SATELLITE SYMPOSIUM

NEW ERA IN NEUTRINO PHYSICS

T.M.U., June 1998

THREE-FLAVOR MIXING and TRIANGLE GRAPHS

E. LISI, INFN Bari, Italy

"... The purpose of the meeting is twofold:

1) To have an extensive discussion on some restricted topics like 3-flavor analyses of various neutrino physics phenomenology, new experimental projects which will start in a few years, etc.

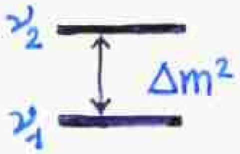
2) to bring young people in Japan to the meeting and tell them how exciting is the field, what is the present understanding of the neutrino physics, and what are the basics to understand them ..."

- Purpose of this seminar (pedagogical):
- UNDERSTANDING THE BASICS OF 3-FLAVOR ANALYSES

• TWO-FLAVOR PARAMETER SPACE :

2-dimensional

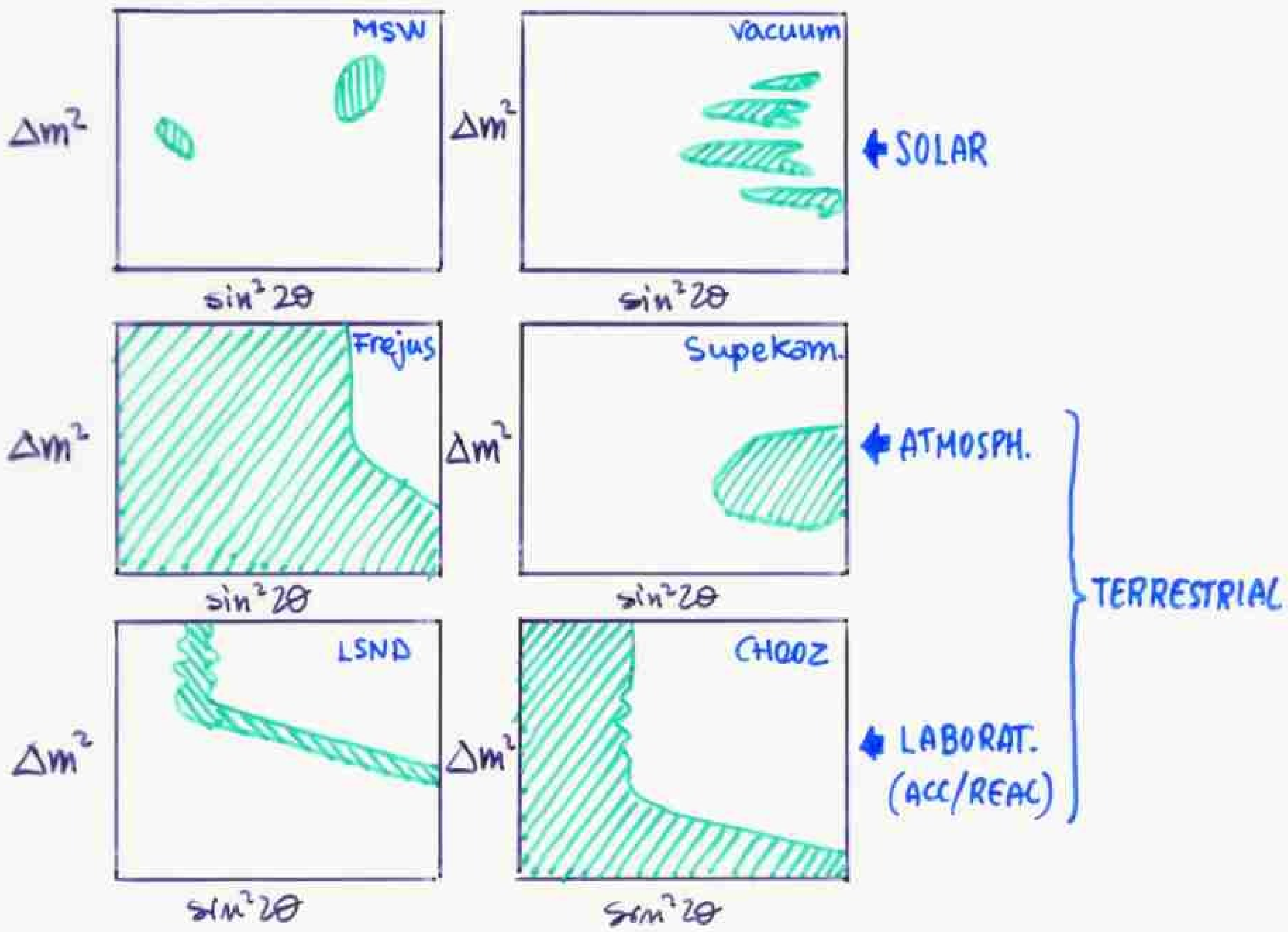
$$(\Delta m^2, \sin^2 2\vartheta)$$



$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$\alpha, \beta = e, \mu, \tau$
$\theta \in [0, \pi/4]$

• USUAL GRAPHICAL REPRESENTATIONS :



... etc.

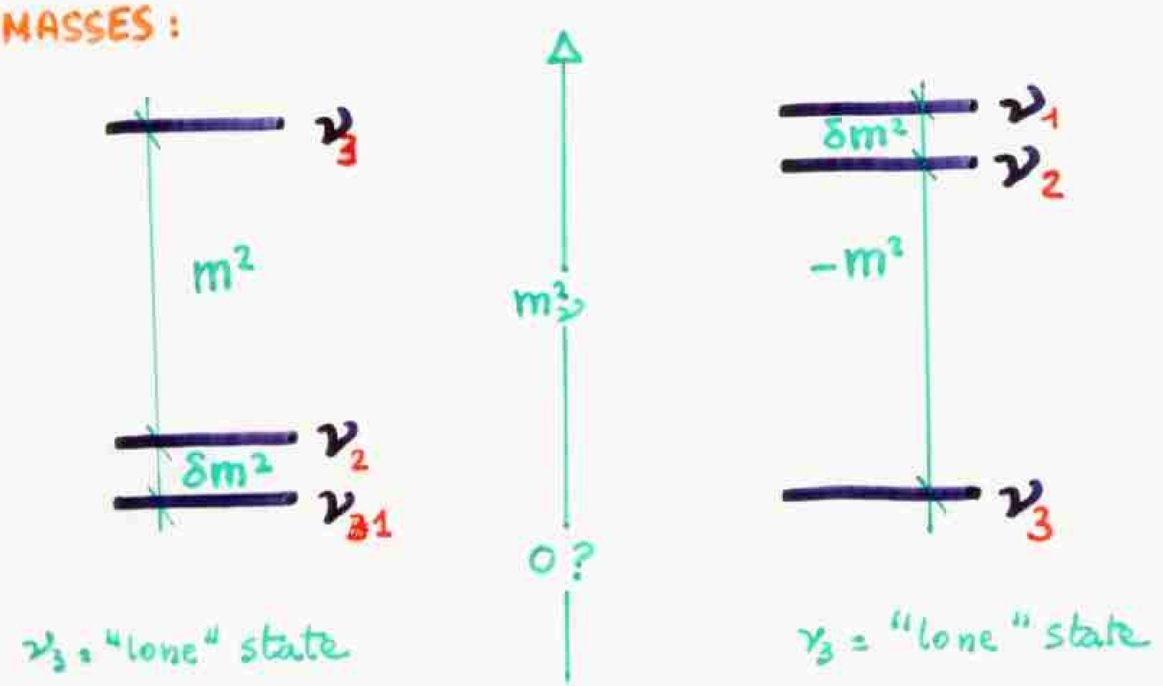
• THREE-FLAVOR PARAMETER SPACE

6-dimensional

$$(\delta m^2, m^2, \omega, \psi, \varphi, \delta_{CP})$$

$\delta m^2 \leq m^2$ by convention
$\omega, \psi, \varphi \in [0, \pi/2]$

• MASSES:



• MIXING:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\psi & s_\psi \\ 0 & -s_\psi & c_\psi \end{pmatrix} \begin{pmatrix} c_\varphi & 0 & s_\varphi \\ 0 & 1 & 0 \\ -s_\varphi & 0 & c_\varphi \end{pmatrix} \begin{pmatrix} c_\omega & s_\omega & 0 \\ -s_\omega & c_\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad \otimes \delta_{CP}$$

• PROBLEM:

Find useful representations of this huge parameter space

NO EASY solution to this problem in the most general cases

However, parameter space reduction is possible under the reasonable hypothesis

$$\delta m^2 \ll m^2$$

motivated by the current ν phenomenology:

$\delta m^2 \lesssim 10^{-4}\ \mathrm{eV}^2$ (solar neutrinos)

$m^2 \gtrsim 10^{-3}\ \mathrm{eV}^2$ (terrestrial neutrinos)

$\downarrow$

$\delta m^2/m^2 \lesssim 1/10$, and often much smaller

- 1st welcome simplification:

  CP violation effects vanish as $\delta m^2/m^2 \to 0$

  (Cabibbo, 1978)

  $\to$ δ_{CP} ~ unobservable

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

mixing matrix $U_{\alpha i}$ real

$\nu/\bar{\nu}$ oscillations indistinguishable (in vacuum)

PARAMETER SPACE REDUCTION FOR SOLAR NEUTRINOS

Only $P(\nu_e \to \nu_e)$ is probed by $\nu_\odot$
(no μ, τ appearance expts !)

Hence, rotations in the (ν_μ, ν_τ) subspace do not change the physics. In particular, redefine ν_μ and ν_τ :

$$\begin{pmatrix} \nu'_\mu \\ \nu'_\tau \end{pmatrix} = \begin{pmatrix} c\psi & -s\psi \\ s\psi & c\psi \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix}$$

so that

$$\begin{pmatrix} \nu_e \\ \nu'_\mu \\ \nu'_\tau \end{pmatrix} = \begin{pmatrix} c\varphi & 0 & s\varphi \\ 0 & 1 & 0 \\ -s\varphi & 0 & c\varphi \end{pmatrix} \begin{pmatrix} c\omega & s\omega & 0 \\ -s\omega & c\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

and only φ and ω are observable.

In the $m^2 \gg \delta m^2$ limit, m^2- driven oscillations are averaged away

$\downarrow$

SOLAR NEUTRINOS PROBE

$$(\delta m^2; \omega, \varphi)$$

IN THE LIMIT $\delta m^2/m^2 \to 0$

PARAMETER SPACE REDUCTION FOR TERRESTRIAL NEUTRINOS

In the limit $\delta m^2 \to 0$, terrestrial (lab. or atmosph.) experiments become insensitive to rotations in the subspace spanned by the quasi-degenerate doublet (ν_1, ν_2). In particular, redefine

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} c\omega & -s\omega \\ s\omega & c\omega \end{pmatrix} \begin{pmatrix} \nu_1' \\ \nu_2' \end{pmatrix}$$

so that

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\psi & s\psi \\ 0 & -s\psi & c\psi \end{pmatrix} \begin{pmatrix} c\varphi & 0 & s\varphi \\ 0 & 1 & 0 \\ -s\varphi & 0 & c\varphi \end{pmatrix} \begin{pmatrix} \nu_1' \\ \nu_2' \\ \nu_3' \end{pmatrix}$$

and only ψ and φ are observable.

