

重イオン衝突における ハドロン相関に関する理論とその展望

京大基研 大西 明

ELPH 研究会 C031

「多彩なビーム実験と多様な理論的手法で迫るハドロン間相互作用」
東北大学 ELPH, Nov.4-5, 2021.



- Introduction
- ハドロン相関関数の理論
 - Koonin-Pratt 公式、LL 模型、結合チャネル
- 最近 & 将来の相関関数によるハドロン間相互作用研究例
 - $N\Xi$ - Λ 、D-p、others
- Summary

なぜハドロン間相互作用？

■ ハドロン物理の面白さ

- 異なる構成粒子・相互作用からくる原子核構造の変化
→ エキゾチック核・ハドロン
- 様々なハドロン間相互作用
→ 一般化した「核力」の理解
- 高密度物質中のハドロン
→ ハイペロン・パズルの解決を目指す
- QCD の対称性
→ フレーバー・カイラル・重クォーク対称性

■ 多彩なビーム実験

- ハドロンビーム、電子ビーム、光子ビーム、重イオンビーム、 ...
(Multi deliverer era)
- ハイパー核、中間子核、中間子原子、粒子スペクトル、相関関数、 ...

■ 多様な理論的手法

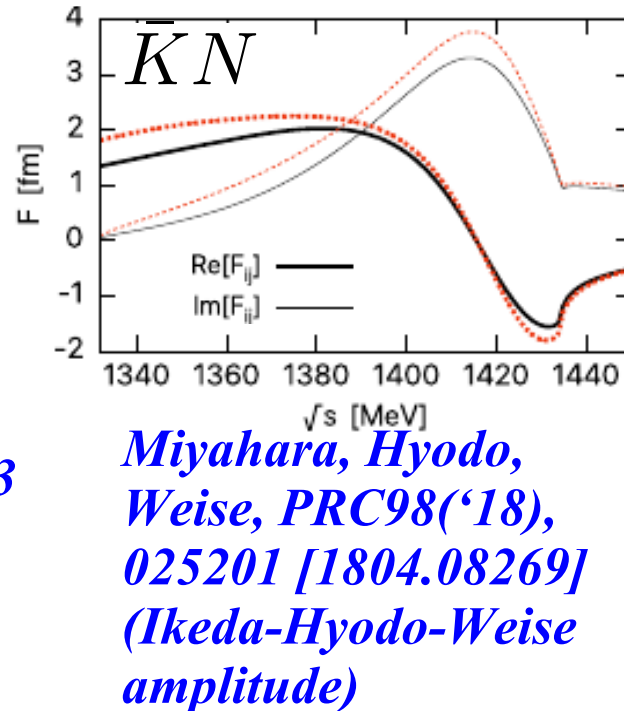
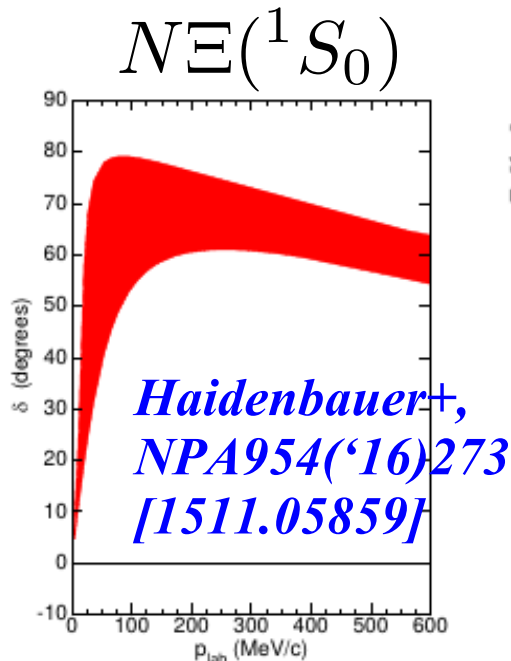
- 中間子交換、クォーク模型、カイラル EFT、格子 QCD、HQCD ...
- 少数系、クラスター、平均場、直接反応、輸送模型、統計模型、 ...

多くの人々が様々な方向から議論できる原子核物理の中心的課題

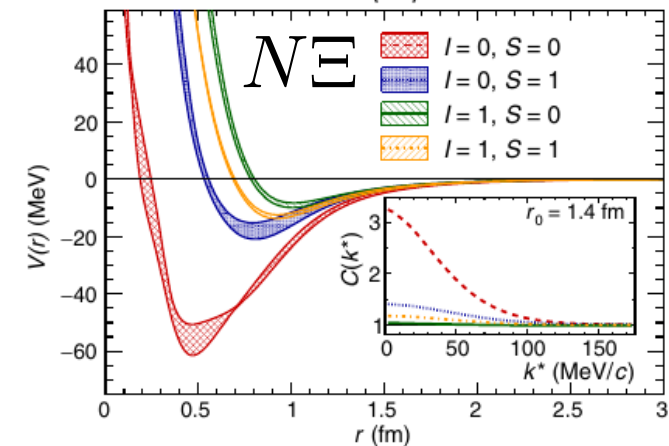
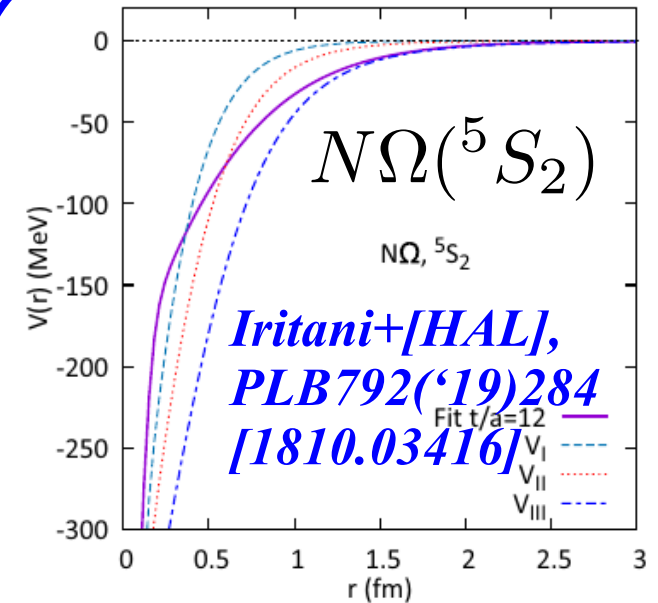
ハドロン間相互作用への理論的アプローチ

- Nuclear force models: meson exch., quark model, ... (need **data**)
- **Ab initio**: chiral EFT, lattice QCD (need **data** or **CPU resources**)

Chiral



Lattice

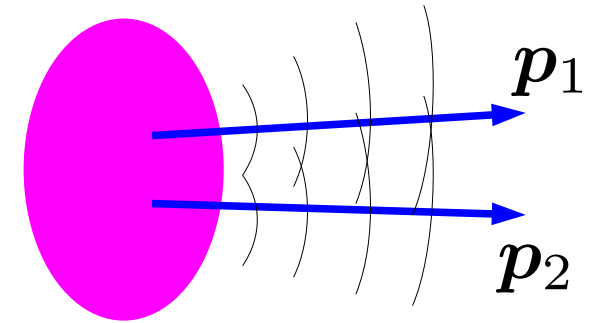


*Sasaki+ [HAL], NPA998
(‘20)121737 [1912.08630]
(taken from ALICE(‘19))*

ハドロン間相互作用への実験的アプローチ

Experimental approaches

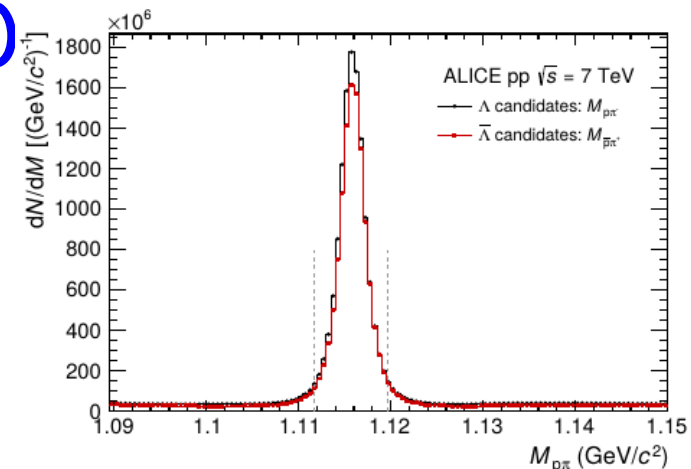
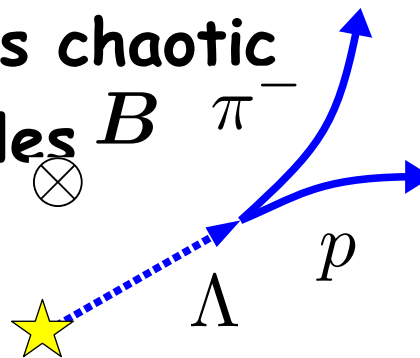
- hh scattering (NN, YN, π N, KN)
- Hadronic nuclei (normal nuclei, hypernuclei, kaonic nuclei)
- Hadronic atom (π^- , K^- , Σ^- , Ξ^- , ...)
- **Femtoscopy**



$$C(\mathbf{p}_1, \mathbf{p}_2) = \frac{N_{12}(\mathbf{p}_1, \mathbf{p}_2)}{N_1(\mathbf{p}_1)N_2(\mathbf{p}_2)}$$

