EFT OF LSS IN MODIFIED GRAVITY AND ITS APPLICATION

(in prep.)

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第10回観測的宇宙論ワークショップ 11/17-20 2021

Introduction

- **One of the origin for the late-time acc.:** Modified Gravity (MG)

Scalar-Tensor theories: f(R)<Horndeski< DHOST

(recent) U-DHOST, …

Horndeski (1974)
Deffayet+ (2011)
Kobayashi+ (2011)
Langlois-Noui (2015)
De Felice+ (2018)
…

- **Non-linear scalar interactions**

Large Scale Structure (LSS) in DHOST *Hirano+ (2018), Hirano+ (2020)*

folded-type bispectrum, **UV divergent** 1-loop power

violation of consistency relation *Lewandowski+ (2019), Lewandowski (2019)*

Introduction

- **One of the origin for the late-time acc.:** Modified Gravity (MG)

Scalar-Tensor theories: f(R)<Horndeski< DHOST *Horndeski (1974)*

Deffayet+ (2011)

(recent) U-DHOST, … *(2011)*

ui (2015)

2018)

This talk

Rescue of UV-divergent 1-loop power

1. we extend the procedure of EFTofLSS to MG
2. its application to DHOST

folded-type bispectrum, **UV divergent** 1-loop power

violation of consistency relation *Lewandowski+ (2019), Lewandowski (2019)*

SPT in LSS

cf) Barnardeau+ (2000)

- non-rela. irrotational dust (DM fluid), inside cosmological horizon
- metric $ds^2 = -(1+2\Phi)dt^2 + (1-2\Psi)d\mathbf{x}^2$
- fluid variables $\delta = \dfrac{\rho - \rho_b}{\rho_b}$, $\theta = \partial_i u^i/(aH)$
- Einstein eq. → Poisson eq. $\partial^2\Phi = \dfrac{3}{2}a^2H^2\Omega_m\delta$
- fluid equations

$$\text{(continuity)}\ \dot{\delta} + \frac{1}{a}\partial_i[(1+\delta)]u^i = 0$$

$$\text{(Eular)}\ \dot{u}_i + Hu_i + \frac{1}{a}u^j\partial_j u_i + \frac{1}{a}\partial_i\Phi = 0$$

- gravity: **Poisson**, non-linearity (**fluid)**

1-loop power spectrum

cf) Barnardeau+ (2000)
Maki+ (1993)

- $\delta(t,\mathbf{p}) = \delta_1(t,\mathbf{p}) + \delta_2(t,\mathbf{p}) + \delta_3(t,\mathbf{p})$

  leading

  $< \delta\delta > = < \delta_1\delta_1 > + < \delta_2\delta_2 > + 2 < \delta_1\delta_3 >$

- $\delta_1(t,\mathbf{p}) = D_+(t)\delta_L(\mathbf{p})$, $\delta_L(\mathbf{p})$: initial density fluc.

$$\delta_2(t,\mathbf{p}) = \frac{D_+^2}{(2\pi)^3}\int d^3k_1^3k_2\delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{p})F_2(\mathbf{k}_1,\mathbf{k}_2)\delta_L(\mathbf{k}_1)\delta_L(\mathbf{k}_2)$$

$$\delta_3(t,\mathbf{p}) = \frac{D_+^3}{(2\pi)^3}\int d^3k_1d^3k_2d^3k_3\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3-\mathbf{p})F_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\delta_L(\mathbf{k}_1)\delta_L(\mathbf{k}_2)\delta_L(\mathbf{k}_3)$$

- $< \delta_1(t,\mathbf{p})\delta_3(t,\mathbf{p}') > = (2\pi)^3\delta_D(\mathbf{p}+\mathbf{p}')P_{13}(t,p)$ $\quad < \delta_L(\mathbf{p})\delta_L(\mathbf{p}') > = (2\pi)^3\delta_D(\mathbf{p}+\mathbf{p}')P_L(p)$

$$P_{13}^{\mathrm{UV}}(t,p) \to \quad -\frac{61}{315}\frac{D_+^4}{(2\pi)^4}p^2P_L(p)\int_{p\ll k} dk\ P_L(k)$$

standard: $P_L(k) \sim k(k \ll k_{\mathrm{eq}}),\ k^{-3}(k_{\mathrm{eq}} \ll k)$ **not diverge**

EFTofLSS in GR

Baumann+ (2010), Carrasco+(2012), Hertzberg (2012)

- **Loop corrections include UV integrals**

SPT is not taken into account of effects of small scale (UV) physics.

e.g.) perturbative expansion may be wrong

Blas+ (2013): linear $\gg$ 3-loop > 2-loop even if on large scales

→ we need to introduce effects of UV physics

- **EFTofLSS** (cf. Wilson's renormalization)

integrating out short modes & introduce effective fluid as UV physics

→ effective fluid can cancel out large (or divergent) terms

= eff. fluid are **counterterms**

EFTofLSS in GR

Baumann+ (2010), Carrasco+(2012), Hertzberg (2012)

