Measuring CP Violation in Long Baseline ν Oscillation Experiments

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I. Introduction
II. Neutrino Oscillation
III. Separation of $\not{CP}$ Effect and Matter Effect
IV. Conclusion

Neutrinos are massive! (1998)

Long Baseline ν. osc. experiments (1999~)
⇒ m_ν's, mixing angles, $\not{T}$ or $\not{CP}$

$$\Delta P \equiv P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) \neq 0 \ ?$$

[Pure $\not{CP}$ effect + Matter effect]

Tanimoto; Arafune, Koike, Sato; Minakata, Nunokawa; Bilenky, Giunti, Grimus; ...

Which δm^2's?

1) LSND experiments $\delta m^2 \sim 1\,\mathrm{eV}^2$
2) Atmospheric ν observations $\delta m^2 \sim 10^{-2\sim-3}\,\mathrm{eV}^2$
3) Solar ν observations (MSW) $\delta m^2 \sim 10^{-4\sim-5}\,\mathrm{eV}^2$
(Vac.) $\delta m^2 \sim 10^{-10}\,\mathrm{eV}^2$

II. Neutrino Oscillation

- Mixing (in vac.) $\nu_\alpha = U_{\alpha i}\nu_i$
  $(e, \mu, \tau) \quad (1, 2, 3)$

  Mass Eigenvalues m_i

$$U = \begin{bmatrix} 1 & & \\ & c_\psi & s_\psi \\ & -s_\psi & c_\psi \end{bmatrix} \begin{bmatrix} 1 & & \\ & 1 & \\ & & e^{i\delta} \end{bmatrix} \begin{bmatrix} c_\phi & & s_\phi \\ & 1 & \\ -s_\phi & & c_\phi \end{bmatrix} \begin{bmatrix} c_\omega & s_\omega & \\ -s_\omega & c_\omega & \\ & & 1 \end{bmatrix}$$

Chau & Keung, '87
Kuo & Pantaleone, '87
Toshev, '89

- Evolution of $\nu \equiv (\nu_e, \nu_\mu, \nu_\tau)^T$

$$i\frac{d\nu}{dx} = H\nu \qquad H = \frac{1}{2E}\left\{ U \begin{pmatrix} 0 & & \\ & \delta m^2_{21} & \\ & & \delta m^2_{31} \end{pmatrix} U^\dagger + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \right\}$$

$$\delta m^2_{ij} \equiv m_i^2 - m_j^2$$

$$a = 2\sqrt{2}\, G_F n_e E = 7.56 \times 10^{-5}\,\mathrm{eV}^2 \left(\frac{\rho}{\mathrm{g\,cm^{-3}}}\right)\left(\frac{E}{\mathrm{GeV}}\right)$$

[Effective mass due to 

]

- Solution

$$\nu(L) = S(L)\nu(0)$$

const. matter density

$$S(L) = T\exp\left[-i\int_0^L dx\, H(x)\right] \simeq e^{-iHL}$$

Osc. Prob. $P(\nu_\alpha \to \nu_\beta ; L, E) = |S(L)_{\beta\alpha}|^2$

$$a \to -a$$
$$\delta \to -\delta \quad (U \to U^*)$$

$$P(\bar{\nu}_\alpha \to \bar{\nu}_\beta ; L, E)$$

2) Disparate δm^2's

Solar ν (MSW): $\delta m^2_{21} \sim (10^{-5} \sim 10^{-4})\,\mathrm{eV}^2$

Atmospheric ν: $\delta m^2_{31} \sim (10^{-3} \sim 10^{-2})\,\mathrm{eV}^2$

Matter Effect: $a \sim 10^{-4}\,\mathrm{eV}^2 \cdot \left(\frac{E}{\mathrm{GeV}}\right)$

$\Rightarrow$ $a,\ \delta m^2_{21} \ll \delta m^2_{31}$

$$H = \frac{1}{2E} U \begin{pmatrix} 0 & & \\ & 0 & \\ & & \delta m^2_{31} \end{pmatrix} U^\dagger + \frac{1}{2E}\left\{ U \begin{pmatrix} 0 & & \\ & \delta m^2_{21} & \\ & & 0 \end{pmatrix} U^\dagger + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \right\}$$

$$\equiv H_0 + H_1$$

$$S(L) \simeq e^{-iH_0L} + e^{-iH_0L}(-i)\int_0^L dx \left(e^{iH_0x} H_1 e^{-iH_0x}\right)$$

