# Critical Properties of Non-Hermitian Correlated Systems

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# Introduction

# Hermitian or non-Hermitian

◆Closed quantum (many-body) systems

Unitary evolution
described by Hermitian Hamiltonian

◆Open quantum (many-body) systems

Dissipative environment
Life time
Gain & loss, etc

Naïvely

Systems with dissipative environment
Effective non-Hermitian description ?

# Effective non-Hermitian description

e.g.

◆Vortex depinning phenomena in superconductors

*N. Hatano and D. Nelson, PRL (1996)*

◆Breakdown of a Mott isnulator

*T. Fukui and N. Kawakami PRB(1998)*

◆Open quantum systems

*C. M. Bender and S. Boettcher, PRL (1998)* (PT symmetry)

*Y. Ashida, S. Furukawa, and M. Ueda, Nat. Commun (2017)*

*K. Kawabata, Y. Ashida, H. Katsura and M. Ueda, PRB (2018)…etc.*

◆PT symmetric systems: Experiments

*A. Guo and G. J. Salamo, PRL (2009)*

*C. E. Ruter et al. Nat. Phys. (2010)*

*A. Regensburger et al. Nature (2010), L. Xiao et al (2017), … etc.*

◆Non-Hermitian topological phases

*K. Esaki, M. Sato, K. Hasebe and M. Kohmoto, PRB(2011), etc*

◆Non-Hermitian perspective of correlated systems

*V. Kozii and Liang Fu (2017), Yoshida et al (2018), etc.*

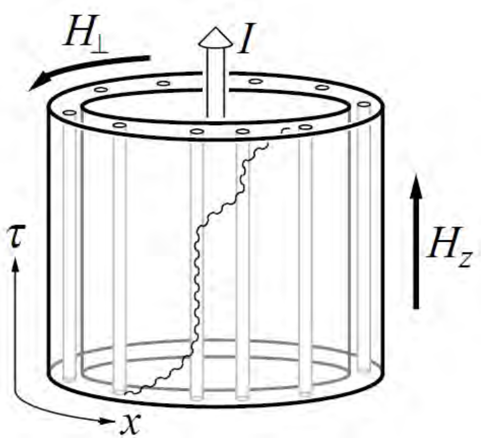
# Localization transitions in non-Hermitian quantum mechanics

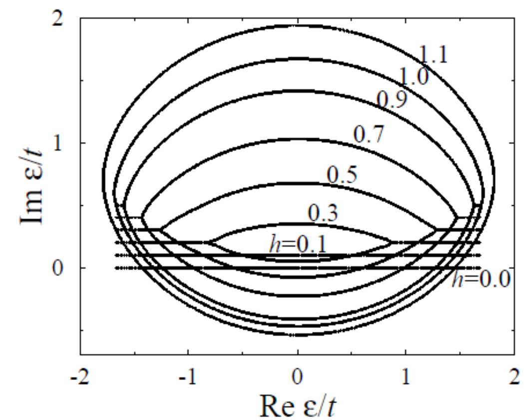
*N. Hatano and D. Nelson, Phys. Rev. Lett. 77, 58 (1996)*

**Vortex: Pinning-depinning transition in superconductors**

columnar defects

Transverse field




Real: localized

Complex: delocalized

Asymmetric hopping

Non-Hermitian Hamiltonian

$$\mathcal{H} \equiv -\frac{t}{2}\sum_{x}\sum_{\nu=1}^{d}\left(e^{h\cdot e_\nu/\hbar} b^\dagger_{x+e_\nu} b_x + e^{-h\cdot e_\nu/\hbar} b^\dagger_x b_{x+e_\nu}\right) + \sum_x V_x b^\dagger_x b_x, \quad (2)$$

Random potential

*One-body problem* ➡ *Many-body problem*

# Breakdown of a Mott insulator:
## Exact solution of non-Hermitian Hubbard model

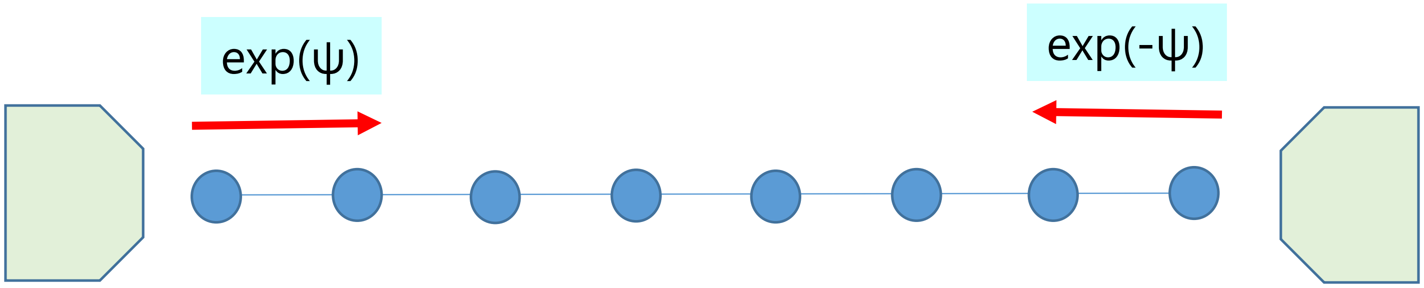
*T. Fukui and NK, Phys. Rev.* ***B58***, *16051 (1998)*

First paper on the Mott breakdown
Non-Hermitian for many-body systems

Asymmetric hopping

*cf Hatano-Nelson for vortex depinning (1996)*

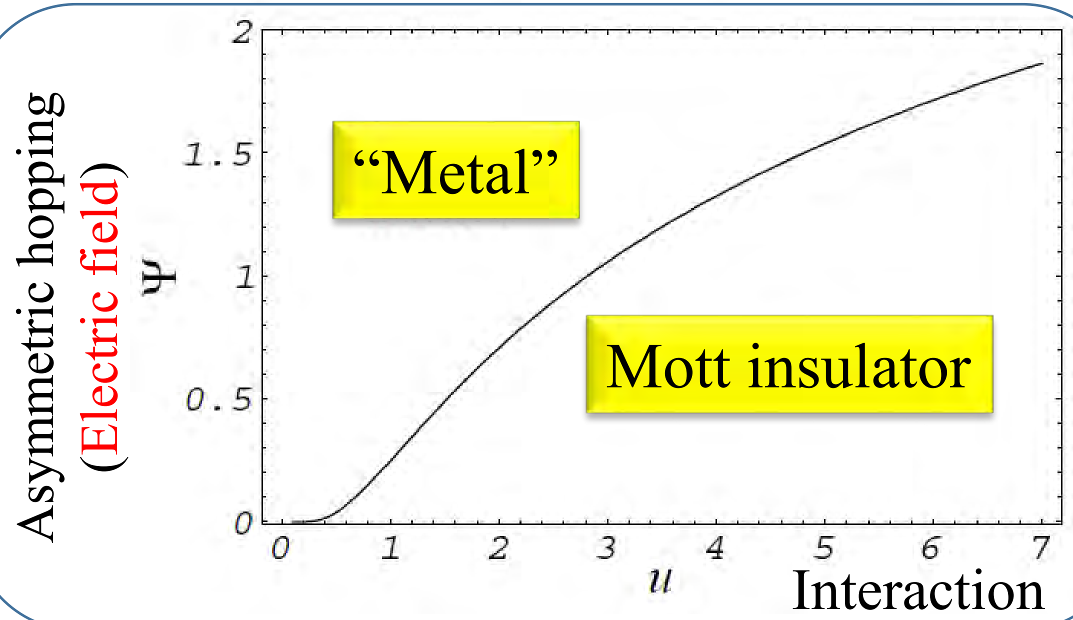
$$H = -t \sum_{j=1}^{L} \sum_{\sigma} \left( e^{-\Psi} c^{\dagger}_{j\sigma} c_{j+1\sigma} + e^{\Psi} c^{\dagger}_{j+1\sigma} c_{j\sigma} \right) + U \sum_{j=1}^{L} n_{j\uparrow} n_{j\downarrow},$$




Oka-Aoki (2010) showed that this breakdown effectively describes
Dielectric breakdown by electric field!

# Breakdown of a Mott insulator:
## Exact solution of non-Hermitian Hubbard model

*T. Fukui and NK, Phys. Rev.* ***B58****, 16051 (1998)*




*Experiment*

"Dielectric breakdown of one-dimensional Mott insulators $Sr_2CuO_3$ and $SrCuO_2$"
Y. Taguchi, T. Matsumoto, and Y. Tokura,
*Phys. Rev. B62, 7015-7018 (2000).*

*Few studies for 20 years:*
*Correlation Effects on Non-Hermitian Systems*
*addressed again recently !*

