# FOUR-NEUTRINO MIXING AND OSCILLATIONS

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NEGATIVE RESULTS:

- REACTOR $\bar{\nu}_e \to \bar{\nu}_e$
- ACCELERATOR $\nu_\mu \to \nu_\mu$
- ACCELERATOR $\nu_\mu \to \nu_e$
- ACCELERATOR $\nu_\mu \to \nu_\tau$

SHORT-BASELINE EXP. (SBL)

- CHOOZ: **LBL** REACTOR $\bar{\nu}_e \to \bar{\nu}_e$

POSITIVE INDICATIONS:

- SOLAR NEUTRINO PROBLEM

$\phi_{\nu_e}^{exp} < \phi_{\nu_e}^{SSM}$ (HOMESTAKE, KAMIOKANDE, GALLEX, SAGE, SUPER-KAMIOKANDE)

$\nu_e \to \nu_\mu$ , $\nu_e \to \nu_\tau$ , $\nu_e \to \nu_s$

- ATMOSPHERIC NEUTRINO ANOMALY

$R \equiv \frac{(\mu/e)_{exp}}{(\mu/e)_{th}} < 1$ (KAMIOKANDE, IMB, SOUDAN, SUPER-KAMIOKANDE)

ATM + CHOOZ $\Rightarrow$ $\nu_\mu \to \nu_\tau$ , $\nu_\mu \to \nu_s$

- LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ TRANSITIONS (SBL)

(WEAKER EVIDENCE OF $\nu_\mu \to \nu_e$ TRANSITIONS)

# 3 INDICATIONS OF $\nu$ OSC. WITH 3 DIFFERENT SCALES OF $\Delta m^2$ $(\Delta m^2_{kj} \equiv m_k^2 - m_j^2)$

$$\Delta m^2_{SUN} \sim 10^{-5}\,eV^2\,(MSW) \text{ OR } 10^{-10}\,eV^2\,(VO)$$

$$\Delta m^2_{ATM} \sim 10^{-3} - 10^{-2}\,eV^2$$

$$\Delta m^2_{LSND} \sim 0.3 - 2\,eV^2$$

## AT LEAST 4 MASSIVE $\nu$'S ARE NEEDED!

$(Z^0 \to \nu\bar{\nu})$ @LEP $\Rightarrow$ $\nu_e, \nu_\mu, \nu_\tau$

FLAVOR NEUTRINOS

IN GENERAL FLAVOR NEUTRINOS ARE NOT MASS EIGENSTATES:

$$\nu_{\alpha L} = \sum_{k=1}^{n} U_{\alpha k}\, \nu_{kL} \qquad \alpha = e, \mu, \tau$$

$$\nu_k \to m_k \qquad k = \underbrace{1, 2, 3, \ldots}_{n}\,?$$

STERILE NEUTRINO

$$n = 4 \Rightarrow \underbrace{\nu_e, \nu_\mu, \nu_\tau, \nu_s}_{\text{FLAVOR BASIS}}$$

# SCHEMES WITH 4 MASSIVE NEUTRINOS:

$$\overbrace{\underbrace{m_1 \ll m_2}_{\text{SUN}} \ll m_3 \ll m_4}^{\text{LSND}}$$

$$\underbrace{m_1 \ll m_2 \ll m_3}_{\text{ATM}}$$

HIERARCHY

$$\underbrace{\overbrace{m_1 \lesssim m_2}^{\text{ATM}} \ll \overbrace{m_3 \lesssim m_4}^{\text{SUN}}}_{\text{LSND}}$$

(A)

$$\overbrace{\underbrace{m_1 \lesssim m_2}_{\text{SUN}} \ll \underbrace{m_3 \lesssim m_4}_{\text{ATM}}}^{}$$

(B)

IN ALL THESE SCHEMES ONLY THE LARGEST $\Delta m^2$ $\left(\Delta m^2_{41} \equiv m_4^2 - m_1^2\right)$ IS RELEVANT FOR OSCILLATIONS IN SHORT-BASELINE EXPERIMENTS (LSND)

# SBL EXPERIMENTS

TRANSITION PROBABILITIES $(\beta \neq \alpha)$

$$P^{(SBL)}_{\nu_\alpha \to \nu_\beta} = A_{\alpha\beta} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

OSCILLATION AMPLITUDE

$$2G \Rightarrow P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

SURVIVAL PROBABILITIES $(\beta = \alpha)$

$$P^{(SBL)}_{\nu_\alpha \to \nu_\alpha} = 1 - B_{\alpha\alpha} \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right)$$

OSCILLATION AMPLITUDE

$$2G \Rightarrow P_{\nu_\alpha \to \nu_\alpha} = 1 - \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

# $4\,\nu$'s WITH A MASS HIERARCHY

$$\overbrace{\underbrace{\underbrace{m_1 \ll m_2}_{\text{SUN}} \ll m_3}_{\text{ATM}} \ll m_4}^{\text{LSND}}$$

OSCILLATION AMPLITUDES IN SBL

$$A_{\alpha\beta} = 4\,|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$ FLAVOR TRANSITIONS

$$B_{\alpha\alpha} = 4\,|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2\right)$$ SURVIVAL

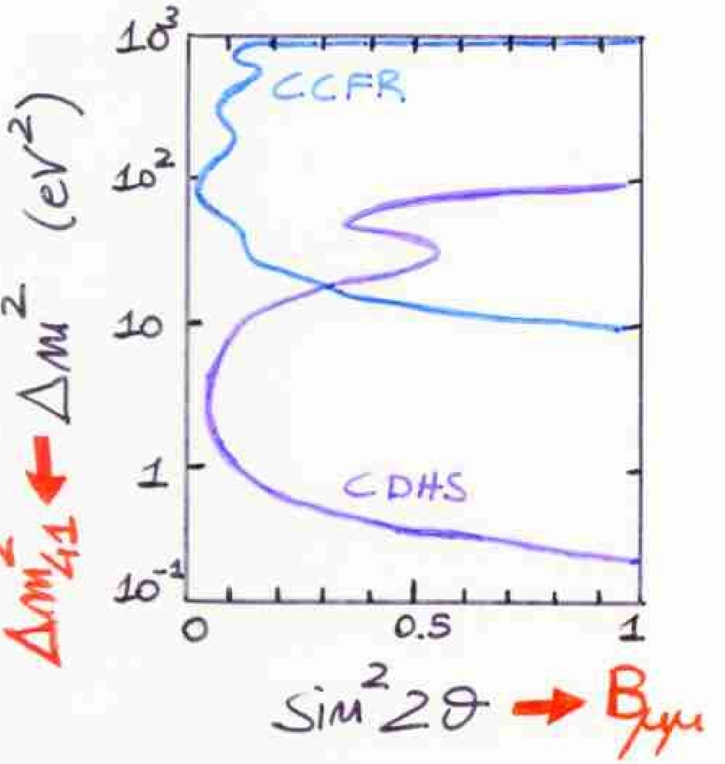
SBL OSCILLATIONS DEPEND ON 4 PARAMETERS:

$$\Delta m^2_{41} \,,\ |U_{e4}|^2,\ |U_{\mu 4}|^2,\ |U_{\tau 4}|^2$$

# SBL DISAPPEARANCE EXP. WITH $\bar{\nu}_e$ AND $\nu_\mu$

BUGEY (1995) $\bar{\nu}_e \to \bar{\nu}_e$ $\Delta m^2 \gtrsim 10^{-2}\,eV^2$

CDHS+CCFR (1984) $\nu_\mu \to \nu_\mu$ $\Delta m^2 \gtrsim 0.2\,eV^2$




FOR EACH POSSIBLE VALUE OF $\Delta m^2_{41}$

$$B_{\mu\mu} \leq B^0_{\mu\mu}$$

$$B_{ee} \leq B^0_{ee}$$

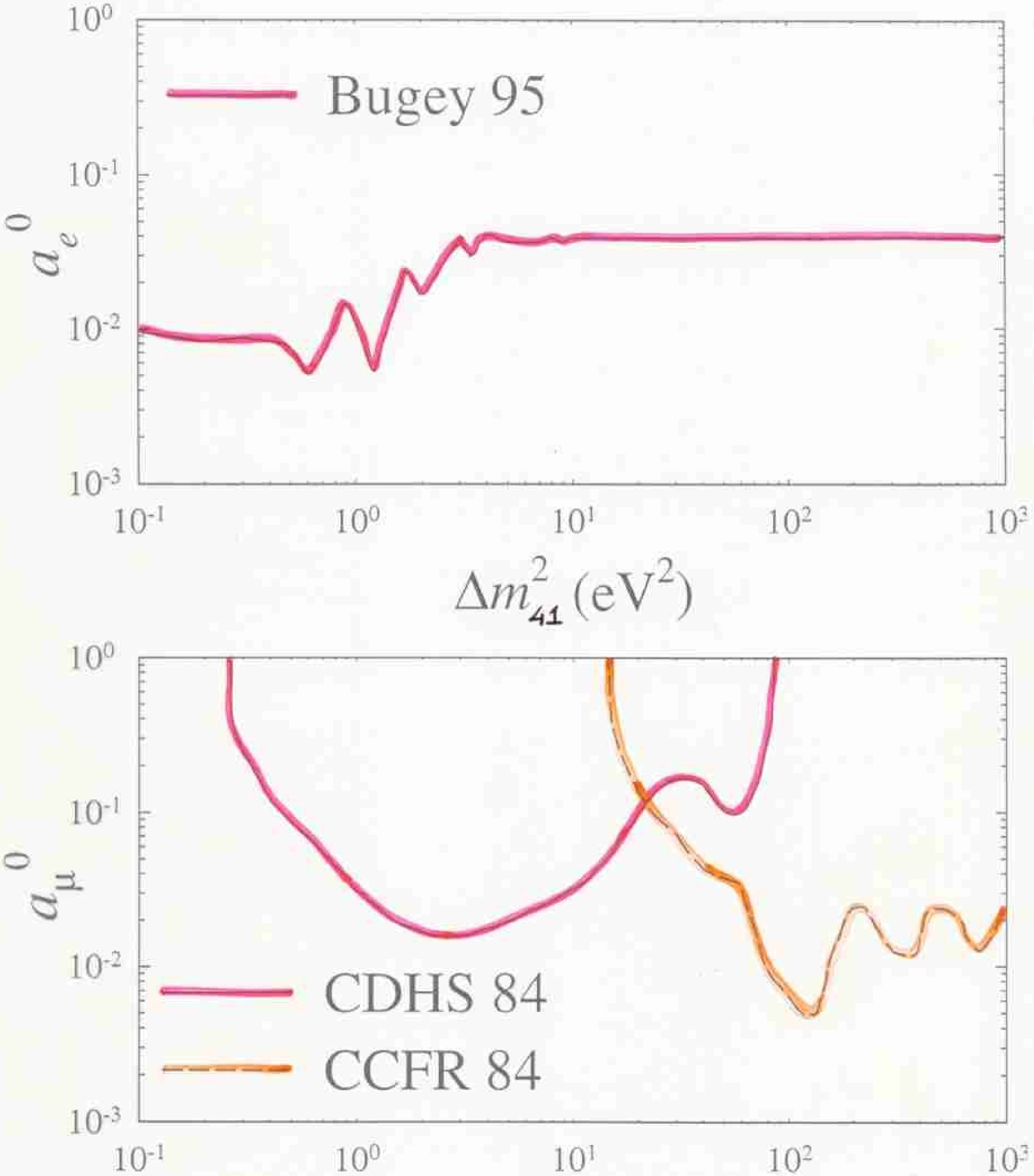
$$B_{\alpha\alpha} = 4|U_{\alpha 4}|^2(1-|U_{\alpha 4}|^2) \Rightarrow |U_{\alpha 4}|^2 = \frac{1}{2}\left(1 \pm \sqrt{1-B_{\alpha\alpha}}\right)$$

$$|U_{\alpha 4}|^2 \leq \frac{1}{2}\left(1-\sqrt{1-B^0_{\alpha\alpha}}\right) \equiv a^0_\alpha$$

