冷中性子物質中における二体、三体アルファクラスター系の構造

Structure of two- and three-alpha systems in cold neutron matter

H. Moriya[1], H. Tajima[2,3], W. Horiuchi[1], K. Iida[3], and E. Nakano[3]

[1]Hokkaido Univ., [2]Univ. of Tokyo, [3]Kochi Univ.

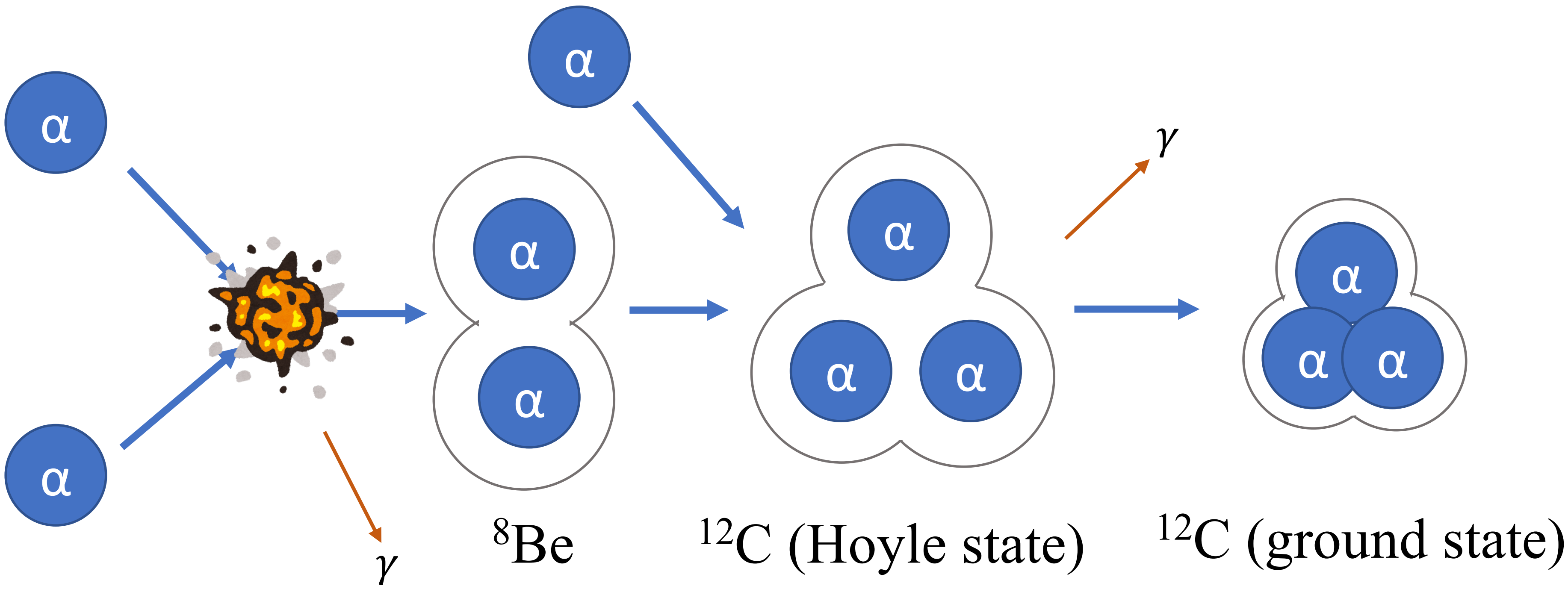
Reference: Phys. Rev. C **104**, 065801

Dec. 9th 2021, 基研研究会 「核力に基づいた原子核の構造と反応」

Alpha clusters in astrophysics

- Alpha clusters (^{4}He nuclei) play an important role in the nuclear structure and the origin of elements
- Particularly, the first 0^+ excited state of ^{12}C, the Hoyle state, is one of the most famous examples F. Hoyle, Astrophys. J. Suppl. Ser. 1, 12 (1954)

Example) Triple alpha process



Three-alpha systems in cold neutron matter

- The accurate description of alpha induced reactions in astrophysical environment can impact on important phenomena

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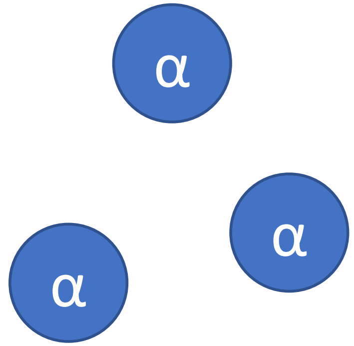
Rev. Mod. Phys. 89, 015007 (2017)



- Those alpha clusters exist <u>in medium</u>

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- We investigate two- and three-alpha systems in medium
  - In thermal plasmas, e.g., X-ray bursting, accreting neutron star

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    - α clusters emerge in a surface region of medium-heavy mass nuclei

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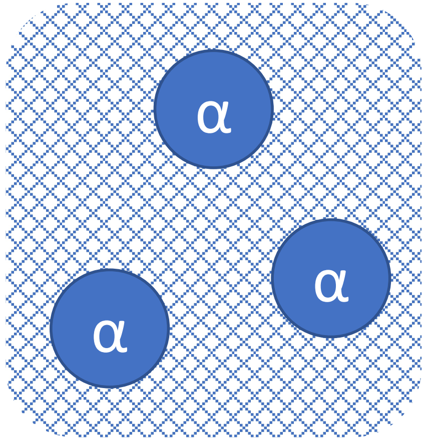