In the $m^2 \gg \delta m^2$ limit, δm^2-driven oscillations are frozen (in vacuum)

$\downarrow$

TERRESTRIAL NEUTRINOS PROBE

$$(m^2, \psi, \varphi)$$

IN THE LIMIT $\delta m^2 / m^2 \to 0$

SUMMARY:

At zeroth order in $\delta m^2/m^2$,

- SOLAR NEUTRINOS PROBE $\quad \delta m^2, \omega, \varphi$
- TERRESTRIAL NEUTRINOS PROBE $\quad m^2, \psi, \varphi$

or, equivalently:

- Solar neutrinos probe $(\delta m^2, U_{e1}, U_{e2}, U_{e3})$, i.e. the mass composition of ν_e:

$$\nu_e = U_{e1}\nu_1 + U_{e2}\nu_2 + U_{e3}\nu_3$$
$$= c_\varphi (c_\omega \nu_1 + s_\omega \nu_2) + s_\varphi \nu_3$$

- Terrestrial neutrinos probe $(m^2, U_{e3}, U_{\mu 3}, U_{\tau 3})$, i.e. the flavor composition of ν_3:

$$\nu_3 = U_{e3}\nu_e + U_{\mu 3}\nu_\mu + U_{\tau 3}\nu_\tau$$
$$= c_\varphi (c_\psi \nu_\tau + s_\psi \nu_\mu) + s_\varphi \nu_e$$

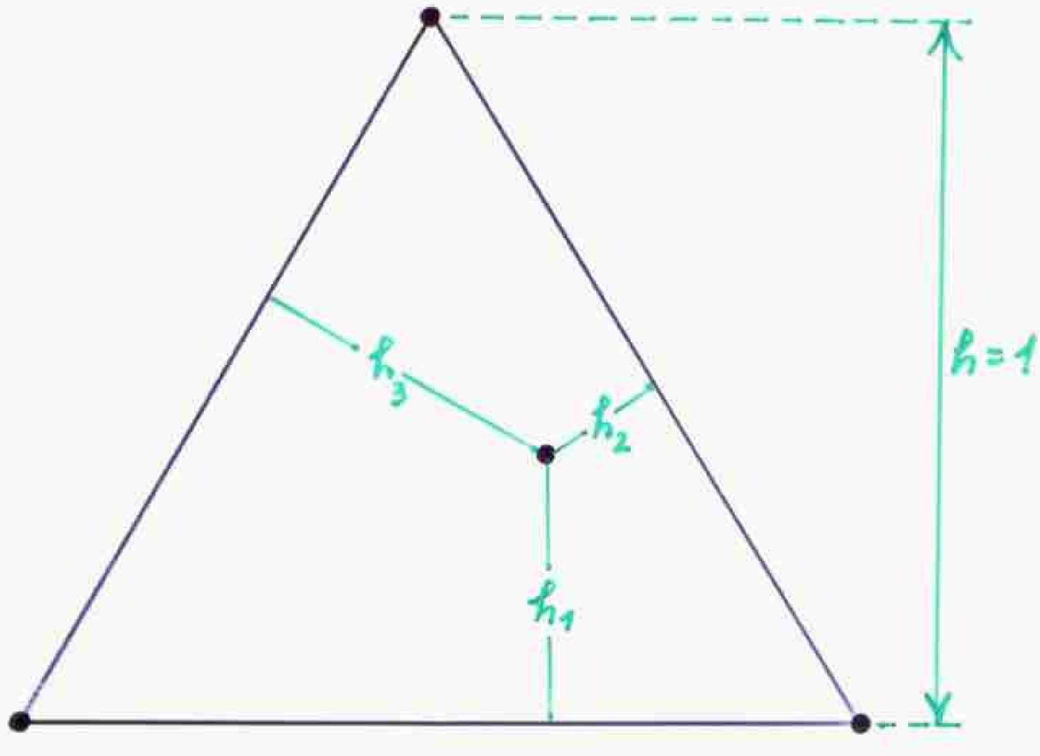
mixing matrix:

TERRESTRIAL

$$\begin{matrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{matrix}$$

SOLAR

Unitarity ($U_{e3}^2 + U_{\mu 3}^2 + U_{\tau 3}^2 = 1$ or $U_{e1}^2 + U_{e2}^2 + U_{e3}^2 = 1$)
can be implemented through
TRIANGLE GRAPHS :

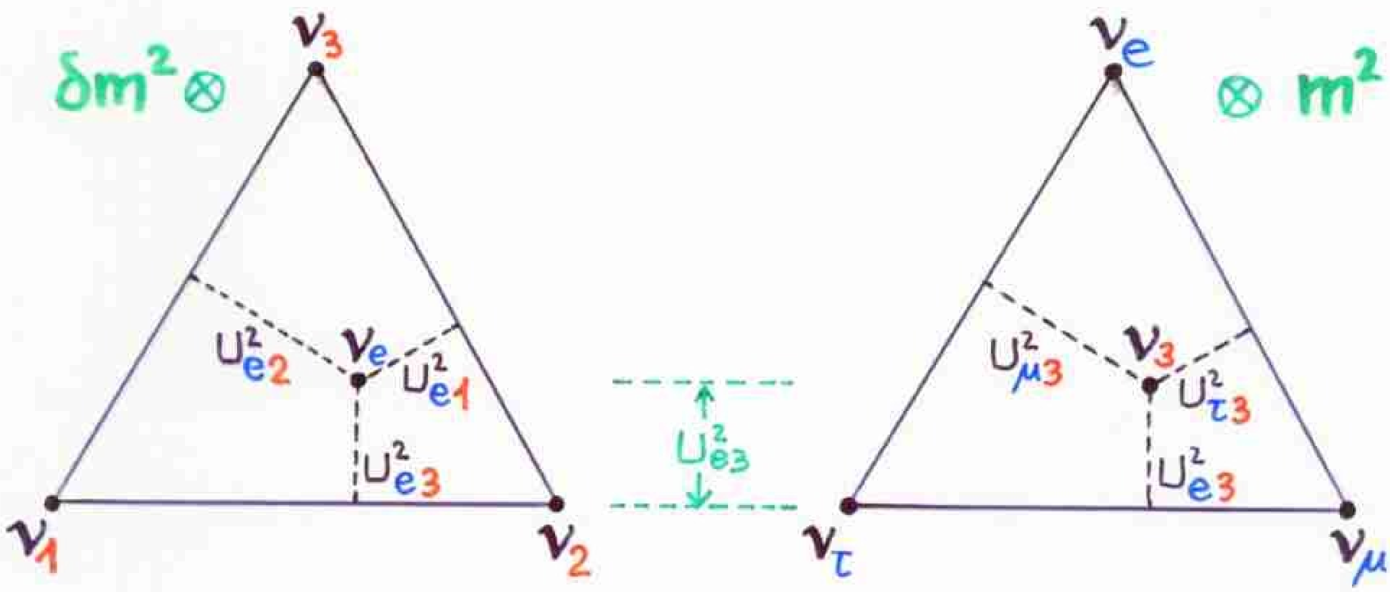


$$h_1 + h_2 + h_3 \equiv h = 1$$

Unitarity suggests the following representations:

SOLAR ν parameter space:

TERRESTRIAL ν parameter space:



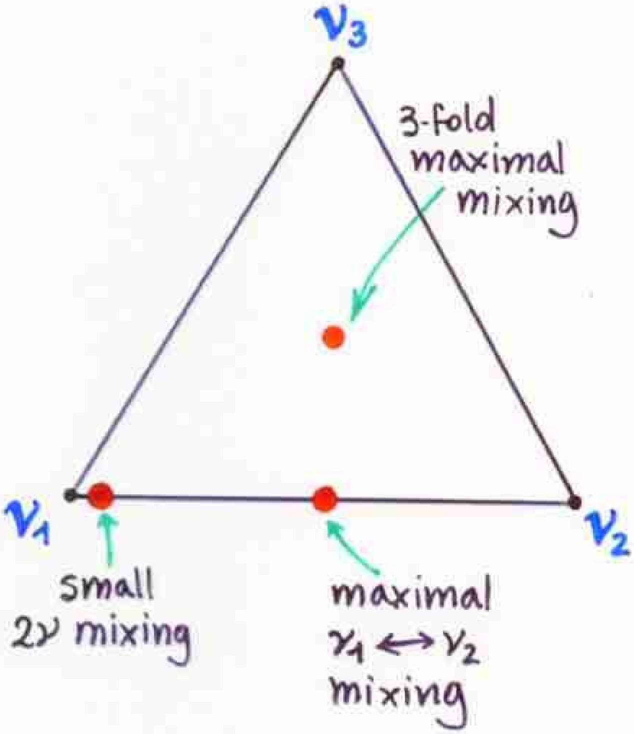
$U_{e1}^2 + U_{e2}^2 + U_{e3}^2 = 1$

$U_{e3}^2 + U_{\mu 3}^2 + U_{\tau 3}^2 = 1$

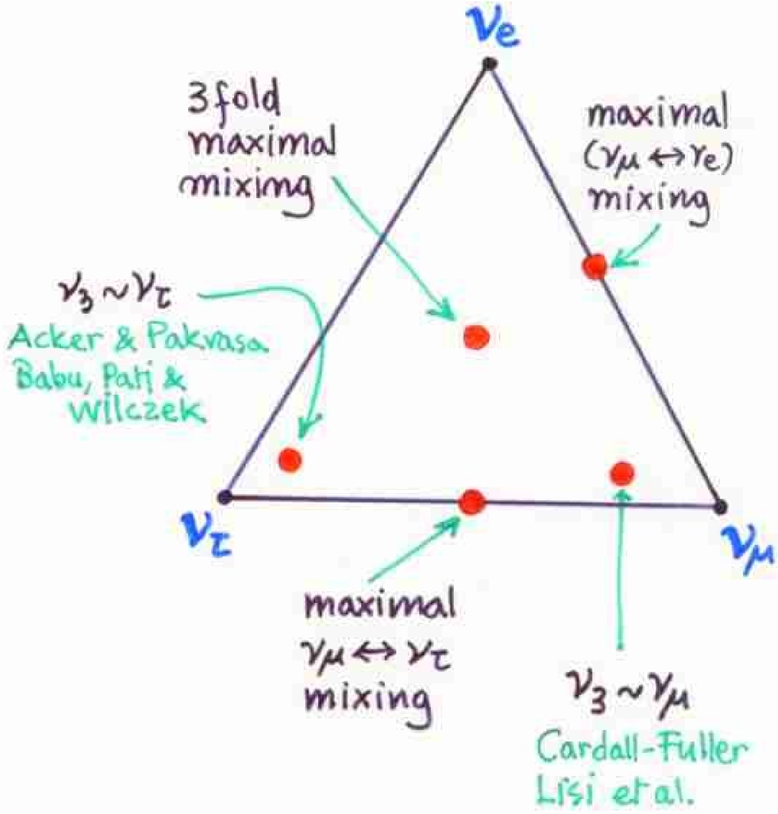
(Lisi, Fogli, Montanino, Scioscia)

INTERESTING SUBCASES:

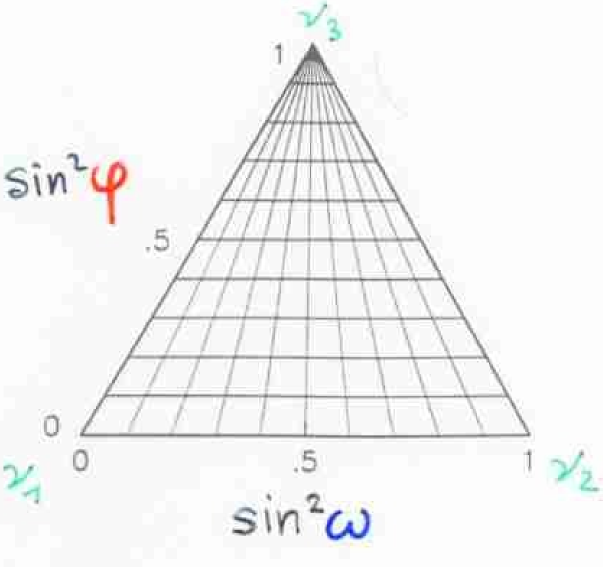
SOLAR ν



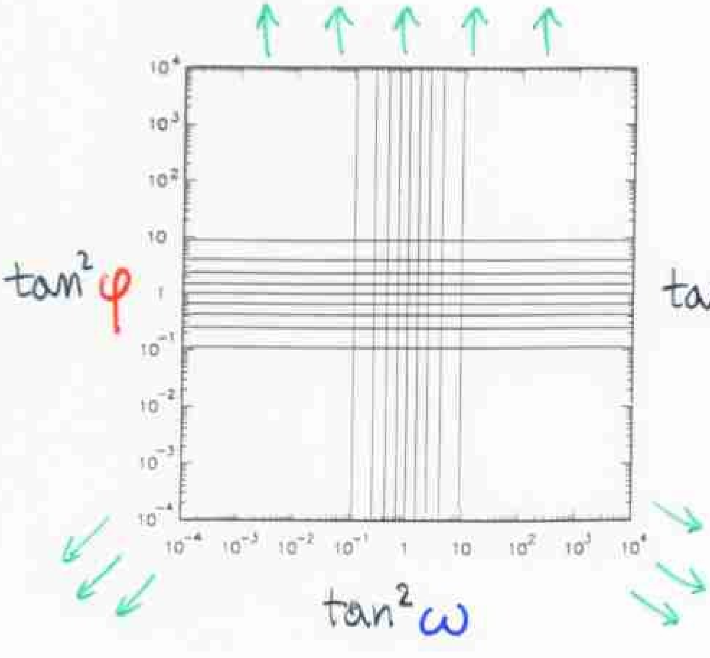
TERRESTRIAL ν



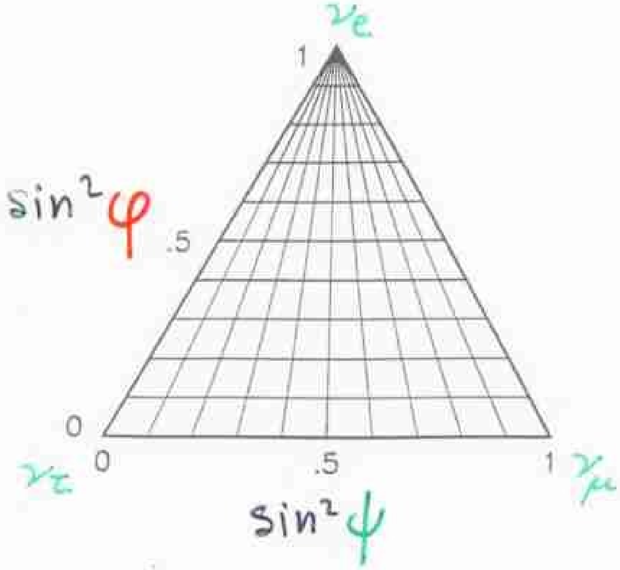
SOLAR



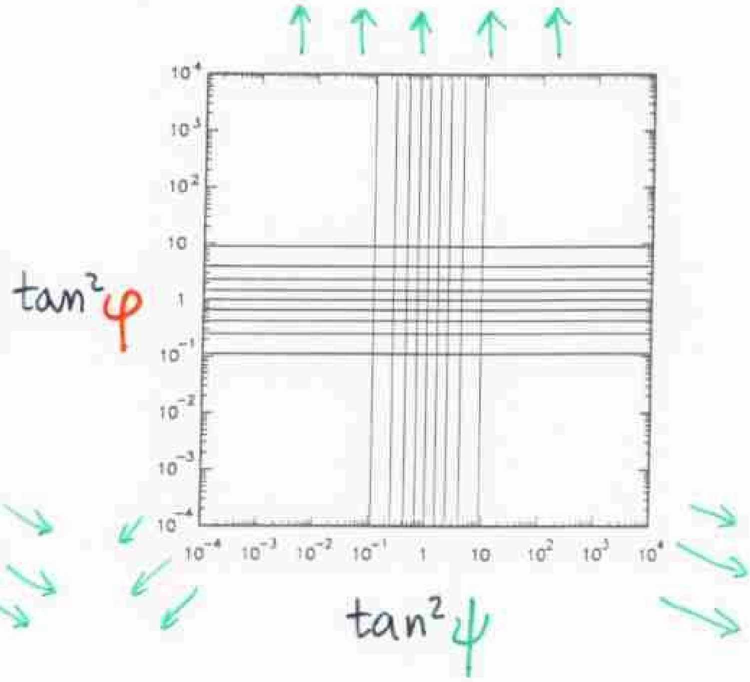
δm^2 fixed



TERRESTRIAL



m^2 fixed



BILOGARITHMIC MIXING/MIXING REPRESENTATION

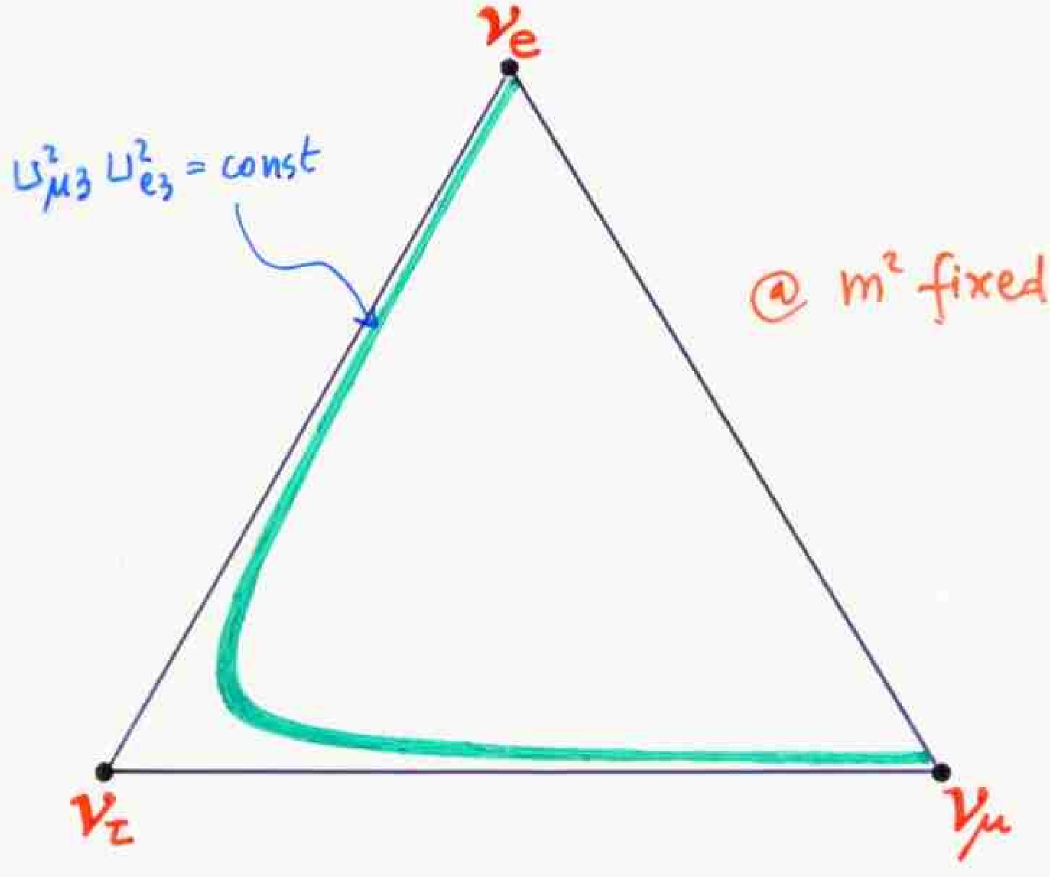
1st EXAMPLE : LSND

$$P_{LSND} = P(\nu_\mu \leftrightarrow \nu_e) \neq 0$$

3ν: $$P_{LSND} = 4\,U_{\mu 3}^2 U_{e3}^2 \sin^2\left(1.27\,\frac{m^2 L}{E}\right) \quad \left(\frac{eV^2\,km}{GeV}\right)$$