DEPEND ON $\Delta m^2$

OR

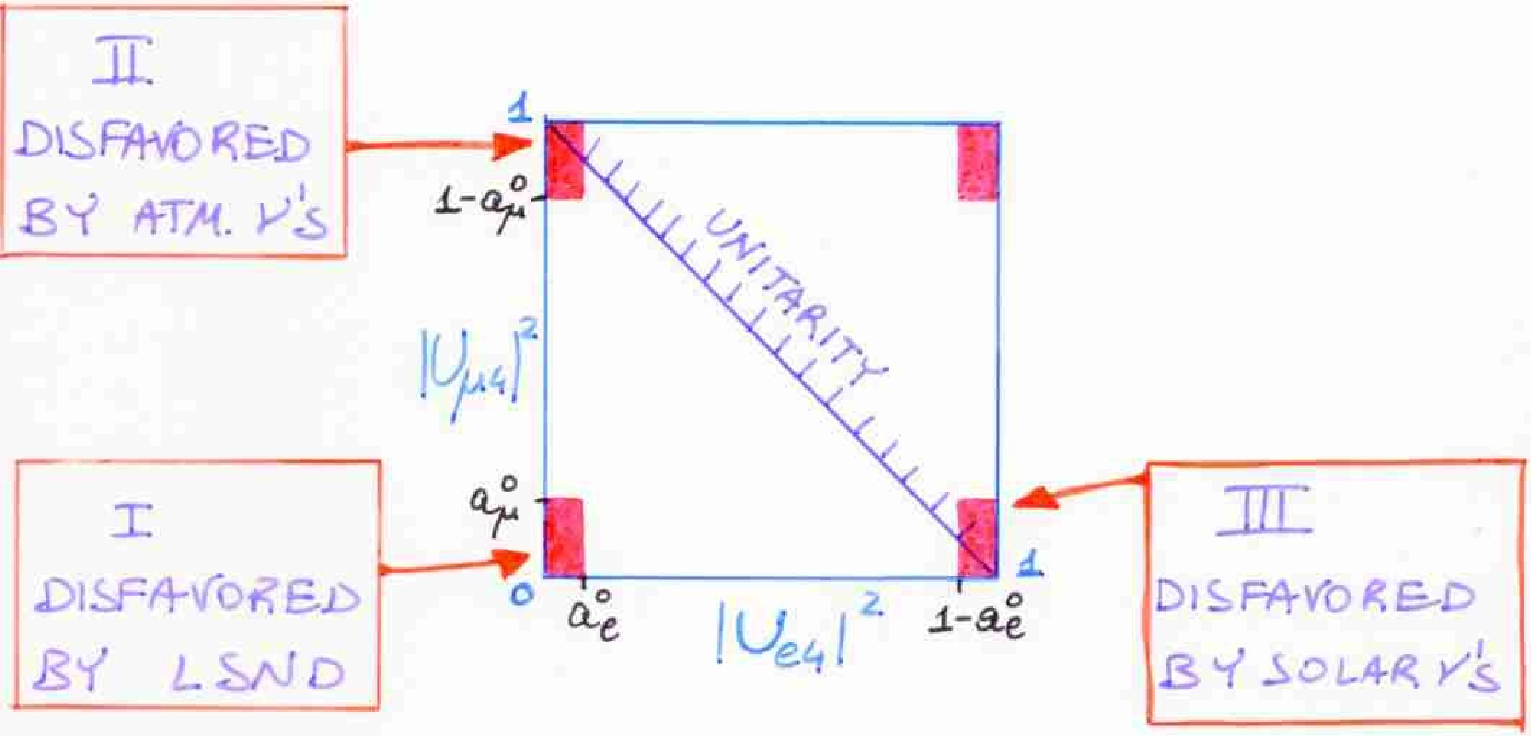
$$|U_{\alpha 4}|^2 \geq \frac{1}{2}\left(1+\sqrt{1-B^0_{\alpha\alpha}}\right) = 1-a^0_\alpha \qquad (\alpha = e, \mu)$$



Bugey 95
$a_e^0$
$10^0$
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-1}$ $10^0$ $10^1$ $10^2$ $10^3$
$\Delta m^2_{41}$ (eV$^2$)
CDHS 84
CCFR 84
$a_\mu^0$
$10^0$
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-1}$ $10^0$ $10^1$ $10^2$ $10^3$
$\Delta m^2_{41}$ (eV$^2$)

$$\left.\begin{array}{c} |U_{e4}|^2 \leq a_e^0 \\ \text{OR} \\ |U_{e4}|^2 \geq 1 - a_e^0 \end{array}\right\} \quad \text{AND} \quad \left\{\begin{array}{c} |U_{\mu 4}|^2 \leq a_\mu^0 \\ \text{OR} \\ |U_{\mu 4}|^2 \geq 1 - a_\mu^0 \end{array}\right.$$

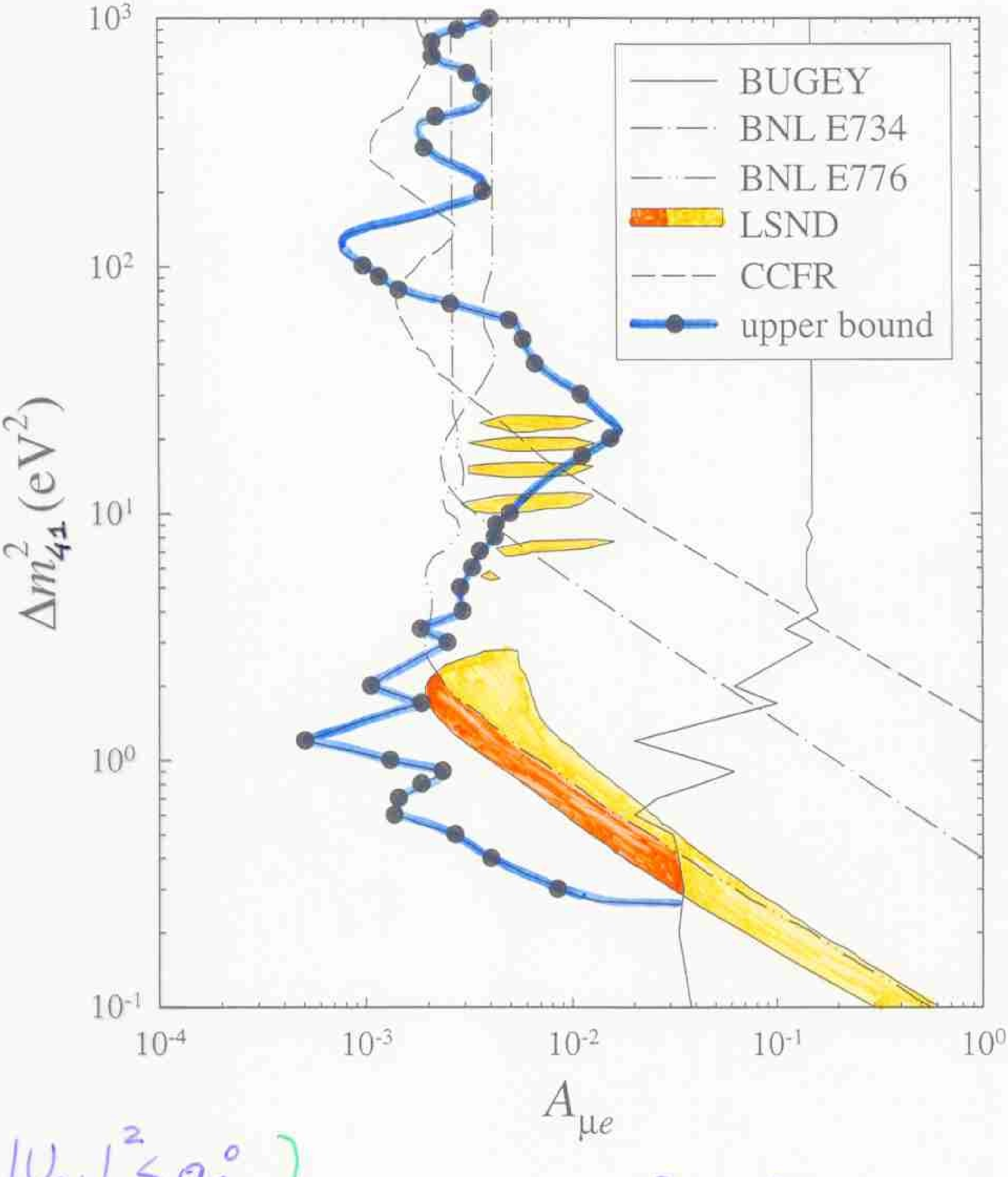
UNITARITY OF $U$ $\rightarrow$ $|U_{e4}|^2 + |U_{\mu 4}|^2 \leq 1$




$$\underbrace{\overbrace{m_1 \ll m_2}^{\text{SUN}} \ll m_3}_{\text{ATM}} \ll m_4$$

**THIS SCHEME IS DISFAVORED BY DATA**

ANALOGOUSLY, ALL THE 4-NEUTRINO SCHEMES WITH A MASS SEPARATED BY THE OTHER THREE BY THE "LSND GAP" ARE DISFAVORED BY THE DATA




$$\left.\begin{array}{l} |U_{e4}|^2 \leq a_e^0 \\ |U_{\mu 4}|^2 \leq a_\mu^0 \end{array}\right\} \Rightarrow A_{\mu e} = 4|U_{e4}|^2 |U_{\mu 4}|^2 \leq 4\, a_e^0 a_\mu^0$$

ONLY 2 SCHEMES WITH 4 NEUTRINOS ARE COMPATIBLE WITH THE RESULTS OF ALL EXP.

(A) $\underbrace{\overbrace{m_1 \lesssim m_2}^{\text{ATM}} \ll \overbrace{m_3 \lesssim m_4}^{\text{SUN}}}_{\text{LSND}}$

(B) $\underbrace{\overbrace{m_1 \lesssim m_2}^{\text{SUN}} \ll \overbrace{m_3 \lesssim m_4}^{\text{ATM}}}_{\text{LSND}}$

SURVIVAL PROBABILITIES IN SBL DISAPPEARANCE EXP.

