カイラル相互作用に基づく現実的殻模型の進展

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7/Dec./2021

Collaborators

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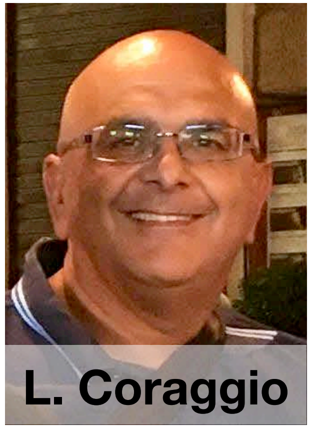
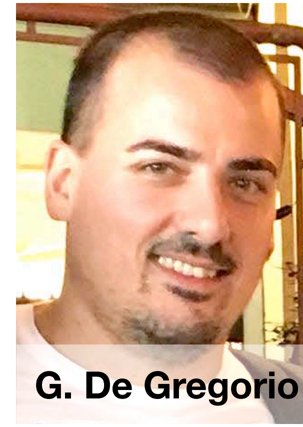
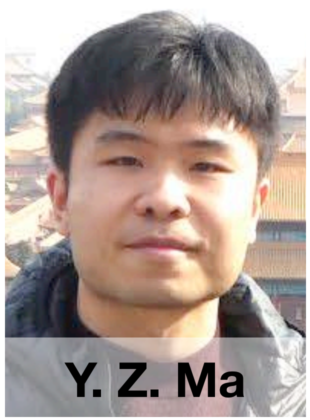




Peking Univ. & South China Normal Univ.



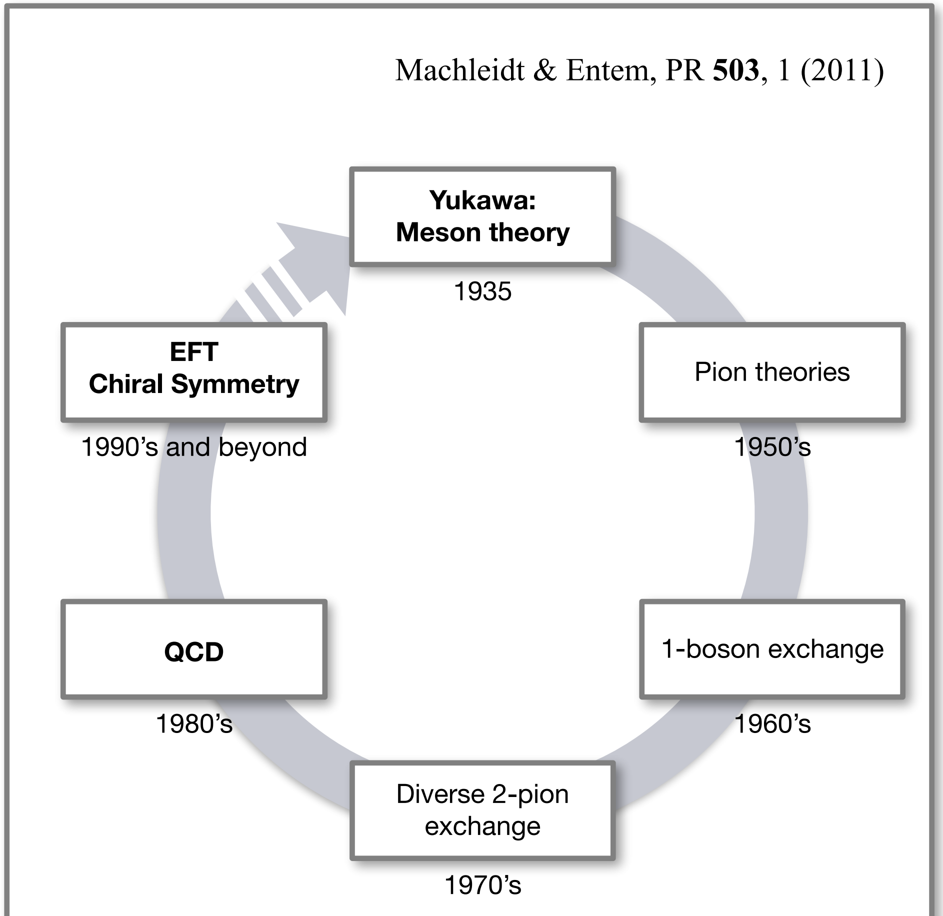
State-of-the-art nuclear force

Motivation

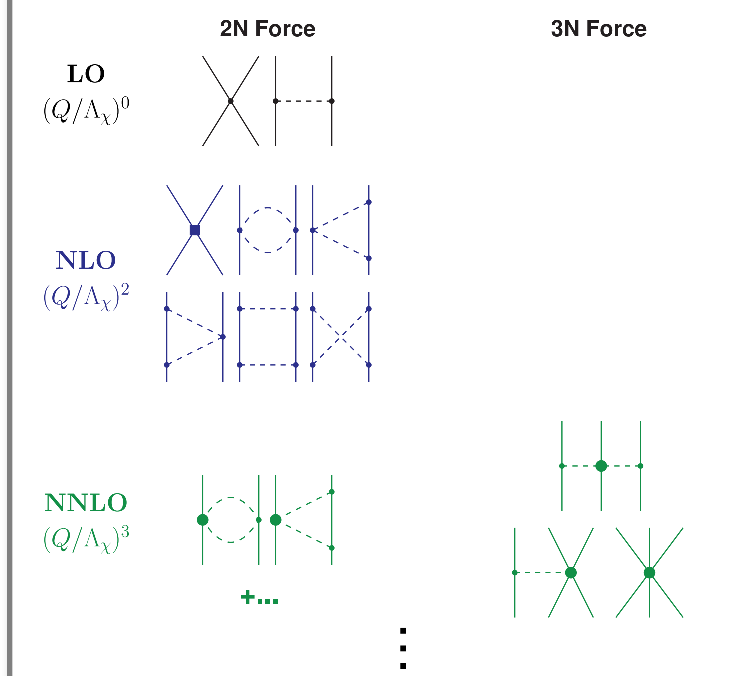
Weinberg, PA **96**, 327 (1979)
Machleidt & Entem, PR **503**, 1 (2011)

Unveil nuclear systems with chiral EFT (πN dynamics)

"circle of history is closing"



Chiral effective field theory



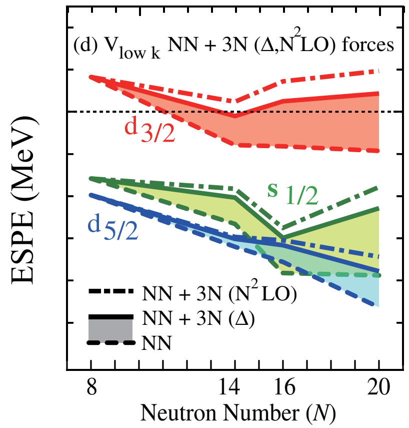
- Hierarchical structure
- Many-body forces
- Realistic force ($\chi^2 \lesssim 1$)

