NEW ENHANCEMENT MECHANISM OF THE EARTH EFFECT IN THE TRANSITIONS OF SOLAR NEUTRINOS CROSSING THE EARTH CORE

S.T. PETCOV

SISSA/INFN - SEZIONE DI TRIESTE, TRIESTE, ITALY, AND
BULGARIAN ACADEMY OF SCIENCES, SOFIA, BULGARIA

"NEUTRINO '98"
4 - 9 JUNE, 1998
TAKAYAMA

"NEW ERA..."
11 - 12 JUNE, 1998
TMU

"New Type" of RESONANCE IN $P_{e2} \equiv P(\nu_2 \to \nu_e)$:

(P_{e2}: accounts for the Earth Effect in $P_\odot(\nu_e \to \nu_e)$ in the case of 2-ν mixing MSW solution of the $\nu_\odot$-problem)

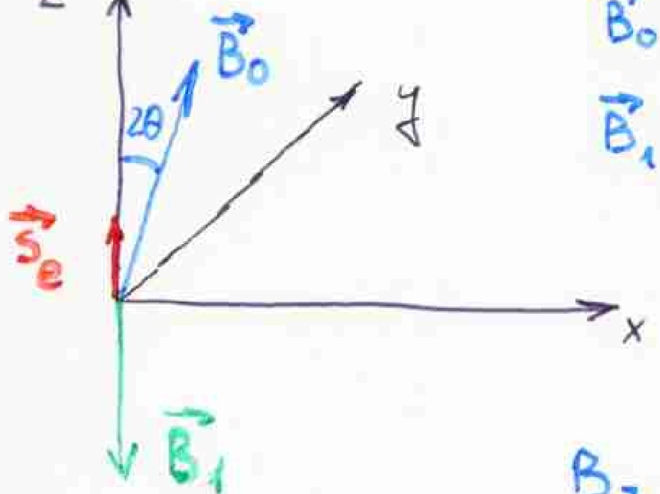
- Not the MSW resonance
- Takes place when

  L_{osc}^{core}, L_{osc}^{man} obey certain constraints

  hence, it is a

  "neutrino oscillation length resonance"
- Similar to the

  electron paramagnetic resonance



$\vec{B}_0$: $B_{0y} = 0$, const;

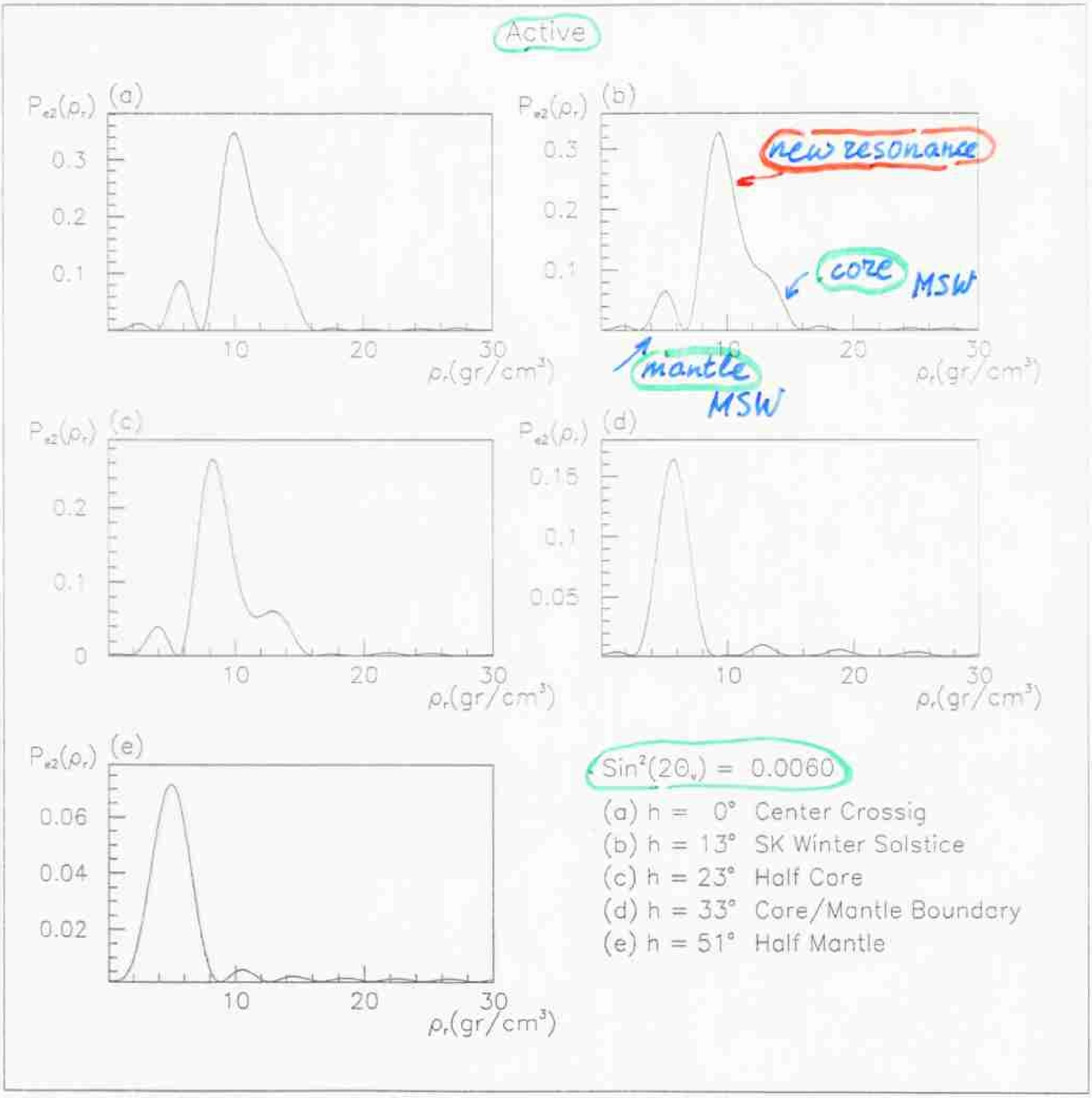
$\vec{B}_1$: $B_{1x} = B_{1y} = 0$, assumes (2) values; changes step-wise

$$B_z = \begin{cases} B_0 \cos 2\theta - B_1 > 0, & t < t_1, \; t_2 \le t \le t_3 \\ B_0 \cos 2\theta - B_1 < 0, & t_1 \le t < t_2 \end{cases}$$

(const.)

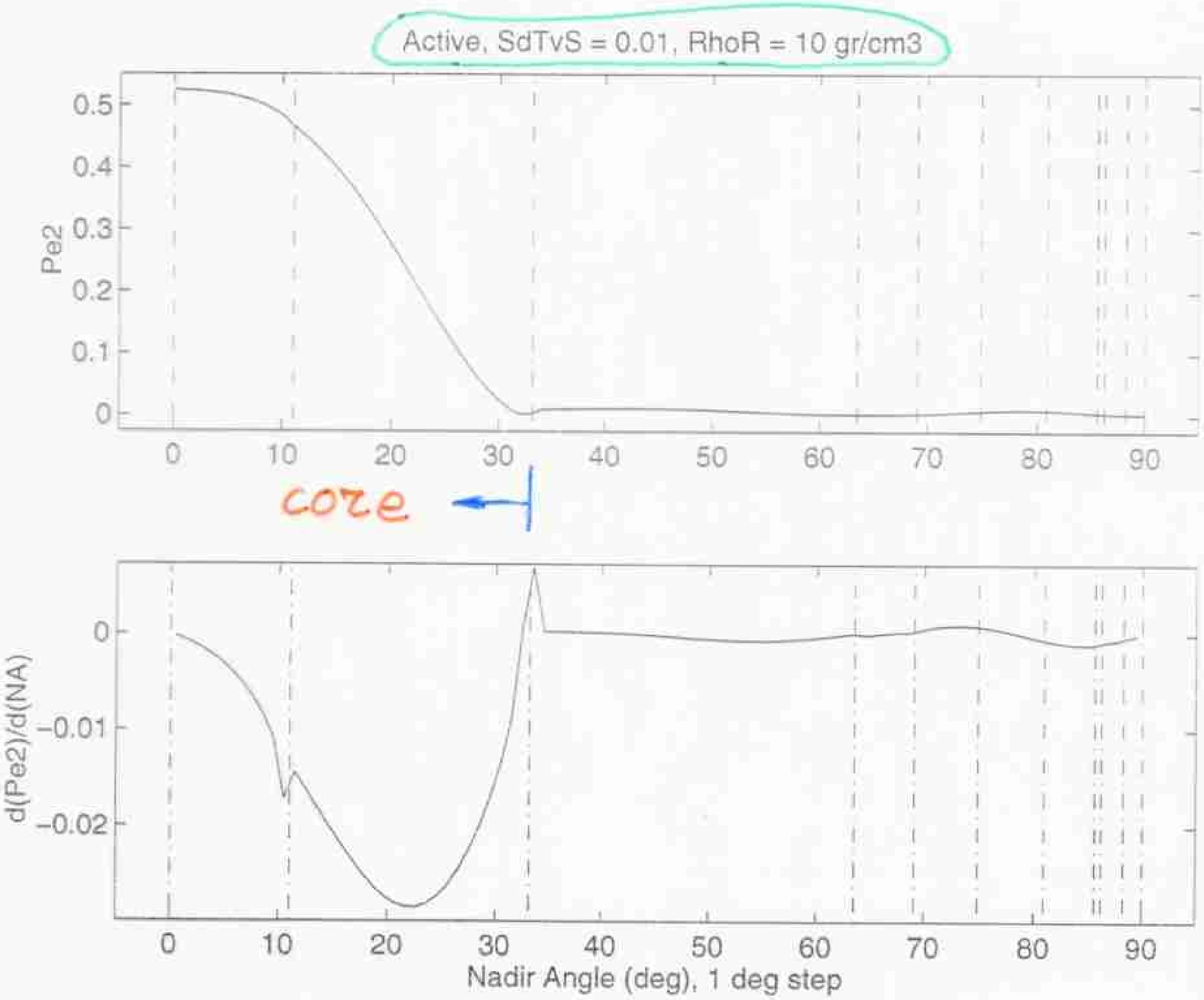
(MARIS, S.T.P., ICHIJOV, '98)

$$E/\Delta m^2 = \frac{6.56 \times 10^6}{0.5\, \rho_2 [g/cm^3]} \cos 2\Theta \quad MeV/eV^2$$