Background medium

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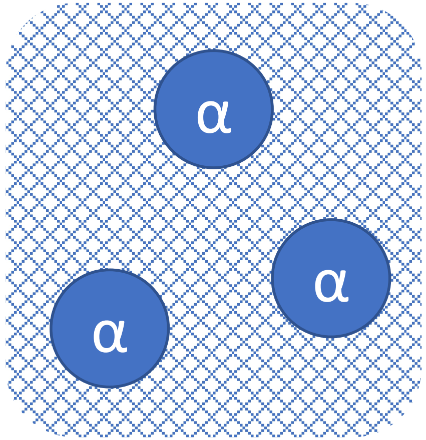

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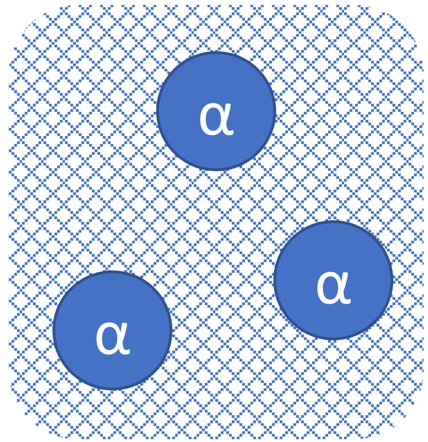
Three-alpha system in a thermal plasmas

- The Coulomb repulsion is screened off at long distances by the surrounding degenerate electrons
- The Coulomb interaction: the Yukawa form

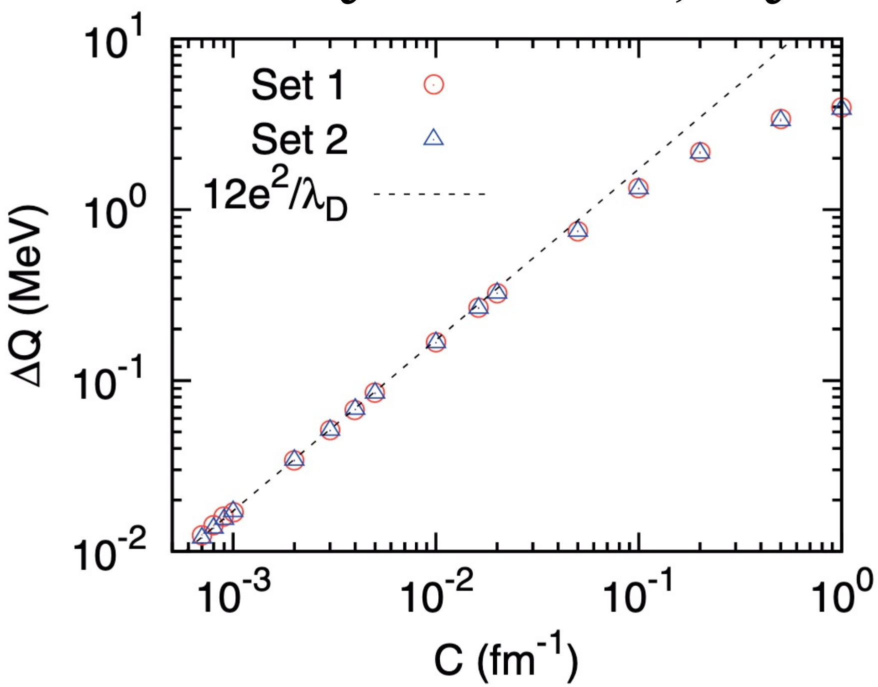
$$V_{ij}^{\mathrm{Coul}} = \frac{4e^2}{r_{ij}} \exp\left(-Cr_{ij}\right).$$

C^{-1}: screening length



Degenerate electrons

- We have performed precise three-alpha calculations and evaluated Q-value shift, ΔQ



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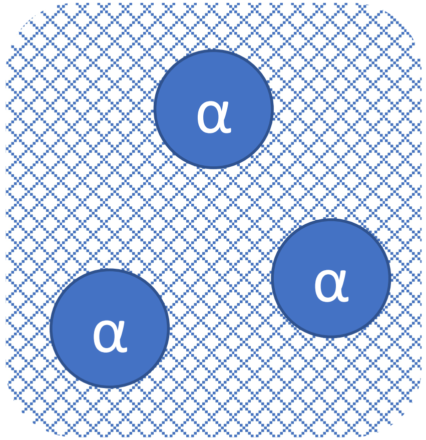

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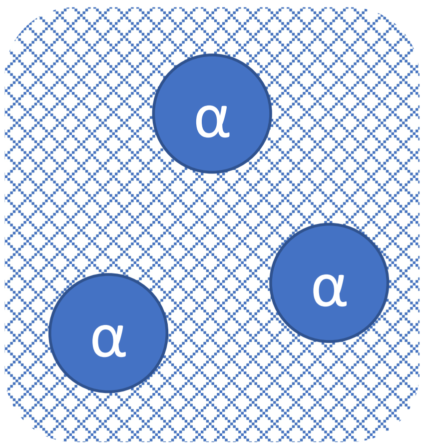
Model & Method