- **separation of fields &introducing coarse-graind fields**

$$\delta = \delta_l + \delta_s \quad , \delta_l = \int d^3x' W_\Lambda(\mathbf{x}, \mathbf{x}')\delta(\mathbf{x}')$$, W_Λ: window function

→ we can rewrite the system about short/long modes

- **Eular eq. has s-s nonlinear interactions** $u^j\partial_j u^i = u_l^j \partial_j u_l^i - \frac{1}{\rho}\partial_j \tau_\Lambda^{ij}$ (s-s)

- **Integrating out small scale ~ counterterms from UV**

→averaging on log-w. BG and **introducing eff. fluid (NS like)**

$$\rho^{-1} < \tau_\Lambda^{ij} >_{\delta_l} = \left(c_s^2 \delta_l - \frac{c_{bv}^2}{aH}\partial_k u_l^k \right)\delta^{ij} - \frac{3}{4}\frac{c_{sv}^2}{aH}\left(\partial_l^i u_l^j + \partial_l^j u_l^i - \frac{2}{3}\partial_k u_l^k \delta^{ij} \right) + \mathcal{O}\left(\frac{\partial^2}{\Lambda^2} \right)$$

→ regarded as source terms (we assume $c = \mathcal{O}(\delta_l^2)$)

EFTofLSS in GR

Baumann+ (2010), Carrasco+(2012), Hertzberg (2012)

- **Standard terms**

$P_L(P)$: δ_L is replaced to δ_l

$P_{13}(t,p)$: δ_L is replaced to δ_l , $\int^{\Lambda} d^3k \;\rightarrow\; P_{13}^{\mathrm{UV}} \sim \text{finite} \times p^2 P_L(p)$

- **EFT terms**

$$\delta_c = p^2 \int^t d\tilde{t} G(t,\tilde{t}) \left(c_s^2 - \frac{c_v^2}{aH} \right) \delta_l \quad \leftarrow \mathcal{O}(\delta_l^3)$$

, G is Green's func. in evolution eq.

$$P_{\delta_1 \delta_c} = p^2 P_L(p) \frac{D_+(t)}{(2\pi)^3} \int^t d\tilde{t} G(t,\tilde{t}) \left(c_s^2 - \frac{c_v^2}{aH} \right) D_+(\tilde{t}) \sim \text{finite} \times p^2 P_L(p)$$

→ EFT terms can cancel out leading contribution on P_{13}^{UV}

Renormalization & parameter fitting

Baumann+ (2010), Carrasco+(2012), Hertzberg (2012)

$$P_{\delta_1\delta_c} = C_{\mathrm{comb}}(t,\Lambda)p^2 P_L(p),\ C_{\mathrm{comb}}(t,\Lambda) := C_{\mathrm{ren}}\left(t,k_{\mathrm{ren}}\right) + C_{\mathrm{ctr}}\left(t,k_{\mathrm{ren}},\Lambda\right)$$

- **Renormalization conditions**

$$P_{13}^{\mathrm{UV}}\left(t,k_{\mathrm{ren}}\right) + C_{\mathrm{ctr}}\left(t,k_{\mathrm{ren}},\Lambda\right)k_{\mathrm{ren}}^2 P_L(k_{\mathrm{ren}}) = 0$$

- **Parameter fitting by obs. or simulations**

$$P_{\mathrm{obs}}(t,k_{\mathrm{ren}}) = D_+^2(t)P_L(k_{\mathrm{ren}}) + \cancel{P_{22}(t,k_{\mathrm{ren}})} + C_{\mathrm{ren}}(t,k_{\mathrm{ren}})k_{\mathrm{ren}}^2 P_L(k_{\mathrm{ren}}) + \cancel{P_{13}^{\mathrm{sub}}(t,k_{\mathrm{ren}})}$$

$$\rightarrow \quad C_{\mathrm{ren}}(t,k_{\mathrm{ren}}) = \frac{P_{\mathrm{obs}}(t,k_{\mathrm{ren}}) - D_+^2(t)P_L(k_{\mathrm{ren}})}{k_{\mathrm{ren}}^2 P_L(k_{\mathrm{ren}})}$$

※ From $P_{\theta\theta}$, we can determine the time-dependece of EFT parameters.

EFTofLSS in MG

- **EFTofLSS in GR**

  s-s interactions come from Euler eq.

