Collective States in medium-heavy and heavy nuclei in Subtracted Second RPA method

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基研研究会

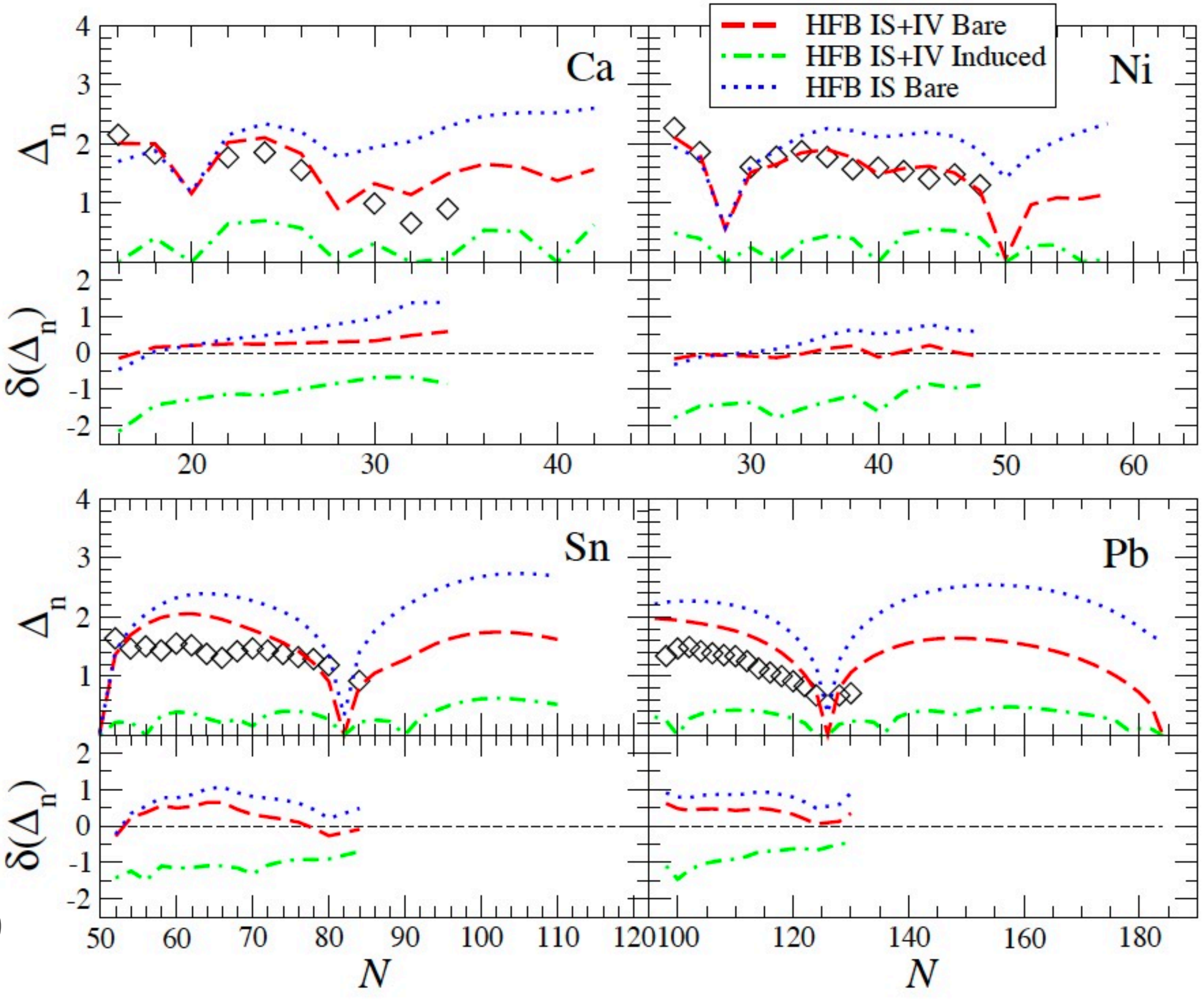
核力に基づいた原子核の構造と反応

2021年12月7日—2021年12月10日
京都大学基礎物理学研究所

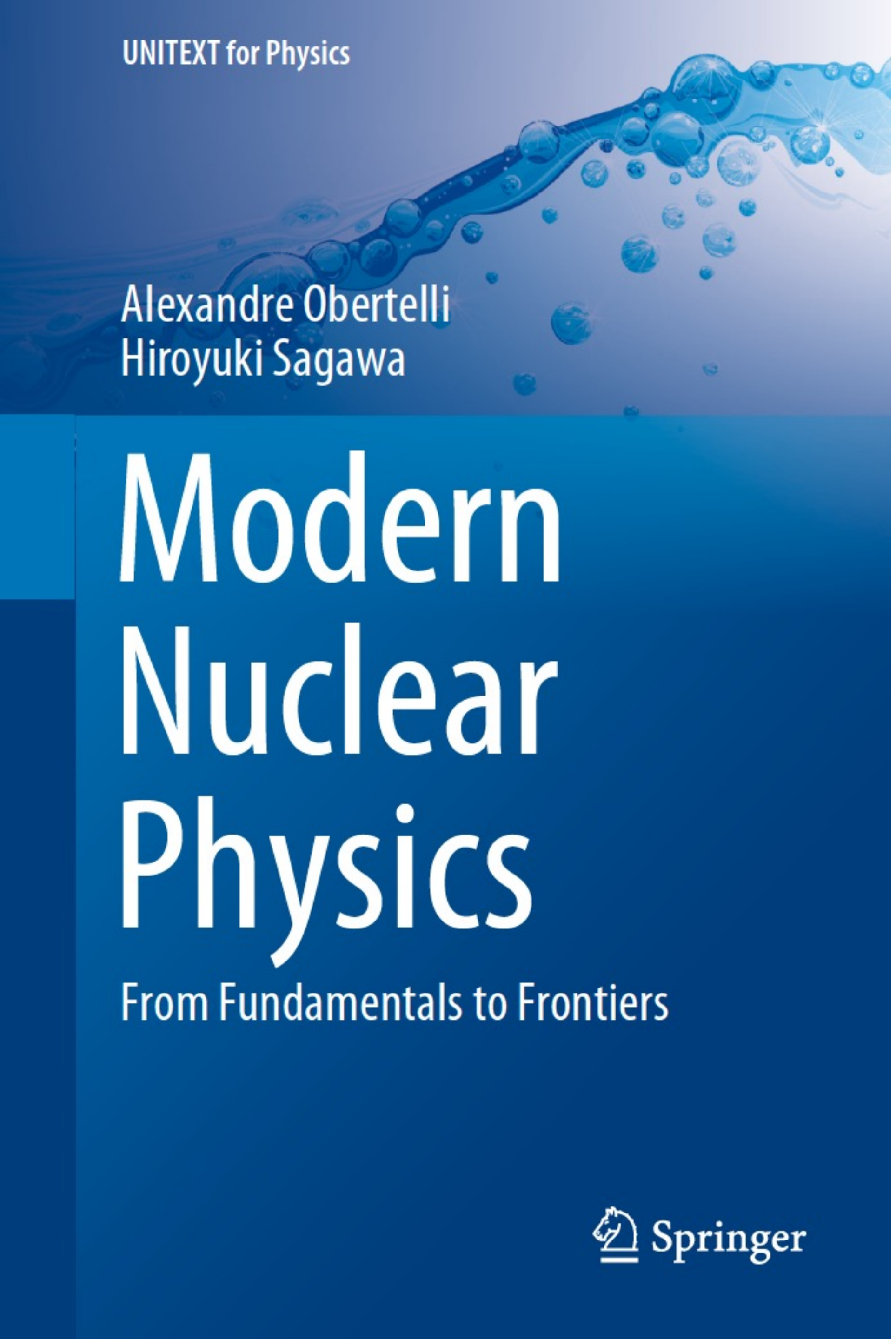
Effective pairing interactions with isospin density dependence
J. Margueron, H. Sagawa, and K. Hagino
Phys. Rev. C **77**, 054309 – Published 14 May 2008