Femtoscopic study of hh interactions

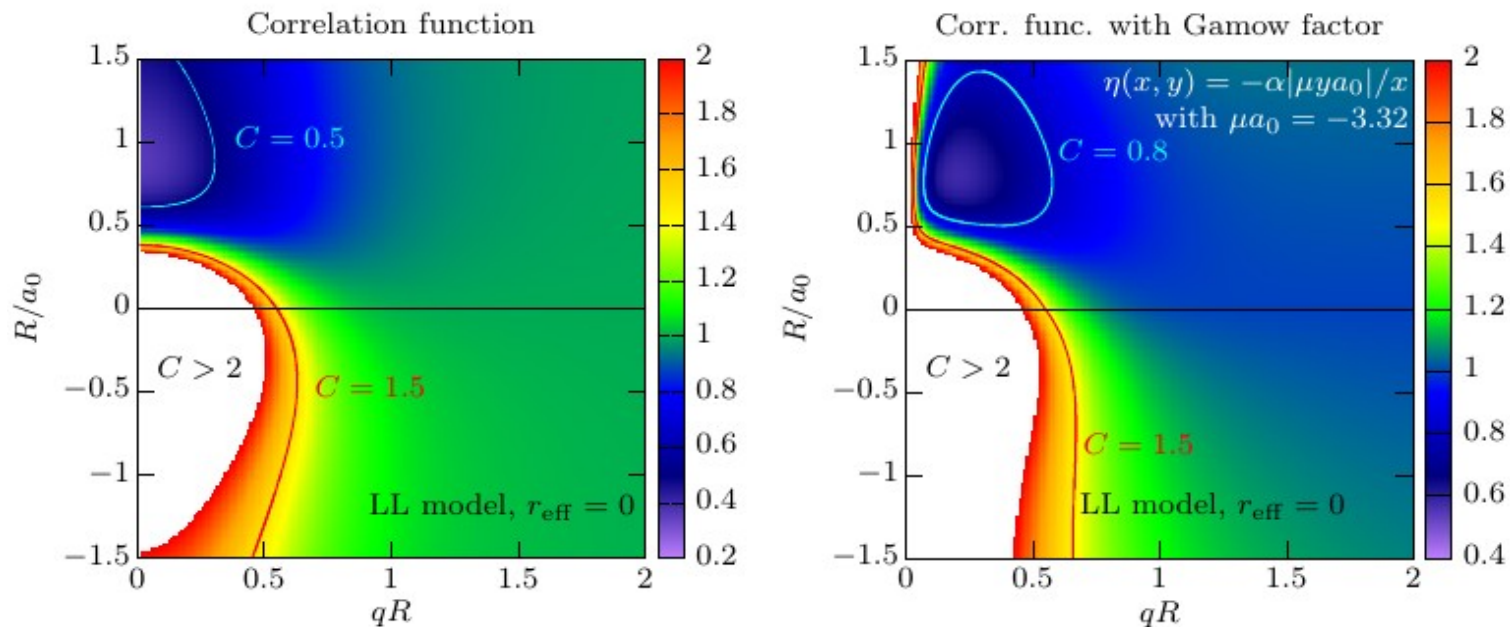
- Applicable to various hh pairs
(NN, YN, KN, DN, YY, Yd, YNN ...)
- Valid when the source is chaotic
- Weakly decaying particles B π^-
→ Good pair purity
- Future measurements: Λ p
Charmed hadron, hNN, ...



ALICE [1805.12455]

相関関数によるハドロン間相互作用の面白い点

- 様々な種類のハドロン間相互作用について実験から情報が得られる
- 相関関数のサイズ依存性 (pp, pA, AA 衝突) から、束縛状態 (ハドロン分子状態) の有無を推定できる



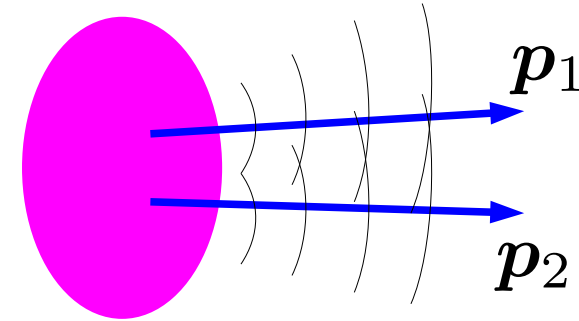
- 重イオン (高エネルギー原子核衝突) 研究者とハドロン研究者の橋渡しができる (多彩なビーム実験!)
- 相関関数自体の理論研究者はまだ少ない (Blue Ocean)

ハドロン相関関数の理論

相関関数の理論 (1): Koonin-Pratt (KP) formula

■ Correlation function (CF)

- CF=convolution of source fn. and $|w.f.|^2$ (Koonin-Pratt formula)



Koonin('77), Pratt+('86), Lednicky+('82)

$$C(\mathbf{p}_1, \mathbf{p}_2) = \frac{N_{12}(\mathbf{p}_1, \mathbf{p}_2)}{N_1(\mathbf{p}_1)N_2(\mathbf{p}_2)} \simeq \int d\mathbf{r} \underbrace{S(\mathbf{r})}_{\text{source fn.}} \underbrace{|\varphi_q(\mathbf{r})|^2}_{\text{relative w.f.}}$$

source fn. relative w.f.

■ Source size from quantum stat. + CF

Hanbury Brown & Twiss ('56); Goldhaber, Goldhaber, Lee, Pais ('60)

■ Hadron-hadron interaction from source size + CF

- CF of non-identical pair from static spherical source

Lednicky, Lyuboshits ('82); Morita, Furumoto, AO ('15)

$$C(\mathbf{q}) = 1 + \int d\mathbf{r} S(r) \{ |\varphi_0(r)|^2 - |j_0(qr)|^2 \} \quad (\varphi_0 = \text{s-wave w.f.})$$

CF shows how much $|\varphi|^2$ is enhanced $\rightarrow V_{hh}$ effects !

相関関数の理論 (2): Lednicky-Lyuboshits (LL) 模型

- Lednicky-Lyuboshits analytic model
(Asymp. w.f.+eff. range corr.+Gaussian source)

Lednicky, Lyuboshits ('82)

$$\psi_0(r) \rightarrow \psi_{\text{asy}}(r) = \frac{e^{-i\delta}}{qr} \sin(qr + \delta) = \mathcal{S}^{-1} \left[\frac{\sin qr}{qr} + f(q) \frac{e^{iqr}}{r} \right]$$

$$\begin{aligned} C_{\text{LL}}(q) &= 1 + \int dr S_{12}(r) (|\psi_{\text{asy}}(r)|^2 - |j_0(qr)|^2) \\ &= 1 + \frac{|f(q)|^2}{2R^2} F_3\left(\frac{r_{\text{eff}}}{R}\right) + \frac{2\text{Re}f(q)}{\sqrt{\pi}R} F_1(2x) - \frac{\text{Im}f(q)}{R} F_2(2x) \end{aligned}$$

($x = qR$, $R = \text{Gaussian size}$, F_1, F_2, F_3 : Known functions)

- 低エネルギー散乱振幅

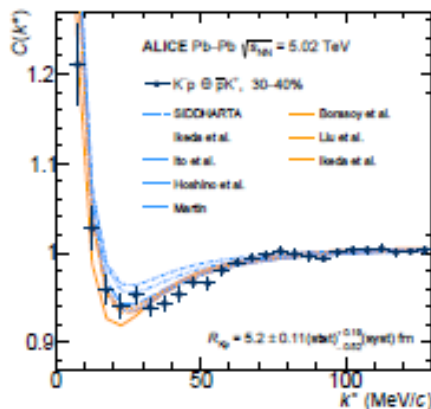
$$q \cot \delta = -\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} q^2 + \mathcal{O}(q^4) \rightarrow f(q) = (q \cot \delta - iq)^{-1}$$

散乱長と有効レンジ (とソースサイズ) があれば相関関数が求まる!

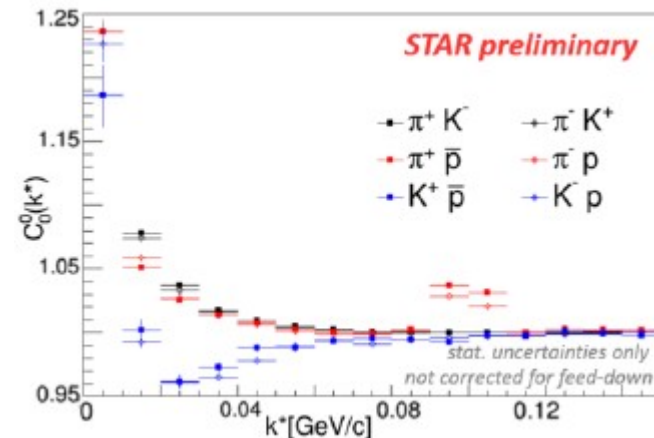
LL 模型を用いた低エネルギー散乱パラメータ決定例

■ K⁻p 相互作用

- 重イオン衝突からの K⁻p 相関関数
- LL 公式から求めた散乱パラメータは SIDDHARTA と無矛盾



*Acharya+[ALICE],
PLB822('21),136708
[2105.05683]*



*Siejka+[STAR, preliminary],
NPA982 ('19)359.*

■ ΦN 相互作用

- pp 13 TeV からの pΦ 相関関数
- LL 公式による散乱パラメータ決定 (本当は J=1/2, 3/2 が混ざっているはずなのだが ...)

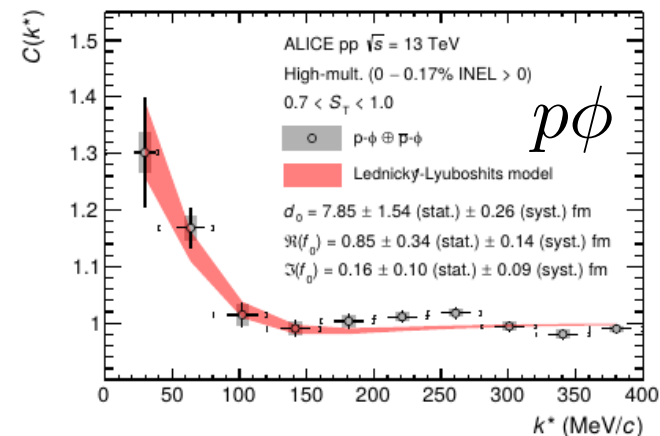
■ ΛΛ 相互作用

- 量子統計性 +LL 公式による fit

→ (a_0, r_{eff}) の領域制限

L. Adamczyk+[STAR], PRL114('15)022301;

S. Acharya+[ALICE], PLB797('19)134822.



*Acharya+[ALICE],
PRL127 ('21) 17,
172301[2105.05578]*

LL 模型を用いた束縛状態の有無の推定 (1)

■ ゼロレンジでの LL 模型

$$r_{\text{eff}} = 0 \rightarrow q \cot \delta = -1/a_0 \rightarrow f(q) = (q \cot \delta - iq)^{-1} = -\frac{R}{R/a_0 + iqR}$$

$$C(x, y) = 1 + \frac{1}{x^2 + y^2} \left[\frac{1}{2} - \frac{2y}{\sqrt{\pi}} F_1(2x) - xF_2(2x) \right] \quad (x = qR, y = R/a_0)$$

