Linear growth of structure in Projected Massive Gravity

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Based on on-going work with Rampei Kimura(WIAS)

KYOTO UNIVERSITY

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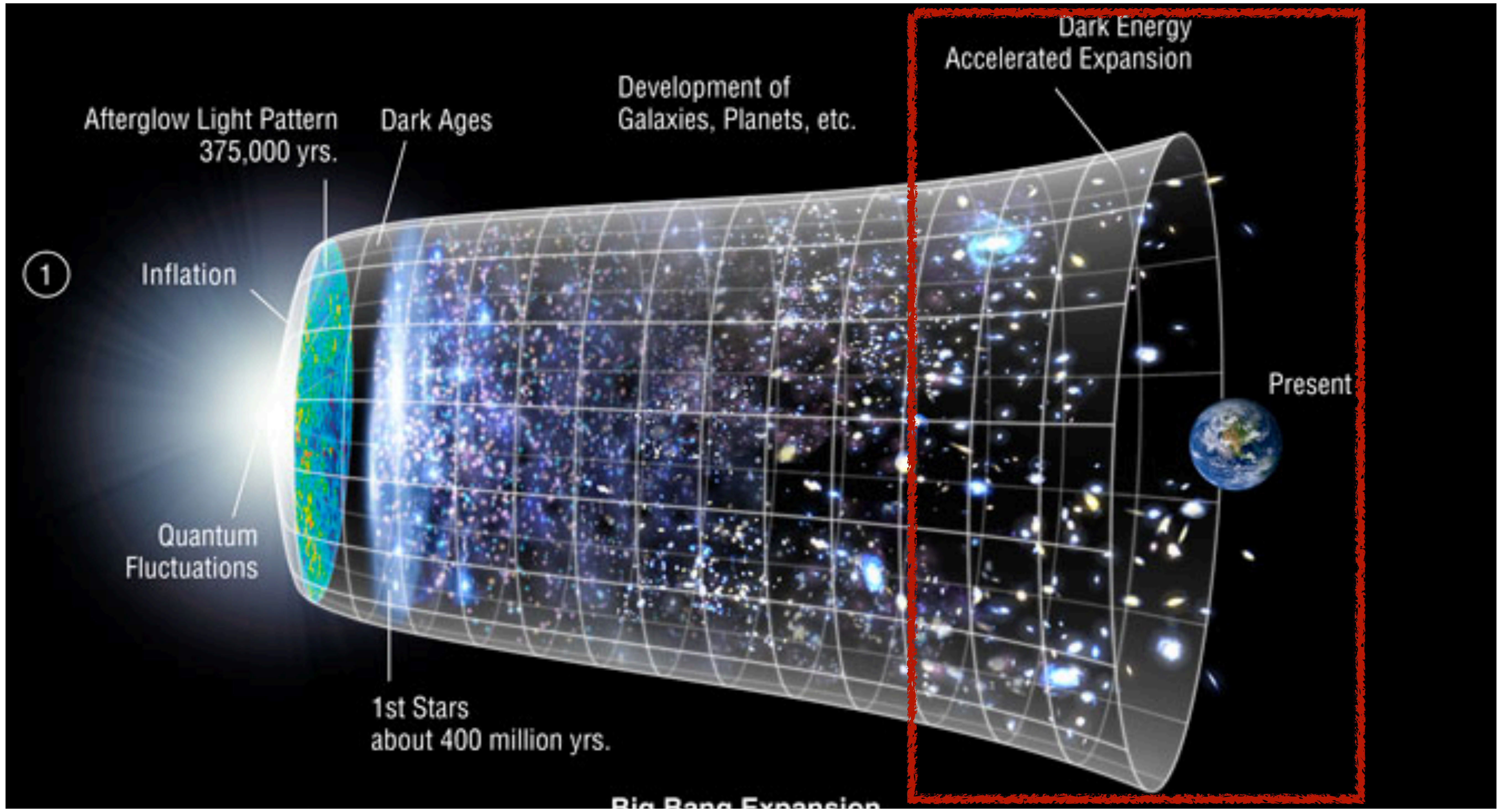
What accelerate the current universe?

Dark energy? Modified gravity?



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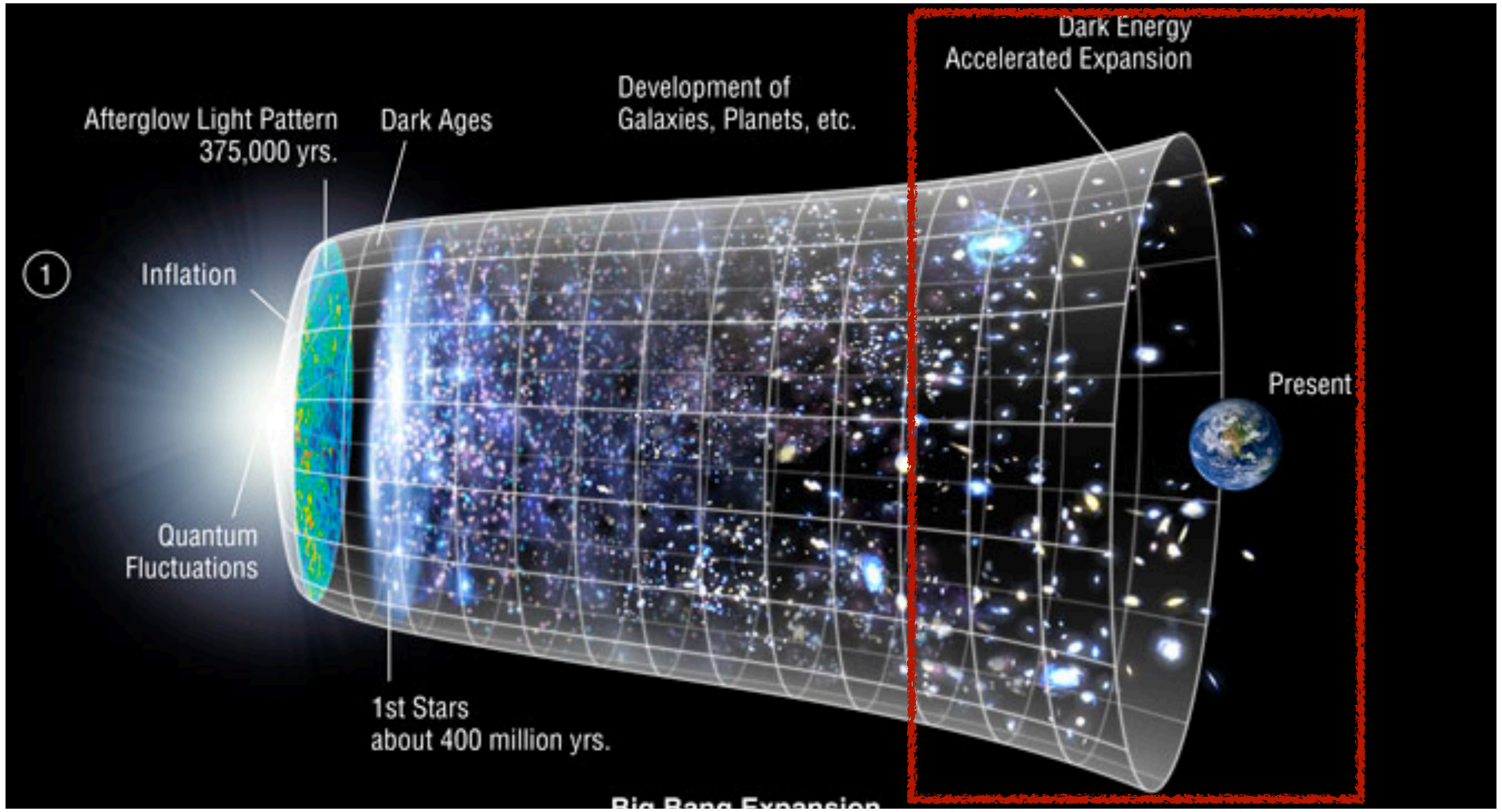

