Novel Quantum States in Condensed Matter 2022, YITP

Majorana-mediated spin transport in Kitaev model at finite temperatures

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Majorana-mediated spin transport in Kitaev model at finite temperature

Colloborators

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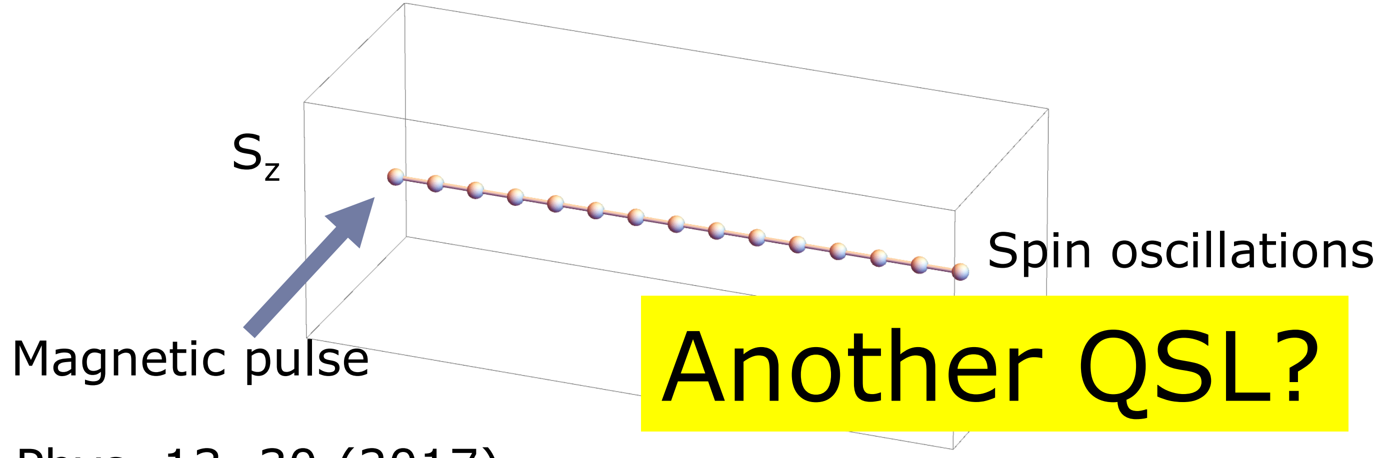
T. Minakawa, Y. Murakami, AK & J. Nasu, Phys. Rev. Lett. 125, 047204 (2020).
H. Taguchi, Y. Murakami & AK, Phys. Rev. B 105,125137 (2022).
那須譲治、古賀昌久, 固体物理 57, 703 (2022)

Spin transport in insulators

- **Ordered states** **YIG**
  - Magnons

- **Quantum Spin liquids** Sr_2CuO_3
  - Spinons
  - holons



Another QSL?

D. Hirobe et al. Nat. Phys. 13, 30 (2017)

Kitaev model

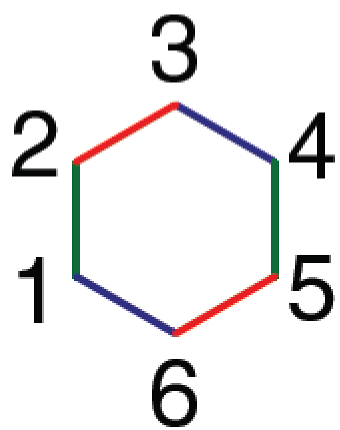
Hamiltonian

Candidate materials
α-$RuCl_3$, A_2IrO_3 (A=Na, Li)

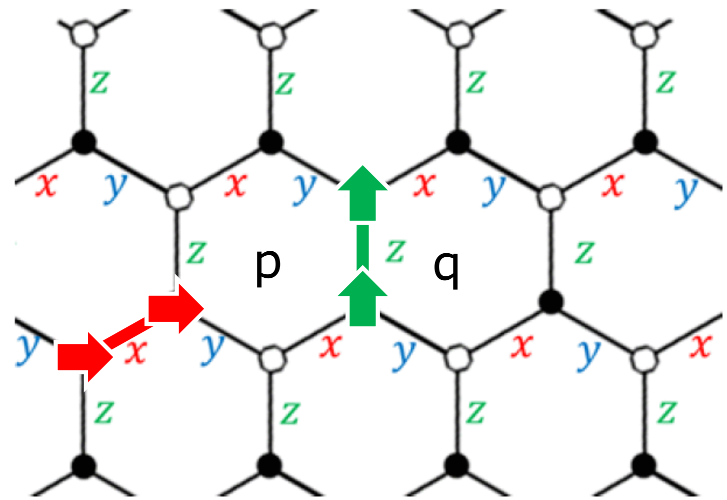
$$H = -J \sum_{\langle i,j \rangle_x} S_i^x S_j^x - J \sum_{\langle i,j \rangle_y} S_i^y S_j^y - J \sum_{\langle i,j \rangle_z} S_i^z S_j^z$$

Local conserved quantity

$$W_p = \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z$$



$[H, W_p] = 0$
$[W_p, W_q] = 0$



Bond frustration

A. Kitaev, Ann. Phys. 321, 2 (2006)

Kitaev model

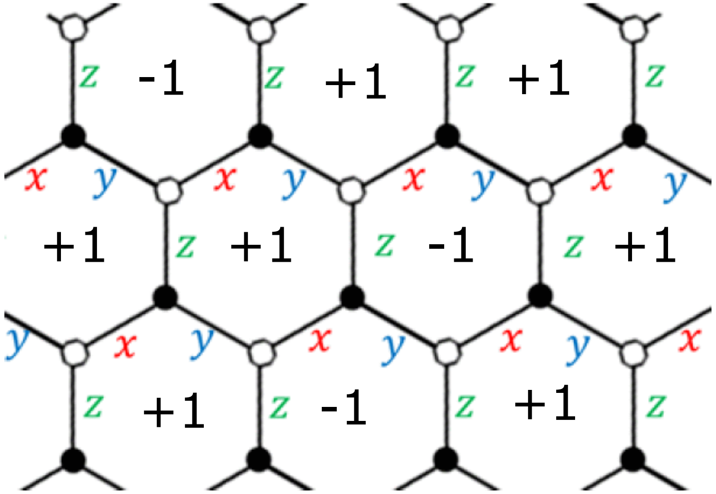
Hamiltonian

$$H = -J \sum_{\langle i,j \rangle_x} S_i^x S_j^x - J \sum_{\langle i,j \rangle_y} S_i^y S_j^y - J \sum_{\langle i,j \rangle_z} S_i^z S_j^z$$

