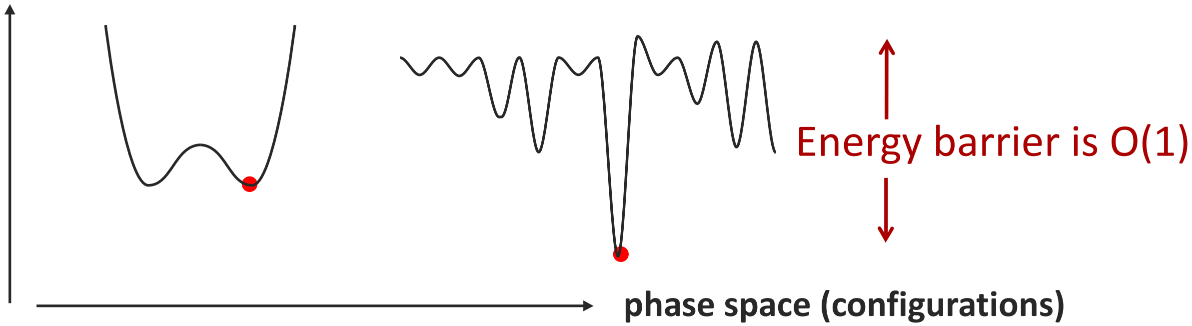


Site dependent random potential
site-dependent random charge occupation

the field conjugate to order parameters

These are not glasses

Energy landscapes



It is ergodic.

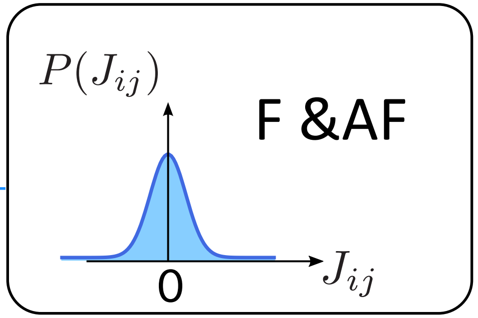
Glasses are nonergodic.

Glasses or not ?

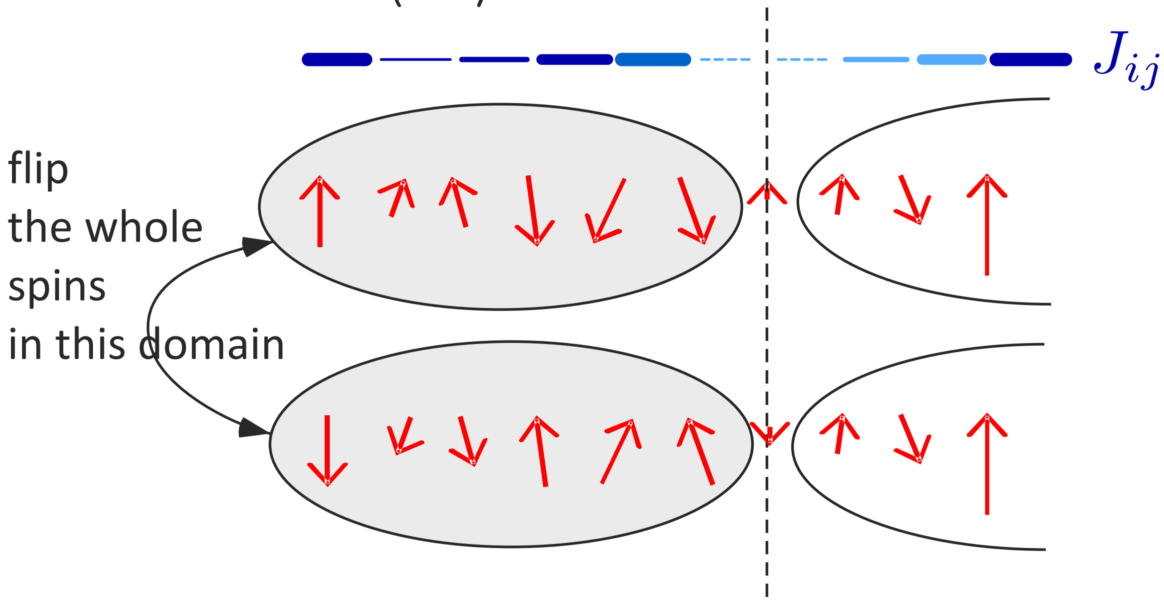
Lattice models for spin glass = Edwards Anderson (EA) model

toy model, **no other interactions than random interactions**.

$$\mathcal{H} = -\sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z$$



The randomness of J_{ij} is NOT coupled to $\langle\sigma_i^z\rangle$: NO apparent reason to freeze $\langle\sigma_i^z\rangle$ randomly.



Time reversal symmetry is preserved in the presence of fluctuation.

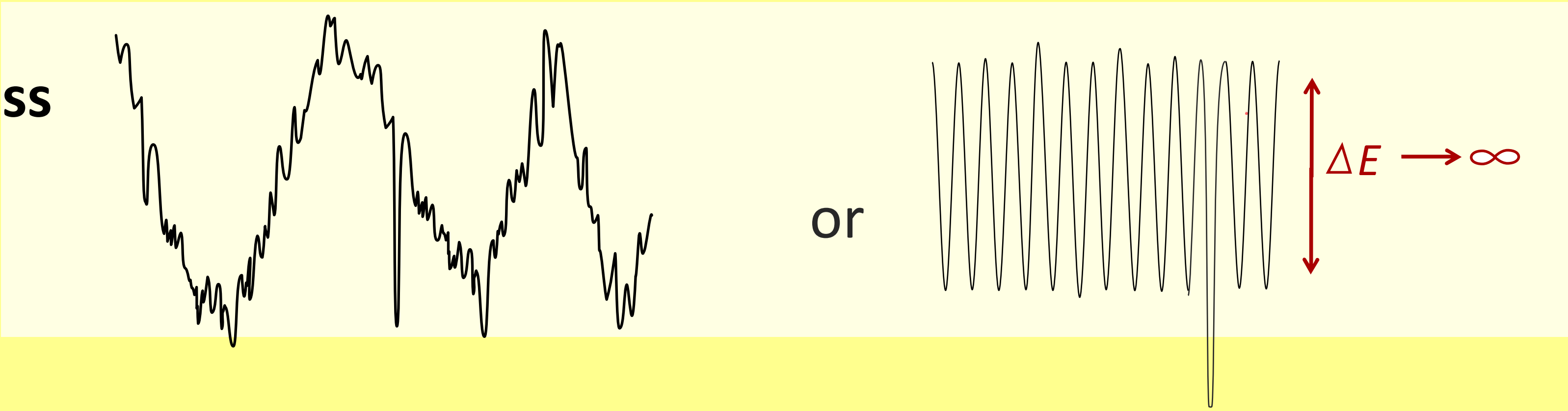
Fluctuation (stronger in 1D and 2D) are the enemy.