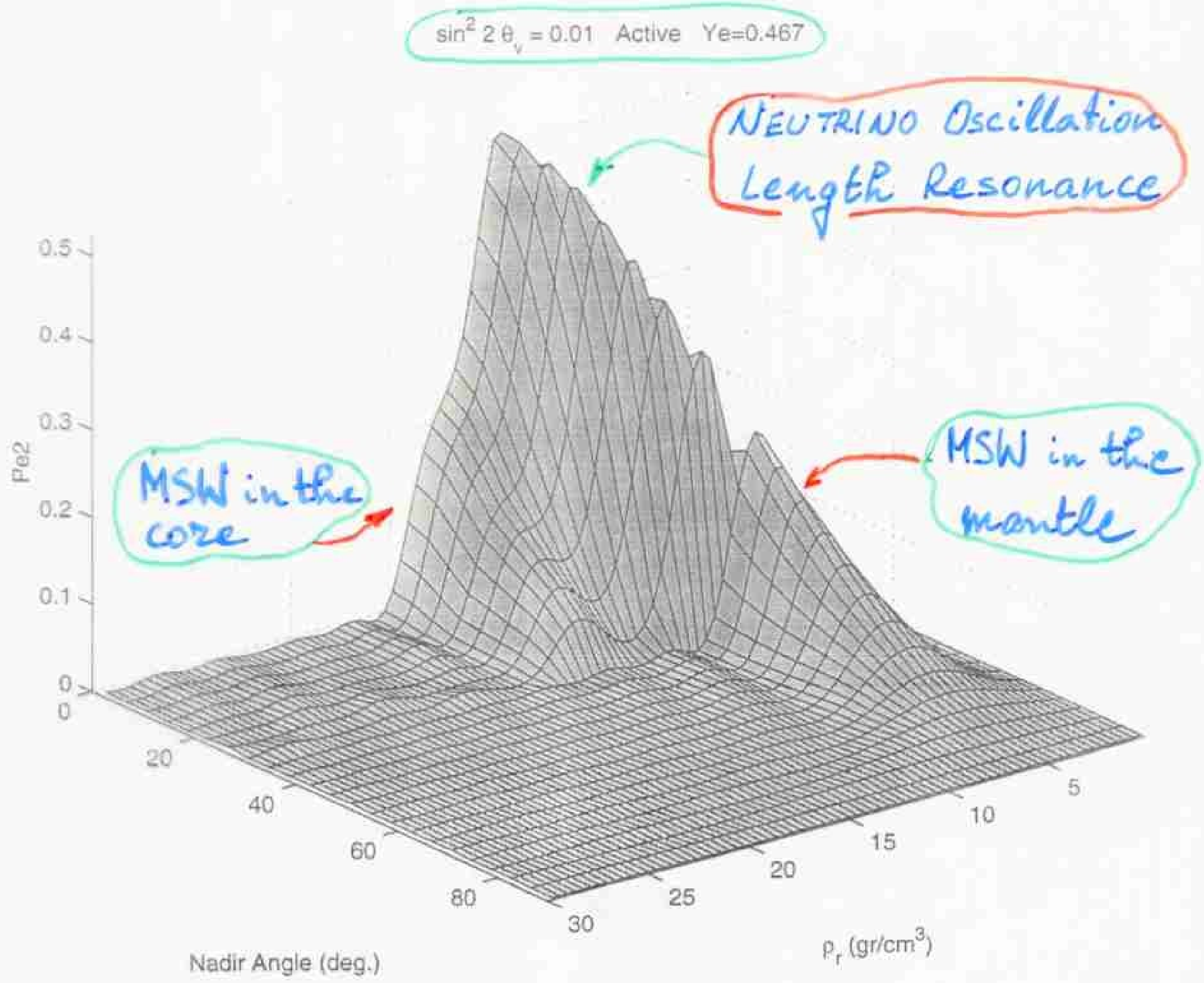
(M. MARIS, S.T.P., M.TCHIJOV '98)

$\nu_e \rightarrow \nu_{\mu(\tau)}$



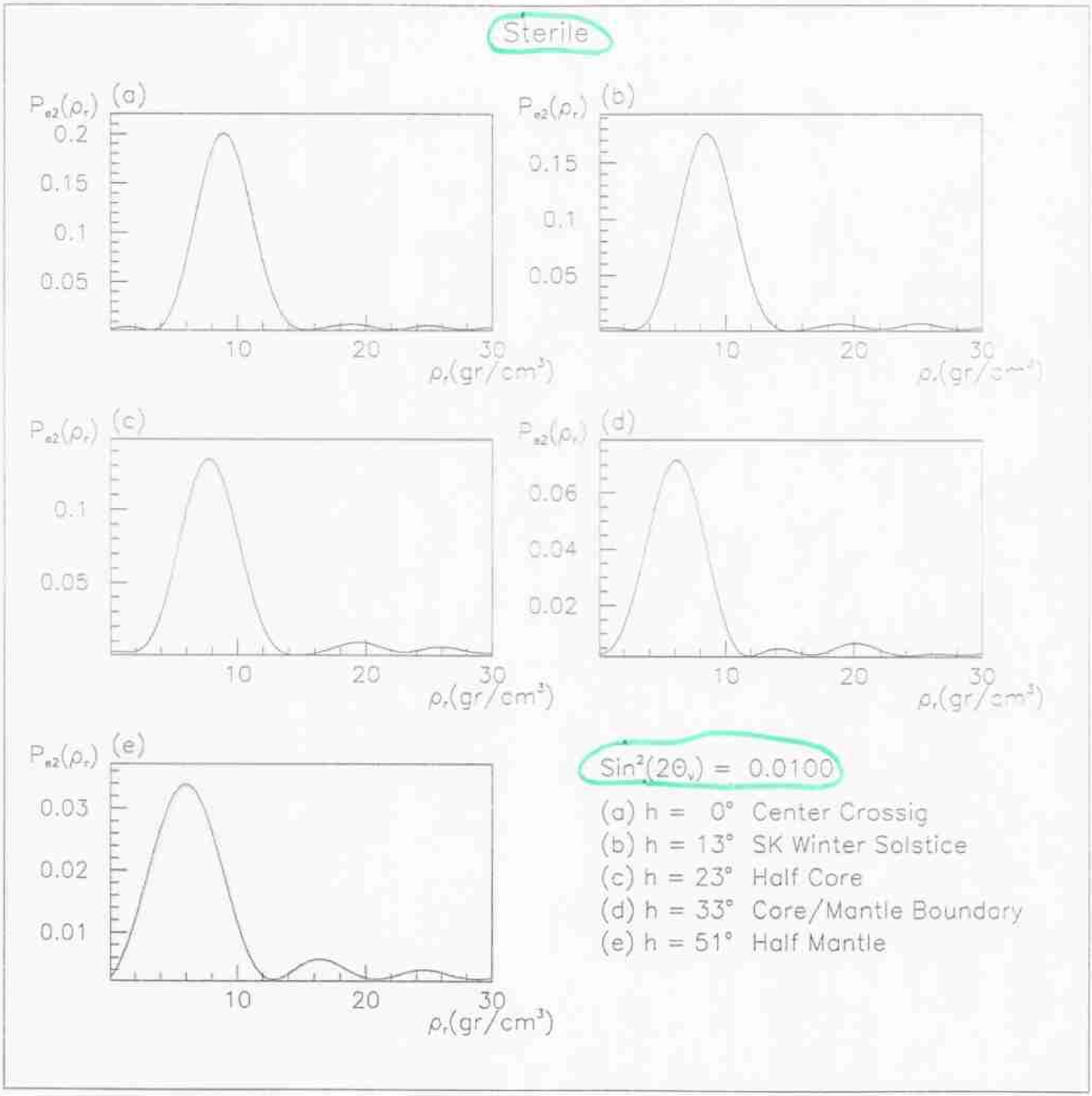
(M. MARIS, S.T.P., M. TCHIJOV, '98)

$\nu_e \rightarrow \nu_{\mu(\tau)}$



(M. MARIS, S.V.P. '98)

$$E/\Delta m^2 = \frac{6.56 \times 10^6}{0.25\ \rho_Z [g/cm^3]}\ MeV/eV^2$$



The neutrino oscillation length resonance in P_{e2}

- exhibits strong dependence on E
- for Δm^2 from the SMA solution region takes place for $E \cong (5-12)$ MeV
- leads in the case of the SMA $\nu_e \to \nu_{\mu(\tau)}$ solution to a ~6 times bigger D-N asymmetry in the "Core" sample of event in the SK detector than the asymmetry in the whole "Night" sample
- is sufficiently wide (it is wider than the MSW resonance)

The resonance takes place in the $\nu_\mu \to \nu_e$ ($\nu_e \to \nu_\mu$) transitions of atmospheric neutrinos as well:

e.g. for $\Delta m^2 \sim 10^{-3}\ eV^2$, $\sin^2 2\theta_{e\mu} \sim (0.01-0.10)$, $h = 0°$

max $P(\nu_\mu \to \nu_e)$ occurs at $E_\nu \cong 1.6$ GeV

(M. MARAS)