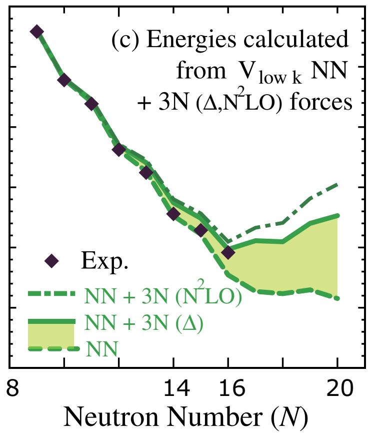
Shell-model studies of chiral 3NF

Oxygen-drip line and 3NF

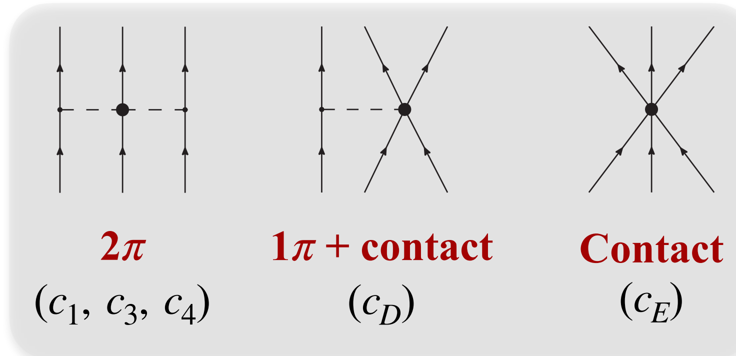
Otsuka +, PRL **105**, 032501 (2010)

The 3NF qualitatively accounts for the oxygen-drip line (^{24}O).





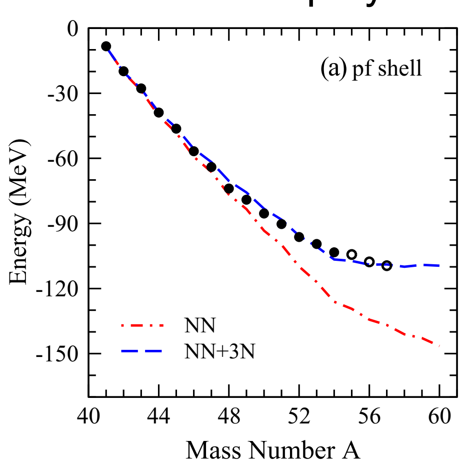
Chiral N^{2}LO 3NF

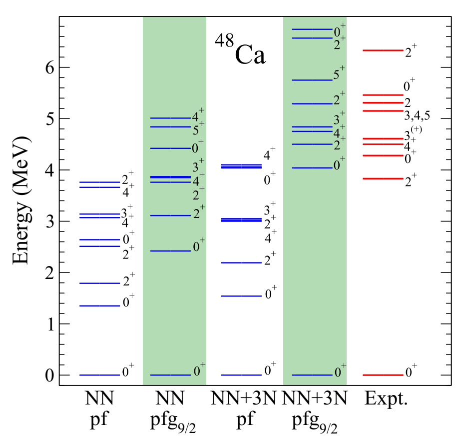


Shell evolution on *pf*-shell

Holt +, PRC **90**, 024312 (2014)

A crucial role played by 3NF for Ca isotopes.





3NF contributions need to be clarified further

In particular the 3NF-magicity relation

Significance

Hierarchical structure
Many-body forces on an equal footing
Precise and hence realistic

I expect to **deepen** and **shed new light on** the understanding of nuclear force and properties of nuclei.

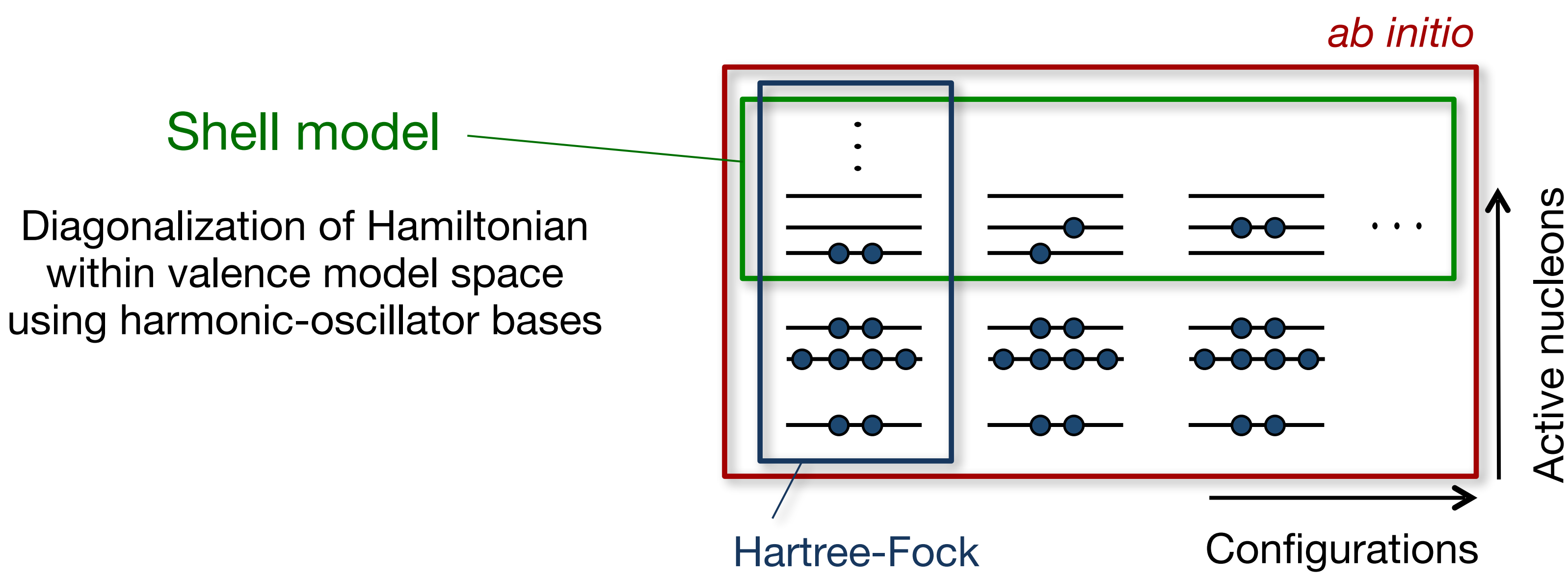
This presentation

1. Spin-orbit splitting and 3NF
2. Drip line of Ca/Ti isotopes
3. Perspectives

Realistic shell model (RSM)

= Shell model with a realistic force

Valence-space diagonalization



Our theoretical framework | Shell-model Hamiltonian

Realistic Hamiltonian (starting point)

$$H = H_{1B} + H_{2B} + H_{3B}$$

- H_{1B}: Single-particle energy
- H_{2B}: Chiral 2NF at N³LO + Coulomb
- H_{3B}: Chiral 3NF at N²LO