Glass transition in theory

- No local order parameter.
  (but have non-local order parameter)

- Breaking of ergodicity.

glass

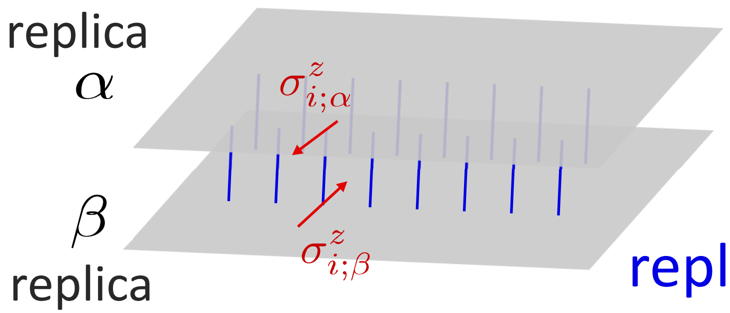


Emergent exponential # of energy minima, O(*N*) energy barrier, breaking ergodicity. relaxation being sensitive to heating/cooling process.

Replica theory

Giorgio Parisi

$$\mathcal{H}_{\alpha+\beta} = \mathcal{H}_\alpha + \mathcal{H}_\beta - \lambda \sum_{i=1}^{N} \sigma_i^\alpha \sigma_i^\beta$$



$\lambda > 0$

replica-coupling
= “external field”

$$\lim_{\lambda \to +0} \lim_{N \to \infty} \left[q_{\alpha\beta} = \frac{1}{N} \sum_{i=1}^{N} \langle \sigma_i^\alpha \sigma_i^\beta \rangle_\lambda \right]$$

replica overlap: NON-local order parameter of spin glass

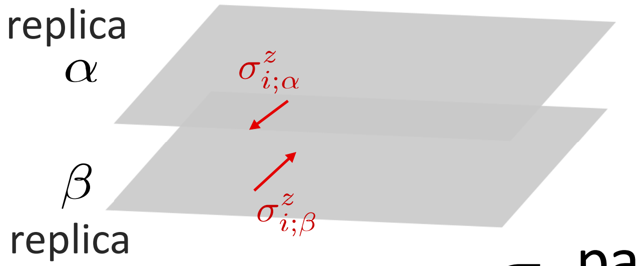
Nonlinear susceptibility = SG susceptibility

$$-\chi_3 = \chi_{SG} = \left. \frac{\partial q_{\alpha\beta}}{\partial \lambda} \right|_{\lambda=0} = -\frac{1}{N} \left. \frac{\partial^2 F_{\alpha+\beta}(N,\lambda)}{\partial \lambda^2} \right|_{\lambda=0}$$

random average

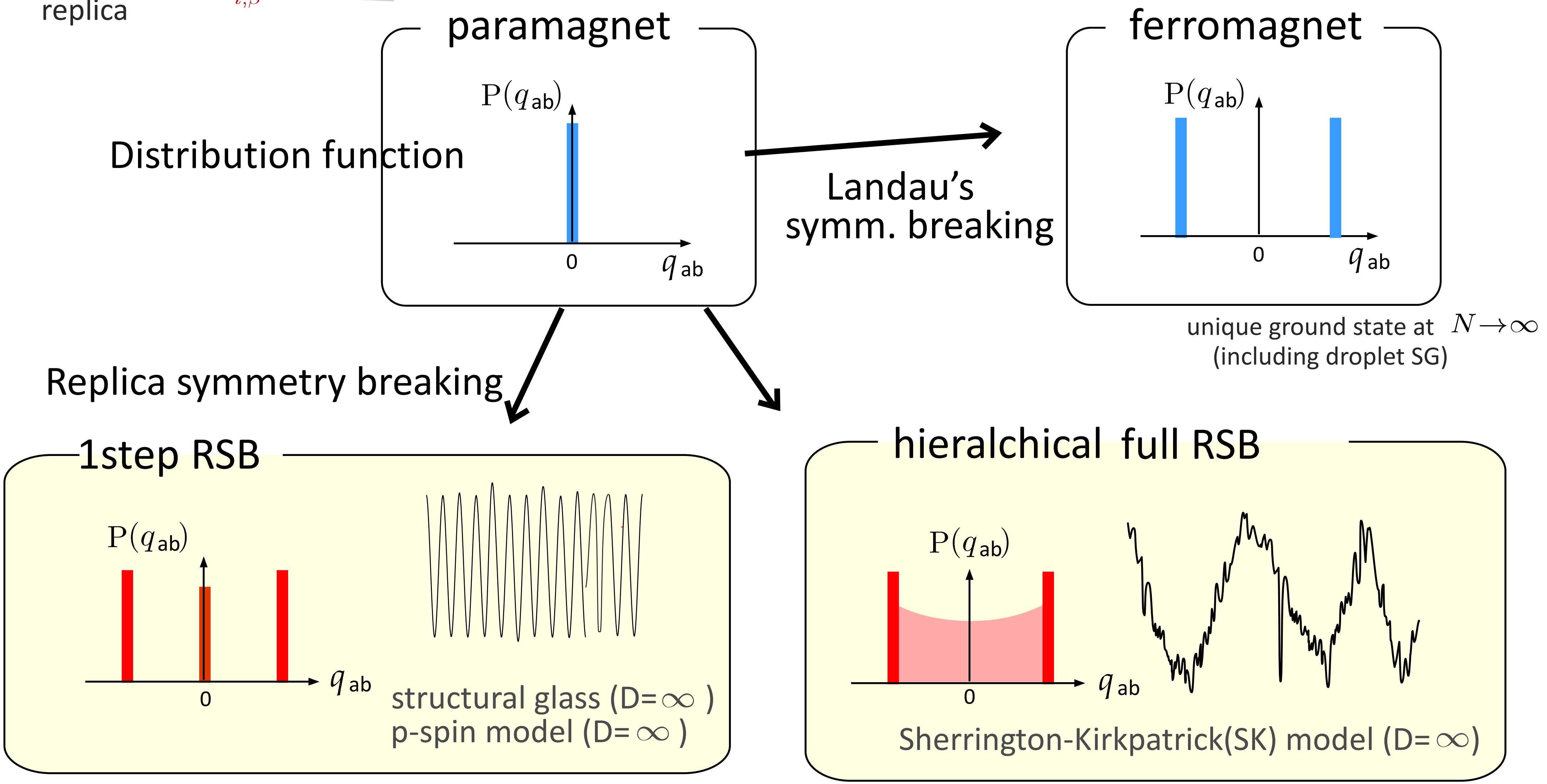
$$= N \left(\langle q_{\alpha\beta}^2 \rangle - \langle q_{\alpha\beta} \rangle^2 \right)_{\lambda=0} = \frac{1}{N} \sum_{i,j} \overline{\left(\langle \sigma_i \sigma_j \rangle^2 - \langle \sigma_i \rangle^2 \langle \sigma_j \rangle^2 \right)_{\lambda=0}}$$