$\rho_c \cong (10 - 13)\, g/cm^3$ over a distance of $R_c = 3486$ km

$\rho_m \cong (3.3 - 5.5)\, g/cm^3$ over a distance of ~ 2885 km

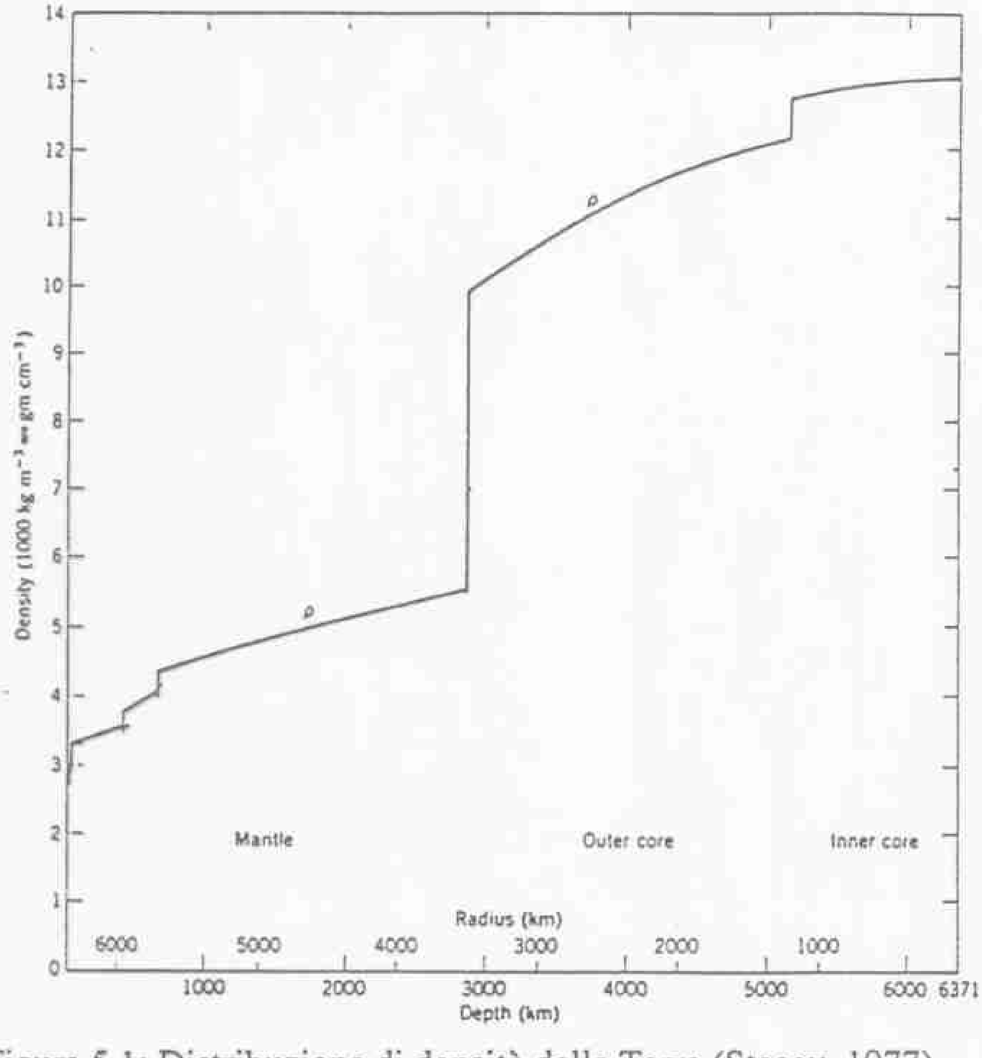


Figura 5.1: Distribuzione di densità della Terra (Stacey, 1977).

The density distribution in the Earth has basically a two-layer structure:

CORE: $\rho_c = \text{const.} = \bar{\rho}_c \cong 11.5\ g/cm^3$

$Y_e^c = \text{const.} \simeq 0.467$

MANTLE: $\rho_m = \text{const.} = \bar{\rho}_m \simeq 4.5\ g/cm^3$

$Y_e^m = \text{const.} = 0.49$

P_{e2} calculated using the two-layer approximation reproduces with high precision (few %) (M. MARIS, S.T.P.; MARIS, LIU, S.T.P., '96)

P_{e2}^{exact}

P_{e2} depends on $E/\Delta m^2, \Theta$ (ρ^{res})

X', θ_m', L_{osc}^m; $X' = R_\oplus \cos h - \sqrt{R_c^2 - R_\oplus^2 \sin^2 h}$

$X'', \theta_m'', L_{osc}^c$: $X'' = 2\sqrt{R_c^2 - R_\oplus^2 \sin^2 h}$

$\left(\Delta E' = \frac{2\pi}{L_{osc}^m} \ ; \ \Delta E'' = \frac{2\pi}{L_{osc}^c} \right)$

$\nu_2 \to \nu_e$: due to ν-oscillations

i) over a distance X' in the mantle with θ_m', L_{osc}^m

ii) over X'' in the core with θ_m'', L_{osc}^c

iii) over X' in the mantle with θ_m', L_{osc}^m

$\nu_2 \to \nu_e$: due to ν-oscillations taking place

1. in the mantle over a distance X' with θ_m', L_{osc}^m.
2. in the core over a distance X'' with θ_m'', L_{osc}^c.
3. in the mantle over a distance X' with θ_m', L_{osc}^c

$$
\begin{aligned}
P_{e2} = \sin^2\theta &+ \frac{1}{2}\left[1-\cos\Delta E''X''\right]\left[\sin^2(2\theta_m''-\theta)-\sin^2\theta\right] \\
&+ \frac{1}{4}\left[1-\cos\Delta E''X''\right]\left[1-\cos\Delta E'X'\right]\left[\sin^2(2\theta_m''-4\theta_m'+\theta) - \sin^2(2\theta_m''-\theta)\right] \\
&- \frac{1}{4}\left[1-\cos\Delta E''X''\right]\left[1-\cos 2\Delta E'X'\right]\left[\sin^2(2\theta_m'-\theta)-\sin^2\theta\right] \times \cos^2(2\theta_m''-2\theta_m') \\
&+ \frac{1}{4}\left[1+\cos\Delta E''X''\right]\left[1-\cos 2\Delta E'X'\right]\left[\sin^2(2\theta_m'-\theta)-\sin^2\theta\right] \\
&+ \frac{1}{2}\sin\Delta E''X''\sin 2\Delta E'X'\left[\sin^2(2\theta_m'-\theta)-\sin^2\theta\right] \times \cos(2\theta_m''-2\theta_m') \\
&+ \frac{1}{4}\left[\cos(\Delta E'X'-\Delta E''X'')-\cos(\Delta E'X'+\Delta E''X'')\right] \times \sin(4\theta_m'-2\theta)\sin(2\theta_m''-2\theta_m')
\end{aligned}
$$

$$\Delta E'X' = 2\pi\frac{X'}{L_{osc}^{man}}, \quad \Delta E''X'' = 2\pi\frac{X''}{L_{osc}^{c}}$$

P_{e2} : two possible maxima corresponding to the MSW resonances in the mantle and in the core

+ a new maximum is present provided

$$\Delta E' X' = \pi(2k+1), \quad \Delta E'' X'' = \pi(2k'+1), \quad k, k' = 0, 1, 2, ...$$

$$\sin^2(2\theta''_m - 4\theta'_m + \theta) - \sin^2\theta > 0,$$

$$\sin(4\theta'_m - 8\theta''_m + 2\theta)\left[\frac{1}{2}\sin(2\theta'_m - 4\theta'_m + \theta)\sin(2\theta''_m - 2\theta'_m) \times \sin(2\theta'_m - \theta) + \sin^2\theta \sin(4\theta''_m - 4\theta'_m)\right] < 0$$

$$\max P_{e2} = \sin^2(2\theta''_m - 4\theta'_m + \theta)$$

In the region of the maximum, when $\Delta E' X' = \pi(2k+1)$,

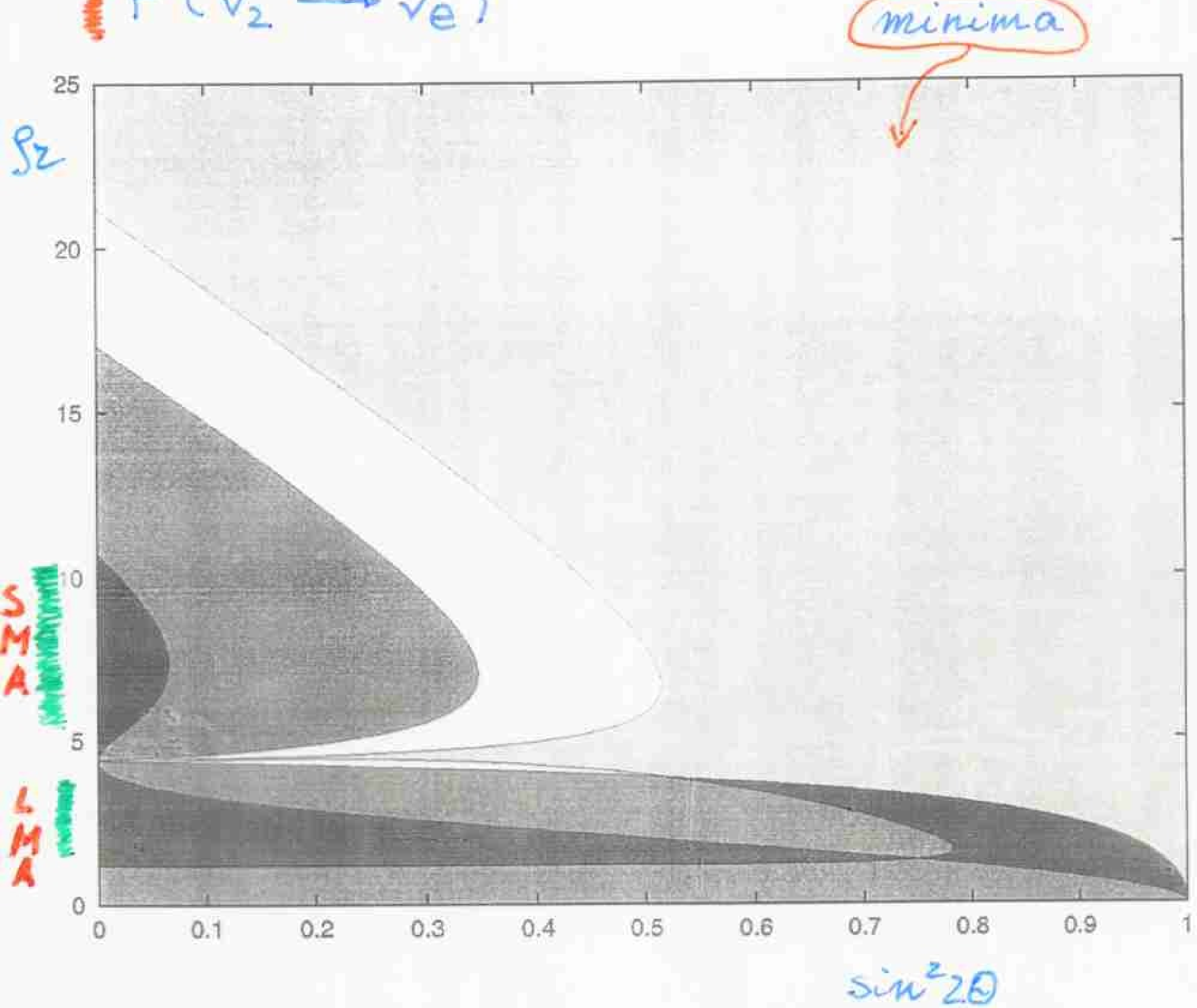
$$P_{e2} = \sin^2\theta + \frac{1}{2}\left[1 - \cos\Delta E'' X''\right]\left[\sin^2(2\theta''_m - 4\theta'_m + \theta) - \sin^2\theta\right]$$