Local conserved quantity

$$W_p = \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z$$

$W_p = \pm 1$



Each eigenstate

$$\psi = \psi(\{w_p\})$$

For all plaquettes

Ground state $W_p = +1$ (Lieb theorem)

Commutation relations

- **W_p=+1**

$$\{\sigma^\alpha, \sigma^\beta\} = 2\delta_{\alpha\beta}$$

$$W_p = \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z$$

$$W_p \sigma_1^x = +\sigma_1^x W_p$$ commute

$$W_p \sigma_1^y = -\sigma_1^y W_p$$

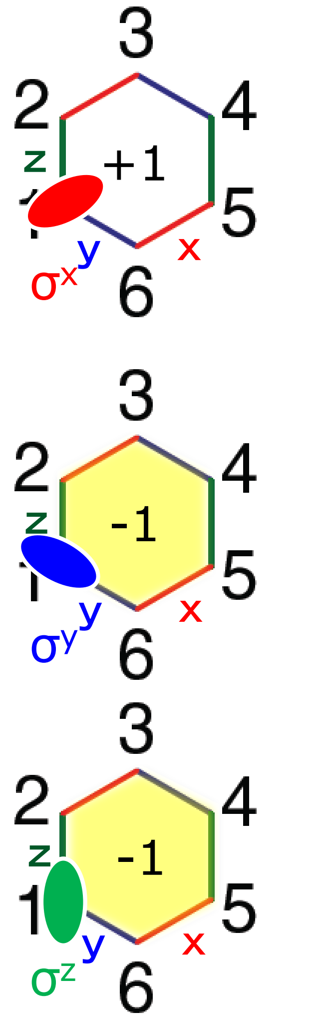
$$W_p \sigma_1^z = -\sigma_1^z W_p$$

anticommute

$$W_p \sigma_1^y |\psi_g\{+1\}\rangle = -\sigma_1^y W_p |\psi_g\{+1\}\rangle$$

$$= -\sigma_1^y |\psi_g\{+1\}\rangle$$

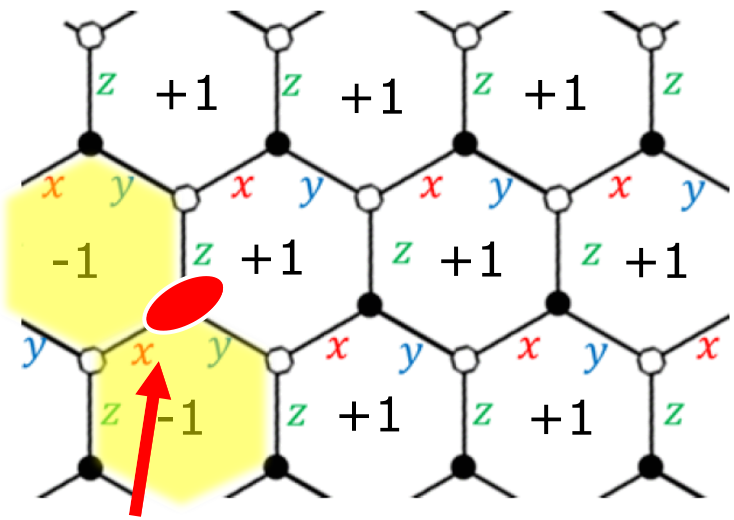
Eigenvalue of Wp



Quantum spin liquid state

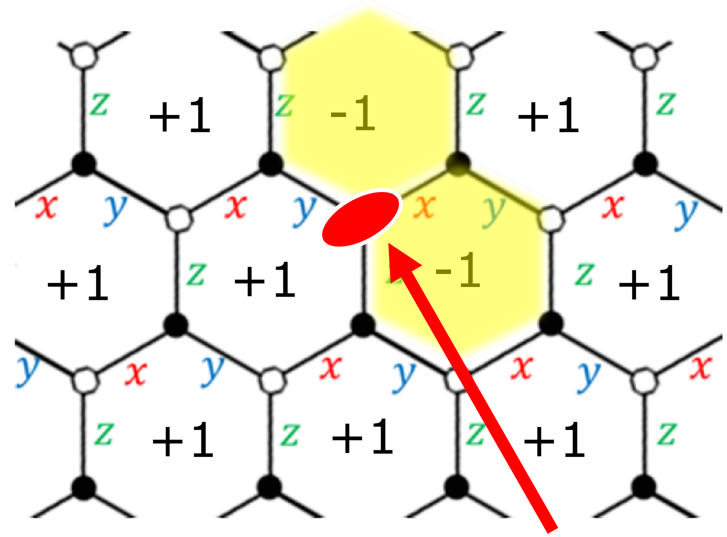
- **W_p=+1**

$$\sigma_i^x|\psi_g\rangle$$



σ_i^x

$$\sigma_j^x|\psi_g\rangle$$



σ_j^x

$$\langle\psi_g|\sigma_i^x\sigma_j^x|\psi_g\rangle = 0$$

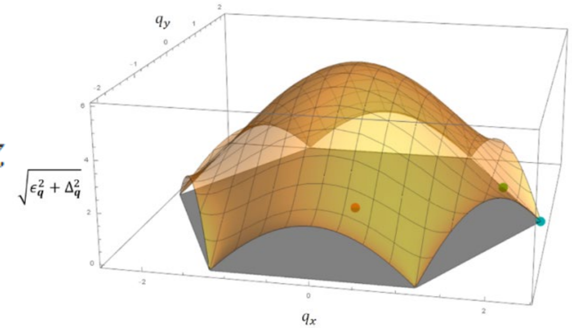
if i,j are not located on the same bond

No long range order

Excitation

■ Hamiltonian

$$H = -J \sum_{\langle i,j \rangle_x} S_i^x S_j^x - J \sum_{\langle i,j \rangle_y} S_i^y S_j^y - J \sum_{\langle i,j \rangle_z} S_i^z S_j^z$$



dispersion

Majorana representation

$$\mathcal{H} = iJ_x \sum_{(jk)_x} c_j c_k - iJ_y \sum_{(jk)_y} c_j c_k - iJ_z \sum_{(jk)_z} \eta_r c_j c_k$$

Free hopping

$\propto W_p = +1$

Kitaev model

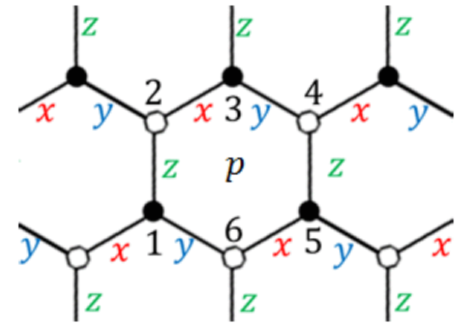
- localized Majorana (flux) $\propto W_p$
- itinerant Majorana c_j

