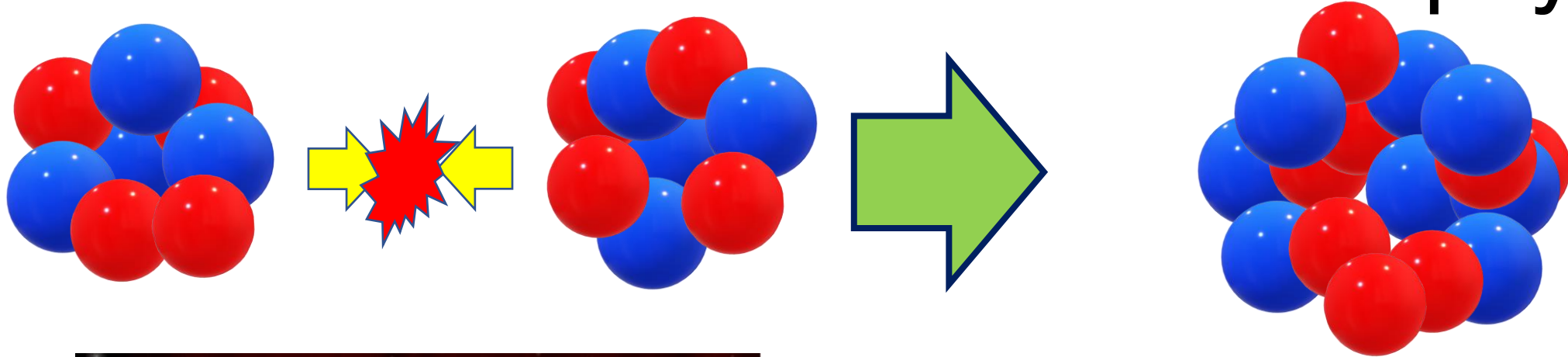


$^{12}\text{C} + ^{12}\text{C}$ Fusion S-factor from a Full-microscopic Nuclear Model

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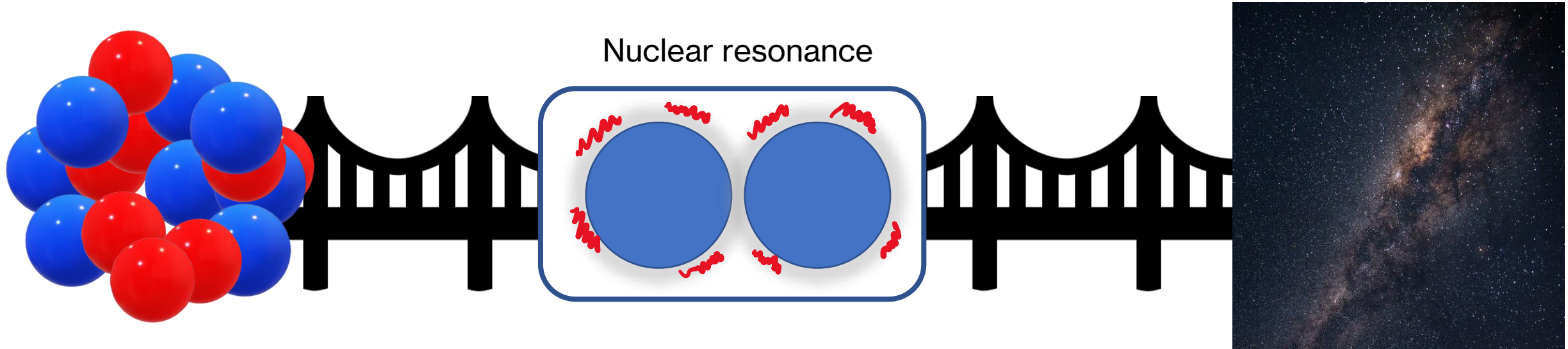
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Thermonuclear fusion and astrophysics



- Thermonuclear fusion is energy source of various astrophysical phenomena.
 - Evolution of stars
 - Supernovae (type Ia)
 - X-ray superbursts
 - Nuclear fusion of accreting matter onto NSs

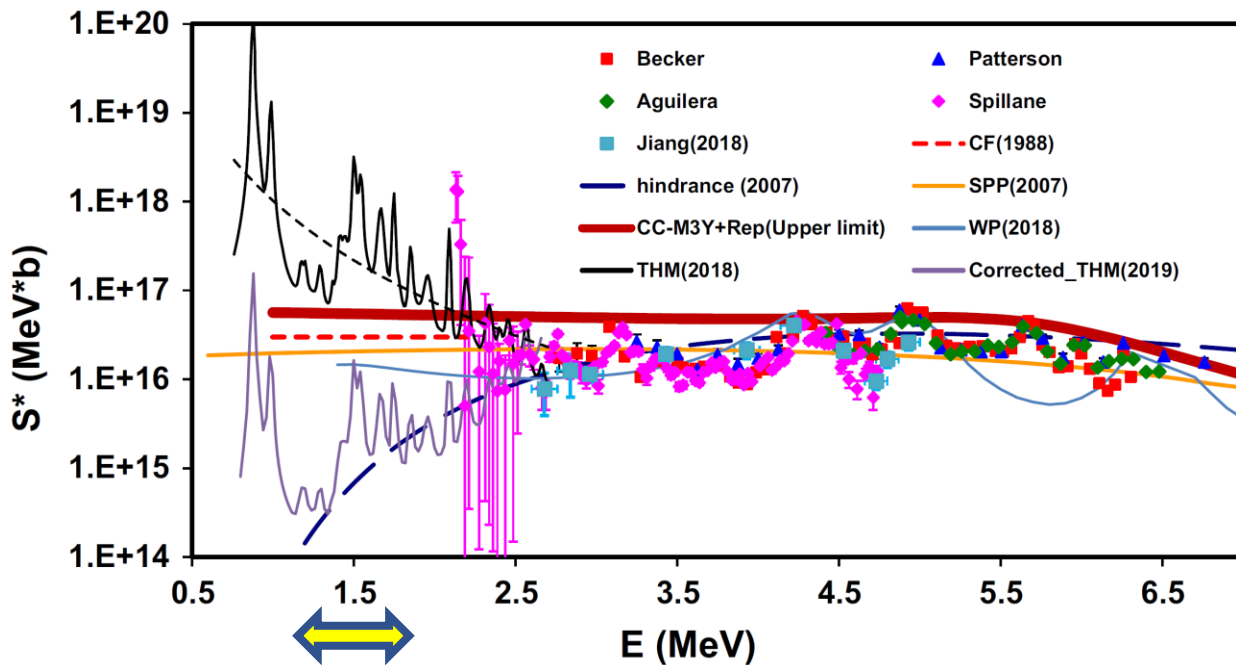
Nuclear resonances as the “bridge” between nuclear physics and astrophysics