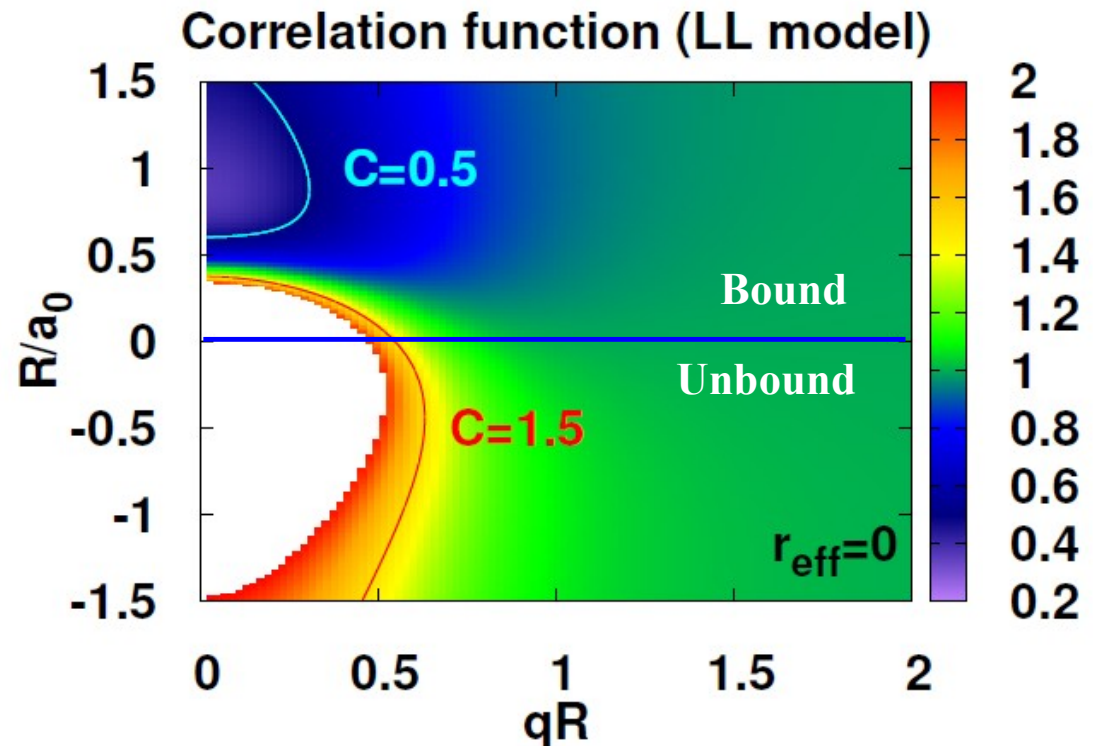
$$= \frac{1}{2} \left(\frac{1}{y} - \frac{2}{\sqrt{\pi}} \right)^2 + 1 - \frac{2}{\pi} \quad (F_1 \rightarrow 1, F_2 \rightarrow 0 \text{ at } x \rightarrow 0)$$

- 束縛状態がある時、
相関関数は抑制される。

$$R/a_0 = \sqrt{\pi}/2 \sim 0.89$$

において最小値

$$1 - 2/\pi \sim 0.36$$



E.g. AO, Morita, Miyahara, Hyodo ('16)

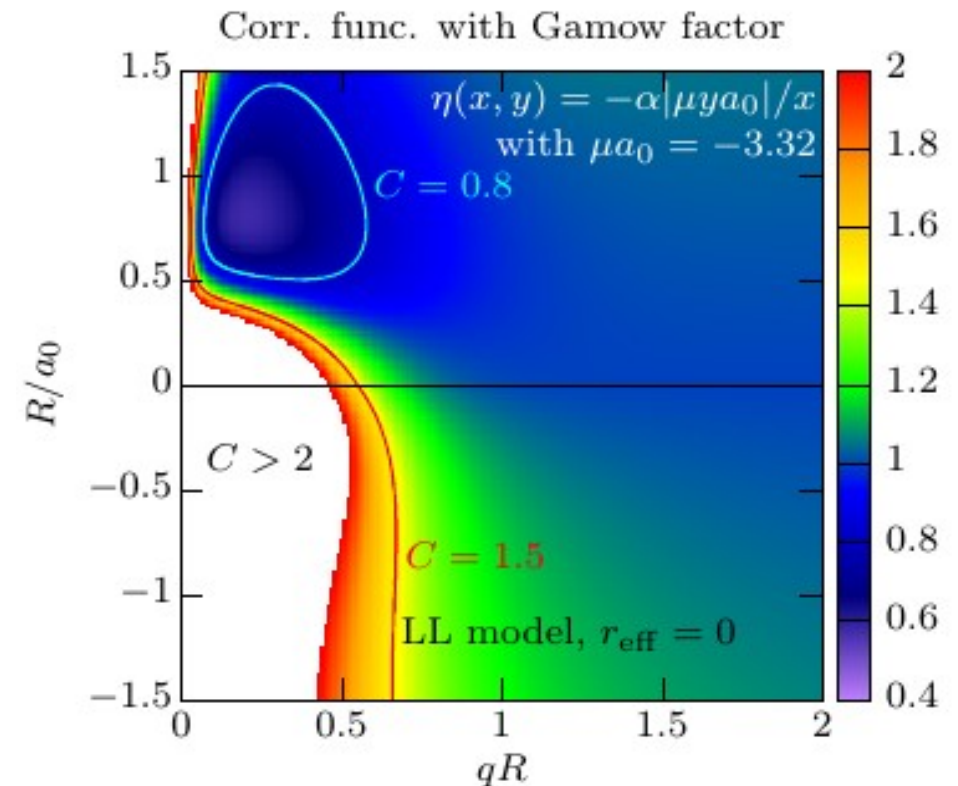
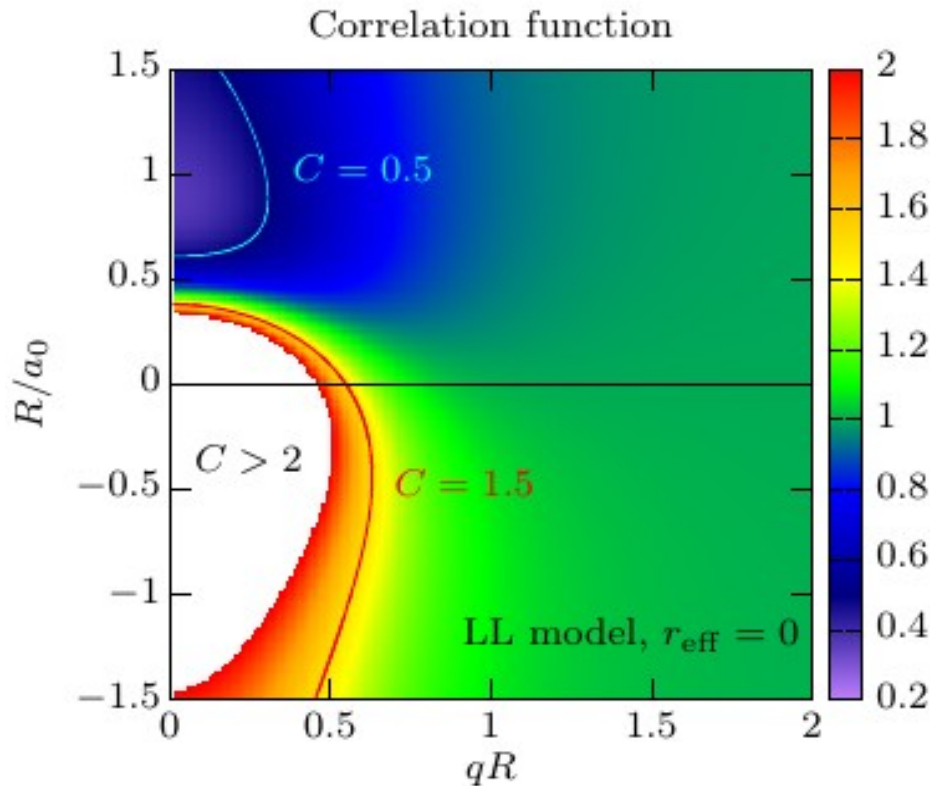
LL 模型を用いた束縛状態の有無の推定 (2)

- クーロン引力がある場合には、束縛状態から dip が現れる。

- クーロン引力による増大が Gamow 因子で表される (近似的) 場合

$$C(q) \rightarrow C_{\text{strong}}(q) \times A_{\text{Gamow}}(\eta)$$

$$A_{\text{Gamow}}(\eta) = 2\eta / (\exp(2\eta) - 1) \quad (\eta = Z_1 Z_2 \mu |y a_0| / x)$$

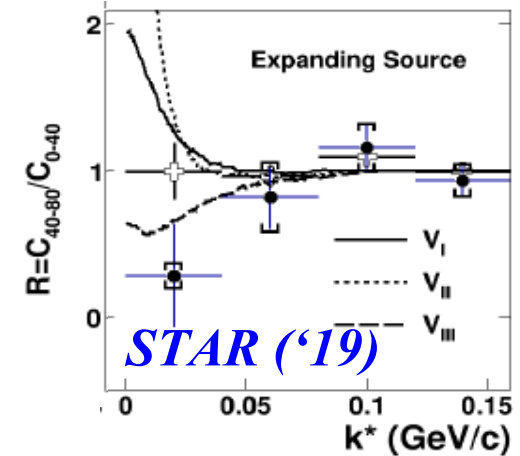
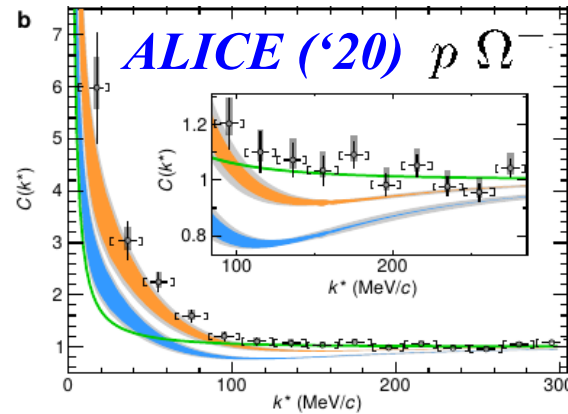


Kamiya+('21), 2108.09644

相関関数による束縛状態存在の示唆の例

■ Ω_p 束縛状態

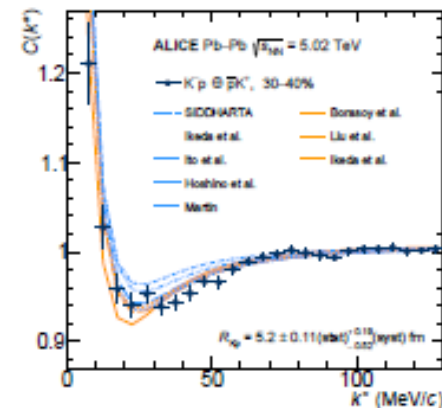
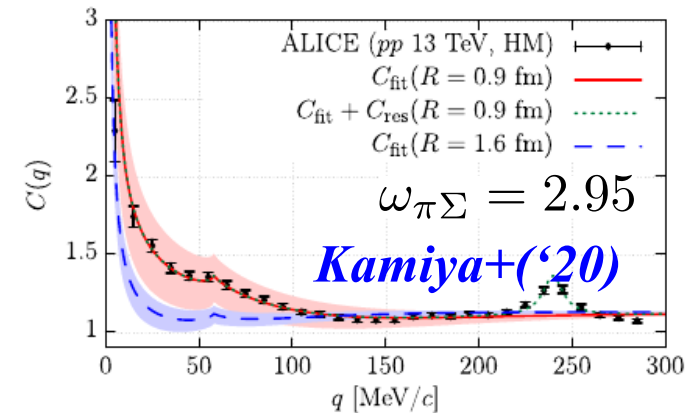
- 格子 QCD は束縛状態を予言
相関関数計算は dip を予言
Iritani+[HAL QCD]('19),
Morita+('20)



- データも重イオン衝突で dip を示す。

■ K^-p 束縛状態

- $\Lambda(1405)$ は $\bar{K}N$ 束縛状態
→ 重イオン衝突で dip
- 散乱パラメータは SIDDARTA と consistent
- pp 衝突では結合チャネル効果から dip は見えない



*Acharya+
[ALICE],
PLB822('21),
136708
[2105.05683]*

相関関数の理論 (3): 結合チャンネル効果

■ 結合チャンネル効果を含む相関関数

- Koonin-Pratt-Lednicky-Lyuboshits-Lyuboshits (KPLLL) 公式

Coupled-channel contributions with $\Psi^{(-)}$ boundary cond.