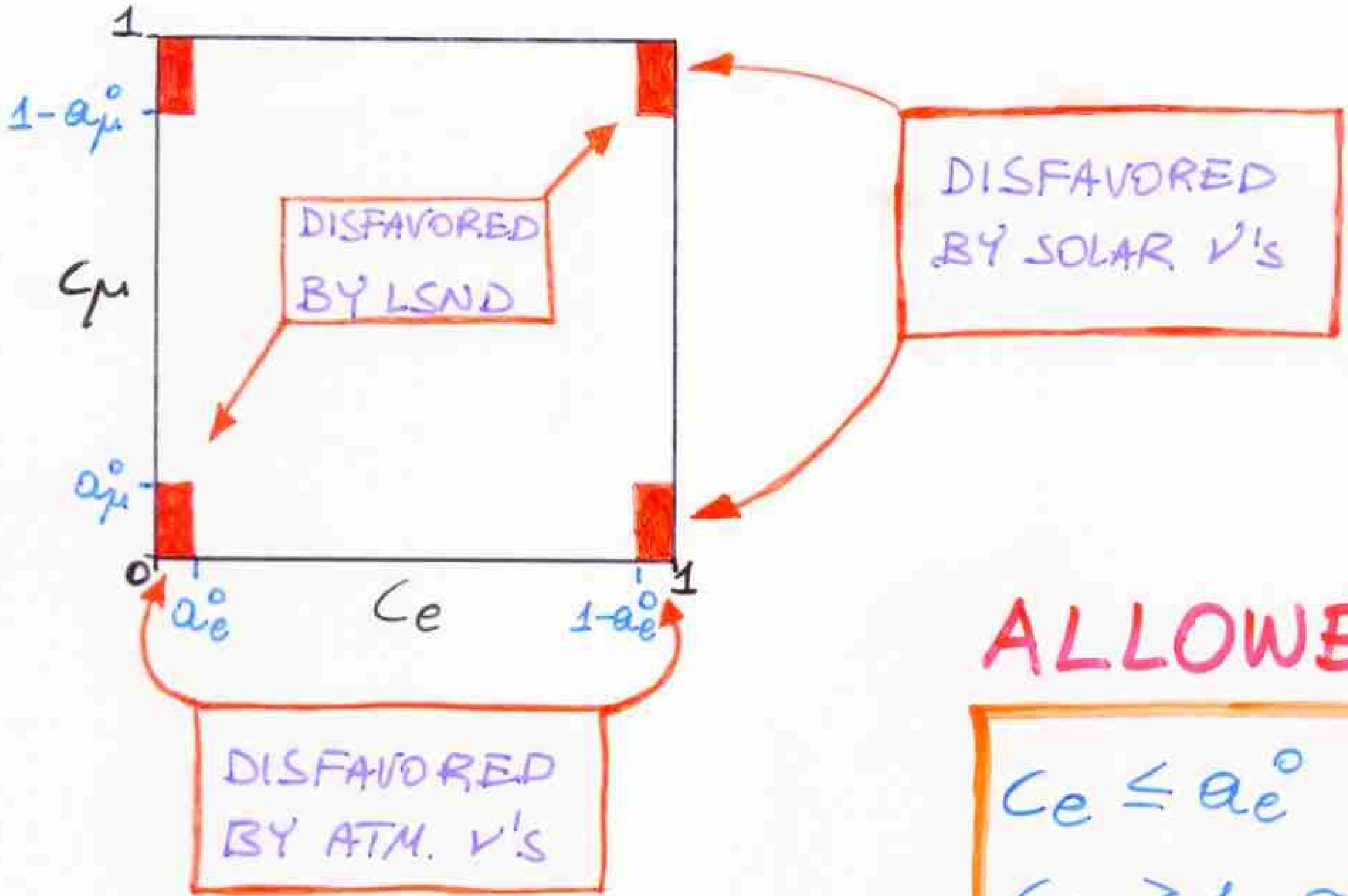
$$P_{\nu_\alpha \to \nu_\alpha} = 1 - B_{\alpha\alpha} \sin^2\left(\frac{\Delta m_{41}^2 L}{4E}\right)$$

$$\Delta m_{41}^2 \equiv m_4^2 - m_1^2 \qquad B_{\alpha\alpha} = c_\alpha (1 - c_\alpha)$$

$c_\alpha = |U_{\alpha 1}|^2 + |U_{\alpha 2}|^2$ IN SCHEME (A)

$c_\alpha = |U_{\alpha 3}|^2 + |U_{\alpha 4}|^2$ IN SCHEME (B)

$$\left.\begin{array}{c} c_e \le a_e^0 \\ \text{OR} \\ c_e \ge 1 - a_e^0 \end{array}\right\} \quad \text{AND} \quad \left\{\begin{array}{c} c_\mu \le a_\mu^0 \\ \text{OR} \\ c_\mu \ge 1 - a_\mu^0 \end{array}\right.$$



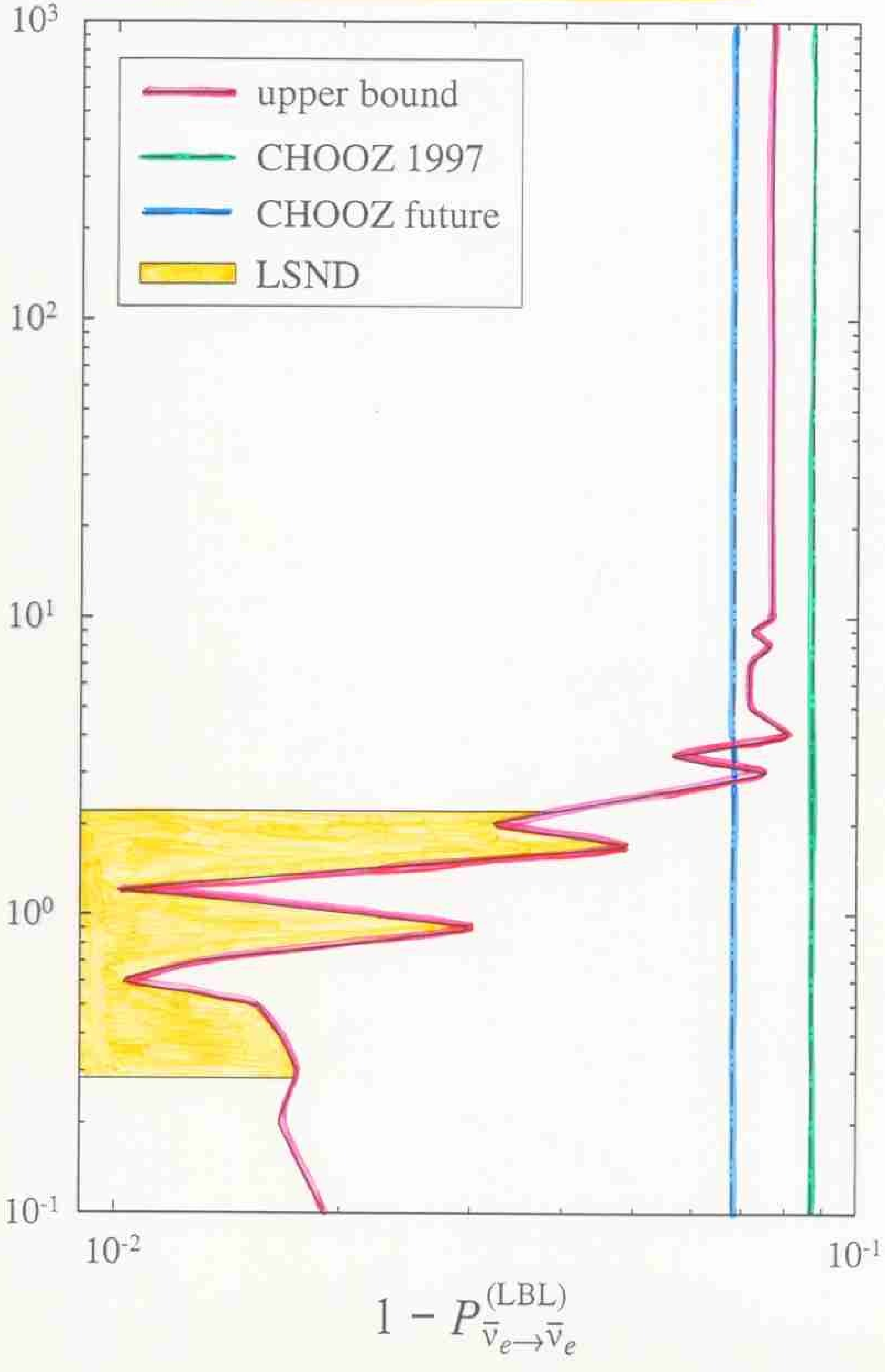

**ALLOWED**

$$\boxed{\begin{array}{l} c_e \le a_e^0 \\ c_\mu \ge 1 - a_\mu^0 \end{array}}$$

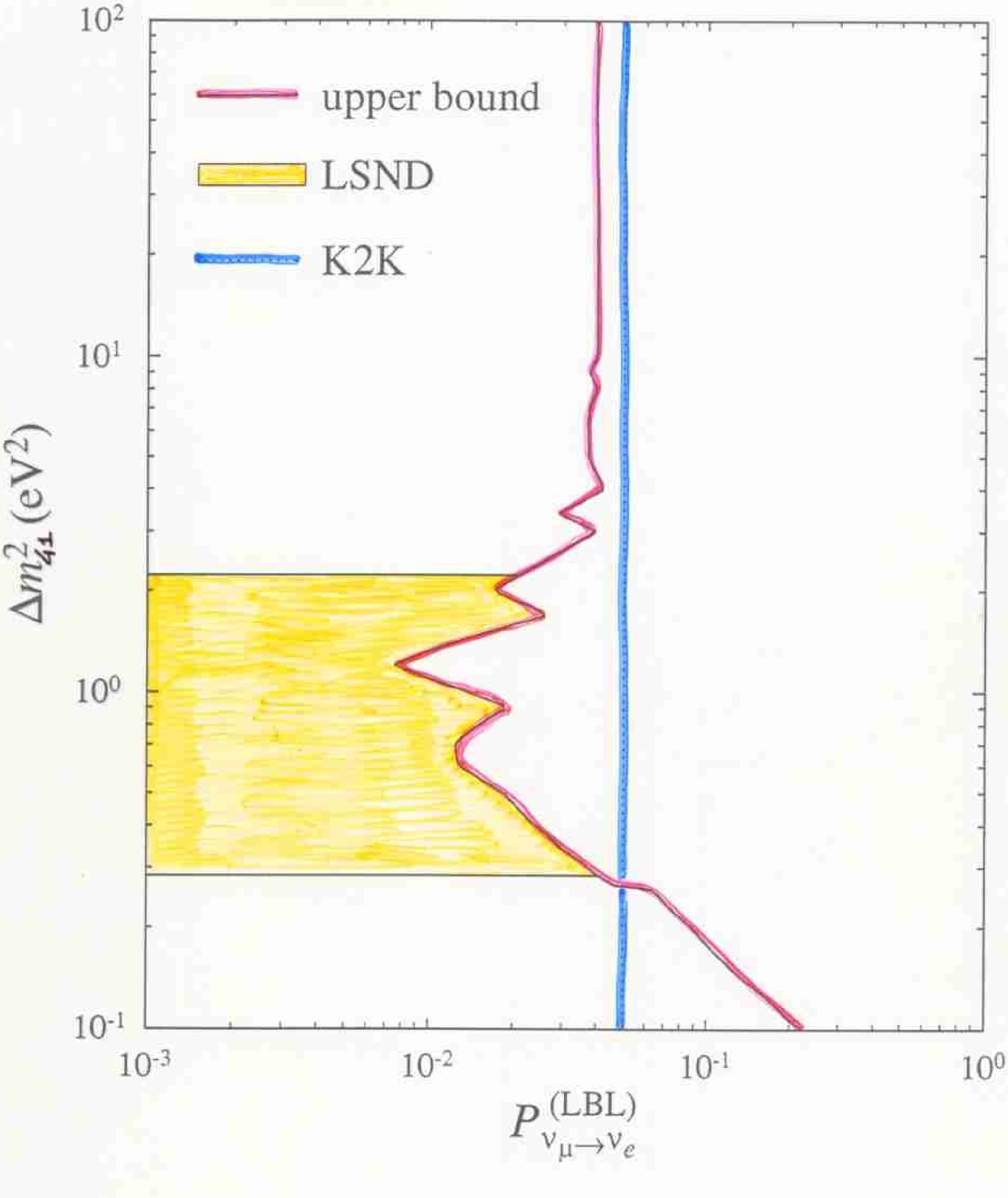
(A) $\underbrace{\overbrace{m_1 \le m_2}^{\text{ATM}} \ll \overbrace{m_3 \lesssim m_4}^{\text{SUN}}}_{\text{LSND}}$ $\qquad c_\alpha = |U_{\alpha 1}|^2 + |U_{\alpha 2}|^2$

(B) $\underbrace{\overbrace{m_1 \lesssim m_2}^{\text{SUN}} \ll \overbrace{m_3 \lesssim m_4}^{\text{ATM}}}_{\text{LSND}}$ $\qquad c_\alpha = |U_{\alpha 3}|^2 + |U_{\alpha 4}|^2$