Replica theory



$$q_{\alpha\beta} = \frac{1}{N}\sum_{i=1}^{N}\langle \sigma_i^{\alpha}\sigma_i^{\beta}\rangle_{\lambda=0} \neq 0 \quad \text{if the two replicas are alike,}$$

$$= 0 \quad \text{totally different}$$

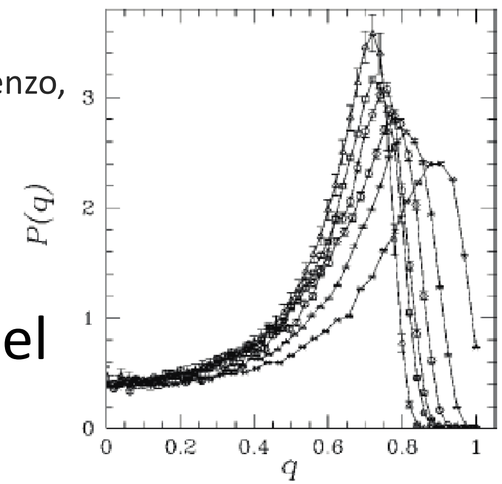


How difficult to find glass transition?

- Replica theory, glass theory are for classical models.
- Replica theory is for D = ∞
- Previously, **lattice models** (D=2,3) could rarely afford true glass transition.
- Even with Edwards-Anderson(EA) toy model, having only random interactions, exhausting effort to establish D=3 SG transition.

Marinari, Parisi and Ruiz-Lorenzo, Phys. Rev. B 58 14852 (1998)

3D EA model



Lattice models (mostly classical)

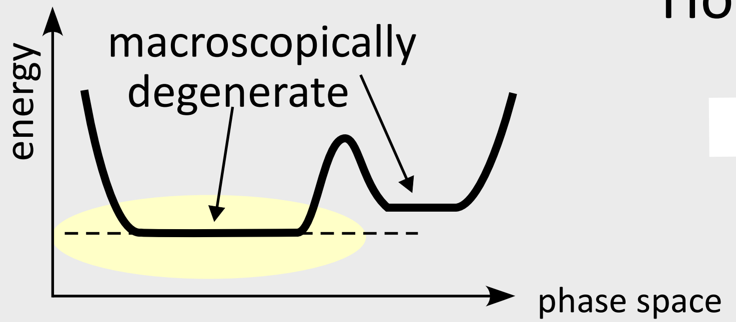
	disorder-free	quenched disorder
D=2	No	~~No~~
D=3	~~No~~	Yes
∞	Yes	Yes

Present talk (quantum) arXiv:2207.07293

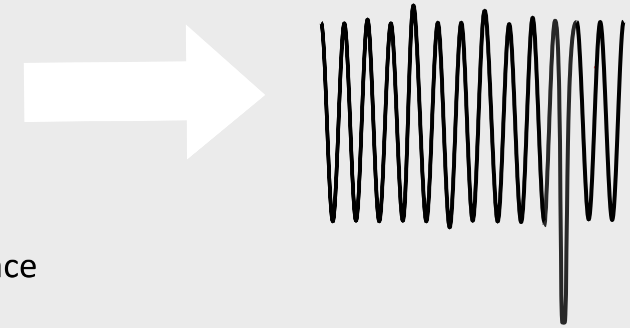
Before our work in 2020 about pyrochlore spin glass.

Mitsumoto Hotta Yoshino PRL 124, 087201 (2020)

Route to find a quantum glass



How can we develop a potential barrier?



frustration will generate a flat energy landscape

What we naively expect is ...

- Need frustration to develop exp-*N* degeneracy.
- Quantum-2D is classsical-3D.
- Maybe we need at least small quenched disorder
  (but it should not directly couple to spin moments, because otherwise
  it is a trivial state and not the glass.)

Finding 2D quantum glass at finite temperature

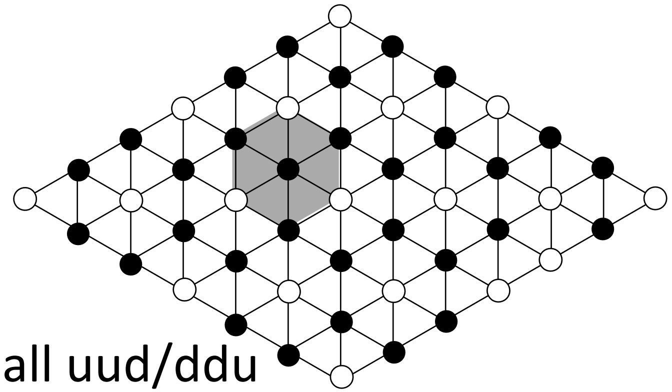
Transverse AF Ising model on a triangular lattice

Quantum + frustration + quenched disorder

Triangular lattice AF Ising model

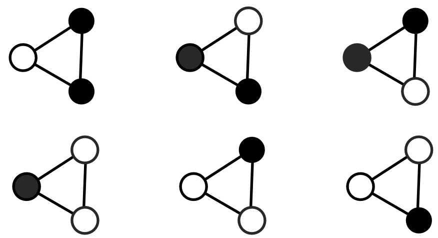
classical

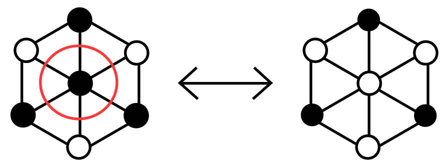
$$\mathcal{H} = \sum_{\langle ij \rangle} J\ \sigma_i^z \sigma_j^z \qquad \sigma_i^z = \begin{cases} 1 & \text{up} \ \bullet \\ -1 & \text{down} \ \circ \end{cases}$$