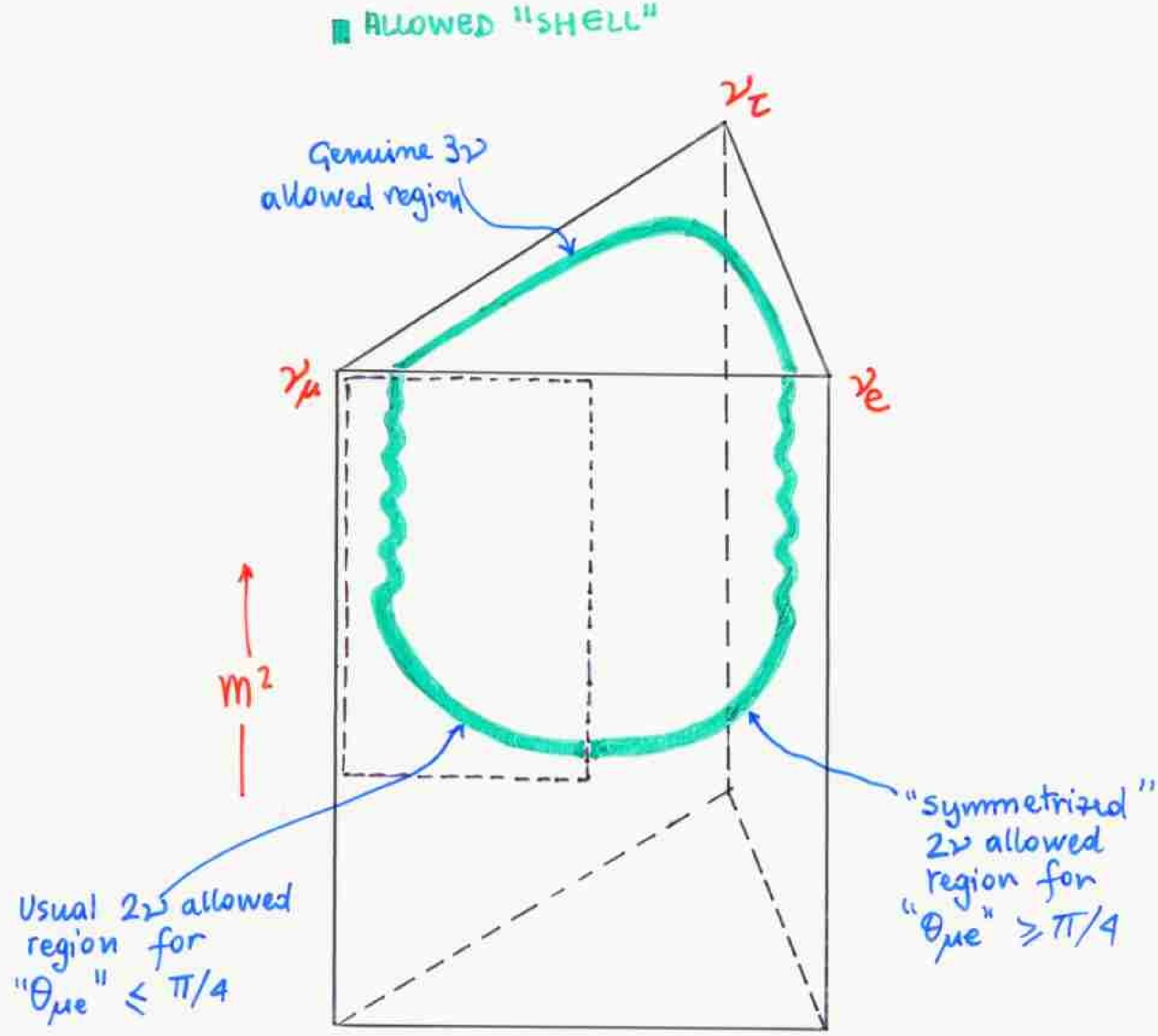
$\downarrow$

■ Region allowed by LSND at fixed m^2 (bounded by P_{LSND} = const. curves)

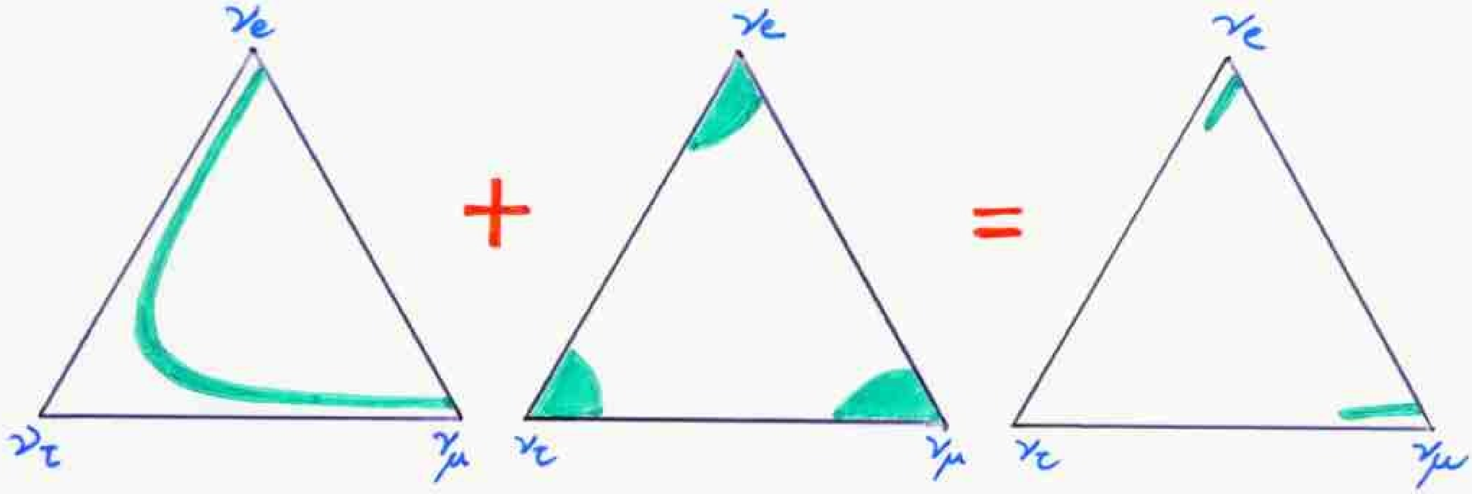


$$\text{“}\sin^2 2\theta_{e\mu}\text{”} \equiv 4\,U_{\mu 3}^2 U_{e3}^2 = 4\, s_\varphi^2 c_\varphi^2 s_\psi^2$$

VIEW OF THE LSND ALLOWED REGION IN THE $(m^2, \omega^2_{\alpha 3})$ SPACE



LSND + negative acc/reac searches = $\nu_3 \sim \nu_e$ or $\nu_3 \sim \nu_\mu$



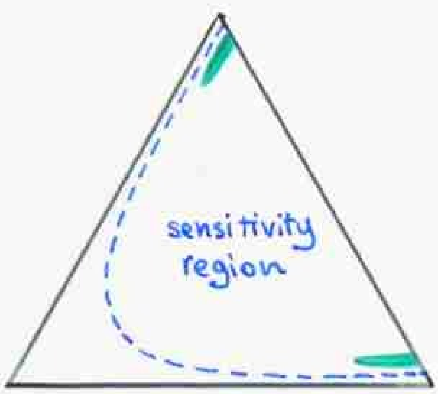
LSND allowed region: $U_{e3}^2 U_{\mu 3}^2 \sim$ const

Regions allowed by negative searches: $\nu_3 \sim \nu_\alpha$ $(\alpha = e, \mu, \tau)$ to suppress oscillations

Combination: ν_3 can be close to either ν_e or ν_μ but <u>not</u> to ν_τ (at 99% CL)

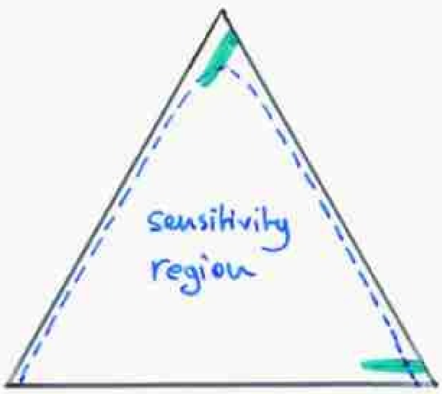
POSSIBLE TESTS AT ACCELERATORS:

$\nu_\mu \leftrightarrow \nu_e$ channel

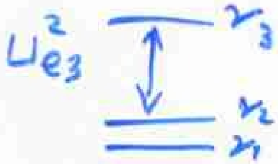


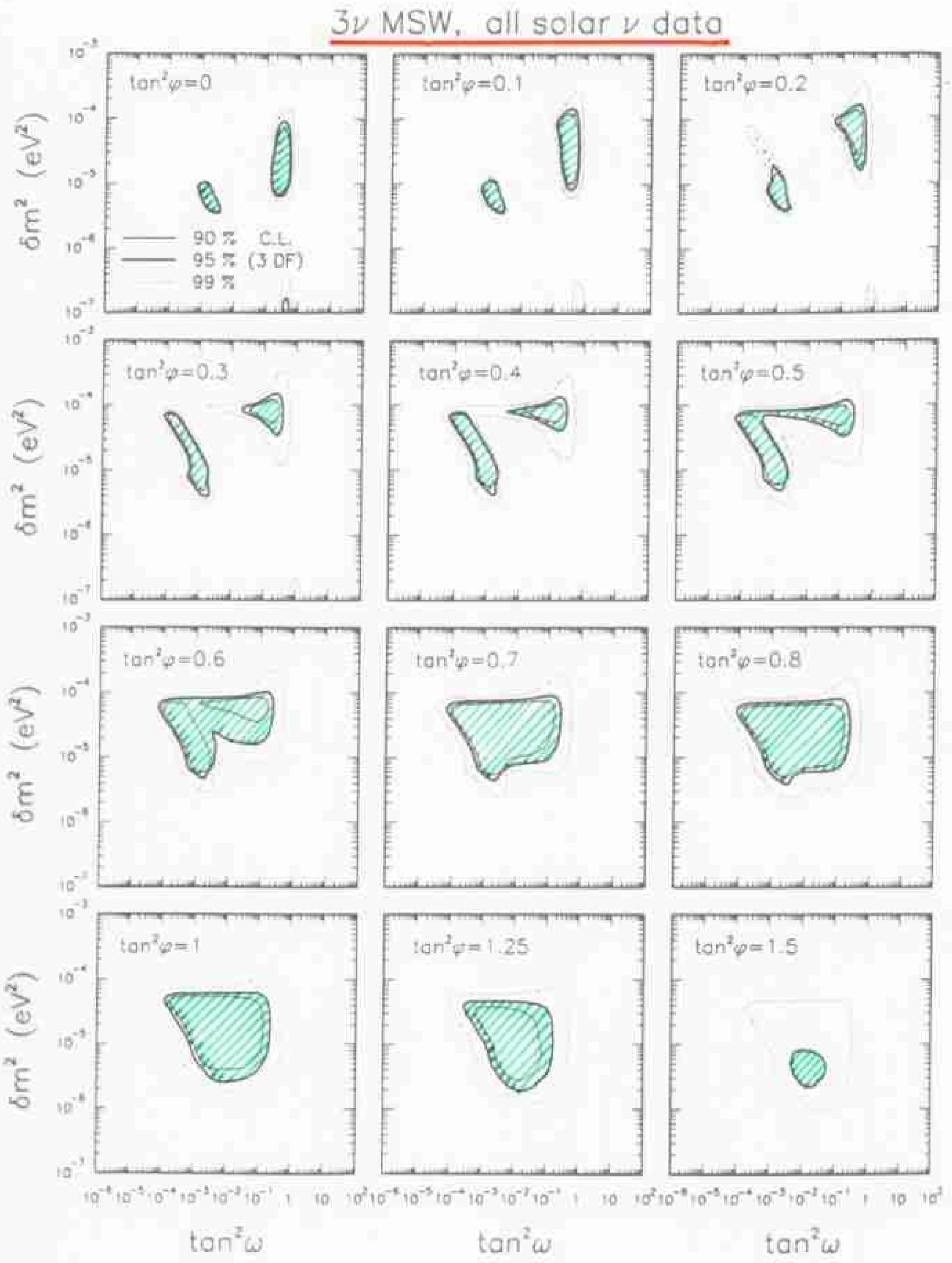
KARMEN
BOONE
CERN expt. ?