N.B.

$$\left.\begin{aligned} \exp\left(-i\frac{\delta m^2_{21}L}{2E}\right) &\simeq 1 - i\frac{\delta m^2_{21}L}{2E} \\ \exp\left(-i\frac{aL}{2E}\right) &\simeq 1 - i\frac{aL}{2E} \end{aligned}\right\} \Rightarrow \begin{aligned} &\frac{\delta m^2_{21}L}{2E} \ll 1 \\ &\frac{aL}{2E} \ll 1 \end{aligned}$$

$$L \ll 5.21 \times 10^4\,\mathrm{km} \cdot \left(\frac{\rho}{\mathrm{g\,cm^{-3}}}\right)^{-1}$$

$$\begin{aligned}\Delta P(\nu_\mu \to \nu_e) &\equiv P(\nu_\mu \to \nu_e; L) - P(\bar{\nu}_\mu \to \bar{\nu}_e; L) \\ &= \Delta P_1(\nu_\mu \to \nu_e) + \Delta P_2(\nu_\mu \to \nu_e) + \Delta P_3(\nu_\mu \to \nu_e)\end{aligned}$$

with

(matter effects)

$$\begin{aligned}\Delta P_1(\nu_\mu \to \nu_e) &= 16\frac{a}{\delta m^2_{31}}\sin^2\frac{\delta m^2_{31}L}{4E}c^2_\phi s^2_\phi s^2_\psi(1-2s^2_\phi), \\ \Delta P_2(\nu_\mu \to \nu_e) &= -4\frac{aL}{2E}\sin\frac{\delta m^2_{31}L}{2E}c^2_\phi s^2_\phi s^2_\psi(1-2s^2_\phi),\end{aligned}$$

and

($\cancel{CP}$ effects)

$$\Delta P_3(\nu_\mu \to \nu_e) = -8\frac{\delta m^2_{21}L}{2E}\sin^2\frac{\delta m^2_{31}L}{4E}s_\delta c^2_\phi s_\phi c_\psi s_\psi c_\omega s_\omega.$$

(N.B. $a \propto E$)

$$\Delta P(\nu_\mu \to \nu_\mu) = 16\frac{a}{\delta m^2_{31}}\left[\sin^2\frac{\delta m^2_{31}L}{4E} - \frac{1}{4}\frac{\delta m^2_{31}L}{2E}\sin\frac{\delta m^2_{31}L}{2E}\right]c^2_\phi s^2_\phi s^2_\psi\left(1-2c^2_\phi s^2_\psi\right)$$