Models of in-medium two- and three-α systems

- The two- and three-α systems in the dilute neutron matter of density lower than about 1/100 of the nuclear saturation density $\sim 0.01\rho_0$ at zero-temperature
  - Neutron gas is in a normal state
  - We ignore neutron-neutron interactions and Pauli principle effect on the structure of α particles by the neutron matter background
  - The zero-temperature approximation can be justified when the temperature is low enough E. Nakano, K. Iida, and W. Horiuchi, Phys. Rev. C 102, 055802 (2020)

- α particles are mobile impurity immersed in the neutron medium



Dilute neutron matter

Our model
- Zero-temperature
- The Pauli principle does not change structure of α particles

In reality…
- Finite temperature
- The Pauli principle affect structure of α particles

Quasiparticle properties of α in a cold neutron matter

- The alpha particles in the neutron matter can be described in terms of Fermi polarons

*F. Chevy and C. Mora, Rep. Prog. Phys. 73, 112401 (2010).
*P. Massignan, M. Zaccanti, and G. M. Bruun, Rep. Prog. Phys. 77, 034401 (2014).
*R. Schmidt, M. Knap, D. A. Ivanov, J.-S. You, M. Cetina, and E. Demler, Rep. Prog. Phys. 81, 024401 (2018).

  – Impurity alpha particles are “dressed” by excitations of majority neutrons via interspecies interactions

An impurity electron in majority atoms

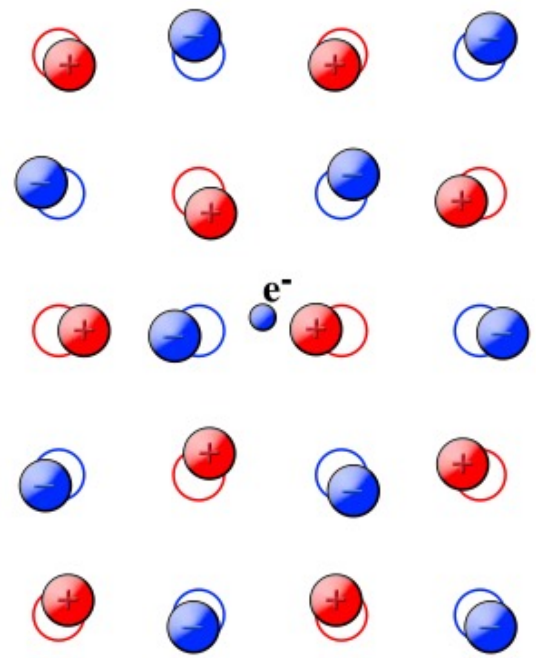
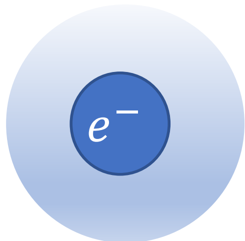


Figure taken from Wikipedia



Electron as a quasiparticle
- effective mass

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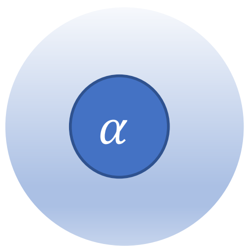
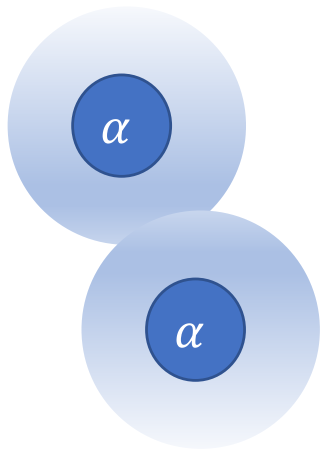
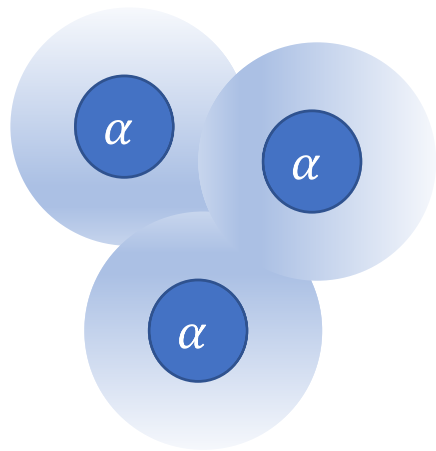


Effective mass M^*



Induced two-α interaction $V_{\mathrm{eff}}^{(2)}$



Induced three-α interaction $V_{\mathrm{eff}}^{(3)}$

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– Impurity alpha particles are "dressed" by excitations of majority neutrons via interspecies interactions