# Critical Properties of Non-Hermitian Correlated Systems

## Contents

*PART I*

1. Non-Hermitian Kondo effect

*Prototype of many-body non-Hermitian systems*

*M. Nakagawa et al. PRL **121**, 203001(2018)*

*PART II*

2. Non-Hermitian Tomonaga-Luttinger liquids

*Quantum XXZ spin chain*

*K. Yamamoto et al. PRB **105**, 205125(2022)*

*PART III*

2. SU(N) Generalization of Dissipative TL liquids

*Haldane's "ideal gas" approach*

*K. Yamamoto et al. arXiv:2207.04395*

*PART I*

# Non-Hermitian Kondo effect in ultracold atoms

M. Nakagawa

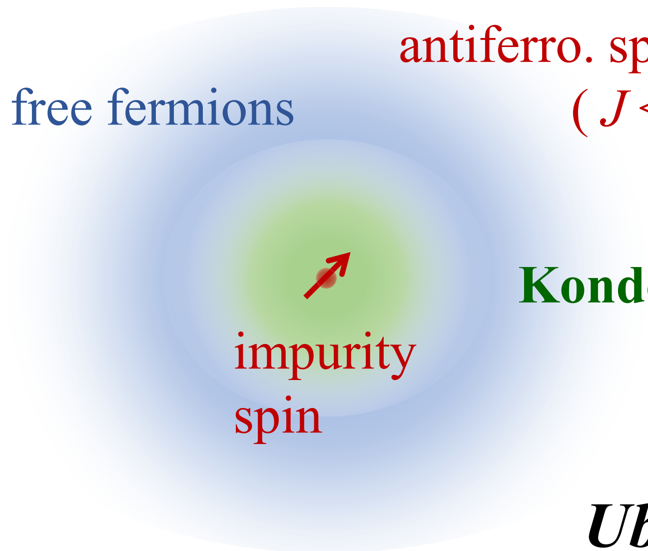
# Kondo Effect

- Paradigmatic example of quantum many-body physics
- A localized impurity spin coupled with free fermions

$$H = \sum_{\boldsymbol{k},\sigma} \varepsilon(\boldsymbol{k}) c^{\dagger}_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma} - J \boldsymbol{S}_{c0} \cdot \boldsymbol{S}_{\mathrm{imp}}$$

**Kondo model**

[J. Kondo, Prog. Theor. Phys. 32, 37 (1964)]




Kondo temperature

$$T_K = D\sqrt{|\rho_0 J|}\exp\left[\frac{1}{\rho_0 J}\right]$$

***Ubiquitous phenomenon***

- ✓ **Dilute magnetic impurities**
- ✓ **Heavy fermions**
- ✓ **Quantum dots**
- ✓ **Cold atoms**

# Alkaline-earth cold atoms

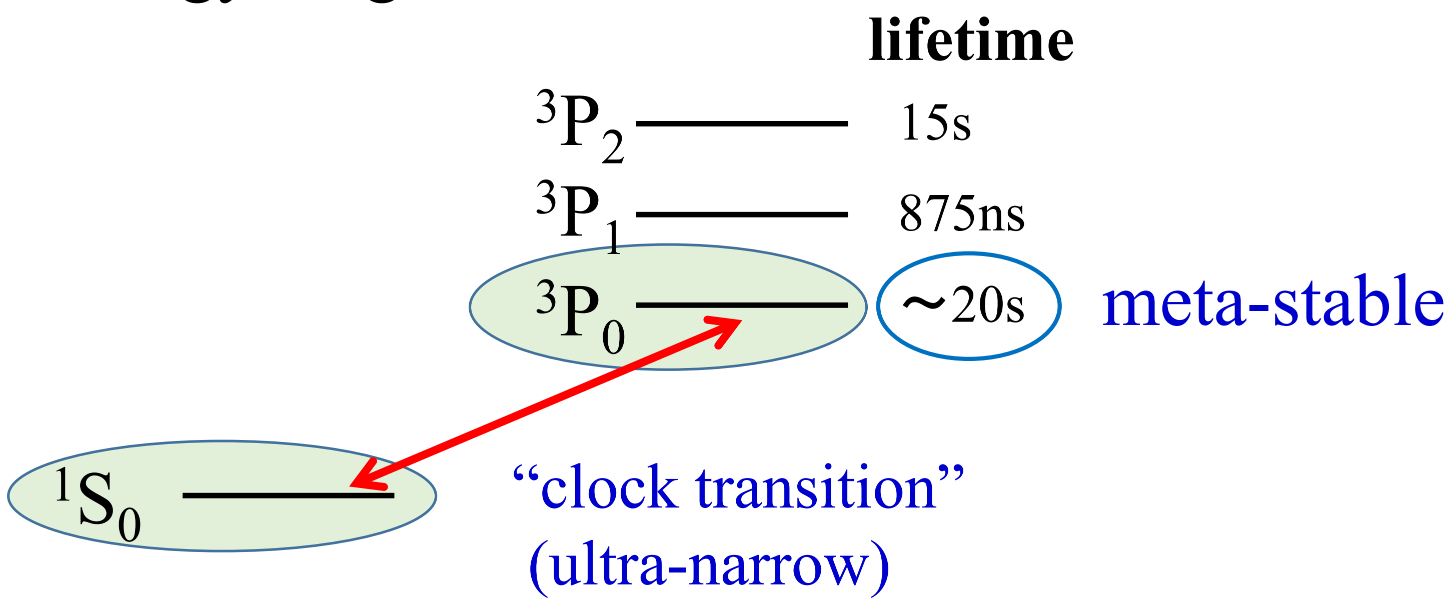
- Two electrons in the outer shell (Ca, Yb, Sr) $^{2S+1}L_J$

*electronic* ground state: $^1S_0$

excited state: $^3P_0$ → meta-stable : "higher-orbital state"

($J$=0 → $J$=0 : forbidden)

- Energy diagram



# Kondo Effect: Ultracold Atoms

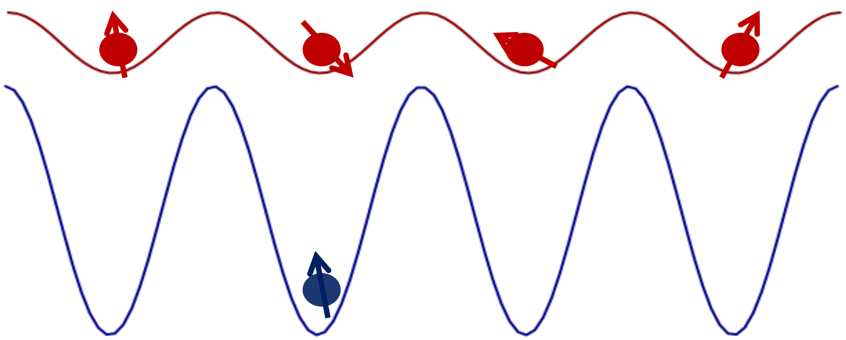
- **Kondo effect in ultracold atoms?**

  Most promising candidate: **alkaline-earth ($^{171}$Yb, $^{173}$Yb, $^{87}$Sr)**

  [Gorshkov *et al*, Nat. Phys. (2010)]

○ **Atomic ground state ($^1S_0$)** → conduction electrons
○ **Metastable excited state ($^3P_0$)** → localized impurity
(spin degrees of freedom : nuclear spin)

- **Difference of polarizability → state-dependent lattice**



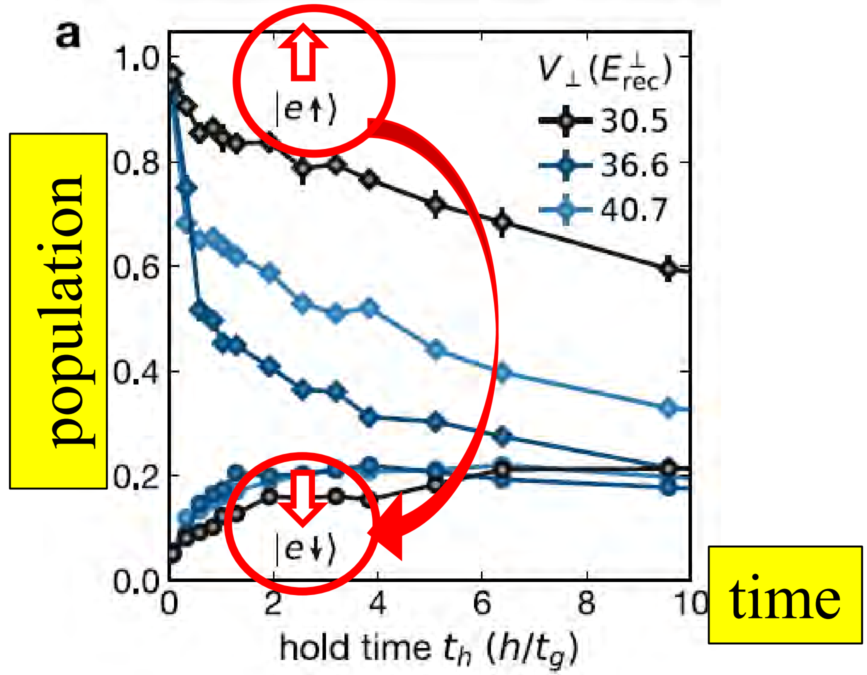
**$^1S_0$ state (g)** : shallow lattice
→ "conduction electrons"

**$^3P_0$ state (e)** : deep lattice
→ "localized impurity"

# Kondo Effect: Ultracold Atoms

## ■ Experimental realization of the "Kondo Hamiltonian"

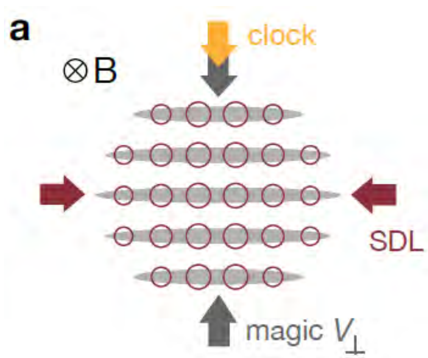
[Riegger *et al.*, PRL 120, 143601 (2018)]




✓$^{173}$Yb, $m_F=\pm 5/2$ (⇧, ⇩)

✓Host atoms: $^1S_0$ state

✓Impurity: metastable $^3P_0$ state




◆Spin-exchange dynamics → observed!