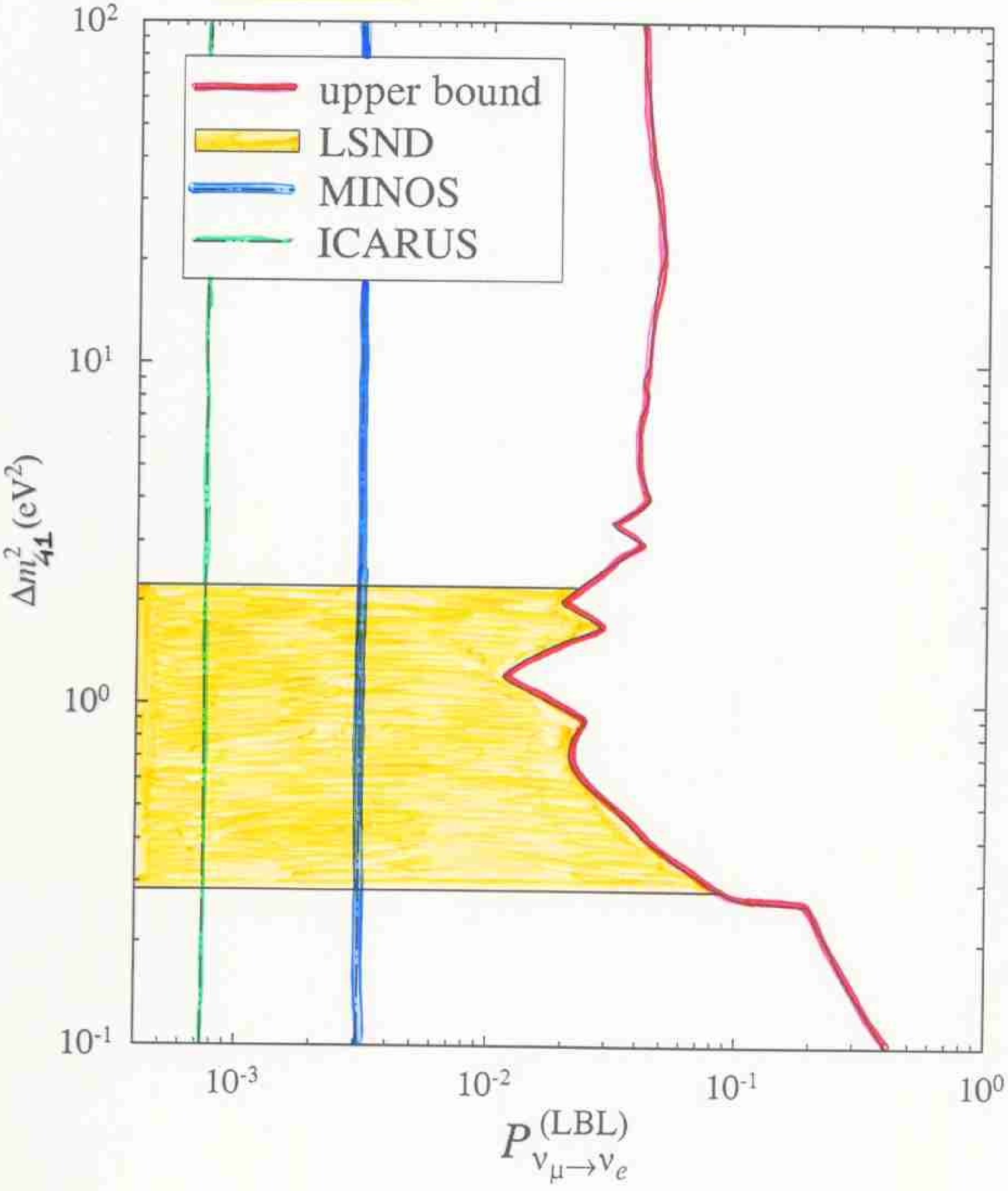
$$\boxed{1 - P^{(LBL)}_{\bar\nu_e \to \bar\nu_e} \le a_e^0 \left(2 - a_e^0\right)}$$



$$P^{(LBL)}_{\nu_\mu \to \nu_e} \leq a^0_e + \frac{1}{4} A^0_{\mu e}$$

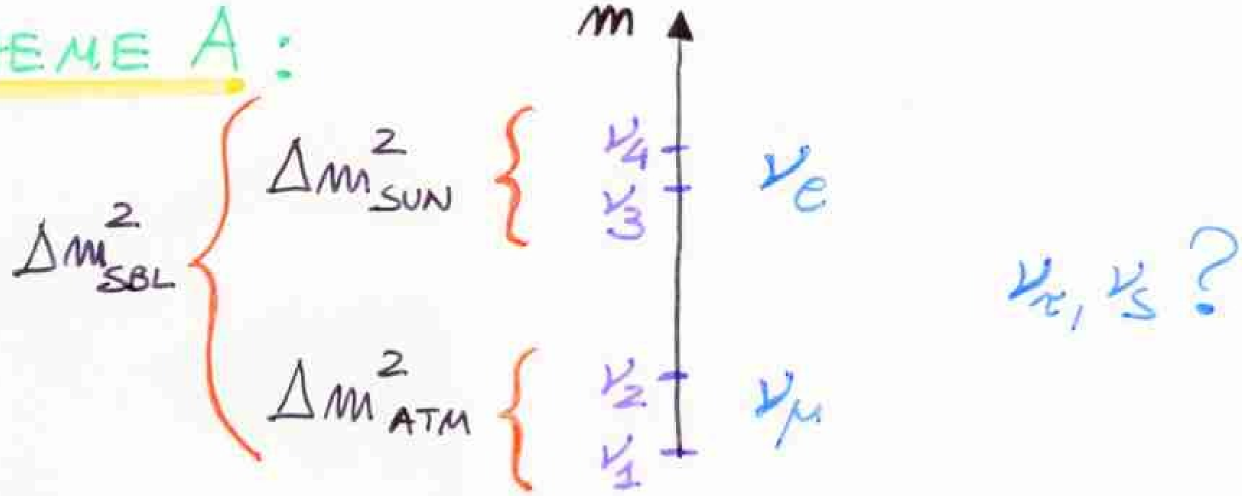


$$\boxed{P^{(LBL)}_{\nu_\mu \to \nu_e} \leq a_e^0 + \frac{1}{4} A_{\mu e}^0}$$

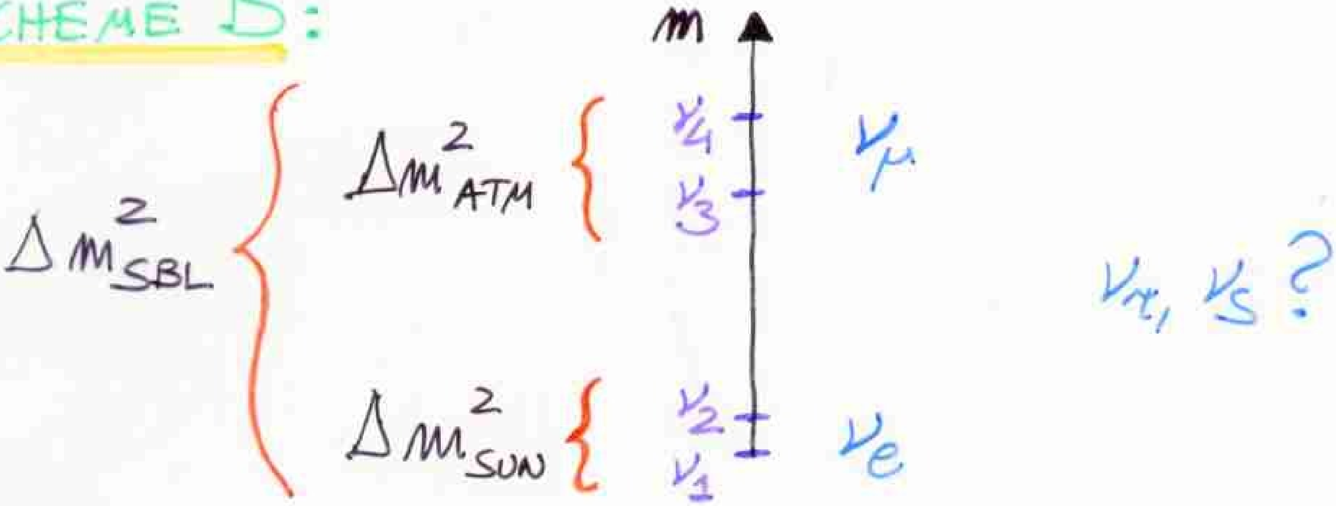


# FROM THE RESULTS OF NEUTRINO OSCILLATION EXPERIMENTS:

SCHEME A:




SCHEME B:




# IMPLICATIONS FOR $(\beta\beta)_{0\nu}$ AND $^3H$ EXP.:

SCHEME A: "HEAVY $\nu_e$" $\begin{cases} \langle m \rangle \lesssim m_4 \\ m(^3H) \simeq m_4 \end{cases}$

SCHEME B: "LIGHT $\nu_e$" $\begin{cases} \langle m \rangle \leq a_e^0\, m_4 \ll m_4 \\ m(^3H) \ll m_4 \end{cases}$

# Big-Bang Nucleosynthesis Constraint on $N_\nu$

$N_\nu \equiv$ NUMBER OF LIGHT NEUTRINOS ($m \ll 1\,\text{MeV}$) IN EQUILIBRIUM AT $T_{DEC} \simeq 2-4\,\text{MeV}$

$\frac{n}{p}$ FREEZES OUT AT $T_f \simeq 0.7\,\text{MeV}$ (IN THE SM)

$$\frac{n}{p} \simeq e^{-\frac{m_n - m_p}{T_f}} \simeq \frac{1}{6} \quad \text{IN THE SM}$$

THE VALUE OF $T_f$ IS DETERMINED BY

$$T_f^5 \sim \Gamma_w(T_f) = H(T_f) \sim g_*^{1/2}\, T_f^2$$

$g_* = 5.5 + 1.75\, N_\nu$

$$\boxed{T_f \propto g_*^{1/6}}$$

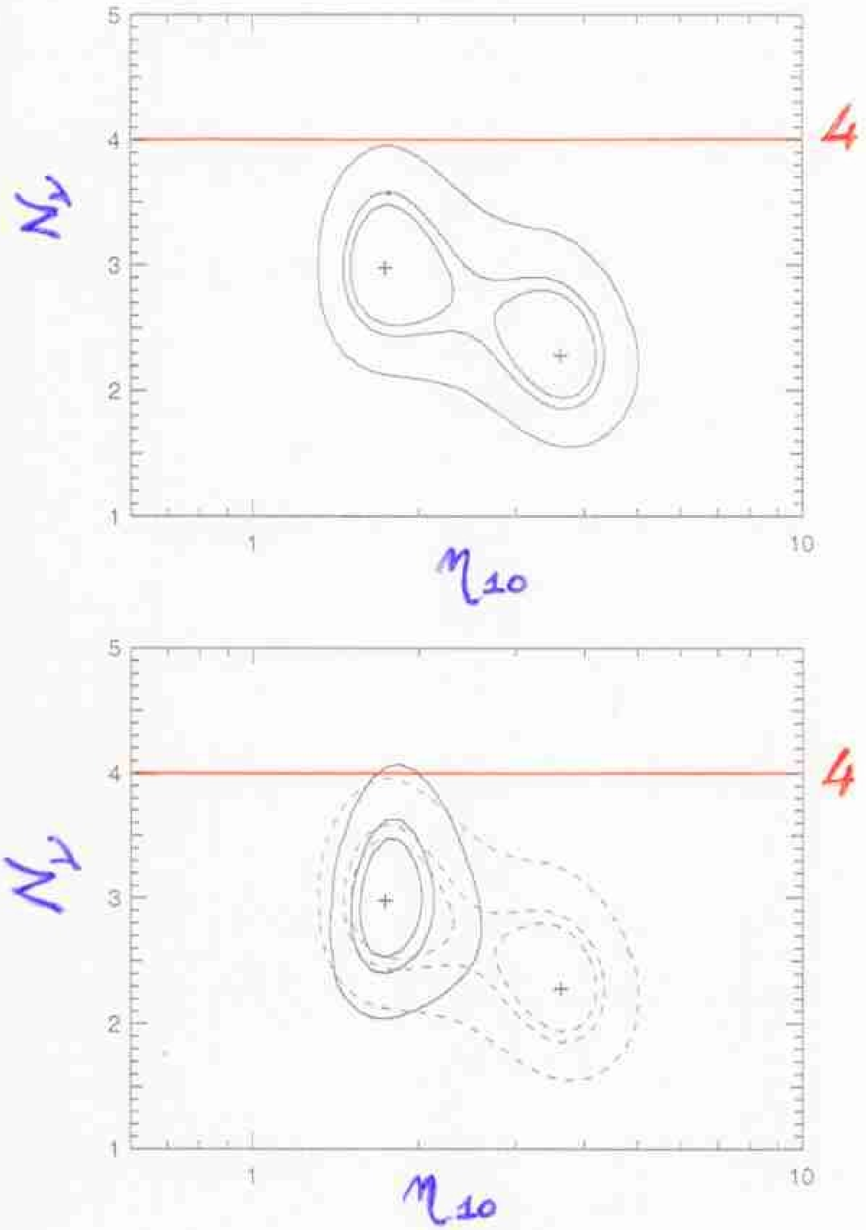
$$N_\nu > 3 \rightarrow T_f > 0.7\,\text{MeV} \rightarrow \frac{n}{p} > \frac{1}{6}$$

$$Y_p \equiv \frac{\text{MASS OF } {}^4He}{\text{TOTAL MASS}} = \frac{4\, \frac{n}{2}}{p+n} = \frac{2\, n/p}{1 + n/p}$$

$$\underbrace{n/p = 1/6}_{T_f} \xrightarrow{\beta} \underbrace{1/7}_{T_{BBN} \simeq 0.1\,\text{MeV}} \longrightarrow Y_p \simeq 0.25$$

# BBN CONSTRAINT ON $N_\nu$

K.A. Olive and D. Thomas, Astropart. Phys. 7, 27 (1997)




Top: Contours in the combined likelihood function for $^4$He and $^7$Li. The contours represent 50% (innermost), 68% and 95% (outermost) confidence levels. The crosses mark the points of maximum likelihood.
Bottom: The equivalent result when D is included.