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Massive gravity

$$\mathcal{L}_{\mathrm{grav}} = \frac{M_{\mathrm{Pl}}^2}{2}\sqrt{-g}R + \mathcal{L}_{\mathrm{mass}}$$

Massive spin 2 field theory

↓

Light mass modifies the law of gravity at long distances

↓

Accelerated expansion without dark energy?

Mass term

$$\mathscr{L}_{\mathrm{grav}} = \frac{M_{\mathrm{Pl}}^2}{2}\sqrt{-g}R + \mathscr{L}_{\mathrm{mass}}$$

[Fierz, Pauli (1939)]

- **Linear massive gravity**
  linear with respect to $h_{\mu\nu} := g_{\mu\nu} - \eta_{\mu\nu}$ in EOM

$$\mathscr{L}_{\mathrm{grav}} = \mathscr{L}_{\mathrm{GR}}^{(2)} - \frac{M_{\mathrm{Pl}}^2 m^2}{2}\eta^{\mu\nu}\eta^{\mu\nu}(h_{\mu\alpha}h_{\nu\beta} - h_{\mu\nu}h_{\alpha\beta})$$

✗ Conflict with the test of gravity in solar system

[van Dam, Veltman, Zakharov (1970)]

Mass term

$$\mathscr{L}_{\rm grav} = \frac{M_{\rm Pl}^2}{2}\sqrt{-g}R + \mathscr{L}_{\rm mass}$$

- **Linear massive gravity**
  linear with respect to $h_{\mu\nu} := g_{\mu\nu} - \eta_{\mu\nu}$ in EOM

[Fierz, Pauli (1939)]

$$\mathscr{L}_{\rm grav} = \mathscr{L}^{(2)}_{\rm GR} - \frac{M_{\rm Pl}^2 m^2}{2}\eta^{\mu\nu}\eta^{\mu\nu}(h_{\mu\alpha}h_{\nu\beta} - h_{\mu\nu}h_{\alpha\beta})$$

✗ Conflict with the test of gravity in solar system

[van Dam, Veltman, Zakharov (1970)]

- **Non-linear massive gravity**
  non-linear with respect to $h_{\mu\nu} := g_{\mu\nu} - f_{\mu\nu}$ in EOM

(fiducial metric: $f_{\mu\nu}$)

$$\mathscr{L}_{\rm grav} = \frac{M_{\rm pl}^2}{2}R[g_{\mu\nu}] - M_{\rm Pl}^2 m^2 U_{\rm mass}[g_{\mu\nu}, f_{\mu\nu}]$$

✗ Most non-linear massive gravity are ghostly···

[Boulware, Deser (1972)]

dRGT massive gravity

✅ **dRGT** theory is the first proposed **ghost-free** non-linear massive gravity. [de Rham, Tolly & Gabadaze, (2010)]

$$\mathcal{L}_{mass}[f, g] = M_{\rm Pl}^2 m_g^2 \left[U_2(\mathcal{K}) + \alpha_3 U_3(\mathcal{K}) + \alpha_4 U_4(\mathcal{K}) \right]$$

U_n: Symmetric polynomials of $\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \left(\sqrt{g^{-1}f}\right)^\mu{}_\nu$

α_n: constant parameters

$$U_2 = \frac{1}{2}([\mathcal{K}]^2 - [\mathcal{K}^2]), \quad U_3 = \frac{1}{6}([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]),$$

$$U_4 = \frac{1}{24}([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4])$$

dRGT massive gravity

✅ **dRGT** theory is the first proposed **ghost-free** non-linear massive gravity. [de Rham, Tolly & Gabadaze, (2010)]

❌ No viable solution for FLRW cosmology in dRGT

- Strongly coupling problem
  [Gümrüolüoğlu, Lin & Mukohyama, (2011)]
- Non-linear instability
  [Gümrüolüoğlu, Lin & Mukohyama, (2012)]

Extension of dRGT

ϕ^a: Stuckelberg fields.

4 scalar like NG boson.

$$\phi^a \to \Lambda^a{}_b \phi^b\,,$$
$$\phi^a \to \phi^a + c^a$$

- Assumptions of dRGT

1. It has Minkowski solution as vacuum solution.
2. No Boulware-Deser ghost at the decoupling limit.
3. ~~It holds internal **Poincaré symmetry**~~. → Remove!

It's not necessary for general massive gravity.

Internal ~~**Lorentz sym.**~~

$$\phi^a \to \Lambda^a{}_b \phi^b, \quad \phi^a \to \phi^a + c^a$$

Phase of massive gravity [Dubovsky (2004)]

Minimal theory of massive gravity [de Felice, Mukohyama(2015)]

etc

Internal ~~**Translation sym.**~~

$$\phi^a \to \Lambda^a{}_b \phi^b, \quad \phi^a \to \phi^a + c^a$$

Generalized massive gravity [De Rham, Keltner, Tolly(2014)]

Projected massive gravity [Gümrükcüoğlu, Kimura, Koyama(2020)]

No strong coupling problem in FLRW!

Projected massive gravity

[Gümrükçüoğlu, Kimura & Koyama(2020)]

Assumption	$\phi^a \to \Lambda^a{}_b \phi^b$, ~~$\phi^a \to \phi^a + c^a$~~	5dof (No BD ghost)

$$S_{\rm grav} = \int d^4x \sqrt{-g} \frac{M_{\rm Pl}^2}{2} \left[G(X) R + \frac{6G'(X)^2}{G(X)} [Y] + m^2 U(X, [Z], [Z^2], [Z^3]) \right]$$

Non-minimal coupling

Counter term

New graviton mass

Projection op.