Hamiltonian of the matter

$$H(x) = \sum_s \int \mathrm{d}r^3 \psi_s^\dagger(r) \frac{-\nabla^2}{2m} \psi_s(r) + \frac{1}{2} \sum_{s,t,s',t'} \iint \mathrm{d}r^3 \mathrm{d}r'^3 \psi_{t'}^\dagger(r') \psi_{s'}^\dagger(r) V_{s't'st}(r-r') \psi_s(r) \psi_t(r') - \frac{\nabla_x^2}{2M}$$

$$+ g \sum_s \int_r \psi_s^\dagger(r) \psi_s(r) \delta(r-x)$$

E. Nakano, K. Iida, and W. Horiuchi, Phys. Rev. C 102, 055802 (2020)

<Low energy limit >

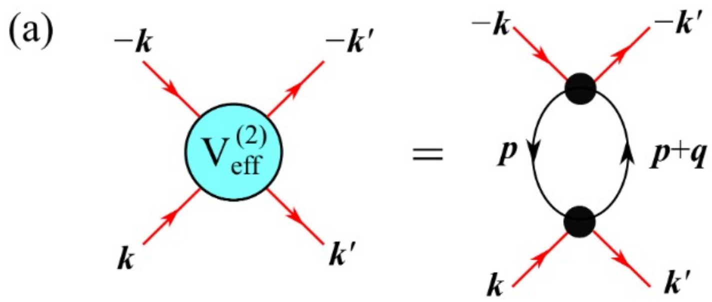
$$g^{-1} = \frac{m_r}{2\pi\hbar^2 a} - \sum_p \frac{2m_r}{p^2}$$

a : $\alpha - n$ scattering length
m_r : reduced mass

Derivation of induced interactions

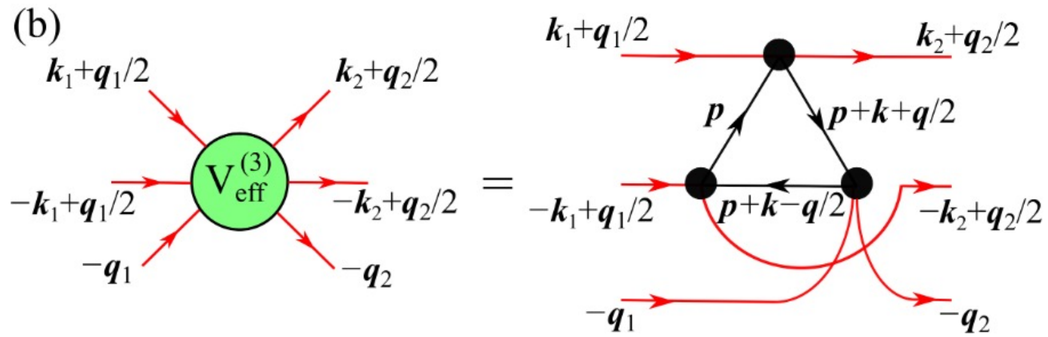
- Diagrammatic derivation of the medium-induced two- and three-body interactions among α particles in a neutron Fermi sea
- Particles exchange their momenta by the induced two- and three-body interactions

Induced two-body interaction



$$V_{\rm eff}^{(2)}(\boldsymbol{q},i\nu_\ell) = -\left(\frac{2\pi\hbar^2 a}{m_r}\right)^2 \times \frac{k_BT}{\hbar^2}\sum_{\sigma=\uparrow,\downarrow}\sum_{\boldsymbol{p},\omega_n} G_\sigma(\boldsymbol{p}+\boldsymbol{q},i\omega_n+i\nu_\ell)G_\sigma(\boldsymbol{p},i\omega_n),$$

Induced three-body interaction



$$V_{\rm eff}^{(3)}(\boldsymbol{k},\boldsymbol{q},i\nu_\ell,i\nu_u) = 2\left(\frac{2\pi\hbar^2 a}{m_r}\right)^3 \times \frac{k_BT}{\hbar^3}\sum_{\sigma=\uparrow,\downarrow}\sum_{\boldsymbol{p},\omega_n} G_\sigma(\boldsymbol{p},i\omega_n)G_\sigma(\boldsymbol{p}+\boldsymbol{k}+\boldsymbol{q}/2,i\omega_n+i\nu_\ell) \times G_\sigma(\boldsymbol{p}+\boldsymbol{k}-\boldsymbol{q}/2,i\omega_n+i\nu_\ell-i\nu_u),$$

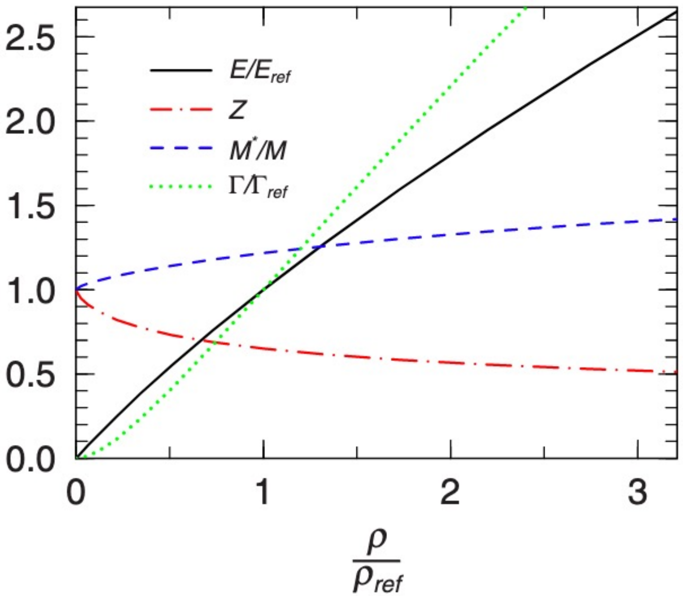
Quasiparticle properties of α in a cold neutron matter