$\nu_\mu \leftrightarrow \nu_\tau$ channel



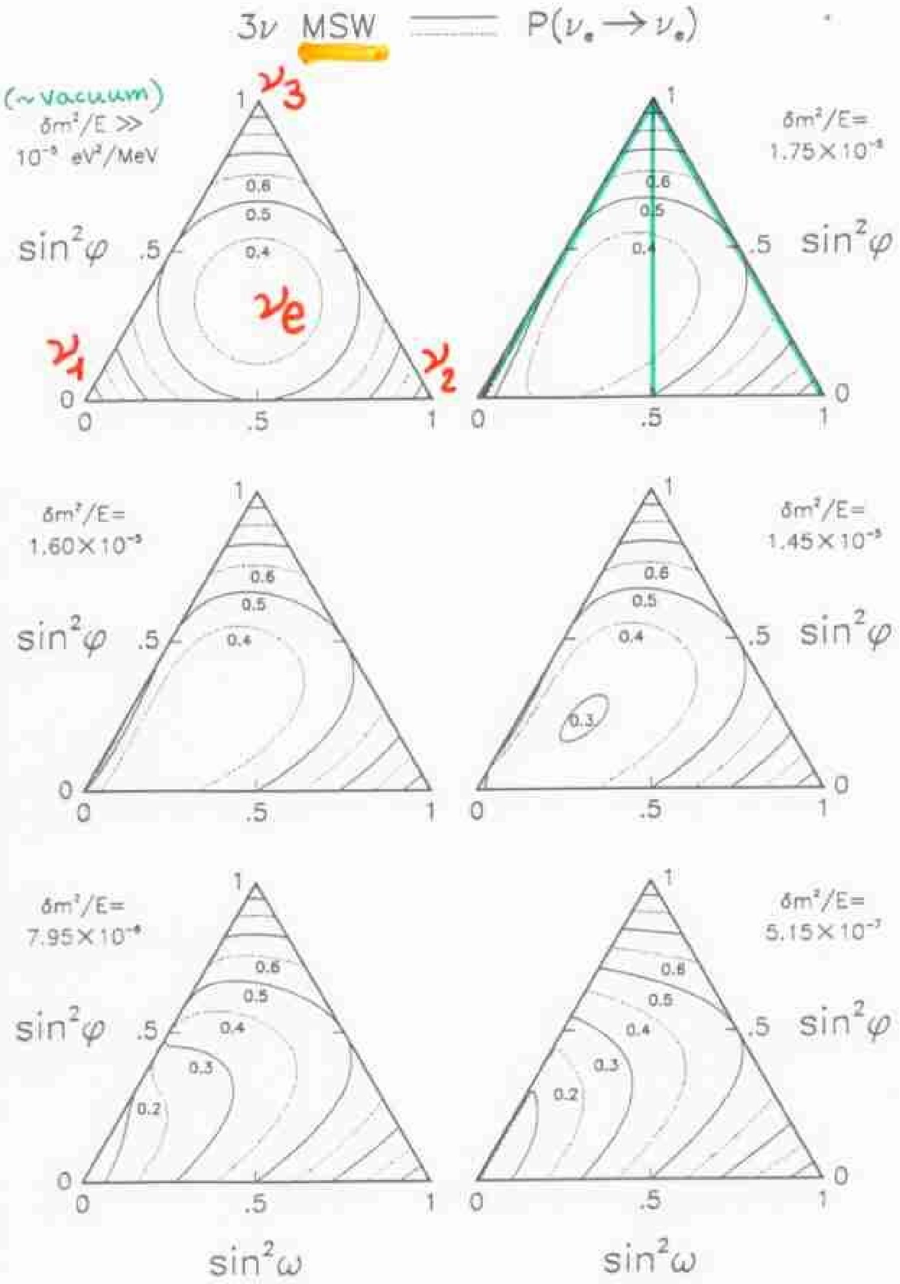
~ CHORUS, NOMAD
COSMOS
TOSCA





$(\delta m^2, \tan^2\omega)$ @ $\tan^2\varphi$ fixed

SOLAR ν_s

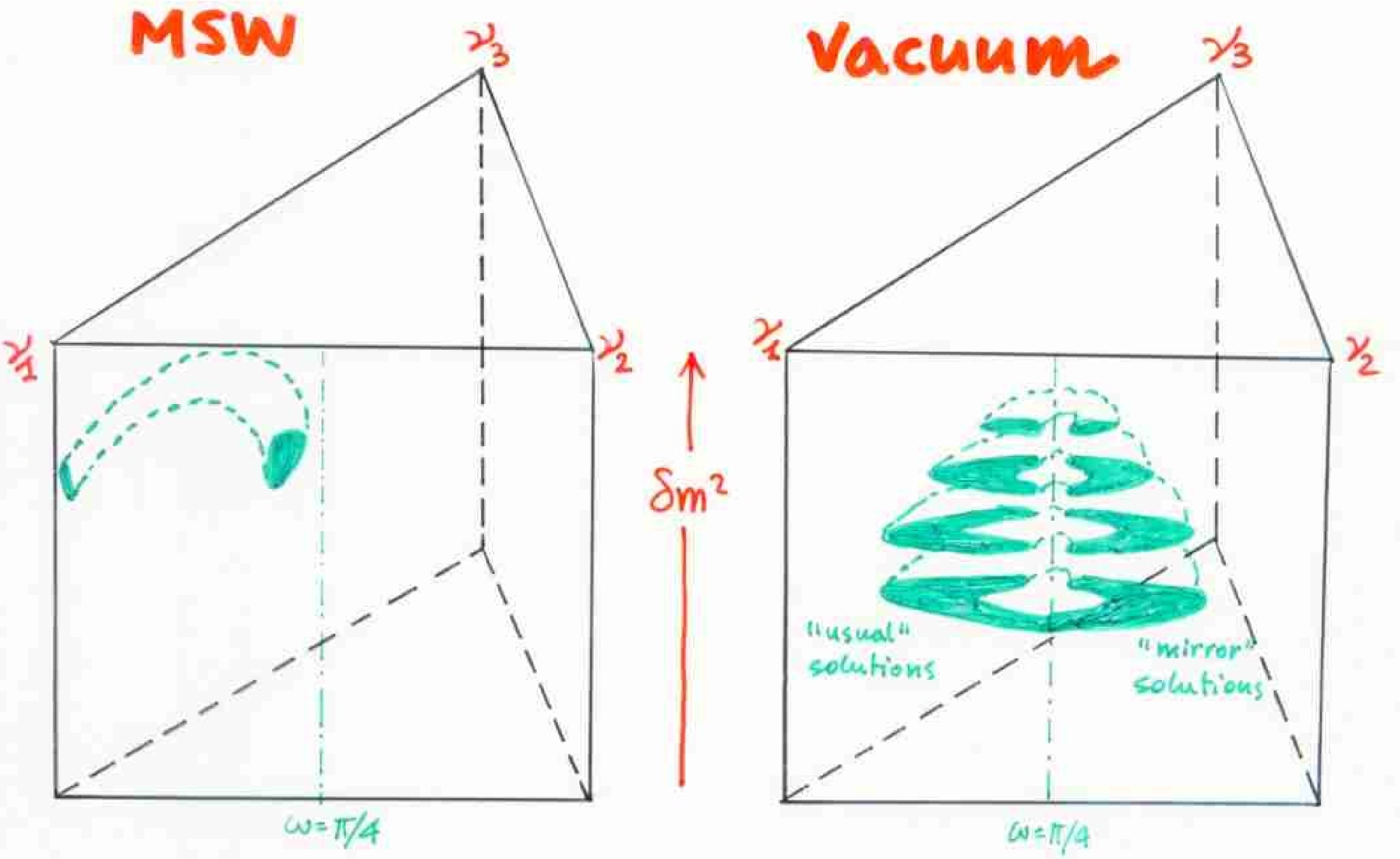


MSW probability contours

Along green lines $(\omega = 0, \pi/4, \pi/2)$: $P_{MSW} = P_{VAC}$

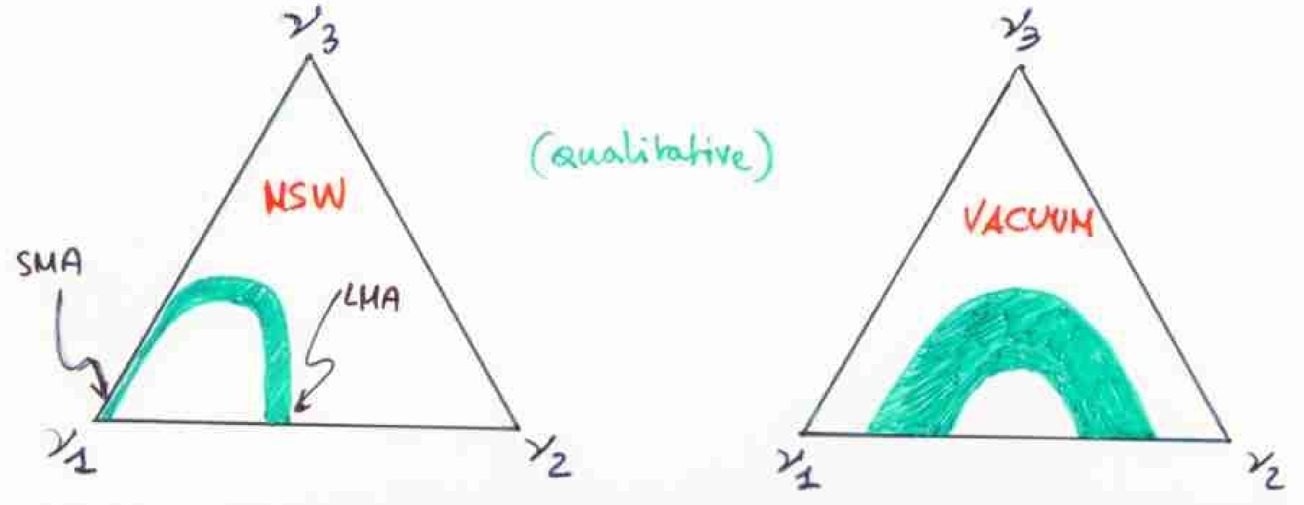
2nd example : Solar ν's

$$\nu_e = U_{e1}\,\nu_1 + U_{e2}\,\nu_2 + U_{e3}\,\nu_3$$



SMALL AND LARGE MIXING MSW SOLUTIONS ARE CONTINUOUSLY CONNECTED

VACUUM SOLUTIONS ARE MIRRORED AND CONNECTED



In any case (MSW or vacuum osc.) large values of ϕ ($\to \pi/2$) are excluded :

$$\phi \to \pi/2 \iff U_{e3}^2 = s_\varphi^2 \to 1 \iff \nu_e \to \nu_3$$

and $P_{solar}(\nu_e \to \nu_e) \to 1$

→ NO SOLAR ν DEFICIT !!!