- The **nuclear fusion reaction rate** is essential for the astrophysical phenomena.
- The **nuclear fusion reaction rate** is obtained from the **fusion reaction cross sections**.
- **Nuclear resonances** enhance the **fusion cross sections** in orders of magnitude.

$^{12}\text{C} + ^{12}\text{C}$ fusion reaction cross section and the Gamow window of X-ray superbursts

$$S^*(E) = E\sigma(E) \exp(2\pi\eta + 0.46 \text{ MeV}^{-1} E)$$



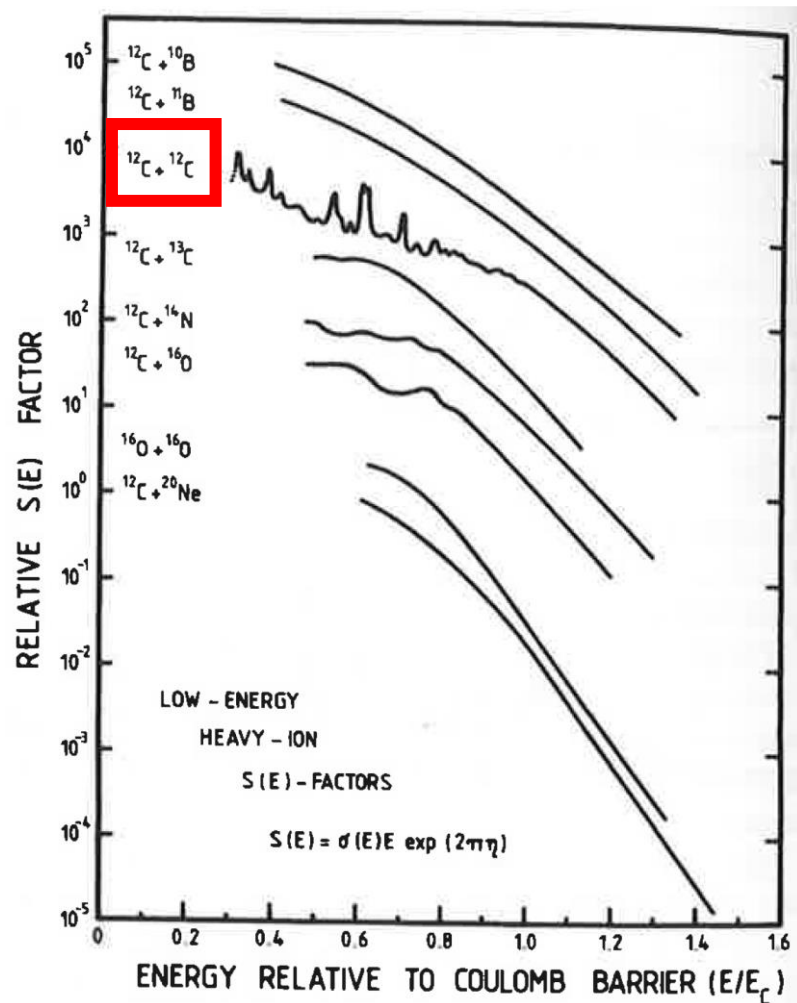
Gamow window
of X-ray superbursts

[R. L. Cooper+, ApJ **702**, 660 (2009)]

[Original fig. : C. Beck et al, EPJA56, 87 (2020)]

- The **resonance around E ~ 1.5 MeV** has been proposed to explain the **X-ray superbursts**.
- Experimental data are insufficient in $E < 2$ MeV.
 - **No direct experiments** in $E < 2$ MeV.
 - The indirect experiment with the THM [Tumino+, Nature (2018)] has **uncertainties** in the analysis.
- The fusion cross sections with $E < 2$ MeV are just **extrapolations** from the higher energy data.
 - CF1988: overestimation
 - Hindrance model
 - The effects of **resonances** are **averaged**.
- Reliable theoretical calculations including **resonant effects** are required.

Resonance-state dominance in $^{12}\text{C} + ^{12}\text{C}$ fusion reaction



- S-factors of $^{12}\text{C} + ^{12}\text{C}$ fusion reaction has a significant peak structure.
- Contributions of **narrow-resonance states** are dominant.