Small mixing:

$$\pi\left\{\frac{1}{X'} + \frac{1}{X''}\right\} \cong \sqrt{2} G_F \left(\bar{\rho}_c Y_e^c - \bar{\rho}_m Y_e^m\right)$$

(M. MARIS, S.T.P., M. TCHIJOV, '98)

$P(\nu_2 \to \nu_e)$



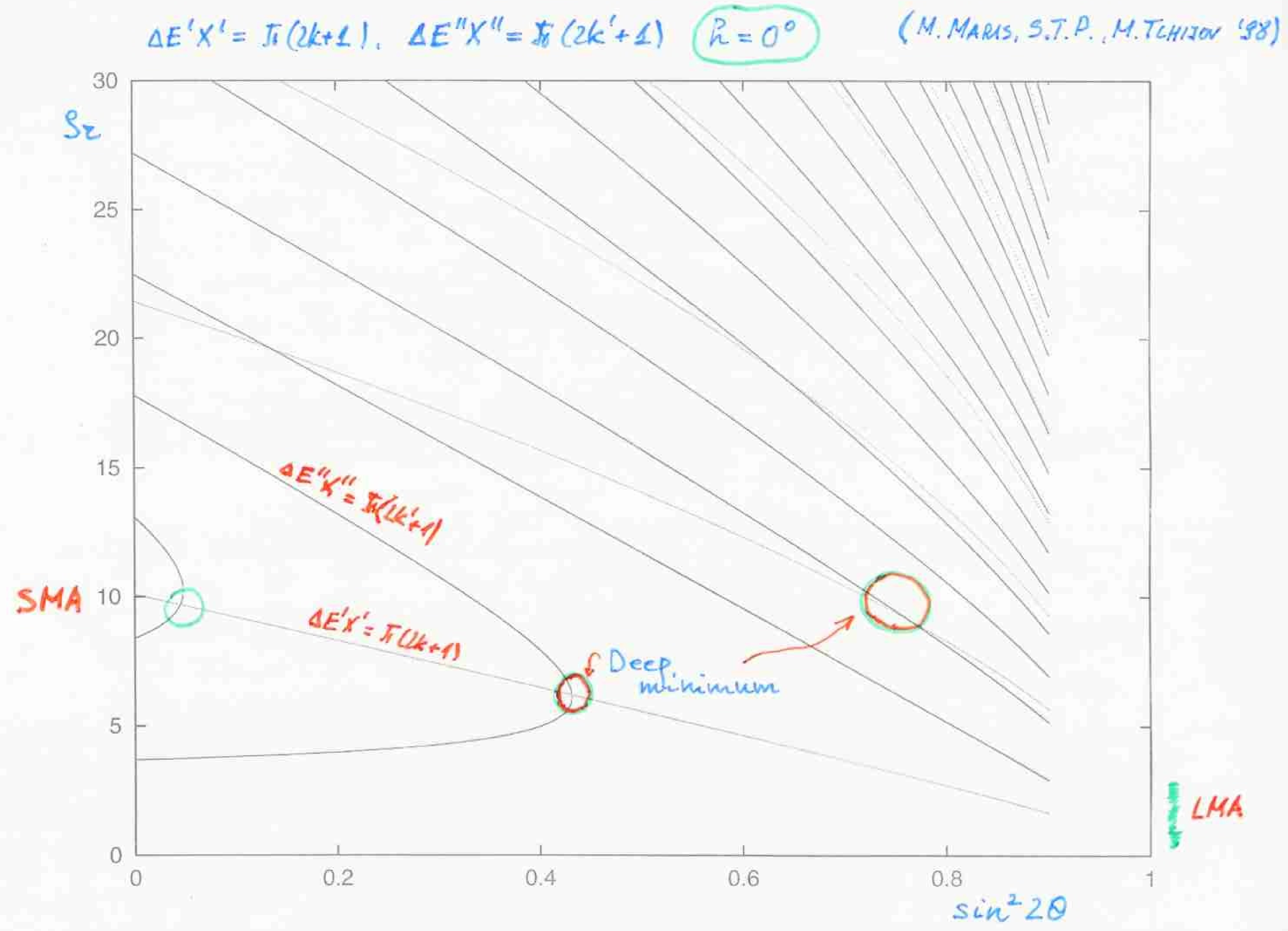
The supplementary ("maximum") conditions are satisfied for

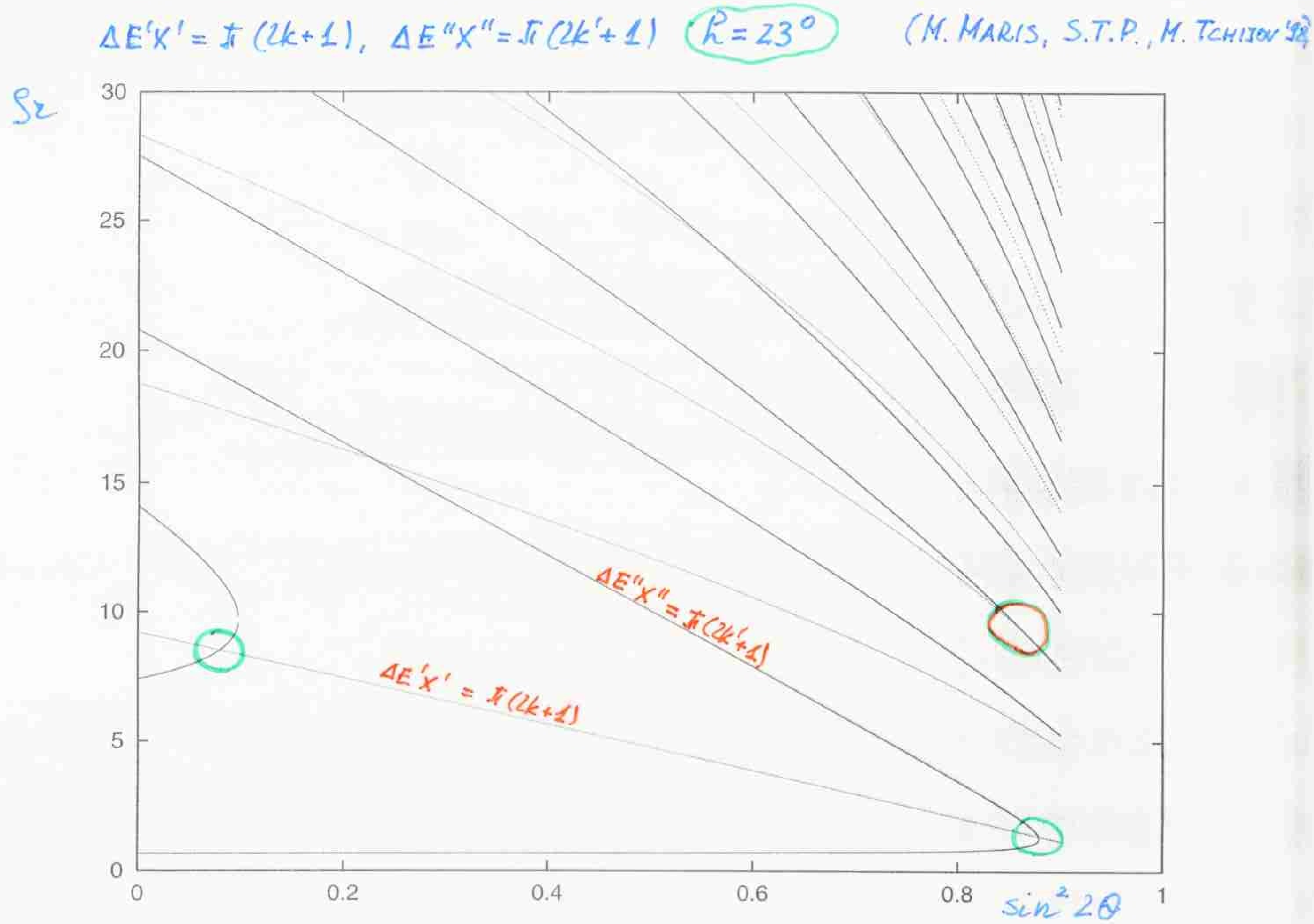
$$\bar{\rho}_m < \rho^{res} < \bar{\rho}_c \qquad \text{SMA}$$

