Neutrino mass spectra
and
parametric (or diffractive like?)
resonance of atmospheric neutrino.

at "New Era in Neutrino Physics" symposium

東京都立大学, June 11-12, 1998.

- parametric resonance.
- about high energy bins of SK spectrum.
- mass spectra and phenomenon.

§. Oscillation solutions to Atmospheric ν problem.

1. $\nu_\mu \longleftrightarrow \nu_e$; 2. $\nu_\mu \longleftrightarrow \nu_\tau$; 3. $\nu_\mu \longleftrightarrow \nu_s$

$$i \frac{d}{dt} \begin{pmatrix} \nu_\mu \\ \nu_i \end{pmatrix} = \begin{bmatrix} 0 & \frac{\sin 2\theta}{4E} \Delta m^2 \\ \frac{\sin 2\theta}{4E} \Delta m^2 & \frac{\cos 2\theta}{2E} \Delta m^2 + \underbrace{\sqrt{2}\, G_F N_i^{eff}}_{V} \end{bmatrix} \times \begin{pmatrix} \nu_\mu \\ \nu_i \end{pmatrix}$$

where $i = e, \tau, s$

and

$$N_i^{eff} = \begin{cases} \frac{1}{2} N_n \, , & i = s \\ N_e \, , & i = e \\ 0 \, , & i = \tau \end{cases}$$

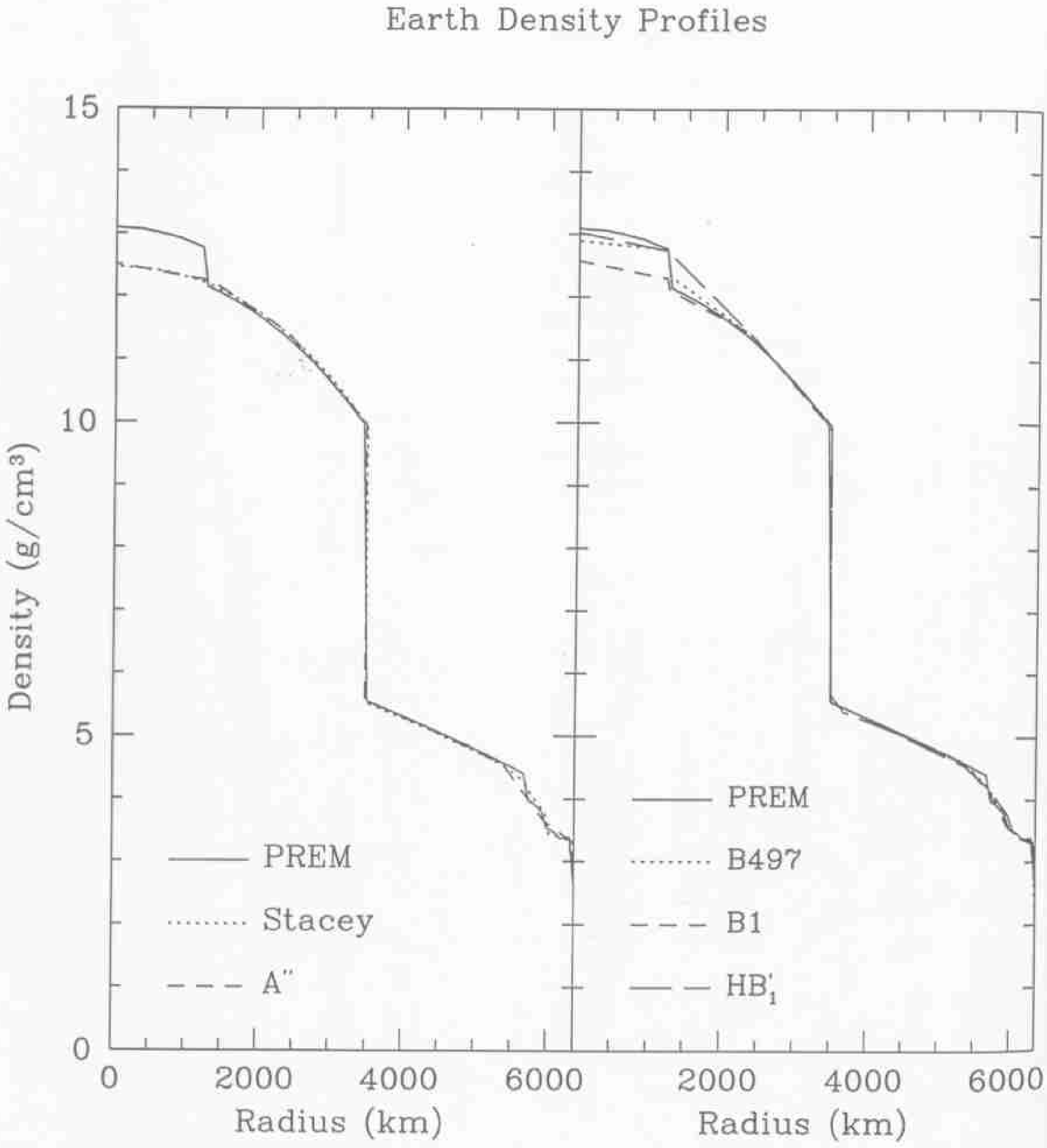
θ : mixing angle between flavor ν and mass ν.

$\Delta m^2 = m_2^2 - m_1^2$

E : neutrino energy.

G_F : Fermi constant.

The density distribution of earth is as fig.



Earth Density Profiles
15
10
5
0
Density (g/cm³)
0
2000
4000
6000
Radius (km)
PREM
Stacey
A''
2000
4000
6000
Radius (km)
PREM
B497
B1
HB'₁

Figure 2

Matter suppress oscillations.

$$M = \begin{pmatrix} 0 & \frac{\sin 2\theta}{4E}\Delta m^2 \\ \underbrace{\frac{\sin 2\theta}{4E}\Delta m^2}_{a} & \underbrace{\frac{\cos 2\theta}{2E}\Delta m^2 + V}_{b} \end{pmatrix}$$

$$tg\, 2\theta_m = \frac{2a}{b}$$

when $E \to \infty$; then $a \to 0$, $b \to V$.