Two energy scales

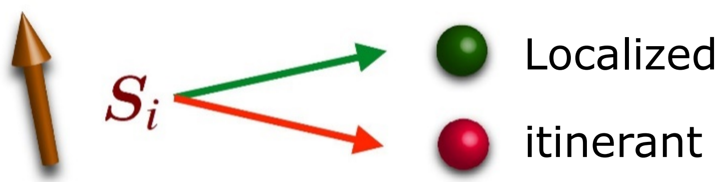
Thermodynamic properties

- **Hamiltonian**

$$H = -J \sum_{\langle i,j \rangle_x} S_i^x S_j^x - J \sum_{\langle i,j \rangle_y} S_i^y S_j^y - J \sum_{\langle i,j \rangle_z} S_i^z S_j^z$$



Spin fractionalization



Local conserved quantity

$$W_p = \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z$$

J. Nasu et al., Phys. Rev. B **92**, 115122 (2015).

Spin transport in the Kitaev quantum spin liquid

Observe itinerant Majoranas in the bulk?

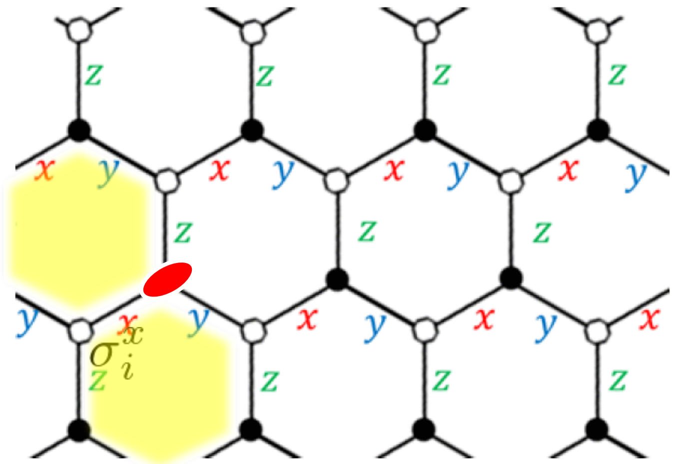
cf. Majorana edge current
in α-$RuCl_3$

Spin correlations in Kitaev model

- **No spin moment**
- **No spin correlations**

$$\langle\psi_g|\sigma_i^x|\psi_g\rangle = 0$$

$$\langle\psi_g|\sigma_i^x\sigma_j^x|\psi_g\rangle = 0$$



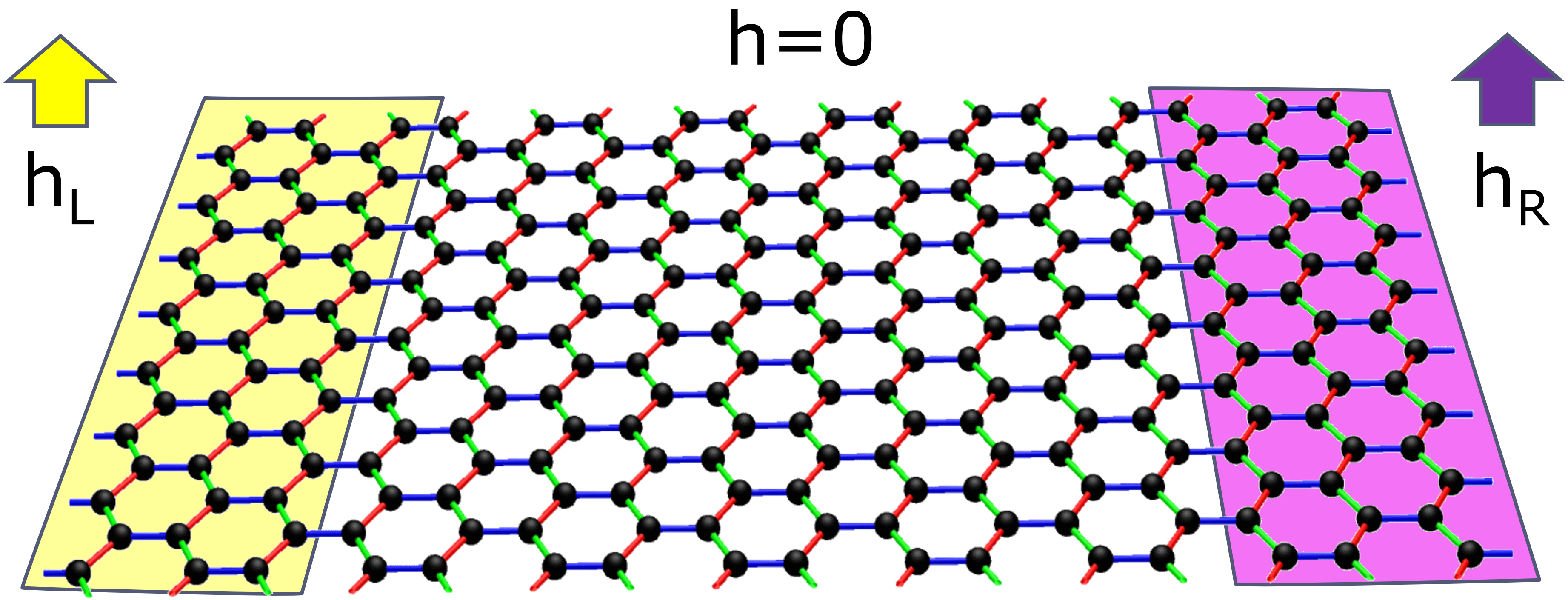
$$\begin{aligned}\langle\psi_g|\sigma_i^x|\psi_g\rangle &= \langle\psi_g|\sigma_i^x W_p|\psi_g\rangle \\ &= -\langle\psi_g|W_p\sigma_i^x|\psi_g\rangle \\ &= -\langle\psi_g|\sigma_i^x|\psi_g\rangle\end{aligned}$$

existence of Wp important

What happens if not ?
eg. field, edges, defects, etc.

Spin transport?