◆Atom losses

**Problem: Kondo effect with atom losses**

# Message of this part

**Quantum many-body physics with inelastic collisions**

⇩

**Atom loss: formulated as an open quantum system**
**→ emergence of non-Hermitian Hamiltonians**

⇩

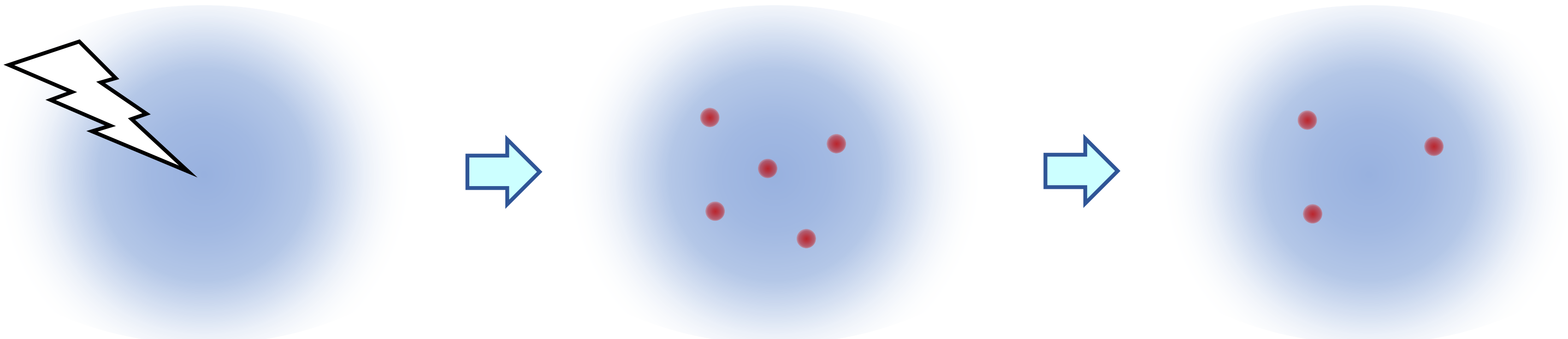
**Quantum many-body physics with non-Hermitian Hamiltonians**

**Non-Hermitian generalization of the Kondo effect**

# Setup

# Setup

## Excitations to $^3P_0$ state (clock transition)



Equilibrium gas of Yb (or Sr)
<u>All atoms are in $^1S_0$ state</u>

$^3P_0$ state as impurities
: Kondo system

Some impurities are lost due to inelastic collisions

⇨ **Measurement of the “surviving” impurities**

⇨ **Non-Hermitian Kondo effect**

# Non-Hermitian Kondo Model

■ Atom loss → described by a **quantum master equation**

[See e.g. Daley, Adv. Phys. 63, 77 (2014)]

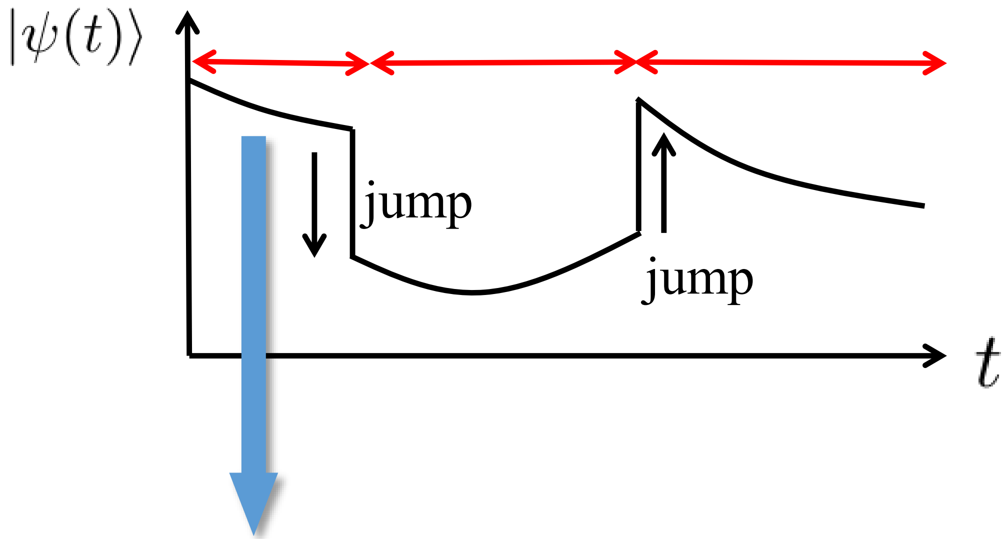
$$\frac{d\rho(t)}{dt} = -i[H,\rho] + \sum_{\alpha=\pm,\uparrow\uparrow,\downarrow\downarrow} \left(L_\alpha \rho L_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\}\right)$$

$$= -i(H_{\text{eff}}\rho - \rho H_{\text{eff}}^\dagger) + \sum_{\alpha=\pm,\uparrow\uparrow,\downarrow\downarrow} L_\alpha \rho L_\alpha^\dagger$$

**Two-body loss event**
: change the particle #

**Non-Hermitian Hamiltonian** : Dynamics between loss events

## Quantum trajectory method




Dynamics in each quantum trajectory

**NH Hamiltonian**

# Non-Hermitian Kondo Model

■ Atom loss → described by a **quantum master equation**

[See e.g. Daley, Adv. Phys. 63, 77 (2014)]

$$\frac{d\rho(t)}{dt} = -i[H,\rho] + \sum_{\alpha=\pm,\uparrow\uparrow,\downarrow\downarrow} \left(L_\alpha \rho L_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\}\right)$$

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**Two-body loss event**
: change the particle #

**Non-Hermitian Hamiltonian** : Dynamics between loss events

**Our interest: "surviving" impurity (projection to lossless dynamics)**
**→ Dynamics is described by the non-Hermitian Hamiltonian!**

**Non-Hermitian Kondo Hamiltonian!**

$$H_{\text{eff}} = H - \frac{i}{2}\sum_\alpha L_\alpha^\dagger L_\alpha = \sum_{\boldsymbol{k},\sigma} \varepsilon(\boldsymbol{k}) c_{\boldsymbol{k}\sigma}^\dagger c_{\boldsymbol{k}\sigma} - J\boldsymbol{S}_{c0}\cdot \boldsymbol{S}_{\text{imp}}$$

**Imaginary int. by "backaction" of projection**

$$(J = J_r + iJ_i)$$

# Results

# Renormalization group

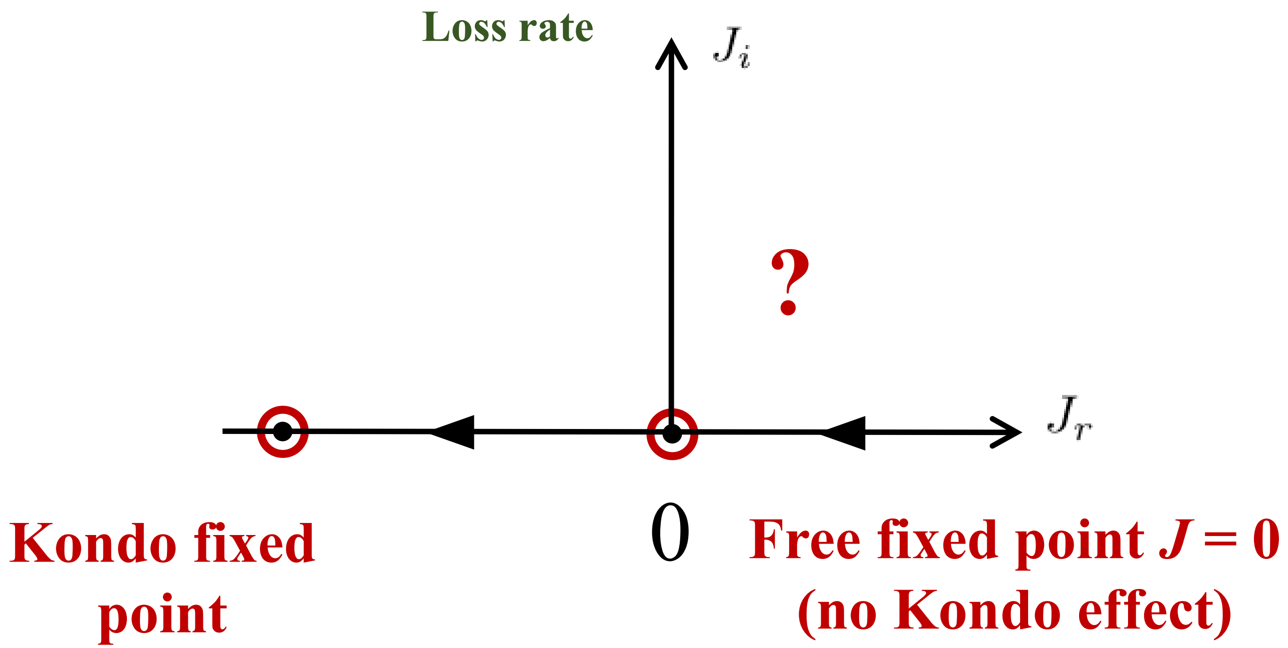
- First approach: renormalization group

$$\frac{dJ}{d\ell} = -\rho_0 J^2 - \frac{\rho_0^2}{2} J^3$$

$$(J = J_r + iJ_i)$$

($\rho_0$: DOS at the Fermi energy)

[Nozières-Blandin, J. Phys. 41, 193 (1980)]