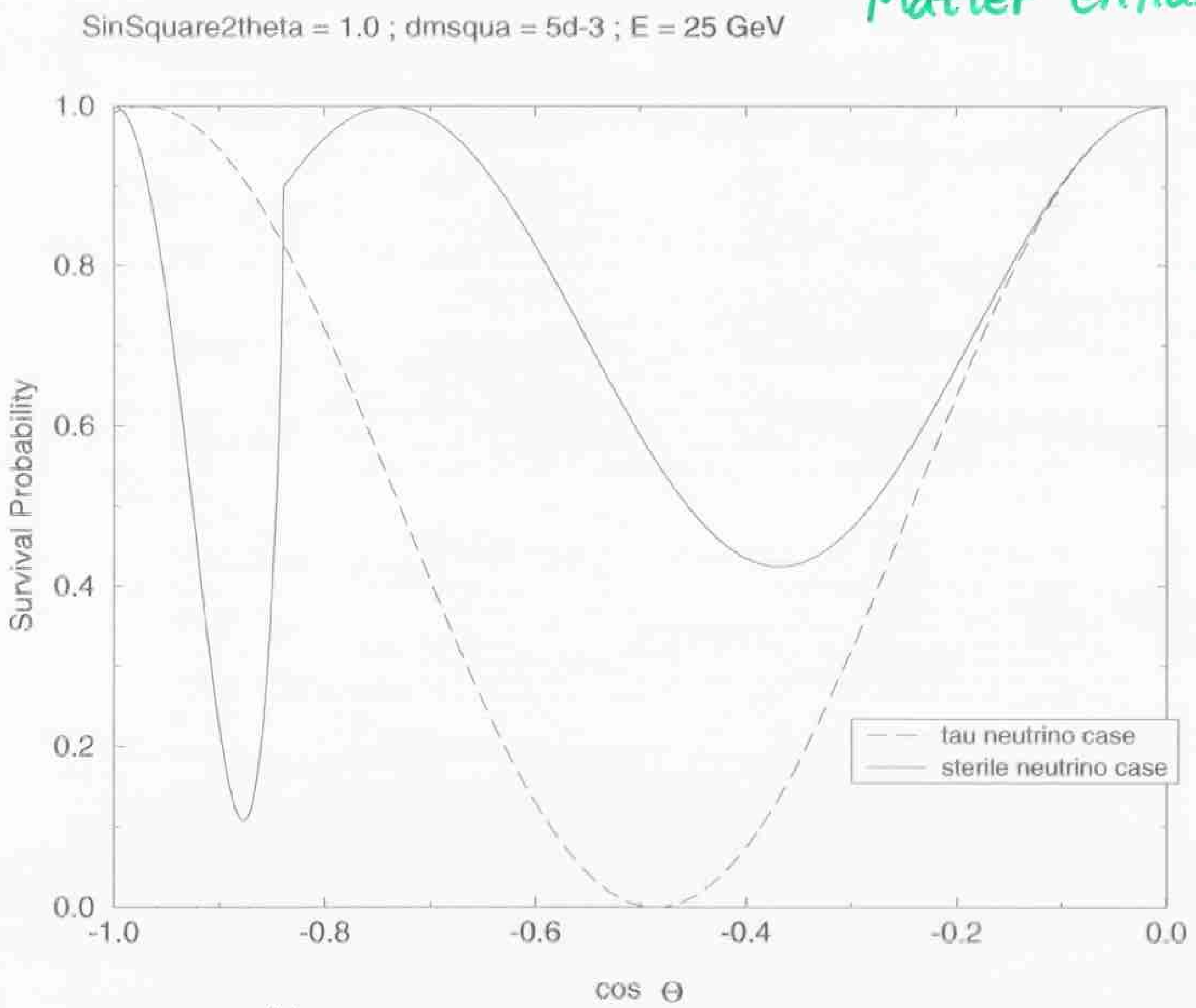
thus, $\sin 2\theta_m \to 0$.

the transition probability

$$P(\nu_\mu \to \nu_x) = \frac{1}{2}\sin^2 2\theta_m (1 - \cos\Phi).$$

※ Except some particular case, like MSW resonance.

Matter enhancement



(b)

How to explain?