Moreover, solar ν data indicate that 2ν fits are better than 3ν

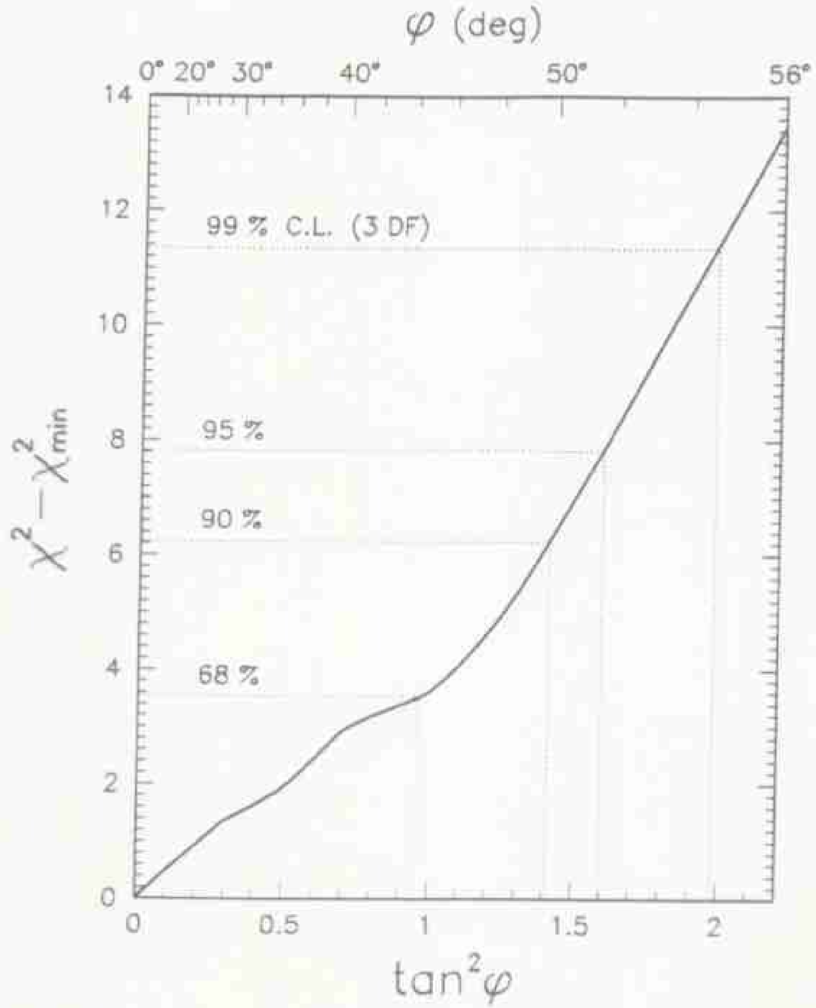
$$\chi^2(2\nu) < \chi^2(3\nu)$$

(MSW: Fogli, E.L., Montanino
vacuum: Berezhiani, Rossi)

although up to $\phi \sim 40°$ (or 50°) is allowed

NONTRIVIAL INDICATION THAT IS WORTH CHECKING WITH THE LATEST DATA

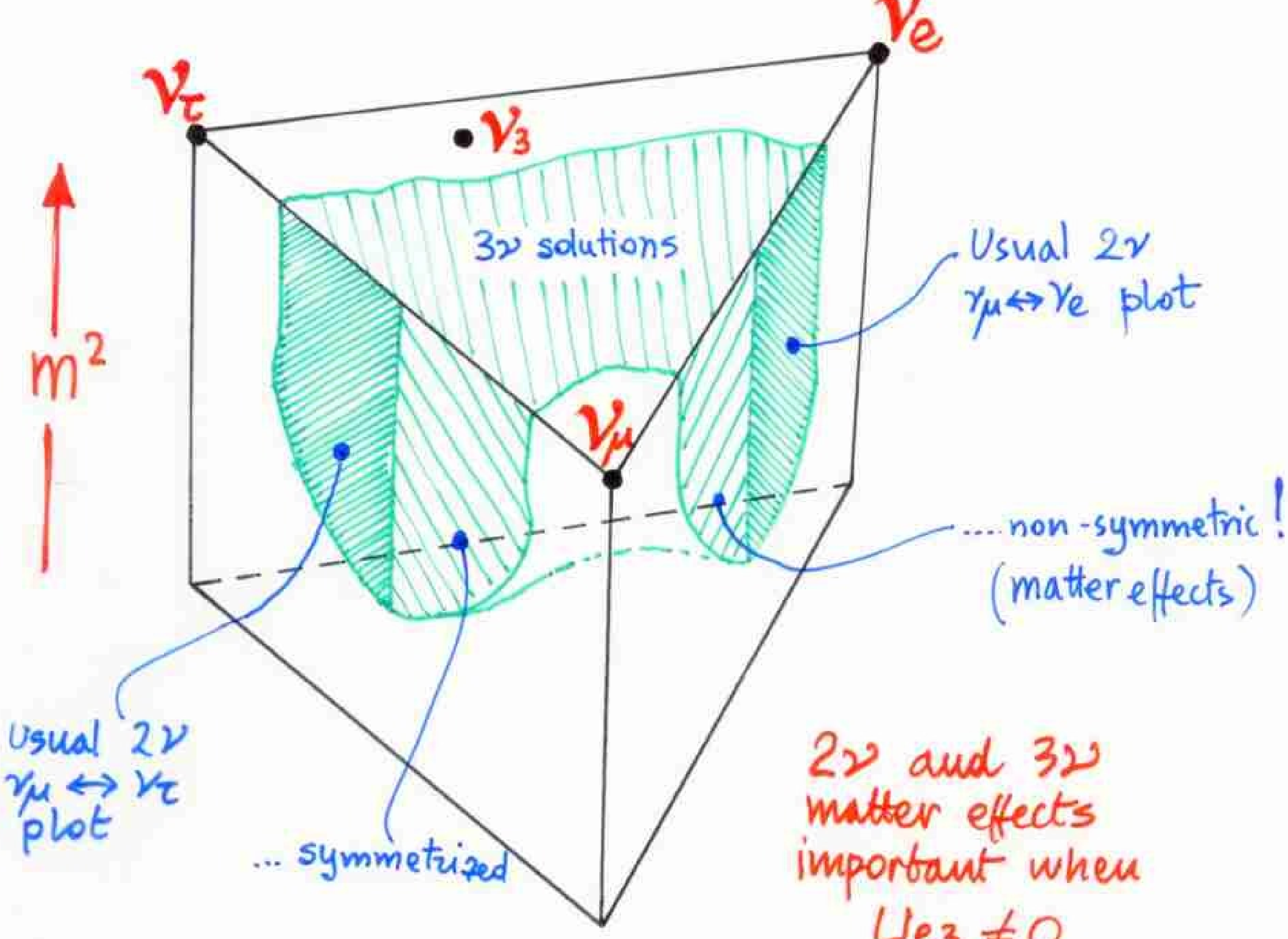
Solar neutrinos,
Pre-Superkamiokande data (MSW)



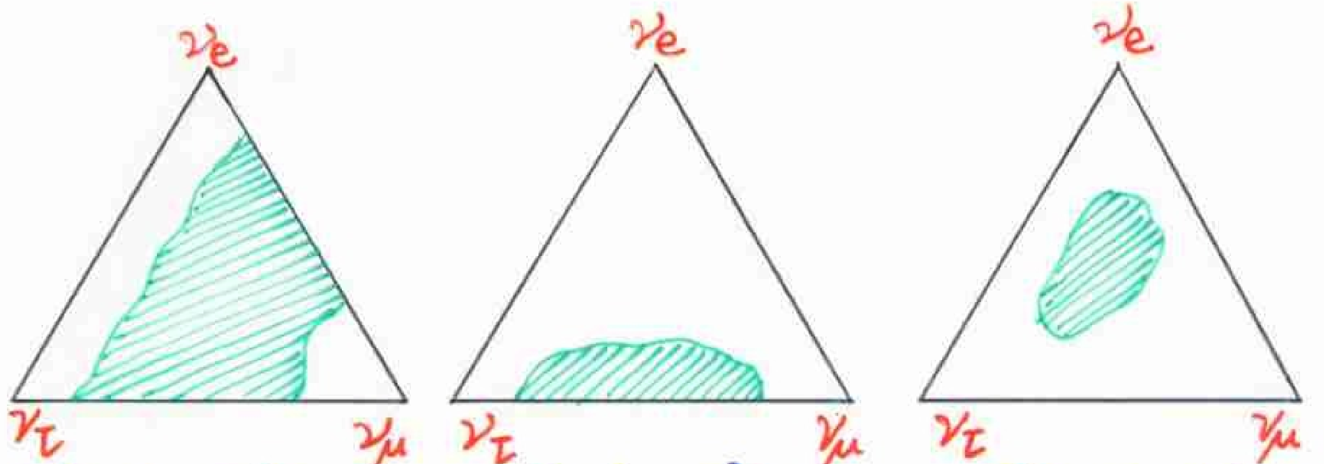
δm^2 and $\tan^2\omega$ unconstrained

3rd example : Atmospheric ν

Parameter space : $(m^2, U_{e3}^2, U_{\mu 3}^2, U_{\tau 3}^2)$



Depending on the DATA and on m^2, several situations are possible, e.g. :

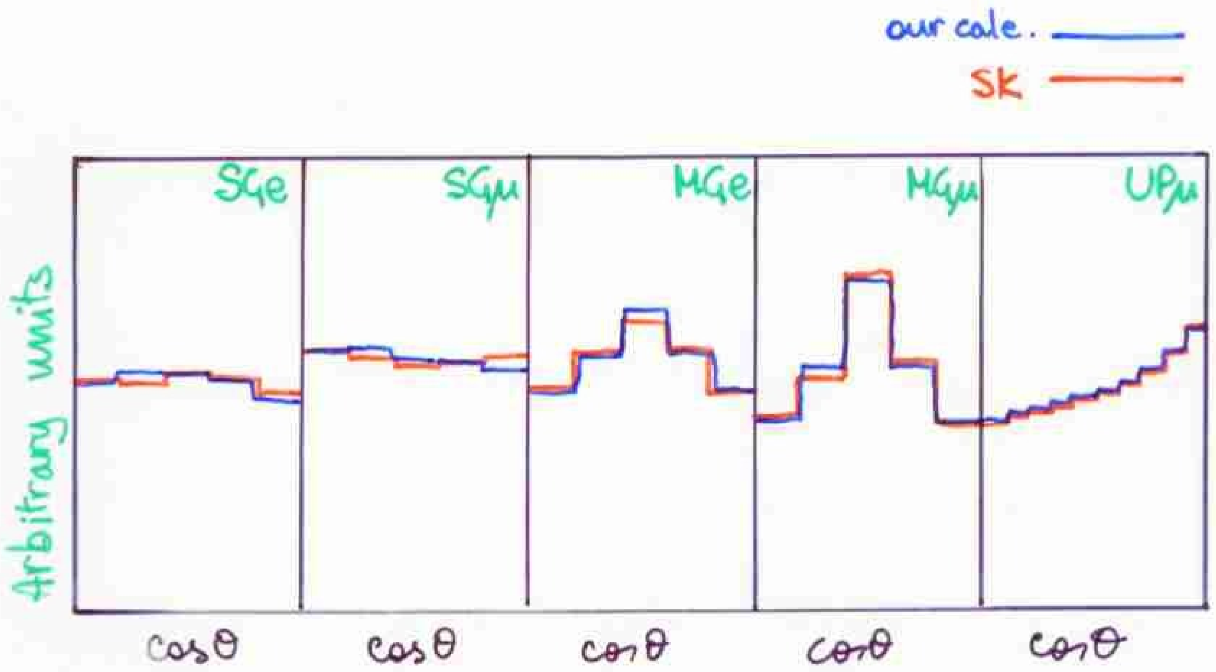


$\chi^2(\mu\tau) \sim \chi^2(\mu e)$ $\qquad$ $\chi^2(\mu\tau) \ll \chi^2(\mu e)$ $\qquad$ $\chi^2(3\nu) \ll \chi^2(2\nu)$