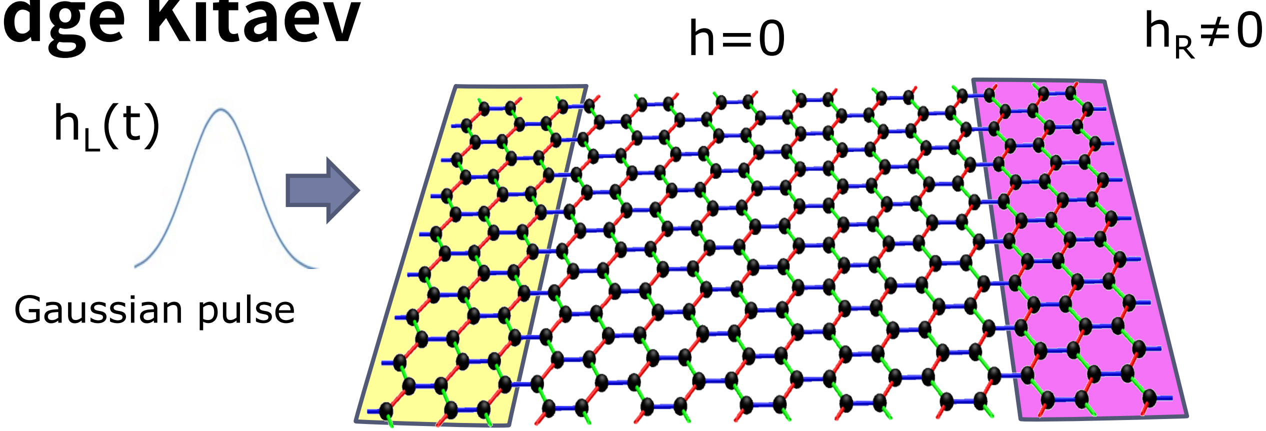
- **Two edges**



$\langle \psi_g | \sigma_i^z | \psi_g \rangle \neq 0$ $\langle \psi_g | \sigma_i^z | \psi_g \rangle = 0$ $\langle \psi_g | \sigma_i^z | \psi_g \rangle \neq 0$

Real-time dynamics

- **zigzag edge Kitaev**



- **Majorana mean-field theory**

$$\mathcal{H}(t) = -\frac{J_K}{4}\sum_{\boldsymbol{r}}\left(i\gamma^A_{\boldsymbol{r}-\boldsymbol{a}+\boldsymbol{b}}\gamma^B_{\boldsymbol{r}} + i\gamma^A_{\boldsymbol{r}+\boldsymbol{b}}\gamma^B_{\boldsymbol{r}}\right) - \frac{J_K}{4}\sum_{\boldsymbol{r}} i\gamma^A_{\boldsymbol{r}}\gamma^B_{\boldsymbol{r}} i\bar{\gamma}^A_{\boldsymbol{r}}\bar{\gamma}^B_{\boldsymbol{r}}$$

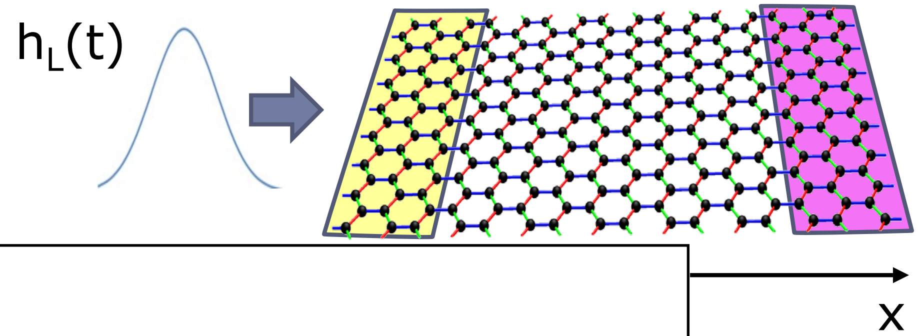
$$-\frac{h_R}{2}\sum_{\boldsymbol{r}\in \mathrm{R}}\left(i\gamma^A_{\boldsymbol{r}}\bar{\gamma}^A_{\boldsymbol{r}} - i\gamma^B_{\boldsymbol{r}}\bar{\gamma}^B_{\boldsymbol{r}}\right) + \frac{h_L(t)}{2}\sum_{\boldsymbol{r}\in \mathrm{L}} i\gamma^B_{\boldsymbol{r}}\bar{\gamma}^B_{\boldsymbol{r}},$$