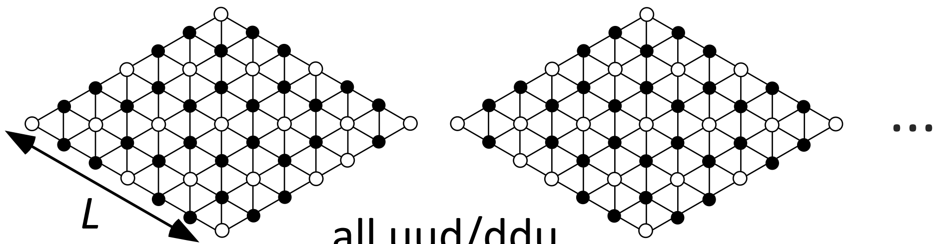


all uud/ddu

uud/ddu has the lowest energy







all uud/ddu

Large # of degenerate states

Because of the frustration,

disordered down to T=0

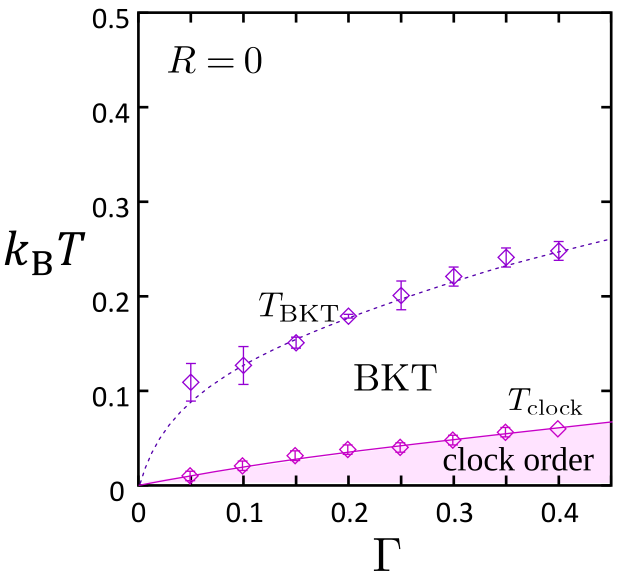
$S = 0.323\ k_B$

Transverse AF Ising model

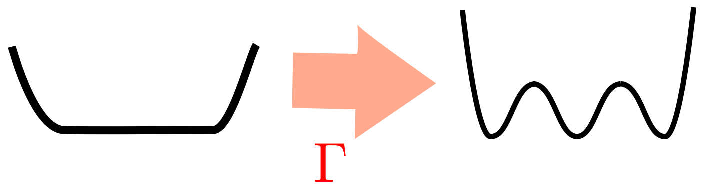
$$\mathcal{H} = \sum_{\langle ij \rangle} J \ \sigma_i^z \sigma_j^z - \Gamma \sum_{i=1}^{N} \sigma_i^x$$

quantum fluctuation added

No randomness



QMC L=24, ... 96

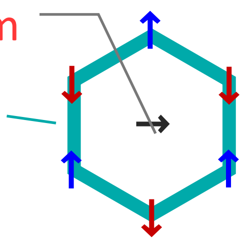


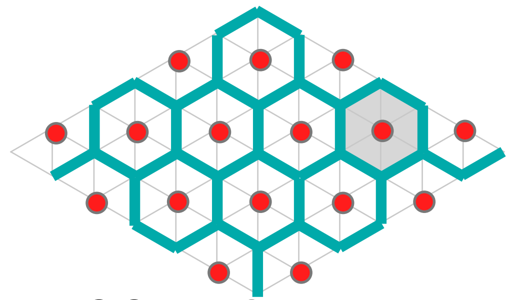
“order-by-disorder” mechanism

Villain (1980)

Clock order

Isakov, Moessner, PRB 68, 104409 (2003)



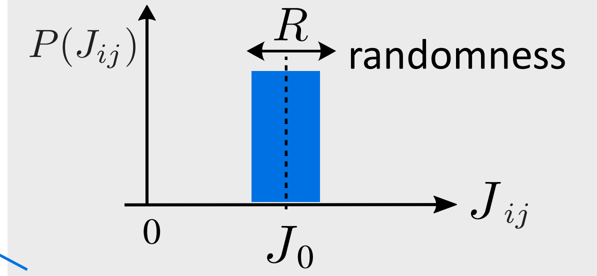


Emergent two degrees of freedom

Transverse AF Ising model + randomness

$$\mathcal{H} = \sum_{\langle ij\rangle} J_{ij}\,\sigma_i^z\sigma_j^z - \Gamma \sum_{i=1}^{N} \sigma_i^x$$