3-body matrix elements

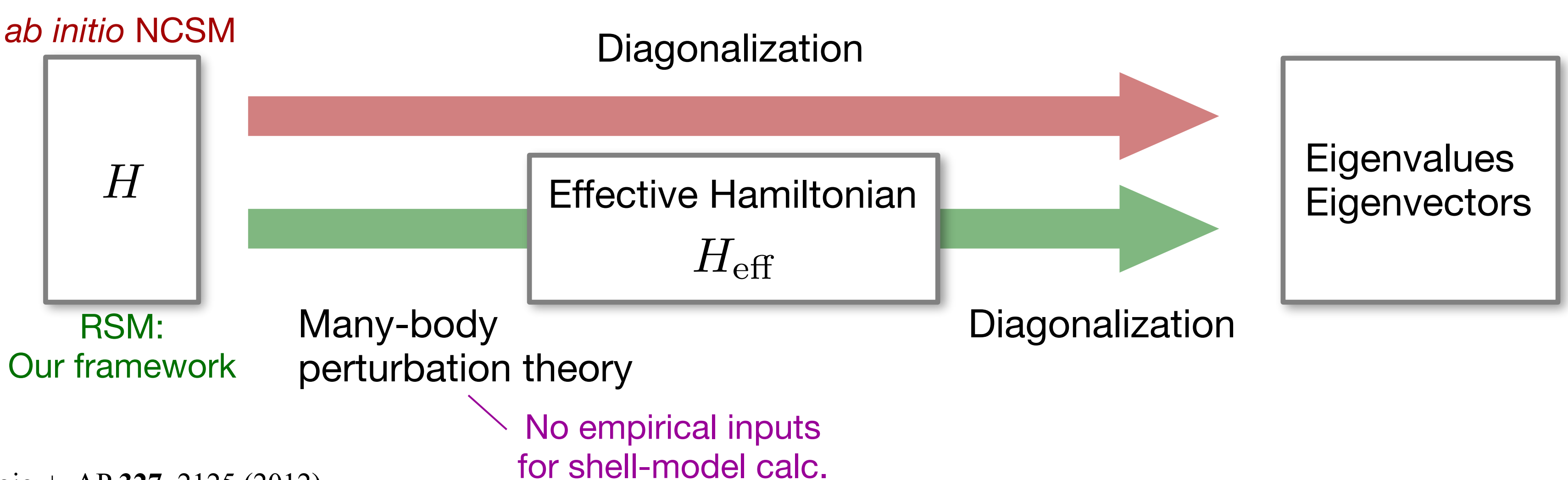
- Our new formalism
- Parallelized code for HPC

Fukui +, PRC **98**, 044305 (2018)

Optional

- Renormalization
- Normal-order approx.

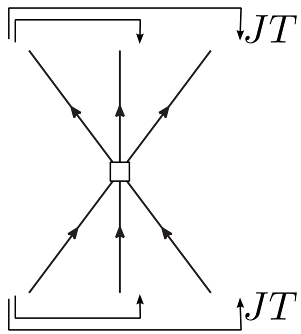
Shell-model framework



Coraggio +, AP **327**, 2125 (2012)

How to compute 3-body matrix elements (3BMEs)

3BMEs



$$= {}_A\Big\langle \big[[\bullet\bullet]\bullet \big]_{JT} \Big| V_{3N} \Big| \big[[\bullet\bullet]\bullet \big]_{JT} \Big\rangle_A$$

Antisymmetrization

Diagonalization of antisymmetrizer

Navrátil +, PRC **61**, 044001 (2000)

Center-of-mass separation

$$\Big| \big[[\bullet\bullet]\bullet \big]_{JT} \Big\rangle_A \rightarrow \Big| \Big[\big|\mathrm{CM}\big\rangle \Big| \;\;\; \Big\rangle_A \Big]_{JT} \Big\rangle$$

Talmi transformation

Talmi, HPA **25**, 185 (1952)
Nogga +, PRC **73** , 064002 (2006)

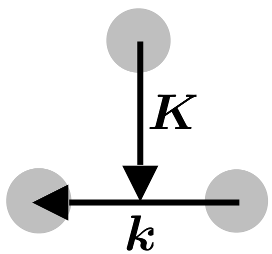
Chiral 3BMEs | Nonlocal regulator

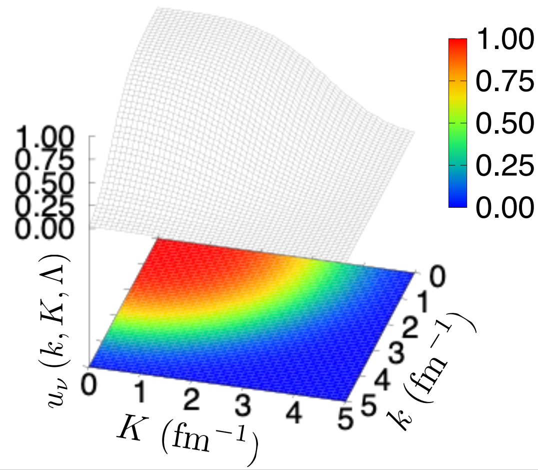
High-momentum truncation by regulator with cutoff Λ

Nonlocal regulator

Epelbaum +, PRC **66**, 064001 (2002)

$$u_\nu(k, K, \Lambda) = \exp\left[-\left(\frac{k^2 + K^2}{2\Lambda^2}\right)^\nu\right]$$



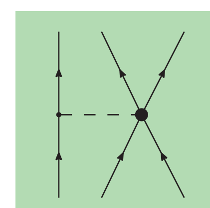


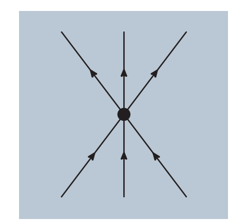
Necessary to retain consistency of 2NF

Nonlocal 3BMEs with HO bases

Pioneering work Navrátil, FBS **41**, 117 (2007)

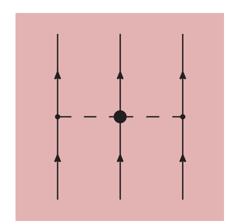
Only for the **1π+contact** and **contact** terms.





Present work Fukui +, PRC **98**, 044305 (2018)

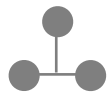
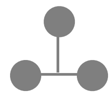
New formalism for **2π terms:**
Triple-fold multipole expansion (brute force method)



Chiral 3BMEs | 2π terms

3BMEs of 2π terms

Fukui +, PRC **98**, 044305 (2018)

$\Big\langle$  $\Big| W_{3N}^{(2\pi)} \Big|$  $\Big\rangle$

$$= \sum_n (\text{coeff.}) \iiiint dk dk' dK dK' g_n(k, k', K, K')$$

23 sums   26 3*nj* symbols, etc. $\propto$ Triple-fold integration

Computationally heavy!

MPI + OpenMP parallelization

MARCONI (CINECA, Italy)

	# of MEs	Time	Memory
***p*-shell**	~800	~30 sec w/ 4 nodes, 48 threads	~500 MB
***sd*-shell**	~20,000	~10 min w/ 60 nodes, 272 threads	~3 GB
***pf*-shell**	~200,000	~5 h w/ 60 nodes, 272 threads	~30 GB

RSM calculations | Numerical details

Low-energy constants ($\Lambda = 500$ MeV)

2NF (N^3LO): Determined from *NN* scattering up to 300 MeV

Entem & Machleidt, PRC **68**, 041001(R) (2003)

3NF (N^2LO): Determined from ^3H- and ^3He binding energies

Navrátil +, PRL **99**, 042501 (2007)

Model space

Standard 1-major shell

(+ a lowest orbit of higher shell if necessary)

Many-body perturbation theory (degenerate)

$$H \rightarrow H_{\mathrm{eff}}$$

2NF: Up to 3rd-order folded-diagram expansion
3NF: Up to 1st-order (normal-order approx.)