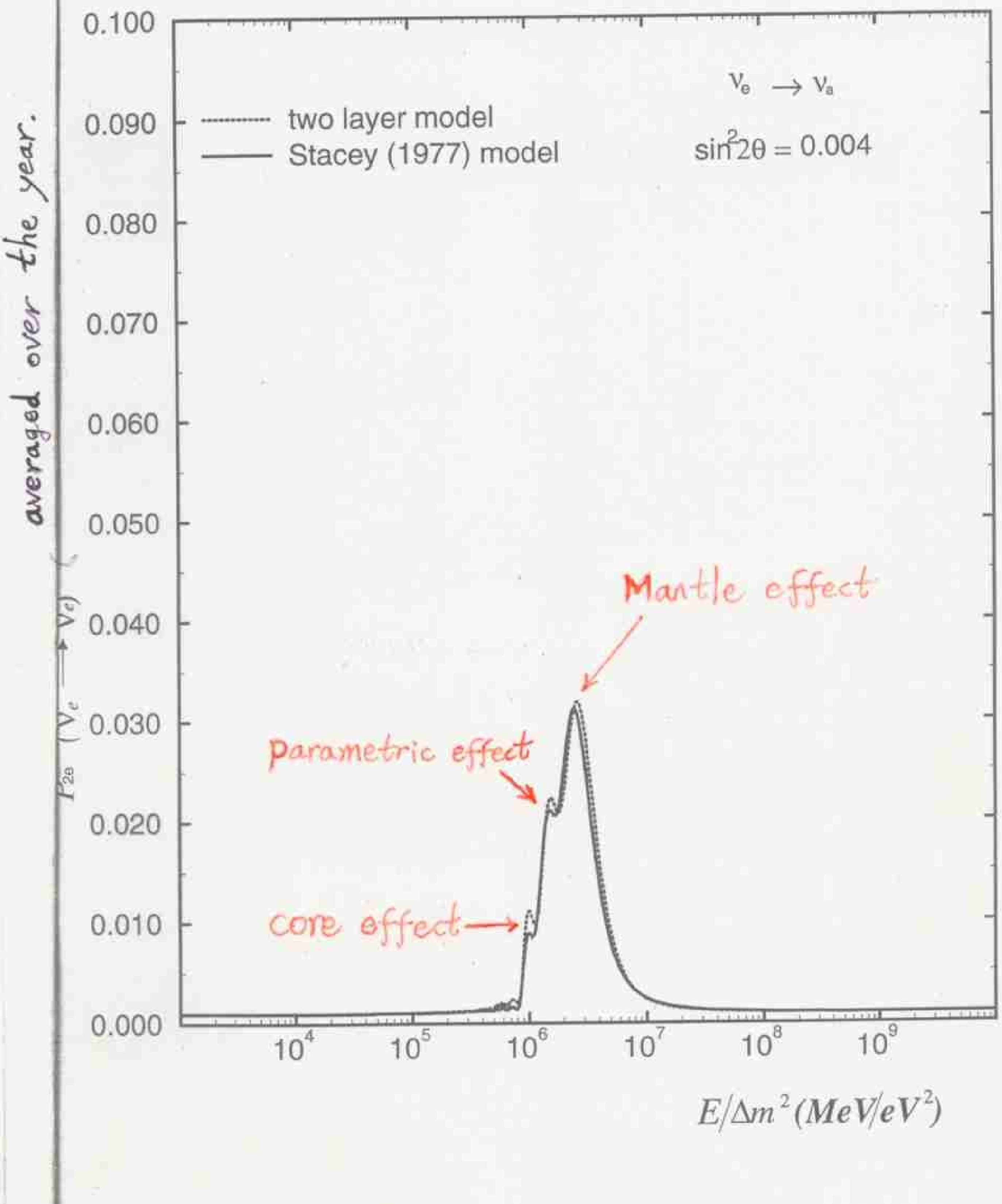
parametric resonan

Liu Smirnov
hep-ph/9712493

Diffractive (D-N

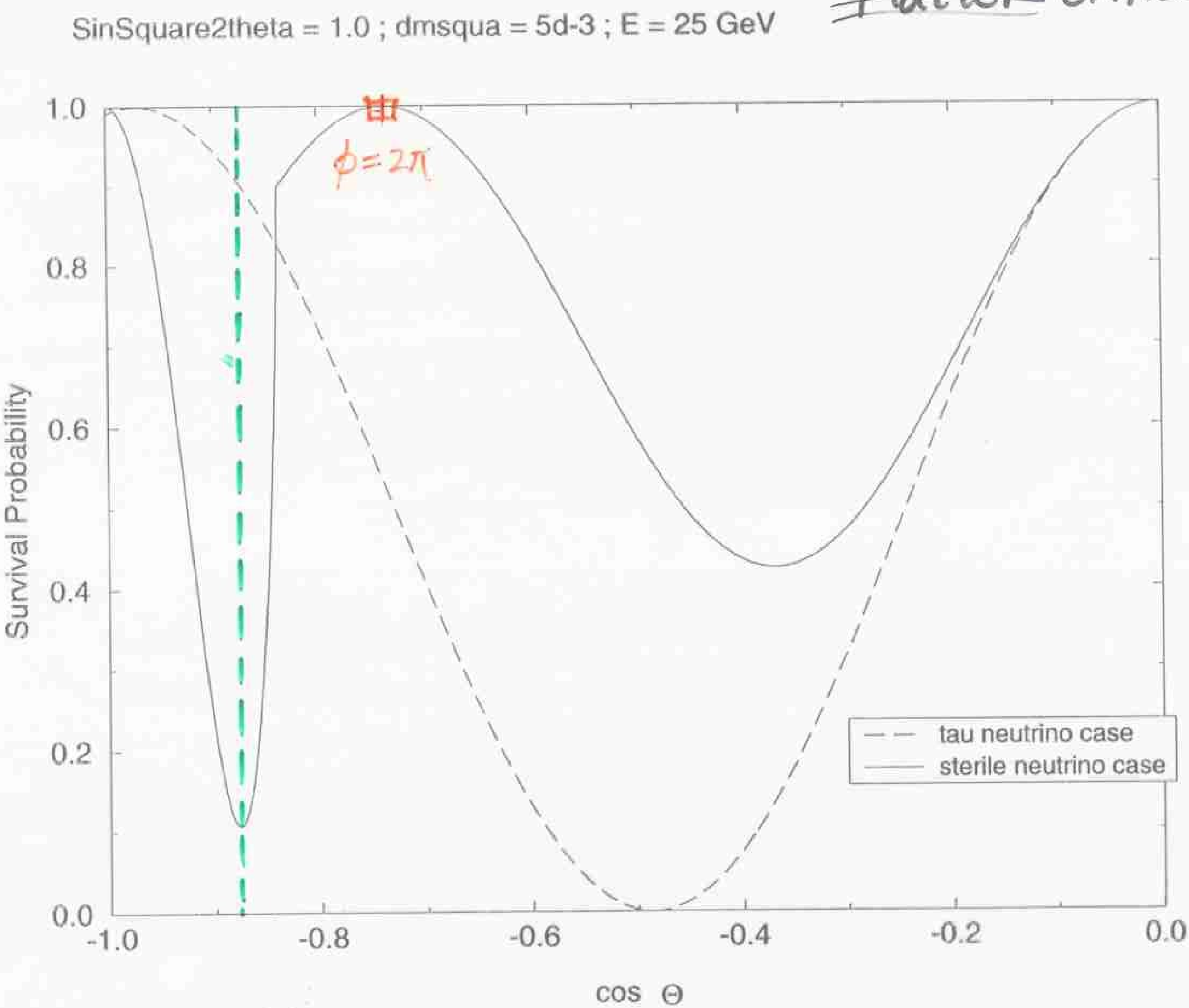
petcov
hep-ph/9805262

Akhmedov
hep-ph/9805272



averaged over the year.
P_{2a} ($\nu_e \longrightarrow \nu_e$)
0.100
0.090
0.080
0.070
0.060
0.050
0.040
0.030
0.020
0.010
0.000
two layer model
Stacey (1977) model
$\nu_e \rightarrow \nu_a$
$\sin^2 2\theta = 0.004$
Mantle effect
parametric effect
core effect
10^4
10^5
10^6
10^7
10^8
10^9
$E/\Delta m^2 (MeV/eV^2)$