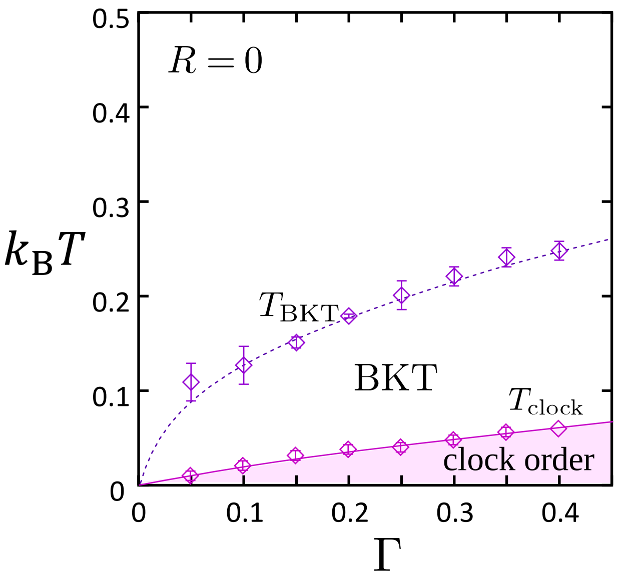
+ small disorder



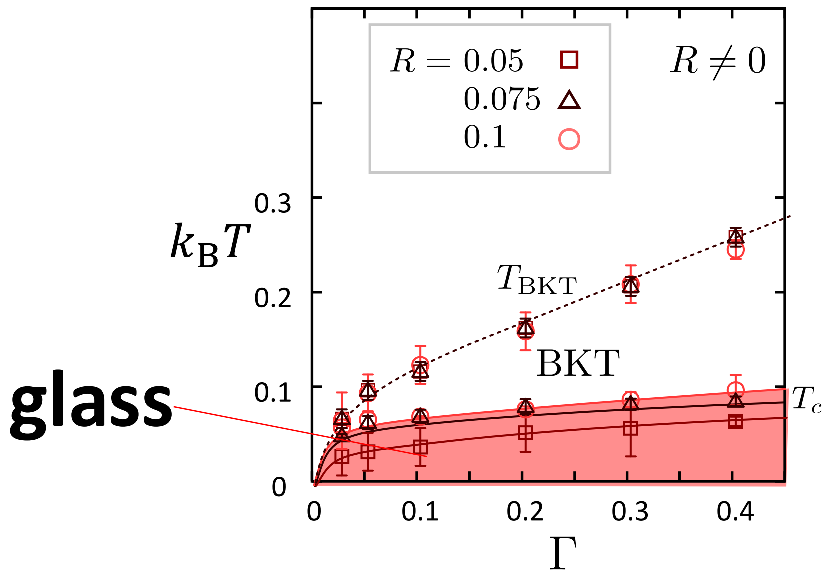
C.f. Edwards Anderson model for spin glass has J_0=0

J_0=1 But mostly antiferromagnetic

No randomness



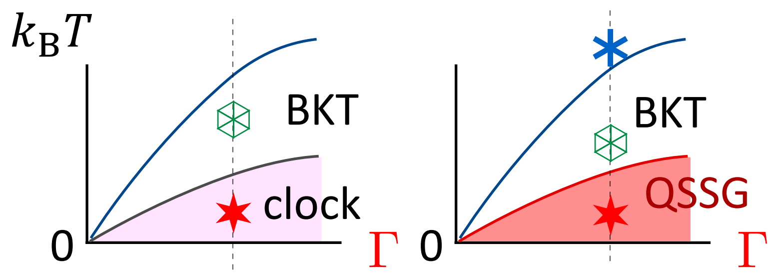
Quenched randomness



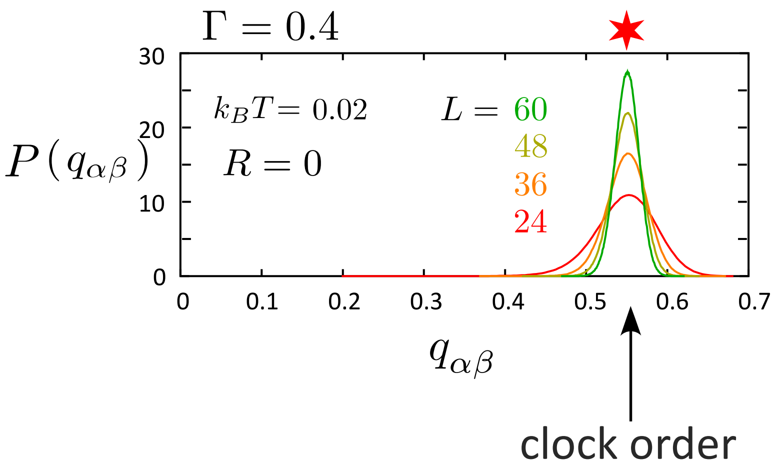
Infinitesimally small randomness generates a glass