$h_R=h_L=0$
exact

- **Exact diagonalization (24 sites)**

Real-time dynamics

- **Change in moment**

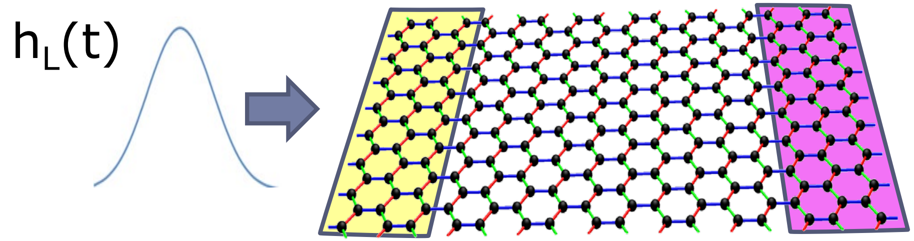


ΔS_z

x

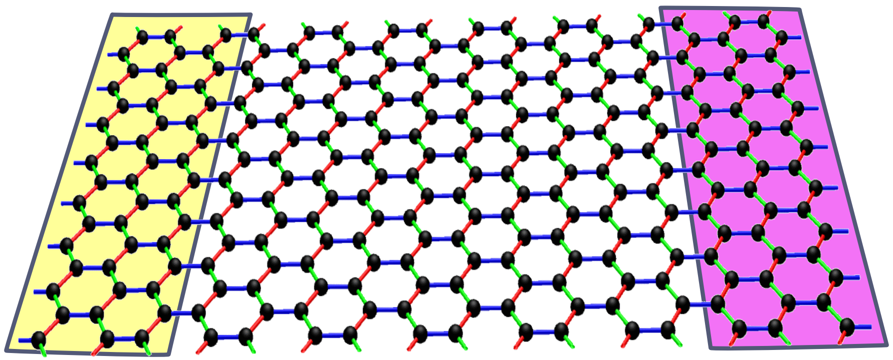
Real-time dynamics

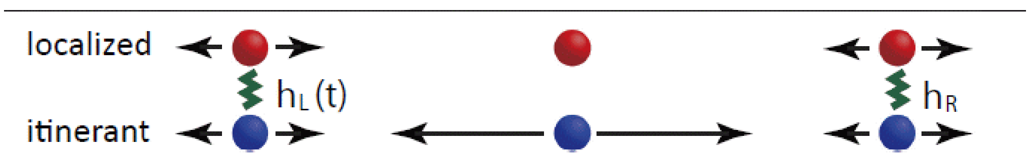
- **Majorana mean-fields**



Role of itinerant Majoranas

- **Majorana-mediated transport**

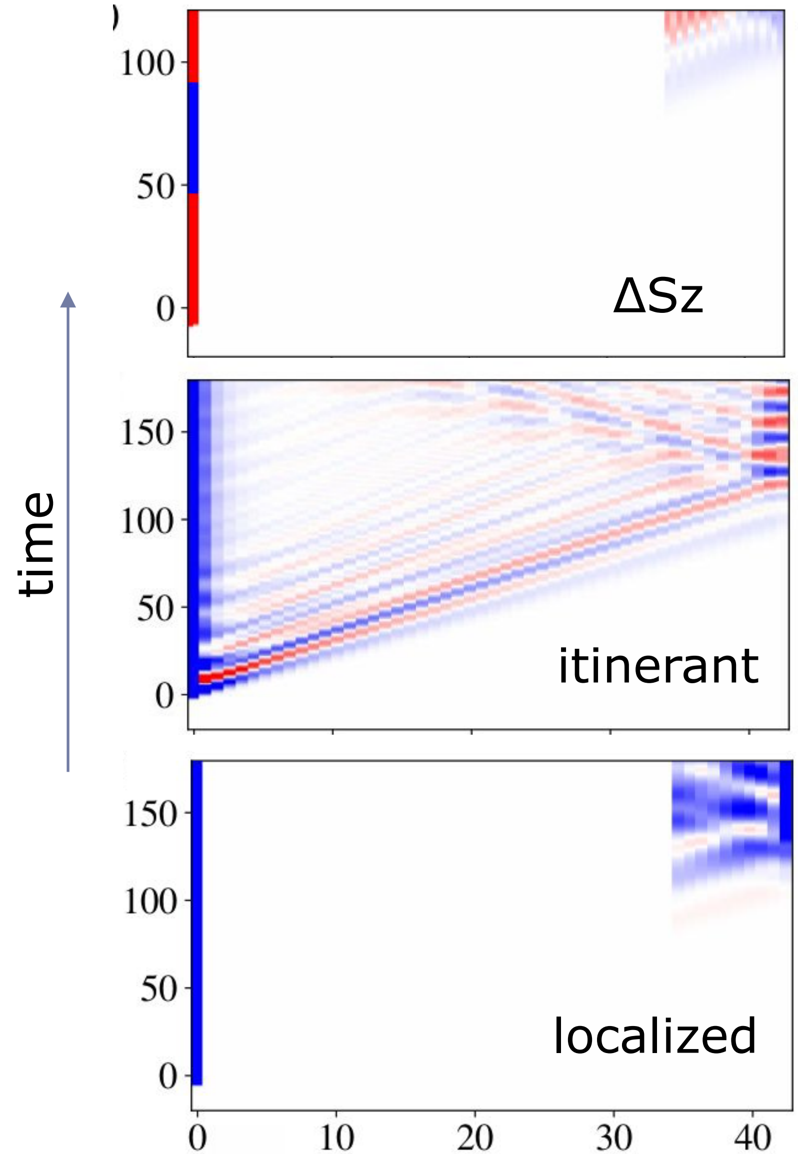




$$S^z \sim c\bar{c}$$

localized $\bar{c}$

itinerant c



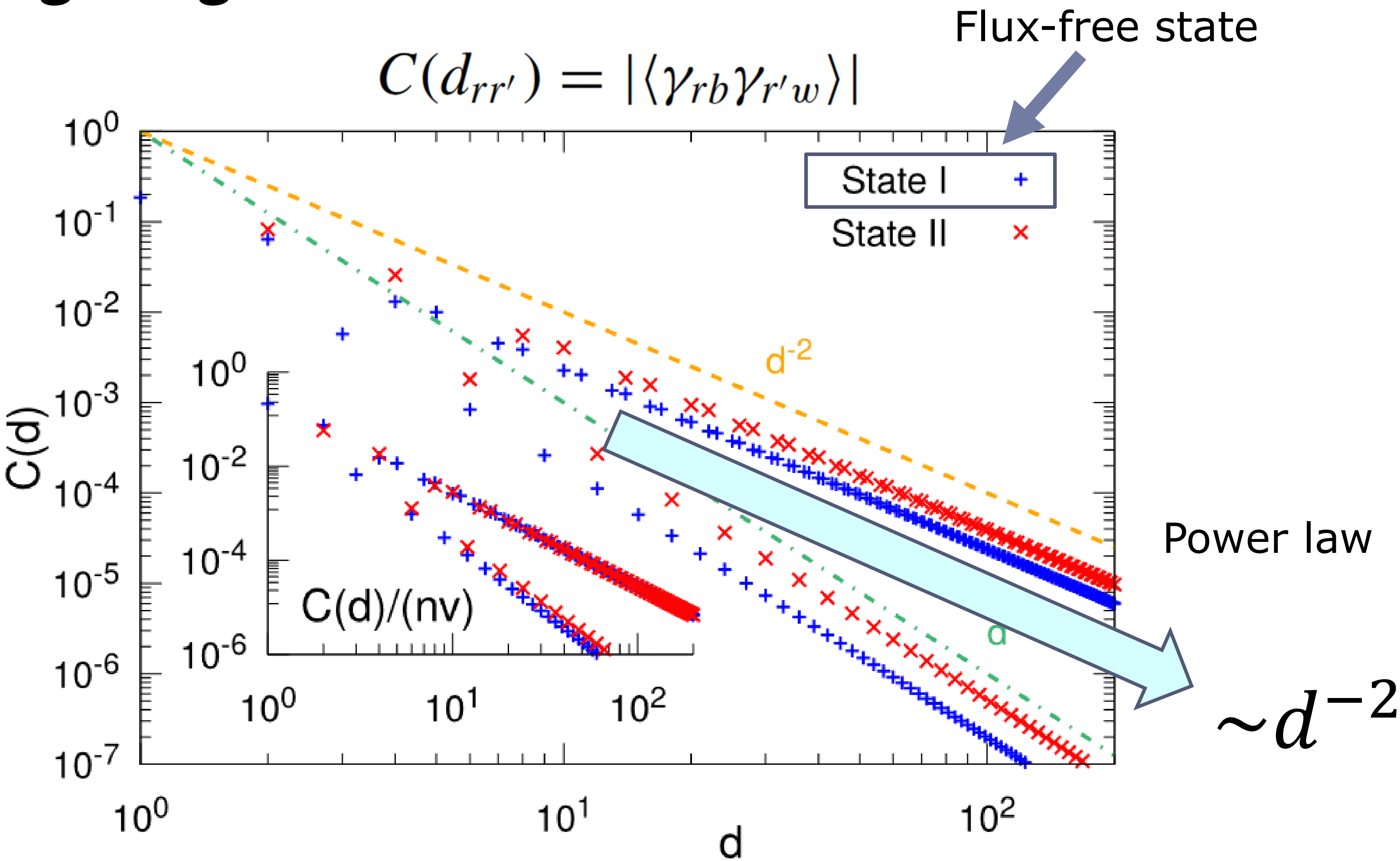
Spinless spin transport

T. Minakawa et al., Phys. Rev. Lett. 125, 047204 (2020).

Majorana correlations

- **Long-range correlations**

$$C(d_{rr'}) = |\langle \gamma_{rb}\gamma_{r'w}\rangle|$$



AK, Y. Murakami, and J. Nasu, Phys. Rev. B 103, 214421 (2021)