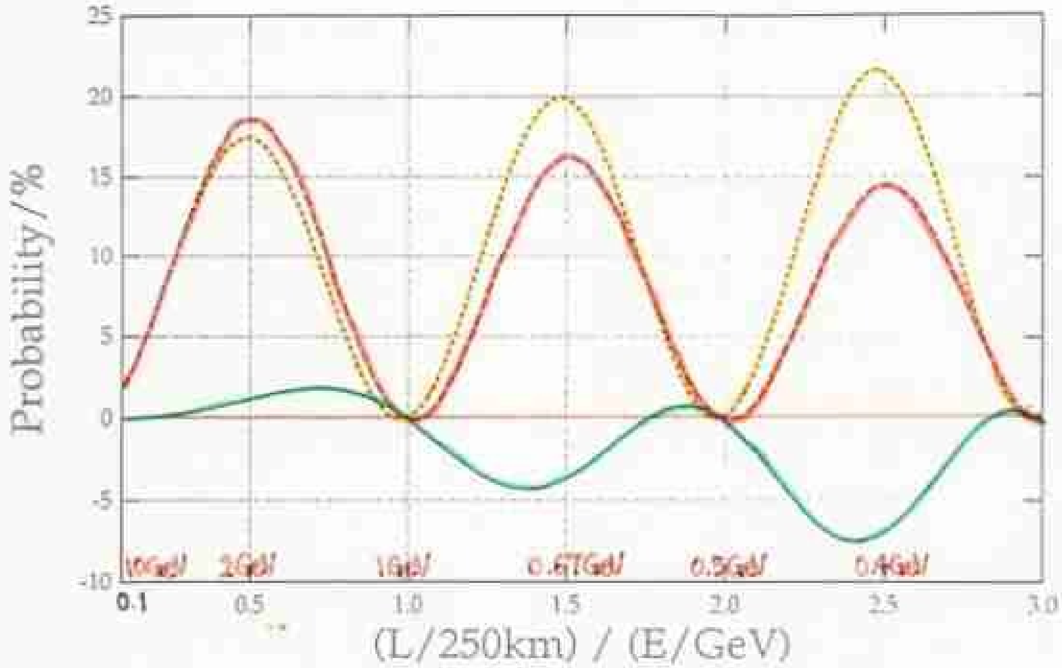
$$\begin{aligned}\Delta P(\nu_\mu \to \nu_\tau) &= -32\frac{a}{\delta m^2_{31}}\left[\sin^2\frac{\delta m^2_{31}L}{4E} - \frac{1}{4}\frac{\delta m^2_{31}L}{2E}\sin\frac{\delta m^2_{31}L}{2E}\right]c^4_\phi s^2_\phi c^2_\psi s^2_\psi \\ &\quad + 8\frac{\delta m^2_{21}L}{2E}\sin^2\frac{\delta m^2_{31}L}{4E}s_\delta c^2_\phi s_\phi c_\psi s_\psi c_\omega s_\omega.\end{aligned}$$

$\delta m_{21}^2 = 10^{-4}\mathrm{eV}^2,\ \delta m_{31}^2 = 10^{-2}\mathrm{eV}^2,$
$\sin\psi = 1/\sqrt{2},\ \sin\omega = 1/2,\ \sin\phi = \sqrt{0.1},$
$\delta = \pi/2,\ \rho = 3\,\mathrm{g\,cm^{-3}}$

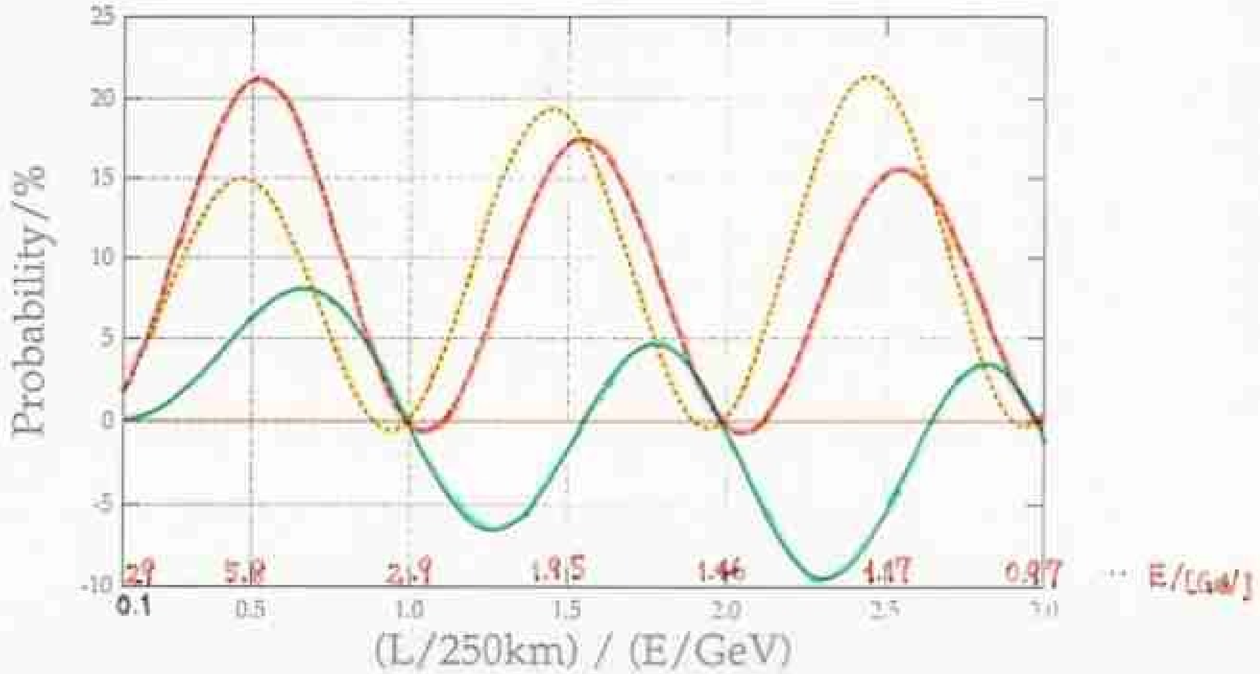
} Fogli et al.