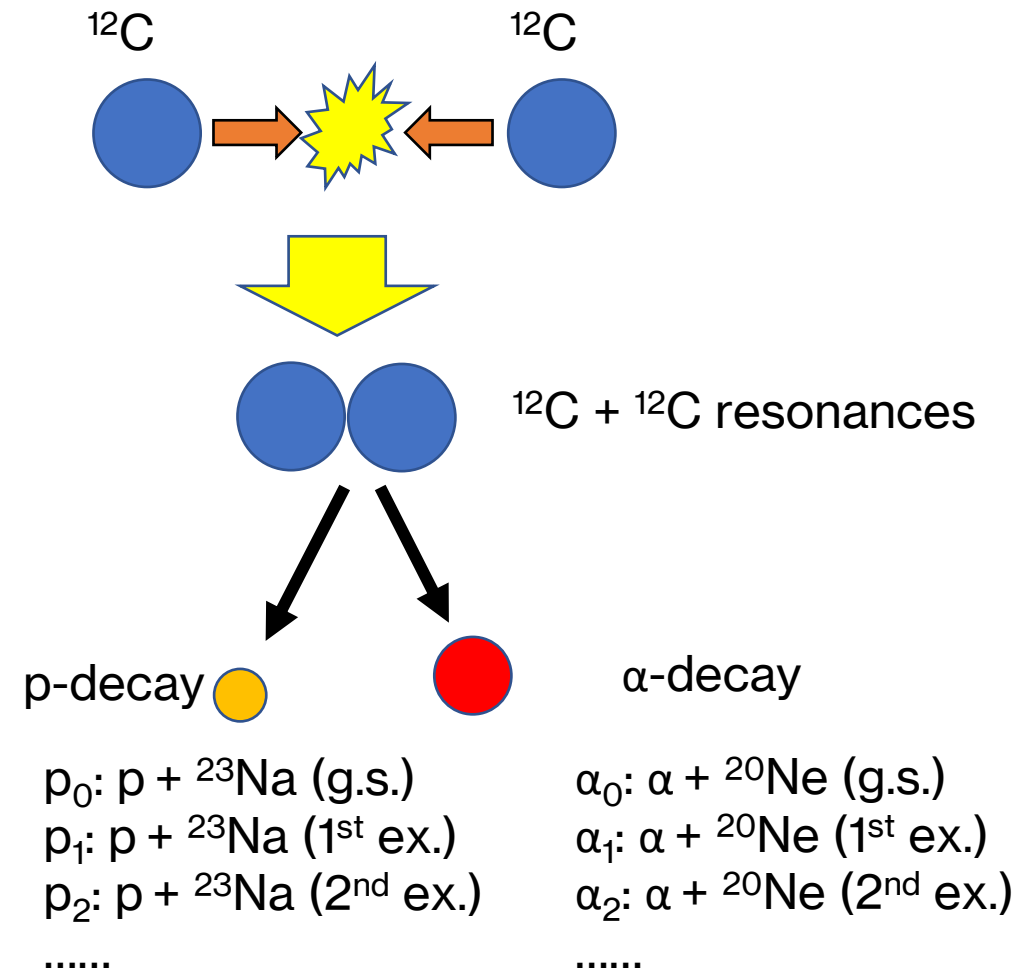
Calculation of $^{12}\text{C} + ^{12}\text{C}$ fusion cross sections

- **Narrow-resonance cross sections** are calculated from the **decay widths** of the **entrance** and **exit** channels.

$$\sigma(E) = \frac{\pi}{k^2} \frac{\Gamma_{ent}\Gamma_{exit}}{(E - E_R)^2 + \Gamma^2/4}$$