- The alpha particles have in-medium effective mass M^*
- The neutron matter induces two- and three-alpha interactions

$$V_{\rm eff}^{(2)}(r) = \frac{m}{8\pi^3\hbar^2}\left(\frac{2\pi\hbar^2 a}{m_r}\right)^2 \frac{(2k_F r)\cos(2k_F r) - \sin(2k_F r)}{r^4}\,\mathrm{erf}\left(\frac{4}{3}\sqrt{\nu}r\right)$$

$$V_{\rm eff}^{(3)}(R) = \frac{m^2}{\pi^2\hbar^4 k_F}\left(\frac{2\pi\hbar^2 a}{m_r}\right)^3 N_\nu e^{-\frac{16}{9}\nu R^2}$$

M. A. Ruderman and C. Kittel, Phys. Rev. 96, 99 (1954); T. Kasuya, Prog. Theor. Phys. 16, 45 (1956); K. Yosida, Phys. Rev. 106, 893 (1957).



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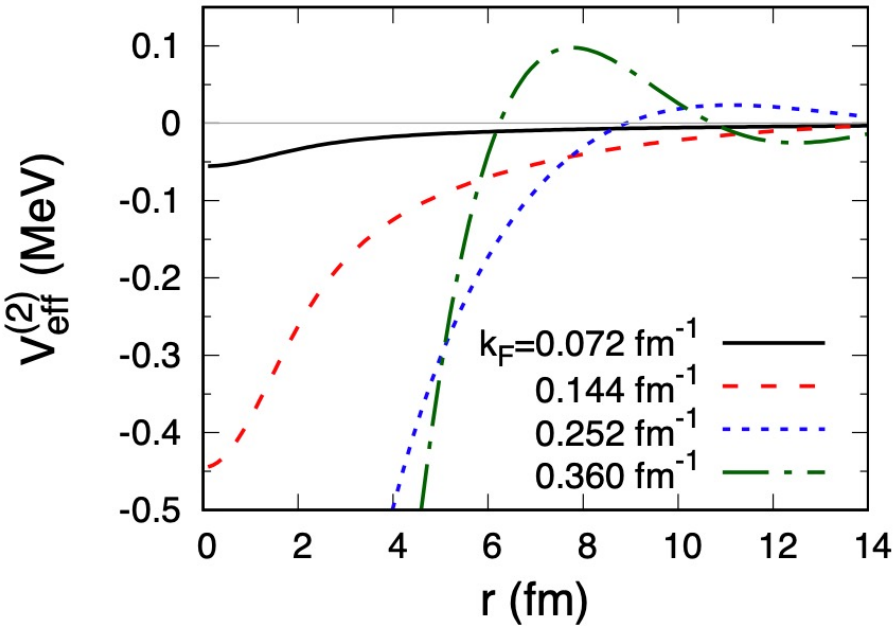

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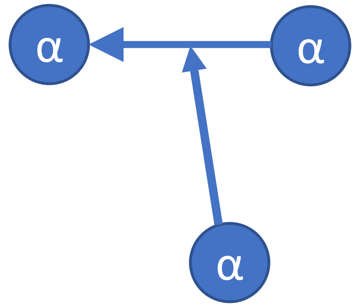
- We employ the orthogonality condition model

$$H = \sum_i \frac{p_i^2}{2M^*} - T_{\rm cm} + \sum_{i>j}\left(V_{\alpha\alpha} + V_{\rm Pauli} + V_{\rm eff}^{(2)}\right) + V_{\alpha\alpha\alpha} + V_{\rm eff}^{(3)}$$



*C. Kurokawa, K. Kato, Phys. Rev. C **71**, 021301 (2005)

*C. Kurokawa, K. Kato, Nucl. Phys. A **792**, 87-101 (2007)

$$V_{\rm Pauli} = \gamma \sum_{nlm\in f} |\phi_{nlm}(ij)\rangle\langle\phi_{nlm}(ij)| \qquad f = \{0S, 1S, 0D\} \qquad \gamma = 10^5$$

*V. I. Kukulin, and V. N. Pomenertsev, Ann. Phys. (N. Y.) **111**, 330 (1978)

Stochastic variational method

- Wave function of the 3α system is obtained by the stochastic variational method with the correlated Gaussian basis

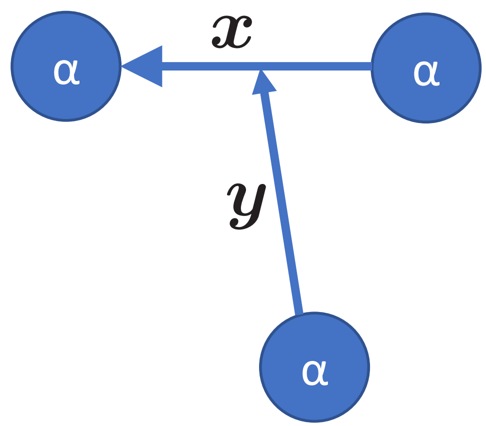
  *K. Varga, and Y. Suzuki, Phys. Rev. C 52, 2885 (1995)
  *Y. Suzuki and K.Varga, *Stochastic Variational Approach to Quantum-Mechanical Few- Body Problems*, Lecture Notes in Physics, Vol. m54 (Springer, Berlin, 1998)