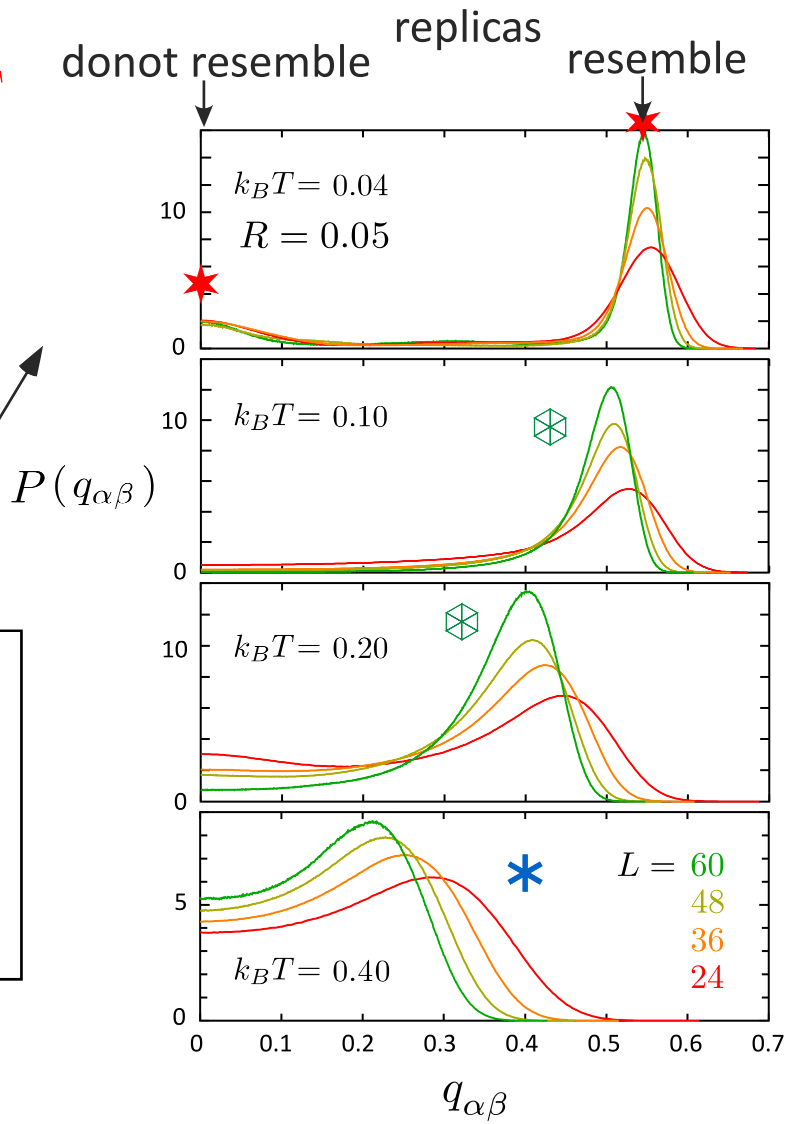
Replica overlap

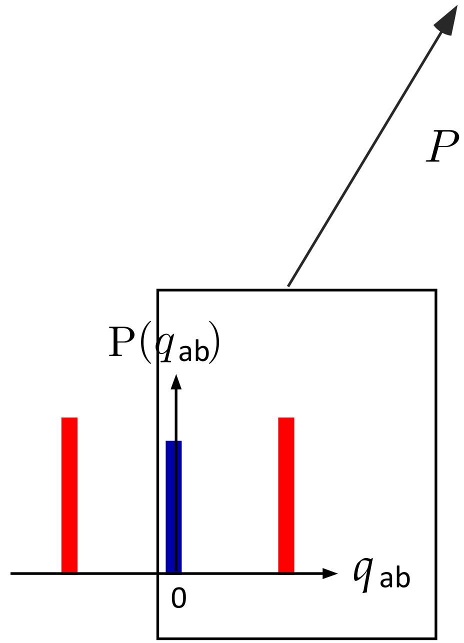
No randomness



Quenched randomness

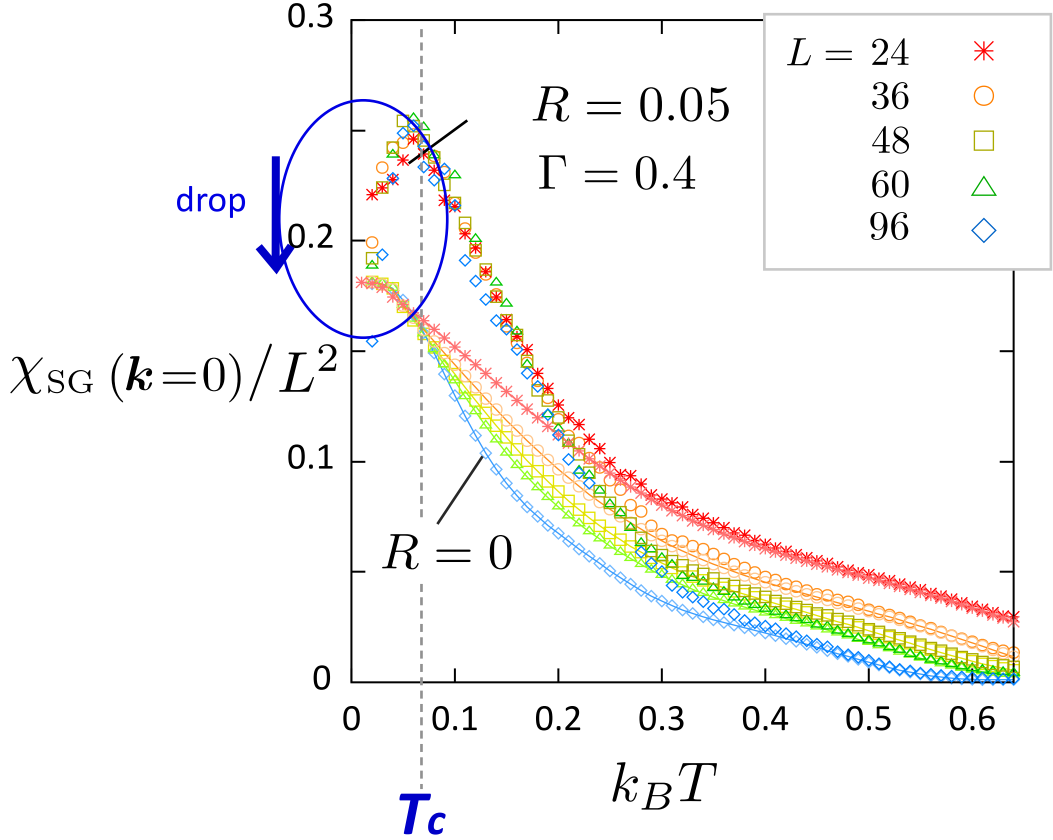


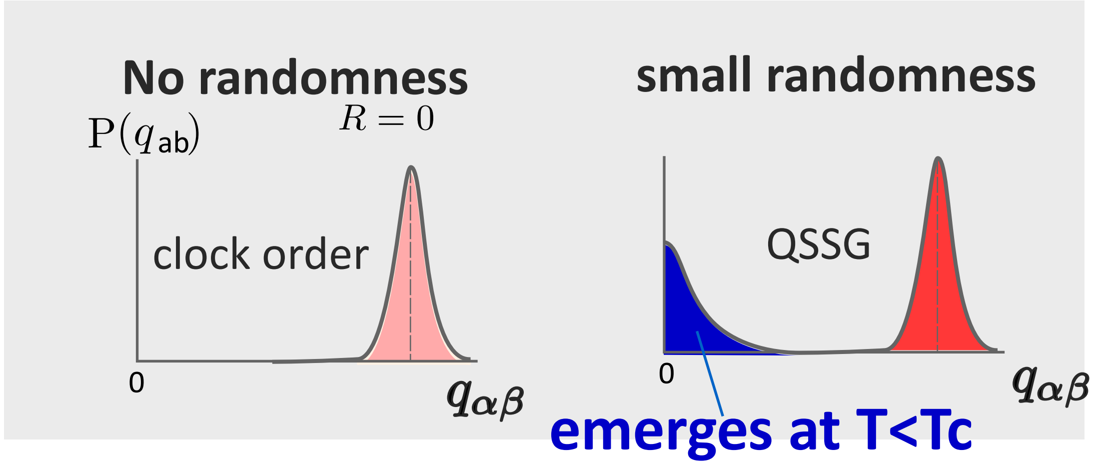




1step RSB ?

Spin-glass susceptibility





$$\chi_{SG}(\boldsymbol{k}=0) = 2\left(\int_0^1 q^2 P(q)dq - \left(\int_0^1 qP(q)dq\right)^2\right)$$

