Textures of Neutrino Mass Matrix with Large Flavor Mixing

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Neutrino Symposium at TMU, Tokyo, 11 June 1998

Contents

Exciting Result of Atmospheric Neutrinos at Super-Kamiokande

Large Flavor Mixing ? $\sin^2 2\theta \sim 1$ maximal

$\Delta m^2 \simeq 10^{-2} \sim 10^{-3}\ \mathrm{eV}^2$

Why is there Large Flavor Mixing in Leptons ?

It may be a clue of the origin of fermion mass matrices.

Are there possible mechanism to derive a Large Mixing Angle from Mass Matrices, which are consistent with Quark Sector ?

$\boxed{\text{yes !!}}$

Many mass matrix models

2 Origin of Large Flavor Mixings

A See - Saw enhancement

$$M_{LL} = m_{LR} M_{RR}^{-1} m_{LR}^{T}$$ (small angle)

Even if m_{LR} and M_{RR} are hierarchical,
M_{LL} could give Large Flavor Mixings
under some conditions.

Smirnov, PRD 48 (1993) 3264; Tanimoto, PL 345 B (1995) 477
Babu, Shafi, PL 294 B (1992) 235 SO(10)

$U_f(1)$, $U_f(2)$ ... flavor Symmetry
Binetruy, Lavignac, Petcov, Ramond, NP 496 B (1997) 3, Ramond ν98

B Type II See - Saw Model

$$\begin{array}{c c} & \begin{array}{cc} L & R \end{array} \\ \begin{array}{c} L \\ R \end{array} & \begin{pmatrix} f v I & m_D \\ m_D^T & M_{RR} \end{pmatrix} \end{array} \qquad m_\nu^{LL} \simeq f I v - \frac{m_D^2}{M_{RR}}$$

Degenerated 3 flavors by Horizontal Symmetry
new Higgs

Lee, Mohapatra, PL 329 B (1994) 463
Ioannissyan, Valle, PL 332 B (1994) 93

Large Flavor Mixing

Quark Mass Matrix Base $\theta_U \ll 1$ $\theta_D \ll 1$

Hierarchical Base

What does See - Saw do? $\begin{pmatrix} 0 & M_{LR} \\ M_{LR}^T & M_{RR} \end{pmatrix}$

1 Dirac Mass Matrix $\overset{M_{LR}}{\Rightarrow}$ Large Mixing

M_{RR} should be simple → See-Saw preserve large mixing

Charged lepton + neutrino (Dirac) + R.G.E

$e^{i\alpha}\theta_{\ell^\pm} + \theta_\nu^D$ Fukugita, M.T. Yanagida

$\theta_{\ell^\pm}$ Sato, Yanagida; Albright, Babu, Barr

θ_ν^D

→ $\sin^2 2\theta_{\mu\tau} \geq 0.8$

2 See - Saw Enhancement — A. Smirnov PRD 48 (1993) 3264; M.T, PL 345B (1995) 477

See-Saw mechanism leads to the enhancement of mixing as well as small neutrino masses

$$\underbrace{e^{i\alpha}\theta_{\ell^\pm}}_{\text{small}} + \underbrace{\theta_\nu^D}_{\text{small}} + \underbrace{\theta_{\text{see-saw}}}_{\text{Large}} \longrightarrow \sin^2 2\theta_{\mu\tau} \geq 0.8$$

physics of Intermediate Scale

A. Smirnov. NP B466 (1996) 25

C Effective Neutrino Mass Matrix with Symmetry

Miss-Matching between M_{ℓ^-} and M_{ν}^{LL}

Democratic Matrix, $S_{3L} \times S_{3R}$, USY .....

D Exotic Fields (with Symmetry)

New Particles are essential for Large Mixing

Zee Model (singlet h^-), Sterile neutrinos

$(SMG)^3 \times U_f(1)$ anti-GUT

E Large Evolution of Mixings by RGE's

$\theta_{GUT} \longrightarrow \theta_{EW}$ $(Y_{\ell^-} \simeq 1 \quad \tan\beta \gg 1)$

Chankowski, Pluciennik ; PL 316B (1993) 312

Babu, Leung, Pantaleone ; PL 319B (1993) 191

Tanimoto ; PL 360B (1995) 41

F

you should add !