Lednicky, Lyuboshits, Lyuboshits, Phys. Atom. Nucl. 61 (1998), 2950;

J. Haudenbauer, NPA981('19) 1 [1808.05049];

Y. Kamiya, T.Hyodo, K.Morita, AO, W.Weise, PRL('20).

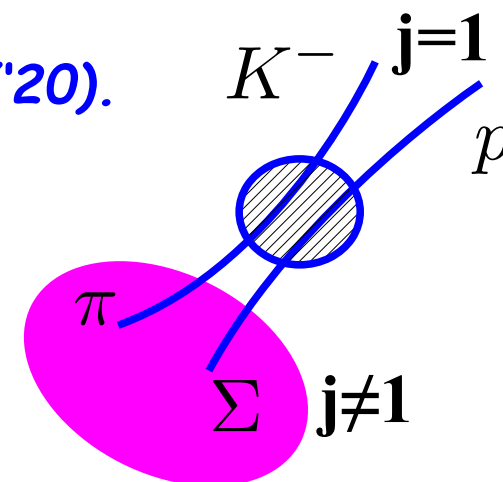
$$C(\mathbf{q}) = \int d\mathbf{r} \sum_j \omega_j S_j(\mathbf{r}) |\Psi_j^{(-)}(\mathbf{r})|^2$$

$$= 1 - \int d\mathbf{r} S_1(r) |j_0(qr)|^2 + \int d\mathbf{r} \sum_j \omega_j S_j(r) |\psi_j^{(-)}(q; r)|^2$$

$$\psi_{j=1}(r) \rightarrow [e^{iqr} + A_1(q)e^{-iqr}] / 2iqr \quad (\omega_1 = 1)$$

$$\psi_{j \neq 1}(r) \rightarrow A_j(q)e^{-iqr} / 2iqr \quad [\Psi^{(-)} \text{ boundary condition}]$$

(No Coulomb case)



$$\omega_j S_j(\mathbf{r}) |\psi_j^{(-)}(q; r)|^2$$

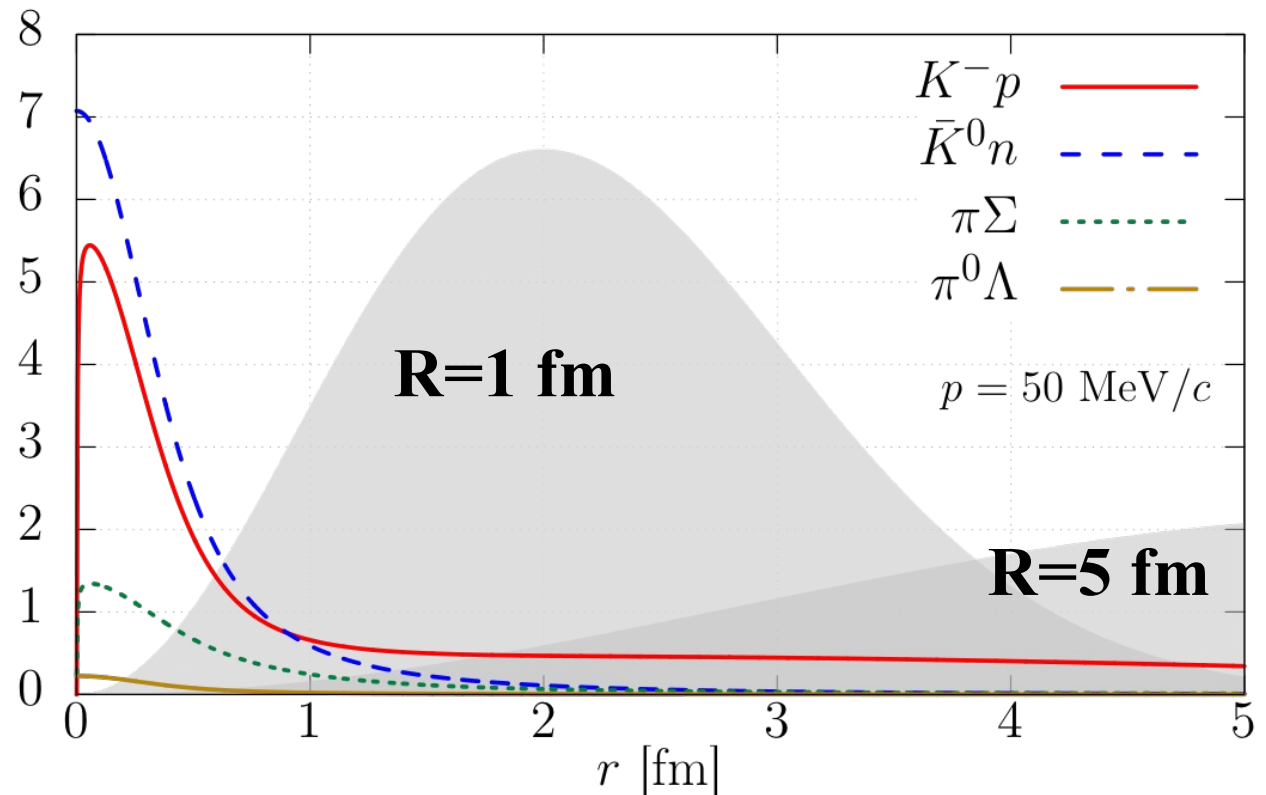
Source weight **Normalized Source fn.**

- Source size R and weight ω_j ($j \neq 1$) are taken as the parameter.

結合チャンネル効果の切り分け

■ 結合チャンネル効果はソースサイズ依存性から推測可能

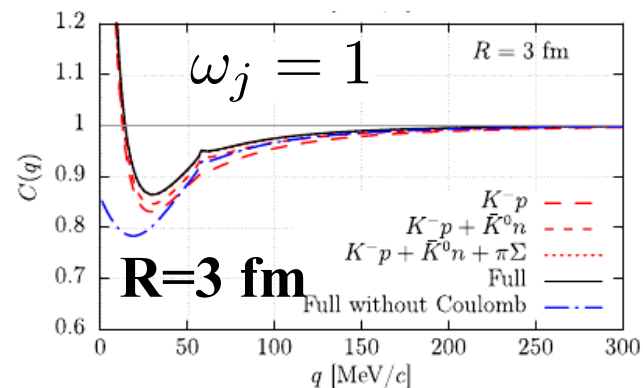
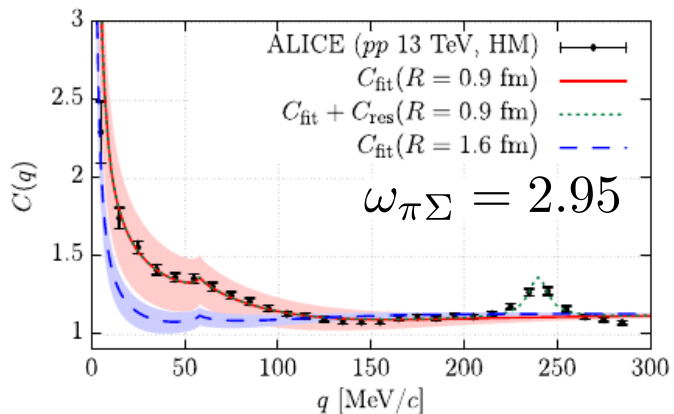
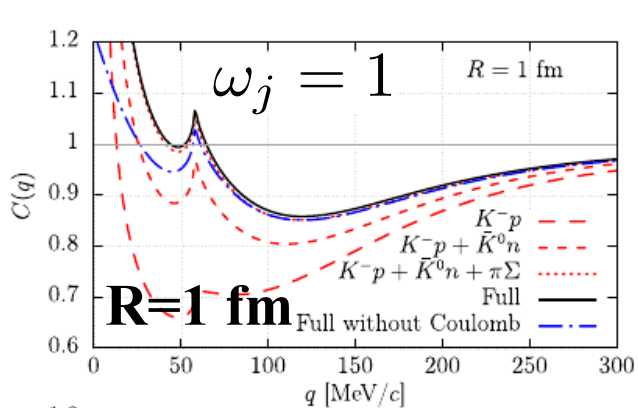
- 結合するチャンネルの波動関数 → 小さな r で有意、遠方で消える
- 大きなソースから散乱パラメータを決め、
小さなソースで結合効果を調べる！



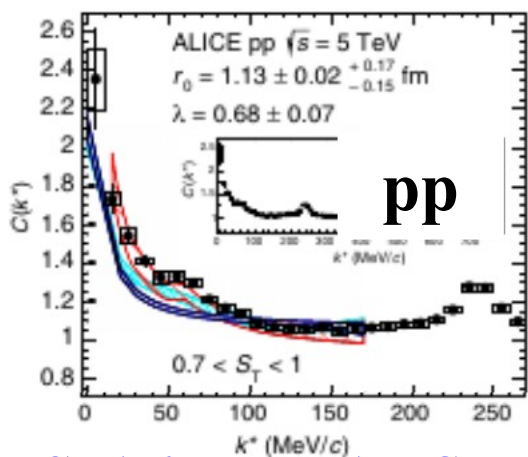
w.f. Kamiya+, arXiv:1911.01041v1

Source Size Dependence of $C(K^- p)$

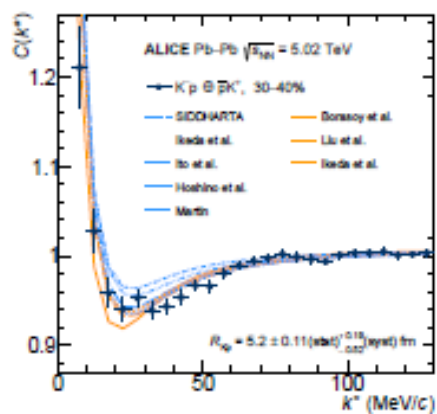
- Coupled-channel effects are suppressed when R is large, and "pure" $K^- p$ wave function may be observed in HIC.



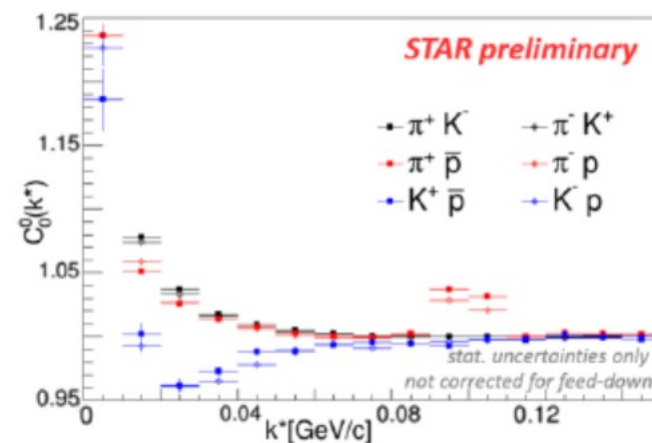
Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501.



S. Acharya+[ALICE], PRL124('20)092301



S. Acharya+[ALICE], PLB822('21)136708



Siejka+[STAR, preliminary], NPA982 ('19)359.