$N_\nu \equiv$ NUMBER OF LIGHT NEUTRINOS ($m \ll 1$ MeV) IN EQUILIBRIUM AT $T_{DEC} \simeq 2$-$4$ MeV

# BBN CONSTRAINTS ON 4-NEUTRINO MIXING ($c_S$)

(OKADA & YASUDA 1996)

$n_{\nu_S} \equiv$ RELATIVE NUMBER DENSITY OF $\nu_S$

FOR $t < t_{DEC}$ ($T > T_{DEC}$)

COLLISION RATES $\propto T^5$

$$\frac{d n_{\nu_S}}{dt} = \frac{1}{2} \sum_{\alpha=e,\mu,\tau} \langle P_{\nu_\alpha \to \nu_S} \rangle_{COLL} \, \Gamma_{\nu_\alpha} (1 - n_{\nu_S})$$

(KAINULAINEN 1990)

VALID FOR

- NON-RESONANT AND ADIABATIC RESONANT TRANSITIONS
- $t_{OSC} \ll t_{COLL} \ll t_{EXP}$ WITH $t_{EXP} = \frac{1}{H}$

$$H \equiv \frac{\dot{R}}{R} \qquad R \propto T^{-1} \qquad H = -\frac{\dot{T}}{T}$$

$$\frac{d n_{\nu_S}}{dT} = -\frac{1}{2HT} \sum_{\alpha=e,\mu,\tau} \langle P_{\nu_\alpha \to \nu_S} \rangle_{COLL} \, \Gamma_{\nu_\alpha} (1 - n_{\nu_S})$$

$$n_{\nu_S}(T_{DEC}) = 1 - e^{-F}$$

(J. CLINE 1992)

$$F \equiv -\int_{T_i}^{T_{DEC}} \frac{1}{2HT} \sum_{\alpha=e,\mu,\tau} \langle P_{\nu_\alpha \to \nu_S} \rangle_{COLL} \, \Gamma_{\nu_\alpha} \, dT$$

$$n_{\nu_s}(T_{DEC}) = 1 - e^{-F}$$

$$N_\nu \leq 3 + \delta N \Rightarrow n_{\nu_s}(T_{DEC}) \leq \delta N$$

$$\boxed{F \leq \left| \ln(1-\delta N) \right|}$$

# CALCULATION OF F $(c_e = 0)$

## SCHEME A:

$$U = \begin{array}{cc} \begin{pmatrix} 0 & 0 & c_\theta & s_\theta \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ s_\varphi s_\chi & -s_\varphi c_\chi & -c_\varphi s_\theta & c_\varphi c_\theta \end{pmatrix} & \begin{array}{l} \leftarrow \nu_e \\ \leftarrow \nu_\mu \\ \leftarrow \nu_\tau \\ \leftarrow \nu_s \end{array} \end{array}$$

## SCHEME B:

$$U = \begin{array}{cc} \begin{pmatrix} c_\theta & s_\theta & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ c_\varphi s_\theta & -c_\varphi c_\theta & -s_\varphi s_\chi & s_\varphi c_\chi \end{pmatrix} & \begin{array}{l} \leftarrow \nu_e \\ \leftarrow \nu_\mu \\ \leftarrow \nu_\tau \\ \leftarrow \nu_s \end{array} \end{array}$$

IN BOTH SCHEMES $c_s = \sin^2\varphi$

NEUTRINO EFFECTIVE POTENTIALS IN THE PRIMORDIAL PLASMA (NOTZOLD & RAFFELT 1988)

$$V_e = -6.02 \frac{G_F E}{M_W^2} T^4 \equiv V \qquad V_\mu = V_\tau = \xi V$$

$$V_s = 0 \qquad \xi = \frac{\cos^2\theta_W}{2+\cos^2\theta_W} \simeq 0.28$$

EFFECTIVE HAMILTONIAN (WEAK BASIS)

$$H_W = \frac{1}{2E} U \,\mathrm{DIAG}(m_1^2, m_2^2, m_3^2, m_4^2)\, U^\dagger$$
$$+ \,\mathrm{DIAG}(V, \xi V, \xi V, 0)$$

SUBTRACT $\left(\frac{m_1^2}{2E} + \xi V\right) \mathbb{1}_{4\times4}$

$$H'_W = \frac{1}{2E} U \,\mathrm{DIAG}(0, \Delta m_{21}^2, \Delta m_{31}^2, \Delta m_{41}^2)\, U^\dagger$$
$$+ \,\mathrm{DIAG}((1-\xi)V, 0, 0, -\xi V)$$

IN THE MASS BASIS

$$H'_M = U^\dagger H'_W U$$
$$= \frac{1}{2E} \,\mathrm{DIAG}(0, \Delta m_{21}^2, \Delta m_{31}^2, \Delta m_{41}^2)$$
$$+ U^\dagger \,\mathrm{DIAG}((1-\xi)V, 0, 0, -\xi V)\, U$$

THE PARTIAL PARAMETERIZATION OF THE MIXING MATRIX IS SUFFICIENT:

$$H'_M = \frac{1}{2E}\,\mathrm{DIAG}\left(0,\ \Delta m^2_{21},\ \Delta m^2_{31},\ \Delta m^2_{41}\right) + U^\dagger\,\mathrm{DIAG}\left((1-\xi)V,\ 0,\ 0,\ -\xi V\right)U$$

(SCHEMEA)

$$\begin{pmatrix} 0 & \cdot & \cdot & -s_\varphi s_\chi \\ 0 & \cdot & \cdot & -s_\varphi c_\chi \\ c_\theta & \cdot & \cdot & -c_\varphi s_\theta \\ s_\theta & \cdot & \cdot & c_\varphi c_\theta \end{pmatrix} \begin{pmatrix} (1-\xi)V & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\xi V \end{pmatrix} \begin{pmatrix} 0 & 0 & c_\theta & s_\theta \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ -s_\varphi s_\chi & -s_\varphi c_\chi & -c_\varphi s_\theta & c_\varphi c_\theta \end{pmatrix} =$$

$$= \begin{pmatrix} -\xi s_\varphi^2 s_\chi^2 & +\xi s_\varphi^2 c_\chi s_\chi & +\xi c_\varphi s_\varphi s_\chi s_\theta & -\xi c_\varphi s_\varphi s_\chi c_\theta \\ +\xi s_\varphi^2 c_\chi s_\chi & -\xi s_\varphi^2 c_\chi^2 & -\xi c_\varphi s_\varphi c_\chi s_\theta & +\xi c_\varphi s_\varphi c_\chi c_\theta \\ +\xi c_\varphi s_\varphi s_\chi s_\theta & -\xi c_\varphi s_\varphi c_\chi s_\theta & (1-\xi)c_\theta^2 - \xi c_\varphi^2 s_\theta^2 & (1-\xi+\xi c_\varphi^2)c_\theta s_\theta \\ -\xi c_\varphi s_\varphi s_\chi c_\theta & +\xi c_\varphi s_\varphi c_\chi c_\theta & (1-\xi+\xi c_\varphi^2)c_\theta s_\theta & (1-\xi)s_\theta^2 - \xi c_\varphi^2 c_\theta^2 \end{pmatrix}$$

$$F = 920 \sqrt{\frac{\Delta m^2_{SBL}}{1\,eV^2}} \begin{Bmatrix} c_s\sqrt{1-c_s} \\ \sqrt{c_s}\,(1-c_s) \end{Bmatrix} \begin{matrix} \leftarrow A \\ \leftarrow B \end{matrix} \leftarrow \begin{Bmatrix} c_s \text{ SMALL} \\ \text{OR LARGE} \end{Bmatrix}$$

$$+\,33 \sqrt{\frac{\Delta m^2_{ATM}}{10^{-2}\,eV^2}} \; \frac{\sin^2 2\chi}{\sqrt{1+\cos 2\chi}} \; c_s^{3/2} \leftarrow \begin{Bmatrix} c_s \\ \text{SMALL} \end{Bmatrix}$$

BBN CONSTRAINT: $\boxed{F \leq |\ln(1-\delta N)|}$

$$c_s \simeq 1 \Rightarrow \begin{cases} (U_{\mu 1}, U_{\mu 2}) \simeq (\cos\chi, \sin\chi) & (A) \\ (U_{\mu 3}, U_{\mu 4}) \simeq (\cos\chi, \sin\chi) & (B) \end{cases}$$

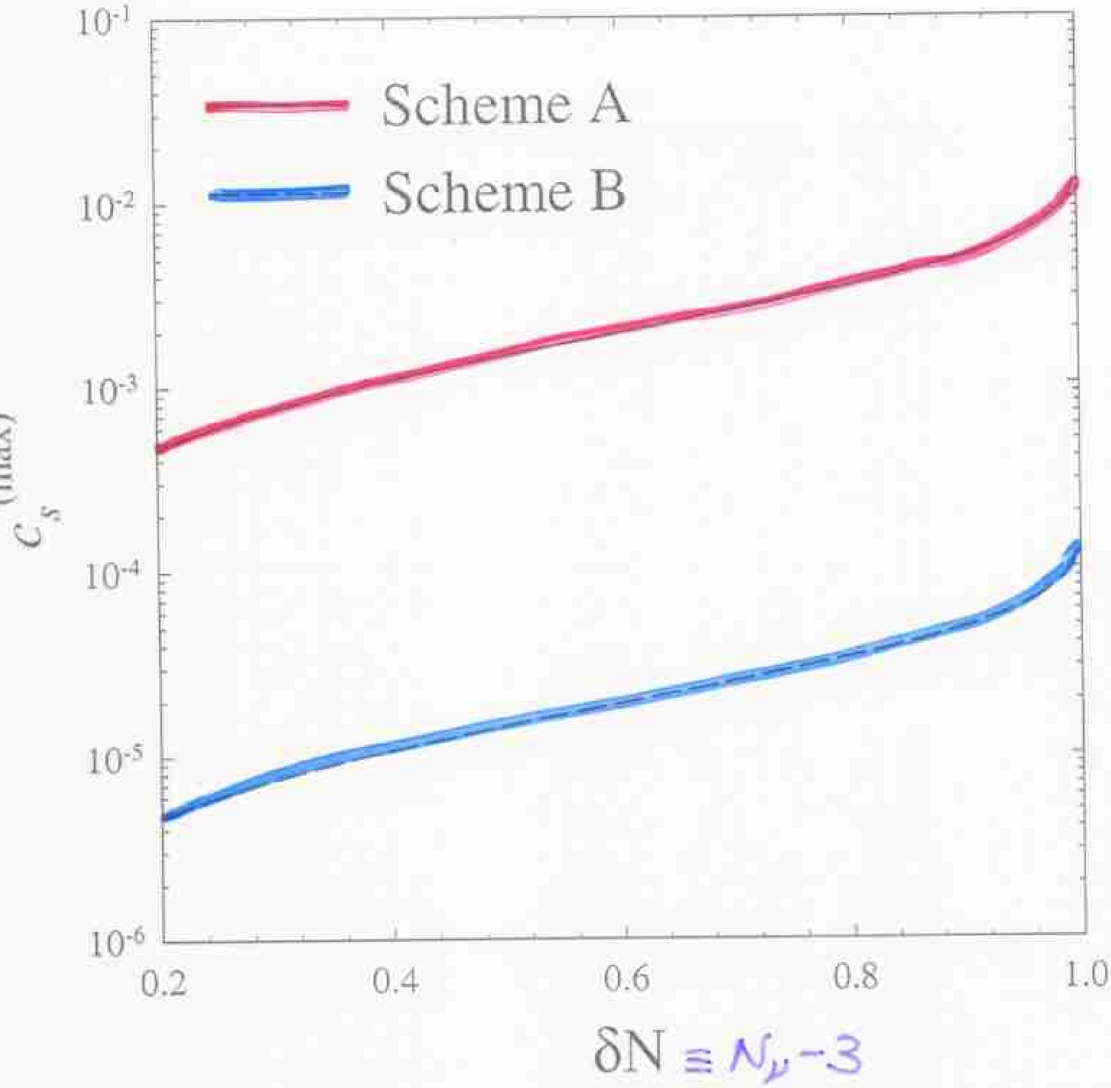
ATMOSPHERIC $\nu$ ANOMALY $\rightarrow$ $\sin^2 2\chi$ CANNOT BE SMALL