$$\langle k|\mathrm{v}_{\tau\tau}|k'\rangle = \frac{1-\mathrm{P}_\sigma}{2}\mathrm{v}_0\,\mathrm{g}_\tau[\rho,\beta]\,\theta(k,k')$$

$$\mathrm{g}^1_\mathrm{n}[\rho,\beta] = 1 - \mathrm{f_s}(\beta)\eta_\mathrm{s}\left(\frac{\rho}{\rho_0}\right)^{\alpha_\mathrm{s}} - \mathrm{f_n}(\beta)\eta_\mathrm{n}\left(\frac{\rho}{\rho_0}\right)^{\alpha_\mathrm{n}}$$



Optimal pair density functional for the description of nuclei with large neutron excess
M. Yamagami, Y. R. Shimizu, and T. Nakatsukasa
Phys. Rev. C **80**, 064301 – Published 3 December 2009



Fusion of theory and experiment.
Modern version of “Subatomic Physics” by
Hans Frauenfelder and Ernest. M. Henley

10 Chapters in all.
742 pages

Number of Illustrations
79 b/w illustrations, 256 illustrations in colour

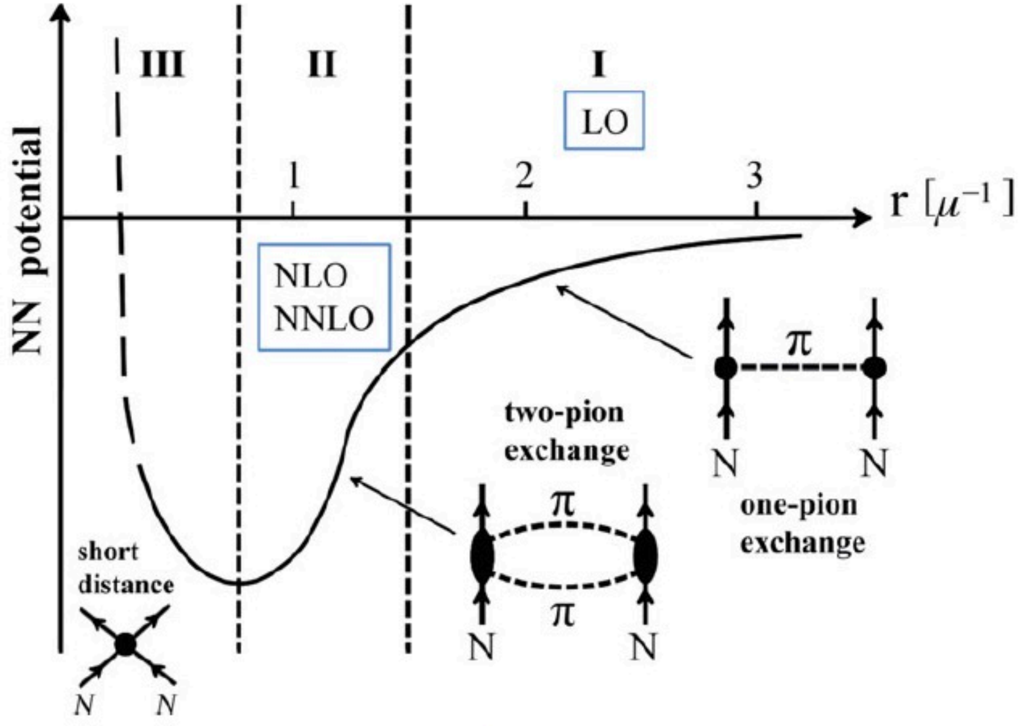


Fig. 2.17 Taketani diagram. Hierarchy of scaling in nuclear forces in pion-exchange potential and in expansion of ChEFT. The unit of distance r is the pion Compton wave length $\mu^{-1} = \hbar/m_\pi\ c \approx$ 1.4 fm