Fourier transform of $q_{\alpha\beta}$-$q_{\alpha\beta}$ correlation

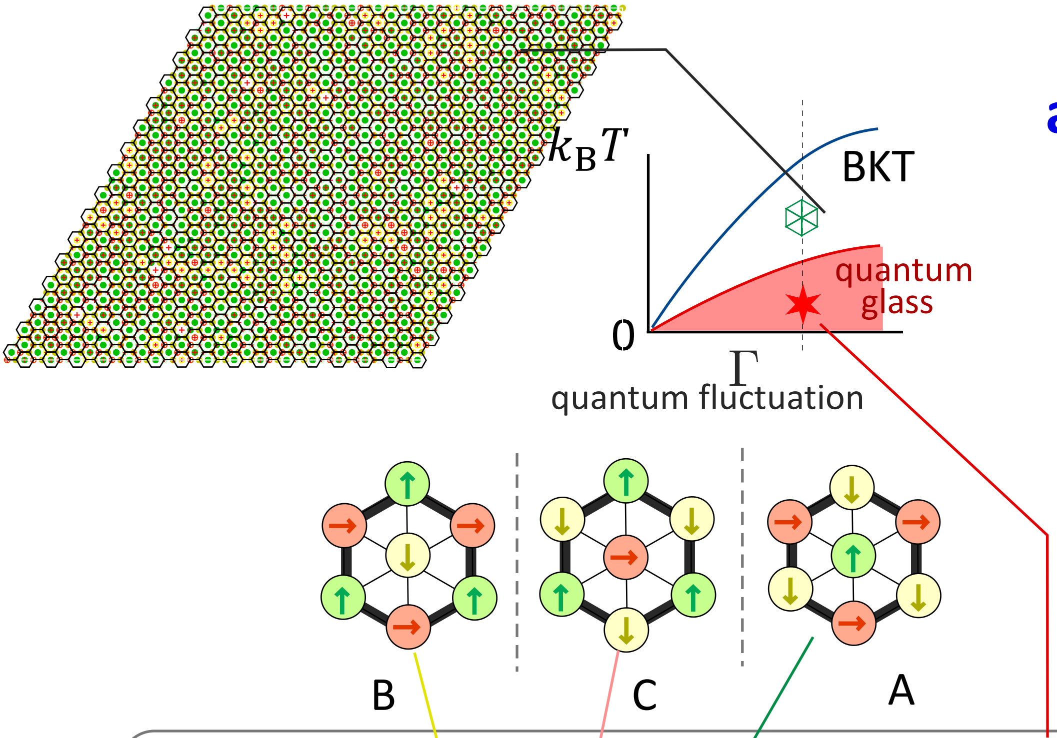
- Small randomness R induces $\chi_{SG}(\boldsymbol{k}=0)$ = order(N) $\xrightarrow{N\to\infty} \infty$

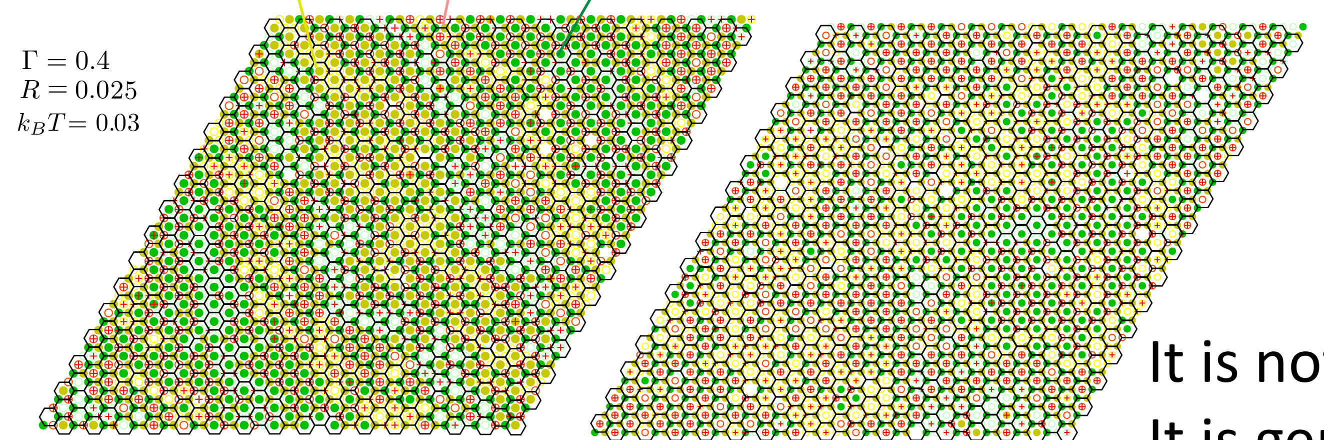
static SG long range order

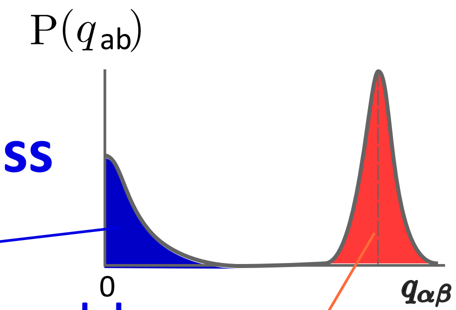
- $\chi_{SG}(\boldsymbol{k}=0)/L^2$ slightly drops at T<Tc . Why ?

because there is an emergent peak in P(q_{ab}=0) = algebraic SG

Domains : source of two component SG







algebraic structural-glass

contribute to $\chi_{SG}(k > 0)$

Replicas do not resemble because there are fluctuating domains.

long range Spin-glass order

χ_{SG} (k=0)

Uniformly glassy spins on the top of robust 3-sublattice structure protected by BKT

It is not a standard type of domain

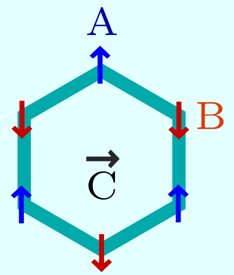
It is generated to gain the energy

Vitrification mechanism

Bond randomness transforms to site random field.

Interplay of quantum + frustration

No randomness

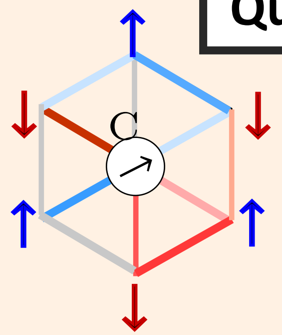


clock order $(m_A, m_B, m_C) = (1, -1,\ 0)$

$$H_{\rm eff}^{\rm C} = \sum_{i \in {\rm A,B}} J_{ij}\ m_i = 0$$

+ bond randomness

Quenched randomness



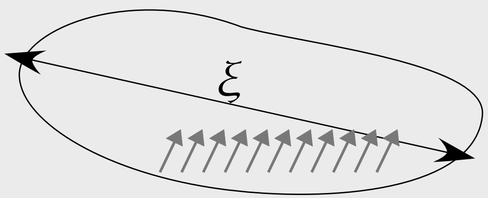
$H_{\rm eff}^{\rm C} \neq 0$ = random field

Imry-Ma (1975) discrete symmetry , uniform J + random field h_i

long range order can be unstable in the presence of arbitrary weak random fields

at dimensions $\leqq d_l$

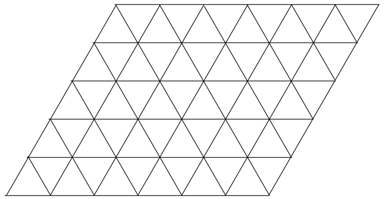
system breaks up into domains.