# Renormalization group

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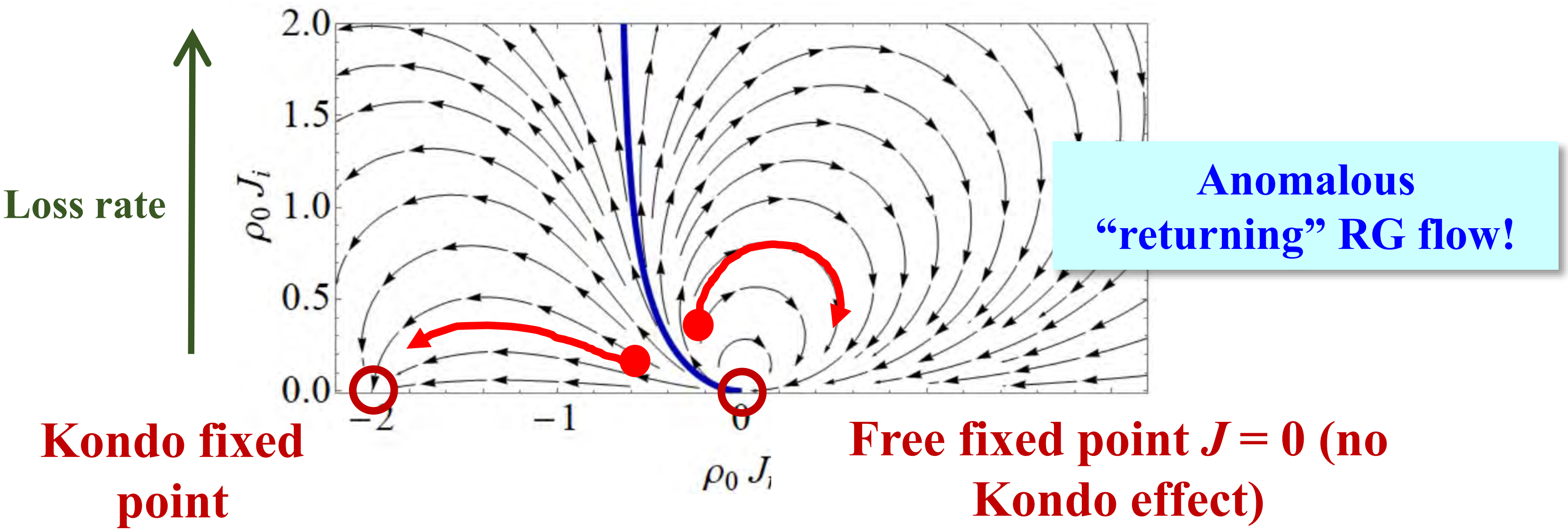
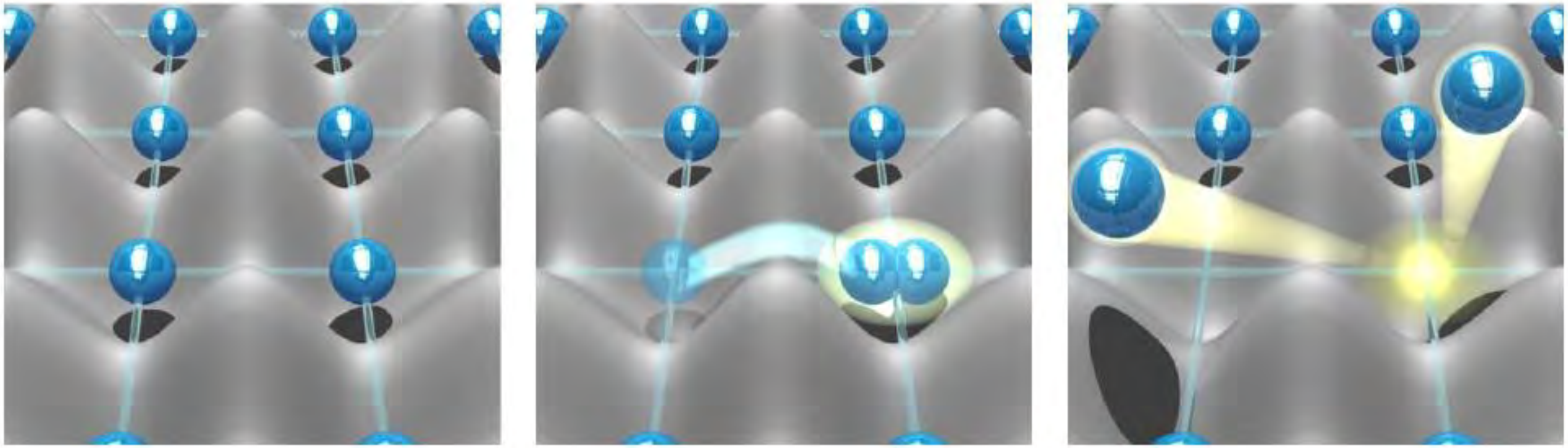



**Non-Hermitian quantum phase transition**
**induced by inelastic scattering**

# Physical interpretation: Quantum Zeno effect

■ Physical picture of the non-Hermitian quantum phase transition

**→ (continuous) quantum Zeno effect**



[Tomita *et al*., Sci. Adv. 3, e1701513 (2017)]

**Particle loss induces effective "repulsion"**

**→ destruction of Kondo singlet**

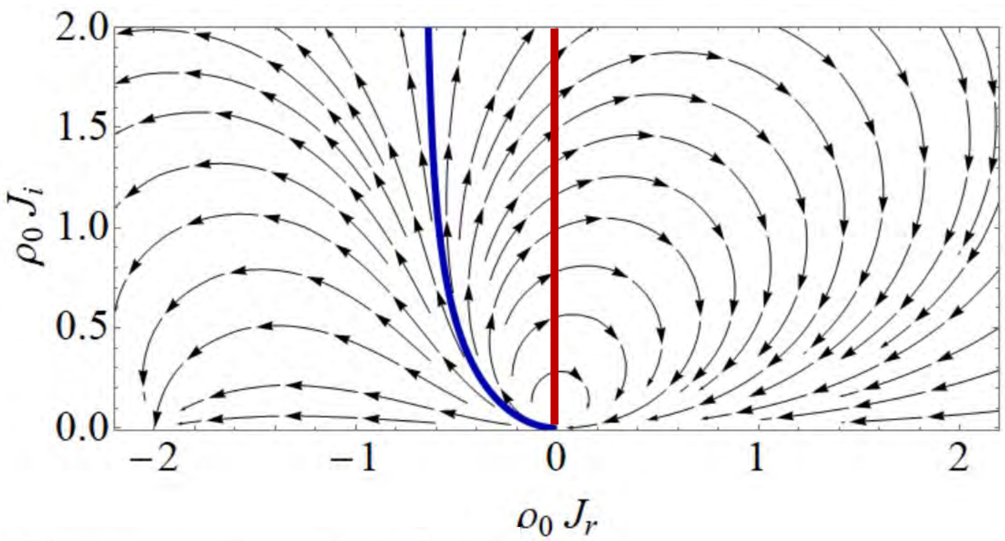
**Competition between Kondo effect & quantum Zeno effect**

# Energy scale

■ Non-Hermitian renormalization group

**Define characteristic scale : $J_r(T_{\mathrm{Kdiss}}) = 0$**

**"reversion" in RG flow**




$$T_{\mathrm{Kdiss}} \simeq \frac{D}{\sqrt{2}}\sqrt{|\rho_0 J|}\exp\left[\frac{J_r}{\rho_0 |J|^2}\right]$$

**Non-Hermitian generalization of Kondo temperature**

# Bethe ansatz exact solution

■ To confirm the RG prediction:

**Exact solution of the non-Hermitian Kondo Hamiltonian**

■ Non-Hermitian generalization of **Bethe ansatz**

[Andrei, PRL (1980), Wiegmann, J. Phys. C (1981)]

$$e^{ik_j L} = e^{-i\pi\rho_0 J/2} \prod_{\alpha=1}^{M} \frac{\lambda_\alpha + i/2}{\lambda_\alpha - i/2} \quad (j = 1, \cdots, N),$$

$$\left(\frac{\lambda_\alpha + i/2}{\lambda_\alpha - i/2}\right)^N \left(\frac{\lambda_\alpha + 1/g + i/2}{\lambda_\alpha + 1/g - i/2}\right) = -\prod_{\beta=1}^{M} \frac{\lambda_\alpha - \lambda_\beta + i}{\lambda_\alpha - \lambda_\beta - i} \quad (\alpha = 1, \cdots, M),$$

$$g = -\tan(\pi\rho_0 J)$$

$k_j \ (j = 1, \cdots, N)$ : momenta of conduction fermions ($N$: # of particles)

$\lambda_\alpha \ (\alpha = 1, \cdots, M)$ : “spin rapidity” of ↓ spin electrons ($M$: # of ↓ spins)

# Bethe ansatz exact solution

## ■ Impurity magnetization $M_i$

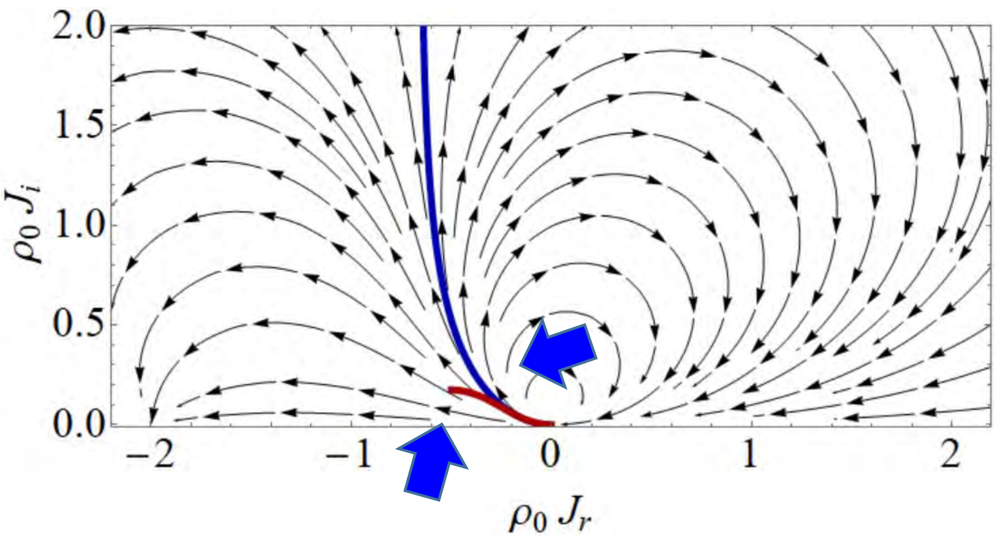
Bethe-ansatz solution of the ground state