Where, $X = \eta_{ab}\phi^a\phi^b$, $Y^\mu{}_\nu = \left(g^{-1}f^{(G)}\right)^\mu{}_\nu$, $Z^\mu{}_\nu := \left(g^{-1}f^{(p)}\right)^\mu{}_\nu$

$$f^{(G)}_{\mu\nu} = \phi_a\phi_b\partial_\mu\phi^a\partial_\nu\phi^b, \quad f^{(p)}_{\mu\nu} := \left(\eta_{ab} - \frac{\phi_a\phi_b}{X}\right)\partial_\mu\phi^a\partial_\nu\phi^b$$

Generalized Massive Gravity also holds the assumption. $\longrightarrow$

[de Rham, Keltner & Tolly, (2014)]
[Gümrükçüoğlu, Kimura & Koyama(2020)]
[Kenna-Allison, Gümrükçüoğlu & Koyama(2020)]
[Kenna-Allison, Kimura, Gümrükçüoğlu & Koyama(2021)]

Our goal

We investigate the linear growth of density fluctuations after matter dominance with projected massive gravity.

$$S = \int d^4x \sqrt{-g} \frac{M_{\rm Pl}^2}{2} \left[G(X)\, R + \frac{6G'(X)^2}{G(X)}[Y] + m^2 U(X, [Z], [Z^2], [Z^3]) \right] + S_{\rm m}[g, \psi] \,,$$

Non-minimal coupling | Counter term | New graviton mass | CDM

Set-up

Back ground equations

- Open FLRW metric

$$ds^2 = -dt^2 + a(t)^2\left(\delta_{ij} - \frac{\kappa\ x^i x^j}{1+\kappa\ x^k x^k}\right)dx^i dx^j$$

- Stückelberg fields

$$\phi^0 = f(t)\sqrt{1+\kappa(x^2+y^2+z^2)}\,, \qquad \phi^i = f(t)\sqrt{\kappa}\,x^i\,,$$

- Background equation

$$3G\left[\left(H+\frac{\dot{G}}{2G}\right)^2 - \frac{\kappa}{a^2}\right] = \frac{\rho}{M_{\rm Pl}^2} + \frac{\rho_g}{M_{\rm Pl}^2}$$

Friedmann equation

$$\dot{\rho}_g + 3H\left(\rho_g + p_g\right) - \frac{\dot{G}}{2G}\left(\rho_g - 3p_g + \rho\right) = 0$$

Stückelberg equation

$$\dot{\rho} + 3H\rho = 0$$

Conservation of CDM

where,

$$T_{\mu\nu}^{\rm mass} := \frac{-2}{\sqrt{-g}}\frac{\delta S_{\rm mass}}{\delta g^{\mu\nu}} = {\rm diag}.\left(\rho_g, p_g, p_g, p_g\right)$$

Effective energy-momentum tensor for the mass term

$$T_{\mu\nu}^{\rm matter} := {\rm diag}.\left(\rho,0,0,0\right)$$

Energy-momentum tensor for the CDM.

Linear perturbations

- Gauge fixing $\delta\phi^a = 0$
- Metric perturbation
$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -(1+2\phi)dt^2 + 2a\partial_i B dt dx^i + a^2(\Omega_{ij} + h_{ij})dx^i dx^j\,,$$
$$h_{ij} = 2\psi\Omega_{ij} + \left(D_i D_j - \frac{1}{3}\Omega_{ij}D_l D^l\right)S\,.$$
- Matter perturbation
$$T^0{}_0 = -(\rho + \delta\rho)\,,\quad T^0{}_i = a\rho(\partial_i B + a\partial_i v)\,,\quad T^i{}_0 = -\rho\partial^i v\,,\quad T^i{}_j = 0$$
- Gauge invariants
$$\Phi = \phi - \dot{\zeta}\,,\qquad \Psi = \psi - H\zeta - \frac{1}{6}D^i D_i S\,,\quad \tilde{v} = v + \frac{1}{2}\dot{S}\,,$$
where, $\zeta \equiv -aB + \frac{1}{2}a^2\dot{S}\,.$

$$\Delta \equiv \frac{\delta\rho - \dot{\rho}\zeta}{\rho} - 3a^2 H\tilde{v}\,,$$
Density perturbation

- Linear EOM can be expressed only with these gauge invariants.

Linear Equation of motion

(0,0)

$$0=3G\left(2H+\frac{\dot G}{G}\right)^2\Phi-\frac{3}{2}G\left(2H+\frac{\dot G}{G}\right)\left(4\dot\Psi+2\frac{\dot G}{G}a\dot B-a^2\frac{\dot G}{G}\ddot S\right)-\left(\frac{4(k^2+3\kappa)}{a^2}G+\frac{6(\rho_g+p_g)}{M_{\rm Pl}^2}\right)\Psi$$
$$+6a^2H\frac{\rho}{M_{\rm Pl}^2}\tilde v+k^2\frac{\rho_g+p_g}{M_{\rm Pl}^2}S+\frac{2\rho}{M_{\rm Pl}^2}\Delta+\left((k^2+3\kappa)\dot G+3a^2\frac{\rho+\rho_g+p_g}{2M_{\rm Pl}^2}\frac{\dot G}{G}\right)\left(\dot S-2\frac{B}{a}\right)$$
$$+\frac{3}{4}a^2G\left(2H+\frac{\dot G}{G}\right)\left[\left\{2\left(H\frac{\dot G}{G}+\frac{\ddot G}{G}\right)-3\frac{\dot G^2}{G^2}\right\}\dot S+2\left(3\frac{\dot G^2}{G^2}-2\frac{\ddot G}{G}\right)\frac{B}{a}\right],$$

(0,i)

$$0=-4G\left[\left(H+\frac{\dot G}{2G}\right)\Phi-\dot\Psi\right]-\frac{2a^2\rho}{M_{\rm Pl}^2}\tilde v+2a\dot G\dot B+a\left(2\ddot G-\frac{3\dot G^2}{G}\right)B-a^2\dot G\ddot S-a^2\frac{\rho_g+p_g}{M_{\rm Pl}^2}\dot S$$
$$-a^2\left(\ddot G+H\dot G-\frac{3\dot G^2}{2G}\right)\dot S,$$

(i,j) trace

$$0=6G\left(2H+\frac{\dot G}{G}\right)\dot\Phi-12G\left(3H+\frac{\dot G}{G}\right)\dot\Psi-\left(4G\frac{k^2-3\kappa}{a^2}+\frac{12p_g}{M_{\rm Pl}^2}\right)\Phi-\left(4G\frac{k^2+3\kappa}{a^2}-6\mathcal{A}\right)\Psi-12G\ddot\Psi$$
$$-k^2\mathcal{A}S+\frac{3a^2}{2}G\left(4\frac{\ddot G}{G}+12H\frac{\dot G}{G}-3\frac{\dot G^2}{G^2}\right)\ddot S-3aG\left(4\frac{\ddot G}{G}+8H\frac{\dot G}{G}-3\frac{\dot G^2}{G^2}\right)\dot B$$
$$-a\left(3\mathcal{C}_B+4\dot G\frac{k^2+3\kappa}{a^2}\right)B+a^2\left(\mathcal{C}_{\dot S}+2\dot G\frac{k^2+3\kappa}{a^2}\right)\dot S+3a\dot G(a\dddot S-2\ddot B),$$