$$1\ g/cm^3 < \rho^{res} < \bar{\rho}_m \qquad \text{LMA}$$

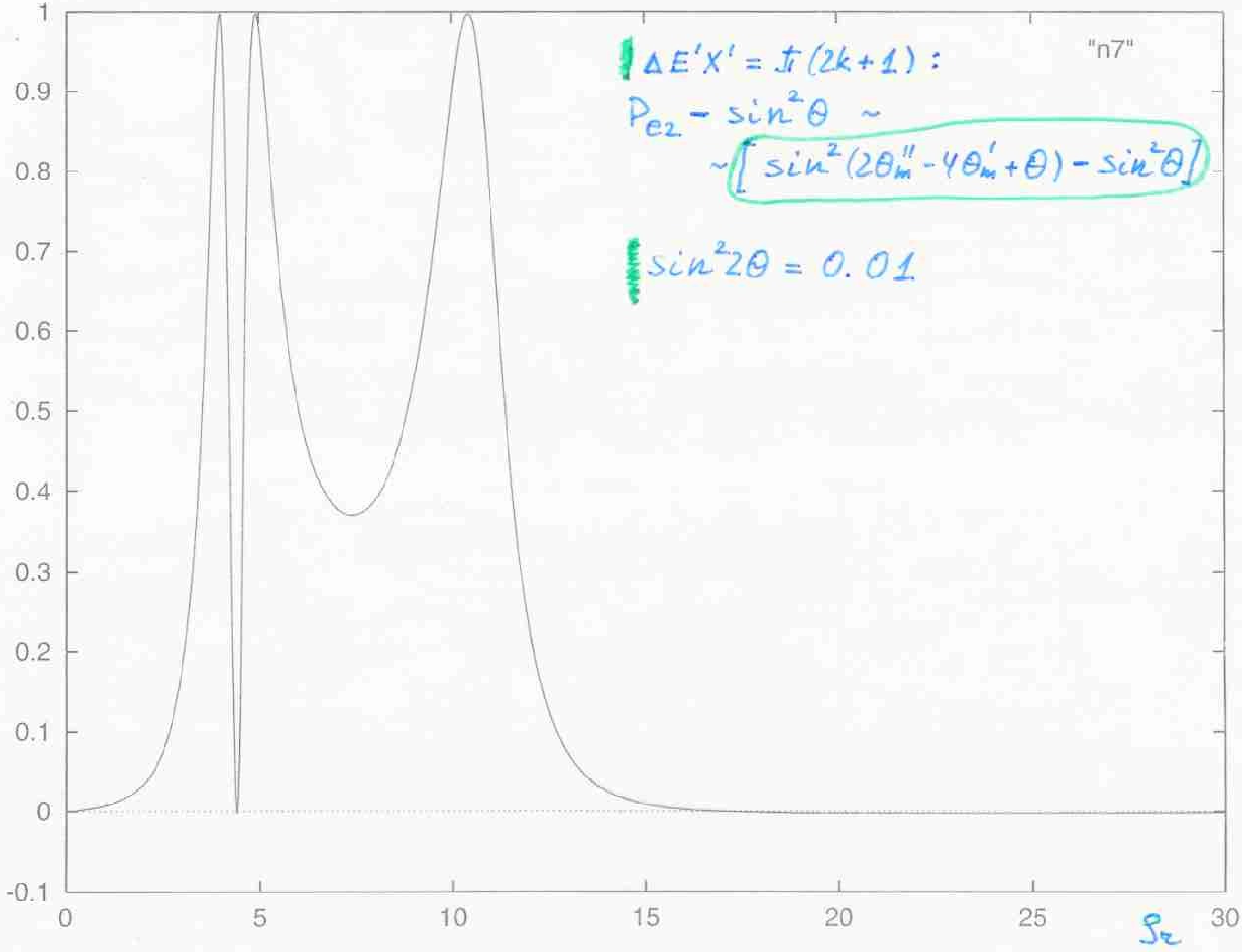
LMA: $\Delta m^2 \gtrsim 10^{-5}\ eV^2 \longrightarrow$

$E \gtrsim 33\ MeV$

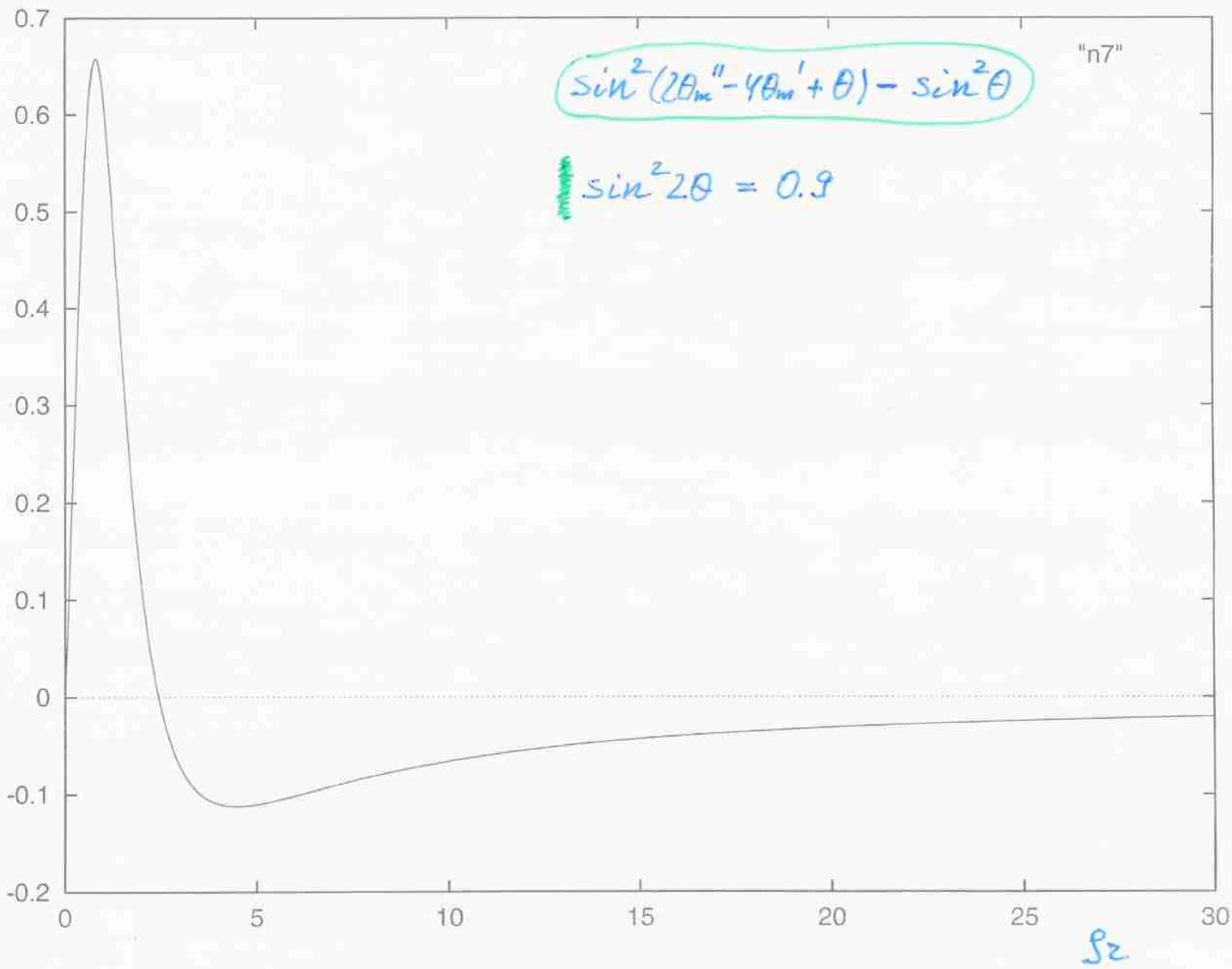

ΔE'X' = π(2k+1), ΔE''X'' = π(2k'+1)
h = 0°
(M. Maris, S.T.P., M.Tchizov '98)
ρ_r
SMA
ΔE''X'' = π(2k'+1)
ΔE'X' = π(2k+1)
Deep minimum
LMA
sin²2θ
30
25
20
15
10
5
0
0.2
0.4
0.6
0.8
1


ΔE'X' = π (2k+1), ΔE"X" = π (2k'+1) h = 23°   (M. MARIS, S.T.P., M. TCHIJOV '98)
30
25
20
15
10
5
0
0.2
0.4
0.6
0.8
1
ΔE"X" = π(2k'+1)
ΔE'X' = π(2k+1)
sin² 2θ

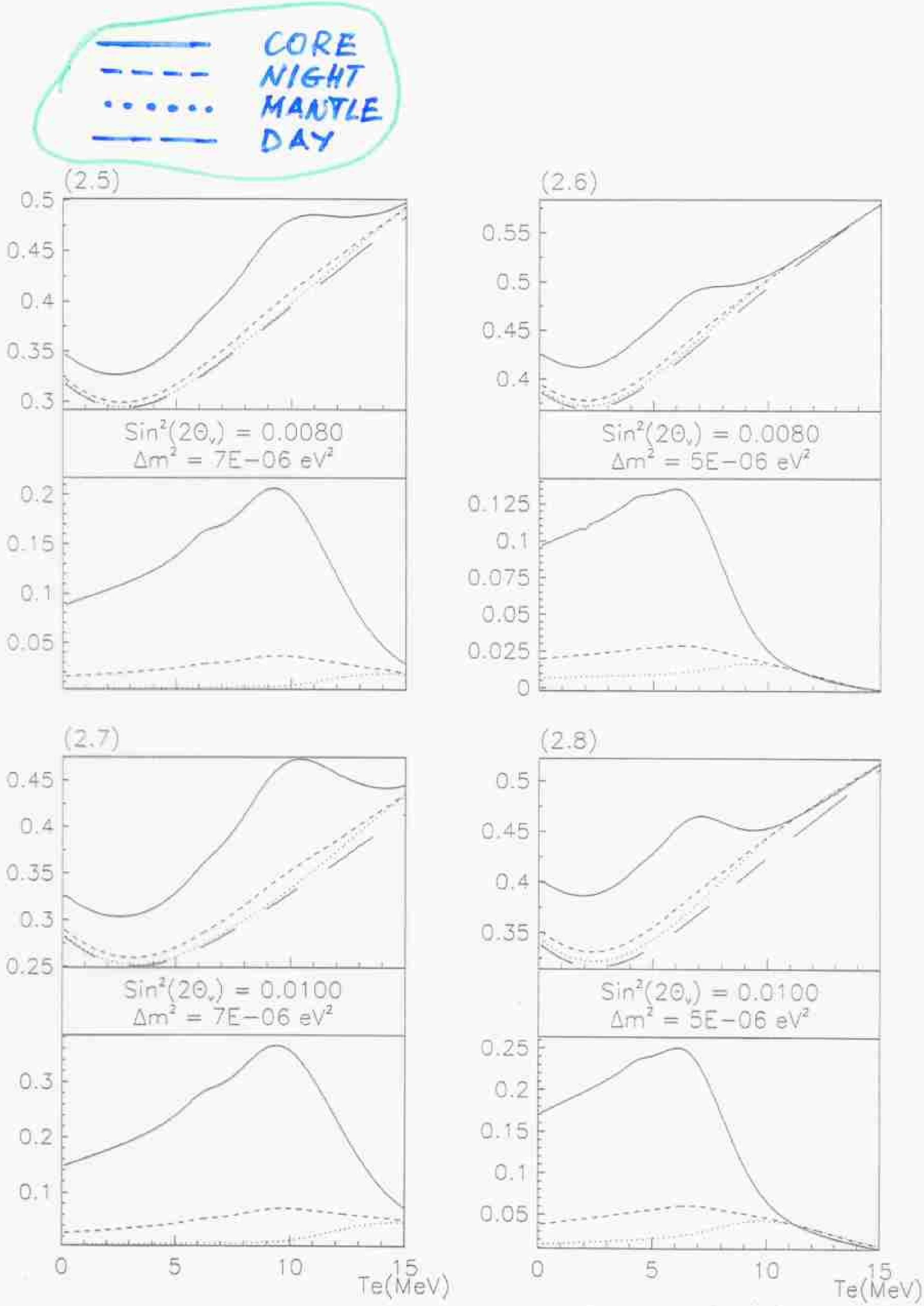
(M. MARIS, S.T.P., M. TCHIJOV '98)