武谷ダイアグアム

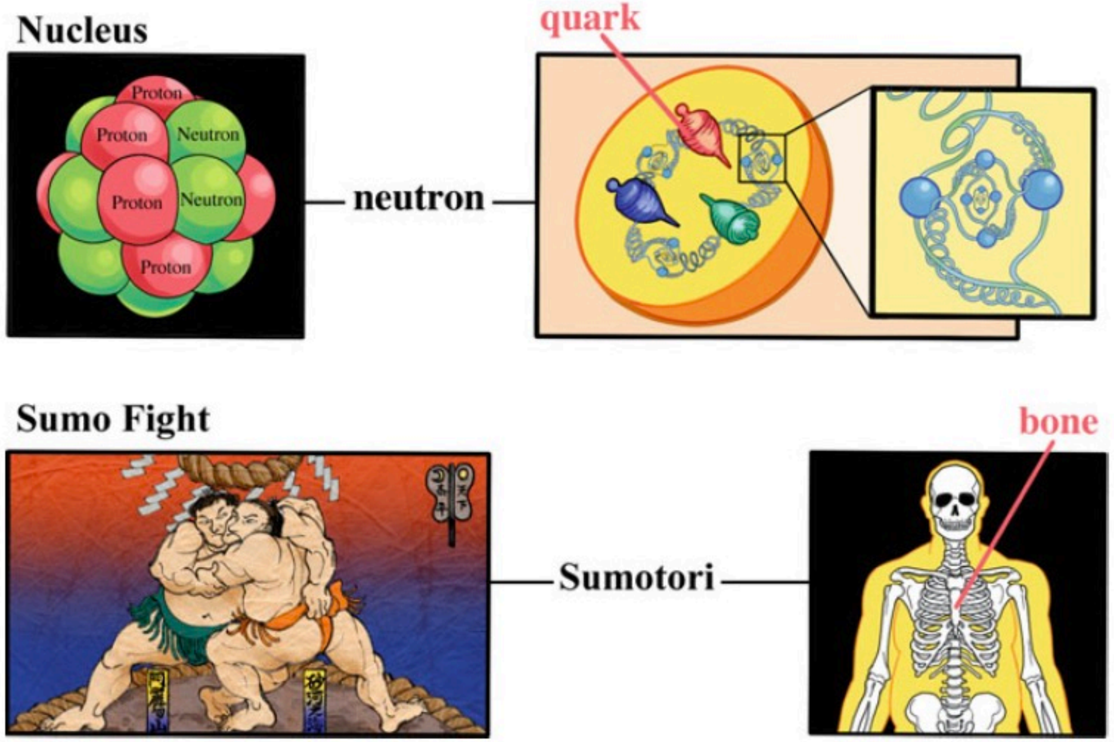


Fig. 2.4 As for a sumo fight which does not require any knowledge of the constituents of the Sumotori (Sumo wrestler), the nucleus can be described in terms of nucleons (protons and neutrons) with no explicit treatment of their internal degrees of freedom, namely quarks and gluons. One can watch, understand and analyze a sumo fight by considering the wrestlers as main entities, without any knowledge of their bones and position of organs in their body. Even though the fights are sometimes violent, fortunately the released energy during the combat is much less than the necessary energy to dismember a fighter. This very same separation of energy scales leads to the emergence of complexity and new phenomena from one scale to another

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Effects of the Skyrme tensor force on 0^+, 2^+, and 3^- states in ^{16}O and ^{40}Ca nuclei with second random phase approximation

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RIKEN, Nishina Center, Wako, Saitama, Japan
[3] *China Institute of Atomic Energy, Beijing 102413, China*

PHYSICAL REVIEW C 103, 054308 (2021)

SRPA（Second RPA based on density functional theory)

P. Papakonstantinou and R. Roth, Phys. Lett. B 671, 356 Phys. Rev. C 81, 024317 (2010).
D. Gambacurta, M. Grasso, and F. Catara, J. Phys. G: Nucl. Part. Phys. 38, 035103 (2011).
D. Gambacurta, M. Grasso, V. De Donno, G. Co’, and F. Catara, Phys. Rev. C 86,021304(R) (2012).
M. Tohyama and P. Schuck, Eur. Phys. J. A 32, 139 (2007).
M. Tohyama, Phys. Rev. C 87, 054330 (2013).

SSRPA (subtracted SRPA): subtract double-counting contribution)

V. I. Tselyaev, Phys. Rev. C 88, 054301 (2013).
P. Papakonstantinou, Phys. Rev. C 90, 024305 (2014).
D. Gambacurta, M. Grasso, and J. Engel, Phys. Rev. C 92, 034303 (2015)
D. Gambacurta, M. Grasso , and J. Engel, PHYS. REV. LETT. 125, 212501 (2020)

RPA ground state is defined as

$$|\Psi\rangle = e^{\hat{S}}|\Phi\rangle,$$

where

$$\hat{S} = \sum_{ph} C_{ph}(t) a_p^\dagger a_h,$$

SRPA operator is

$$\hat{S} = \sum_{ph} C_{ph}(t) a_p^\dagger a_h + \frac{1}{2} \sum_{php'h'} \hat{C}_{pp'hh'}(t) a_p^\dagger a_{p'}^\dagger a_h a_{h'}.$$

The basic idea is the same as the coupled cluster model with s- and d- pairs.

Hohenberg- Kohn (HK) theorem [29] and the Kohn-Sham (KS) procedure

the energy-density functional $E[\rho]$ is universal: in the presence of an additional local Hermitian operator $\lambda Q(\mathbf{r})$, with λ an arbitrary constant, $E[\rho]$ is modified in a simple way,

$$E[\rho] \to E_\lambda[\rho] = E[\rho] + \lambda \int d\mathbf{r}\, Q(\mathbf{r})\rho(\mathbf{r}). \qquad (1)$$

system's density changes from the unperturbed ground-state density ρ_0 to a new one, ρ_λ, given by

$$\rho_\lambda = \rho_0 + \lambda \int d\mathbf{r}\, R(\omega = 0, \mathbf{r}, \mathbf{r}')Q(\mathbf{r}), \qquad (2)$$

In the KS approach, the fundamental philosophy or the essential assumption is EDF(in our case Skyrme interaction) is exact to produce the binding energy and the ground state density

In the adiabatic limit, $\omega \to 0$ $\qquad R(\omega = 0) = R_{\mathrm{KS}}^{\mathrm{RPA}}$,

since a small amplitude limit of TDHF is RPA.