最近 & 将来の相関関数による ハドロン間相互作用研究

- ΞN - $\Lambda\Lambda$ interactions
- Charmed hadron interactions
- Others

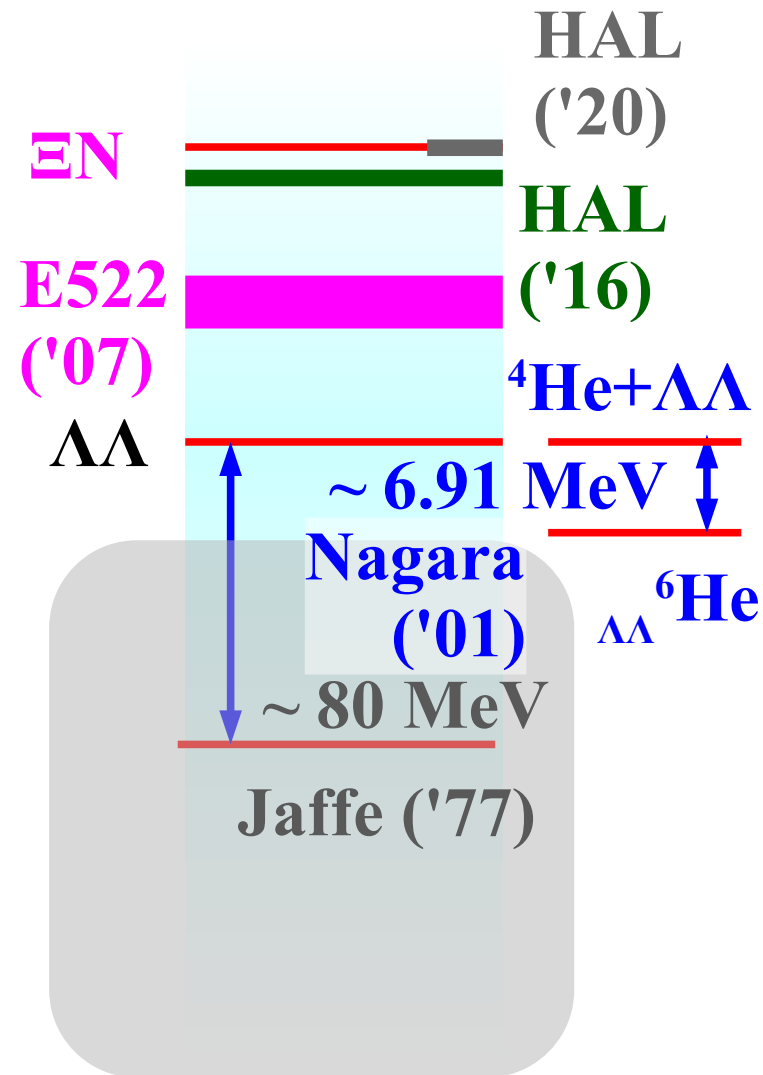
H dibaryon state, to be bound or not to be bound ?

■ H-dibaryon: 6-quark state (uuddss)

- Prediction: *R.L.Jaffe, PRL38(1977)195*
- Ruled-out by double Λ hypernucleus
Takahashi et al., PRL87('01) 212502
- Resonance or Bound "H" ?
Yoon et al.(KEK-E522)+AO ('07)

■ Lattice QCD results

- Bound (below $\Lambda\Lambda$ threshold):
HALQCD('11), NPLQCD('11,'13), Mainz('19)
(heavier quark mass or SU(3) limit)
- Resonance (Bound state of $N\Xi$):
HAL QCD ('16,18) (HAL preliminary)
- Virtual Pole (around $N\Xi$ threshold)
HAL QCD ('20) (almost physical m_q)



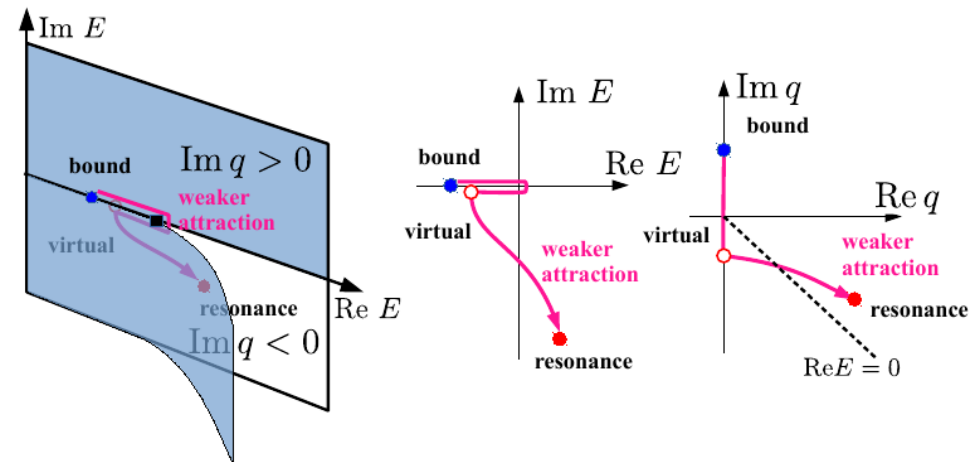
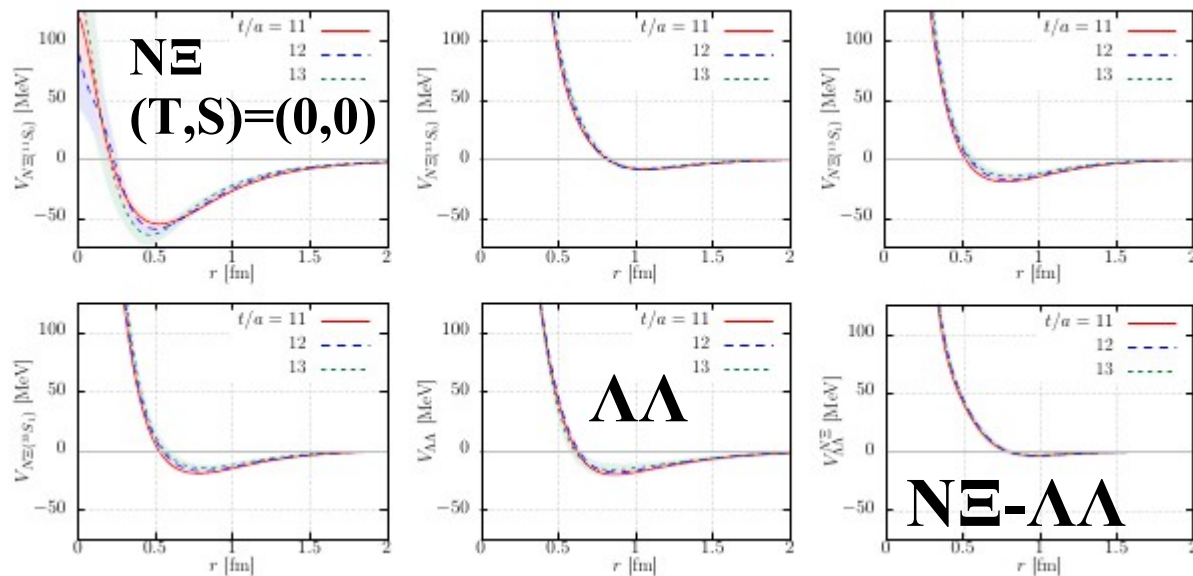
We examine LQCD $N\Xi$ - $\Lambda\Lambda$ potential and discuss H using CF !

$N\Xi-\Lambda$ potential from Lattice QCD

- $N\Xi-\Lambda$ potential at almost physical quark mass ($m_\pi=146$ MeV) by HAL QCD Collaboration

K. Sasaki et al. [HAL QCD Collab.], NPA 998 ('20) 121737 (1912.08630)

- Strong attraction in $(T,S)=(0,0)$ of $N\Xi$
- Weak attraction in $\Lambda\Lambda$ (Coupling with $N\Xi$ causes $\Lambda\Lambda$ attraction)
- **There is no bound state in $N\Xi-\Lambda$ system (except for Ξ^- atom), but there is a virtual pole around the $N\Xi$ threshold (3.93 MeV below $n\Xi^0$ threshold) on the irrelevant Riemann sheet, (+, -, +) [relevant=(-, +, +)] (sign of Im (eigenmomentum))**

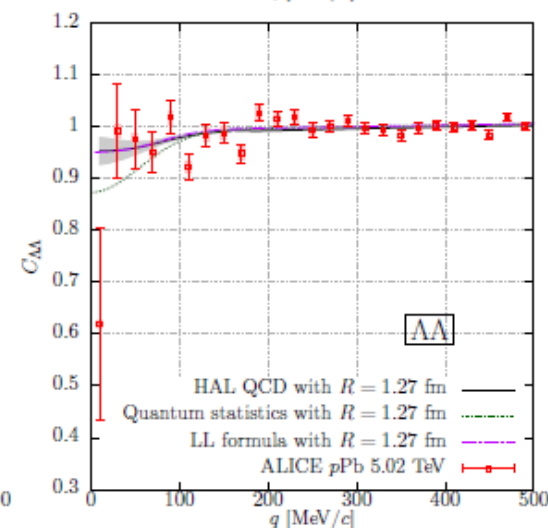
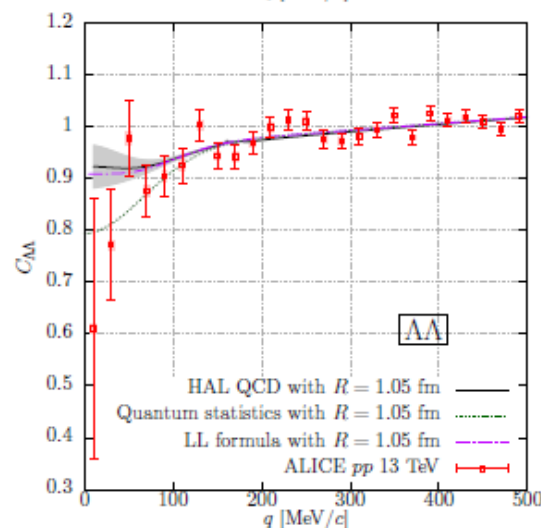
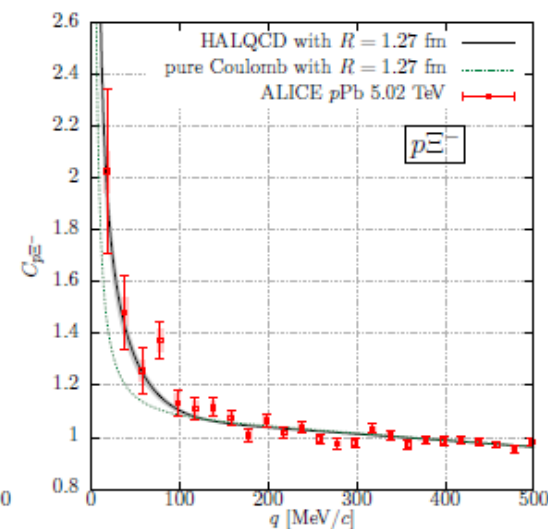
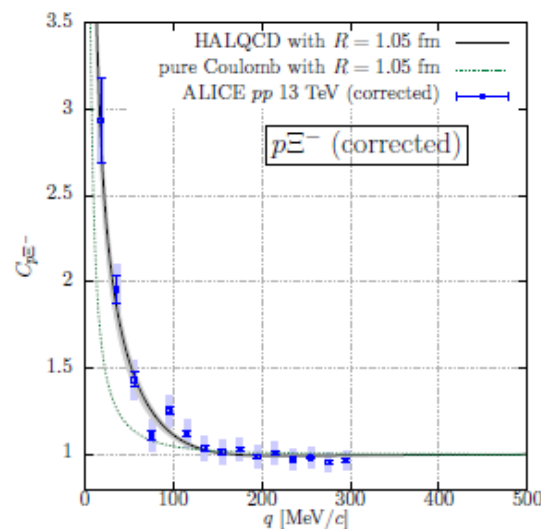


pp & pA 衝突からの Ξ^-p & $\Lambda\Lambda$ 相関関数 (1)

■ Correlation function data from pp and pA collisions

S. Acharya et al. [ALICE], PLB 797('19)134822 ($\Lambda\Lambda$); PRL123('19)112002 (Ξ^-p from pA); Nature 588('20)232 (Ξ^-p from pp).