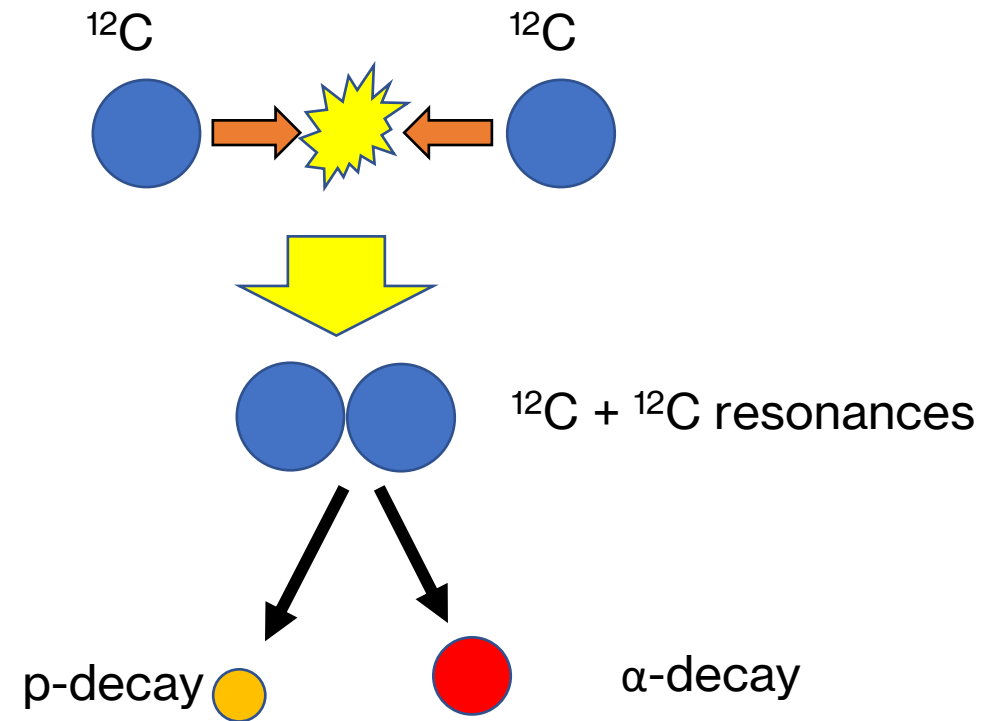
- Channels

- Entrance: $^{12}\text{C} + ^{12}\text{C}$
- Exit: $\alpha + ^{20}\text{Ne}$, $p + ^{23}\text{Na}$



Why full-microscopic?

- $^{12}\text{C} + ^{12}\text{C} \rightarrow ^{24}\text{Mg}^* \rightarrow \alpha + ^{20}\text{Ne}$
 $\rightarrow ^{24}\text{Mg}^* \rightarrow \text{p} + ^{23}\text{Na}$
- Rearrangement of many particles.
- Strong channel couplings.
- By using a **full-microscopic nuclear model**, we can treat them with a **nucleon-nucleon interaction**.
- Phenomenological coupling potentials are not necessary.



Framework:

Antisymmetrized molecular dynamics (AMD)

- Slater determinant of deformed Gaussian wave packets

$$|\Phi\rangle = \mathcal{A}|\varphi_1, \varphi_2, \dots, \varphi_A\rangle$$

$$\varphi_i(\mathbf{r}) = \exp\left[-\frac{1}{2}(\mathbf{r} - \mathbf{z}_i) \cdot \mathbf{M}(\mathbf{r} - \mathbf{z}_i)\right] \otimes \sigma_i \otimes \tau_i$$

- Coupling of the entrance and exit channels are treated by linear combination of basis wave functions.

$$|\Psi\rangle = c_{12} \overset{^{12}\text{C} + ^{12}\text{C}}{\text{blue circles}} + c_4 \overset{\alpha + ^{20}\text{Ne}}{\text{red and yellow circles}} + c_{24} \overset{\text{deformed}}{\text{purple circle}} \quad (\text{Gogny D1S})$$

- Fusion and fission dynamics are also treated by linear combination of wave functions of various inter-nuclear distance.

$$|\text{red and yellow circles}\rangle = d_1 |\text{overlapping red and yellow circles}\rangle + d_2 |\text{separated red and yellow circles}\rangle + d_3 |\text{red and yellow circles}\rangle + \dots$$

- Diagonalization of Hamiltonian with the Gogny D1S effective interaction (density functional).

Decay widths

- The Breit-Wigner formula

$$\sigma(E) = \frac{\pi}{k^2} \frac{\Gamma_{ent} \Gamma_{exit}}{(E - E_R)^2 + \Gamma^2/4}$$

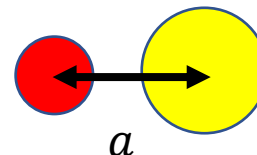
- The R-matrix theory

$$\Gamma_{C_1+C_2} = \frac{2ka}{F_l(ka)^2 + G_l(ka)^2} \frac{3\hbar^2}{2\mu a^2} \theta_{C_1+C_2}^2$$

$$Q = \frac{k^2}{2\mu}$$

a : channel radius

(Dimensionless) Reduced width
amplitude at channel radius a
Probability of existence of clusters



Laplace expansion method
[Chiba+, PTEP (2017)]