By using a time-dependent Kohn-Sham procedure as a time dependent Hohenberg Kohn (HK) theorem (known as Runge-Gross theorem) gives the energy dependent response function

$$R(\omega,\mathbf{r},\mathbf{r}') = R^0_{\mathrm{KS}}(\omega,\mathbf{r},\mathbf{r}') + \int d\mathbf{r}_1 d\mathbf{r}_2 \times R^0_{\mathrm{KS}}(\omega,\mathbf{r},\mathbf{r}_1)V(\omega,\mathbf{r}_1,\mathbf{r}_2)R(\omega,\mathbf{r}_2,\mathbf{r}'),$$

where R^0_{KS} is the bare Kohn-Sham (mean-field) response and $V(\omega)$ is a frequency-dependent effective interaction obtained from the time-dependent energy-density functional $\mathcal{E}[\rho(t),t]$.

But since $R^{\mathrm{RPA}}_{\mathrm{KS}}$ is correct (as correct as the Skyrme functional, anyway) in the adiabatic limit, we must modify the SRPA so that it gives the RPA response at $\omega = 0$.

SRPA and RPA effective interactions by $U(\omega)$,

$$U(\omega) \equiv V^{\mathrm{SRPA}}(\omega) - V^{\mathrm{RPA}}(\omega) \rightarrow 0 \qquad \omega \rightarrow 0$$

$$Q_\nu^\dagger = \sum_{ph} (X_{ph}^\nu a_p^\dagger a_h - Y_{ph}^\nu a_h^\dagger a_p) + \sum_{\substack{p_1<p_2 \\ h_1<h_2}} (X_{p_1p_2h_1h_2}^\nu a_{p_1}^\dagger a_{p_2}^\dagger a_{h_2} a_{h_1} - Y_{p_1p_2h_1h_2}^\nu a_{h_1}^\dagger a_{h_2}^\dagger a_{p_2} a_{p_1})$$

RPA equation.

$$\begin{bmatrix} A & B \\ -B^* & -A^* \end{bmatrix} \begin{bmatrix} X^\nu \\ Y^\nu \end{bmatrix} = \hbar\omega_\nu \begin{bmatrix} X^\nu \\ Y^\nu \end{bmatrix}$$

Where

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$$

$$X = \begin{pmatrix} X_1^\nu \\ X_2^\nu \end{pmatrix}, Y = \begin{pmatrix} Y_1^\nu \\ Y_2^\nu \end{pmatrix}$$

$$\begin{aligned} A_{11} =& A_{ph;p'h'} \\ =& < HF|[a_h^\dagger a_p, [H, a_{p'}^\dagger a_{h'}]]|HF > \\ =& (E_p - E_h)\delta_{pp'}\delta_{hh'} + \bar{V}_{ph'hp'} \end{aligned}$$

$$\begin{aligned} B_{11} =& B_{ph;p'h'} \\ =& - < HF|[a_h^\dagger a_p, [H, a_{h'}^\dagger a_{p'}]]|HF > \\ =& \bar{V}_{pp'hh'} \end{aligned}$$

$$\begin{aligned} A_{12} =& A_{ph;p_1p_2h_1h_2} \\ =& < HF|[a_h^\dagger a_p, [H, a_{p_1}^\dagger a_{p_2}^\dagger a_{h_2} a_{h_1}]]|HF > \\ =& U(h_1h_2)\bar{V}_{p_1p_2ph_2}\delta_{hh_1} - U(p_1p_2)\bar{V}_{hp_2h_1h_2}\delta_{pp_1} \end{aligned}$$

$U(h_1h_2)$ is an anti-symmtrizer.

$$\begin{aligned} A_{22} =& A_{p_1p_2h_1h_2;p_1'p_2'h_1'h_2'} \\ =& < HF|[a_{h_1}^\dagger a_{h_2}^\dagger a_{p_2} a_{p_1}, [H, a_{p_1'}^\dagger a_{p_2'}^\dagger a_{h_2'} a_{h_1'}]]|HF > \\ =& (E_{p_1} + E_{p_2} - E_{h_1} - E_{h_2})U(p_1p_2)U(h_1h_2) \\ & \times \delta_{p_1p_1'}\delta_{p_2p_2'}\delta_{h_1h_1'}\delta_{h_2h_2'} \\ & + U(h_1h_2)\bar{V}_{p_1p_2p_1'p_2'}\delta_{h_1h_1'}\delta_{h_2h_2'} \\ & + U(p_1p_2)\bar{V}_{h_1h_2h_1'h_2'}\delta_{p_1p_1'}\delta_{p_2p_2'} \\ & - U(p_1p_2)U(h_1h_2)U(p_1'p_2')U(h_1'h_2') \\ & \times \bar{V}_{p_1h_1'p_1'h_1}\delta_{p_2p_2'}\delta_{h_2h_2'} \end{aligned}$$

In SRPA with subtraction procedure (SSRPA), A_{11} and B_{11} are modified.

$$\begin{aligned} A^S_{11'} =& A_{11'} + \sum_2 A_{12}(A_{22})^{-1}A_{21'} + \sum_2 B_{12}(A_{22})^{-1}B_{21'} \\ B^S_{11'} =& B_{11'} + \sum_2 A_{12}(A_{22})^{-1}B_{21'} + \sum_2 B_{12}(A_{22})^{-1}A_{21'} \end{aligned}$$

$$\mathcal{A}^S_F = \begin{pmatrix} A_{11'} + \sum_{2,2'} A_{12}(A_{22'})^{-1}A_{2'1'} + \sum_{2,2'} B_{12}(A_{22'})^{-1}B_{2'1'} & A_{12} \\ A_{21} & A_{22'} \end{pmatrix},$$

$$\mathcal{B}^S_F = \begin{pmatrix} B_{11'} + \sum_{2,2'} A_{12}(A_{22'})^{-1}B_{2'1'} + \sum_{2,2'} B_{12}(A_{22'})^{-1}A_{2'1'} & B_{12} \\ B_{21} & 0 \end{pmatrix}.$$

If the coupling amongst the 2p-2h configuration is neglected, A_{22} will becomes diagonal, this approximation is denoted by SRPAD. In SRPAD, A_{22} is calculated by:

$$A^D_{22} = \delta_{p_1p'_1}\delta_{p_2p'_2}\delta_{h_1h'_1}\delta_{h_2h'_2}(E_{p_1} + E_{p_2} - E_{h_1} - E_{h_2}) \quad (8)$$