- $CF(\Xi^-p)$ is enhanced at low q .
→ Coulomb+Att. pot.
- $CF(\Lambda\Lambda)$ is enhanced from quantum stat. result.
→ Weakly attractive pot.

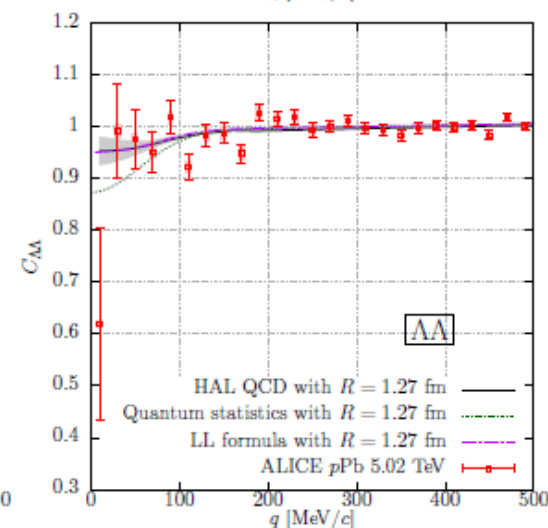
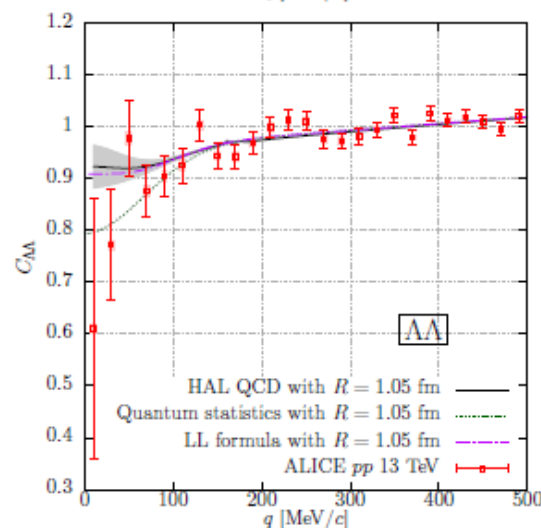
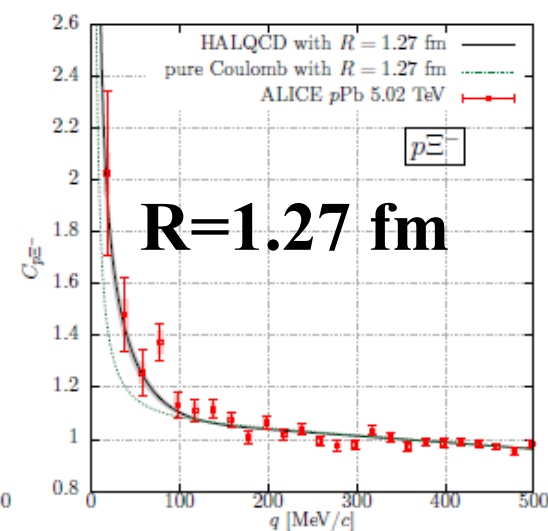
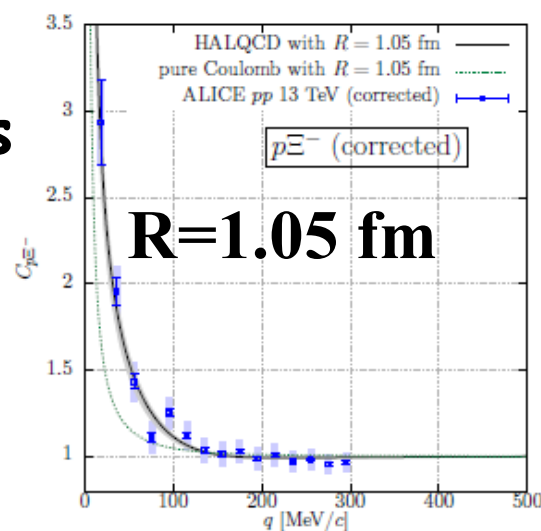


pp & pA 衝突からの Ξ^- & $\Lambda\Lambda$ 相関関数 (2)

- Correlation function with coupled-channel effects using lattice QCD potential

Y.Kamiya, K.Sasaki, T.Fukui, T. Hyodo, K.Morita, K.Ogata, A.Ohnishi, T. Hatsuda, arXiv:2108.09644.

- Source size in pp collisions is obtained from χ^2 fit. (Consistent with ALICE analysis result)
- R from pA is given from scaling.
- Coupling effects are small (w's do not change results much)
- HAL QCD potential well explains the data !



Ξ^- correlation function

*T. Hatsuda, K. Morita, AO, K. Sasaki, NPA967('17)856.
(heavier quark mass, $I=0$ only, w/o CC effects)*

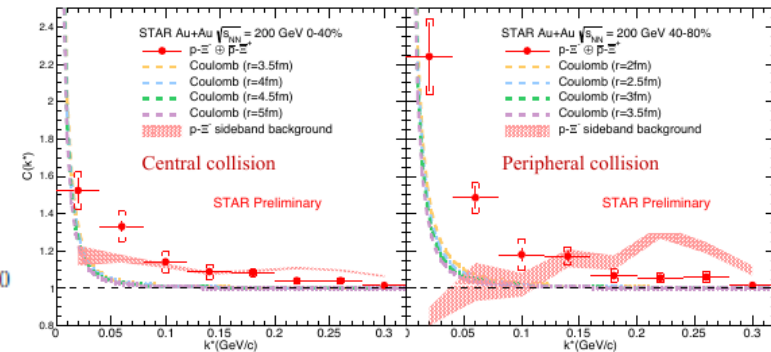
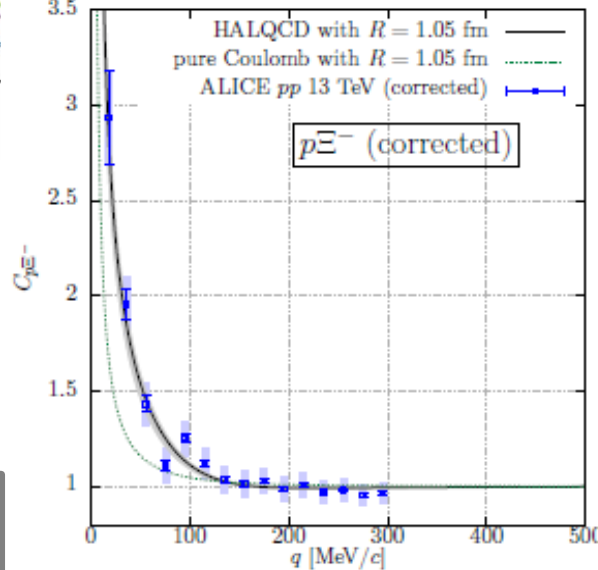
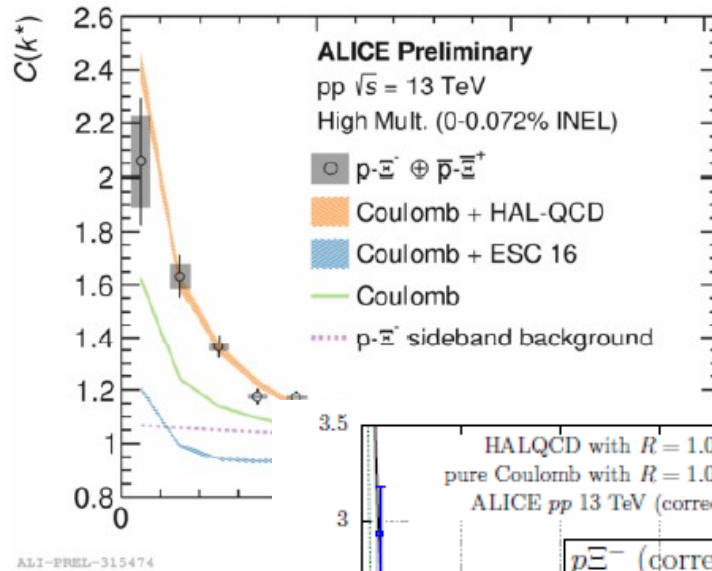
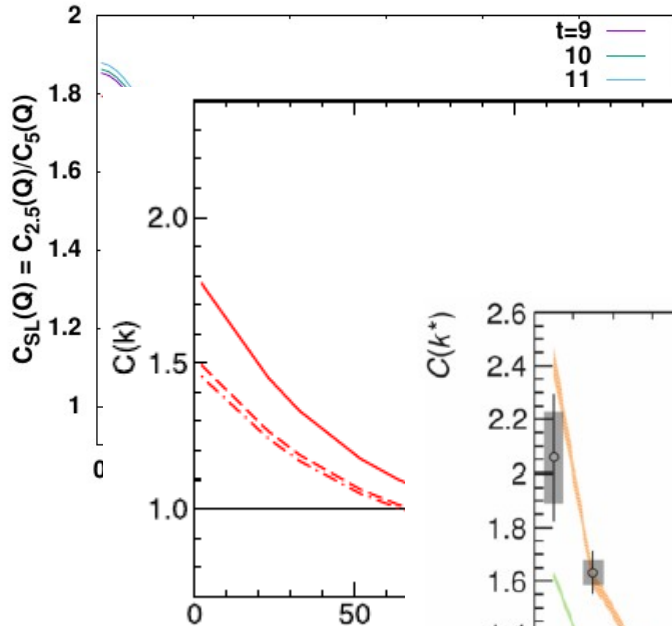
*J. Haidenbauer, NPA981('19)1.
(NLO(600), w/ CC effects, w/o Coulomb)
(w/ Coulomb, it will be comparable with data.)*

