カルシウム**62, 72**のハロー構造に関する３体模型からの示唆

基研研究会「核力に基づいた原子核の構造と反応」
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京都大学基礎物理学研究所

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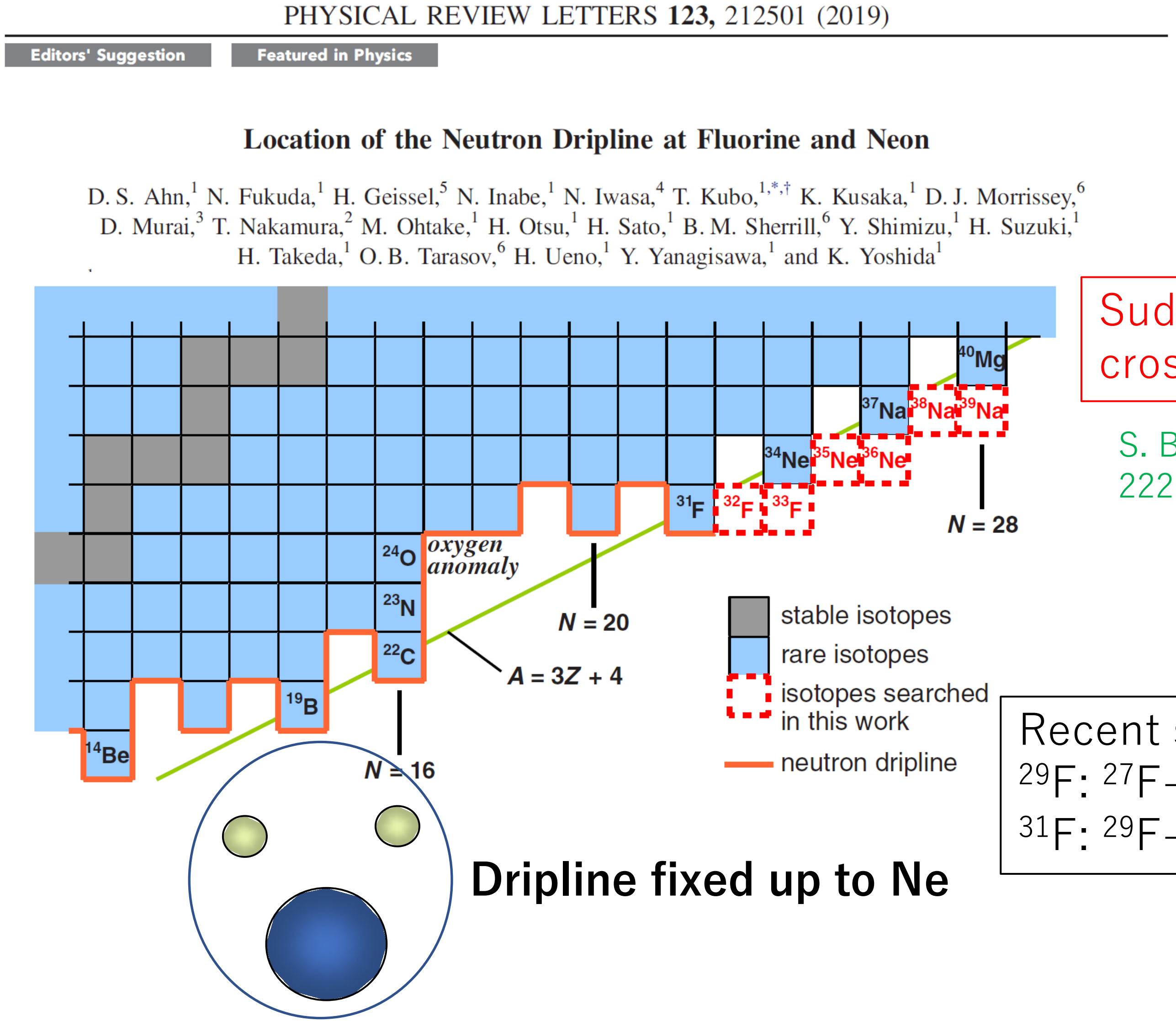
WH, Y. Suzuki, M. A. Shalchi, L. Tomio, arXiv: 2112.12923

Exploring heavier halo nuclei



PHYSICAL REVIEW LETTERS **123,** 212501 (2019)

Editors' Suggestion

Featured in Physics

Location of the Neutron Dripline at Fluorine and Neon

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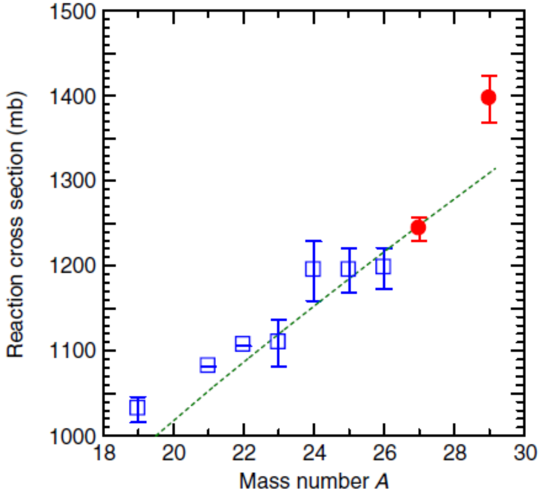
Dripline fixed up to Ne