The transition operators of the spin-independent modes are:

$$F_\lambda^{IS} = \sum r_i^n Y_{\lambda 0}(r_i)$$
$$F_\lambda^{IV} = \sum r_i^n Y_{\lambda 0}(r_i)\tau_z(i) \tag{10}$$

$$B(E_\lambda) = |\sum_{ph} b_{ph}(E_\lambda)|^2 = |\sum_{ph}(X_{ph}^\lambda + (-1)^J Y_{ph}^\lambda)F_{ph}^\lambda|^2$$

$$\sum_{ph}(|X_{ph}^\nu|^2 - |Y_{ph}^\nu|^2) + \sum_{p_1p_2h_1h_2}(|X_{p_1p_2h_1h_2}^\nu|^2 - |Y_{p_1p_2h_1h_2}^\nu|^2)$$
$$= n_1 + n_2 = 1$$

$$m_1 = \sum_\nu \hbar\omega_\nu| < \nu|F|0 > |^2,$$

can be calculated analytically [86],

$$m_1 = \begin{cases} \frac{4}{4\pi}\frac{\hbar^2}{2m}A < r^2 >, & \lambda = 0 \\ \frac{50}{4\pi}\frac{\hbar^2}{2m}A < r^2 >, & \lambda = 2 \end{cases}$$

TABLE I: Isoscalar EWSR m_1 obtained in the analytic formula (15), RPA and SRPA calculations for ^{16}O and ^{40}Ca with SGII interaction.

^{16}O			
State	analytic one	RPA	SRPA
0^+	676.37	673.87	673.87
2^+	8454.65	8375.43	8375.43

^{40}Ca			
State	analytic one	RPA	SRPA
0^+	2889.16	2879.92	2879.92
2^+	36114.5	35934.4	35934.4

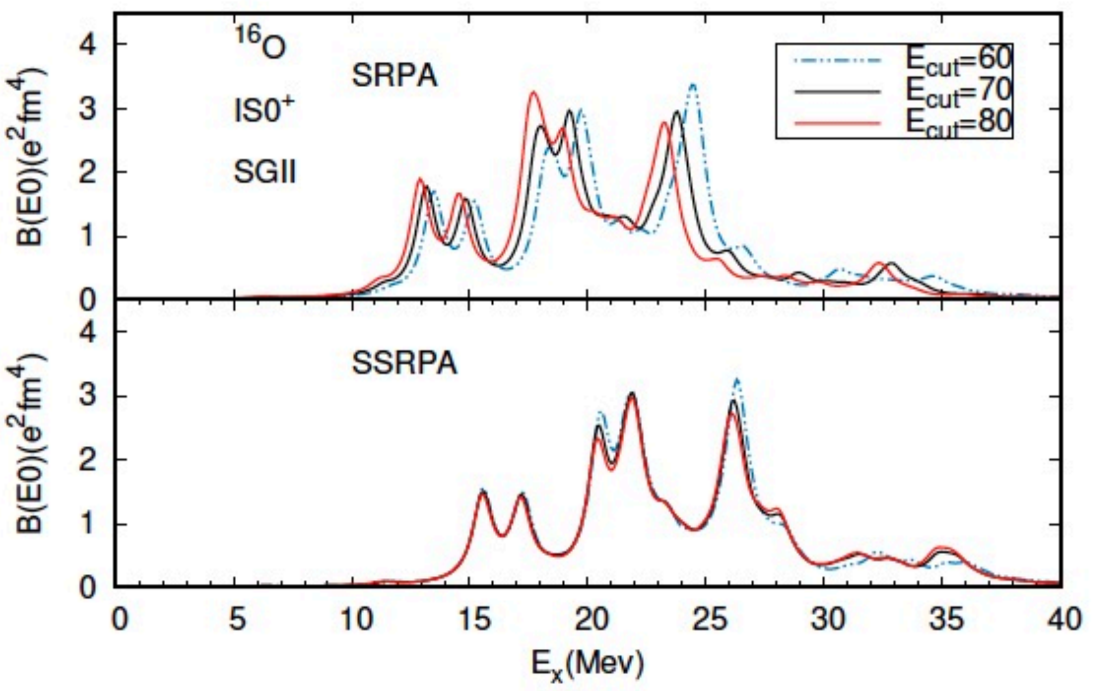

FIG. 1: IS 0^+ strength distributions for ^{16}O calculated by SRPA (upper panel)and SSRPA (lower panel) by SGII interaction with 2p-2h energy cutoff 60, 70, and 80 MeV. See the text for more details.