How stable ?
Majorana-mediated spin transport

Stability of the spin transport

- **Majorana-mediated spin transport**
  - **Spin fractionalization**
    - **Itinerant Majorana fermions**
    - **fluxes**

1. Heisenberg and/or Γ terms
2. Thermal fluctuations
3. Anisotropy in Ising exchanges
4. Flux configurations

1. Heisenberg term

Realistic Materials

Na_2IrO_3, α-$RuCl_3$

Heisenberg Gamma

$$H = \sum_{\langle ij \rangle_\alpha} \left\{ K S_i^\alpha S_j^\alpha + J \sum_\beta S_i^\beta S_j^\beta + \Gamma \sum_{\bar{\beta} \neq \beta \neq \alpha} S_i^\beta S_j^{\bar{\beta}} \right\},$$

Kitaev

No local symmetry Wp

Majorana-mediated spin transport robust?

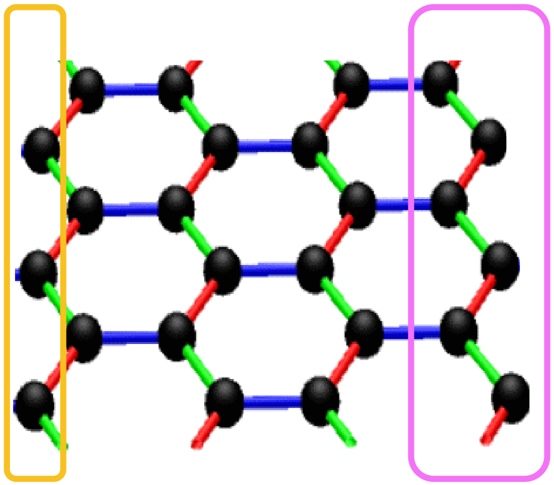
J. Chaloupka et al., Phys. Rev. Lett. **105**, 027204 (2010).
T. Suzuki et al., Phys. Rev. B **92**, 184411 (2015).
Y. Yamaji et al., Phys. Rev. B **93**, 174425 (2016).