Halo structure studies: ^{6}He, ^{11}Li, ^{14}Be, ^{19}B, ^{22}C,…

I. Tanihata et al., Phys. Rev. Lett. 55, 2676 (1985)

Sudden increase of interaction cross section is observed at ^{29}F

S. Bagchi et al., Phys. Rev. Lett. 124, 222504 (2020)



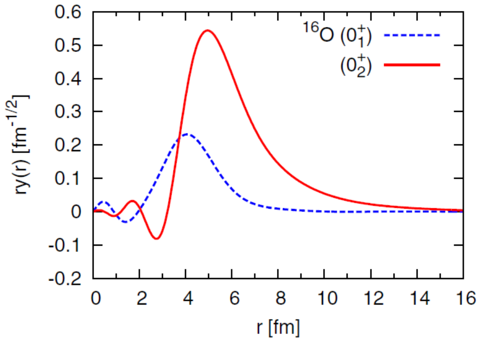
Recent structure studies by three-body models

^{29}F: ^{27}F+n+n J. Singh et al., Phys. Rev. C 101, 024310 (2020)

^{31}F: ^{29}F+n+n H. Masui, WH, M. Kimura, Phys. Rev. C 101, 041303(R) (2020)

Demand of heavier core+few-nucleon models

Core+few-nucleon model with a heavy core



- Use of explicitly correlated basis expansion
  - Limited to light nuclei
    - ^{20}C+n+n: Halo structure of ^{22}C WH, Y. Suzuki, Phys. Rev. C 74, 034311 (2006)
    - ^{12}C+n+n+p+p: alpha clustering in the first excited 0^+ state in ^{16}O

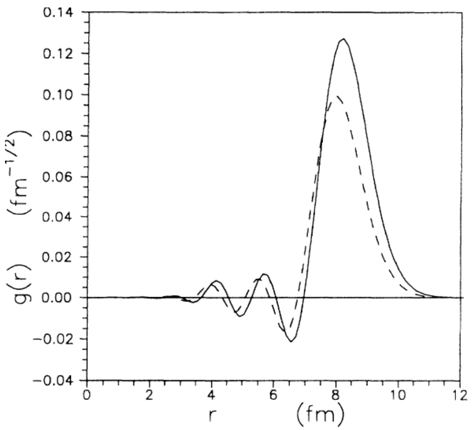
WH, Y. Suzuki, Phys. Rev. C 89, 011304(R) (2014)

Alpha clustering phenomena in Sn isotopes

J. Tanaka et al., Science 371, 260 (2021)

- **Difficulties** for describing nucleon orbits around a heavy core
  - Many forbidden occupied states
  - Correlated nucleon motion far from the origin
    - Halo structure
    - Clustering near the surface of the core

→ the needs for efficiently describing...
- Small (nodal) amplitude in the core
- Large amplitude near the nuclear surface



K. Varga, R.J. Liotta, Phys. Rev. C 50, R1292 (1994)

Purpose of this work

- To establish an efficient way to describe
  - Short-ranged nodal behavior
  - Enhanced amplitude near the nuclear surface

  Ordinary Gaussian → Forbidden-state-Free Locally-Peaked Gaussian (FFLPG)

  $r^l \exp(-ar^2)$ → LPG $r^{2k+l} \exp(-ar^2)$ → FFLPG (present study)

  Y. Suzuki, WH, Phys. Rev. C 95, 044320 (2017)
  Y. Suzuki, Phys. Rev. C 101, 014002 (2020)

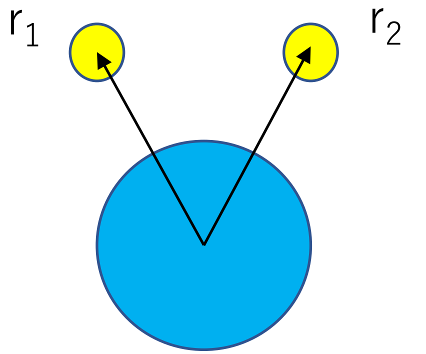
  WH, Y. Suzuki, M. A. Shalchi, L. Tomio, in prep.

- To clarify the condition that the halo structure emerges for $^{62,\,72}$Ca
  - Touchstone nuclei for extremely neutron-rich Ca isotopes
  - **FFLPG: Easy application to various nuclear states**
    - Ground and **excited** states