(i,j) traceless

$$0=-2(\Phi+\Psi)-2a\frac{\dot G}{G}B+\frac{a^2}{G}(M_{\rm GW}^2S+\dot G\dot S),$$

Linear Einstein equation

$$0=\left[2(k^2+3\kappa)+\frac{3a^2}{G}\frac{\rho+\rho_g+p_g}{M_{\rm Pl}^2}-3a^2\left(4H^2+\frac{\dot G}{G}H+\frac{3\dot G^2}{2G^2}-\frac{\ddot G}{G}\right)\right]\tilde v-2(3a^2H\dot{\tilde v}+\dot\Delta+3\dot\Psi),$$
$$0=\Phi+a^2(2H\tilde v+\dot{\tilde v}),$$

Conservation law of CDM

Models

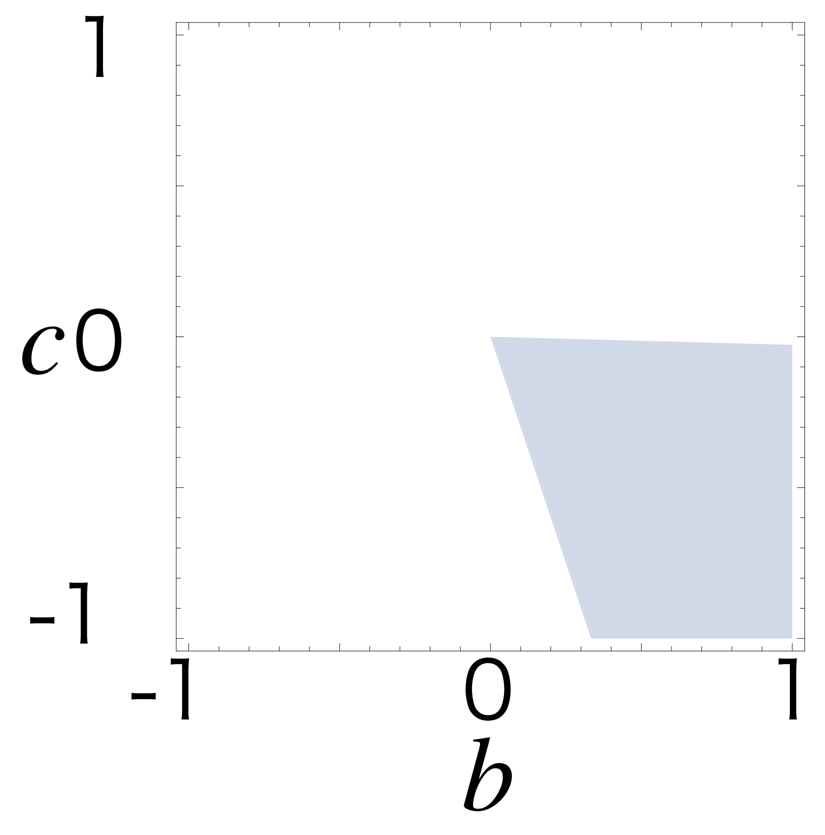
$$S_{\rm PMG} = \int d^4x\sqrt{-g}\frac{M_{\rm Pl}^2}{2}\left[G(X)R + \frac{6G'(X)^2}{G(X)}[Y] + m^2 U(X,[Z],[Z^2],[Z^3])\right]$$

- In order to solve, we specify arbitrary functions.

Non-minimal coupling	Mass potential
$G = 1$	$U = \left(a_1 + a_2 m^2 X\right)[Z] + b[Z]^2 + c[Z^2]$

- The model has a stable parameter regions without ghosts, tachyons, and gradient instability!

$$a_1 < 0 \ \wedge \ a_2 < 0$$



Result

Background solution: Stückelberg field

- Background equation

$$3G\left[\left(H+\frac{\dot G}{2G}\right)^2-\frac{\kappa}{a^2}\right]=\frac{\rho}{M_{\rm Pl}^2}+\frac{\rho_g}{M_{\rm Pl}^2}$$ Friedmann equation

$$\dot\rho_g+3H\left(\rho_g+p_g\right)-\frac{\dot G}{2G}\left(\rho_g-3p_g+\rho\right)=0$$ Stückelberg equation

$$\dot\rho+3H\rho=0$$ Conservation of CDM

→

Model: $G=1,\quad U=(a_1+m^2a_2X)[Z]+b[Z]^2+c[Z^2]$

Stückelberg equation $\quad \xi := \frac{\sqrt{\kappa}f}{a}$

$$\xi(\dot\xi+H\xi)\left[a_1\kappa+2(-a_2m^2a^2+(3b+c)\Omega_\kappa)\xi^2\right]=0$$

- **Solutions**

$\xi=0$	$\xi=\frac{\rm const.}{a}$	$\xi=\sqrt{\frac{a_1\kappa}{2a_2m^2a^2-2\kappa(3b_1+c_1)}}$
$\rho_g=0$	$\rho_g=\frac{\rm const.}{a^2}+\frac{\rm const.}{a^4}$	$\rho_g=\frac{3a_1^2m^2M_{\rm pl}^2\kappa}{8(3b+c)\kappa-8a_2m^2a^2}$
	radiation+curvature	dynamical Λ

Hubble parameter and equation of state

The solution of the Stückelberg eq.

$$\xi = \sqrt{\frac{a_1\kappa}{2a_2m^2a^2 - 2\kappa(3b_1 + c_1)}}$$

↓

Friedmann equation

$$\left(\frac{H(a)}{H_0}\right)^2 = \frac{8a^2a_2\mu^2\Omega_{\rm m} + a^3(a_1^2 - 8a_2)\mu^2\Omega_\kappa + 8(3b + c)\Omega_{\rm m} + 8a(3b + c)\Omega_\kappa^2}{8a_2\mu^2a^5 - 8(3b + c)\Omega_\kappa a^3}$$

Equation of state for mass term

$$w = \frac{p_g}{\rho_g} = -\frac{3\Omega_\kappa(3b_1 + c_1) - a_2\mu^2a^2}{3\Omega_\kappa(3b_1 + c_1) - 3a_2\mu^2a^2}$$

where

$$m = \mu H_0\,,$$

$$\kappa = H_0^2\Omega_\kappa\,,$$

$$\rho = \frac{3H_0^2M_{\rm pl}^2\Omega_m}{a^3}$$

Normalization

At present time,

$$H(a)\big|_{a=1} = H_0\,,$$
$$w(a)\big|_{a=1} = w_0\,,$$

$$a_1\mu = -\sqrt{\frac{-16\Omega_\Lambda}{1+3w_0}(3b+c)}$$

$$a_2\mu^2 = \frac{3(w_0+1)}{3w_0+1}\Omega_\kappa(3b+c)$$

parameters

$$m = \mu H_0\,,$$
$$\kappa = H_0^2\Omega_\kappa\,,$$
$$\rho = \frac{3H_0^2M_{\rm pl}^2\Omega_m}{a^3}$$
$$\Omega_\Lambda := 1 - \Omega_{\rm m} - \Omega_\kappa$$

Background is parametrized only by $\{w_0, H_0, \Omega_{\rm m}, \Omega_\kappa\}$.