$P(\nu_\mu \to \nu_e)$
$P(\bar{\nu}_\mu \to \bar{\nu}_e)$
ΔP

KEK/Super-Kamiokande (L = 250 km)



Minos (L = 730 km)



III. Separating CP Effect and Matter Effect

A. Patterns of Envelope

Recall $a \propto E$:

$\Delta P_1/L$, $\Delta P_2/L$ and ΔP_3 are functions of L/E

➡ Common zeroes, Different envelopes

- Only 1 detector is needed.
- Measurement of ΔP over wide energy range

B. Comparison of Experiments with Different L's

ΔP is function of L/E apart from a

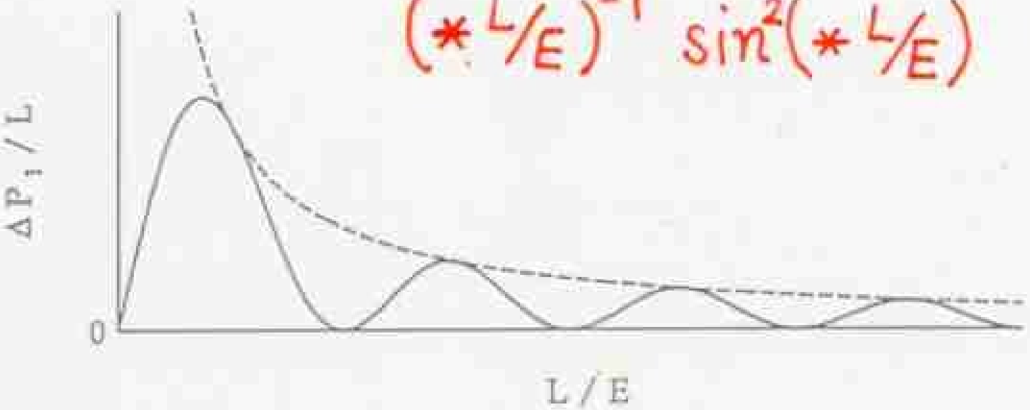
➡ $\left[\Delta P(\nu_\mu \to \nu_e ; L_1, E_1) - \Delta P(\nu_\mu \to \nu_e ; L_2, E_2)\right]_{\frac{L_1}{E_1} = \frac{L_2}{E_2}}$

is due to the matter effect.

$$\boxed{\begin{aligned}&\Delta P_3(\nu_\mu \to \nu_e ; L_1)\\ &= \left[\Delta P(\nu_\mu \to \nu_e ; L_1) - \frac{L_1}{L_2 - L_1}\left\{\Delta P(\nu_\mu \to \nu_e ; L_2) - \Delta P(\nu_\mu \to \nu_e ; L_1)\right\}\right]_{L/E = \mathrm{const.}}\end{aligned}}$$

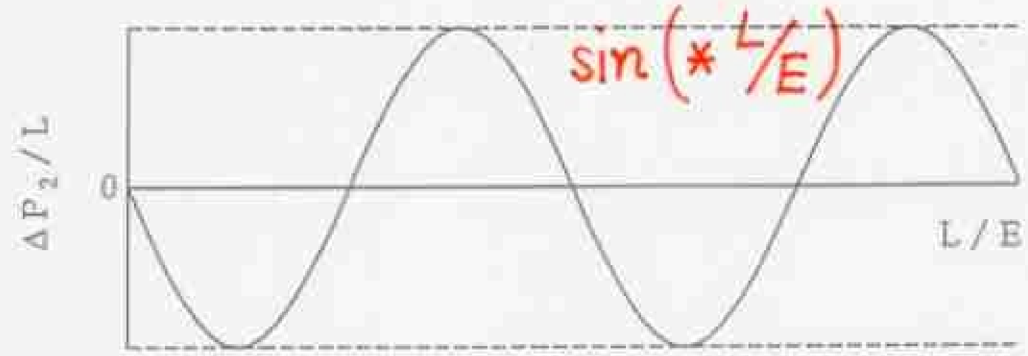
- Need not measure ΔP in the low energy range.