THEREFORE FROM THE **ATM** TERM OF F

$\boxed{\text{BBN} \rightarrow c_s \text{ IS SMALL !}}$

FROM THE **SBL** TERM OF F IN SCHEME A:

$$\boxed{c_s \leq 1.1 \times 10^{-3} \left(\frac{\Delta m^2_{SBL}}{1\,eV^2}\right)^{-1/2} |\ln(1-\delta N)|}$$



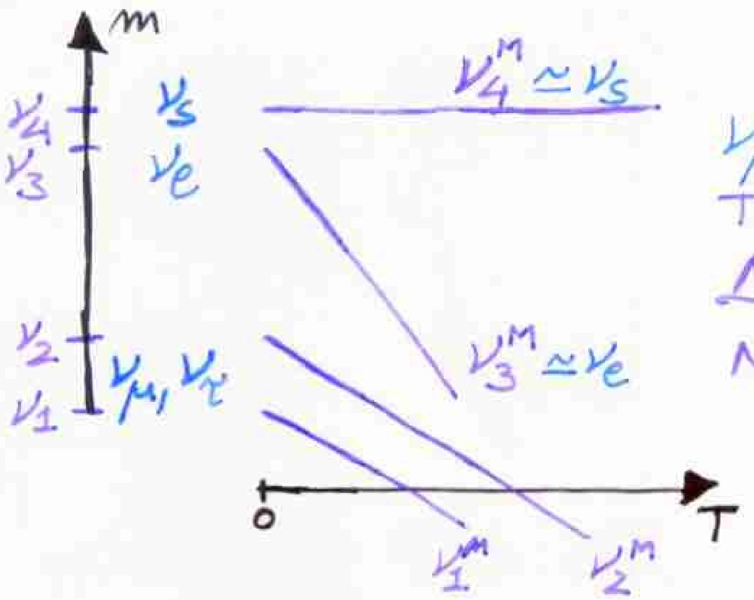

**Scheme A**: $\quad c_s \leq 1.1 \times 10^{-3} \left( \frac{\Delta m^2_{\text{SBL}}}{1\,\text{eV}^2} \right)^{-1/2} |\ln(1-\delta N)|$

**Scheme B**: $\quad c_s \leq 1.1 \times 10^{-5} \left( \frac{\Delta m^2_{\text{SBL}}}{1\,\text{eV}^2} \right)^{-1/2} |\ln(1-\delta N)|$

**LSND+Bugey** $\quad \Longrightarrow \quad \Delta m^2_{\text{SBL}} \gtrsim 0.27\,\text{eV}^2$

# SCHEME A

ATM TERM OF F $\rightarrow$ $C_S$ IS SMALL




$\nu_\mu \to \nu_S$ AND $\nu_\tau \to \nu_S$ TRANSITIONS DUE TO $\Delta m^2_{SBL} \equiv \Delta m^2_{41}$ ARE NON-RESONANT

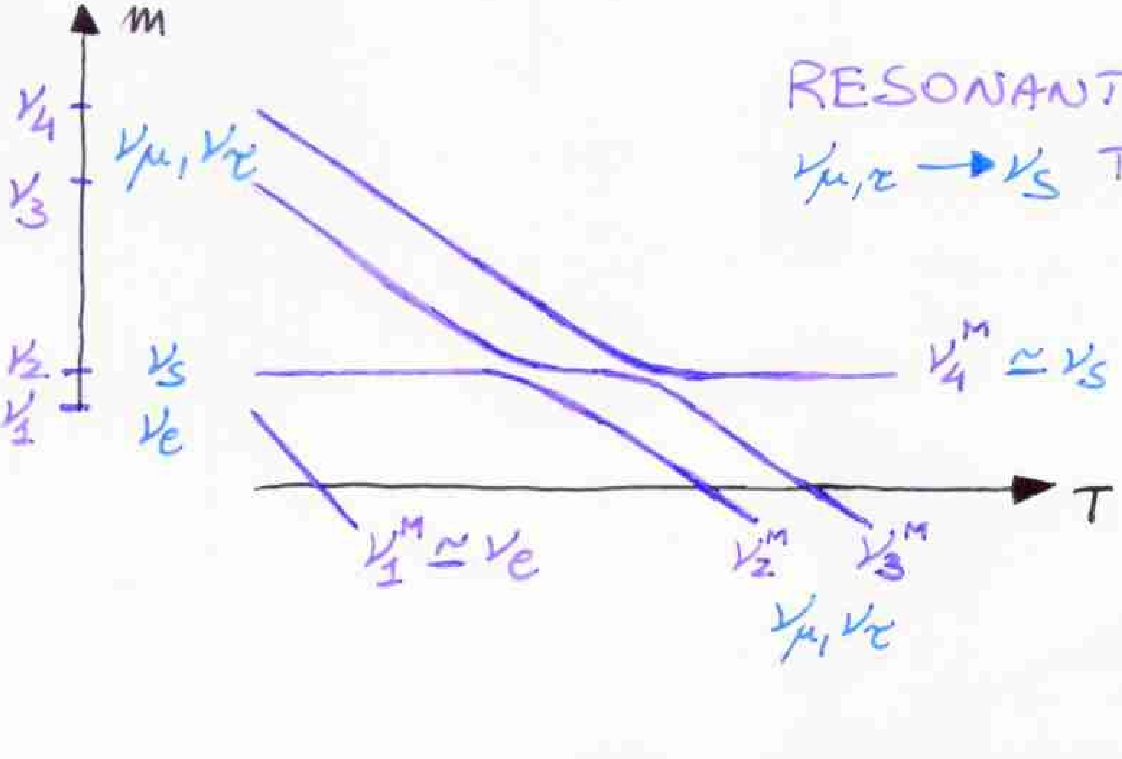
$\downarrow$

THE SBL TERM OF F IS CORRECT AND GIVES THE BBN BOUND FOR $C_S$

# SCHEME B

ATM TERM OF F $\rightarrow$ $C_S$ IS SMALL

RESONANT $\nu_{\mu,\tau} \to \nu_S$ TRANSITIONS!



RESONANCE TEMPERATURE:

$$T_{RES} \simeq 16 \left(\frac{\Delta m^2_{SBL}}{1\,eV^2}\right)^{1/6} MeV \gtrsim 13\ MeV$$

$\Delta m^2 \gtrsim 0.27\ eV^2$ FROM LSND

(ENQVIST, KAINULAINEN & MAALAMPI 1990)

$$n_{\nu_s} \simeq \frac{1}{2} - \left(\frac{1}{2} - P_{LZ}\right) \cos 2\varphi_b \cos 2\varphi_a \simeq 1 - P_{LZ}$$

$\cos 2\varphi_b \simeq 1$ FOR $T \gg T_{RES}$

$\cos 2\varphi_a \simeq \cos 2\varphi = 2c_s - 1 \simeq -1$

$$P_{LZ} = e^{-Q}$$

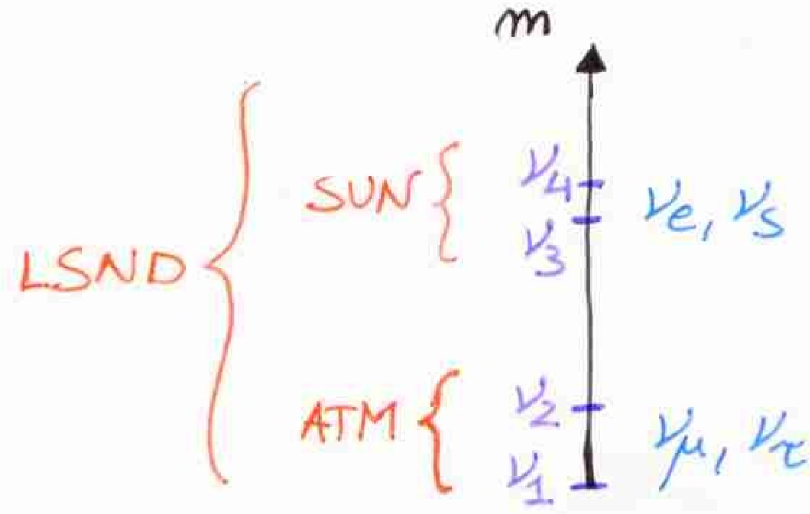
$$Q \simeq 9.2 \times 10^4 \, \frac{c_s(1-c_s)}{|1-2c_s|^{3/2}} \sqrt{\frac{\Delta m^2_{SBL}}{1\,eV^2}}$$

$$n_{\nu_s} \leq \delta N \longleftrightarrow Q \leq |\ln(1-\delta N)|$$

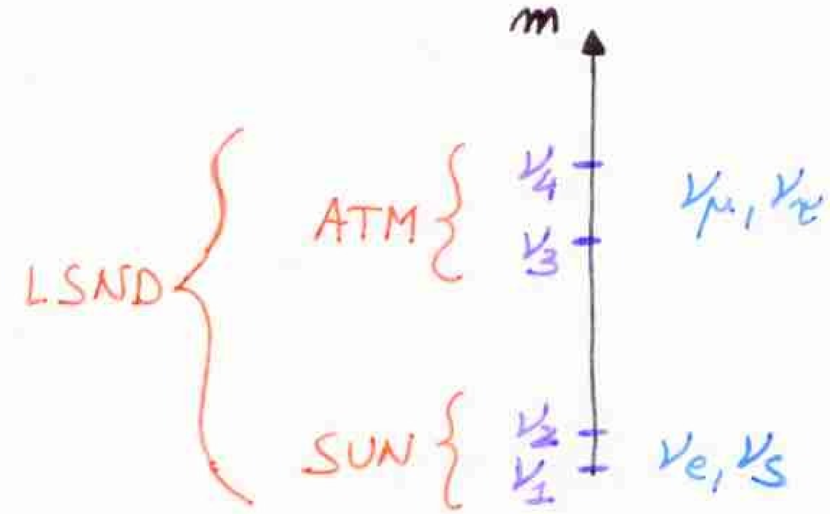
$$c_s \leq 1.1 \times 10^{-5} \left(\frac{\Delta m^2_{SBL}}{1\,eV^2}\right)^{-1/2} |\ln(1-\delta N)|$$

$$\boxed{\text{BBN } (N_\nu < 4) \longrightarrow c_s \ll 1}$$

SCHEME A:



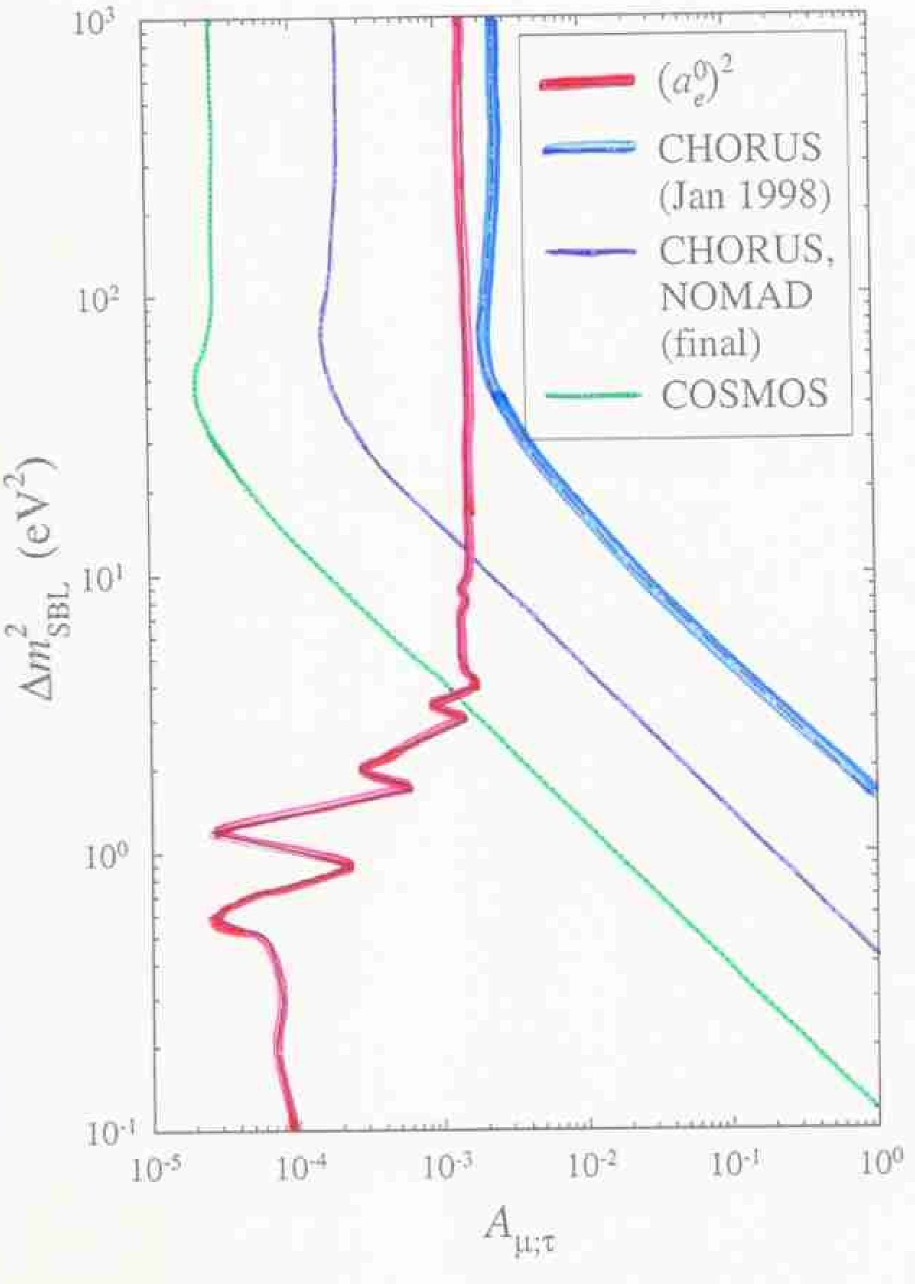

SCHEME B:



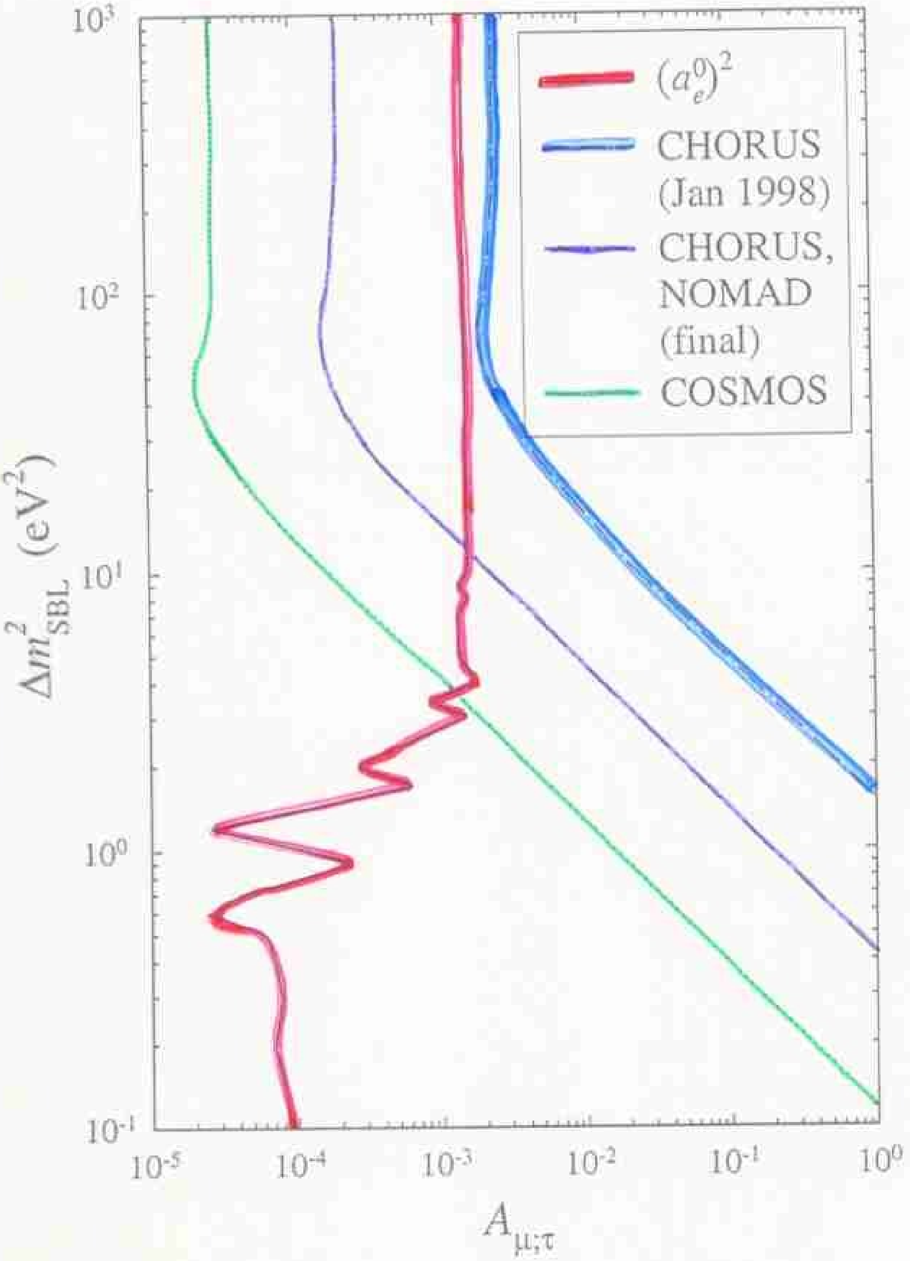

SOLAR $\nu$'S : $\nu_e \to \nu_s$ (SMALL MIX. MSW SOL.)

ATMOSPHERIC AND LBL $\nu$'S : $\nu_\mu \to \nu_\tau$

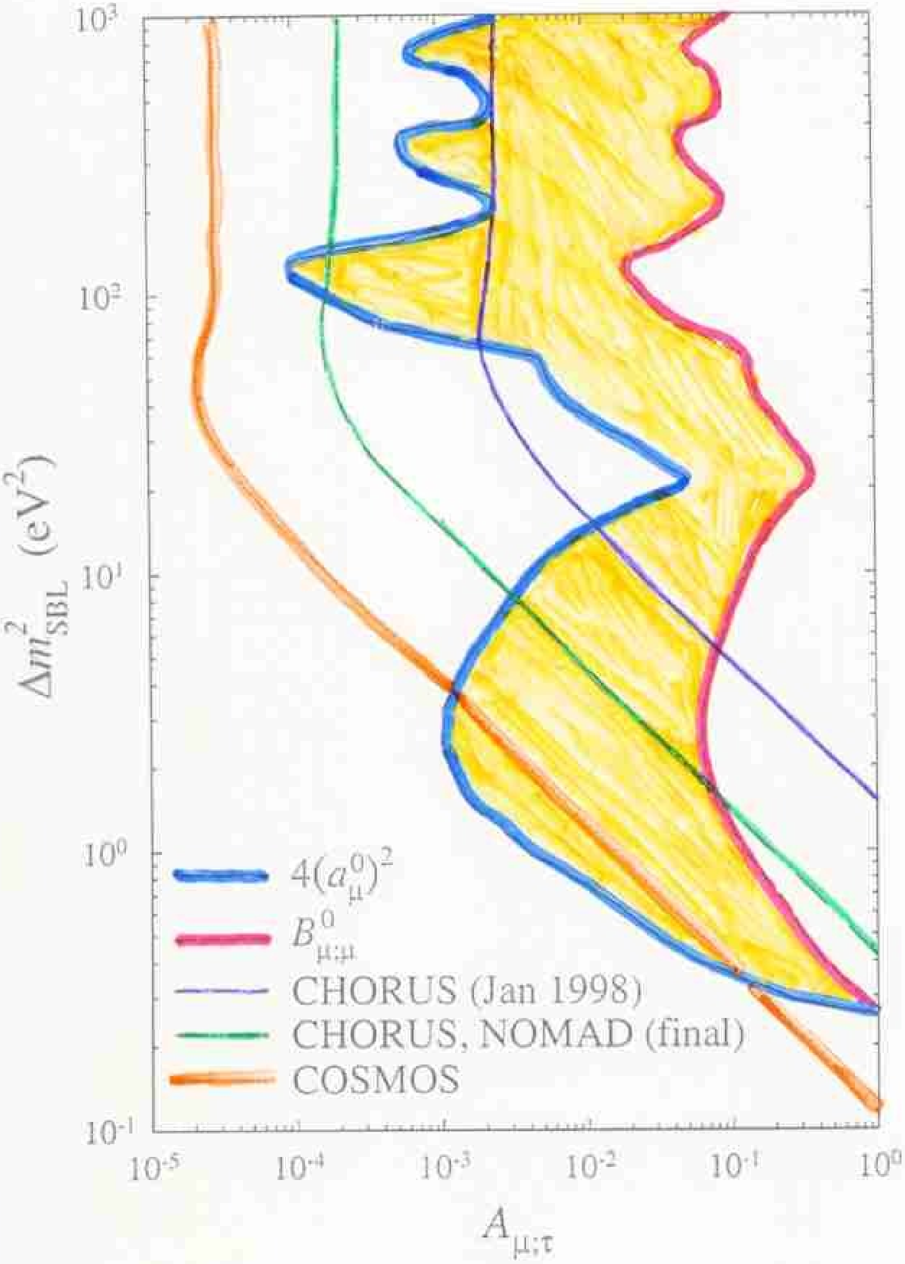
SBL EXP. : $\nu_\mu \to \nu_\tau$ AND $\nu_e \to \nu_s$ TRANSITIONS ARE STRONGLY SUPPRESSED