The mass parameter μ is canceled.

Equation of state for graviton mass term

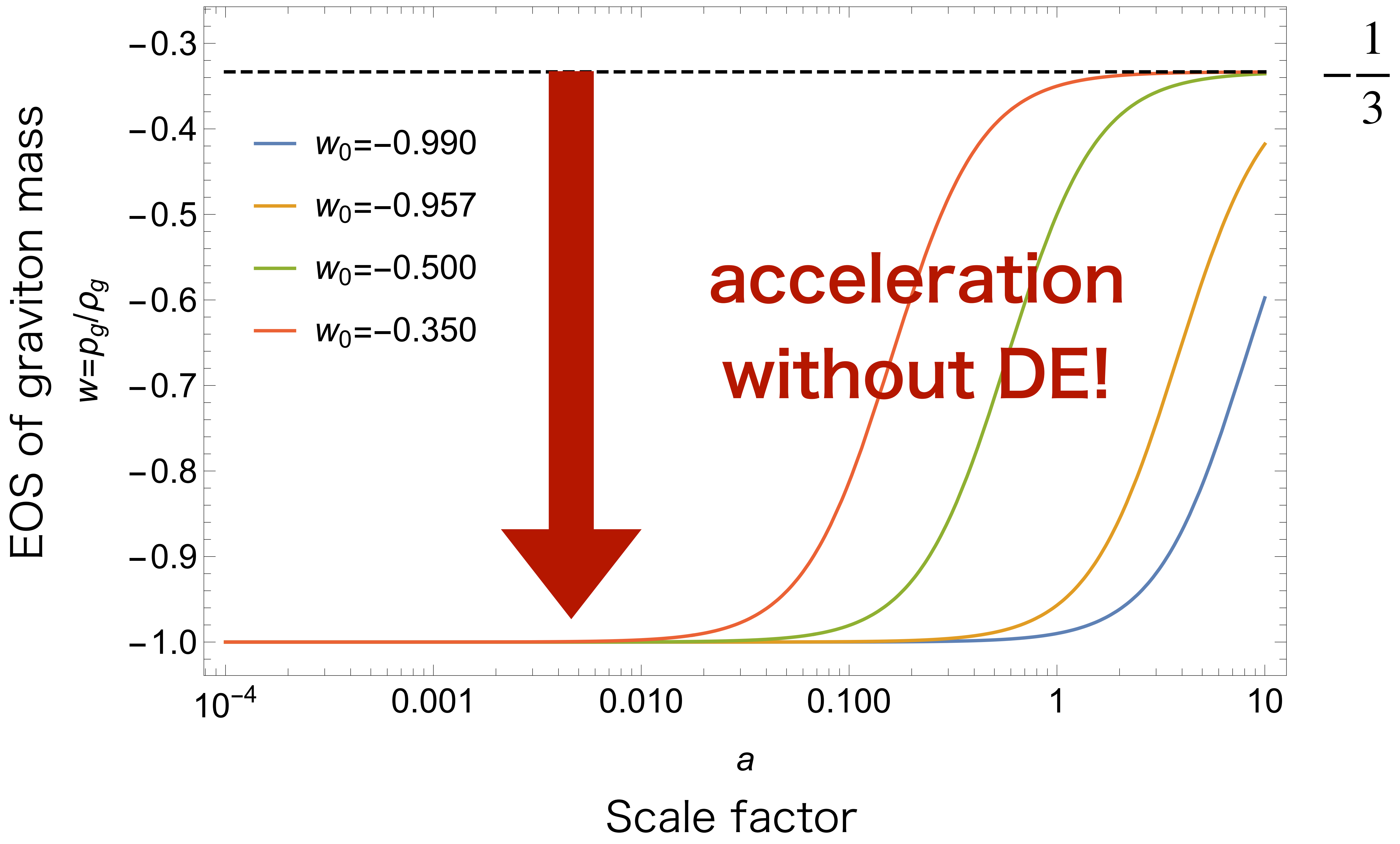
$$w(a) := \frac{p_g}{\rho_g} = \frac{1+3w_0-(1+w_0)a^2}{-1-3w_0+3(1+w_0)a^2}$$

Hubble parameter

$$\left(\frac{H(a)}{H_0}\right)^2 = \frac{\Omega_{\rm m}}{a^3} + \frac{\Omega_\kappa}{a^2} + \frac{2\Omega_\Lambda}{-(1+3w_0)+a^2(1+w_0)}$$