G.L. Fogli, E.L., A. Marrone, G. Scioscia

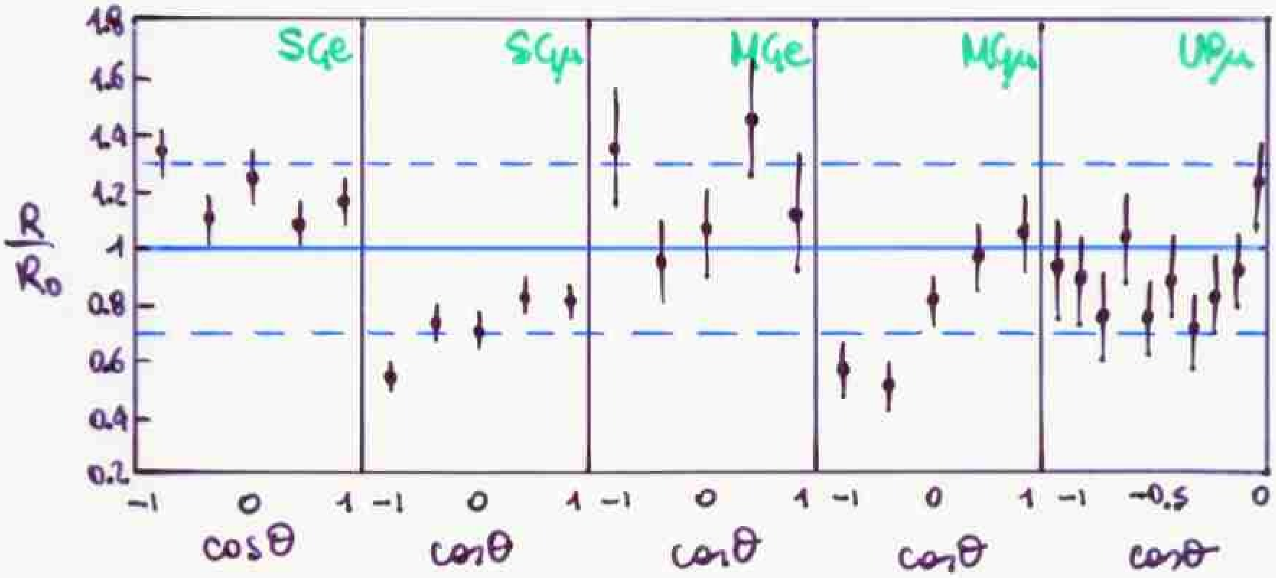
– Preliminary – (~ 1y work)

NO-OSCILLATION CHECK OF S-K ANGULAR DISTRIBUTIONS



+ checks for oscillation points
+ $\theta_{\nu e}, \theta_{\nu\ell} \neq 0$ at different energies
+ ....
+ $P(\nu_\alpha \to \nu_\beta)$ and $P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$ calculated numerically for the Earth density profile @ generic $m_1, m_2, m_3, \omega, \varphi, \psi$

SK 400 day

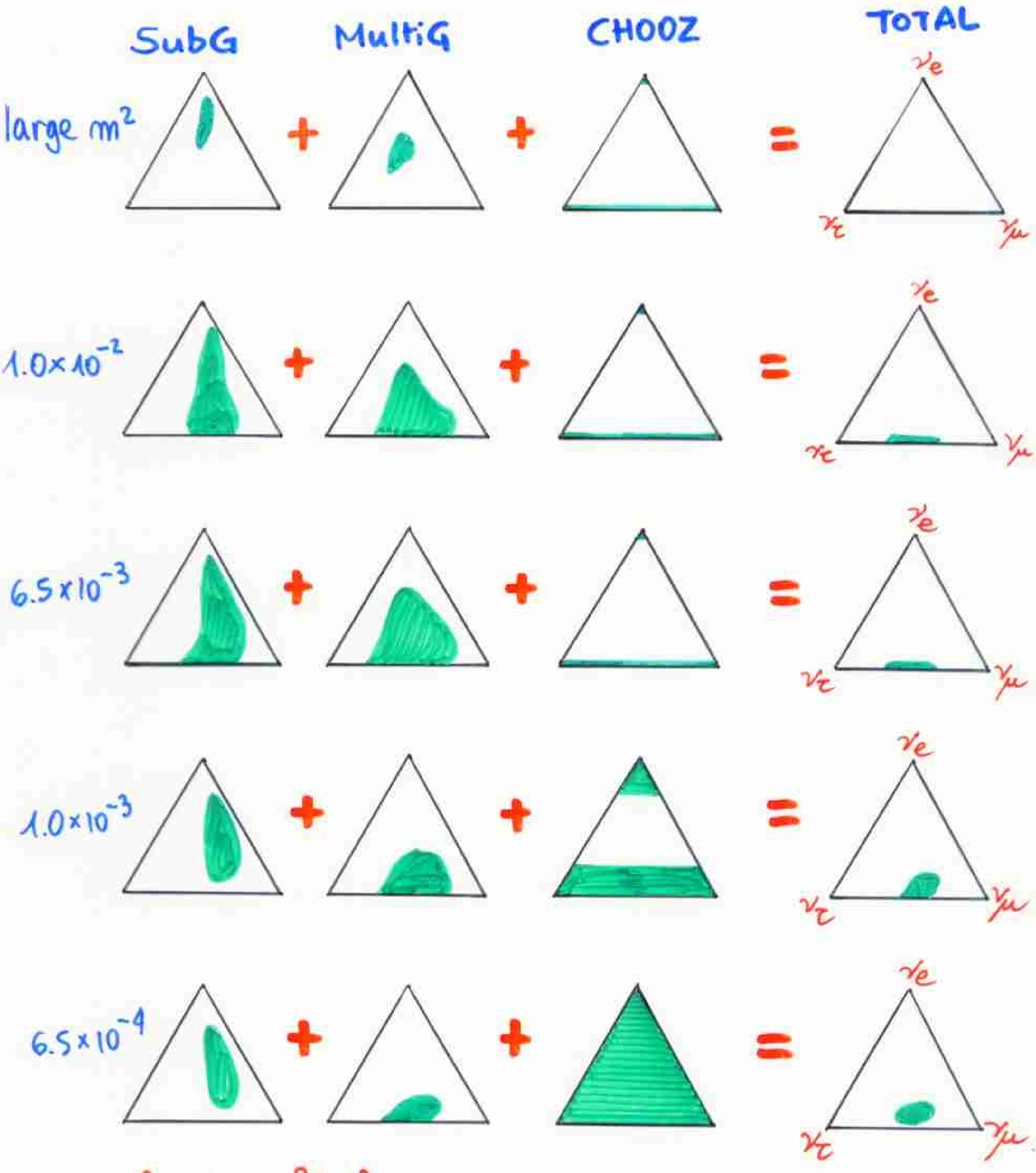


Theory = 1 ± 0.3 in this plot

Superkamiokande 400 day DATA
PRELIMINARY ANALYSIS

(Fogli, E.L., Marrone, Scioscia)

We include generic 3ν matter effects numerically (≥ 1994)



$\chi^2(3\nu) \lesseqgtr \chi^2(2\nu)$? NOT CLEARLY ESTABLISHED BY 400d SK DATA

$U_{e3} \neq 0$ leads to a much richer phenomenology and helps fitting SK data.

$U_{e3} \neq 0$ → several new effects that do not show the "canonical" **L/E** dependence

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One such effect is due to the fact that

$$\frac{\overset{(-)}{\nu}_\mu}{\overset{(-)}{\nu}_e} \text{ increases for } \begin{cases} \text{high energies} \\ \cos\theta \to \pm 1 \quad (\text{vertical}) \end{cases}$$

→ THE ATMOSPHERIC NEUTRINO BEAM IS RICHER IN $\overset{(-)}{\nu}_\mu$'s at high energy and along the vertical

Best place to look for $\nu_\mu \to \nu_e$ oscillations!

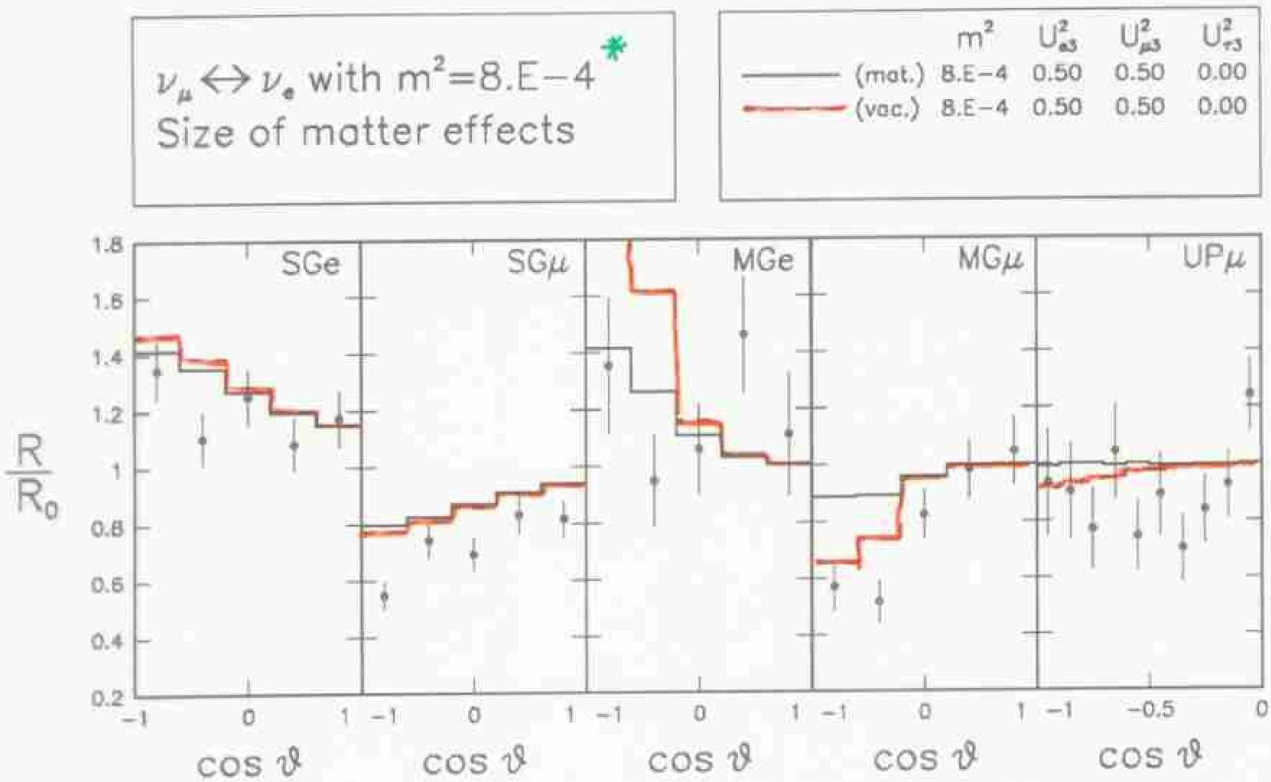
$U_{e3} \neq 0$ ➡ $\nu_\mu \leftrightarrow \nu_e$ open ➡ e-like rate increases

effect larger as E increases and $\cos\theta \to \pm 1$!!

SPECTRAL DEFORMATION NOT DEPENDENT ON L/E !!! (even in vacuum)

PRELIMIN.

Example of 2ν matter effects
(not dependent on L/E !)