Dripline of Ca isotopes, **under debate**

- No experimental information for N>40
  - Existence, confirmed up to ^{60}Ca O.B. Tarasov et al., Phys. Rev. Lett. 121, 022501 (2018))
  - Mass measurement up to ^{57}Ca S. Michimasa et al., Phys. Rev. Lett. 121, 022506 (2018)
    - Interaction cross section ^{51}Ca M. Tanaka et al., Phys. Rev. Lett. 124, 102501 (2020)
- Theory
  - Dripline around ^{60}Ca G. Hagen et al., Phys. Rev. Lett. 109, 032502 (2012), G. Hagen et al., Phys. Rev. Lett. 111, 132501 (2013)
    - 2n halo of ^{62}Ca
  - Dripline around ^{70}Ca
    - EDF J. Erler et al., Nature 486, 509 (2012), C. Forssen et al., Phys. Scr. T152, 014022 (2013) and references therein
    - Bayesian analysis L. Neufcourt et al., Phys. Rev. Lett. 122, 062502 (2019)
    - Discussion on Efimov physics in ^{72}Ca D. Hove et al., Phys. Rev. Lett. 120, 052502 (2018)
    - Recent shell model based on chiral int. L. Coraggio et al., Phys. Rev. C 102, 054326 (2020)
    - Recent ab initio calculation S.R. Stroberg et al., Phys. Rev. Lett. 126, 022501 (2021)

Hamiltonian for core+2n system



Core: mass number A

Standard three-body Hamiltonian with cluster-orbital shell model

Y. Suzuki and K. Ikeda, Phys. Rev. C 38, 410 (1988)

$$H = \sum_{i=1}^{2} \underline{(T_i + V_i)} + \underline{\frac{1}{Am}\boldsymbol{p}_1 \cdot \boldsymbol{p}_2} + \underline{v_{12}}.$$

Kinetic, n-core interaction

Recoil term

n-n interaction

Minnesota pot. D.R. Thompson et al. Nucl. Phys. A 286, 53 (1977)

- Variational calculation with non-orthogonal basis expansion $\Psi_{JM} = \sum_{i=1}^{K} c_i \Phi_{JM}(\alpha_i),$
  → generalized eigenvalue problem $\mathcal{H}\boldsymbol{c} = E\mathcal{B}\boldsymbol{c},$

$$\mathcal{H}_{ij} = \langle \Phi(\alpha_i) | H | \Phi(\alpha_j) \rangle\,,\ \mathcal{B}_{ij} = \langle \Phi(\alpha_i) | \Phi(\alpha_j) \rangle\,.$$

"Good" basis needed

Valence neutron orbits **should be orthogonal** to all the occupied (forbidden) states in the core

- Pseudopotential method V.I. Kukulin, V.N. Pomerantsev, Ann. Phys. (NY), 111, 330 (1978)
  - λ value typically $>10^4$ → numerically unstable

$$H \to H + \lambda \sum_{i=1}^{2} P_F(i)$$

- Forbidden-state free basis (This work)
  - **No pseudo-potential is introduced**

Forbidden-state-Free Locally-Peaked Gaussian (FFLPG) approach

- Single-particle (sp) state $\phi^a_{kljm} = \phi^a_{kl}(r)\left[Y_l(\hat{\boldsymbol{r}})\chi_{1/2}\right]_{jm},$
- LPG basis $$\phi^a_{kl}(r) = N_{kl}\left(\frac{a^3}{\pi}\right)^{\frac{1}{4}}(\sqrt{a}r)^{2k+l}\exp\left(-\frac{1}{2}ar^2\right) \qquad N_{kl} = \sqrt{\frac{2^{2k+l+2}}{(4k+2l+1)!!}}$$
- Projection operator $$P_F = \sum_{n'l'j'\in F}\sum_{m'=-j'}^{j'}\left|\psi_{n'l'j'm'}\right\rangle\left\langle\psi_{n'l'j'm'}\right|$$

ψ^ν_{nljm} : Harmonic-oscillator wave function for simplicity

- **FFLPG basis**

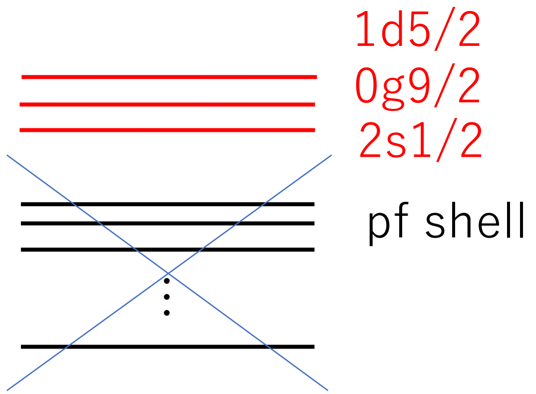
^{61}Ca case (N=40 closed): up to pf-shell (10 Orbits)
^{71}Ca case: additional forbidden 0g9/2 orbit

$$\begin{aligned}\bar{\phi}^a_{kljm} &= (1-P_F)\phi^a_{kljm}\\ &= \phi^a_{kljm} - \sum_{n'\in F}\left\langle\psi^\nu_{n'l}\middle|\phi^a_{kl}\right\rangle\psi^\nu_{n'ljm}\end{aligned}$$

Antisymmetrized two-neutron basis

$$\begin{aligned}&\Phi_{JM}(a_1k_1l_1j_1;a_2k_2l_2j_2)\\ &= \frac{1}{\sqrt{2}}(1-P_{12})\left\{\left[\bar{\phi}^{a_1}_{k_1l_1j_1}(1)\bar{\phi}^{a_2}_{k_2l_2j_2}(2)\right]_{JM}\right\}\end{aligned}$$