◆ $|\mathrm{Im}(1/\tan(\pi\rho_0 J))| < 1/2$

⇨ $M_i = 0$ **Kondo singlet solution**

◆ $|\mathrm{Im}(1/\tan(\pi\rho_0 J))| > 1/2$

⇨ $M_i = 1/2$ **Non-Kondo solution!!**




**Blue: critical line by RG**

**Red: critical line by Bethe ansatz**

**Good agreement between RG & Bethe ansatz in the weak-coupling regime**

# Part I: Summary

■ Kondo effect in ultracold alkaline-earth atoms

→ **Non-Hermitian generalization of the Kondo problem**

■ Non-Hermitian Kondo effect

✓Transition from Kondo to non-Kondo @ critical inelastic scattering

✓Non-Hermitian phase transition → **no analog in equilibrium systems**

✓Exact solution by Bethe ansatz

**Prototypical Example of**
**Non-Hermitian Correlated Phenomena**

Reference:

*M. Nakagawa , N. K., and M. Ueda, PRL (2018)*

*PART II*

# Universal Properties of Dissipative Tomonaga-Luttinger Liquids

K. Yamamoto

M. Nakagawa

● *K. Yamamoto et al. PRB* ***105****, 205125(2022)*

# Universal properties of 1D dissipative systems

**Question addressed in this section**

- 1D quantum critical systems: Hermitian (Unitary)

  well understood by Conformal Field Theory

> *Ex. Spin chain, Hubabrd model, etc*
> Tomonaga-Luttinger liquid *c=1 CFT*

**How about the Non-Unitary (Non-Hermitian) case ?**

- Some problems are well understood

> *Ex. Yang–Lee edge singularity in Ising model*
> **Negative** central charge:
> $c = -\frac{22}{5}$ $c_{\rm eff} = c - 24\Delta$ $c_{\rm eff} = \frac{2}{5}$

**Little is known for CFT with complex energy spectrum**

# Universal properties of 1D dissipative systems

**Message in this section**

**Dissipative Tomonaga-Luttinger liquid**

Non-Hermitian XXZ spin chain

Complex energy spectrum

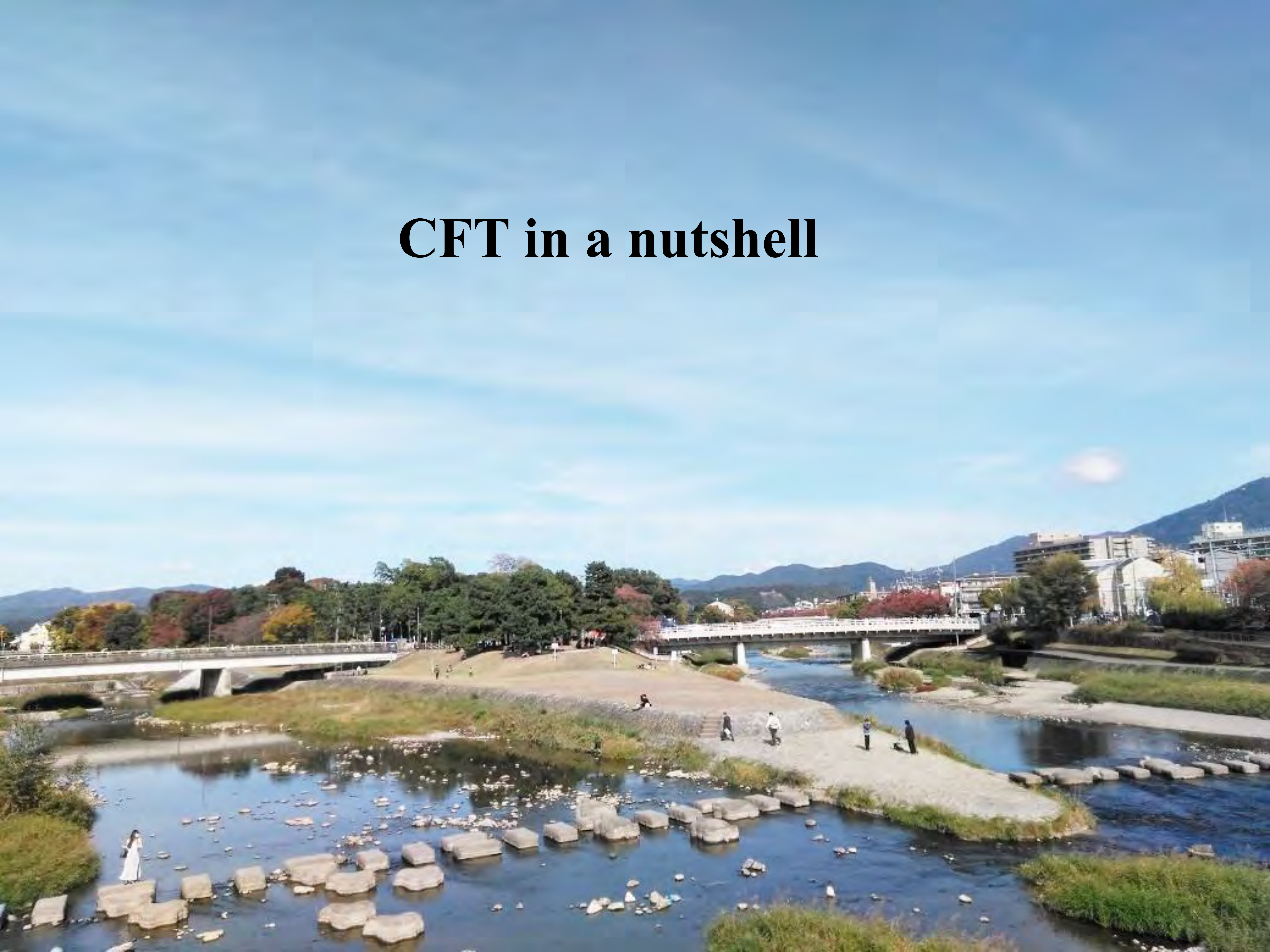
**We find**

**Complex generalization of c=1 CFT**

Conformal tower for complex spectrum

Universal properties for correlation functions

# CFT in a nutshell



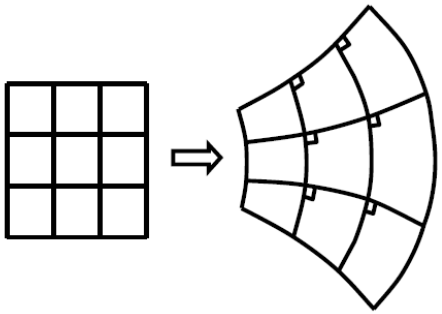
# CFT in a nutshell

## Critical phenomena in 1+1 dimensions

**Conformal symmetry**

Local scale invariance

Infinite # of generators



**Virasoro algebra**

$$[L_m, L_n] = (m-n)L_{n+m} + \frac{c}{12}m(m-1)(m+1)\delta_{m+n,0}$$

**Central charge**, universality class

$L_m$: conformal generators

Example $\begin{cases} c = \frac{1}{2}: \text{Ising model,} \\ c = 1: \text{Tomonaga-Luttinger liquids} \end{cases}$

# CFT in a nutshell

**Primary field**

$$\phi(z,\bar{z}) = \left(\frac{dw}{dz}\right)^{\Delta^+} \left(\frac{d\bar{w}}{d\bar{z}}\right)^{\Delta^-} \tilde{\phi}(w,\bar{w})$$

**Correlation function**

$$\langle \phi(z,\bar{z})\phi(z',\bar{z}')\rangle = |z-z'|^{-2\Delta^+}|\bar{z}-\bar{z}'|^{-2\Delta^-}$$

**$\Delta^+, \Delta^-$: Conformal dimensions**

**Verma module**

$$|\phi\rangle = \lim_{z,\bar{z}\to 0} \phi(z,\bar{z})|0\rangle, \quad L_n|0\rangle = 0 \quad (n \geq -1)$$