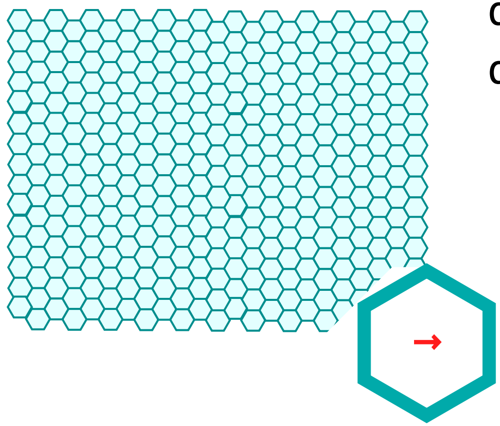
$$E(\xi) \sim J\,\xi^{d-1} - h\,\xi^{d/2} < 0 \ \rightarrow\ d_l = 2$$

proof : Imbrie(1984) Aizenmann-Wehr (1989)

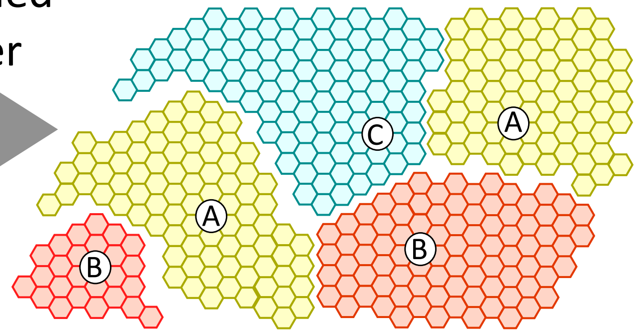
Vitrification mechanism



quantum
fluctuation



quenched
disorder



single component
(spin) system

quantum liquid component
classical solid component

bond-disorder induces random fields

frustration
: magnetic order suppressed
quantum fluctuation

order-by-disorder

honeycomb lattice center spins

Two-types of emergent degrees of freedom coorperatively form
long range ordered spin glass + algebraic structural glass

Some analogies

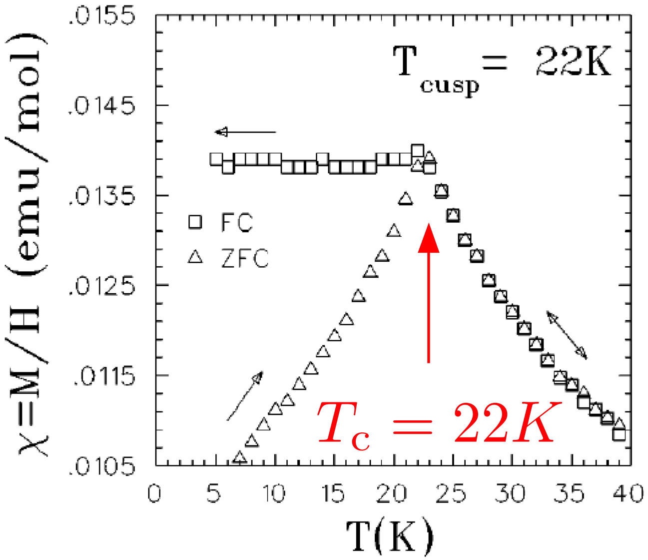
Mitsumoto Hotta Yoshino ,
PRL **124**, 087201 (2020)
PRR **4**, 033157 (2022)

3D Spin glass transition without quenched randomness

: 20 years of difficult problem

No randomness

$Y_2Mo_2O_7$ Gingras, et al (1997)



thermodynamic glass transition

Theory on slow dynamics

Rau ,Gingras (2016)

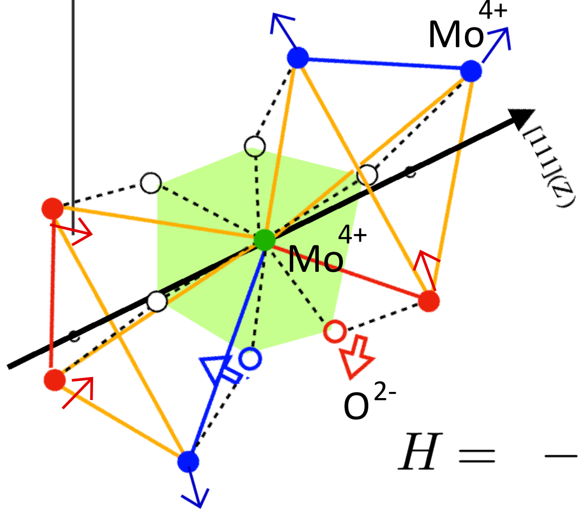
Udagawa, Jaubert, Castelnovo, Moessner (2016)

With quenched disorder

Saunders- Chalker(2007)

Shinaoka-Tomita-Motome (2011)

Motivated by Thygesen, et.al. PRL**118**, 167201 (2017)



Jahn-Teller Ice of lattice displacements

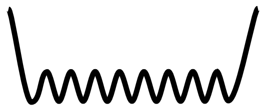
"good glass-former"

lattice spin

$$H = -3\epsilon \sum_{ij} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + \sum_{\langle ij \rangle} J_{\boldsymbol{\sigma}_i,\boldsymbol{\sigma}_j} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Random interations dynamically generated by disorderd ice-lattice displacements

Jahn-Teller lattice ice



\+ **frustrated spins**

→ **glass transition**

Summary

Frozen glass ⟷ underlying physics may have something in common. ⟷ unfrozen: quantum spin liquids, classical spin liquid, MBL, etc.

- Materials show phenomenological glass behavior, but the true glasses are not easy to find in theory.

- Glass is a ergodicity breaking: order parameter= a non-local replica overlap.
- We found a route to form a quantum 2D(=classical 3D) glass.

  × Fluctuations enhanced in low dimensions.

  ○ Frustration to generate massively competing states.

  ○ At least very small quenched randomness.

  ◎ When two degrees of freedom which are both frustrated, couple, ...

- What is new for our QSSG as a phenomenon?

  Two existing "orders": static uniform long range glass order
  algebraically decaying glass