  → eff. fluid becomes counterterms

- **EFTofLSS in MG** **Hirano, Fujita in prep.**

  s-s interactions come from gravitational field eqs.

$$\partial^2\Phi + \sigma_{\Phi\gamma}[(\partial^2\Phi)^2 - (\partial_i\partial_j\Phi)^2] + \sigma_{\Phi\alpha}[(\partial^2\Phi)^2 + \partial_i\Phi\partial^2\partial_j\Phi] = \frac{3}{2}a^2H^2\Omega_m\delta$$

EFTofLSS in MG

- **EFTofLSS in GR**

s-s interactions come from Euler eq.

→ eff. fluid becomes counterterms

- **EFTofLSS in MG** **Hirano, Fujita in prep.**

given by MG model

s-s interactions come from gravitational field eqs.

$$\partial^2\Phi + \sigma_{\Phi\gamma}[(\partial^2\Phi)^2 - (\partial_i\partial_j\Phi)^2] + \sigma_{\Phi\alpha}[(\partial^2\Phi)^2 + \partial_i\Phi\partial^2\partial_j\Phi] = \frac{3}{2}a^2H^2\Omega_m\delta$$

long →

$$\partial^2\Phi_l \quad +c_2\partial^2\Phi_l + c_3\frac{\partial^2}{a^2\Lambda^2}\partial^2\Phi_l \quad = \frac{3}{2}a^2H^2\Omega_m\delta_l$$

new source terms!

where $c_2 = \mathcal{O}(\delta_l^2)$ and $c_3 = \mathcal{O}(\delta_l)$ in order to be counterterms

Application to DHOST

- **counterterms in MG**

$$P_{\delta_1\delta_{c2}} = P_L(p)\frac{D_+}{(2\pi)^3}\int^t d\tilde{t}G(t,\tilde{t})\left(c_2\frac{3}{2}a^2H^2\Omega_m D_+\right) = \text{finite} \times P_L(p)$$

$$P_{\delta_1\delta_{c3}} = p^2P_L(p)\frac{D_+}{(2\pi)^3}\int^t d\tilde{t}G(t,\tilde{t})\left(c_3\frac{3}{2}\frac{H^2}{\Lambda^2}\Omega_m D_+\right) = \text{finite} \times p^2P_L(p)$$

- **UV-divergent 1-loop power** *Hirano+ (2020)*

UV divergent

$$P_{13}(t,p) \to \quad P_L(p)\frac{D_+^4}{(2\pi)^4}\int_{p\ll k}^{\Lambda} dk\ P_L(k)\left(d_{\mathrm{DHOST}}k^2 + d_{\mathrm{GR,Horn}}p^2\right)$$

(standard: $P_L(k) \sim k(k \ll k_{\mathrm{eq}}),\ k^{-3}(k_{\mathrm{eq}} \ll k)$)

→ $P_{\delta_1\delta_{c2}}$ can cancel first term and $P_{\delta_1\delta_{c3}}$ can cancel second one!

Renormalization & EFT paras.

$$P_{\delta_1\delta_{c2}} = C_2(t,\Lambda)P_L(p), \quad C_i(t,\Lambda) := C_i^{\rm ren}\left(t, k_{\rm ren}\right) + C_i^{\rm ctr}\left(t, k_{\rm ren}, \Lambda\right)$$

$$P_{\delta_1\delta_{c3}} = C_3(t,\Lambda)p^2 P_L(p)$$

- **Renormalization conditions**

$$P_L(k_{\rm ren})\frac{D_+^4}{(2\pi)^4}\int_{k_{\rm ren}\ll p}^{\Lambda} dp\ P_L(p)\ d_{\rm DHOST}p^2 + C_2^{\rm ctr}\left(t, k_{\rm ren}, \Lambda\right) P_L(k_{\rm ren}) = 0$$

$$k_{\rm ren}^2 P_L(k_{\rm ren})\frac{D_+^4}{(2\pi)^4}\int_{k_{\rm ren}\ll p}^{\Lambda} dp\ P_L(p) d_{\rm GR,Horn} + C_3^{\rm ctr}\left(t, k_{\rm ren}, \Lambda\right) k_{\rm ren}^2 P_L(k_{\rm ren}) = 0$$

- **Running relations** (cf. Callan-Symanzik equation)

$$C_i^{ctr'}(k_{\rm ren}) + [C_i^{ctr}(k_{\rm ren}) + A(t)]\frac{P_L'(k_{\rm ren})}{P_L(k_{\rm ren})} = 0 \quad \rightarrow \quad C_i^{ctr}(k_{\rm ren}) \propto 1/k_{\rm ren}$$

- **EFT parameters** $C_2,\ C_3$ cannot be determined independently.

Summary

- Loop corrections include UV integral

  This leads to some problem inSPT

  → EFTofLSS introduce effectively UV physics

  and EFT terms can cancel out problematic terms

- we extend the procedure of EFTofLSS in GR to MG

  counterterms come from non-linearity of gravitational field eqs.

→ similar to that in GR, EFT terms can cancel out problematic terms

in particular, UV divergence in DHOST can be cancelled out!