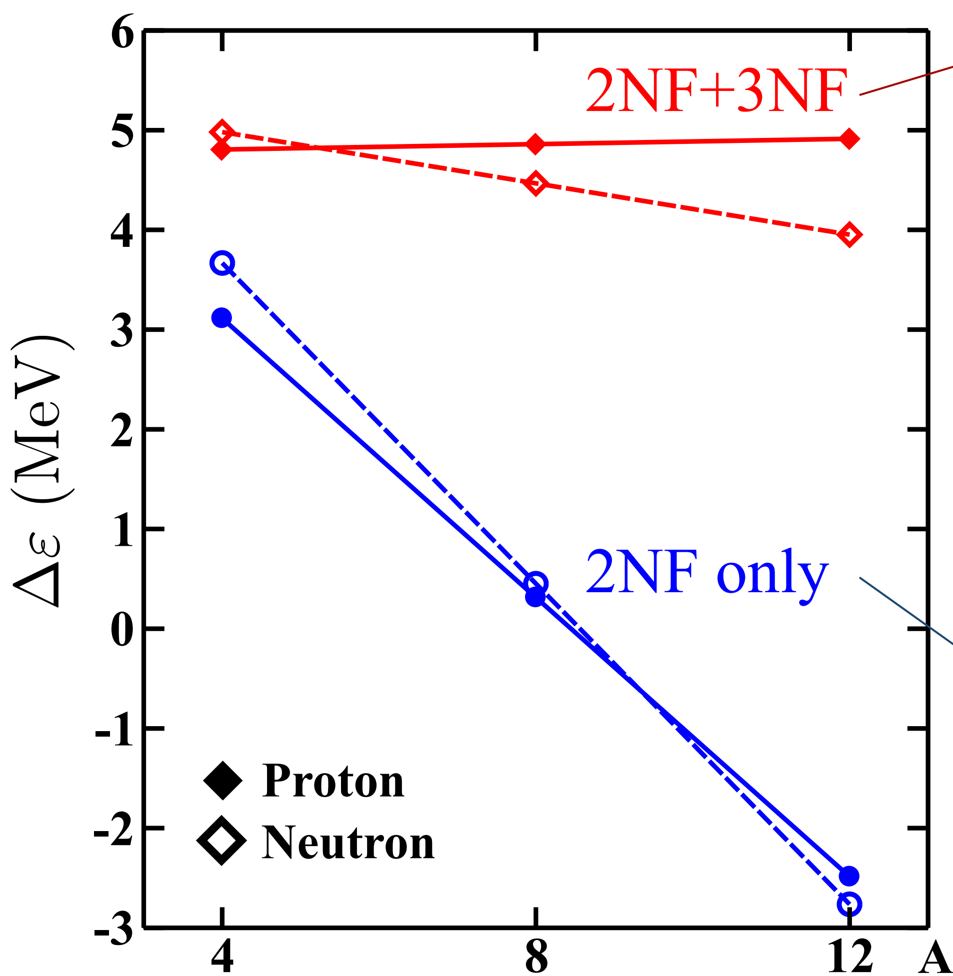
Coraggio + AP **327**, 2125 (2012)
Roth +, PRL **109** , 052501 (2012)

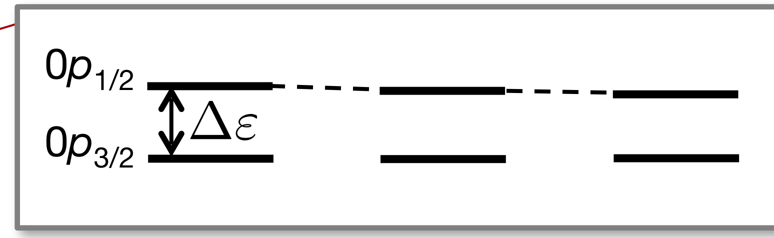
Closure property | *p*-shell nuclei

Effective single-particle energies

Fukui +, PRC **98**, 044305 (2018)

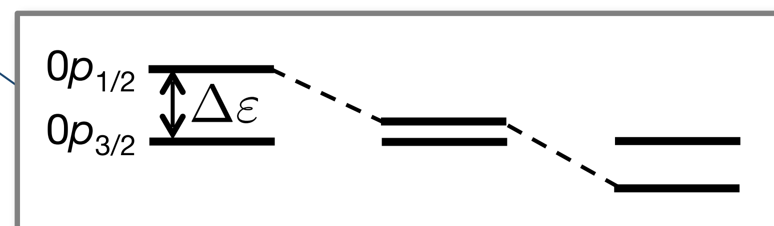
= SPE modified by average 2NF (+3NF)





$\Delta\varepsilon$ is almost constant.

→ **Better closure properties**

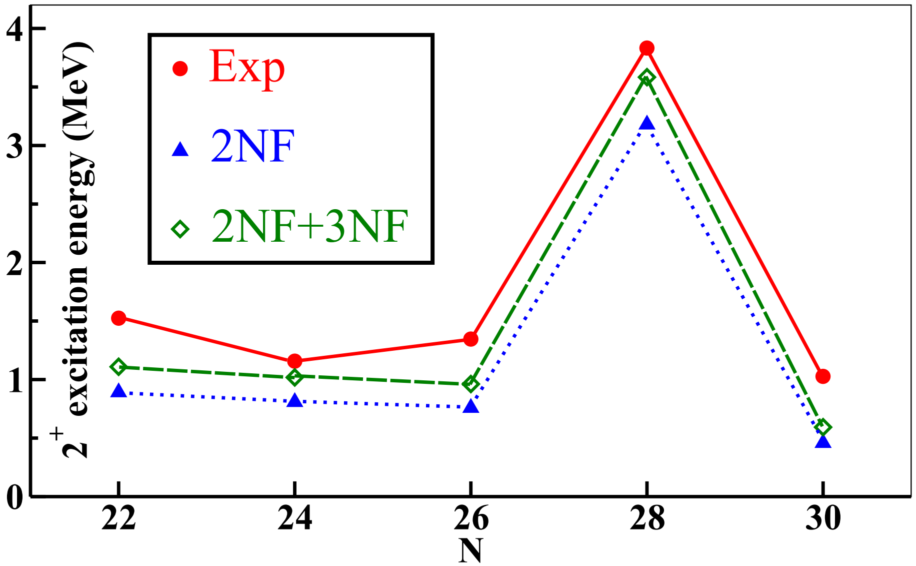


$\Delta\varepsilon$ decreases with A, and the two orbits become inverted.