~~Matter~~ enhancement



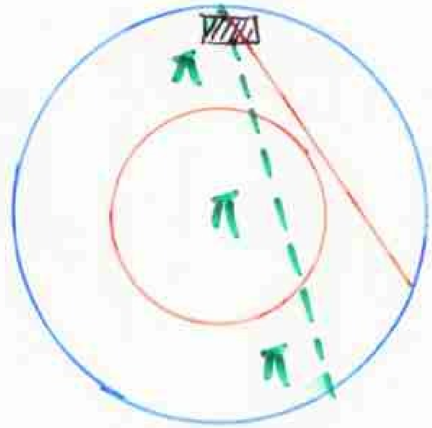
(b)

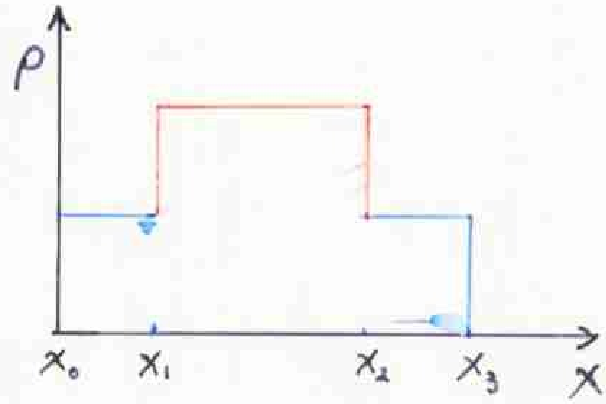
For two layer density model of the earth.

define:

θ_{m_1} : mixing angle in mantle

θ_{m_2} : mixing angle in the core





The survival amplitude is

$$A(\nu_\mu \leftrightarrow \nu_\mu) = 1 + (e^{-i\Delta M_1 2 x_1} - 1)\times$$
$$\times\left[1 + (e^{-i\Delta M_2 x_2} - 1)\cdot\cos^2(\theta_{m_1} - \theta_{m_2})\right]\cdot\sin^2\theta_{m_1} +$$
$$+(e^{-i\Delta M_2} - 1)\sin^2\theta_{m_2} + (e^{-i\Delta M_1 x_1} - 1)(e^{-i\Delta M_2 x_2} - 1)\times(-\tfrac{1}{2})\times$$
$$\times\sin 2\theta_{m_1}\sin 2(\theta_{m_1} - \theta_{m_2}).$$

where $\Delta M_{1(2)} = \frac{\Delta m^2}{2E}\sqrt{\left(1 - \frac{N^{eff}_{1(2)}}{N^{res}}\right)^2\cos^2 2\theta + \sin^2 2\theta}$

$$P(\nu_\mu \to \nu_\mu) = \left|A(\nu_\mu \to \nu_\mu)\right|^2$$

Qualitative treatment:

The oscillation lenth in matter

$$L_m = \frac{1}{\sqrt{\frac{1}{L_\nu^2} + \left(\frac{V}{2\pi}\right)^2}}$$

when $E \to \infty$,

$$L_m \to \frac{2\pi}{V} \qquad \text{indep. of } E$$

for $E_\nu = 30$ GeV (upward going muon)

$$L_m = 0.7 \frac{2\pi}{V} = 0.7\, L_m (E = \infty).$$

So, for E : [30 GeV — ∞]

$$L_m = (0.7 \sim 1) \cdot \frac{2\pi}{V}.$$

it is unsensitive to E. (see fig.)

Our effect is significant at the ν energy :

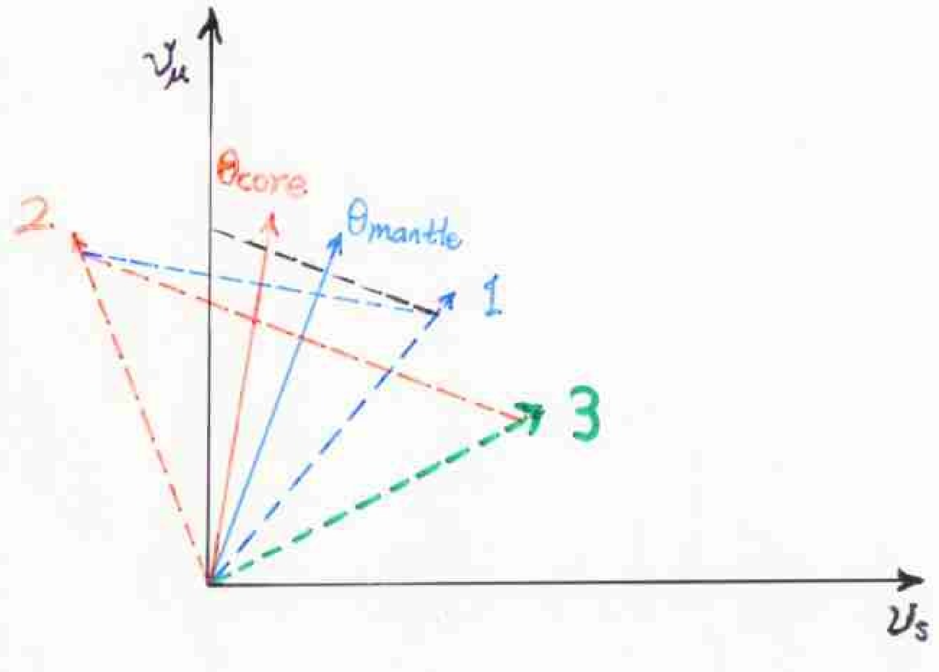
$$20 \text{ GeV} \lesssim E_\nu \lesssim 70 \text{ GeV}$$