- Correlated Gaussian
  - Correlations between particles are included explicitly
  - Basis functions are fully symmetrized

    *H. Moriya, W. Horiuchi, J. Casal, and L. Fortunato, Few-Body Syst. 62, 46 (2021)
    *Lai Hnin Phyu, H. Moriya, W. Horiuchi, K. Iida, K. Noda, and M. T. Yamashita, Prog. Theor. Exp. Phys. 2020, 093D01 (2020)

$$\Psi_{3\alpha}(\boldsymbol{x},\boldsymbol{y}) = \sum_k c_k \phi_k(\boldsymbol{x},\boldsymbol{y})$$

$$\phi_k(\boldsymbol{x},\boldsymbol{y}) = \mathcal{S}\exp\left(-\frac{1}{2}\begin{pmatrix}\boldsymbol{x} & \boldsymbol{y}\end{pmatrix}\underbrace{\begin{pmatrix}A_{k,11} & A_{k,12}\\ A_{k,21} & A_{k,22}\end{pmatrix}}_{A_k}\begin{pmatrix}\boldsymbol{x}\\ \boldsymbol{y}\end{pmatrix}\right)$$



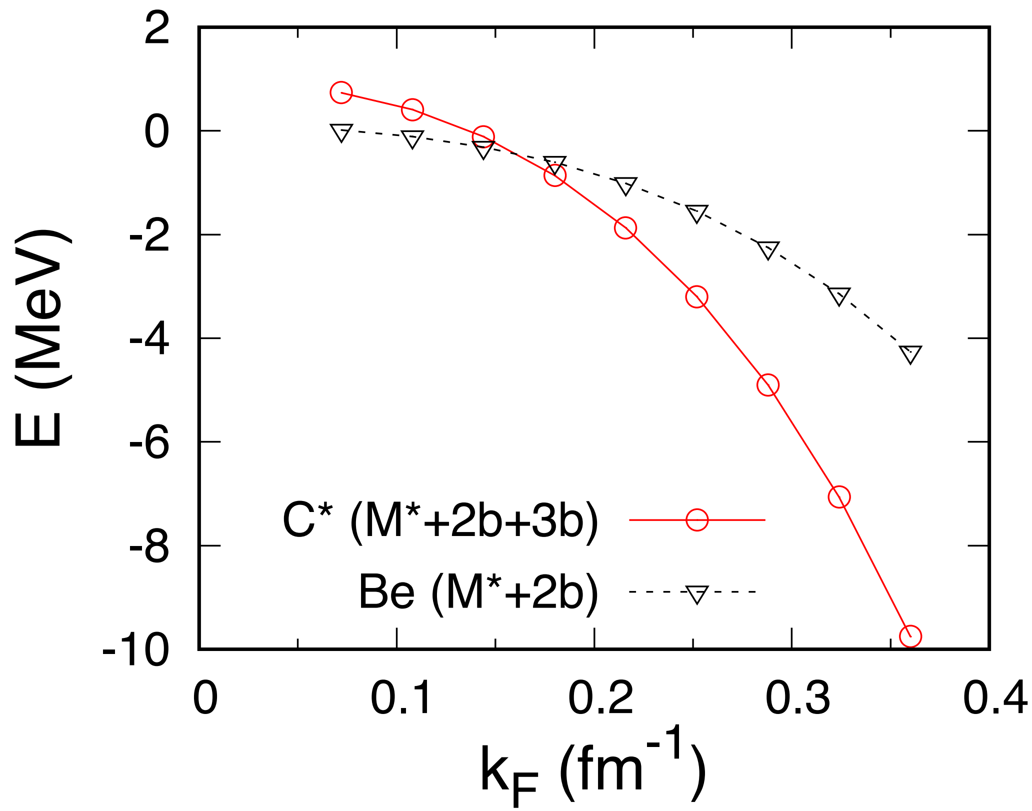
Jacobi relative coordinate

- Determined the variational parameters by solving generalized eigenvalue problem

Results

^{8}Be and Hoyle state become bound states

- ^{8}Be and Hoyle state become bound in the neutron matter with $k_F \geq 0.08$ fm^{-1} and $k_F \geq 0.16$ fm^{-1} respectively
- This would be a significant impact on the modeling of matter in stellar collapse and neutron star mergers M. Oertel, M. Hempel, T. Klaˈhn, and S. Typel, Rev. Mod. Phys. 89, 015007 (2017)
  - Those results would change reaction rates for nucleosynthesis