(M. MARIS, S.T.P., M. TCHIJOV '98)



(M.MARIS, S.T.P. '97)



(M. MARIS, S.T.P. '97)

Table II. D - N Asymmetries for the Super - Kamiokande Detector.

				$Y_e = 0.467$				$Y_e = 0.500$			
				$A^s_{D-N} \times 100$			$\frac{\lvert A^C_{D-N}\rvert}{\lvert A^N_{D-N}\rvert}$	$A^s_{D-N} \times 100$			$\frac{\lvert A^C_{D-N}\rvert}{\lvert A^N_{D-N}\rvert}$
N.	$\sin^2 2\theta_V$	Δm^2	f_B	*Night*	*Core*	*Mantle*		*Night*	*Core*	*Mantle*	
1	0.0008	9.0e-6	0.4	-0.09	-0.54	-0.01	6.24	-0.12	-0.75	-0.01	6.25
2	0.0008	7.0e-6	0.4	-0.21	-1.26	-0.04	6.00	-0.22	-1.35	-0.04	6.14
3	0.0008	5.0e-6	0.4	-0.37	-1.29	-0.23	3.44	-0.35	-1.14	-0.23	3.26
4	0.0010	9.0e-5	0.4	3e-3	4e-3	3e-3	1.25	3e-3	4e-3	3e-3	1.33
5	0.0010	7.0e-6	0.4	-0.25	-1.50	-0.04	6.01	-0.26	-1.59	-0.04	6.12
6	0.0010	5.0e-6	0.4	-0.45	-1.52	-0.27	3.39	-0.43	-1.35	-0.27	3.14
7	0.0020	1.0e-5	0.5	-0.07	-0.41	-0.01	6.00	-0.10	-0.62	-0.01	6.20
8	0.0020	7.0e-6	0.5	-0.35	-2.10	-0.07	5.98	-0.36	-2.18	-0.07	6.06
9	0.0020	5.0e-6	0.5	-0.71	-2.30	-0.45	3.23	-0.67	-2.02	-0.45	3.01
10	0.0040	1.0e-5	1.0	0.15	0.79	0.04	5.42	0.22	1.28	0.04	5.82
11	0.0040	7.0e-6	0.7	-0.04	-0.12	-0.02	3.48	0.01	0.21	-0.02	21.00
12	0.0040	5.0e-6	0.7	-0.59	-1.36	-0.46	2.31	-0.56	-1.12	-0.47	2.00
13	0.0060	1.0e-5	1.5	0.72	3.98	0.17	5.56	1.05	6.20	0.17	5.90
14	0.0060	7.0e-6	1.0	1.06	6.46	0.17	6.13	1.26	7.60	0.17	6.03
15	0.0060	5.0e-6	0.7	0.47	2.93	0.06	6.19	0.46	2.82	0.06	6.13
16	0.0080	1.0e-5	1.5	1.63	8.93	0.37	5.47	2.36	13.59	0.37	5.76
17	0.0080	7.0e-6	1.5	3.04	17.00	0.53	5.59	3.39	19.10	0.53	5.63
18	0.0080	5.0e-6	1.0	2.55	10.26	1.22	4.02	2.44	9.54	1.22	3.91
19	0.0100	7.0e-6	1.5	5.72	29.85	1.05	5.22	6.28	32.85	1.05	5.23
20	0.0100	5.0e-6	1.0	5.60	19.84	3.03	3.54	5.36	18.38	3.03	3.43
21	0.0130	5.0e-6	1.5	11.69	36.33	6.89	3.11	11.21	33.72	6.89	3.01
22	0.3000	1.5e-5	2.0	10.73	13.25	10.31	1.24	11.03	15.29	10.30	1.39
23	0.3000	2.0e-5	2.0	7.64	9.60	7.31	1.26	7.79	10.66	7.31	1.37
24	0.3000	3.0e-5	2.0	4.74	5.54	4.61	1.17	4.78	5.85	4.61	1.22
25	0.3000	4.0e-5	2.0	3.29	3.93	3.18	1.20	3.31	4.07	3.19	1.23
26	0.4800	3.0e-5	1.5	5.95	6.81	5.81	1.15	5.99	7.08	5.81	1.18
27	0.4800	5.0e-5	1.5	2.98	3.50	2.89	1.17	2.99	3.57	2.89	1.19
28	0.5000	2.0e-5	1.5	9.48	10.97	9.24	1.16	9.60	11.79	9.23	1.23
29	0.5600	1.0e-5	1.5	23.65	34.9	21.64	1.48	27.50	40.77	25.11	1.48
30	0.6000	8.0e-5	1.0	1.42	1.64	1.38	1.15	1.42	1.65	1.38	1.16
31	0.7000	3.0e-5	1.0	6.54	7.26	6.41	1.11	6.56	7.43	6.41	1.13
32	0.7000	5.0e-5	1.0	3.37	3.90	3.29	1.16	3.38	3.95	3.29	1.17
33	0.7700	2.0e-5	1.0	9.89	10.27	9.83	1.04	9.94	10.59	9.83	1.07
34	0.8000	1.3e-4	0.7	0.57	0.69	0.55	1.21	0.57	0.69	0.55	1.21
35	0.9000	4.0e-5	0.7	4.53	5.12	4.42	1.13	4.53	5.17	4.42	1.14
36	0.9000	1.0e-4	0.7	1.15	1.33	1.13	1.15	1.15	1.33	1.13	1.16

(M. MARIS, S.T.P. '97)

$$A^{MP}_{D-N} = \frac{N-D}{\frac{1}{2}(N+D)} = -2A^{SK}_{D-N}$$

SK :

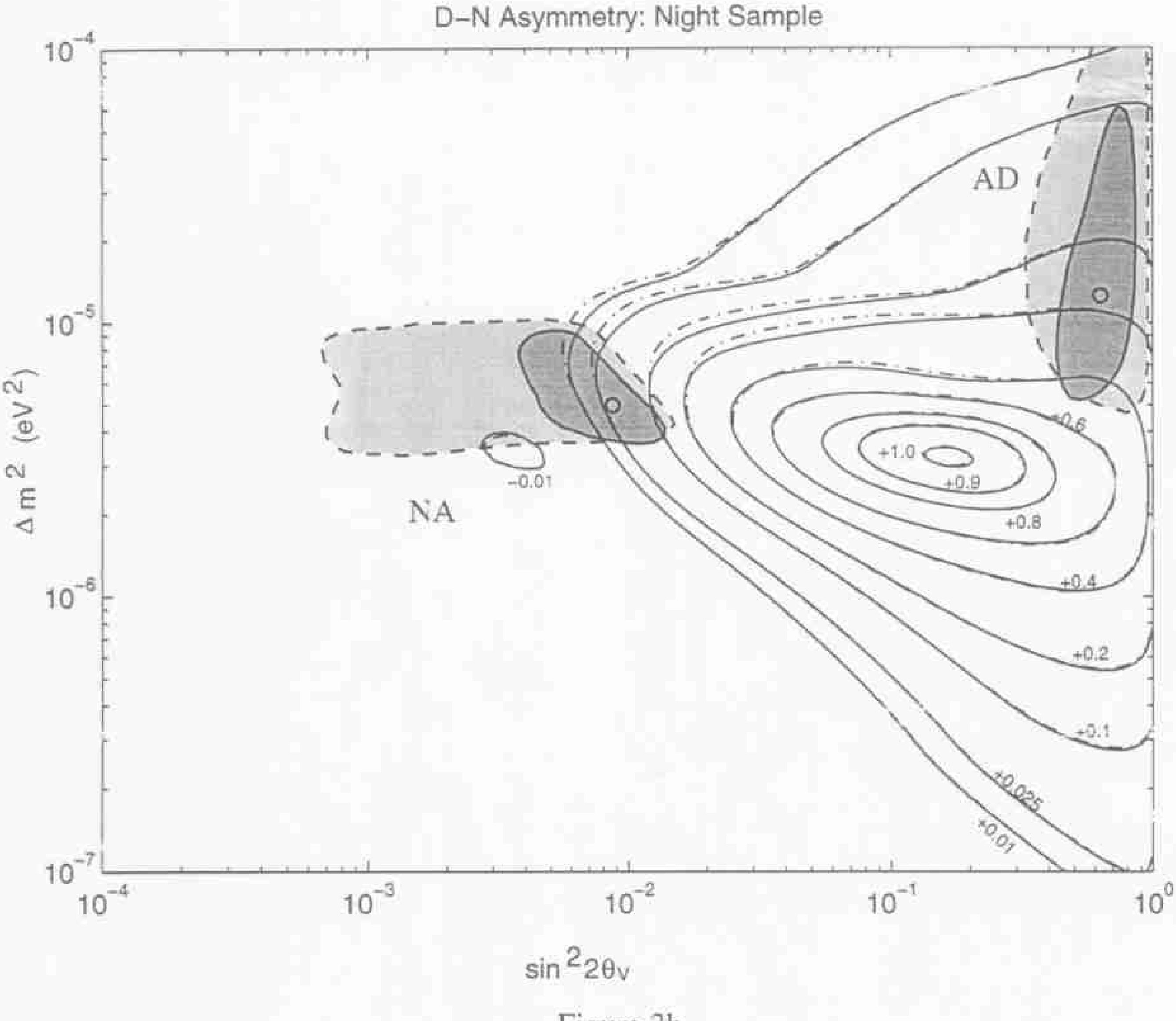


Figure 3b

(M. MARIS, S.T.P. '97)