The phase is determined by

$$\Phi = 2\pi \int \frac{dx}{L_m} \approx \int V \, dx$$

Maximal transition (resonance) condition is:

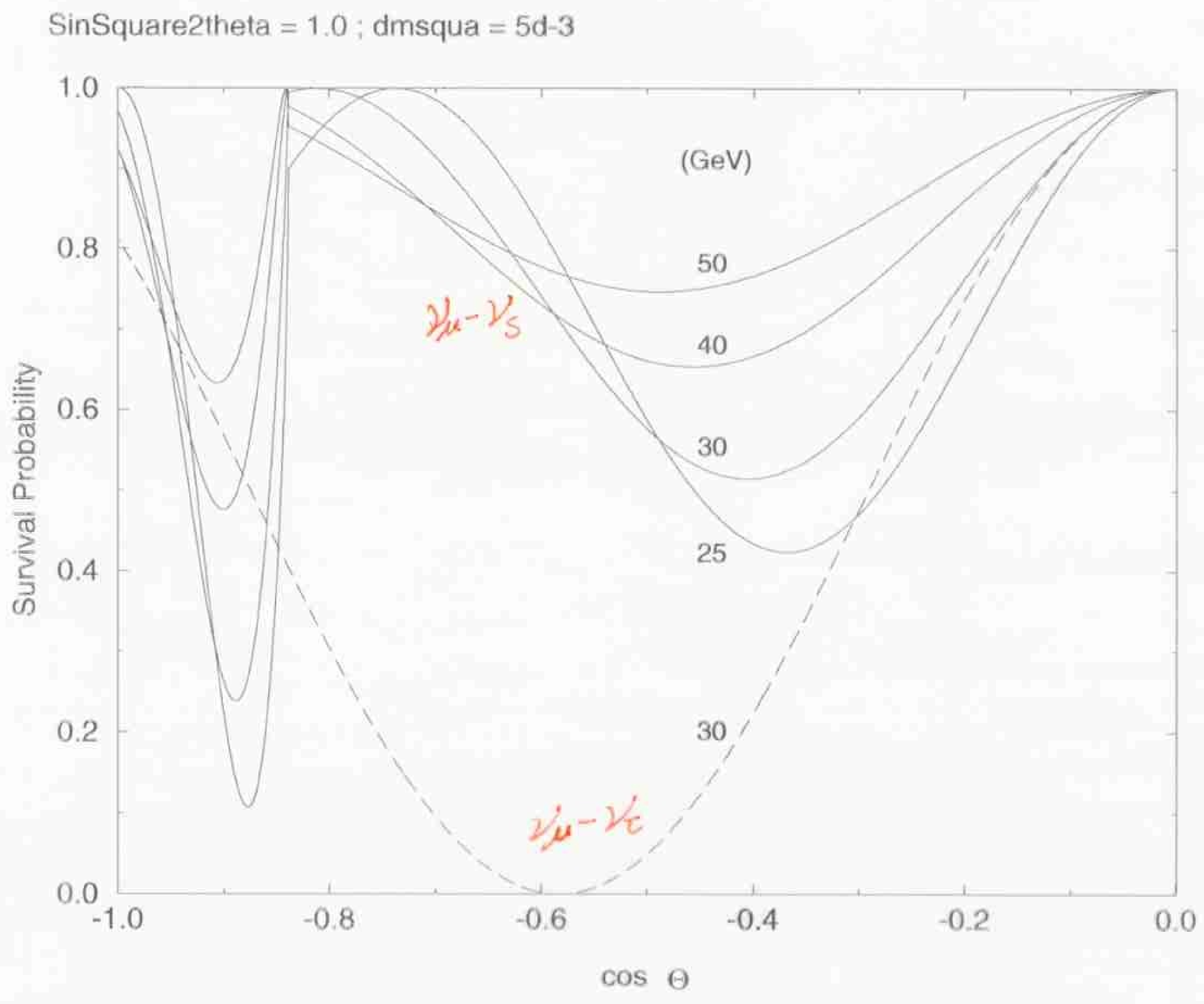
$$\Phi_{mantle1} \approx \Phi_{core} \approx \Phi_{mantle2} \approx \pi.$$



Graphic representation.

$$\theta_3 = 4\,\theta_{man} - 2\,\theta_{core}$$

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2(4\,\theta_{man} - 2\,\theta_{core})$$


SinSquare2theta = 1.0 ; dmsqua = 5d-3
Survival Probability
1.0
0.8
0.6
0.4
0.2
0.0
-1.0
-0.8
-0.6
-0.4
-0.2
0.0
cos Θ
(GeV)
50
40
30
25
30
$\nu_\mu - \nu_s$
$\nu_\mu - \nu_\tau$

parametric enhancement of ν crossing the earth core solve the ν_{atm} - problem via small mixing ?