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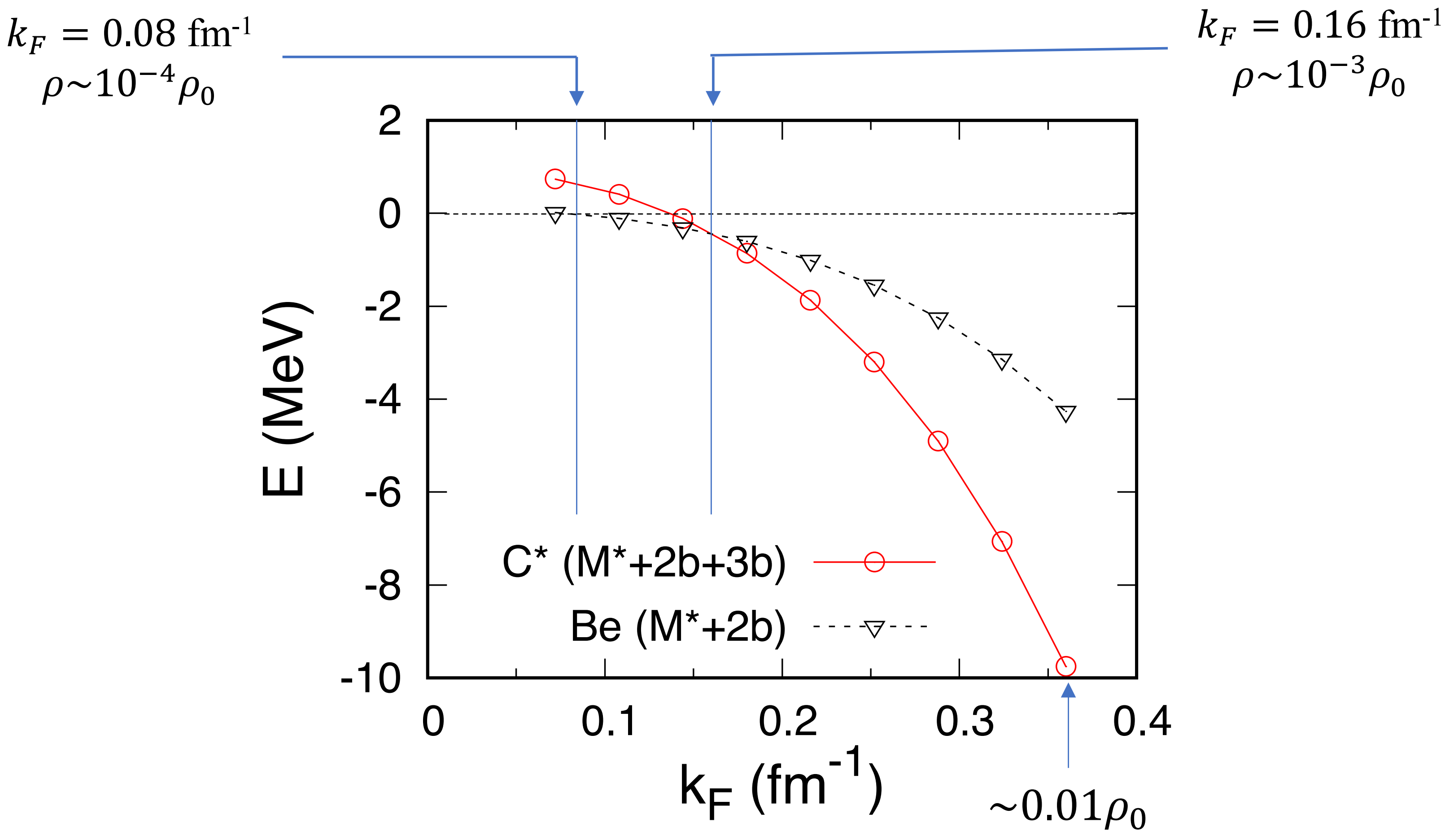
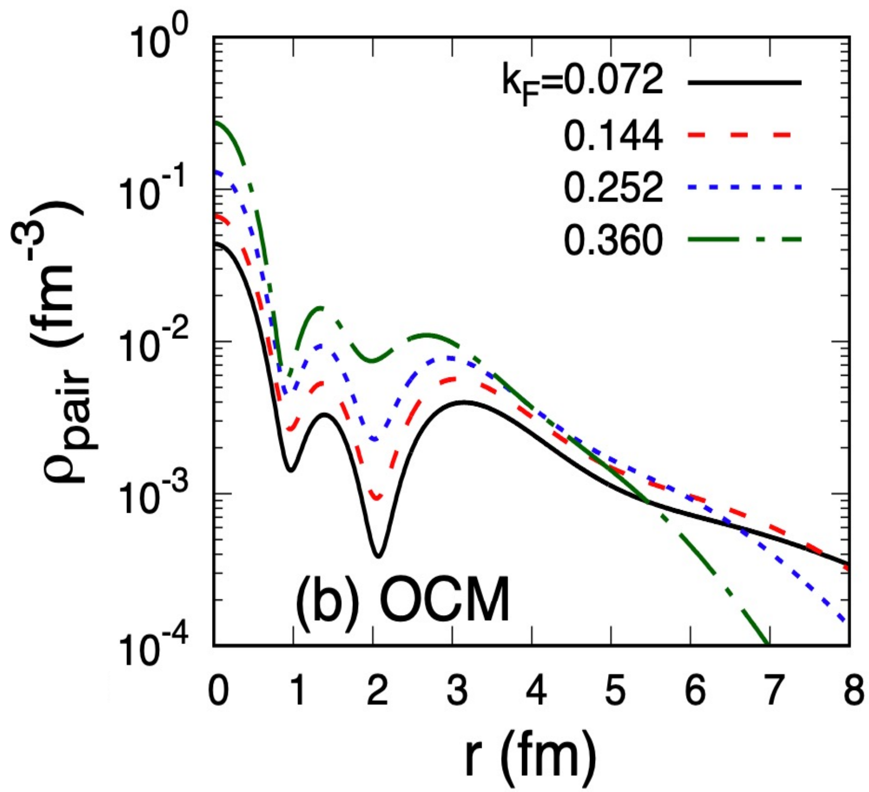
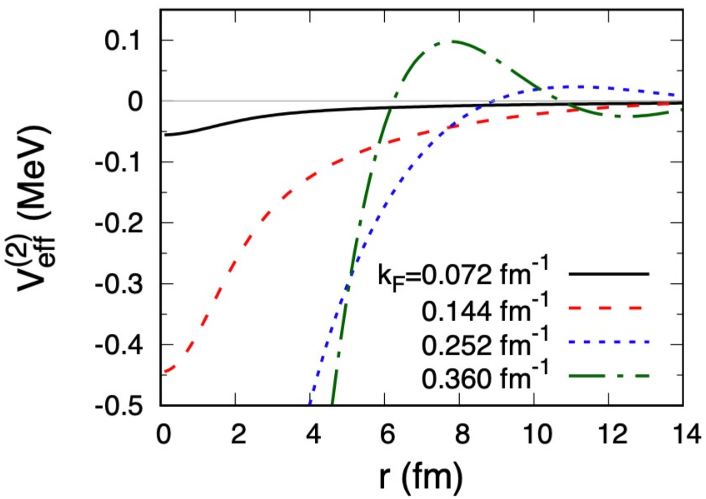