$\sin^2 2\theta_{vac} \sim 1 \quad \rightarrow \quad \sin^2 2\theta_{mat} < 1$

(suppression of oscillation)

*Below Chooz bounds

+ GENUINELY 3ν EFFECTS IN MATTER

Under some conditions:

$$\begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} \\ P_{\mu e} & P_{\mu\mu} & P_{\mu\tau} \\ P_{\tau e} & P_{\tau\mu} & P_{\tau\tau} \end{pmatrix}_{matter} \simeq \begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} \\ P_{\mu e} & P_{\mu\mu} & P_{\mu\tau} \\ P_{\tau e} & P_{\tau\mu} & P_{\tau\tau} \end{pmatrix}_{vac} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\delta P & +\delta P \\ 0 & +\delta P & -\delta P \end{pmatrix}$$

$$\delta P = 4\,\frac{U^2_{e3}\,U^2_{\mu 3}\,U^2_{\tau 3}}{(1-U^2_{e3})^2}\,\sin^2\Big(\underbrace{\frac{G_F}{\sqrt{2}}\,N_e\,(1-U^2_{e3})\,L}_{\equiv\ 2.47\,(1-U^2_{e3})\,\frac{N_e}{mol/cm^3}\cos\theta}\Big)$$

3ν large phase!

$\delta P = 0$ for pure 2ν oscillations
no L/E dependence!

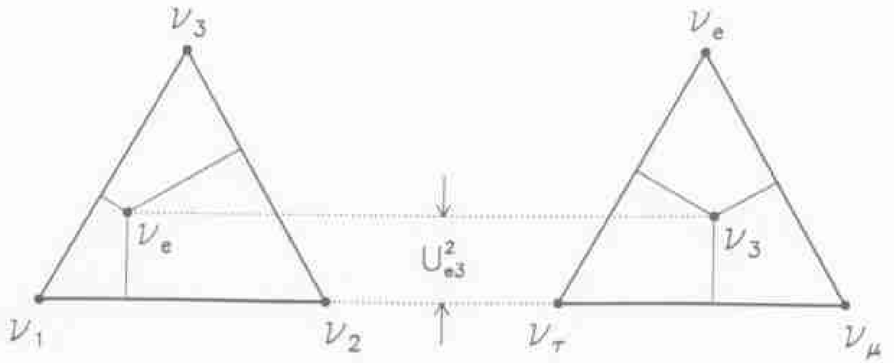
Within Chooz bounds: no effect on $\left(\frac{U}{D}\right)_e$

few % effect on $\left(\frac{U}{D}\right)_\mu$

Fogli, E.L., Marrone, Montanino
hep-ph/9711421, to appear in PLB

Solar ν	Terrestrial ν
$\nu_e = U_{e1}\nu_1 + U_{e2}\nu_2 + U_{e3}\nu_3$	$\nu_3 = U_{e3}\nu_e + U_{\mu 3}\nu_\mu + U_{\tau 3}\nu_\tau$
$\delta m^2 = m_2^2 - m_1^2$	$m^2 = m_3^2 - m_{1,2}^2$

$(\delta m^2 \ll m^2)$



ν CKM elements probed:

$$\left\| \begin{matrix} \underline{e1} & \underline{e2} & \underline{e3} \\ \mu 1 & \mu 2 & \mu 3 \\ \tau 1 & \tau 2 & \tau 3 \end{matrix} \right\| \qquad \left\| \begin{matrix} e1 & e2 & \underline{e3} \\ \mu 1 & \mu 2 & \underline{\mu 3} \\ \tau 1 & \tau 2 & \underline{\tau 3} \end{matrix} \right\|$$

Parameter space:

$$(\delta m^2, \omega, \varphi) \qquad (m^2, \psi, \varphi)$$

$U_{e1}^2 = \cos^2\varphi \cos^2\omega$
$U_{e2}^2 = \cos^2\varphi \sin^2\omega$

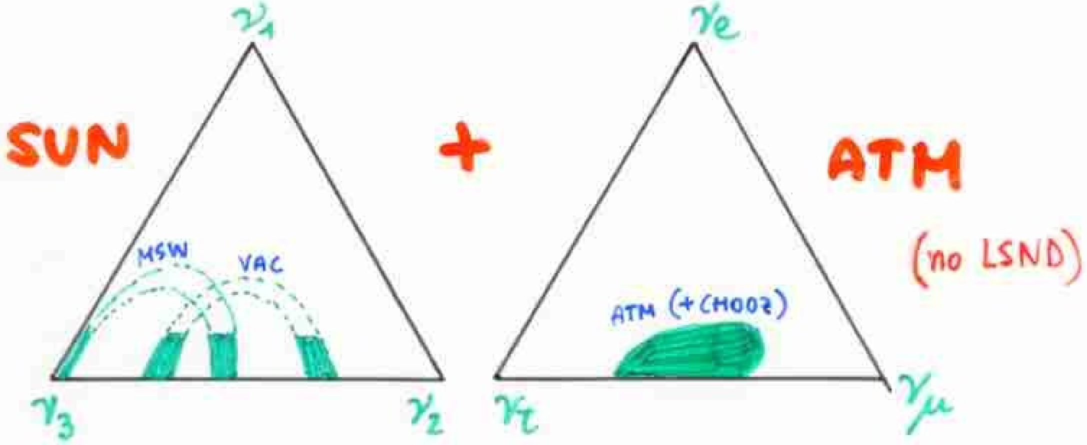
$U_{e3}^2 = \sin^2\varphi$

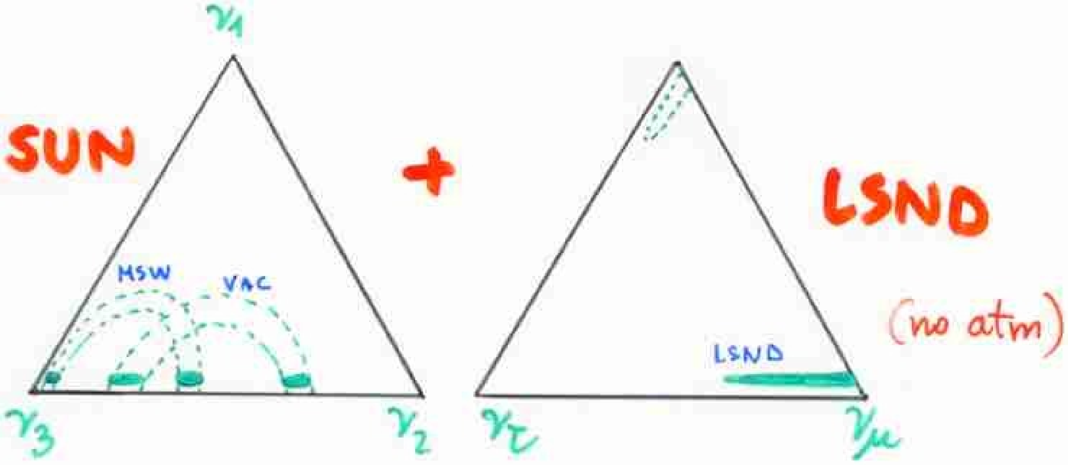
$U_{\mu 3}^2 = \cos^2\varphi \sin^2\psi$
$U_{\tau 3}^2 = \cos^2\varphi \cos^2\psi$

$0 < \omega, \varphi, \psi < \pi/2$: δ_{CP} unobservable

CONCLUSIONS :

ONLY TWO POSSIBLE "SOLUTIONS" IN THREE-FLAVOR (HIERARCHICAL) SCHEMES :



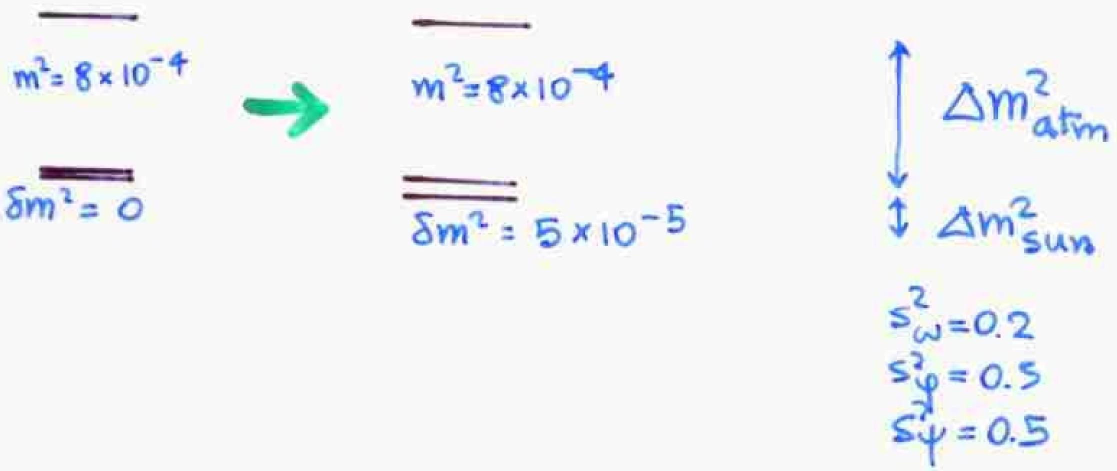


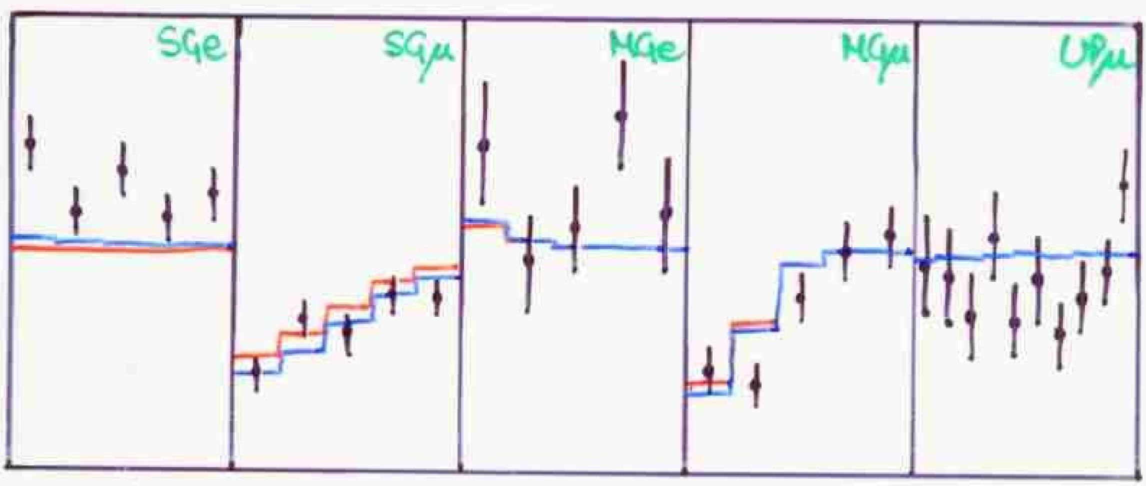
U_{e3} can be nonnegligible in case 1.

Tests : - Prove that U_{e3} is constrained by solar data (or atm. data) only

- Probe U_{e3} with lab. expts in the $\nu_\mu \to \nu_e$ channel

... THE NEXT FRONTIER : SOLAR ν MASS EFFECT IN ATMOSPHERIC NEUTRINO OSCILLATIONS ?





— $\delta m^2 = 0$

— $\delta m^2 = 5\times10^{-5}$