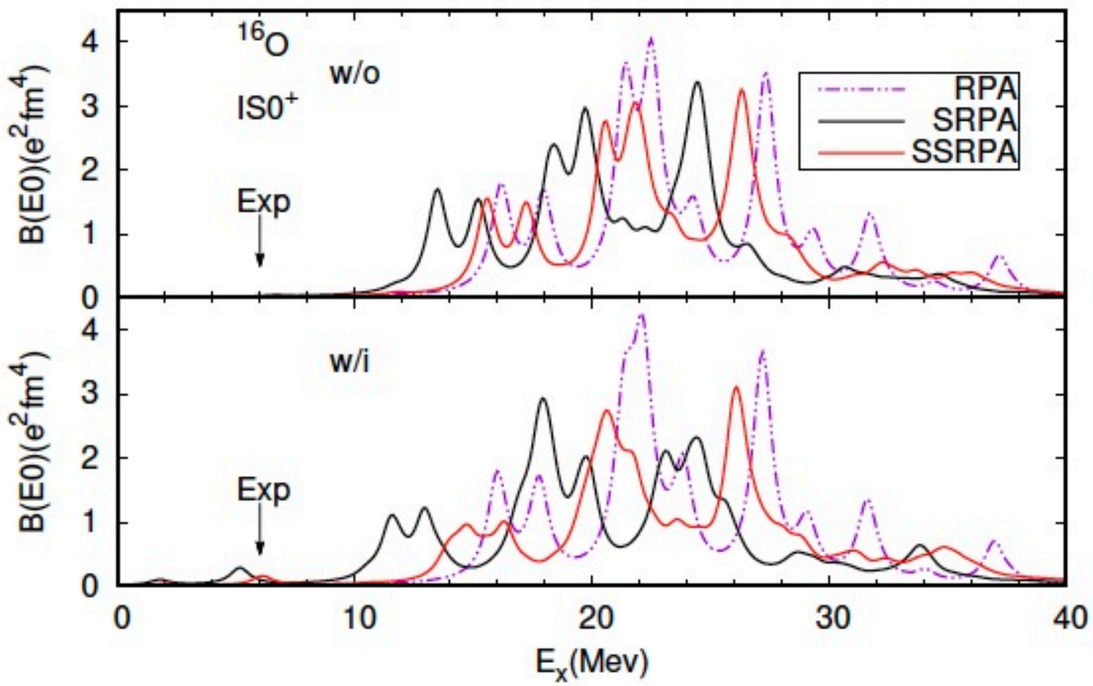

FIG. 2: IS 0^+ strength distributions in ^{16}O. The results calculated without and with tensor interaction are shown in the upper and lower panels, respectively. Results of RPA, SRPA, and SSRPA are labelled by purple dash lines, black solid lines, and red solid lines, respectively. The lowest state measured by experiment [3] is represented by an arrow.

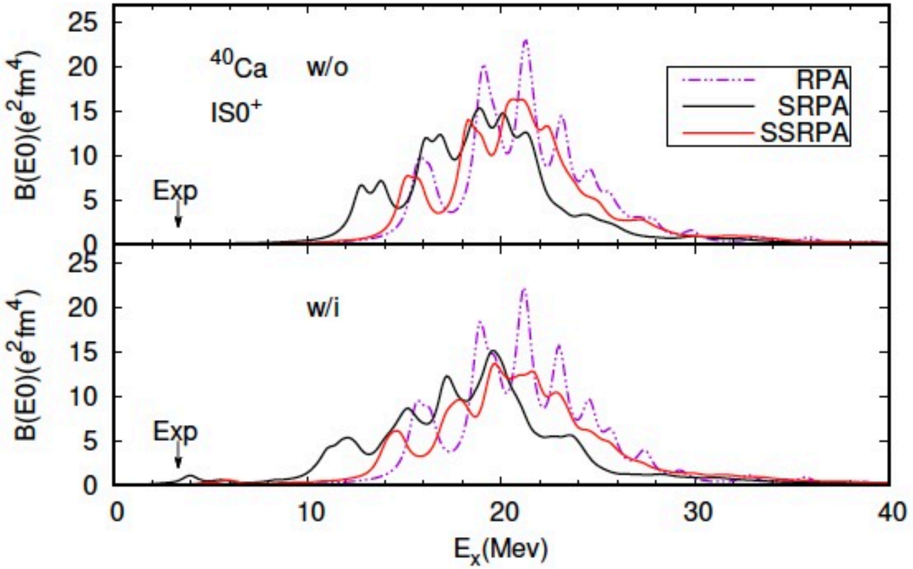

FIG. 4: The same as Fig. 2, but for IS 0^+ in ^{40}Ca nucleus, The experimental data is taken from Ref.[5].

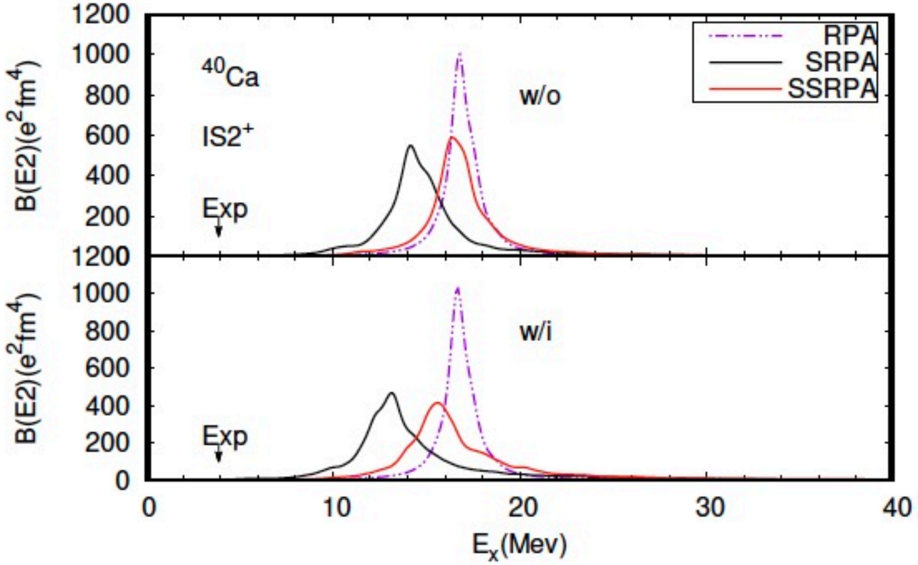

FIG. 9: The same as Fig. 4, but for IS 2^+ in ^{40}Ca. Experimental data is taken from Ref.[8].

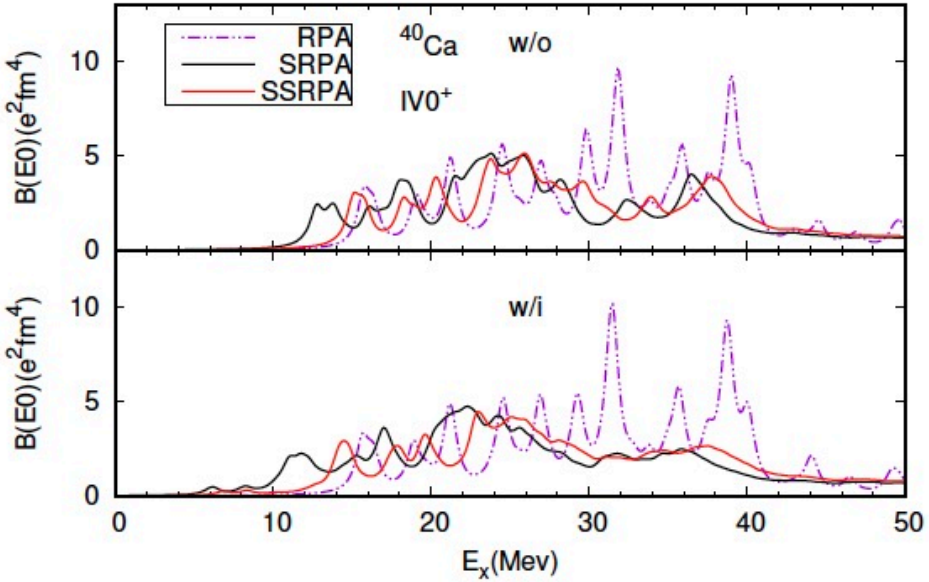

FIG. 5: The same as Fig. 4, but for IV 0^+ in ^{40}Ca.