Choice of n-^{60}Ca potential



- Phenomenological WS potential

$$V(r) = -V_0 f(r) + V_1 r_0^2 \frac{1}{r}\frac{df(r)}{dr}(\boldsymbol{\ell}\cdot\boldsymbol{s}),$$
$$f(r) = \left[1+\exp\left(r-R_{\mathrm{WS}}\right)/a_{\mathrm{WS}}\right]^{-1}$$

a_{WS}=0.67 fm, R_{WS} =1.27$A^{1/3}$ fm

- Following the theoretical work
  - 2s1/2 energy is set to be -0.01 MeV
  - Vary V_1 to change 0g9/2 energy

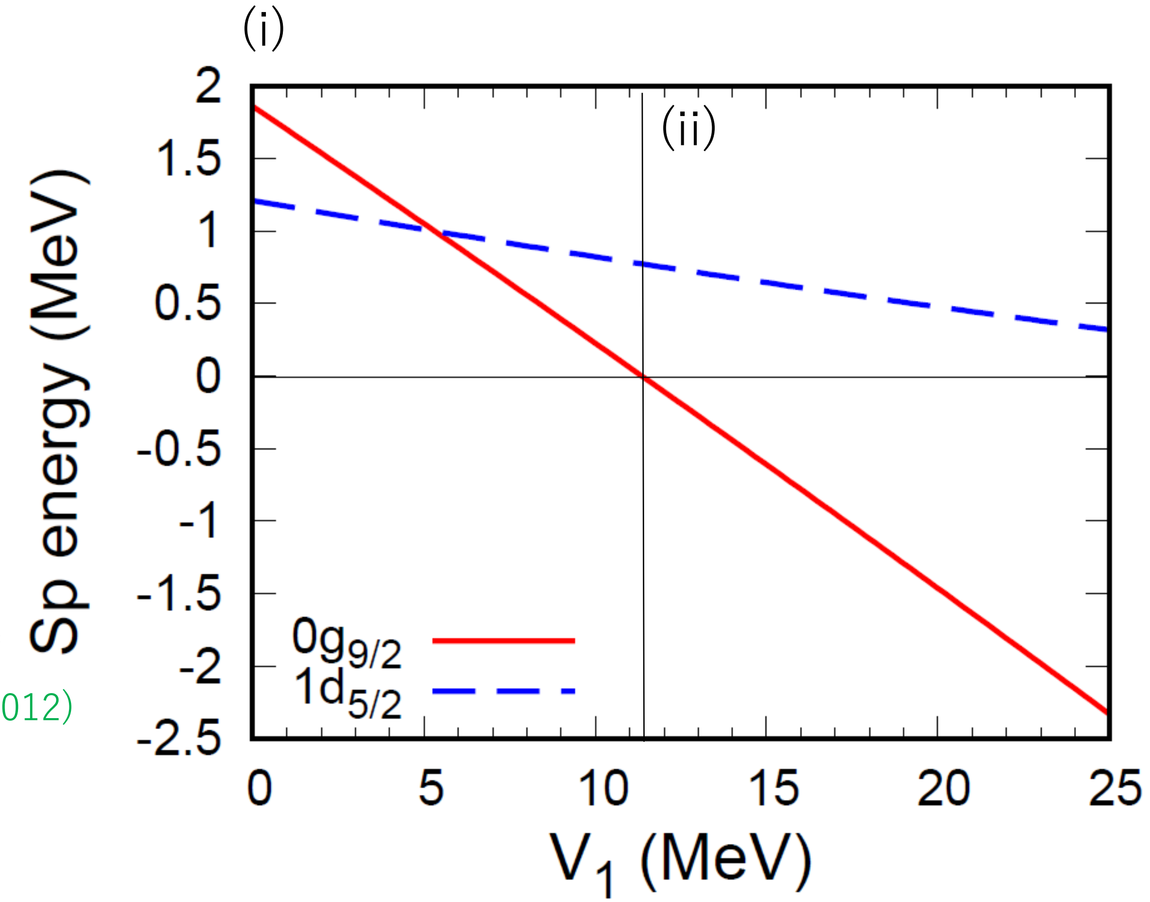
(i)Vanishing spin-orbit limit V_1～0

- Simulates the results of the coupled-cluster*

*G. Hagen et al., Phys. Rev. Lett. 109, 032502 (2012)

(ii)Degenerate sg limit V_1～11 MeV

- ε (0g9/2)～ ε (2s1/2)



Tests of FFLPG basis expansion

- Energy convergence only with $l_1=l_2=0$ ($V_1=0$)

$$\Phi_{JM}(a_1k_1l_1j_1;a_2k_2l_2j_2) = \frac{1}{\sqrt{2}}(1-P_{12})\left\{\left[\bar{\phi}^{a_1}_{k_1l_1j_1}(1)\bar{\phi}^{a_2}_{k_2l_2j_2}(2)\right]_{JM}\right\}$$