Closure property | *pf*-shell nuclei

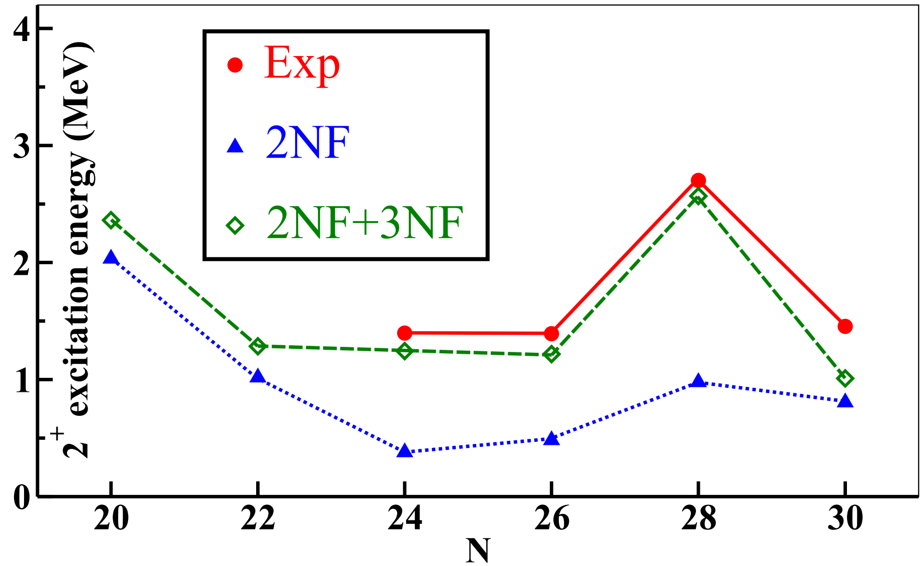
Ca isotopes

- Even **2NF** reasonably accounts for **experimental** behavior.



Ni isotopes

- **2NF** fails but **3NF** plays an important role to explain **experimental** data.



Ma +, PRC **100**, 034324 (2019)

Spin-orbit splitting and 3NF

Why is the 3NF effect drastic in Ni and not in Ca?

Naively…

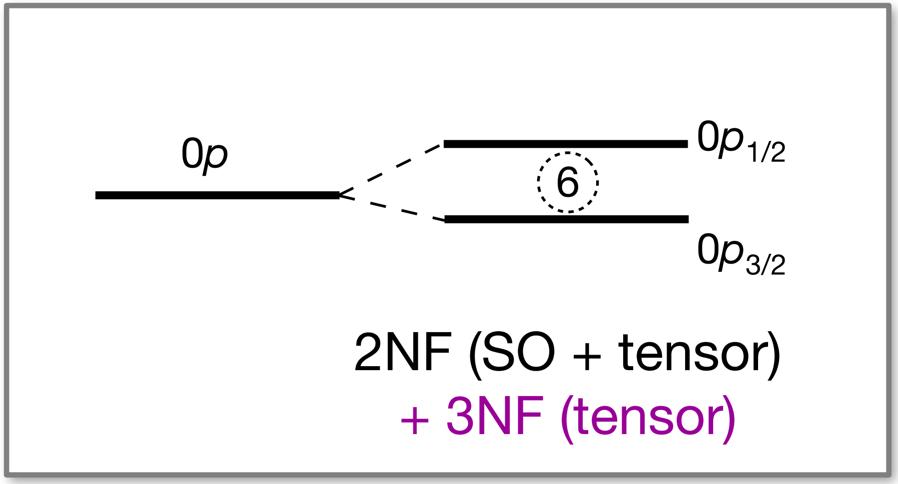
Neutron-proton interaction induced by the 2π term in Ni is more relevant than that in Ca.

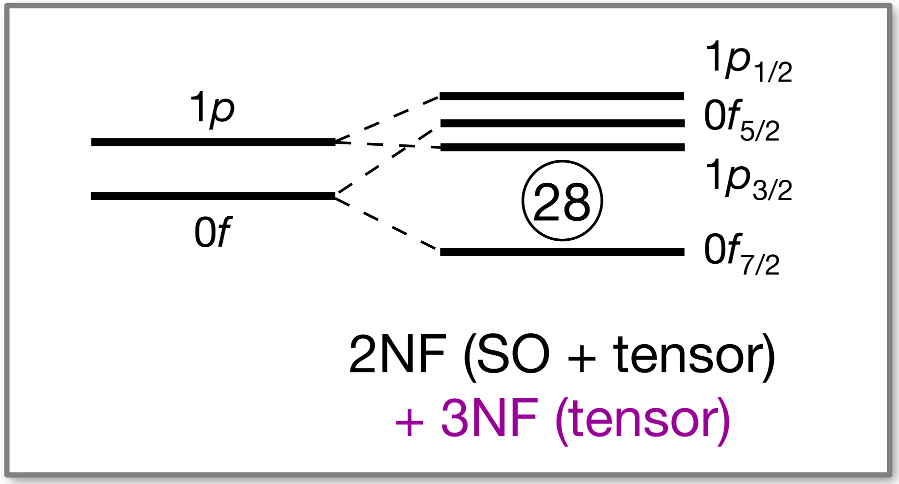
The c_4 term of 2π exchange has the operator

$$[\boldsymbol{\tau}_a \times \boldsymbol{\tau}_b] \cdot \boldsymbol{\tau}_c$$

which vanishes for identical particles.

Spin-orbit splitting stabilized by 3NF





Tensor-force contribution of 3NF: Under investigation

Spin-orbit splitting and 3NF

Why is the 3NF effect drastic in Ni and not in Ca?

Progress of Theoretical Physics, Vol. 17, No. 3, March 1957

Spin-Orbit Coupling in Heavy Nuclei

Jun-ichi FUJITA and Hironari MIYAZAWA

Department of Physics, University of Tokyo, Tokyo

(Received October 27, 1956)

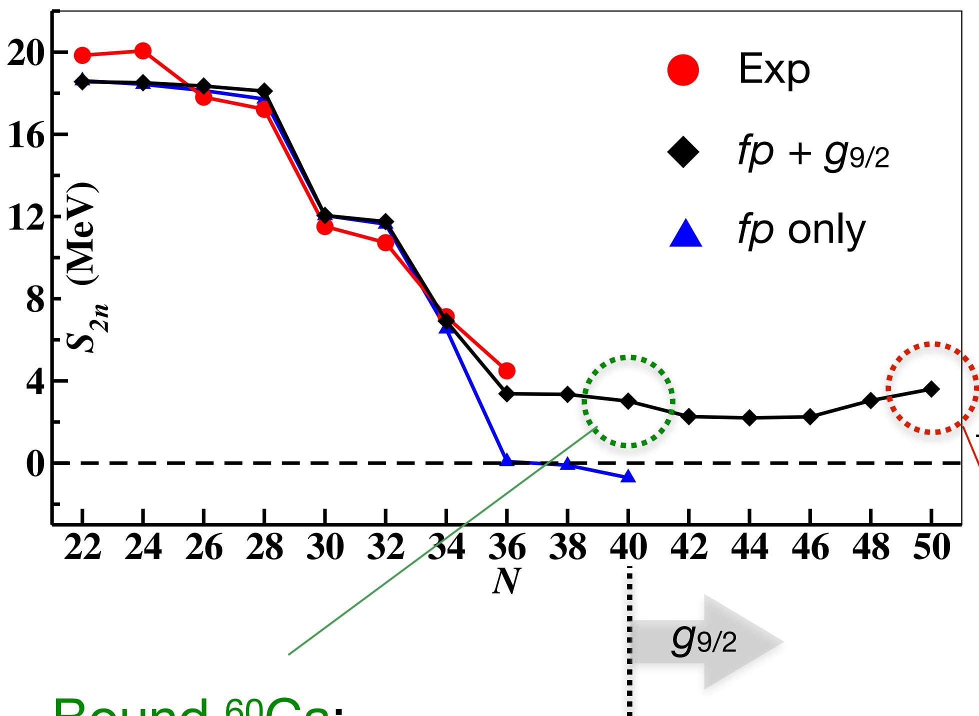
In the preceding paper we have calculated the three-body forces in the static approximation. Using the result a strong spin-orbit coupling, compared with the Thomas term, is derived in this paper. Though it is not sufficient to explain the observed spin-orbit coupling for itself, we expect that a considerable part of the nuclear spin-orbit interaction should be due to the many-body forces.