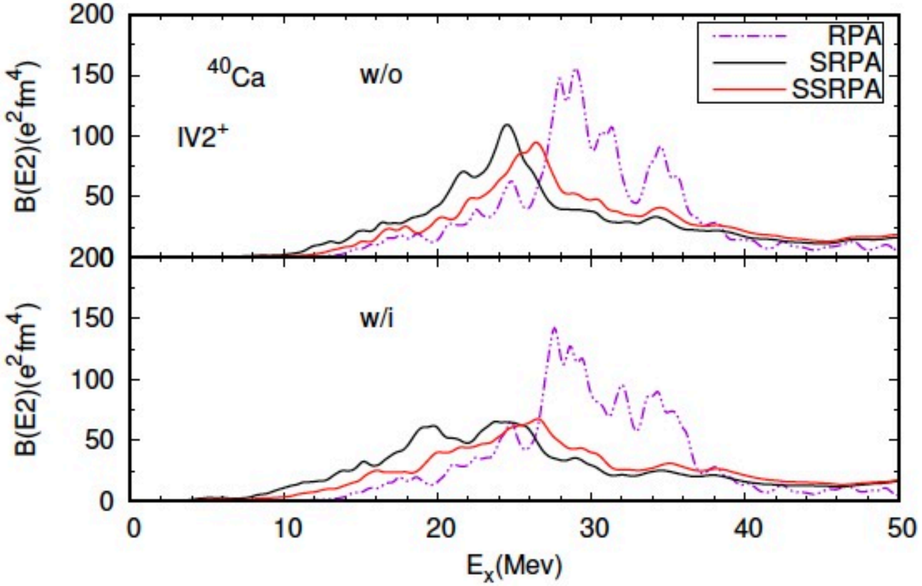

FIG. 11: The same as Fig. 10, but for IV 2^+ in ^{40}Ca.

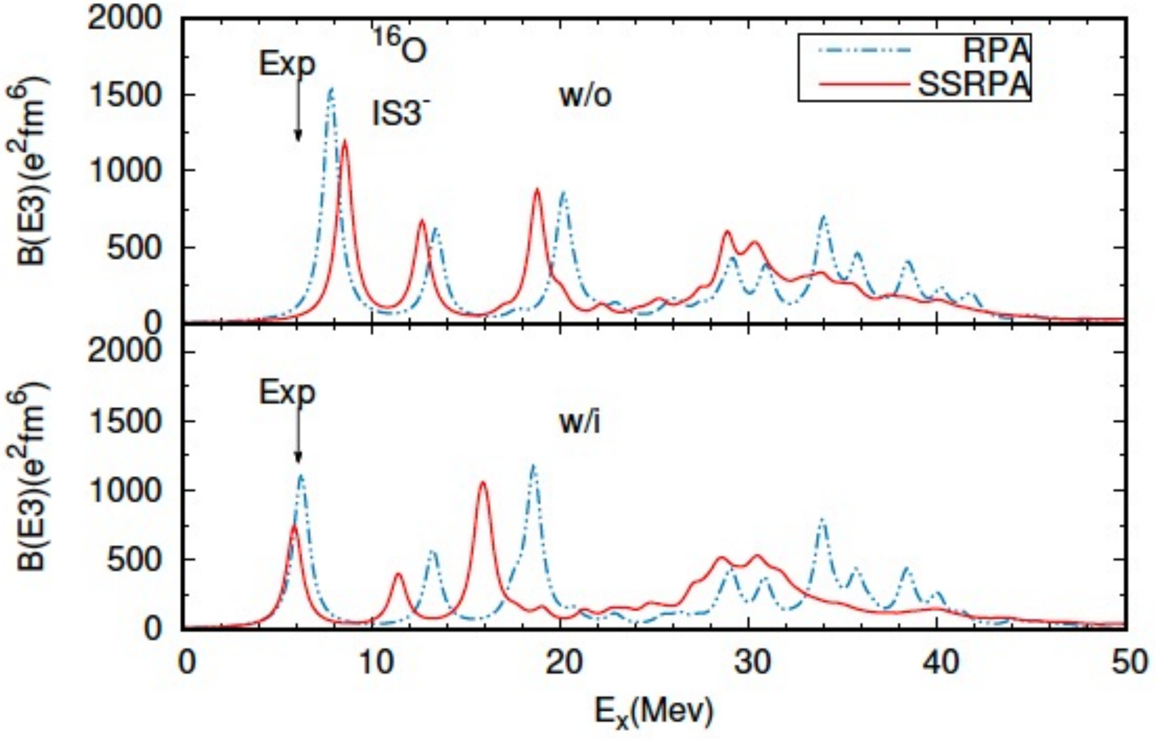

FIG. 12: Isoscalar octupole strength distributions in ^{16}O calculated without tensor(upper panel) and with tensor(lower panel). Blue dash lines are RPA result, while red solid lines are SSRPA result. Experimental data is taken from *****.