---

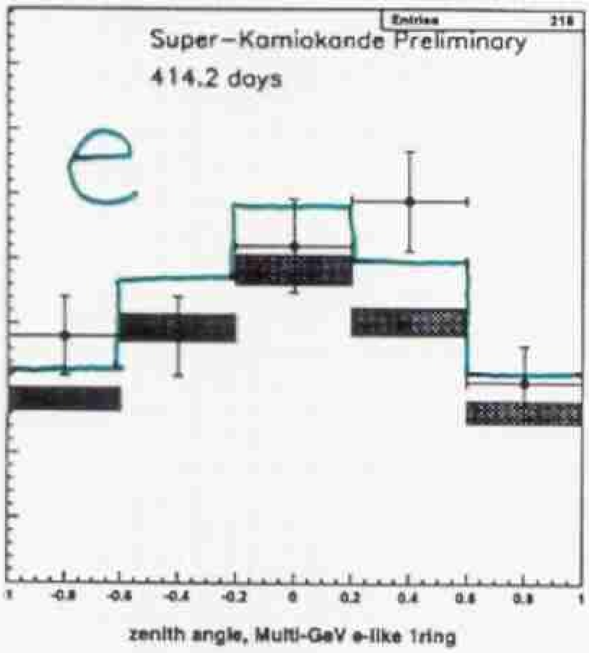
advantage: Chooz doesn't exclude small mixing $\nu_\mu - \nu_e$ channel.

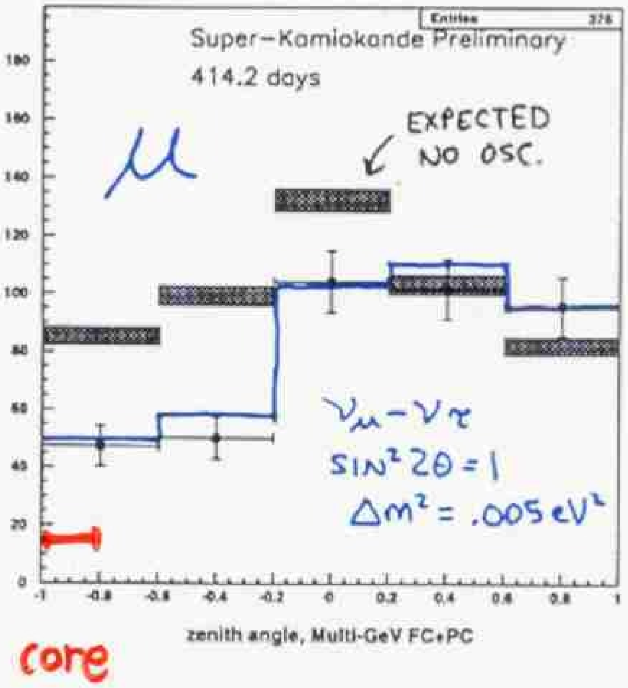
but: $\nu_\mu - \nu_e$ or $\nu_\mu - \nu_s$?

1. SuperK e-like zenith pictures exclude $\nu_\mu - \nu_e$ case

2. deficit of ν_μ at $-0.6 \lesssim \cos\Theta \lesssim 0$ exclude this simple scheme.

"MULTI-GeV" + PARTIALLY CONTAINED
(FULLY CONTAINED)





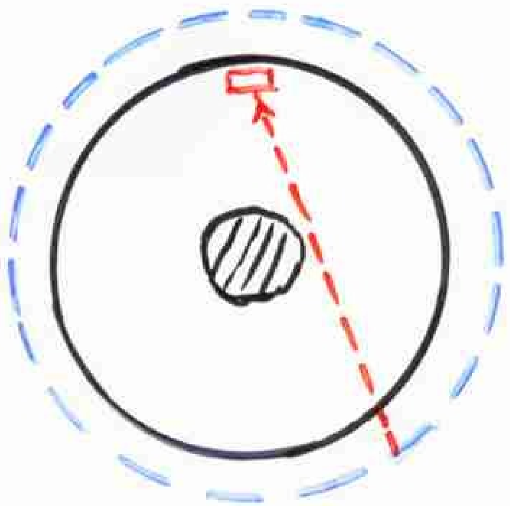
$$P(\nu_\mu - \nu_\mu) = 1 - \frac{1}{2}\sin^2 2\theta_m \cdot (1 - \cos\Delta)$$

$$\approx 1.$$

parametric enhancement via earth mantle?

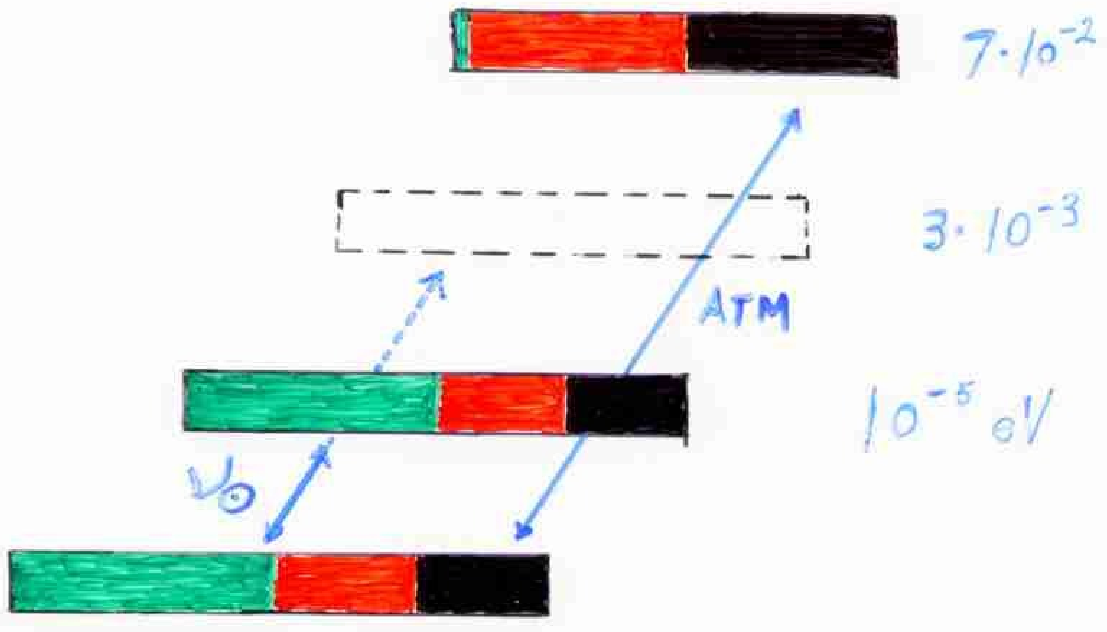
$\phi_1 \gtrsim \pi$, neutrino phase in the air (≥ 10 km)

$\phi_2 \approx (2k+1)\pi$, phase in the mantle.