Parameters optimized

by the stochastic variational method

K. Varga and Y. Suzuki, PRC52, 2885 (1995).

LPG

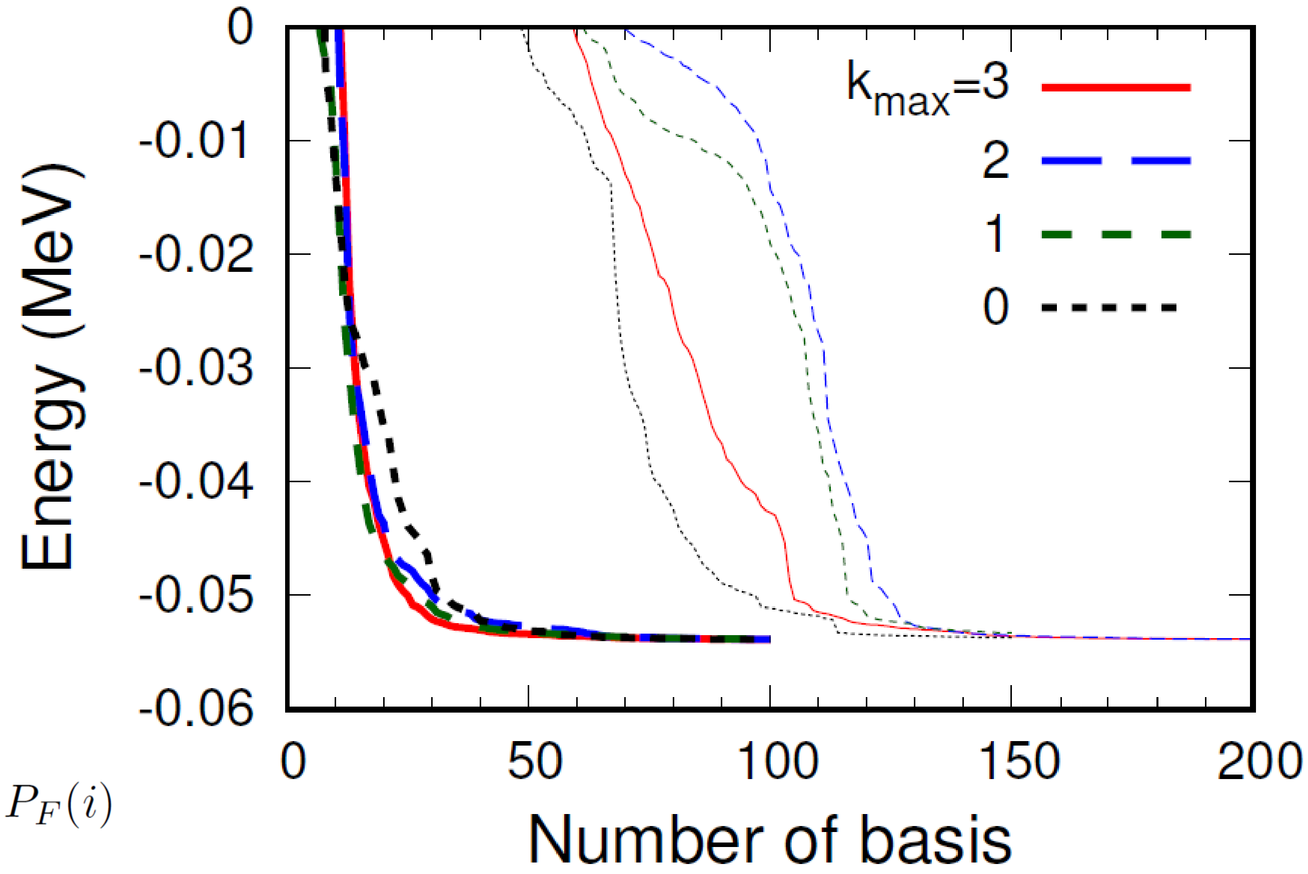
$$\phi^a_{kl}(r) = N_{kl}\left(\frac{a^3}{\pi}\right)^{\frac{1}{4}}(\sqrt{a}r)^{2k+l}\exp\left(-\frac{1}{2}ar^2\right)$$

FFLPG

$$\bar{\phi}^a_{kljm} = \phi^a_{kljm} - \sum_{n'\in F}\langle\psi^\nu_{n'l}|\phi^a_{kl}\rangle\,\psi^\nu_{n'ljm}.$$

Comparison with the LPG basis
using the pseudo-potential method
(Thin curves)