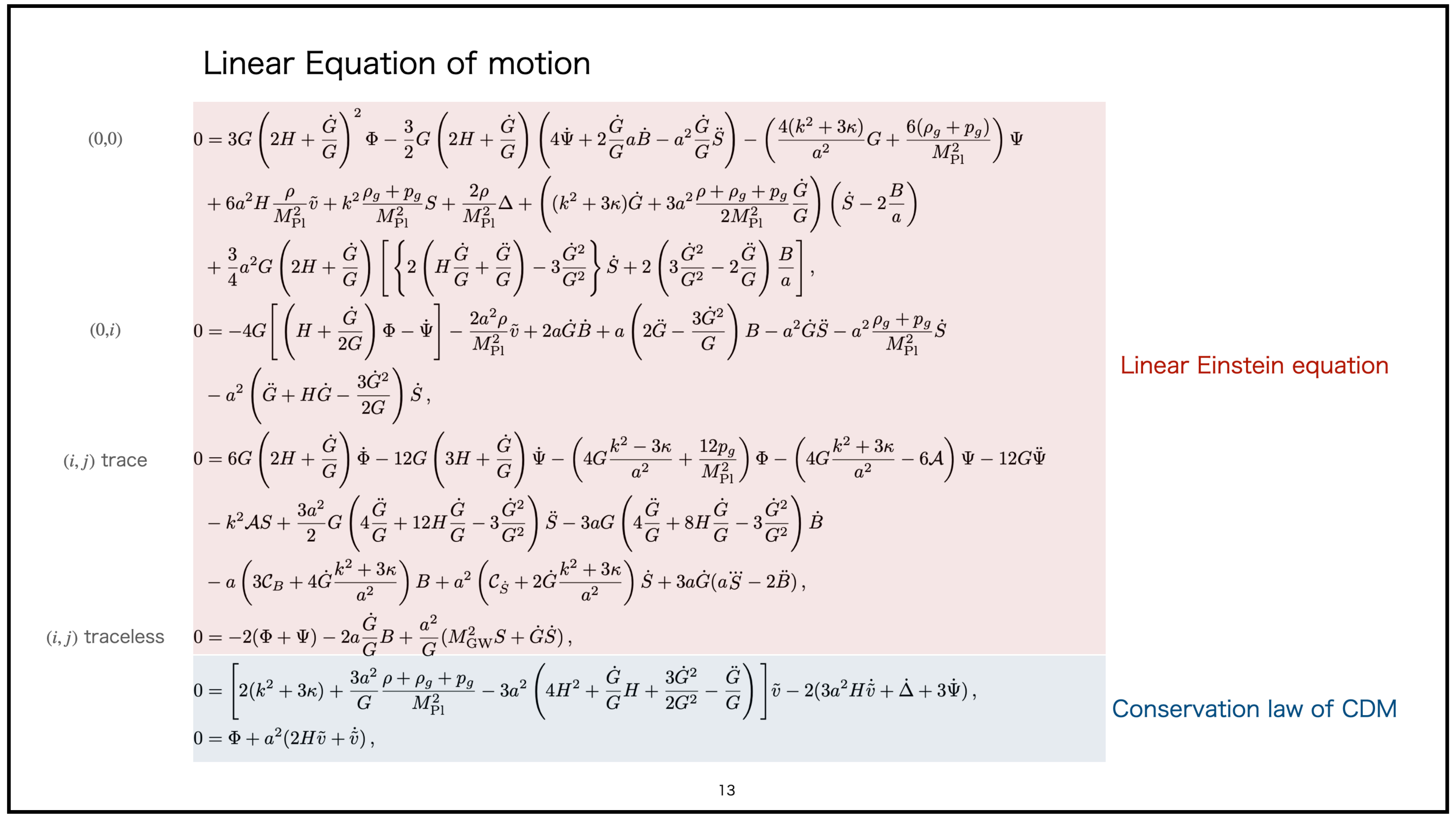
Self-acceleration

$$w(a) := \frac{p_g}{\rho_g} = \frac{1 + 3w_0 - (1 + w_0)a^2}{-1 - 3w_0 + 3(1 + w_0)a^2}$$



Linear growth (quasi-static-limit)

sub-horizon behavior



Linear Equation of motion

(0,0)

$$0 = 3G\left(2H+\frac{\dot G}{G}\right)^2\Phi - \frac{3}{2}G\left(2H+\frac{\dot G}{G}\right)\left(4\dot\Psi + 2\frac{\dot G}{G}a\dot B - a^2\frac{\dot G}{G}\ddot S\right) - \left(\frac{4(k^2+3\kappa)}{a^2}G + \frac{6(\rho_g+p_g)}{M_{\rm Pl}^2}\right)\Psi$$
$$+6a^2H\frac{\rho}{M_{\rm Pl}^2}\tilde v + k^2\frac{\rho_g+p_g}{M_{\rm Pl}^2}S + \frac{2\rho}{M_{\rm Pl}^2}\Delta + \left((k^2+3\kappa)\dot G + 3a^2\frac{\rho+\rho_g+p_g}{2M_{\rm Pl}^2}\frac{\dot G}{G}\right)\left(\dot S - 2\frac{B}{a}\right)$$
$$+\frac{3}{4}a^2G\left(2H+\frac{\dot G}{G}\right)\left[\left\{2\left(H\frac{\dot G}{G}+\frac{\ddot G}{G}\right) - 3\frac{\dot G^2}{G^2}\right\}\dot S + 2\left(3\frac{\dot G^2}{G^2} - 2\frac{\ddot G}{G}\right)\frac{B}{a}\right],$$

(0,i)

$$0 = -4G\left[\left(H+\frac{\dot G}{2G}\right)\Phi - \dot\Psi\right] - \frac{2a^2\rho}{M_{\rm Pl}^2}\tilde v + 2a\dot G\dot B + a\left(2\ddot G - \frac{3\dot G^2}{G}\right)B - a^2\dot G\ddot S - a^2\frac{\rho_g+p_g}{M_{\rm Pl}^2}\dot S$$
$$-a^2\left(\ddot G + H\dot G - \frac{3\dot G^2}{2G}\right)\dot S,$$

(i, j) trace

$$0 = 6G\left(2H+\frac{\dot G}{G}\right)\dot\Phi - 12G\left(3H+\frac{\dot G}{G}\right)\dot\Psi - \left(4G\frac{k^2-3\kappa}{a^2} + \frac{12p_g}{M_{\rm Pl}^2}\right)\Phi - \left(4G\frac{k^2+3\kappa}{a^2} - 6\mathcal{A}\right)\Psi - 12G\ddot\Psi$$
$$-k^2\mathcal{A}S + \frac{3a^2}{2}G\left(4\frac{\ddot G}{G} + 12H\frac{\dot G}{G} - 3\frac{\dot G^2}{G^2}\right)\ddot S - 3aG\left(4\frac{\ddot G}{G} + 8H\frac{\dot G}{G} - 3\frac{\dot G^2}{G^2}\right)\dot B$$
$$-a\left(3\mathcal{C}_B + 4\dot G\frac{k^2+3\kappa}{a^2}\right)B + a^2\left(\mathcal{C}_{\dot S} + 2\dot G\frac{k^2+3\kappa}{a^2}\right)\dot S + 3a\dot G(a\dddot S - 2\ddot B),$$

(i, j) traceless

$$0 = -2(\Phi+\Psi) - 2a\frac{\dot G}{G}B + \frac{a^2}{G}(M_{\rm GW}^2S + \dot G\dot S),$$

Linear Einstein equation

$$0 = \left[2(k^2+3\kappa) + \frac{3a^2}{G}\frac{\rho+\rho_g+p_g}{M_{\rm Pl}^2} - 3a^2\left(4H^2 + \frac{\dot G}{G}H + \frac{3\dot G^2}{2G^2} - \frac{\ddot G}{G}\right)\right]\tilde v - 2(3a^2H\dot{\tilde v} + \dot\Delta + 3\dot\Psi),$$
$$0 = \Phi + a^2(2H\tilde v + \dot{\tilde v}),$$