$\frac{\Delta P_1}{L}$



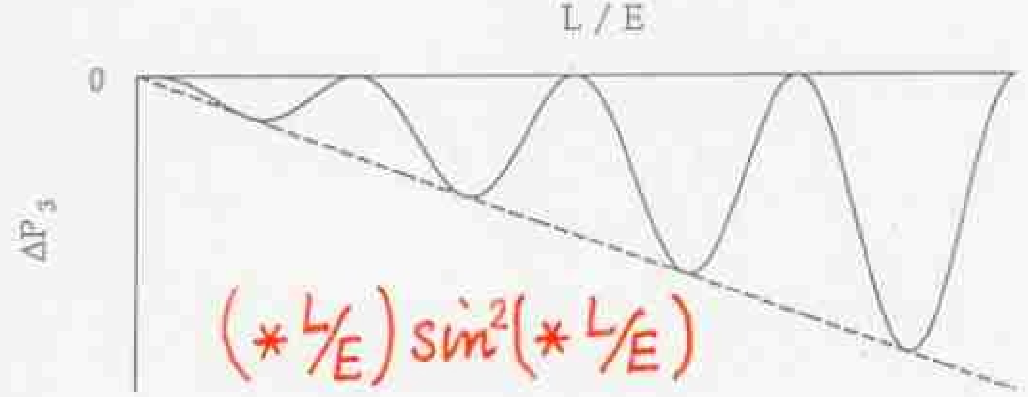
(a) Matter effect term $\Delta P_1(\nu_\mu \to \nu_e)$ divided by L for $c_\phi^2 s_\phi^2 s_\psi^2(1-2s_\phi^2) > 0$. The envelope decreases monotonously with L/E.