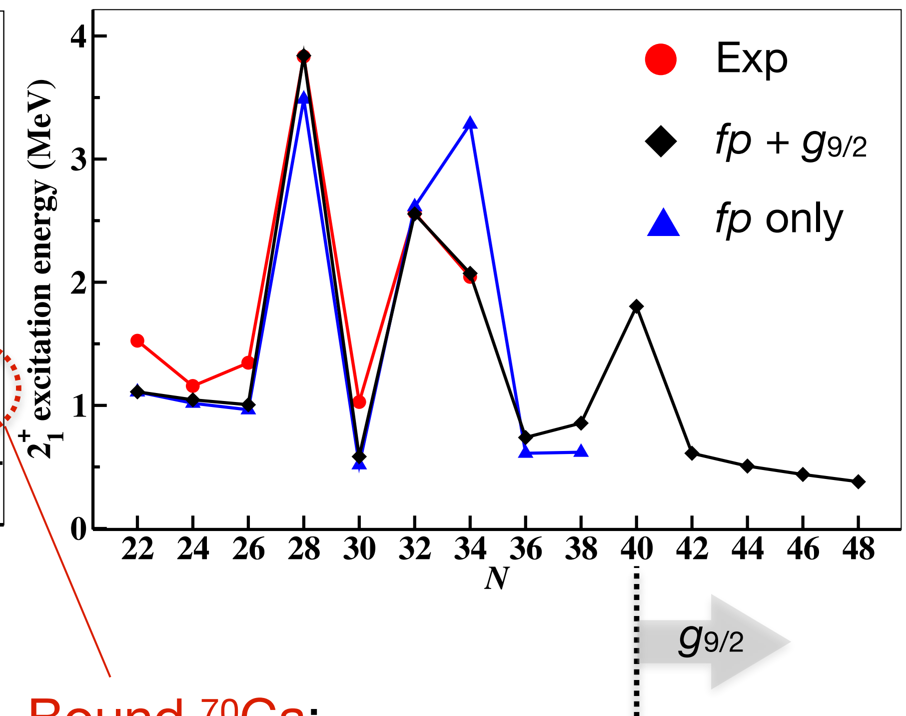
Tensor-force contribution of 3NF: Under investigation

Towards Ca-drip line | Beyond 1-major shell

Ground and low-lying structure

Coraggio+, PRC **102**, 054326 (2020)





Bound ^{60}Ca:
Consistent with experiment

Tarasov +, PRL **121**, 022501 (2018)

Bound ^{70}Ca:
Consistent with other predictions

- Density functional theories
- Bayesian analysis
- Valence-space in-medium SRG

We also predict bound ^{70}Ti.

Coraggio+, Phys. Rev. C **104**, 054304 (2021)

Kortelainen +, PRC **85**, 024304 (2012), Goriely +, PRC **88**, 024308 (2013), Wang +, PLB **734**, 215 (2014)
Neufcourt +, PRL **122**, 062502 (2019), Stroberg +, PRL **126**, 022501 (2021)

My daydream | Physics with Δ explicitly addressed

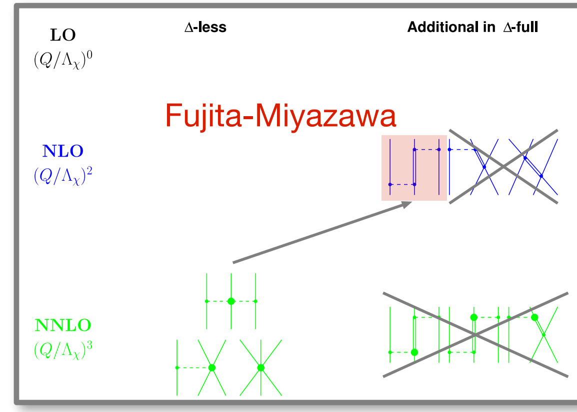
Why Δ isobar?

- **3NF contributes almost everywhere**
  (already shown)

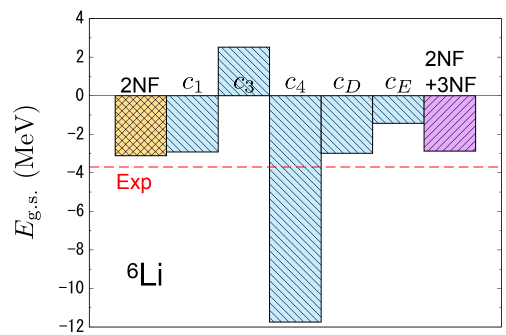
- **3NF is 2π-exchange dominant**
  - In Δ-full chiral EFT, the 2π-exchange 3NF appears at lower order (NLO), separated from the contact terms.
  - Confirmed numerically within the realistic shell model.

Machleidt & Entem, PR **503**, 1 (2011)

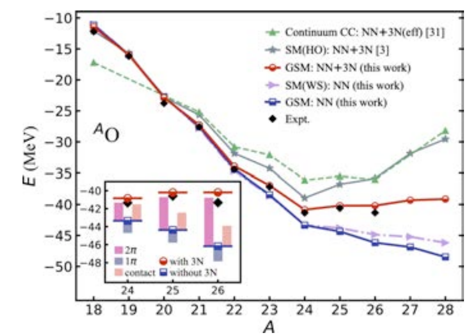
Δ-full-chiral 3NFs



Fukui +, EPJWC **223**, 01018 (2019)

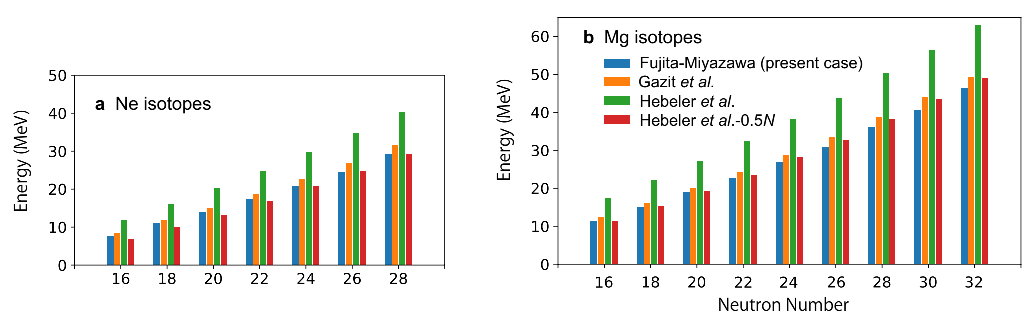


Ma +, PLB **802**, 135257 (2020)



- **Fujita-Miyazawa force can approximate chiral-EFT 3NF**

Fujita & Miyazawa, PTP **17**, 360 (1957)
Tsunoda +, Nature **587**, 66 (2020)



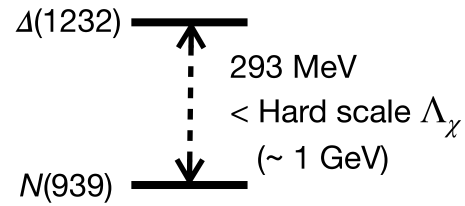
My daydream | Physics with Δ explicitly addressed

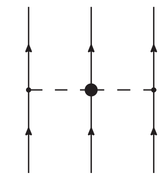
How does Δ contribute?