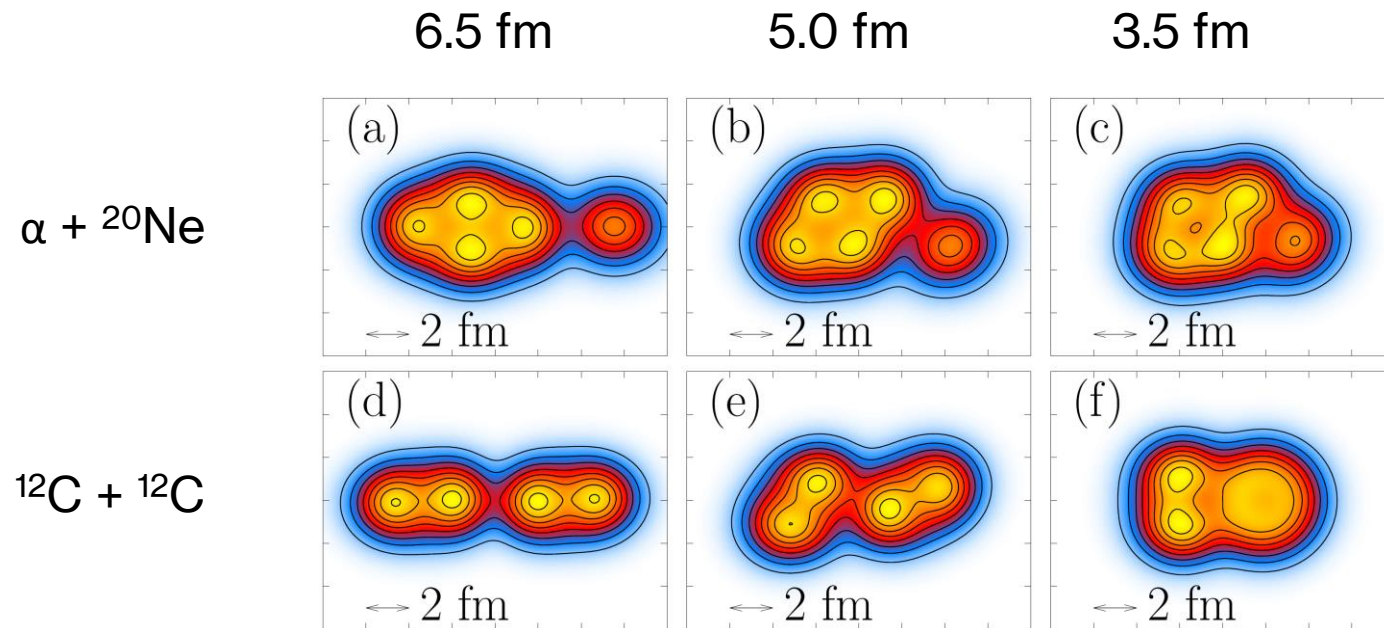
$$\Gamma = \Gamma_\alpha + \Gamma_p$$

$$\Gamma_\alpha = \left(\frac{\Gamma_\alpha}{\Gamma_{\alpha 1}} \right) \cdot \Gamma_{\alpha 1}$$

$$\Gamma_p = \left(\frac{\Gamma_p}{\Gamma_{p 1}} \right) \cdot \Gamma_{p 1}$$

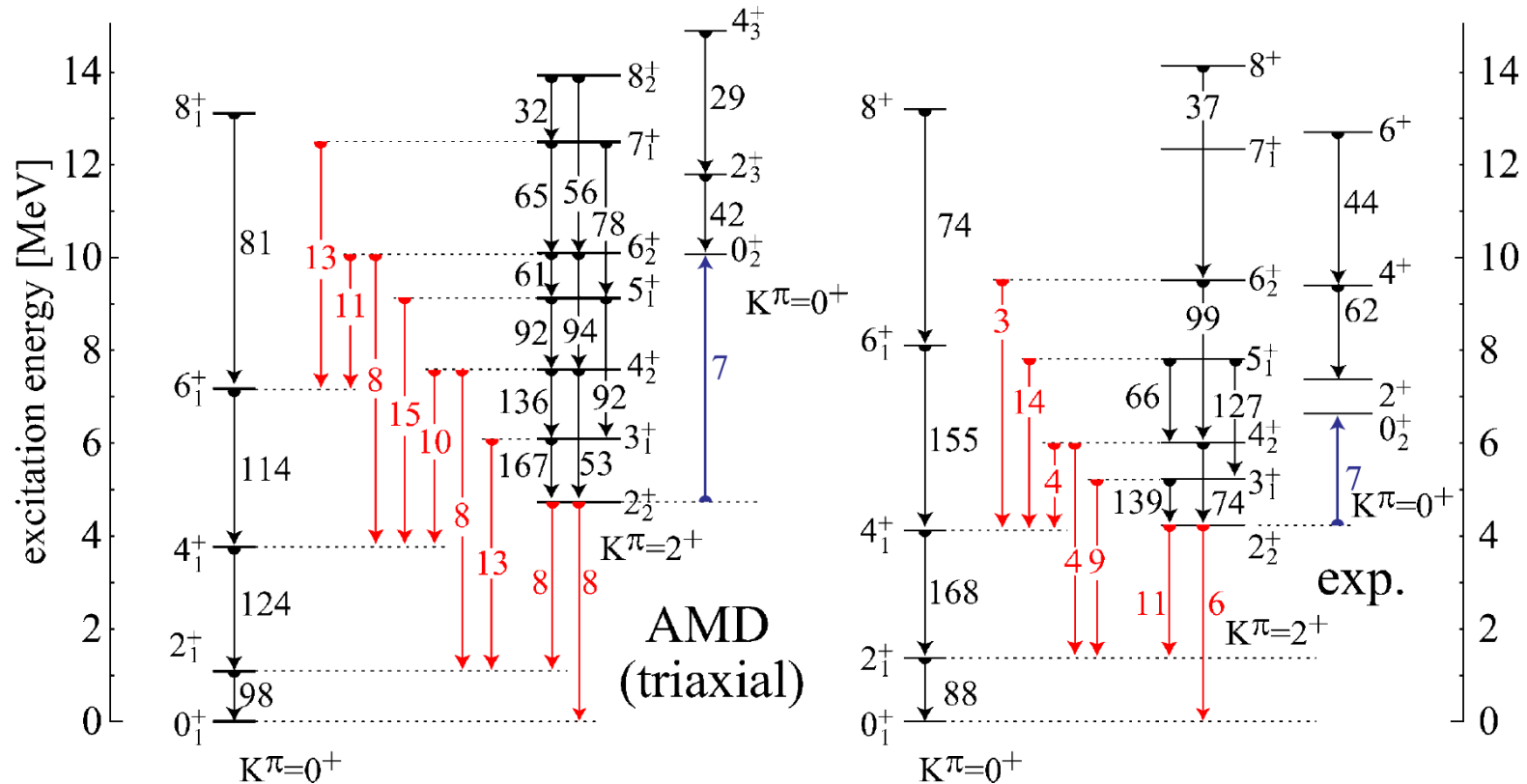
exp. data by Becker+, Z. Phys. 303, 305 (1981)