$\frac{\Delta P_2}{L}$

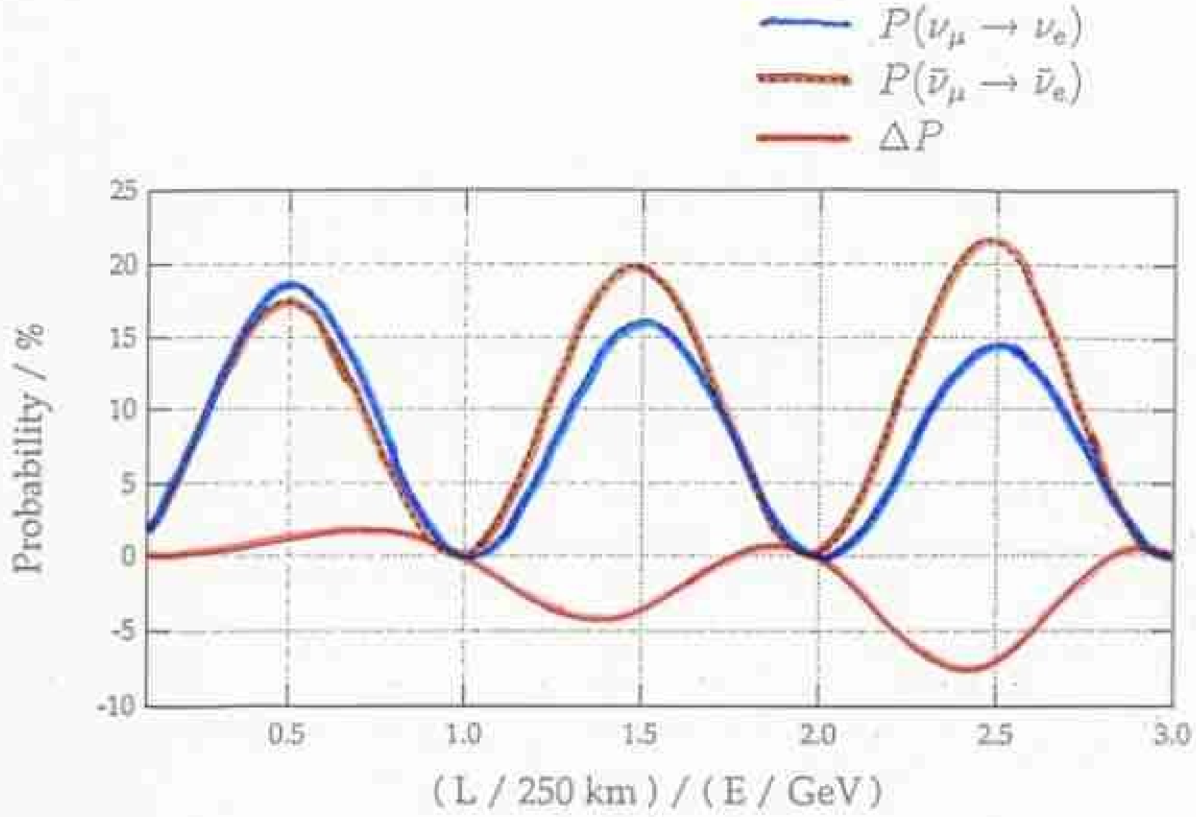


(b) Matter effect term $\Delta P_2(\nu_\mu \to \nu_e)$ divided by L for $c_\phi^2 s_\phi^2 s_\psi^2(1-2s_\phi^2) > 0$. The envelope is flat.

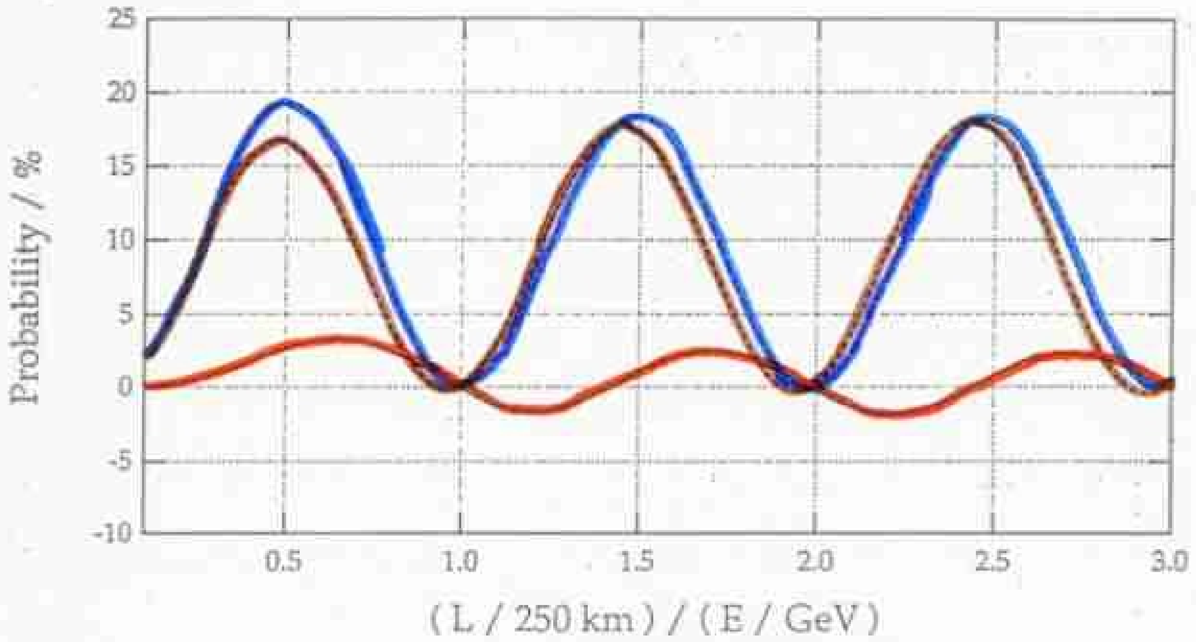
ΔP_3



(c) CP-violation effect term $\Delta P_3(\nu_\mu \to \nu_e)$ for $s_\delta c_\phi^2 s_\phi c_\psi s_\psi c_\omega s_\omega > 0$. The envelope increases linearly with L/E.