$$H \to H + \lambda\sum_{i=1}^{2}P_F(i)$$

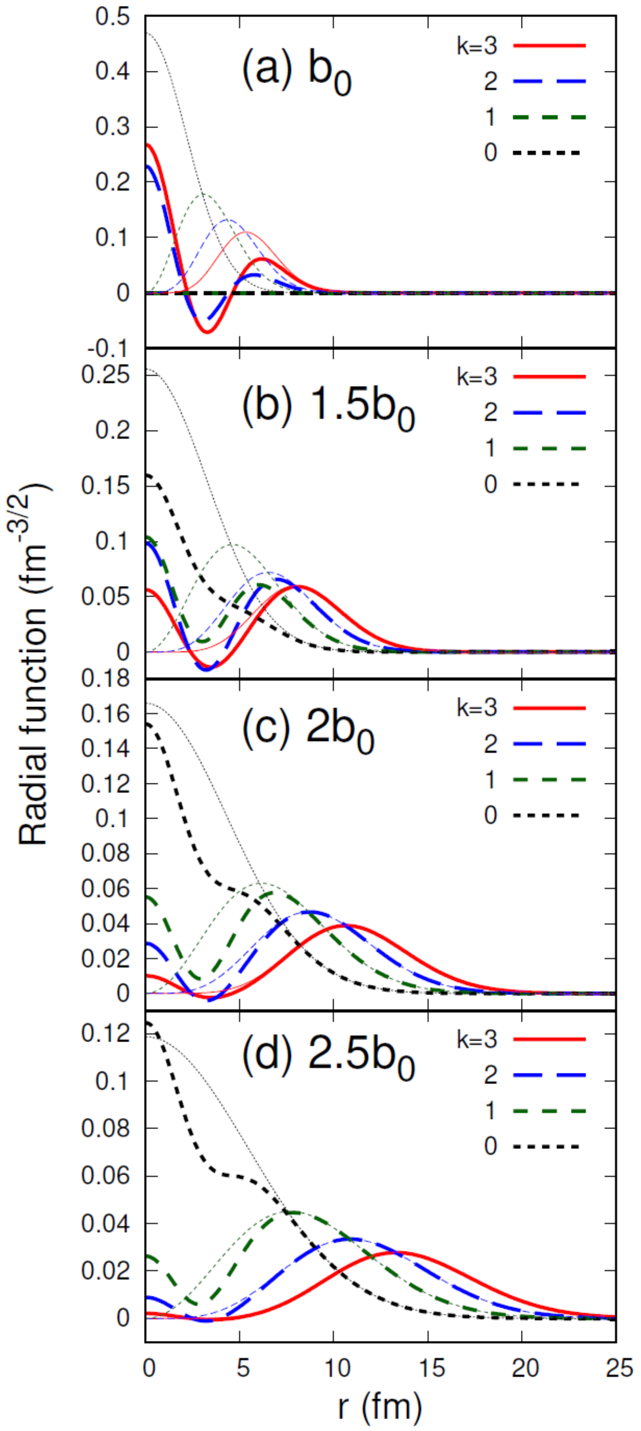


Fast and numerically stable results

FFLPG vs LPG

- $b=b_0(=1/\nu^{1/2})$
  - k=0,1 forbidden states
    - The amplitude is zero
  - k>1, the radial function includes nodes
- $b>b_0$
  - Peak of LPG $r = \sqrt{(2\bar{k}+l)/a}$
  - Radial nodes are taken into account
  - Identical to LPG for large b and k

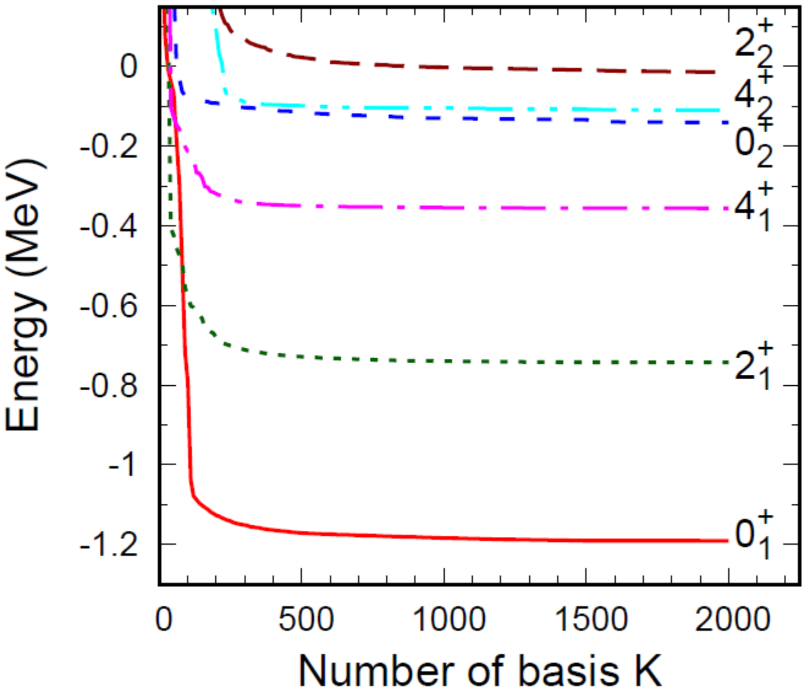
FFLPG: Advantageous to describing large amplitudes near the nuclear surface with short-ranged nodal behavior