*D. L. Mihairov+[ALICE], NPA 1005 ('21)121760 (QM2019). (Nijmegen pot. does not explain the data. w/o CC)
Acharya+(ALICE), Nature ('20)*

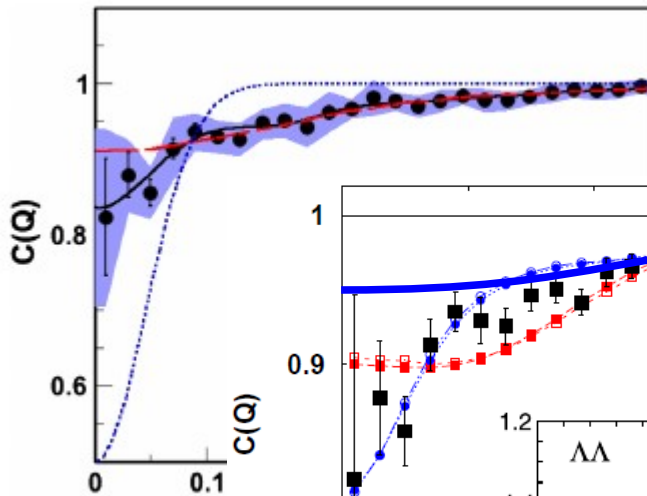
*K. Mi+(STAR, preliminary),
Au+Au 200 AGeV, APS2021.
(No Dip at larger R)*

*Kamiya+, 2108.09644
w/ Lattice BB pot. at phys. m_q
CC effects with $\Lambda\Lambda$.*

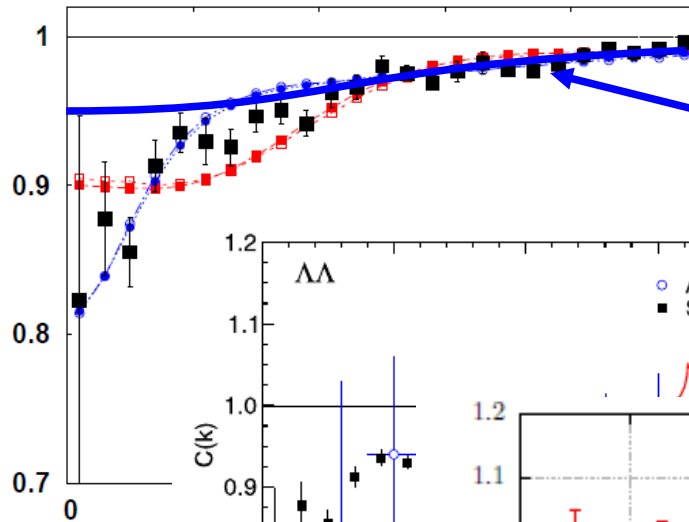
**There is no signal
of bound state.**



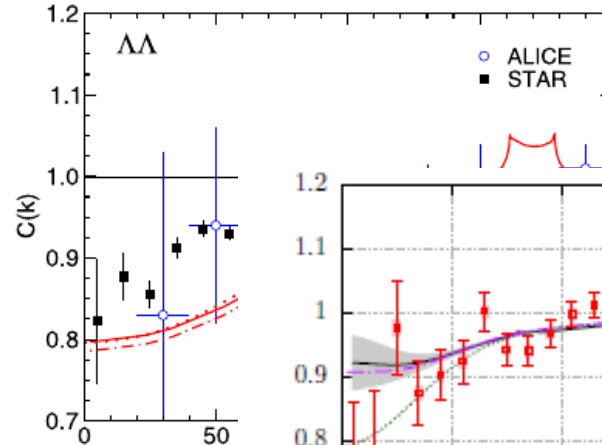
$\Lambda\Lambda$ correlation function



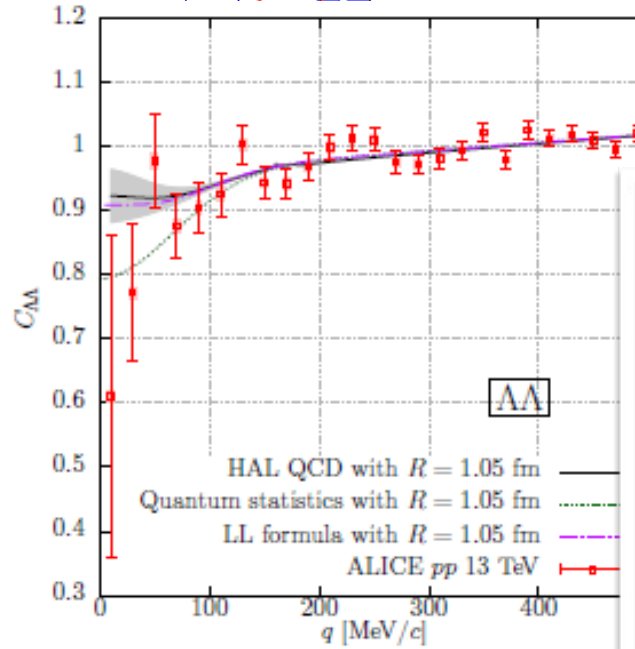
*Adamczyk+[STAR], PRL114('15)022301
(Residual source $R \sim 0.5$ fm was assumed.)*



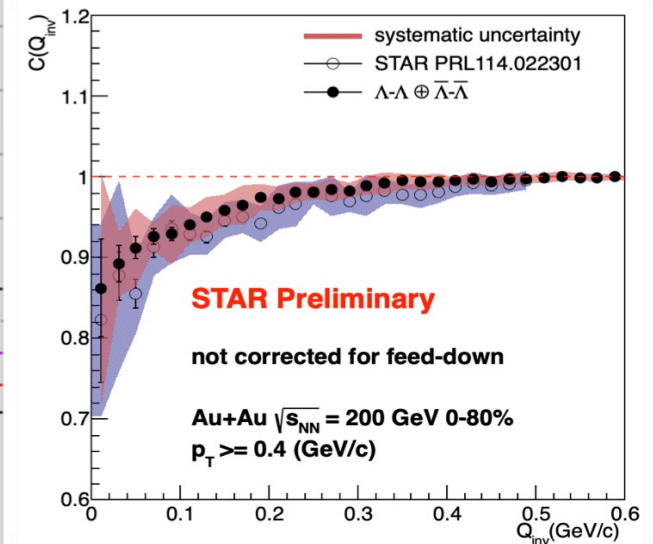
*Morita, Furumoto, AO, PRC91('15)
024916. (Res.Source + flow)*



*J. Haidenbauer, NPA981('19)1.
(NLO600)*



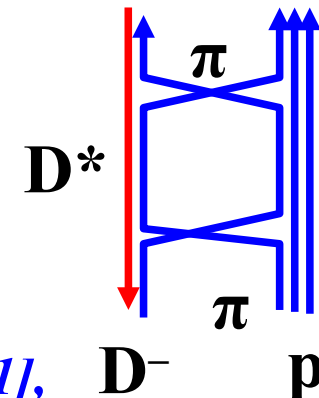
*Moe Isshiki+[STAR],
preliminary.*



*Kamiya+, 2108.09644
w/ Lattice BB pot. at phys. m_q
CC effects with $\Lambda\Lambda$.*

Charmed Hadron Interactions (1)

- $C(q)$ including a charmed hadron
 - Extremely important in recent hadron physics.
- $D^-(\bar{c}d)$ - $p(uud)$ correlation
 - Probes $\Theta_c(\bar{c}\text{-}ud\text{-}ud)$ state (replace s in $\Theta(\bar{s}\text{-}ud\text{-}ud)$ with \bar{c})
D. O. Riska, N. N. Scoccola, PLB299('93)338 (pred.); A. Aktaset+ [H1], PLB588('04)17 (positive); J. M. Linket+ [FOCUS], PLB622('05)229 (negative).
 - Proposed potentials generally predict weak or repulsive interaction.
Hofmann, Lutz ('05) (repulsive); Haidenbauer+('07) (repulsive); Yamaguchi+('11) (att., w/ bs); Fontoura+('13) (repulsive)
 - Attraction from two pion exchange *S. Yasui, K. Sudoh, PRD80('09)034008.*
 - Easy to calculate the potential in LQCD. *Y. Ikeda et al. (private comm.)*

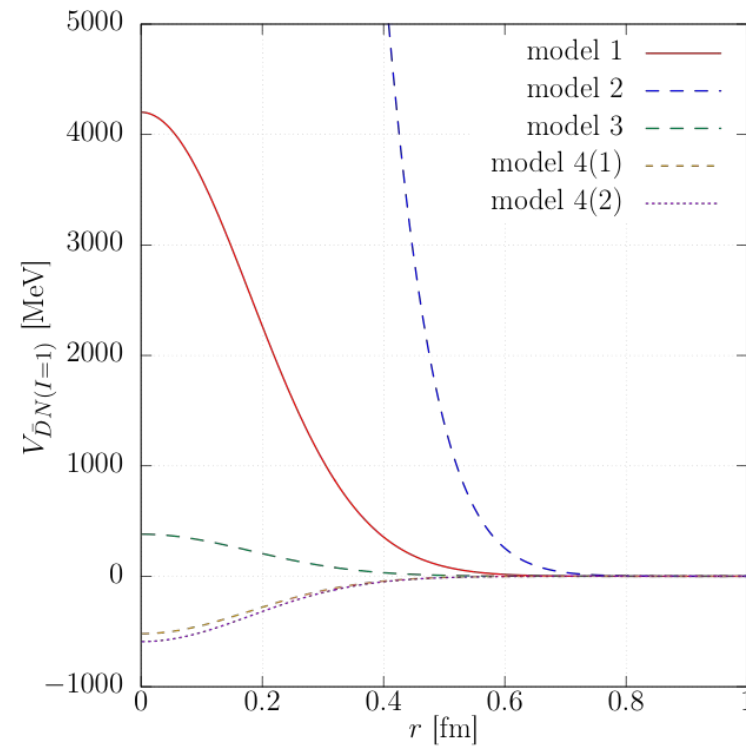
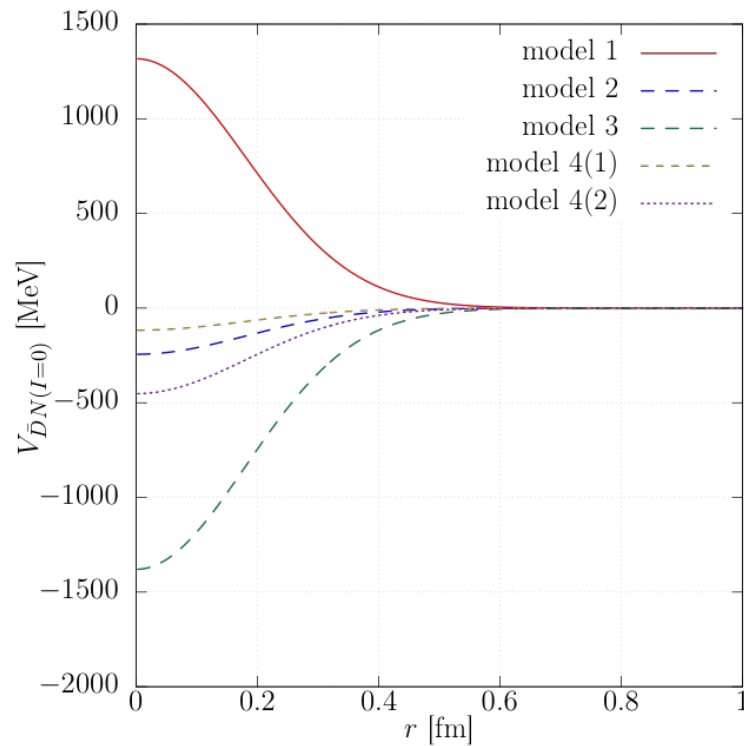