**conformal tower**

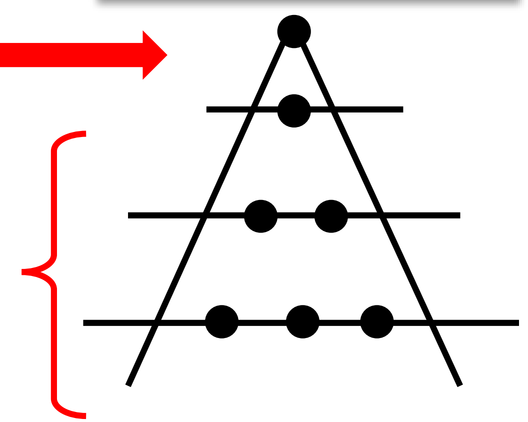
$$L_0|\phi\rangle = \Delta^+|\phi\rangle$$

$$L_n|\phi\rangle = 0 \quad n > 0$$

**Primary field: highest weight**

$$L_{-n}|\phi\rangle$$

**secondary fields**

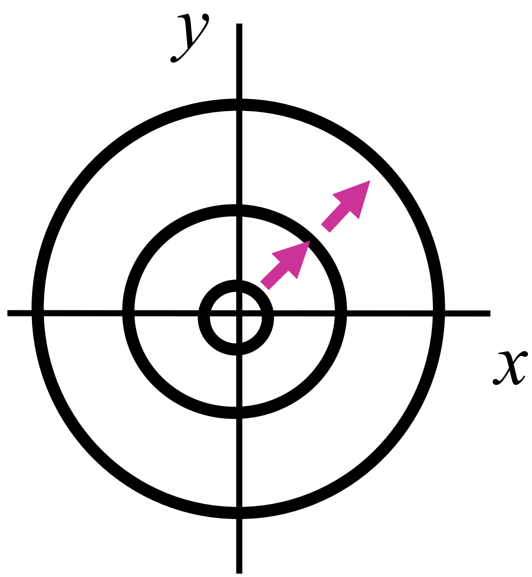


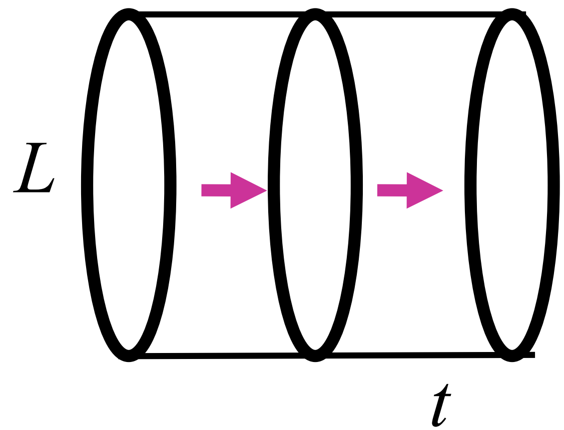
# CFT in a nutshell

How to obtain ? $(c, \Delta)$

Conformal transformation

$z \longrightarrow (L/2\pi)\log z$

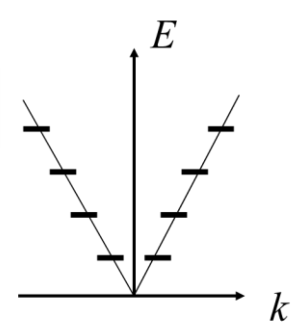



**Scale transformation** ⟺ **Time evolution of 1D ring**

**Central charge c**
**Conformal dimensions Δ**
**Conformal tower**

⟺ **Energy spectrum of H**

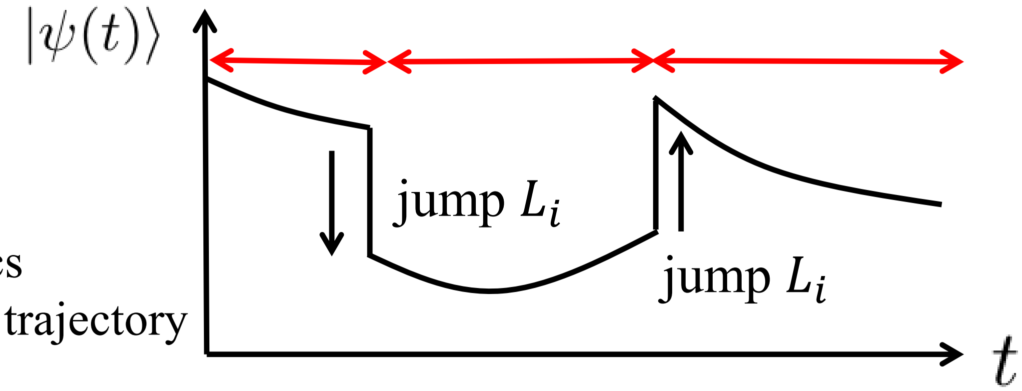



Field theory — Bridge — Microscopic model

# Non-Hermitian XXZ Model

# Ultracold gas with two-body dissipation

## Quantum trajectory method




Dynamics
in each quantum trajectory

$L_i$: jump operator

$$H_{\text{eff}} = H - \frac{i}{2}\gamma \sum_i L_i^\dagger L_i$$

**In our study**

## Two-component Bose-Hubbard model

$$H_{\text{eff}} = -t_h \sum_{j\sigma} (b_{j+1\sigma}^\dagger b_{j\sigma} + \text{H.c.}) + \sum_j \underbrace{(U_{\uparrow\downarrow} - i\gamma_{\uparrow\downarrow})}_{\text{complex}} n_{j\uparrow} n_{j\downarrow} + \sum_{j\sigma} \underbrace{\frac{U_{\sigma\sigma} - i\gamma_{\sigma\sigma}}{2}}_{\text{complex}} n_{j\sigma}(n_{j\sigma} - 1)$$

$$L_{j\sigma\sigma'} = \sqrt{\gamma_{\sigma\sigma'}} b_{j\sigma} b_{j\sigma'}$$

**Two-body loss** of particles

Complex energy (decay)

# Ultracold gas with two-body dissipation

■**1D two-component Bose-Hubbard model + Two-body loss**

$$H_{\text{eff}} = -t_h \sum_{j\sigma} (b_{j+1\sigma}^{\dagger} b_{j\sigma} + \text{H.c.}) + \sum_j (U_{\uparrow\downarrow} - i\gamma_{\uparrow\downarrow}) n_{j\uparrow} n_{j\downarrow} + \sum_{j\sigma} \frac{U_{\sigma\sigma} - i\gamma_{\sigma\sigma}}{2} n_{j\sigma}(n_{j\sigma} - 1)$$

Strongly correlated regime $U_{\sigma\sigma'} \gg t_h$

Second-order perturbation theory

■**NH XXZ model ( $J_{eff}^{\perp} < 0$ )**

$$H_{\text{eff}} = (J_{\text{eff}}^{\perp} + i\Gamma^{\perp}) \sum_j (S_{j+1}^x S_j^x + S_{j+1}^y S_j^y + \Delta_\gamma S_{j+1}^z S_j^z)$$

We see the **longest surviving state** in the long-time limit (**largest imaginary part** of energy)

map

■**NH XXZ model ( $J > 0$ )**

$$H_{\text{eff}}^{\text{XXZ}} = \frac{J}{2} \sum_j (S_{j+1}^{+} S_j^{-} + S_{j+1}^{-} S_j^{+}) + J \Delta_\gamma \sum_j S_{j+1}^z S_j^z$$

complex ($\Delta_\gamma$)

**Ground state** with the **smallest real part of energy**

We seek for universal properties (correlation functions, finite-size scaling)

# Finite-size scaling analysis in CFT

## CFT analysis of excitation spectra

■ **Excitation energy in a finite system**

<u>PBC</u>

$$\Delta E_{\mathrm{PBC}} = \frac{2\pi \tilde{u}}{L}\left[\frac{1}{4\tilde{K}}(\Delta N)^2 + \tilde{K}(\Delta D)^2 + n^+ + n^-\right]$$

- Complex velocity $\tilde{u}$
- Complex TL parameter $\tilde{K}$

## Conformal dimensions

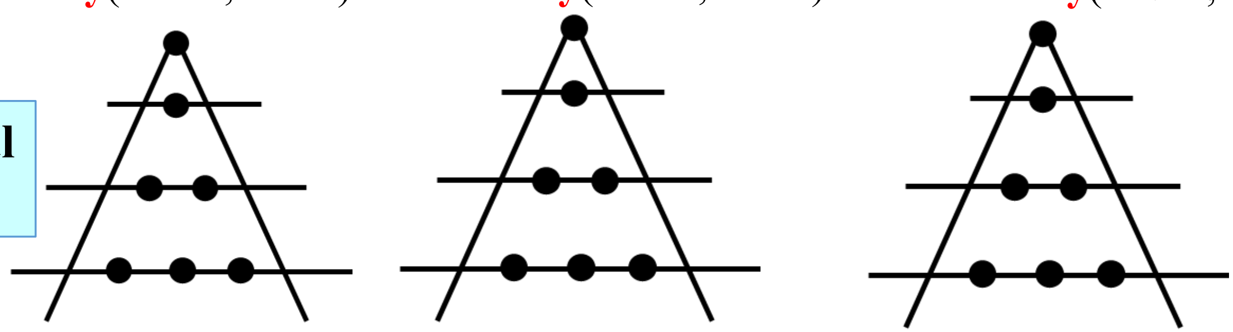
$$\Delta^{\pm}_{\mathrm{CFT}} = \frac{1}{2}\left(\frac{\Delta N}{2\sqrt{\tilde{K}}} \pm \Delta D\sqrt{\tilde{K}}\right)^2 + n^{\pm}$$

$\Delta N, \Delta D \in \mathbb{Z}$

$n^+, n^-$: nonnegative integers

**Primary**(ΔN=1,ΔD=0) **Primary**(ΔN=0,ΔD=1) **Primary**(ΔN=1,ΔD=1)

**Conformal tower**



. . .