?

If accept the spectrum of SK.



or degenerate masses.

- spectrum distortion.

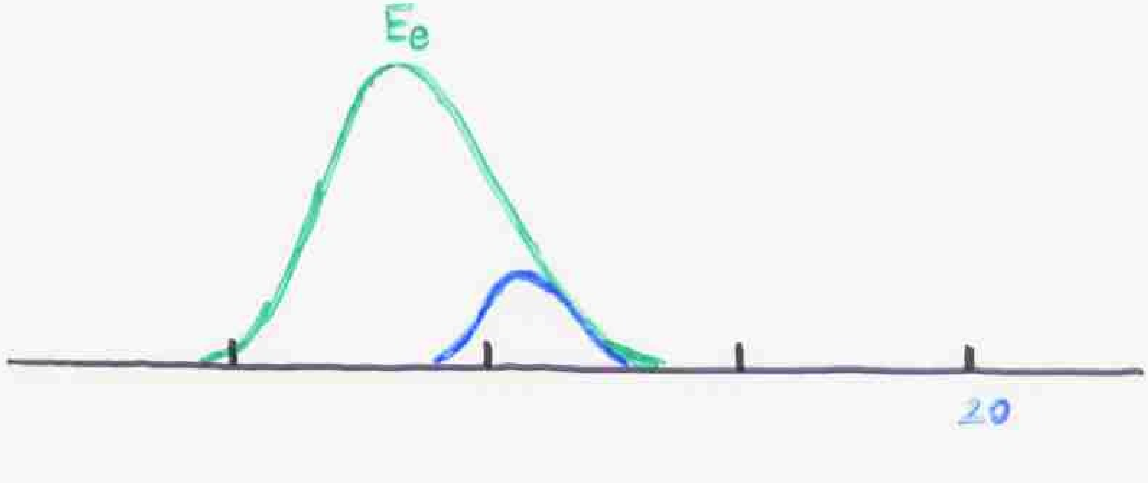
$$S(E_{vis}) = \int dE_e \cdot f(E_{vis}, E_e) \cdot T(E_e) \cdot W(E_{vis})$$

$$bin(i) = \int_{E_i}^{E_i + \Delta E} S(E_{vis})\, dE_{vis}.$$

Babu, Smirnov, Q.Y.L.

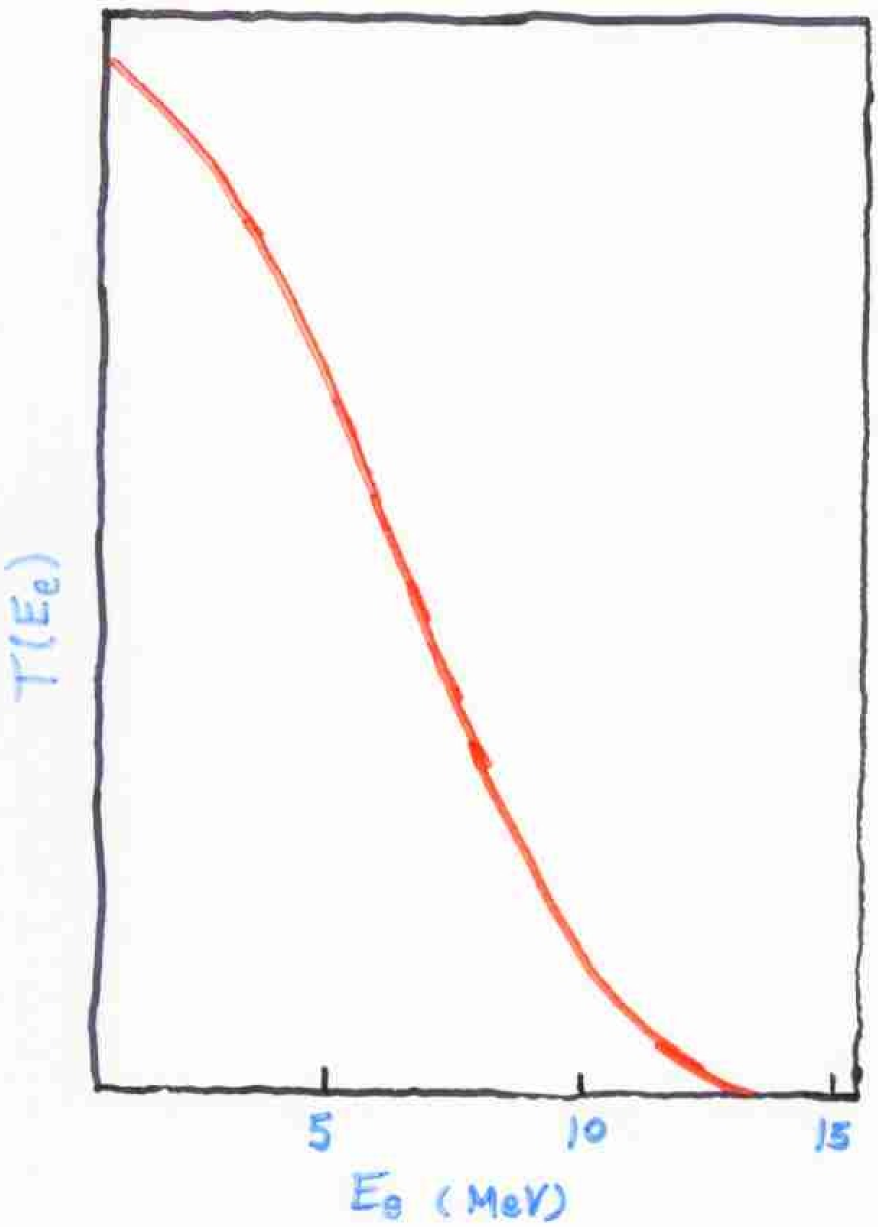
$$f(E_{vis}, E_e) \doteq \frac{1}{\sqrt{2\pi}\, E_e\, \sigma(E_e)} \cdot \exp\left[-\left(\frac{E_{vis} - E_e}{\sqrt{2}\, E_e\, \sigma(E_e)}\right)^2\right]$$

or M. C.