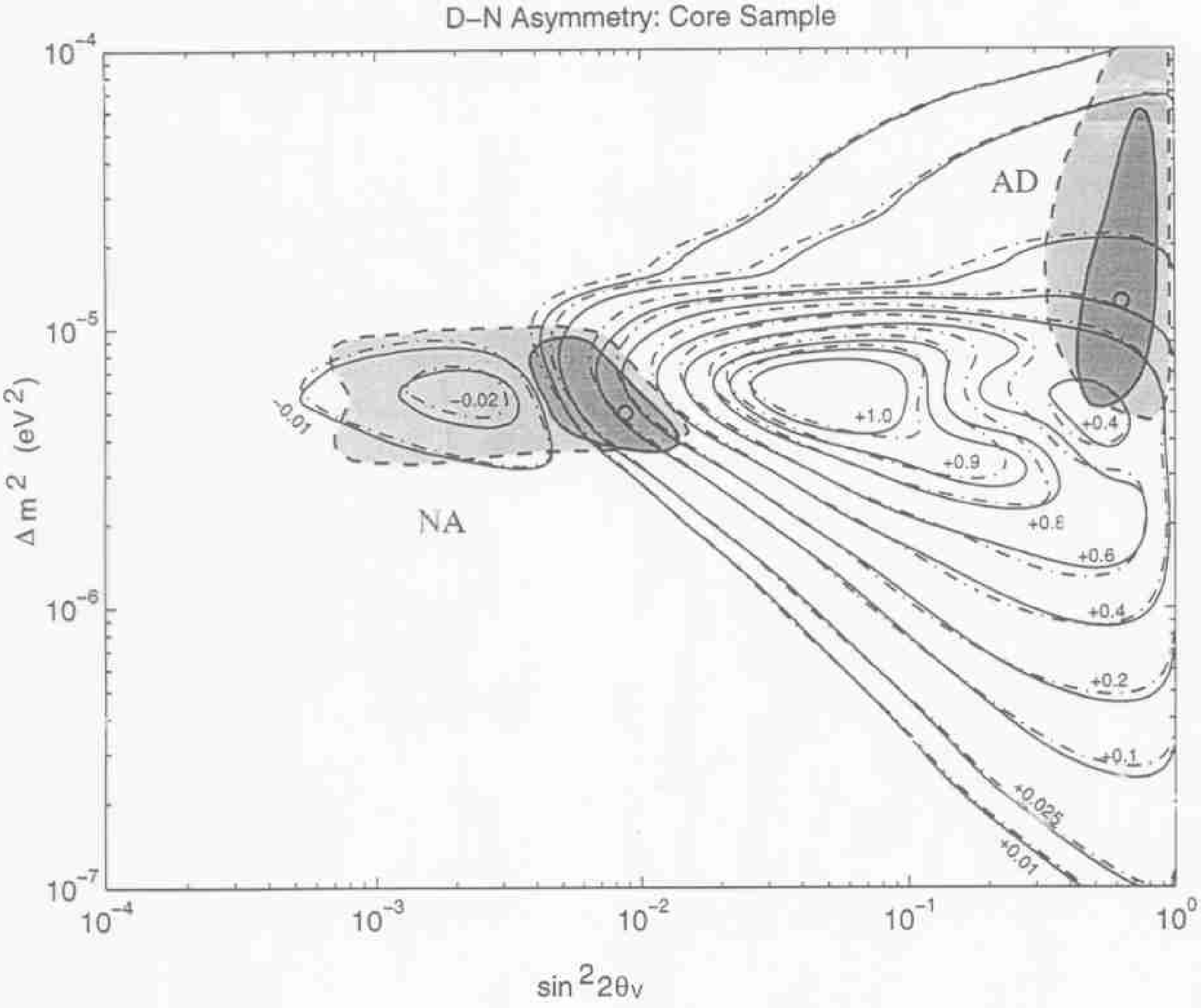


Figure 3c

The same resonance is present in the

$$\nu_\mu \to \nu_e \, , \, \nu_e \to \nu_\mu$$

transitions of atmospheric neutrinos crossing the Earth core.

S.T.P., REPORT SISSA 31/98/EP
(hep-ph/9805262)
M. MARIS, S.T.P., M. TCHIJOV,
Report SISSA 53/98/EP
(to be released)

$$P(\nu_\mu \to \nu_e) = P(\nu_2 \to \nu_e)\Big|_{\substack{\theta = 0 \\ \theta'_m \neq 0,\ \theta''_m \neq 0}}$$

Conditions for the oscillation length resonance in $P(\nu_\mu \to \nu_e)$:

$$\Delta E' X' = \pi(2k+1), \quad \Delta E'' X'' = \pi(2k'+1), \quad k, k' = 0, 1, \ldots$$

maxima

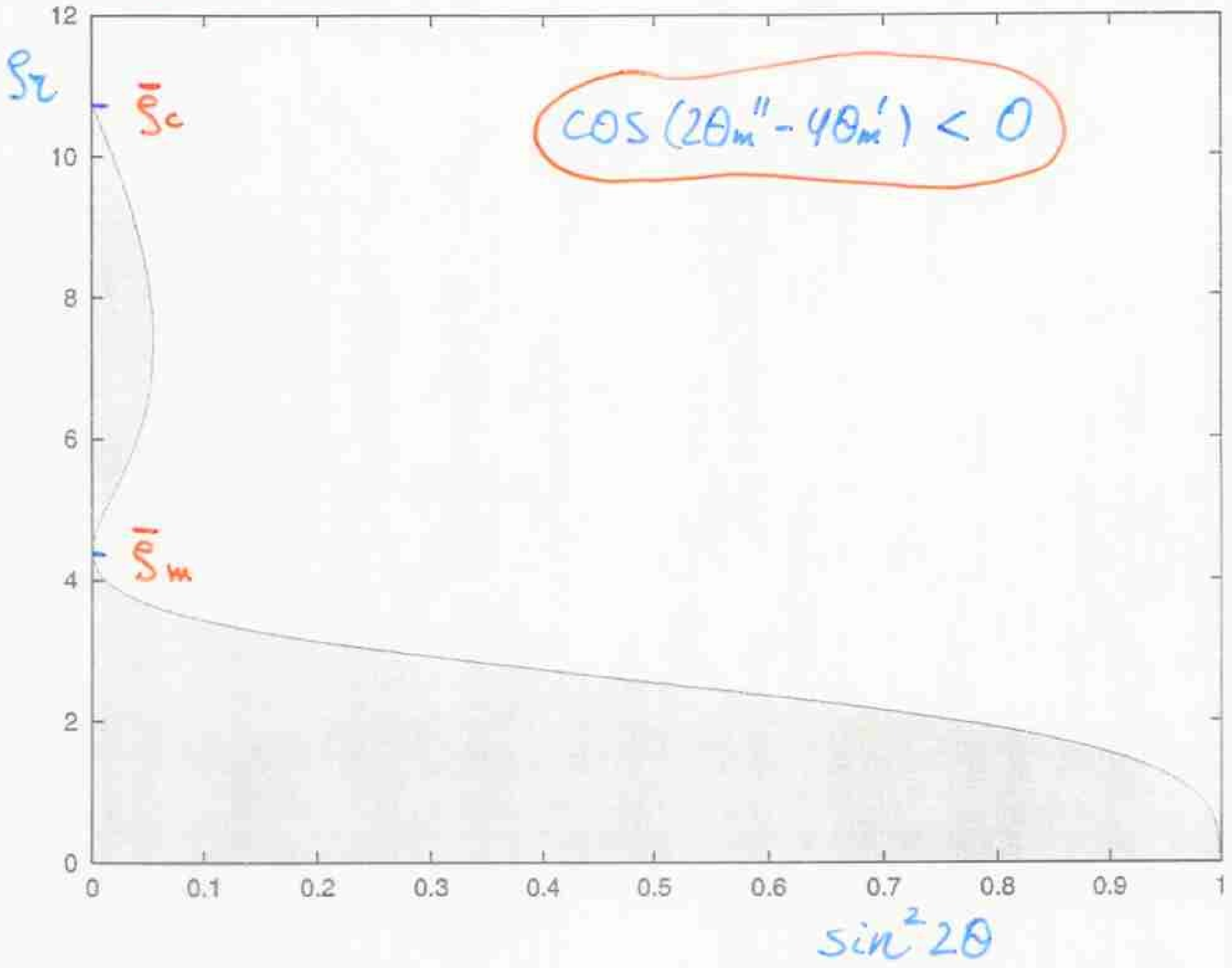
$\sin^2(2\theta''_m - 4\theta'_m) > 0$ – always fulfilled