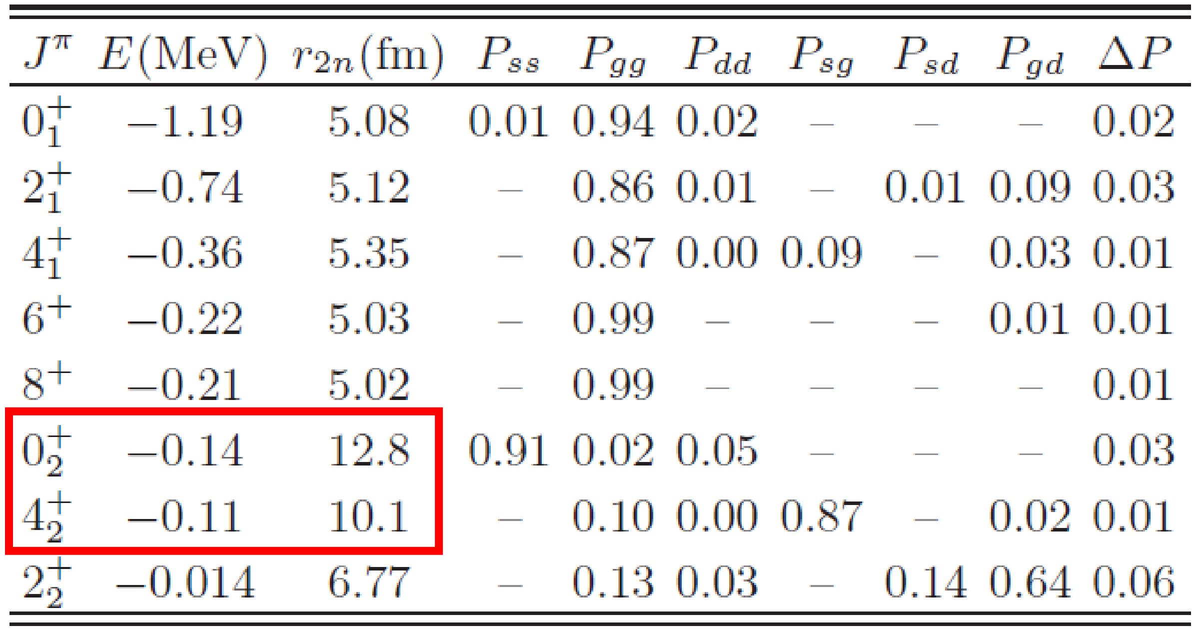


Application to ^{62}Ca

$$\Phi_{JM}(a_1k_1l_1j_1;a_2k_2l_2j_2)$$
$$=\frac{1}{\sqrt{2}}(1-P_{12})\left\{\left[\bar{\phi}^{a_1}_{k_1l_1j_1}(1)\bar{\phi}^{a_2}_{k_2l_2j_2}(2)\right]_{JM}\right\}$$

Stochastically selected within
$b=1/a^{1/2}=[0, 40]$ fm, $l=0$-10, $k=0$-3





J^π	E(MeV)	r_{2n}(fm)	P_{ss}	P_{gg}	P_{dd}	P_{sg}	P_{sd}	P_{gd}	ΔP
0_1^+	-1.19	5.08	0.01	0.94	0.02	–	–	–	0.02
2_1^+	-0.74	5.12	–	0.86	0.01	–	0.01	0.09	0.03
4_1^+	-0.36	5.35	–	0.87	0.00	0.09	–	0.03	0.01
6^+	-0.22	5.03	–	0.99	–	–	–	0.01	0.01
8^+	-0.21	5.02	–	0.99	–	–	–	–	0.01
0_2^+	-0.14	12.8	0.91	0.02	0.05	–	–	–	0.03
4_2^+	-0.11	10.1	–	0.10	0.00	0.87	–	0.02	0.01
2_2^+	-0.014	6.77	–	0.13	0.03	–	0.14	0.64	0.06

P_{xy}: occupation number

8 states are found with degenerate limit
ε (0g9/2)～ ε (2s1/2)

- $(0g9/2)^2$ produces compact $0^+,\cdots,8^+$ states
- 0_2^+: **2n halo** $(2s1/2)^2$
- 4_2^+: **1n halo** $(2s1/2)(0g9/2)$