Conservation law of CDM

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quasi-static limit

$$\xrightarrow{\quad\quad}$$

$$k \gg H$$

$$S = 0$$

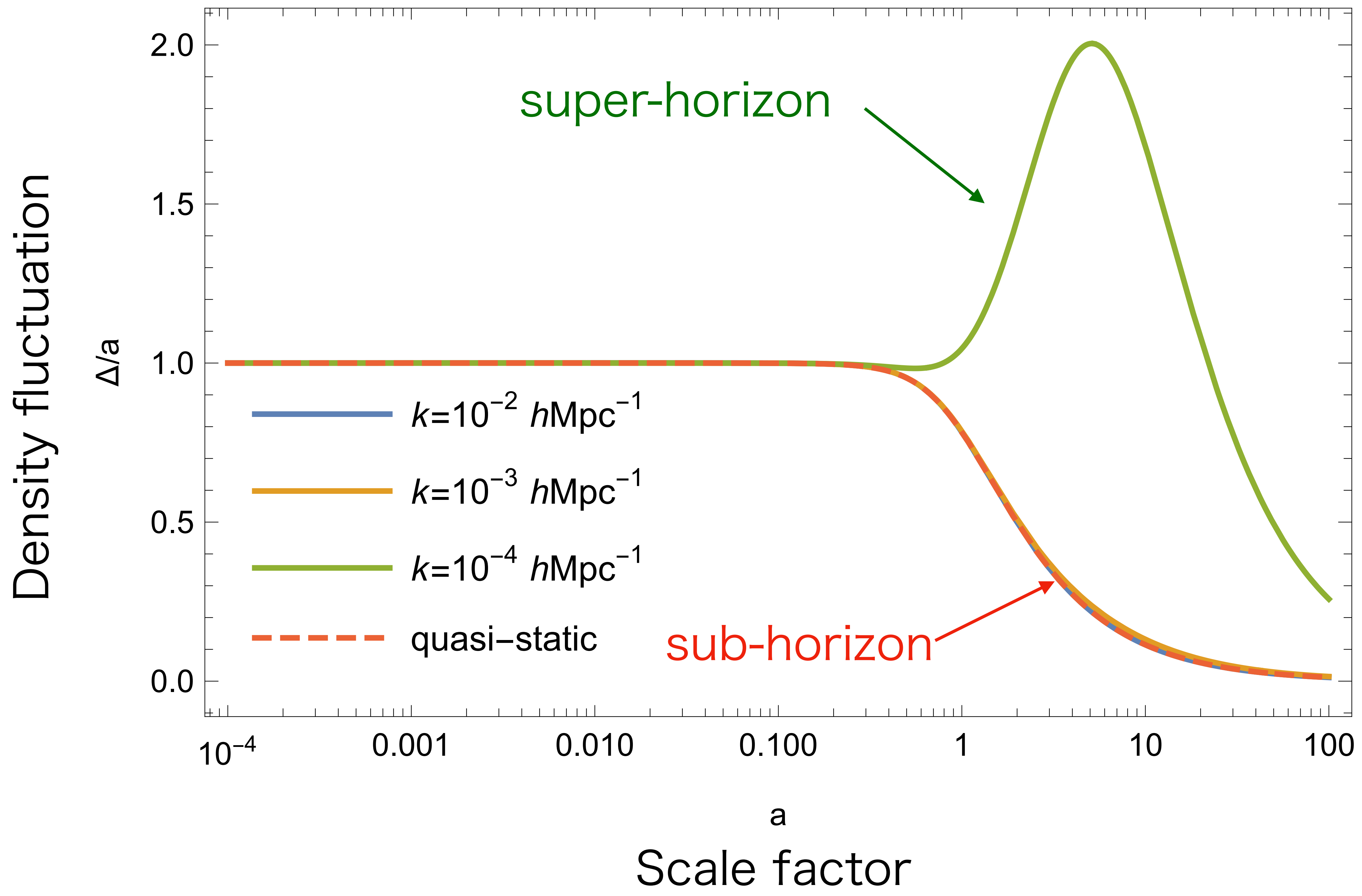
No additional dof!

$$\ddot\Delta + 2H\dot\Delta - \frac{\rho}{2M_{\rm Pl}^2}\Delta = 0$$

Same as ΛCDM!

At sub-horizon, **GR recovers even without non-linear screening!**

Linear growth (full numerical)