**Complex generalization of $c = 1$ conformal field theory**

# Finite-size scaling analysis in CFT

## CFT analysis of excitation spectra

■ **Excitation energy in a finite system**

PBC

$$\Delta E_{\mathrm{PBC}} = \frac{2\pi\tilde{u}}{L}\left[\frac{1}{4\tilde{K}}(\Delta N)^2 + \tilde{K}(\Delta D)^2 + n^+ + n^-\right]$$

- Complex velocity $\tilde{u}$
- Complex TL parameter $\tilde{K}$

## Conformal dimensions

$$\Delta^{\pm}_{\mathrm{CFT}} = \frac{1}{2}\left(\frac{\Delta N}{2\sqrt{\tilde{K}}} \pm \Delta D\sqrt{\tilde{K}}\right)^2 + n^{\pm}$$

$\Delta N, \Delta D \in \mathbb{Z}$

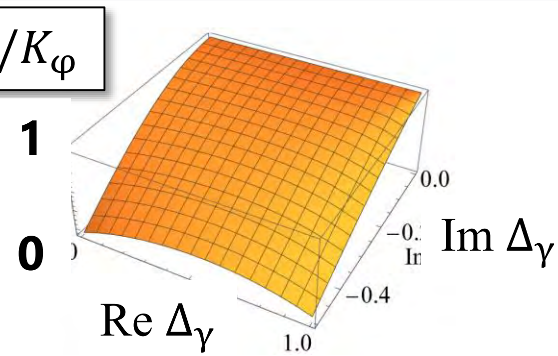
$n^+, n^-$: nonnegative integers

**CFT analysis with the Bethe ansatz**

■ **TL parameter**

$$\tilde{K} = \frac{\pi}{2(\pi - \arccos\Delta_\gamma)}$$

$$K_\theta = \mathrm{Re}\tilde{K} \qquad \frac{1}{K_\phi} = \mathrm{Re}\frac{1}{\tilde{K}}$$




deviates from 1

(Hermitian limit $K_\theta/K_\phi \to 1$)

# Correlation functions

## ■NH XXZ model

complex

$$H_{\rm eff}^{\rm XXZ} = \frac{J}{2}\sum_j (S_{j+1}^+ S_j^- + S_{j+1}^- S_j^+) + J\Delta_\gamma \sum_j S_{j+1}^z S_j^z$$

**Long-distance behavior**

## ■Bosonization

$$S^z(x) = -\frac{1}{\pi}\nabla\phi(x) + \frac{(-1)^x}{\pi\alpha}\cos(2\phi(x)),$$

$$S^+(x) = \frac{e^{-i\theta(x)}}{\sqrt{2\pi\alpha}}\left((-1)^x + \cos(2\phi(x))\right),$$

Bosonic fields
with commutation relations

$$[\phi(x_1), \nabla\theta(x_2)] = i\pi\delta(x_2 - x_1)$$

## ■NH Sine-Gordon Hamiltonian

complex

$$H_{\rm eff}^{\rm sG} = H_{\rm eff}^{\rm TL} - \frac{2\tilde{g}_3}{(2\pi\alpha)^2}\int dx \cos(4\phi(x))$$

## ■NH Tomonaga-Luttinger Hamiltonian (Gaussian)

complex

$$H_{\rm eff}^{\rm TL} = \frac{1}{2\pi}\int dx \left[\tilde{u}\tilde{K}(\nabla\theta(x))^2 + \frac{\tilde{u}}{\tilde{K}}(\nabla\phi(x))^2\right]$$

In the **massless regime**, the model is described by the **NH TL Hamiltonian**

# Correlation functions

**■NH TL model (massless regime)**

$$H_{\text{eff}}^{\text{TL}} = \frac{1}{2\pi}\int dx\left[\tilde{u}\tilde{K}(\nabla\theta(x))^2 + \frac{\tilde{u}}{\tilde{K}}(\nabla\phi(x))^2\right]$$

complex

**Right** $H_{\text{eff}}^{\text{TL}}|\Psi^R\rangle = E|\Psi^R\rangle$

**Left** $H_{\text{eff}}^{\text{TL}\dagger}|\Psi^L\rangle = E^*|\Psi^L\rangle$

**Expectation values by right or left ?**

## [1] Biorthogonal correlation functions ($c$ = 1 CFT)

$$_L\langle S^z(x,0)S^z(0,0)\rangle_R = -\frac{\tilde{K}}{2\pi^2}\frac{1}{x^2} + \tilde{C}_2(-1)^x\left(\frac{1}{x}\right)^{2\tilde{K}}$$

$$_L\langle S^+(x,0)S^-(0,0)\rangle_R = \tilde{C}_3\left(\frac{1}{x}\right)^{2\tilde{K}+\frac{1}{2\tilde{K}}} + \tilde{C}_4(-1)^x\left(\frac{1}{x}\right)^{\frac{1}{2\tilde{K}}}$$

**Complex-valued** TL parameter $\tilde{K}$

## [2] Right-state correlation functions (observable)

$$_R\langle S^z(x)S^z(0)\rangle_R = -\frac{K_\phi}{2\pi^2}\frac{1}{x^2} + C_2(-1)^x\left(\frac{1}{x}\right)^{2K_\phi}$$

$$_R\langle S^+(x)S^-(0)\rangle_R = C_3\left(\frac{1}{x}\right)^{2K_\phi+\frac{1}{2K_\theta}} + C_4(-1)^x\left(\frac{1}{x}\right)^{\frac{1}{2K_\theta}}$$

Real part of (the reciprocal of) $\tilde{K}$

$$\frac{1}{K_\phi} = \mathrm{Re}\frac{1}{\tilde{K}}$$

$$K_\theta = \mathrm{Re}\tilde{K}$$

*cf* Ashida et al. (2016)

Both correlation functions are characterized by the **complex-valued TL parameter** $\tilde{K}$

# NH-DMRG results

**NH-DMRG algorithm**

Density matrix for truncation of eigenstates

$$\rho_i = \frac{1}{2}\widehat{\mathrm{Tr}}\left\{|\psi_i\rangle_{LL}\langle\psi_i| + |\psi_i\rangle_{RR}\langle\psi_i|\right\}$$

→ We apply this algorithm to **open quantum systems**

**TL parameter:** calculating $\Delta E_{\mathrm{spin}}$ (spin change) and $\Delta E_{\mathrm{spectral}}$ (spin conserved)

| *TL parameter* | *Velocity of excitations* |
|---|---|
| $\tilde{K} = \frac{\Delta E_{\mathrm{spectral}}}{2\Delta E_{\mathrm{spin}}}$ | $\tilde{u} = \frac{L\Delta E_{\mathrm{spectral}}}{\pi}$ |

→ Comparison with the Bethe-ansatz results

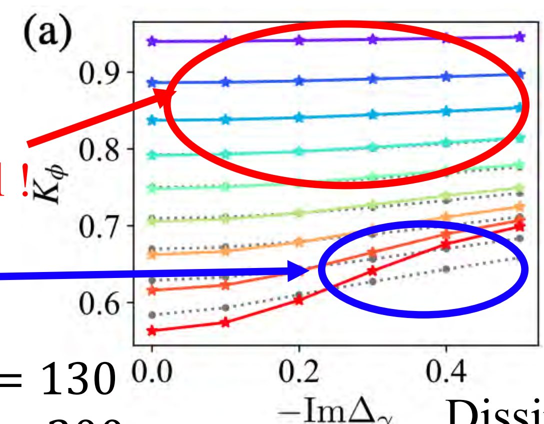
**Results for $K_\phi$**

$$\frac{1}{K_\phi} = \mathrm{Re}\frac{1}{\tilde{K}}$$

**NH-DMRG** (color) and the **Bethe ansatz** (gray) agree well !

Discrepancy for large dissipation

→ **Finite-size effect becomes serious** near the AF transition point **(Re $\Delta_\gamma$~1)**




$L = 130$
$m = 200$

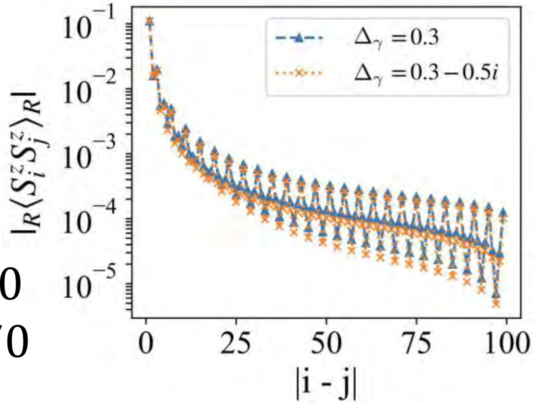
Re$\Delta_\gamma$
small (0.1)
large (0.9)

Dissipation

# NH-DMRG results

## NH-DMRG results of correlation functions

$L = 100$
$m = 170$




Field theory

$$_R\langle S^z(x)S^z(0)\rangle_R = -\frac{K_\phi}{2\pi^2}\frac{1}{x^2} + C_2(-1)^x\left(\frac{1}{x}\right)^{2K_\phi}$$

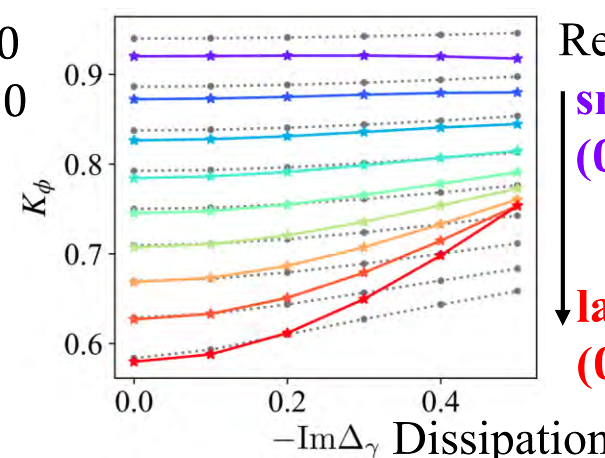
We perform the fitting for $K_\phi$ and $C_2$

## Two-parameter fitting with $K_\phi$ and $C_2$ in **correlation functions**

**Qualitatively the same** as the finite-size scaling results of the excitation spectrum

$L = 100$
$m = 170$

NH-DMRG (color)
Bethe ansatz (gray)