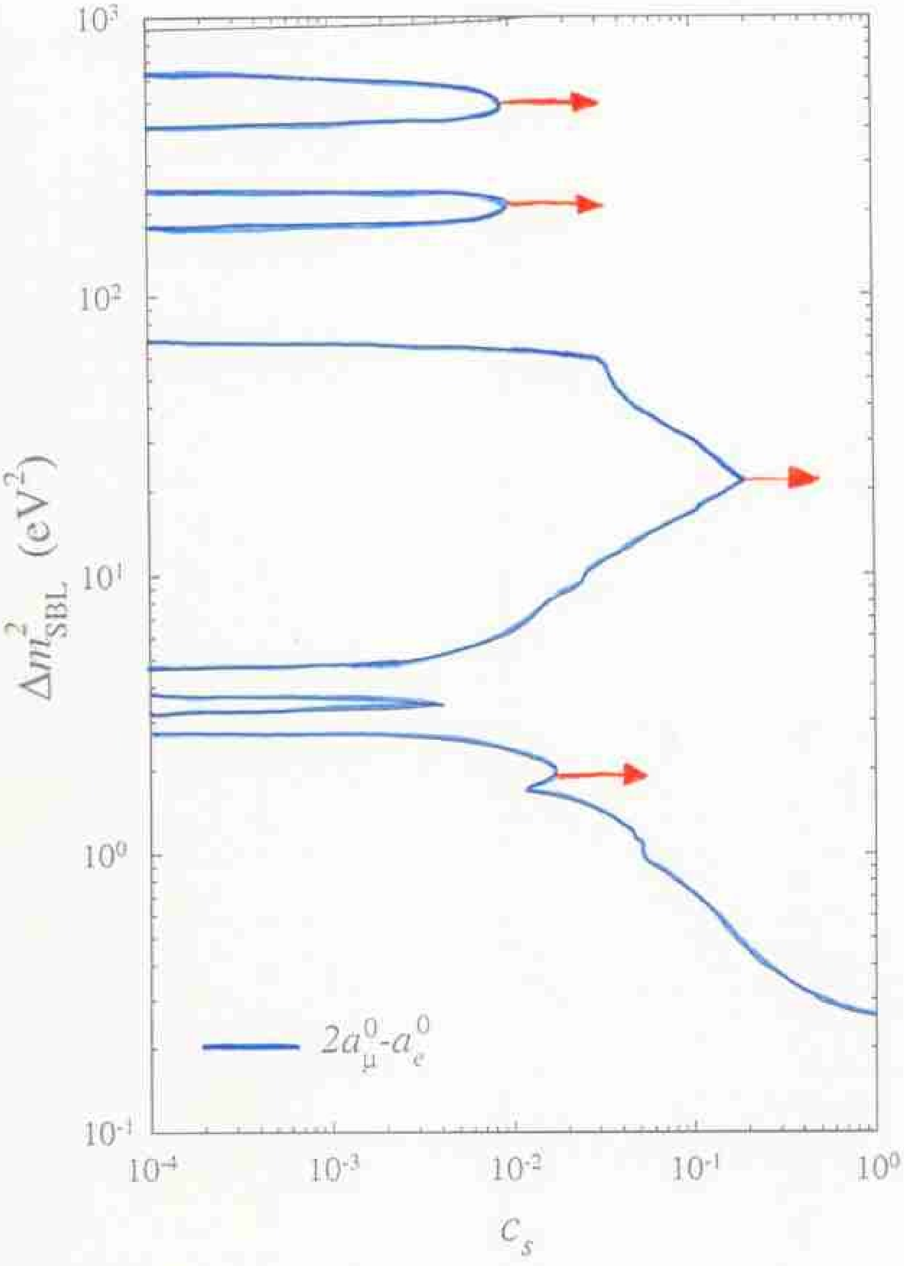

$$\left.\begin{array}{l} c_e \leq a_e^0 \\ c_\mu \geq 1 - a_\mu^0 \\ c_s \ll 1 \end{array}\right\} \quad \Longrightarrow \quad \boxed{A_{\mu;\tau} \leq (a_e^0)^2}$$




$$\left.\begin{array}{l} c_e \leq a_e^0 \\ c_\mu \geq 1 - a_\mu^0 \\ c_s \ll 1 \end{array}\right\} \quad \Longrightarrow \quad \boxed{A_{\mu;\tau} \leq (a_e^0)^2}$$




$$A_{\mu;\tau} > 4(a^0_\mu)^2 \qquad \Longrightarrow \qquad c_s \geq 2a^0_\mu - a^0_e$$




$$A_{\mu;\tau} > 4(a^0_\mu)^2 \qquad \Longrightarrow \qquad c_s \geq 2a^0_\mu - a^0_e$$

$$A_{\mu;\tau} > A^{(min)}_{\mu;\tau} \qquad \Longrightarrow \qquad c_s \geq \frac{A^{(min)}_{\mu;\tau}}{4a^0_\mu} - a^0_e$$

(BETTER IF $A^{(min)}_{\mu;\tau} > 8(a^0_\mu)^2$)

THE AMPLITUDE OF $\nu_e \to \nu_s$ TRANSITIONS IN SBL EXP. IS STRONGLY SUPPRESSED:

$$A_{es} \leq 4\, c_e\, c_s \leq 4\, a_e^0\, c_s^{(MAX)}$$

SINCE $B_{ee} = A_{\mu e} + A_{e\tau} + A_{es}$

$$A_{es} \ll 1 \;\longrightarrow\; \boxed{B_{ee} = A_{\mu e} + A_{e\tau}}$$

MEASURABLE RELATION

IF IT WILL BE FOUND THAT

$$B_{ee} > A_{\mu e} + A_{e\tau}$$

THEN

$$A_{es} = B_{ee} - A_{\mu e} - A_{e\tau} > 0$$

$$\boxed{c_s \geq 1 - \frac{A_{\mu e} + A_{e\tau}}{B_{ee}}}$$

INCOMPATIBLE WITH BBN UPPER BOUND FOR $c_s$

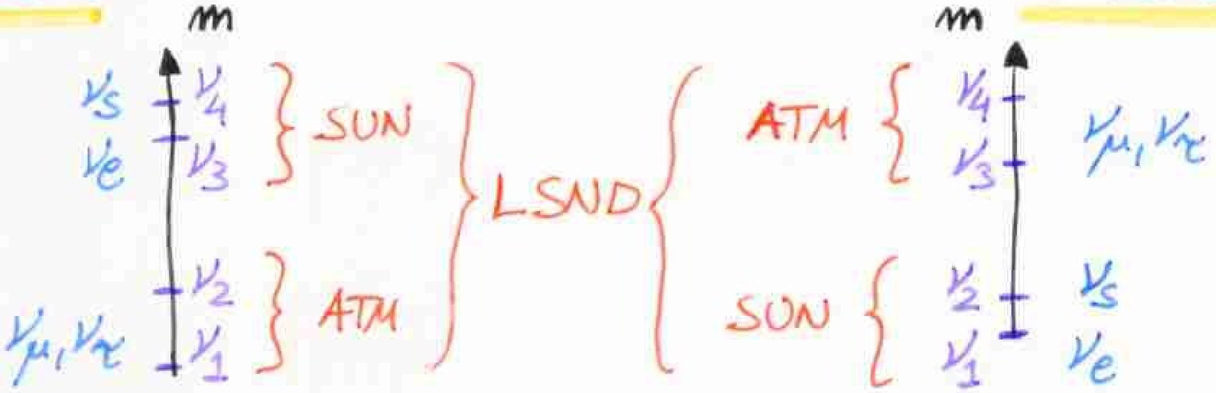
EXAMPLE:

$$\left.\begin{array}{l} B_{ee} \simeq 10^{-2} \\ A_{\mu e} \simeq 5\times 10^{-3} \\ A_{\mu\tau} \ll A_{\mu e} \end{array}\right\} \longrightarrow c_s \gtrsim 0.5$$

# CONCLUSIONS

FROM THE RESULTS OF ALL NEUTRINO OSCILLATION EXPERIMENTS AND BBN ($N_\nu < 4$) WE HAVE TWO FAVORED SCHEMES OF $\nu$ MIXING:

SCHEME A | SCHEME B



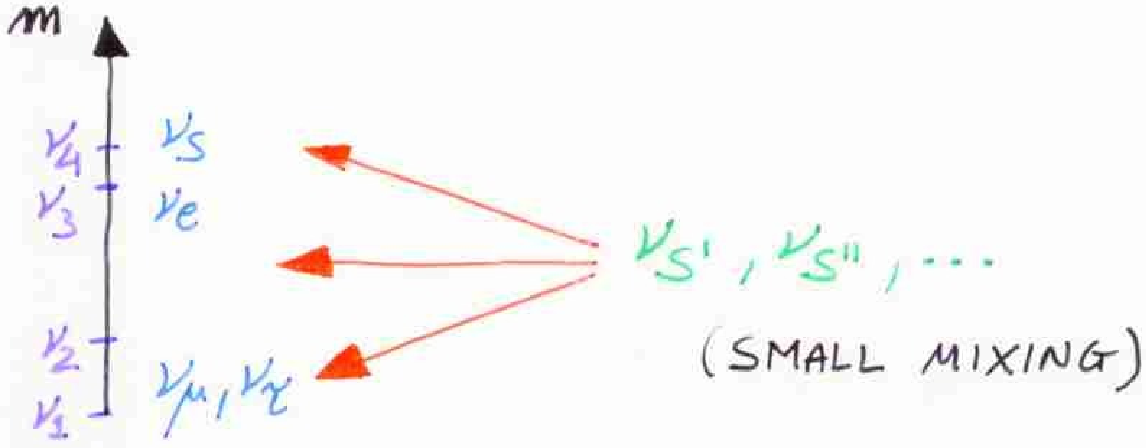

NEUTRINO MIXING MATRIX (SCHEME A)

$$U \simeq \begin{pmatrix} 0 & 0 & c_\theta & s_\theta \\ c_\gamma & s_\gamma & 0 & 0 \\ -s_\gamma & c_\gamma & 0 & 0 \\ 0 & 0 & -s_\theta & c_\theta \end{pmatrix} \begin{array}{l} \leftarrow \nu_e \\ \leftarrow \nu_\mu \\ \leftarrow \nu_\tau \\ \leftarrow \nu_s \end{array} \qquad (\theta \ll 1)$$

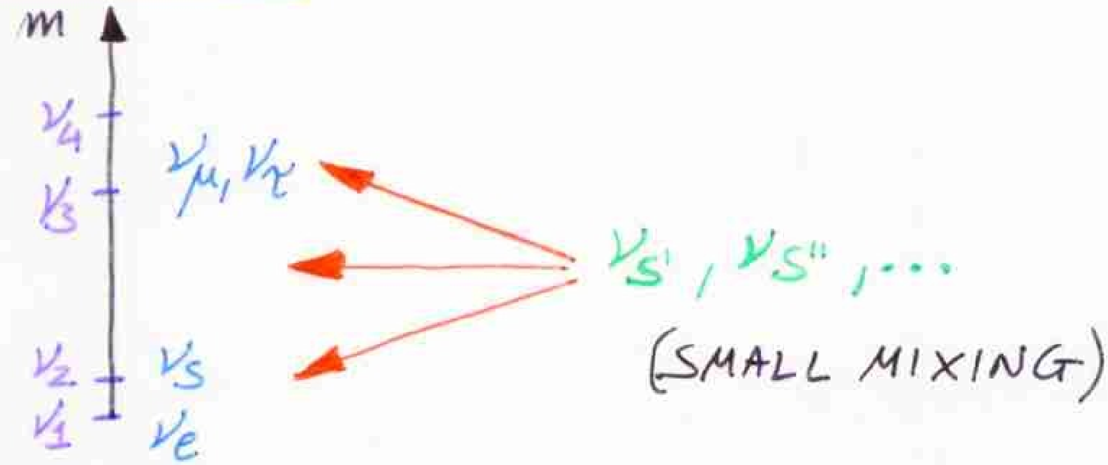
SMALL MIXING ANGLE MSW SOLUTION OF SOLAR $\nu$ PROB.

$$U \simeq \begin{pmatrix} 0 & 0 & 1 & 0 \\ c_\gamma & s_\gamma & 0 & 0 \\ -s_\gamma & c_\gamma & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

## Scheme A




## Scheme B




# MORE THAN FOUR NEUTRINOS?