(a) The oscillation probabilities as functions of L/E for $\delta = \pi/2$.



(b) The oscillation probabilities as functions of L/E for $\delta = 0$.

K2K

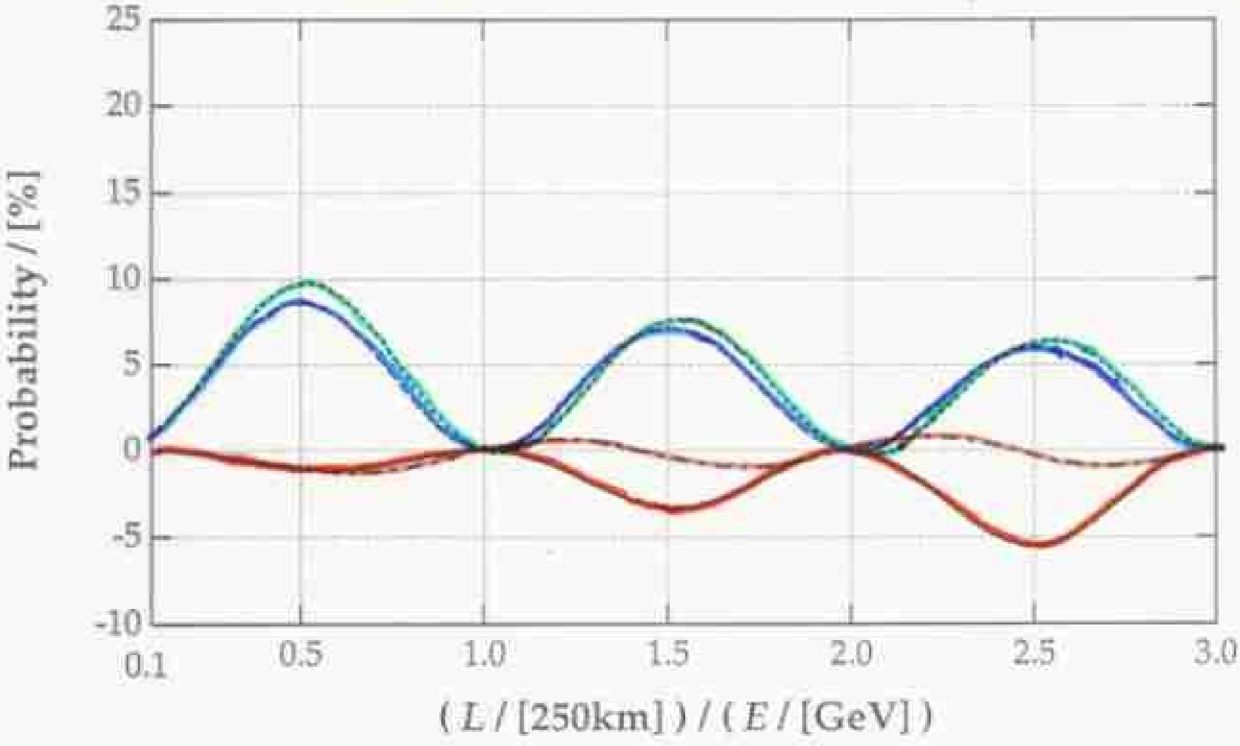
Minos

K2K- Minos

CP violation in K2K

$\delta m^2_{21} = 10^{-4}\mathrm{eV}^2$, $\delta m^2_{31} = 10^{-2}\mathrm{eV}^2$,
$\sin\psi = 1/\sqrt{2}$, $\sin\omega = 1/2$, $\sin\phi = \sqrt{0.18}$,
2ϕ
$\delta = \pi/2$, $\rho = 2.34\,\mathrm{g\,cm^{-3}}$

Fogli *et al.*
+
Narajan *et al.*
(CHOOZ)



IV. Summary

- Possibility to detect $\not{CP}$ effect ($\sim 5\%$) respecting Solar ν (MSW) and Atmospheric ν.

- Separating $\not{CP}$ effect and matter effect — 2 methods

(1) Measure ΔP over wide energy range
— Only 1 detector is needed.

(2) Compare experiments with different L and E, but same L/E
— Need not measure L/E in the low energy range.