Δ-full chiral EFT (intermediate st.)

- Δ(1232): Relatively small excitation energy
- Unnaturally large LECs (c_1, c_3, c_4) move to reasonable values
- Improves convergence but contains more

Ordonez +, PRL **72**, 1982 (1994)
Krebs +, EPJA **32**, 127 (2007)



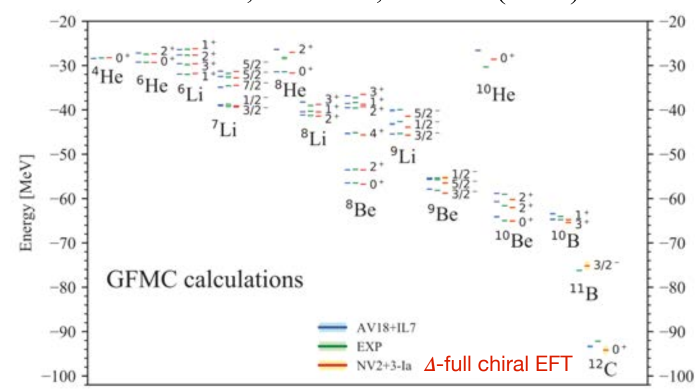


2π (c_1, c_3, c_4)

Spectra of light nuclei

N²LO (2NF+3NF) +N3LO contacts
in quantum Monte Carlo calculations

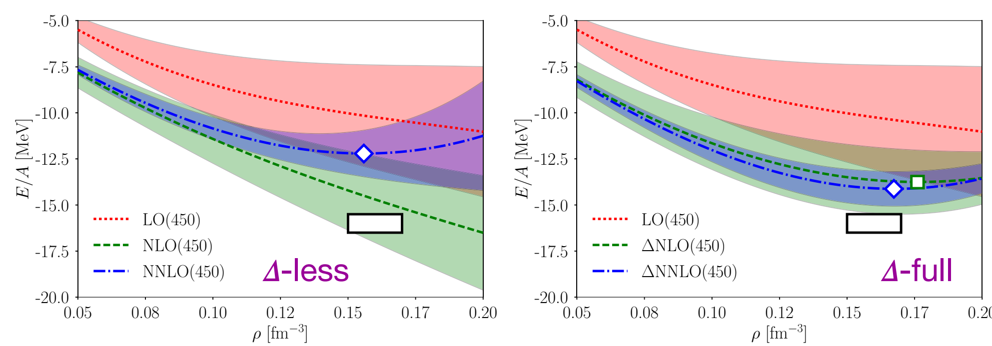
Piarulli +, PRL **120**, 052503 (2018)



Nuclear-matter saturation

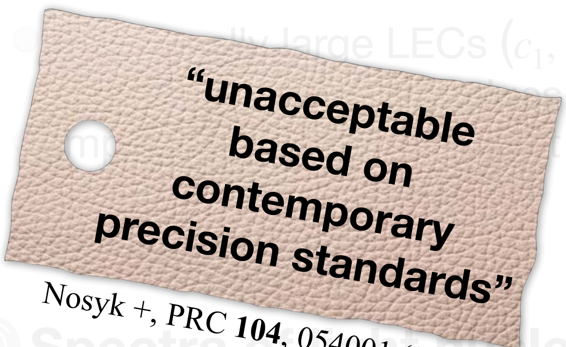
N²LO (2NF+3NF)
in coupled-cluster calculations

Ekström +, PRC **97**, 024332 (2018)

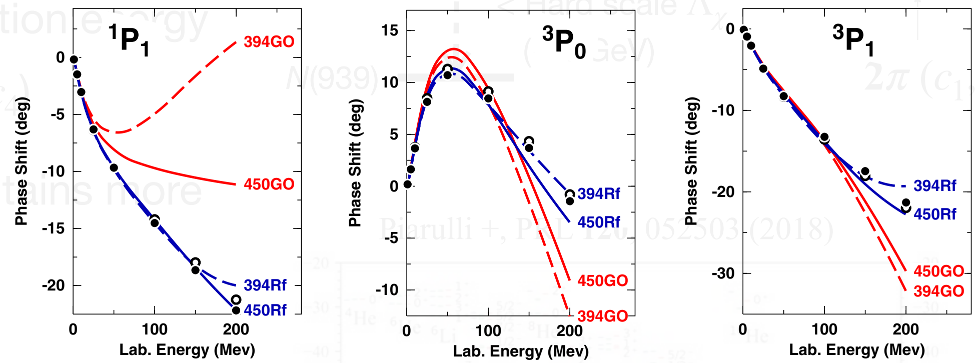


Δ may be relevant!

My daydream | Physics with Δ explicitly addressed



Nosyk +, PRC **104**, 054001 (2021)

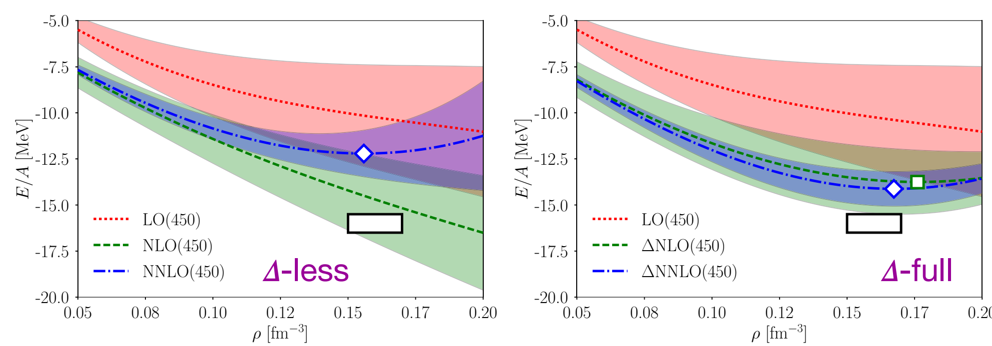


Inaccurate phase shift, and thus, very large χ^2

Nuclear-matter saturation

N²LO (2NF+3NF)
in coupled-cluster calculations

Ekström +, PRC **97**, 024332 (2018)



My daydream | Physics with Δ explicitly addressed

Δ probability per nucleon

Theoretical studies

- Shell model + meson potentials — Horlacher & Arenhövel, NPA **300**, 348 (1978)
  - ^{4}He: a few% (Δ), ~1% ($\Delta\Delta$)
  - ^{16}O: a few% (Δ), < 1% ($\Delta\Delta$)
- Coupled channels/Brueckner + meson potentials — Anastasio +, NPA **322**, 369 (1979)
  - Deuteron: < 1% ($\Delta\Delta$)
  - ^{16}O: a few% (Δ and $\Delta\Delta$)
  - Matter: a few% (Δ and $\Delta\Delta$) increasing with density

Experimental studies

- Δ-knockout (inclusive) from ^{9}Be induced by 1-GeV-proton — Amelin +, PLB **337**, 261 (1994)
  - ^{9}Be: < 1% (Δ)
- $(\pi, \pi p)$ at 500 MeV — Morris +, PLB **419**, 25 (1998)
  - ^{12}C, ^{13}C, ^{90}Zr, ^{208}Pb: a few% (Δ)
- $(\gamma, \pi p)$ at energy up to 1120 MeV — Huber +, PRC, **62**, 044001 (2000); Bystritsky +, JETPL **73**, 453 (2001); Bystritsky +, NPA **705**, 55 (2002)
  - ^{12}C: < 1% (Δ)
  - ^{3}He: a few% (Δ)

My daydream | Physics with Δ explicitly addressed

Δ probability per nucleon

Theoretical studies

- Shell model + meson potentials — Horlacher & Arenhövel, NPA **300**, 348 (1978)
- Cou ... 79)

Exp

- Δ-k ... indu
- $(\pi, \pi$ ...
- $(\gamma, \pi p)$ at energy up to 1120 MeV — Huber +, PRC, **62**, 044001 (2000); Bystritsky +, JETPL **73**, 453 (2001); Bystritsky +, NPA **705**, 55 (2002)

  ^{12}C: < 1% (Δ)

  ^{3}He: a few% (Δ)

Are these percentages large?