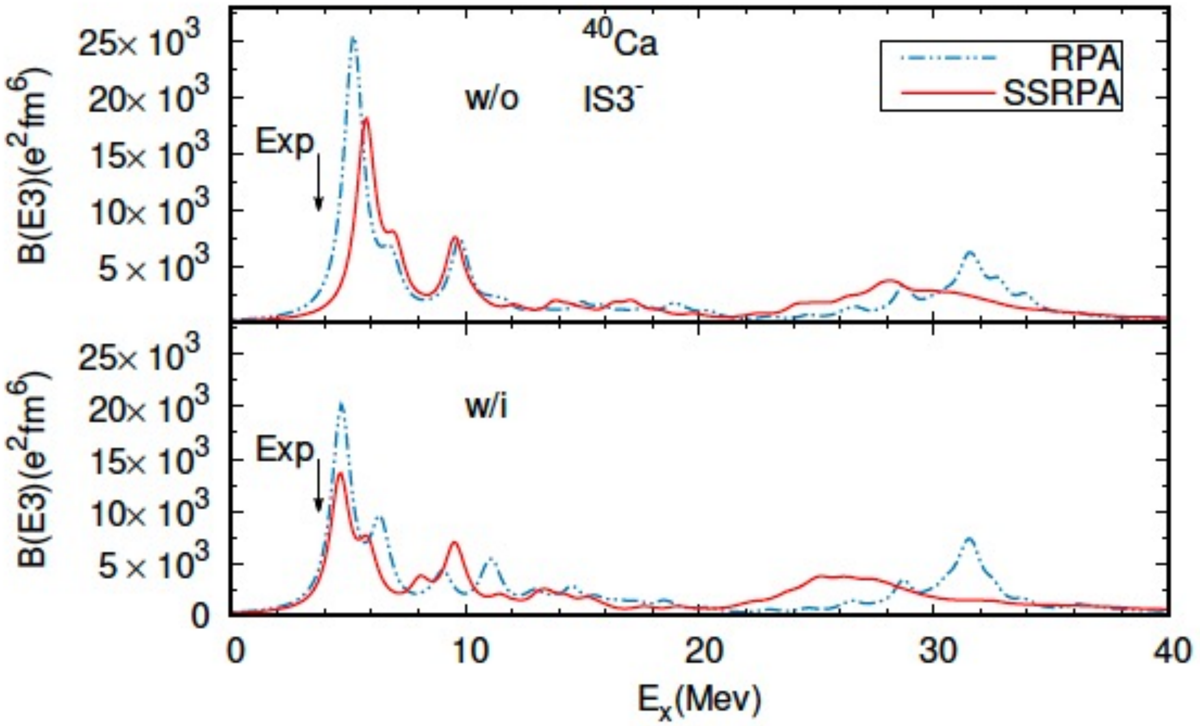

FIG. 14: IS3$^-$ in ^{40}Ca without(upper panel) with(lower panel) tensor force. Blue lines are RPA results, while red lines are SSRPA results. Experimental data is taken from *****.

Gamow-Teller states and 2particle-2hole configurations

D. F. Bertsch and I.Hamamoto, Phys. Rev. C 26, 1323 (1982).

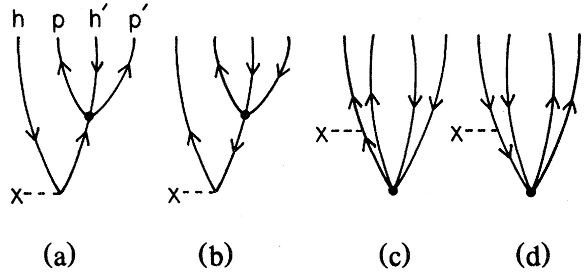

FIG. 3. Four types of amplitude included in the actual calculation. (a) should of course also include the graph with h and h' interchanged.

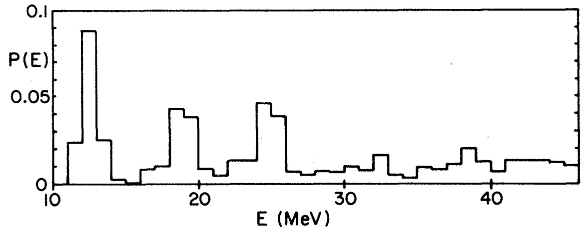

FIG. 4. Calculated strength distribution $P(E)$ for the Gamow-Teller operator in ^{90}Zr. Energies are measured with respect to the ground state of ^{90}Nb.

TABLE I. Contributions to Gamow-Teller strength in the region 10–45 MeV excitation in ^{90}Zr, $\int_{10}^{45} P(E)\,dE$, with $P(E)$ defined in Eq. (4). The partial sums need not add to the total because of possible coherence of amplitudes.

$\int P$	Graphs (a) + (b)	Graphs (c) + (d)	Total
Tensor	0.13	0.06	0.20
Central	0.25	0.15	0.36
Total	0.38	0.20	0.56

Spreading of the Gamow-Teller Resonance in ^{90}Nb and ^{208}Bi
Nguyen Dinh Dang, Akito Arima, Toshio Suzuki, and Shuhei Yamaji
PRL79, 1638 (1997)

PHYSICAL REVIEW LETTERS **125**, 212501 (2020)

Gamow-Teller Strength in ^{48}Ca and ^{78}Ni with the Charge-Exchange Subtracted Second Random-Phase Approximation

D. Gambacurta,[1] M. Grasso,[2] and J. Engel[3]

Summary

SS RPA(subtracted second RPA) model is applied to describe collective states of medium-heavy and heavy nuclei.

Low-monopole states are affected by the tensor force and get a better agreement with experimental data.

Gamow-Teller states of ^{90}Zr and ^{208}Pb are also studied by SSRPA and 2p-2states make a larger spreading width on top of the proper excitation energies compared with experimental ones.

Quenching: ^{48}Ca 20-35% Ex<20 MeV
^{208}Pb 20-30% Ex<25 MeV

Future perspectives

Ab initio EDF to apply SSRPA

Nuclear energy density functionals grounded in ab initio calculations
Authors: F. Marino, C. Barbieri, G. Colò, A. Lovato, F. Pederiva, X. Roca-Maza, E. Vigezzi

Phys. Rev. C 104, 024315 (2021)

Toward ab initio determination of charge symmetry breaking strength of Skyrme functionals
Authors: Tomoya Naito, Gianluca Colò, Haozhao Liang, Xavier Roca-Maza, Hiroyuki Sagawa

arXiv:2107.14436 (2021)

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GT sum rule quenching :Two-body currents