Gaining binding energy by shrinking

- The induced two-body interaction $V_{\mathrm{eff}}^{(2)}$ always works for binding systems deeper
  - Shrinking the average distance between α particles, which is consistent with a microscopic $\alpha + \alpha + n$ cluster model calculation*
- The amplitude in internal region of the pair density become larger with denser neutron matter

$$\rho_{\mathrm{pair}} = \left\langle \frac{\delta(|\boldsymbol{r}_1 - \boldsymbol{r}_2| - r)}{4\pi r^2} \right\rangle$$





*M. Lyu, Z. Ren, B. Zhou, Y. Funaki, H. Horiuchi, G. Ropke, P. Schuck, A. Tohsaki, C. Xu, and T. Yamada, Phys. Rev. C 91, 014313 (2015)

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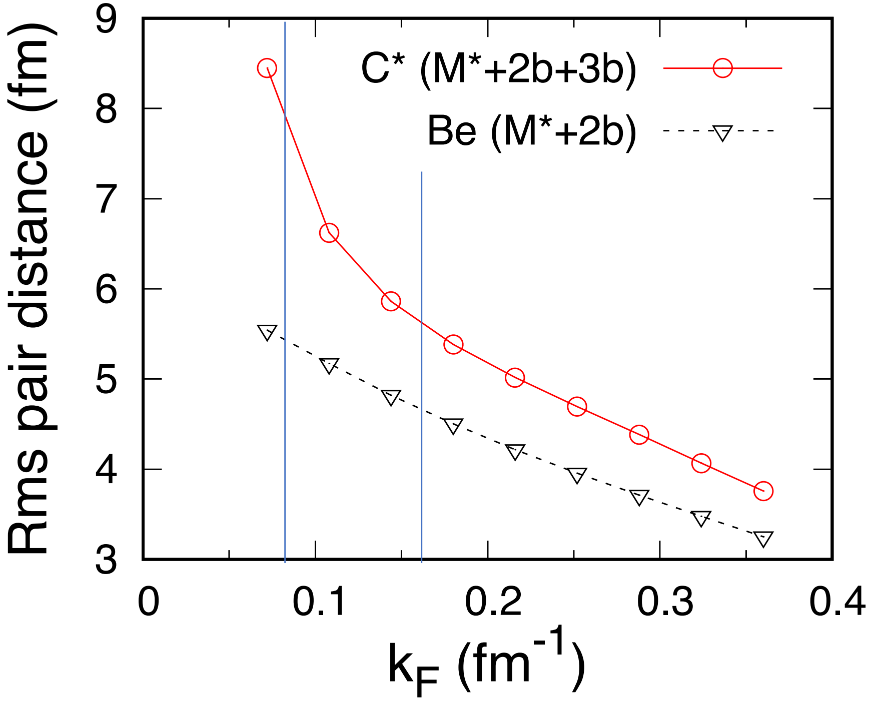
Rms pair distance $= \int dr\, r^2 \rho_{pair}(r)$



Conclusion and prospects

- We have pointed out the possibility that ^{8}Be and Hoyle resonant states become bound in cold neutron matter with $k_F \geq 0.08, 0.16$ fm^{-1}
- The presence of those light nuclear ingredients as bound states would give a significant impact on the modeling of matter in stellar collapse and neutron star mergers M. Oertel, M. Hempel, T. Klähn, and S. Typel, Rev. Mod. Phys. 89, 015007 (2017)
- Induced in-medium interactions have to be realized in finite nuclear systems with well developed alpha clusters, e.g., 2α + Xn and 3α + Xn
- Ignored effects of the Pauli principle on the alpha clusters would work as a repulsion
  - Counteracting binding of ^{8}Be and Hoyle state