Machleidt & Entem, PR **503**, 1 (2011)

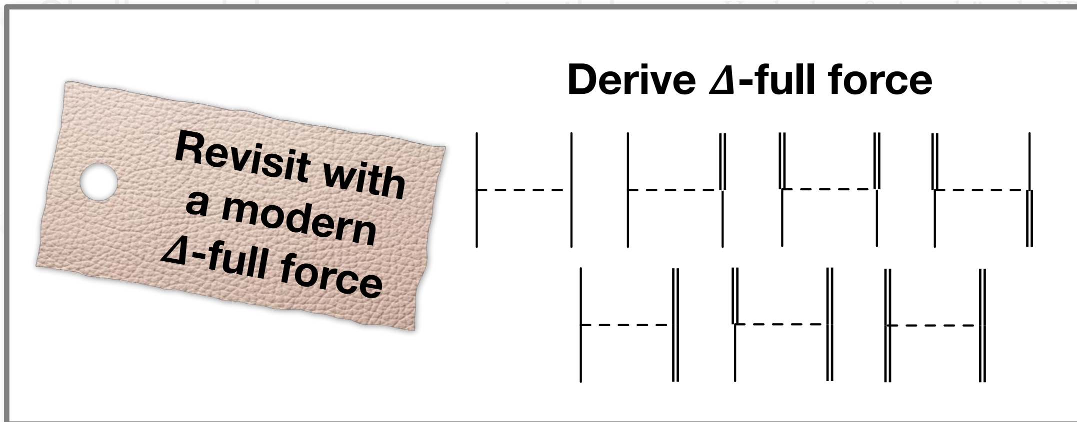
	Idaho N^{3}LO [68] (500)	Juelich N^{3}LO [171] (550/600)	CD-Bonn [13]	AV18 [174]	Empirical[a]
B_d (MeV)	2.224575	2.218279	2.224575	2.224575	2.224575(9)
A_S (fm$^{-1/2}$)	0.8843	0.8820	0.8846	0.8850	0.8846(9)
η	0.0256	0.0254	0.0256	0.0250	0.0256(4)
r_d (fm)	1.975	1.977	1.966	1.967	1.97535(85)
Q (fm^2)	0.275	0.266	0.270	0.270	0.2859(3)
P_D (%)	4.51	3.28	4.85	5.76	

Deuteron D-state probability (a few%):
Crucial tensor contributions

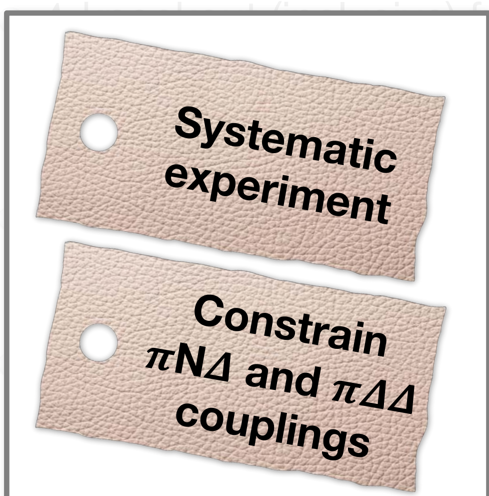
Δ probability of a few% could be large enough.

Δ probability per nucleon

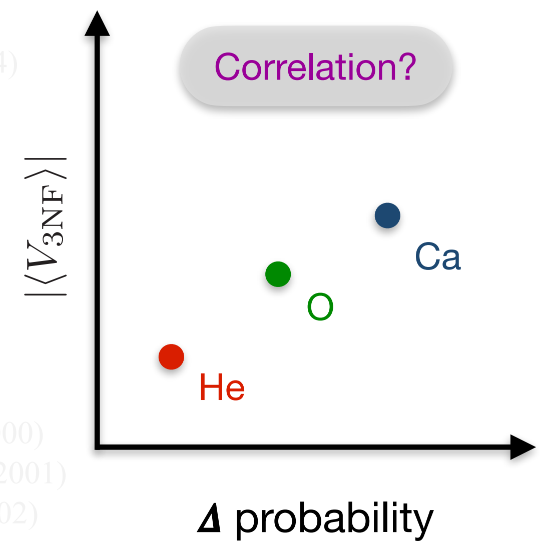
Theoretical studies



Experimental studies



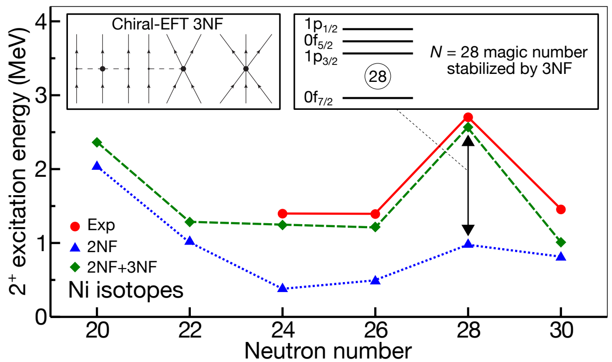
Huber +, PRC, **62**, 044001 (2000)
Bystritsky +, JETPL **73**, 453 (2001)
Bystritsky +, NPA **705**, 55 (2002)



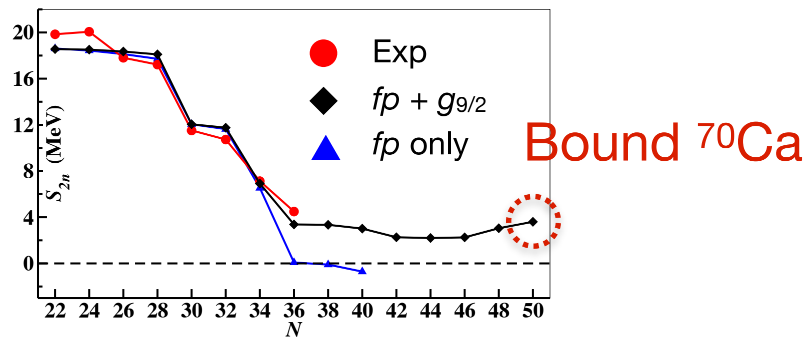
Summary

Realistic shell model with chiral EFT

Spin-orbit splitting stabilized by 3NF



Ca- and Ti-drip line



Daydream

Δ degree of freedom