Model	$a_{DN}^{l=0}$	$a_{DN}^{l=1}$	a_D	
SU(4) contact [185]	-0.16	-0.26	-0.24	1. Hoffmann, Lutz ('05)
Meson exchange [194]	0.07	-0.45	-0.32	2. Haidenbauer+ ('07)
Pion exchange [192]	-4.38	-0.07	-1.15	3. Yamaguchi+('11)
Chiral quark model [219]	0.03-0.16	-(0.20-0.25)	-(0.14-0.15)	4. Fontoura+('13)

Taken from Hosaka, Hyodo, Sudoh, Yamaguchi, Yasui, PPNP96('17)88

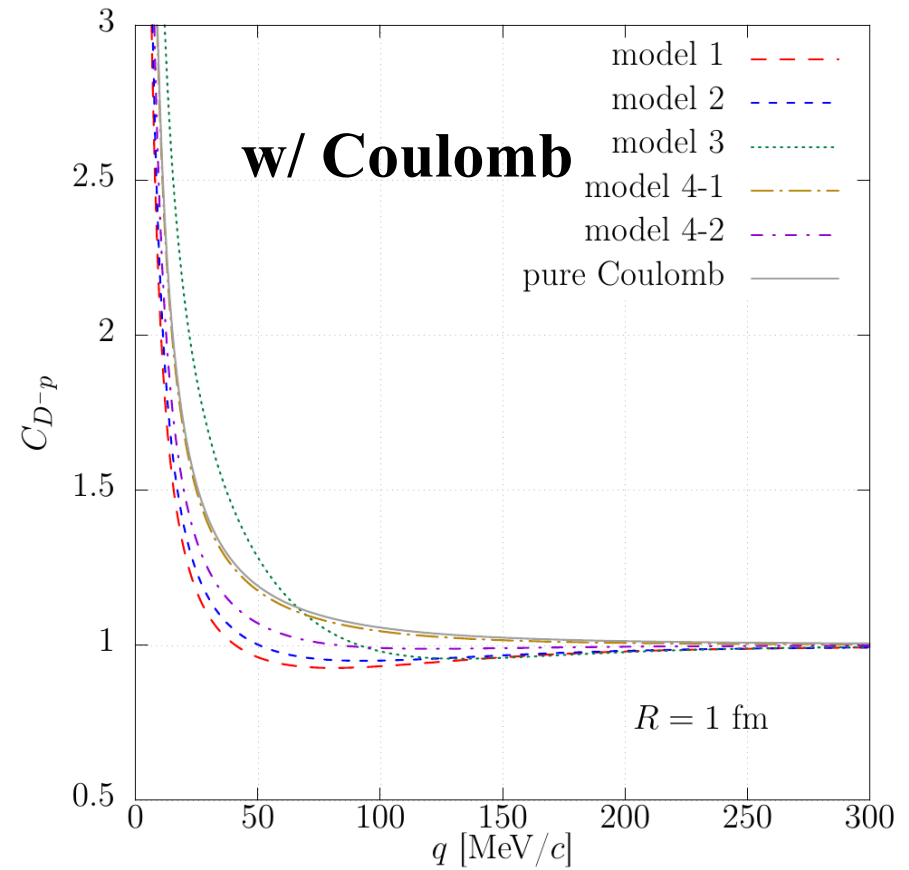
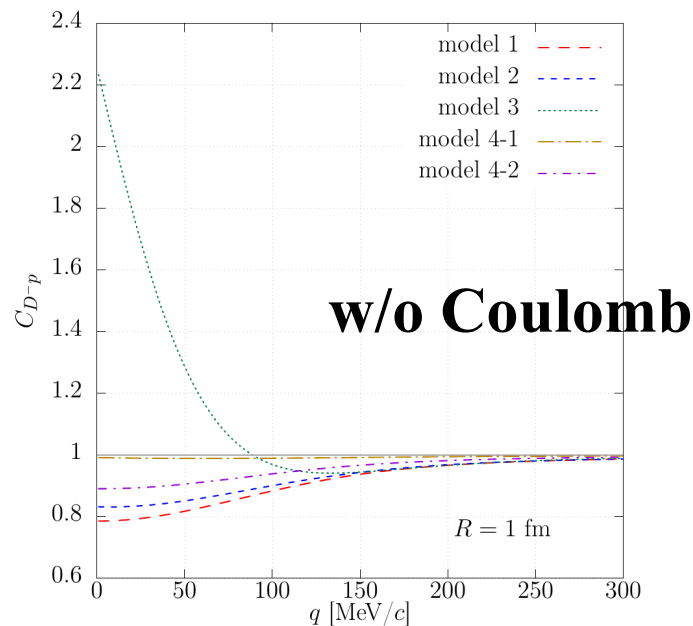
Charmed Hadron Interactions (2)

- Charm を含むハドロン間相互作用の予言は大きく違う！
 - 多くの理論は斥力 or 浅い引力
 - π 交換を取り入れた場合のみ強い引力あり



Charmed Hadron Interactions (3)

- $D^-(\bar{c}d)$ - $p(uud)$ CFs from proposed scattering length
Kamiya, Hyodo, AO (in prog.)
 - One-range Gaussian potential strength is fitted to proposed a_0 with the range of ρ meson exchange.
 - Measurable difference is found.



Data will discriminate these potentials !

Recent & Near-Future Correlation Functions

- $\bar{p}p, p\bar{\Lambda}$ E.g. A. Kisiel [ALICE], Acta Phys.Polon.Supp. 6 ('13)519

- $K^\pm K_s^0$ S.Acharya+ [ALICE], PLB774 ('17)64 [1705.04929]

→ Slightly suppressed at low q

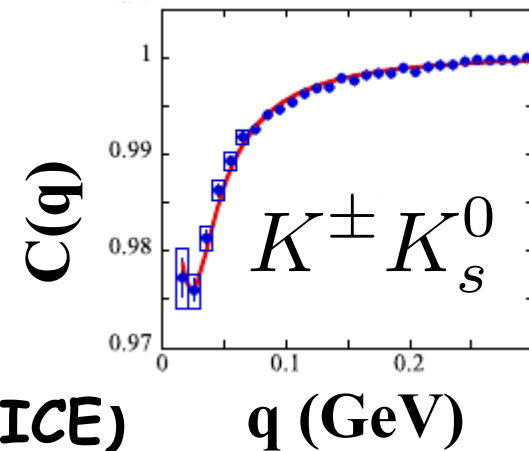
Tetraquark component of a_0 meson

- $p\Lambda$ [2104.04427], $p\phi$ [2105.05578],

$p\bar{\Lambda}, \Lambda\bar{\Lambda}$ [2105.05190], $p\Sigma^0$ ['20 [1910.14407]] (ALICE)

- pD^\pm (in prog.) Scatt. length is strongly model dependent.

→ To be discriminated by experiment !



Model	$a_{DN}^{l=0}$	$a_{DN}^{l=1}$	a_D
SU(4) contact [185]	-0.16	-0.26	-0.24
Meson exchange [194]	0.07	-0.45	-0.32
Pion exchange [192]	-4.38	-0.07	-1.15
Chiral quark model [219]	0.03-0.16	-(0.20-0.25)	-(0.14-0.15)

Hofmann+('05)
Haidenbauer+('07)
Yamaguchi+('11)
Fontoura+('13)

- deuteron-hadron CF

S. Mrówczyński and P. Słoń, Acta Phys.Polon.B51('20)1739 [1904.08320]; F. Etminan, M. M. Firoozabadi, [1908.11484]; J. Haidenbauer, PRC102('20)034001 [2005.05012]; K.Ogata, T.Fukui, Y.Kamiya, AO [2103.00100].

相関関数によるハドロン間相互作用研究の展望

pK^-
Chiral CC pot.
(examined)
Bound state
(favored)

$p\Xi^-$
Lattice QCD CC
pot. (examined)
Bound state
(disfavored)

$p\Omega$
Lattice QCD pot.
 $J=2$ (examined)
Bound state
(favored)

	n	p	K^-	K^+	π^-	π^+	Λ	Σ	Ξ^-	Ω^-	D^-	D^+	K_s	$+\alpha$
n														
p		O	O	O	Δ	Δ	O	O	O	O	O	O		
K^-		O	O	O	O	O							O	
K^+		O	O	O	O	O							O	
π^-		Δ	O	O	O	O								
π^+		Δ	O	O	O	O								
Λ		O					O							
Σ		O												
Ξ^-		O												
Ω^-		O												
D^-		O												
D^+		O												
K_s				O	O									
$+\alpha$														



pD^\pm
Chamed
hadron-
nucleon
interaction
(work in
prog.)

$K^\pm K_s^0$
Tetraquark
component
in a_0 meson

$\Lambda\Lambda$
Scattering pars. (a_0, r_{eff})
(constrained)
Bound state (disfavored)

Summary

- 相関関数を用いたハドロン間相互作用の研究 (Femtoscopic study of HHI) は様々なハドロン対について適用でき、束縛状態の有無などエキゾチックハドロン構造研究にも有用。
 - 今後数年で 100 対程度のハドロン間相互作用の情報が得られる (?)
 - ただし低運動量領域にのみシグナルがあるため、
s 波相互作用しか見えない
 - 興味あるハドロン対の「散乱長」をください。
- 多くの宿題はあります。
 - データから直接散乱パラメータは得られないか？
(現時点では限られた対のみで可能)
 - ソース関数が静的なガウス関数は簡単すぎる！
(現時点の精度で 1 次元相関関数ではサイズ調整で対応可と期待)
 - 3 体相関関数から 3 体力は引き出せるのか？
- Stay tuned !

Thank you for your attention !

Other bound states ?

■ $\Lambda\text{-N}\Xi$

- $C_{\Lambda\Lambda}(q)$ in AA(RHIC) and pp(LHC) are similar (No b.s. below $\Lambda\Lambda$).
- LQCD predicts a virtual pole near $N\Xi$ threshold, which can be detected as the cusp in $C_{\Lambda\Lambda}(q)$.

NLO(600) potential predicts the same.
(The fate of H particle)

*K. Sasaki+[HAL QCD], NPA998('20)121737;
Y. Kamiya+, 2108.09644; Haidenbauer('19).*

■ $\bar{K}N$

- $\Lambda(1405)$ is believed to be the bound state of $\bar{K}N$, and "dip" is expected at $R \sim a_0$.
 - However, Coulomb and coupled-channel effects modify the dip-like behavior.
- Kamiya+ ('20).*