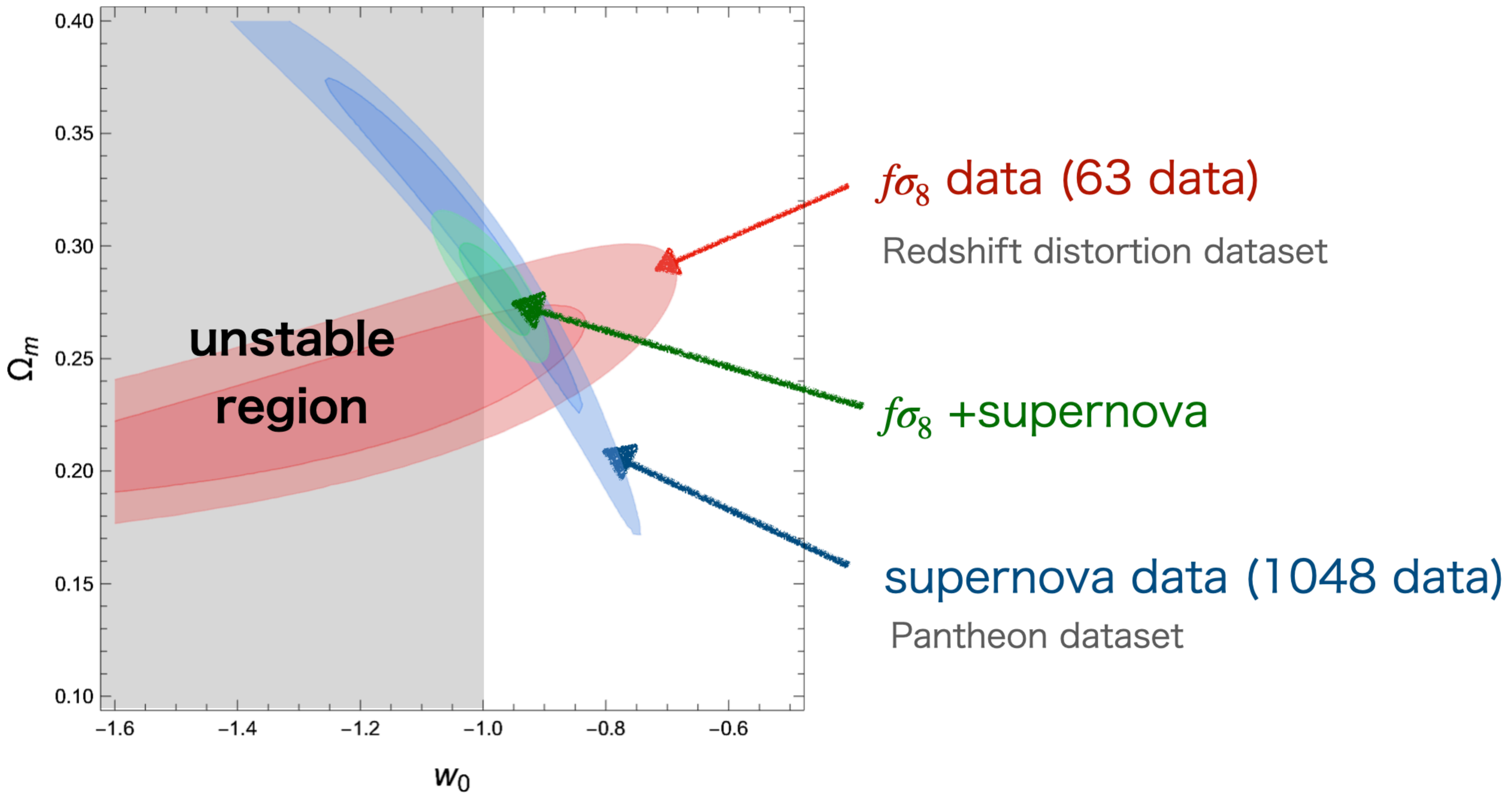
Model Parameter

$$w_0 = -0.9$$
$$b = 1, c = -1$$

- Super-horizon→Gravitational modification is **appeared**.
- Sub-horizon→Gravitational modification is **suppressed**.

Current constraint

w_0	$\Omega_{\rm m}$	Ω_κ	σ_8
$-0.978^{+0.297}_{-2.67}$	$0.280^{\,0.176}_{-0.156}$	$0.0255^{+0.579}_{-0.767}$	$0.789^{\,+1.268}_{-0.0711}$



There exists a theoretically and observationally consistent parameters!

Non-minimal coupling model

Model2	$G = \left(1 + m^2 g X\right)$	$U = a_1[Z] + b[Z]^2 + c[Z^2]$
Model3	$G = \left(1 + m^2 g X\right)^2$	$U = a_1[Z] + b[Z]^2 + c[Z^2]$

- **Model2**
  There are no parameters that are stable for all $a(t) > 0$.
  The same behavior is observed when higher order terms are included. (We didn't cover all models.)

- **Model3**
  There are parameter regions that are stable at least in the limit of $a \to 0$ and $a \to \infty$.
  We examine the middle region numerically, but no stable region was found. (We didn't cover all models.)

Quasi-static-limit for non-minimal coupling model

sub-horizon behavior of $G \neq 1$ model

Linear Equation of motion

(0,0)

$$0 = 3G\left(2H+\frac{\dot G}{G}\right)^2\Phi - \frac{3}{2}G\left(2H+\frac{\dot G}{G}\right)\left(4\dot\Psi + 2\frac{\dot G}{G}a\dot B - a^2\frac{\dot G}{G}\ddot S\right) - \left(\frac{4(k^2+3\kappa)}{a^2}G + \frac{6(\rho_g+p_g)}{M_{\rm Pl}^2}\right)\Psi$$
$$+ 6a^2H\frac{\rho}{M_{\rm Pl}^2}\tilde v + k^2\frac{\rho_g+p_g}{M_{\rm Pl}^2}S + \frac{2\rho}{M_{\rm Pl}^2}\Delta + \left((k^2+3\kappa)\dot G + 3a^2\frac{\rho+\rho_g+p_g}{2M_{\rm Pl}^2}\frac{\dot G}{G}\right)\left(\dot S - 2\frac{B}{a}\right)$$
$$+ \frac{3}{4}a^2G\left(2H+\frac{\dot G}{G}\right)\left[\left\{2\left(H\frac{\dot G}{G}+\frac{\ddot G}{G}\right) - 3\frac{\dot G^2}{G^2}\right\}\dot S + 2\left(3\frac{\dot G^2}{G^2} - 2\frac{\ddot G}{G}\right)\frac{B}{a}\right],$$

(0,i)

$$0 = -4G\left[\left(H+\frac{\dot G}{2G}\right)\Phi - \dot\Psi\right] - \frac{2a^2\rho}{M_{\rm Pl}^2}\tilde v + 2a\dot G\dot B + a\left(2\ddot G - \frac{3\dot G^2}{G}\right)B - a^2\dot G\ddot S - a^2\frac{\rho_g+p_g}{M_{\rm Pl}^2}\dot S$$
$$- a^2\left(\ddot G + H\dot G - \frac{3\dot G^2}{2G}\right)\dot S,$$

Linear Einstein equation

(i,j) trace

$$0 = 6G\left(2H+\frac{\dot G}{G}\right)\dot\Phi - 12G\left(3H+\frac{\dot G}{G}\right)\dot\Psi - \left(4G\frac{k^2-3\kappa}{a^2} + \frac{12p_g}{M_{\rm Pl}^2}\right)\Phi - \left(4G\frac{k^2+3\kappa}{a^2} - 6\mathcal{A}\right)\Psi - 12G\ddot\Psi$$
$$- k^2\mathcal{A}S + \frac{3a^2}{2}G\left(4\frac{\ddot G}{G} + 12H\frac{\dot G}{G} - 3\frac{\dot G^2}{G^2}\right)\ddot S - 3aG\left(4\frac{\ddot G}{G} + 8H\frac{\dot G}{G} - 3\frac{\dot G^2}{G^2}\right)\dot B$$
$$- a\left(3\mathcal{C}_B + 4\dot G\frac{k^2+3\kappa}{a^2}\right)B + a^2\left(\mathcal{C}_{\dot S} + 2\dot G\frac{k^2+3\kappa}{a^2}\right)\dot S + 3a\dot G(a\dddot S - 2\ddot B),$$

(i,j) traceless

$$0 = -2(\Phi+\Psi) - 2a\frac{\dot G}{G}B + \frac{a^2}{G}(M_{\rm GW}^2S + \dot G\dot S),$$

$$0 = \left[2(k^2+3\kappa) + \frac{3a^2}{G}\frac{\rho+\rho_g+p_g}{M_{\rm Pl}^2} - 3a^2\left(4H^2 + \frac{\dot G}{G}H + \frac{3\dot G^2}{2G^2} - \frac{\ddot G}{G}\right)\right]\tilde v - 2(3a^2H\dot{\tilde v} + \dot\Delta + 3\dot\Psi),$$
$$0 = \Phi + a^2(2H\tilde v + \dot{\tilde v}),$$

Conservation law of CDM

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quasi-static limit

$\longrightarrow$

$k \gg H$

$$S = 0$$

$$\Delta = 0$$

No structure?

Is the limit taken correctly?

It should be easy to find out by numerical calculations, but we can't be sure because we haven't found a stable parameter region yet...

Summary

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We investigate the linear growth of structure in self-accelerating solution of **Projected Massive Gravity**.

- Background is parametrized only by $\{w_0, H_0, \Omega_{\rm m}, \Omega_\kappa\}$.
- At sub-horizon, GR recovers even without non-linear screening.
- There exists a theoretically and observationally consistent parameters.