Coexistence of 2n and 1n halo structure

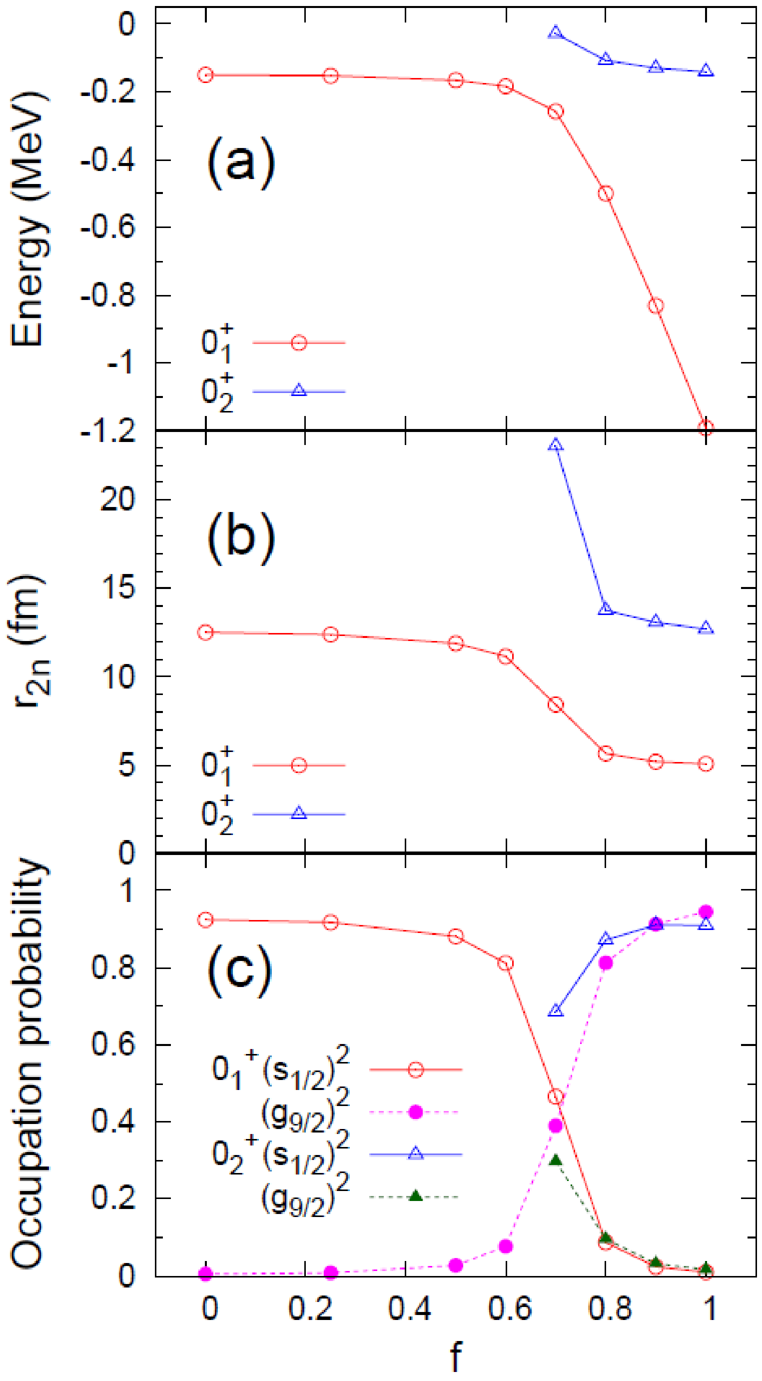
Structure changes

- Vary V_1 as f V_1 with f=[0, 1]
  - f=0: vanishing spin-orbit limit
    - Only two-neutron halo exist
  - f<0.7 level crossing
    - Pairing antihalo effect
      - ε (0g9/2) > ε (2s1/2)
      - $v_{12}[(0g9/2)^2] > v_{12}[(2s1/2)^2]$
  - f=1: degenerate sg limit
    - Nonhalo ground state
    - Two- and one-neutron halo structure coexists

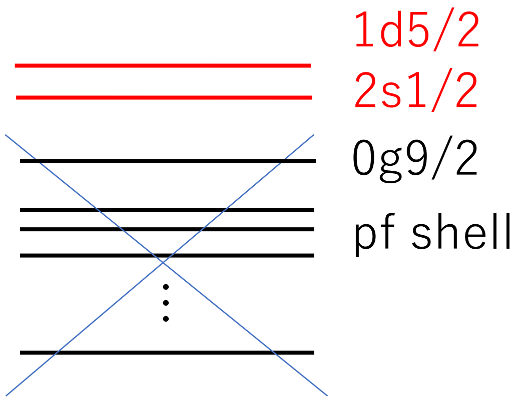
G. Hagen et al., Phys. Rev. Lett. 109, 032502 (2012)

H. Masui, WH, and M. Kimura, PRC101, 041303 (R) (2020)

Experimental information highly desired



Implications for ^{72}Ca



- No information
- Similar model can be considered
  - ε (2s1/2)=-0.01
  - Absence of 0g9/2: Competition between 2s1/2 and 1d5/2 orbits

    If V_1 is strong enough like the degenerate limit
    - **2n halo** of 0_2^+ $(2s1/2)^2$
    - 2^+ and 3^+ doublet **1n halo** states with (2s1/2)(1d5/2) orbits

From this phenomenological three-body model approach

・The degenerate case is unlikely

too strong V_1 (more than 2 times than standard value) is needed to realize it

2n Halo structure emerges if the 2s1/2 orbit is close to the threshold

Summary and prospects

- Forbidden-state-Free Locally-Peaked Gaussian expansion
  - Short-ranged nodal behavior
  - Large amplitude near the nuclear surface
  - Fast and numerically stable convergence
  - **Easy extension to core plus few-nucleon models involving a heavy core nucleus**

- Application to ^{62,72}Ca
  - Competition of 2s1/2 and 0g9/2 (1d5/2) orbits
    - Vanishing spin-orbit limit: Two-neutron halo structure for the ground state
    - Degenerate sg limit: Two- and one-neutron halo structure can coexist in spectrum
  - Implication to possible two-neutron halo structure in ^{72}Ca

- Future applications include
  - ^{212}Po: ^{208}Po+n+n+p+p, improved results than K. Varga, R.J. Liotta, PRC 50, R1292 (1994)
  - ^{104}Te: ^{100}Sn+n+n+p+p
  - Sn isotopes, role of neutron excess J. Tanaka et al., Science 371, 260 (2021)

WH, Y. Suzuki, M. A. Shalchi, L. Tomio, arXiv: 2112.12923