$\cos(2\theta''_m - 4\theta'_m) < 0$ – important

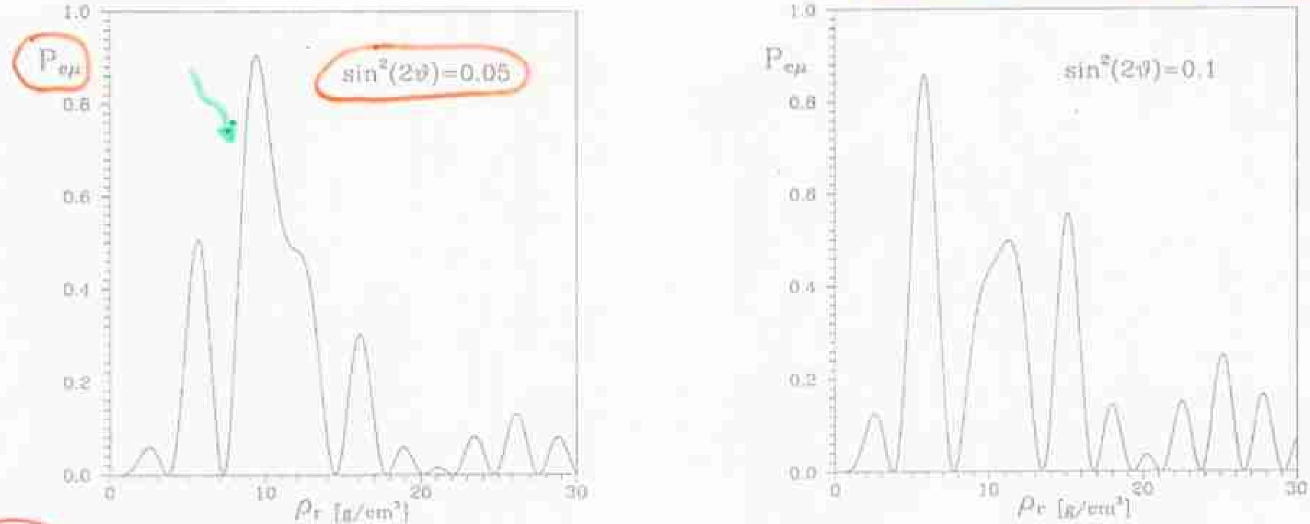
(no possibility of deep minima)

(M. MARIS, S.T.P., M. TCHIJOV, '98)

$P(\nu_\mu \to \nu_e)$



(M. MARIS, S.T.P., M. TCHIZOV '98)



$\Delta m^2_{atm} \simeq 10^{-3}\,eV^2$ { $E_{max} = 1.46$ GeV, $E_{SR} \cong 1.05$ GeV

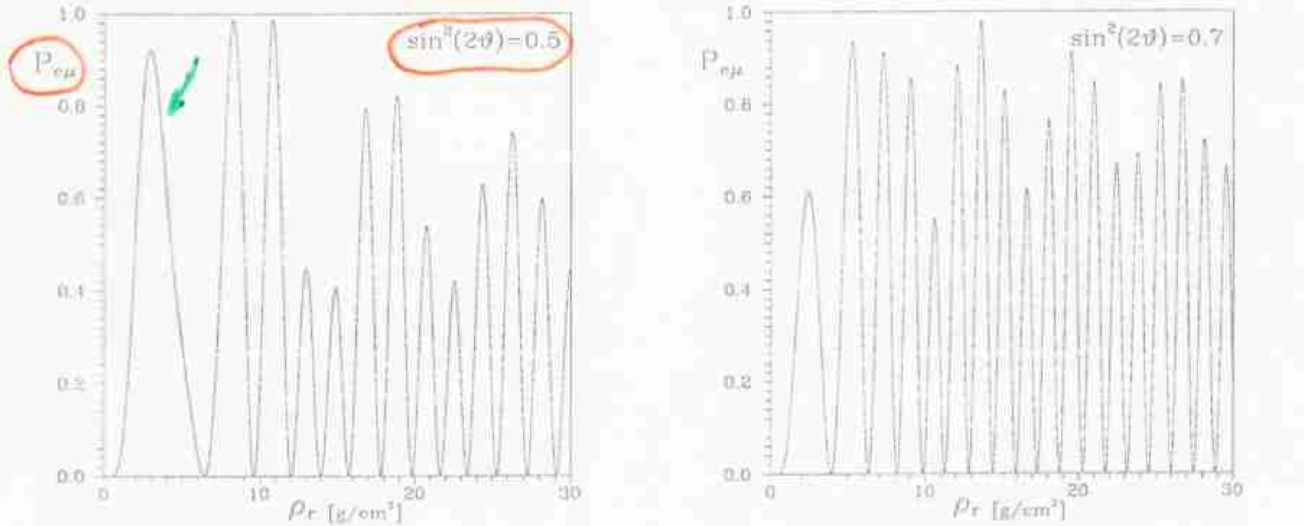


Figure 5a: h=0

Assume:

$$U \cong \begin{pmatrix} 1 & \varepsilon & \varepsilon' \\ -\frac{(\varepsilon+\varepsilon')}{\sqrt{2}} & \frac{1+\varepsilon'}{\sqrt{2}} & \frac{1-\varepsilon'}{\sqrt{2}} \\ \frac{\varepsilon-\varepsilon'}{\sqrt{2}} & -\frac{1-\varepsilon'}{\sqrt{2}} & \frac{1+\varepsilon'}{\sqrt{2}} \end{pmatrix}$$

$\varepsilon^2, \varepsilon'^2, \varepsilon\varepsilon'$ – small

$\nu_\odot$: $\Delta m^2_{21} \sim (4-8)\cdot 10^{-6}\ eV^2$

$$\sin^2 2\theta_\odot \cong 4\frac{|U_{e1}|^2 |U_{e2}|^2}{(|U_{e1}|^2+|U_{e2}|^2)^2} \cong 4\varepsilon^2 \cong (6-8)\cdot 10^{-3}$$

ν_{atm}: $\Delta m^2_{31} \cong 10^{-3}\ eV^2$

$P(\nu_\mu \to \nu_\tau): 4|U_{\mu 3}|^2 |U_{\tau 3}|^2 \cong 1-\varepsilon'^2 \simeq 0.975$

$P(\nu_\mu \to \nu_e): 4|U_{e3}|^2 |U_{\mu 3}|^2 \cong 2\varepsilon'^2 \cong 5\cdot 10^{-2}$

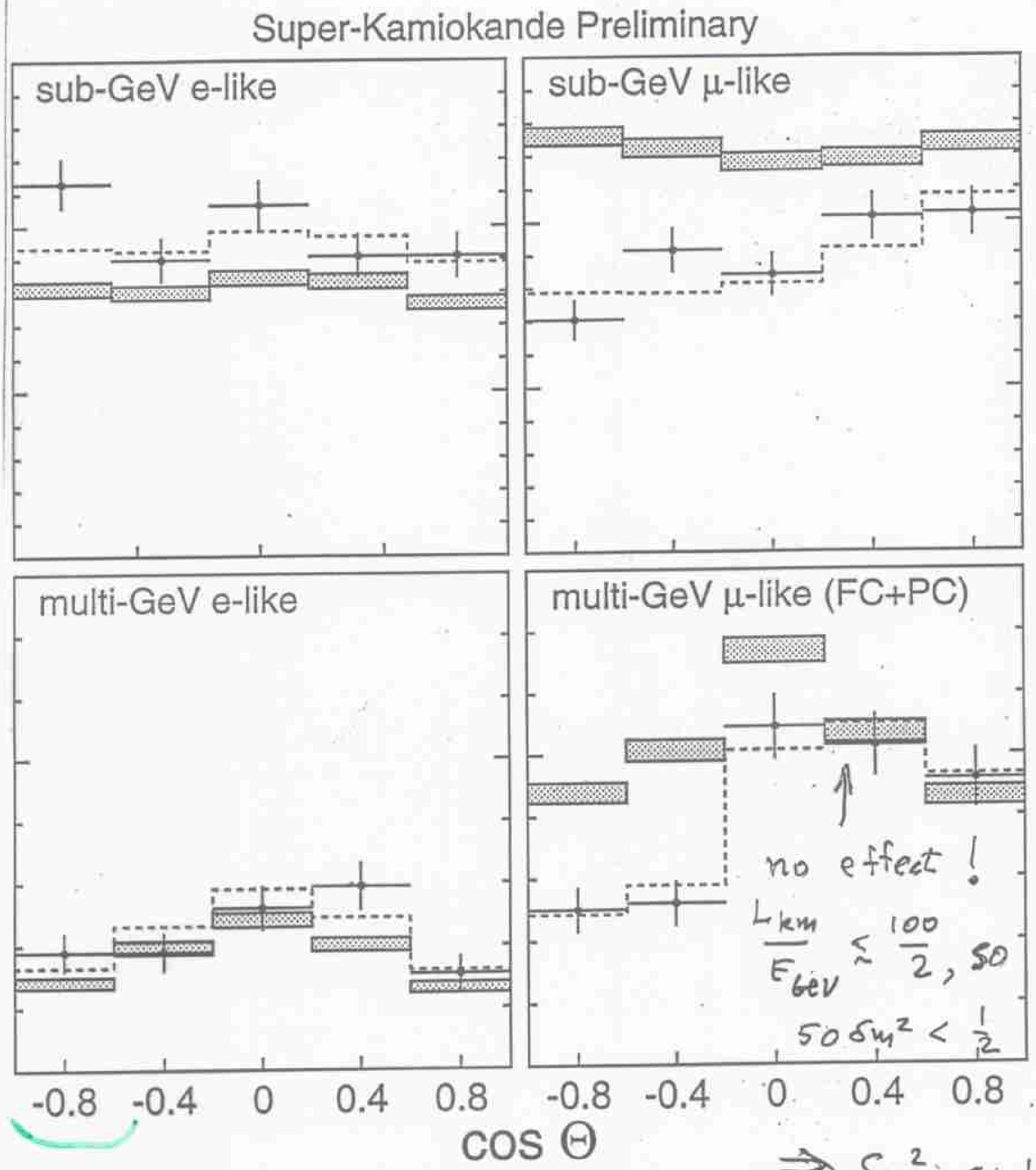
$h = 0°$: $\rho_r^{max} \simeq 9\ g/cm^3$, $E_{max} \simeq 1.46$ GeV

max $P(\nu_\mu \to \nu_e) \cong 0.9$

"shoulder" at $\rho_r \simeq 12.5\ g/cm^3$, $E_{sh} \simeq 1.05$ GeV

ν_e $\frac{\text{data}}{\text{calc.}} \sim 1.20$ ν_μ $\frac{\text{data}}{\text{calc}} \sim 0.75$ T. Gaisser ν'98

but large (20%) uncertainty in norm. of calc.



$\Rightarrow \delta m^2 \lesssim 10^{-2}$