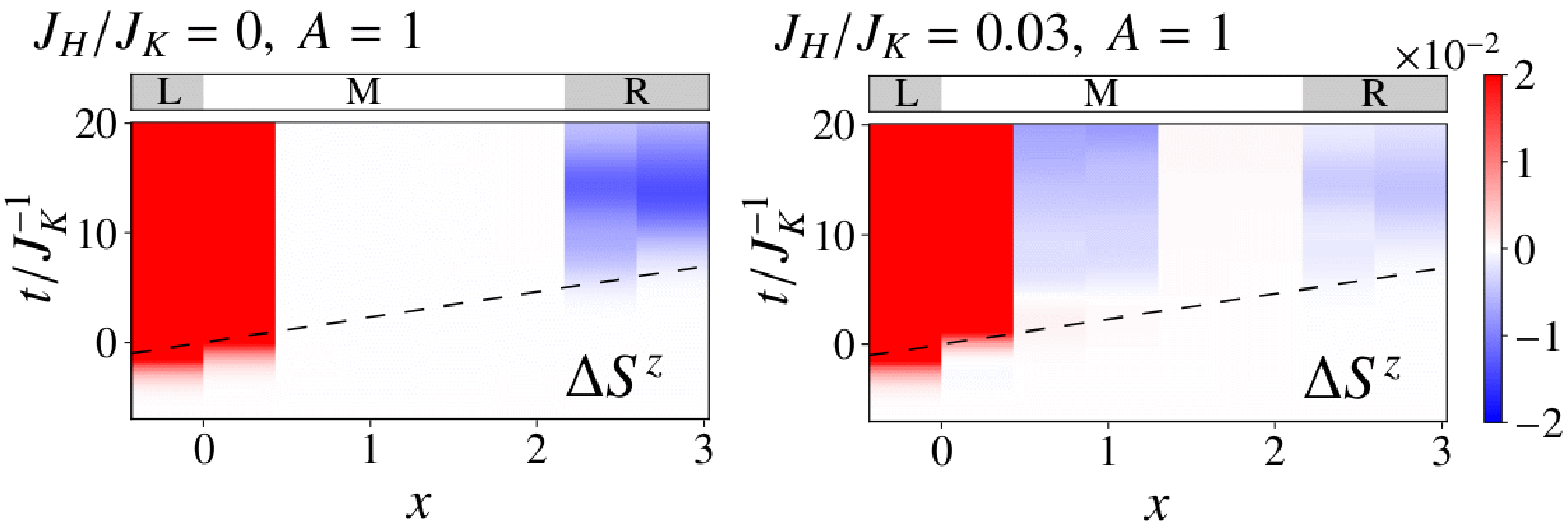
Heisenberg interaction

- **Hamiltonian**

$$\mathcal{H}(t) + J_H \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

- **No local symmetry W_p**
  - Exact diagonalization 24sites

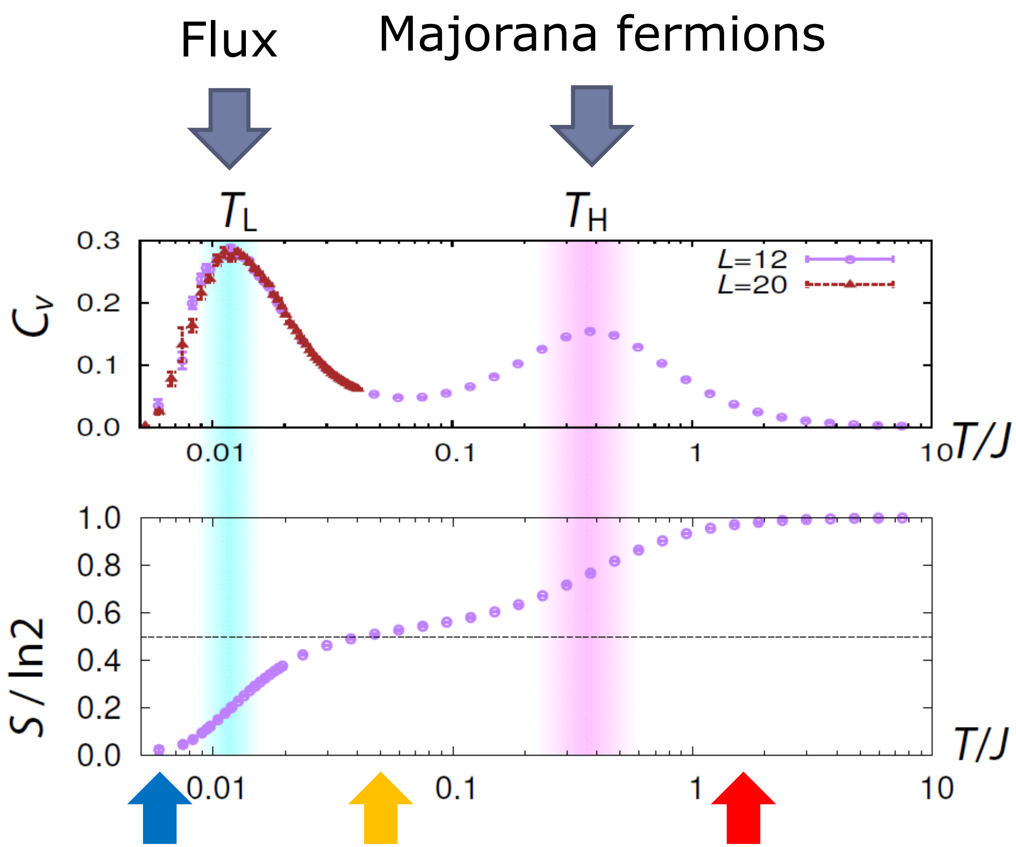


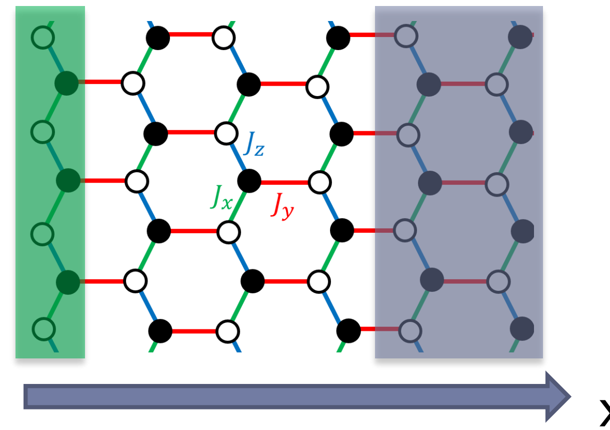


T. Minakawa et al., Phys. Rev. Lett. 125, 047204 (2020).

2. Thermal fluctuations

■ Specific heat



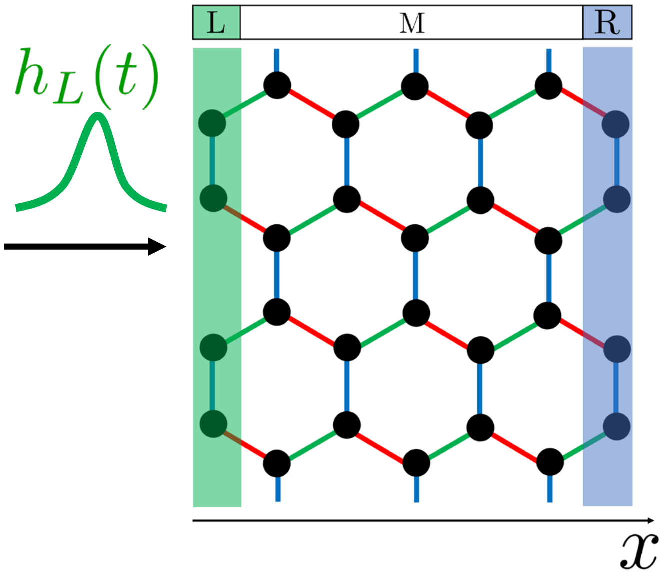


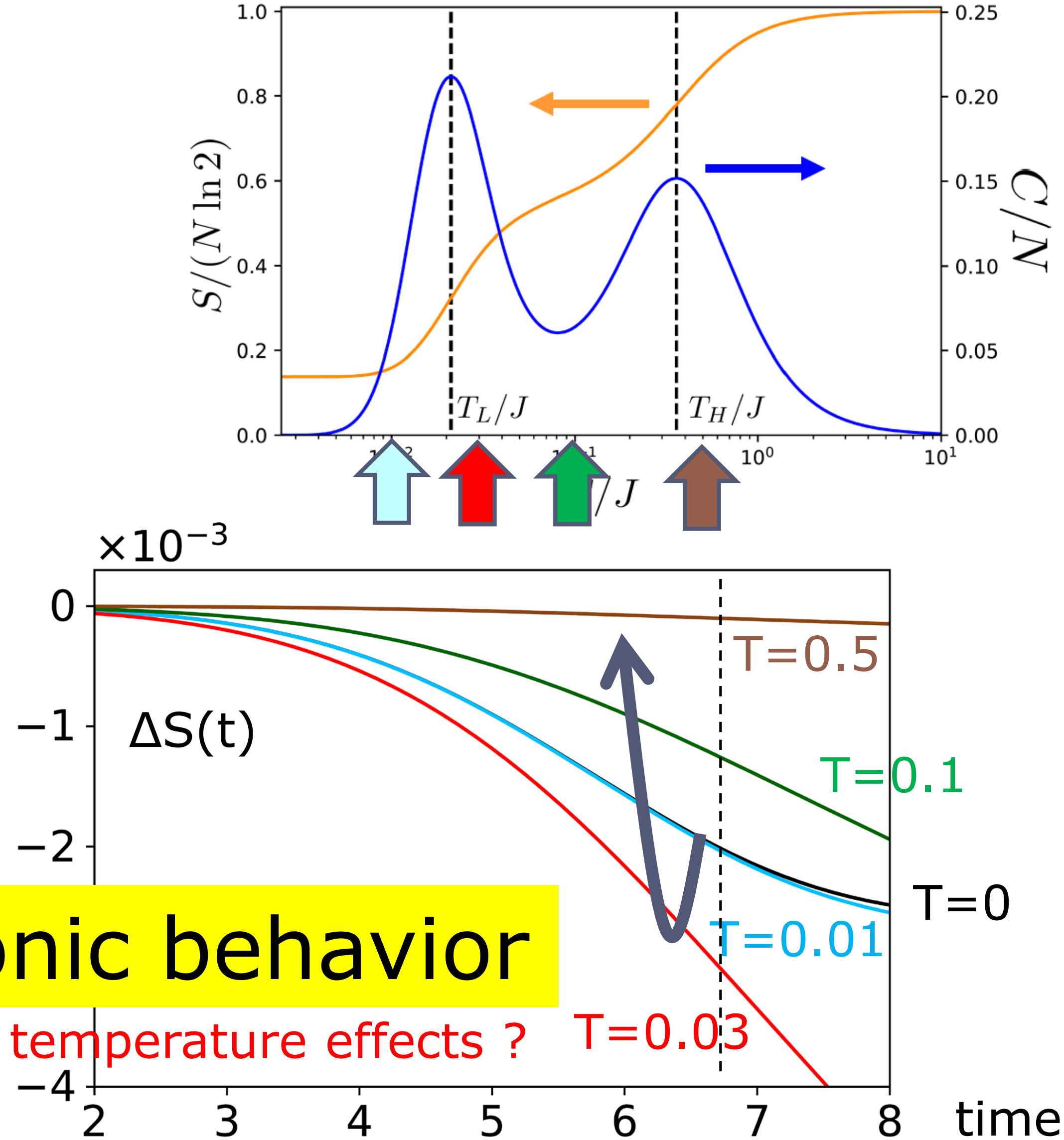
What happens?