Re$\Delta_\gamma$
**small (0.1)**
**large (0.9)**
Dissipation

However... fitting accuracy is low due to

Effect of the edges (system size is limited)

Severe convergence problem due to **non-Hermiticity**

**NH-DMRG agree rather well with the analytical results !**

# Summary of Part II

## ● Universal Properties of Dissipative Tomonaga-Luttinger liquid

**Non-unitary CFT with complex spectra: little is known**

## ● Model : NH XXZ model

### ■ Two-types of correlation functions

$$ {}_L\langle S^+(x,0)S^-(0,0)\rangle_R = \tilde{C}_3\left(\frac{1}{x}\right)^{2\tilde{K}+\frac{1}{2\tilde{K}}} + \tilde{C}_4(-1)^x\left(\frac{1}{x}\right)^{\frac{1}{2\tilde{K}}} $$

$$ {}_R\langle S^+(x)S^-(0)\rangle_R = C_3\left(\frac{1}{x}\right)^{2K_\phi+\frac{1}{2K_\theta}} + C_4(-1)^x\left(\frac{1}{x}\right)^{\frac{1}{2K_\theta}} $$

**Universal scaling**

Complex TL parameter $\tilde{K}$

$$ K_\theta = \mathrm{Re}\tilde{K} $$

$$ \frac{1}{K_\phi} = \mathrm{Re}\frac{1}{\tilde{K}} $$

### ■ Finite-size scaling analysis

$$ E_L(0) = Le_\infty(0) - \frac{\pi\tilde{u}c}{6L} $$

$$ \Delta E_{\mathrm{PBC}} = \frac{2\pi\tilde{u}}{L}\left[\frac{1}{4\tilde{K}}(\Delta N)^2 + \tilde{K}(\Delta D)^2 + n^+ + n^-\right] $$

$$ \Delta^{\pm}_{\mathrm{CFT}} = \frac{1}{2}\left(\frac{\Delta N}{2\sqrt{\tilde{K}}} \pm \Delta D\sqrt{\tilde{K}}\right)^2 + n^{\pm} $$

**Complex generalization of $c = 1$ CFT**

*PART III*

# SU(N) Generalization of Dissipative Tomonaga-Luttinger Liquids

- *K. Yamamoto and N. Kawakami. arXiv:2207.04395*

K. Yamamoto

# Generalization to SU($N$) spin symmetry

Experimental advances in **ultracold atoms** with **SU($N$) symmetry**
e.g. SU($N$) Hubbard models

→ **Universal properties with SU($N$) symmetry**

**Haldane's ideal-gas description**

- Powerful method to obtain **universal properties** of **1D critical systems**
- Critical properties of **SU(N) Hubbard** models can be extracted

**In our study**

**NH SU(N) Calogero-Sutherland model**

Ha and Haldane PRB 1992

complex (dissipation)

spin exchange operator

$$H_{\mathrm{eff}} = -\frac{1}{2}\sum_{i=1}^{M}\frac{\partial^2}{\partial x_i^2} + \sum_{i<j} D(x_i - x_j)^{-2}\tilde{\lambda}'(\tilde{\lambda}' + P_{ij}^{\sigma})$$

***$1/x^2$ long-range interaction***

→ **"Ideal gas description"** of **universal properties** in 1D

# Ideal-gas description

## Single-component case

Sutherland, 1971

$$H_{\text{eff}} = -\sum_{j=1}^{M} \frac{\partial^2}{\partial x_j^2} + \sum_{j>l} V(x_j - x_l)$$

$$V(x) = \frac{\tilde{g}\pi^2}{L^2}\left[\sin\left(\frac{\pi x}{L}\right)\right]^{-2}$$

***$1/x^2$ interaction***

**Ground state**

$$\Psi_g = \prod_{j>l} \left|\sin\frac{\pi(x_j - x_l)}{L}\right|^{\tilde{\lambda}-s} \left(\sin\frac{\pi(x_j - x_l)}{L}\right)^s$$

s=1 fermion, s=0 boson

→ **Two-body phase shift** $\theta(k) = \pi(\tilde{\lambda} - 1)\text{sgn}^*(k)$

**Step-function two-body phase shift *$\theta(k)$*:**

Haldane, 1991 →

Interaction effect : **level repulsion** $\tilde{\lambda}$

**" Ideal gas" obeying**

**"Fractional exclusion statistics"**

↓

**Universal** properties in 1D quantum many-body systems

# Ideal-gas description

**Excitation energy: SU(N) model**

**Ideal gas approach**

$$\Delta E = (\pi\tilde{v}/L)(\boldsymbol{m}^t\boldsymbol{D}\boldsymbol{m}/2 + 2\boldsymbol{d}^t\boldsymbol{D}^{-1}\boldsymbol{d})$$

$\boldsymbol{d},\boldsymbol{m}$ : quantum numbers

$$\boldsymbol{D} = \begin{pmatrix} \tilde{\lambda}'+1 & -1 & & & \\ -1 & 2 & -1 & \mathbf{0} & \\ & -1 & 2 & \ddots & \\ & \mathbf{0} & \ddots & \ddots & -1 \\ & & & -1 & 2 \end{pmatrix}$$

**Spin sector (real)**

SU(N) Cartan matrix

c=N-1

**SU(N) Kac-Moody algebra**

**Charge sector**

**Complex generalization** of **c=1 U(1) Gaussian CFT**

NH generalization of **spin-charge separation**

# Critical exponents

## Two-kinds of parameters for charge sector

$$\frac{1}{K_\rho^\phi} = \mathrm{Re}\left[\frac{1}{\tilde{K}_\rho}\right] \qquad K_\rho^\theta = \mathrm{Re}[\tilde{K}_\rho]$$

➡ **Dissipation** only affects the **charge exponent**

## Charge-density correlator

$$_R\langle\rho(x)\rho(0)\rangle_R \simeq \sum_{j=1}^{N} A_j \cos[2(N-j+1)k_F x] x^{-\beta_j} + \frac{A_0}{x^2}$$

$$\beta_j = \frac{2(N-j+1)(j-1)}{N} + \frac{2(N-j+1)^2}{N} K_\rho^\phi$$

## Fermion correlator

$$_R\langle c_\sigma^\dagger(x) c_\sigma(0)\rangle_R \simeq C_1 \cos(k_F x) x^{-\eta_F}$$

$$\eta_F = \frac{N-1}{N} + \frac{1}{2NK_\rho^\theta} + \frac{K_\rho^\phi}{2N}$$

## Boson correlator

$$_R\langle b_\sigma^\dagger(x) b_\sigma(0)\rangle_R \simeq B_1 x^{-\eta_B}$$

$$\eta_B = \frac{N-1}{2N} + \frac{1}{2NK_\rho^\theta}$$

**Dissipation effects** are described by two **real TL parameters**

# Towards experimental realization

**Universal Properties obtained by "ideal gas approach"**

➡ can be applied to a variety of SU(N) systems

**Hubbard model + Two-body loss**

complex (dissipation)

$$H_{\mathrm{eff}}^{\mathrm{Hubbard}} = -t\sum_{j\sigma}(c_{j\sigma}^{\dagger}c_{j+1\sigma} + \mathrm{H.c.}) + \tilde{U}\sum_{j} n_{j\uparrow}n_{j\downarrow}$$

➡ Universal properties of metallic phases are described by **complex U(1)×SU(2) TL liquids**

◆ **Candidate** ➡ Hubbard model w/ ultracold alkaline-earth-like atoms **$^{173}Yb$**

◆ **Dissipation** ➡ Introduction of two-body loss by **photoassociation**

On the other hand…

◆ **Realization of NH system** ➡ Postselection by **quantum-gas microscopy**

# Summary of Part III

SU(N) Interacting systems:

$$\Delta E = (\pi\tilde{v}/L)(\boldsymbol{m}^t\boldsymbol{D}\boldsymbol{m}/2 + 2\boldsymbol{d}^t\boldsymbol{D}^{-1}\boldsymbol{d})$$

NH generalization of **spin-charge separation**

$$\boldsymbol{D} = \begin{pmatrix} \tilde{\lambda}'+1 & -1 & & & \\ -1 & 2 & -1 & \mathbf{0} & \\ & -1 & 2 & \ddots & \\ & \mathbf{0} & \ddots & \ddots & -1 \\ & & & -1 & 2 \end{pmatrix}$$

**Spin sector (real)**

SU(N) Cartan matrix

c=N-1 level-1 SU(N) Kac-Moody algebra

**Charge sector** (**complex** due to **dissipation**)

**Complex generalization** of c=1 U(1) Gaussian CFT

**Universal scaling relations**

# Outlook

Critical Properties of Non-Hermitian Correlated Systems

◆CFT classification of non-Hermitian critical systems

◆CFT description of measurement-induced transitions

What is the " effective central charge"?

Primary fields, Virasoro tower structure ?

etc.

# Summary

## Critical Properties of Non-Hermitian Correlated Systems

*PART I*

1. Non-Hermitian Kondo effect

*Prototype of many-body non-Hermitian systems*

*M. Nakagawa et al. PRL **121**, 203001(2018)*

*PART II*

2. Non-Hermitian Tomonaga-Luttinger liquids

*Quantum XXZ spin chain*

*K. Yamamoto et al. PRB **105**, 205125(2022)*

*PART III*

2. SU(N) Generalization of Dissipative TL liquids

*Haldane's "Ideal Gas" Approach*

*K. Yamamoto et al. arXiv:2207.04395*

# Thank you very much for organizing “real workshop” !

NQS2011 Kawakami (chair)
NQS2014
NQS2017
NQS2022