Examples of the basis wave functions of the multi-configuration mixing



- Coupling of the entrance (${}^{12}\text{C} + {}^{12}\text{C}$) and exit ($\alpha + {}^{20}\text{Ne}$) channels
- Rotation of clusters

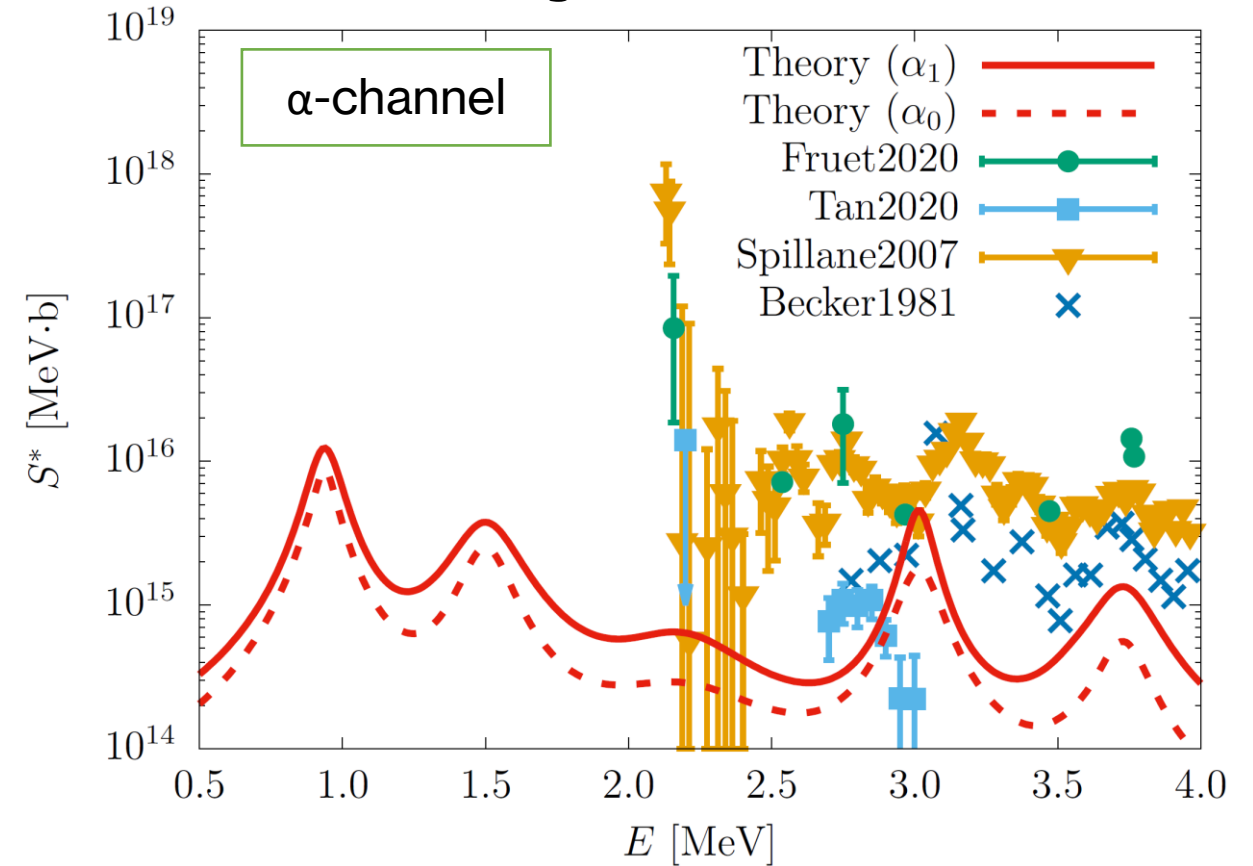
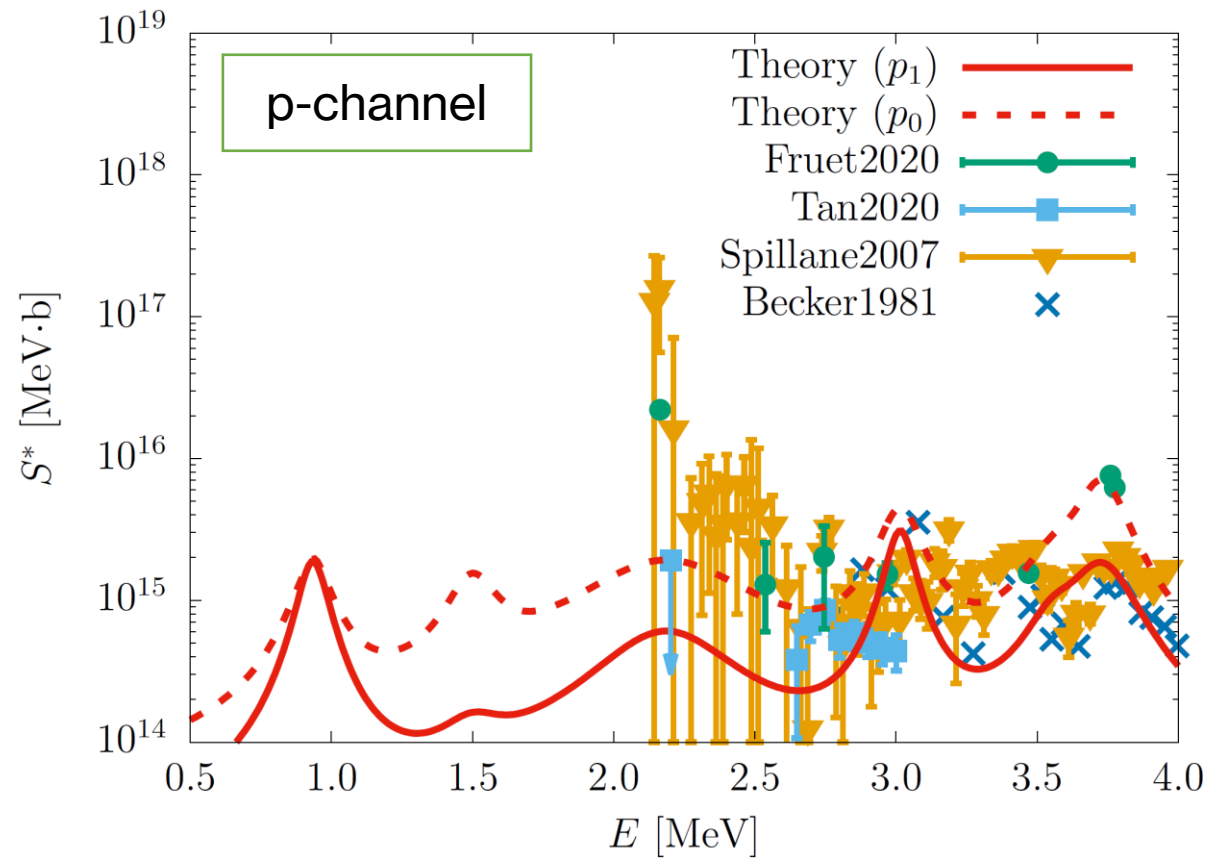
Low-lying states of ^{24}Mg