qualitative

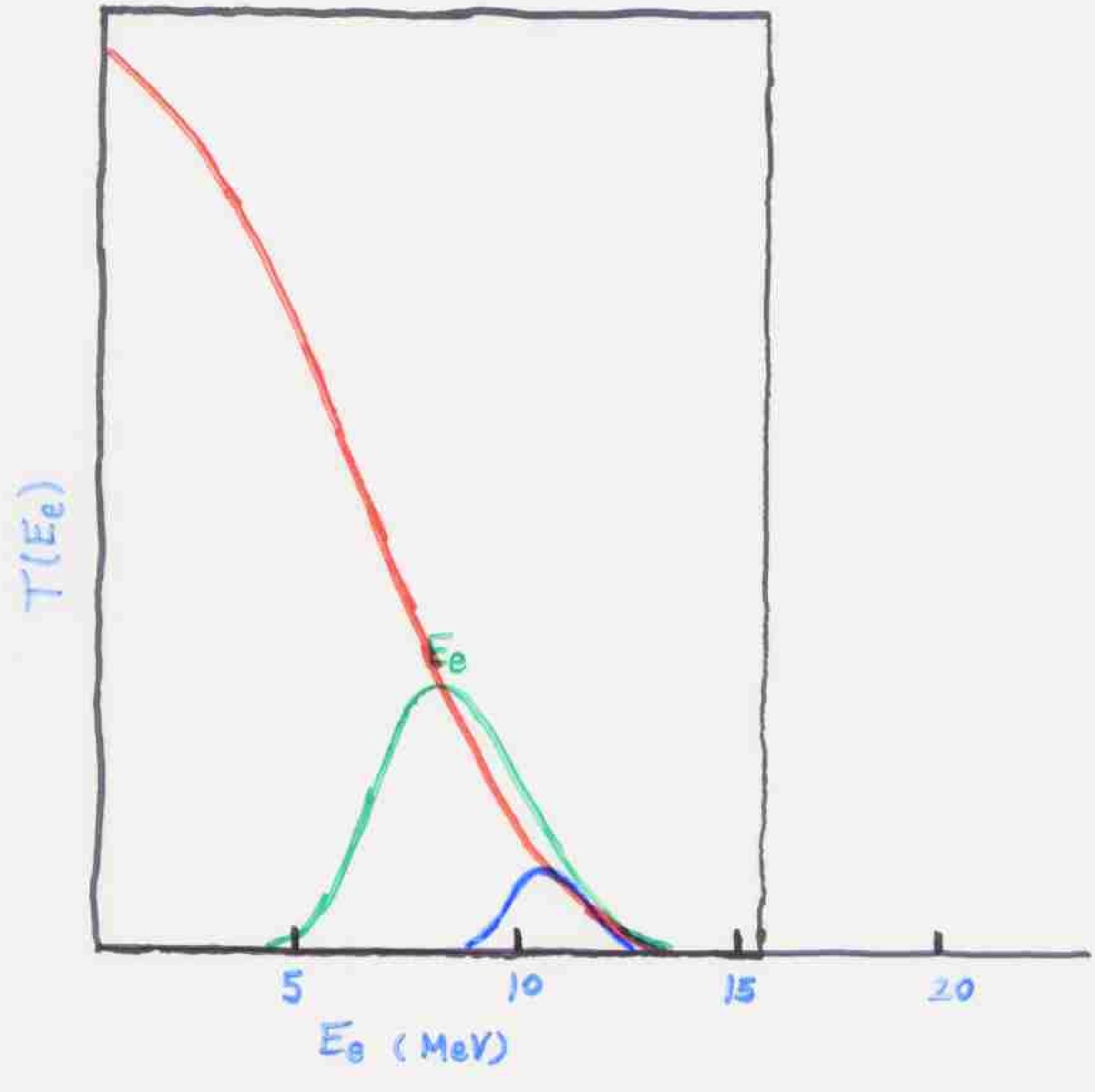
$$T(E_e) = \int_{E_e - \frac{m_e}{2}} dE_\nu \cdot \Phi(E_\nu) \cdot \left[P(E_\nu) \cdot \frac{d\sigma_{\nu_e}}{dE_e}(E_e, E_\nu) + (1 - P(E_\nu)) \frac{d\sigma_{\nu_\mu}}{dE_e}(E_e, E_\nu) \right]$$



$$T(E_e) = \int_{E_e - \frac{m_e}{2}} dE_\nu \cdot \Phi(E_\nu) \cdot \left[P(E_\nu) \cdot \frac{d\sigma_{\nu e}}{dE_e}(E_e, E_\nu) + (1 - P(E_\nu)) \frac{d\sigma_{\nu \mu}}{dE_e}(E_e, E_\nu) \right]$$

or M.C.

$$f(E_{vis}, E_e) \doteq \frac{1}{\sqrt{2\pi}\, E_e\, \sigma(E_e)} \cdot \exp\left[- \left(\frac{E_{vis} - E_e}{\sqrt{2}\, E_e\, \sigma(E_e)} \right)^2 \right]$$



qualitative

- The last 3 bins are from tails of $f(E,E)$.

  More than 3σ tails?

- Different energy resolutions due to different positions, effect tails?

Conclusion

- Is the strong deformation of SK solar neutrino spectrum due to the tail of Energy resolution function ?

- Neutrino cross the core of the Earth may give an important feature of the Oscillation. (parametric resonance / or other names ?)

- $\nu_\mu - \nu_s$ oscillations is still a good candidate of atmospheric ν oscillations.

- Some mass spectra are still possible.

- 2ν parametric resonance via earth core is excluded.