J. Nasu et al., Phys. Rev. B **92**, 115122 (2015).

Temperature dependence (TPQ 28sites)

- **R region**



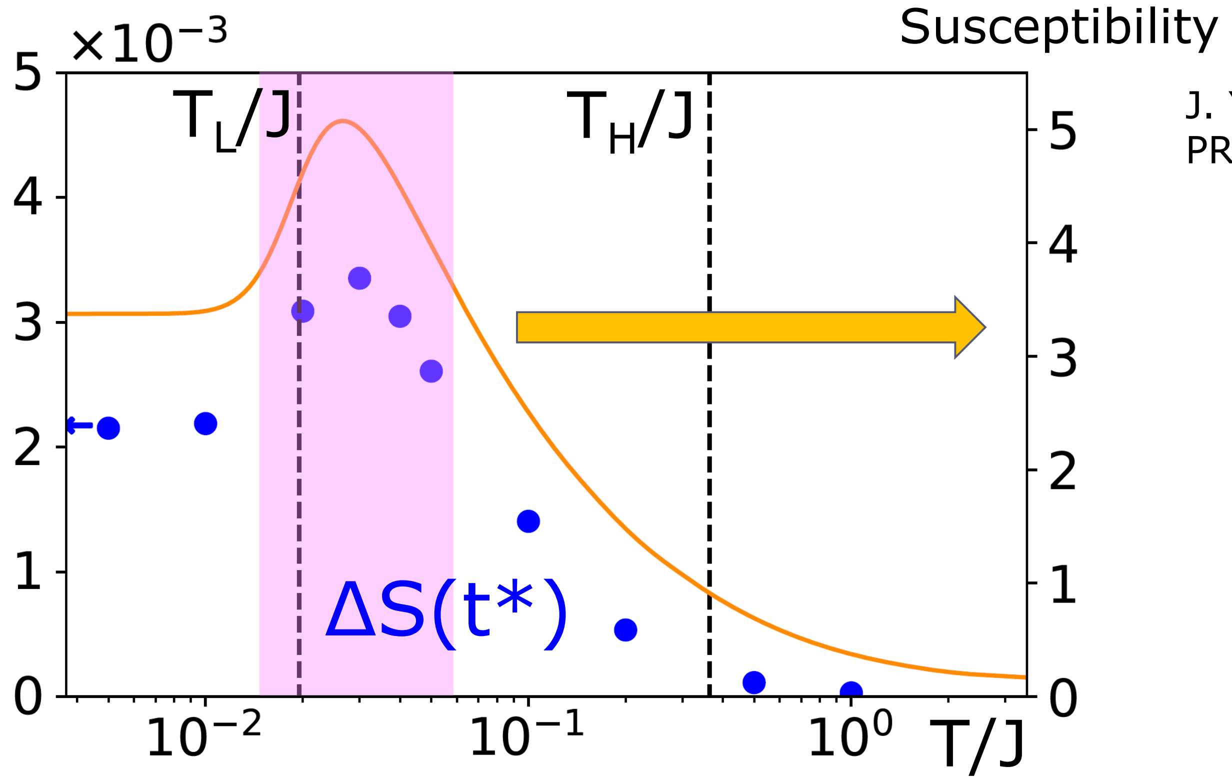


Non-monotonic behavior

cf. simple temperature effects ?

Spin oscillation at finite T

- **Majorana mediated spin transport**



J. Yoshitake et al.,
PRL 117, 157203 (2016).

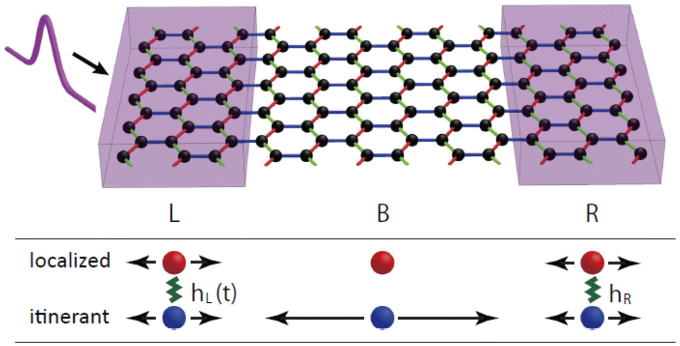
Nonmonotonic behavior ～ Susceptibility

Spin oscillations above T_L larger than those at T=0

H. Taguchi, Y. Murakami & AK, Phys. Rev. B 105,125137 (2022).

Summary

- **Kitaev model**
  - **Transport properties**
    - **Real-time dynamics**
      - **Majorana mean-field theory**
      - **Exact diagonalization**
    - **Majorana-mediated spin transport**
  - **Similar spin transport also found in the S=1 Kitaev model**



cf. Majorana edge current
Y. Kasahara et al., Nature **559**, 227 (2018).

T. Minakawa et al., Phys. Rev. Lett. 125, 047204 (2020).
H. Taguchi, Y. Murakami & AK, Phys. Rev. B 105,125137 (2022).
AK et al., J. Phys. Soc. Jpn. 89, 033701 (2020).
那須譲治、古賀昌久, 固体物理 57, 703 (2022)