[M. Kimura et al, PTP127, 287 (2012)]

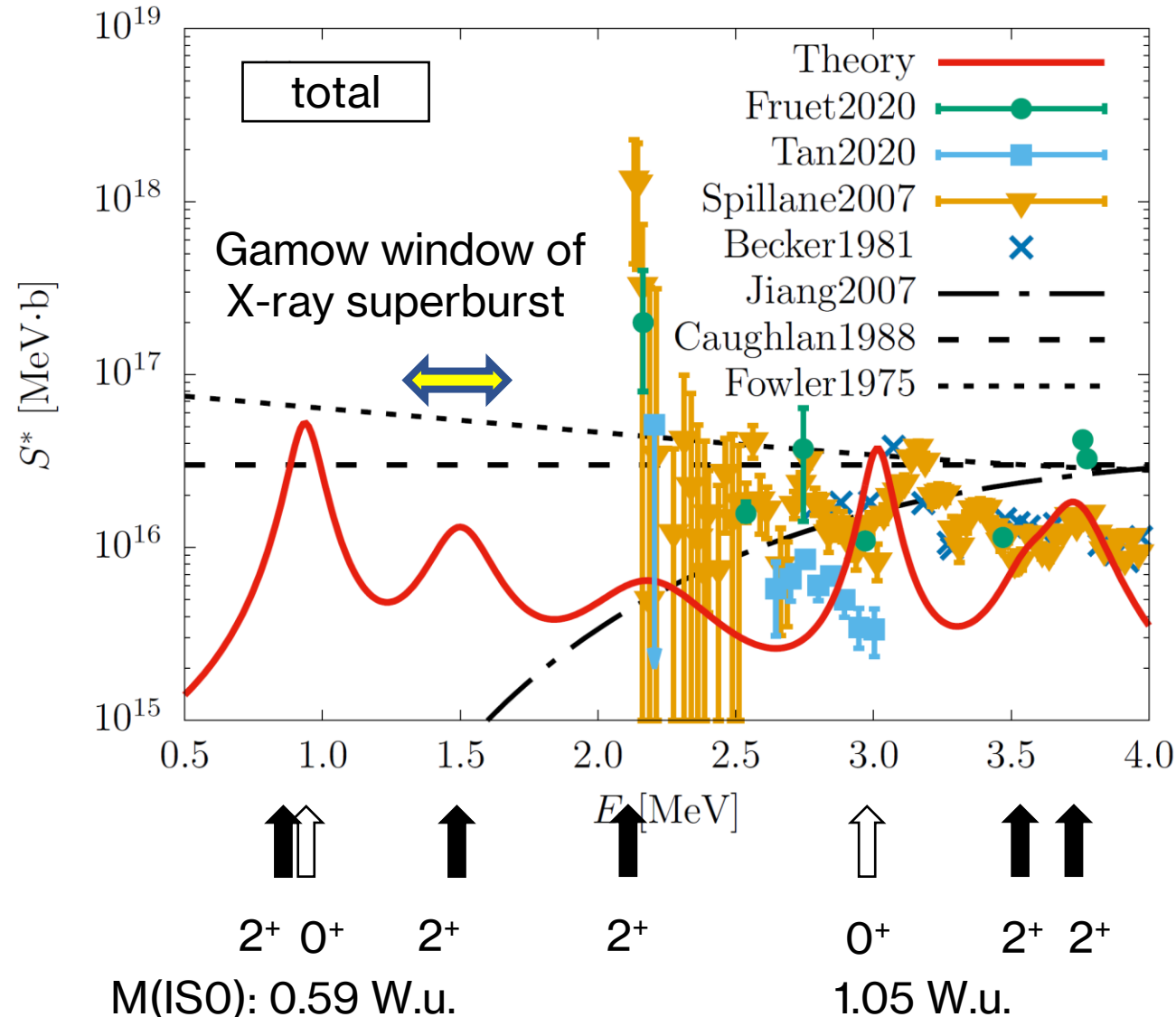
- Low-lying states of ^{24}Mg are well reproduced.
- Framework of AMD with Gogny D1S force works well for ^{24}Mg .

S factors of the p and α -decay channels



- Existence of the 2.1-MeV and 3.8-MeV resonances are reproduced.
- The S_{p_1} factors are reproduced well at the $E > 3$ MeV region.
- The S_{α_1} factors are underestimated.

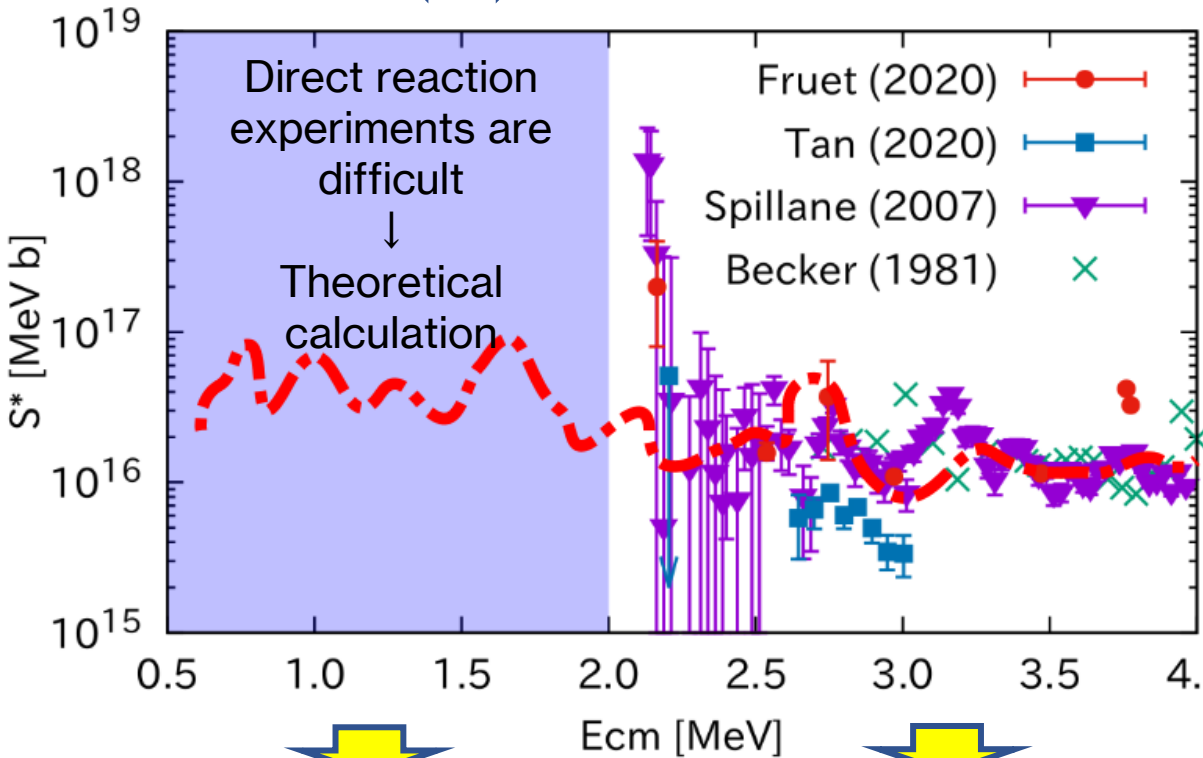
Astrophysical S factor



- The 1.5-MeV resonance is predicted.
 - Related to **X-ray superburst**?
- Theoretical S factors are **much larger** than the **hindrance extrapolation** (Jiang2007) in $E < 2$ MeV.
 - Contributions of **resonances** are **dominant**.
- We plan to compare with inelastic scattering data to check the reliability of the wave functions.
 - Matrix elements (ISO, ...)
 - Spin and parity
 - Branching ratio

Strategy to the reliable fusion cross sections

Gamow window of X-ray superburst



1. Calculate **wave functions of resonances** and **fusion cross sections**.
2. Check the reliability of wave functions with comparison with experimental data.
 - **Fusion cross sections** by direct fusion reactions
 - **Transition strengths** of ISO, ..., by inelastic scattering

Other systems

- $^{12}\text{C} + ^{16}\text{O}$
- $^{16}\text{O} + ^{16}\text{O}$
-

Inelastic scattering (ISO, ...)

Fusion cross sections by direct reaction experiments

Summary

- We have investigated the properties of the $^{12}\text{C} + ^{12}\text{C}$ resonances using the antisymmetrized molecular dynamics.
- We have treated the coupling of the entrance ($^{12}\text{C} + ^{12}\text{C}$) and exit channels ($\alpha + ^{20}\text{Ne}$) explicitly and rotation of clusters.
- The astrophysical S factors are reasonably reproduced.
- Low-energy resonance states with $E < 2$ MeV are predicted, which may be important for X-ray superbursts.
- We plan to compare with inelastic scattering data to check the reliability of the wave functions of the $^{12}\text{C} + ^{12}\text{C}$ resonances with $E < 2$ MeV.
- Our method can apply to other reactions such as $^{16}\text{O} + ^{16}\text{O}$, $^{12}\text{C} + ^{16}\text{O}$, and so on.