G

⋮

Naturalness of Mass Matrix

Peccei – Wang (1996)

$m_Q = U_Q^T \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} U_Q$

Example

$$m_Q \simeq \begin{pmatrix} m_1 + \sin\theta_Q^2 m_2 & \sin\theta_Q m_2 \\ \sin\theta_Q m_2 & m_2 \end{pmatrix} \qquad \theta_c \equiv \theta_d - \theta_u$$

$$m_D \sim \begin{pmatrix} \underset{\smile}{a}\lambda^2 + \lambda^2 & \lambda \\ \lambda & 1 \end{pmatrix} m_D^0 \qquad m_d : m_s = \underline{a\lambda^2} : \underline{1}$$

$$\theta_d \simeq \lambda$$

$$m_U \sim \begin{pmatrix} \underset{\smile}{b}\lambda^4 + \lambda^4 & \lambda^2 \\ \lambda^2 & 1 \end{pmatrix} m_U^0 \qquad m_u : m_c = \underline{b\lambda^4} : \underline{1}$$

$$\theta_u \simeq \lambda^2$$

natural

$$a = -1 \Rightarrow \theta_d \simeq \sqrt{\frac{m_d}{m_s}}$$

If

$$m_U \sim \begin{pmatrix} \underline{b\lambda^4} + \lambda^2 & \lambda \\ \lambda & 1 \end{pmatrix} m_U^0 \; ; \quad \theta_u \simeq \lambda$$

unnatural

$$\theta_c = O(\lambda) - O(\lambda) = O(\lambda)$$

What is Natural Neutrino Mass Matrix ?

M. Matsuda, M. T

Testable Mass Matrix Model

$U_{\mu 3} \sim$ Large Many Models

$\underbrace{U_{e2},\ U_{e3}}_{\text{predictions}}$ should be tested !

Scheme of Neutrino Mass Hierarchy

Starting Point

Atmospheric Neutrino Anomaly at S-Kam

$$\Delta m^2_{\mathrm{Atm}} = 10^{-3} \sim 10^{-2}\mathrm{eV}^2$$
$$\sin^2 2\theta \simeq 1$$

And we take into account

Solar Neutrino Data

$\Delta m^2_{\odot} = (0.3 \sim 1.2) \times 10^{-5}\mathrm{eV}^2$

MSW small angle $\sin^2 2\theta_{\odot} \simeq (0.1 \sim 2) \times 10^{-2}$

MSW large angle $\sin^2 2\theta_{\odot} \simeq 0.65 \sim 0.85$

$\Delta m^2_{\odot} = (5 \sim 8) \times 10^{-11}\mathrm{eV}^2$

Just so $\sin^2 2\theta_{\odot} \simeq 0.8 \sim 1$

LSND Data

$$\Delta m^2_{\mathrm{LSND}} = 0.3 \sim 2\mathrm{eV}^2$$
$$\sin^2 2\theta_{\mathrm{LSND}} \simeq (0.2 \sim 3) \times 10^{-2}$$

Scheme A: Sacrifice LSND Data

$$\begin{aligned} \Delta m_{31}^2 \simeq \Delta m_{32}^2 \simeq &\ \Delta m_{\mathrm{Atm}}^2 \\ \Delta m_{21}^2 \simeq &\ \Delta m_{\odot}^2 \end{aligned}$$

Natural Mass Hierarchy: $m_3 \simeq m_2 \simeq m_1$

Scheme B: Include LSND Data with a Sterile Neutrino ν_s

$$\begin{aligned} \Delta m_{31}^2 \simeq \Delta m_{21}^2 \simeq &\ \Delta m_{\mathrm{LSND}}^2 \\ \Delta m_{32}^2 \simeq &\ \Delta m_{\mathrm{Atm}}^2 \\ \Delta m_{1s}^2 \simeq &\ \Delta m_{\odot}^2 \end{aligned}$$

Natural Mass Hierarchy: $m_3 \simeq m_2 \gg m_1, m_s$

Define $\mathbf{3} \times \mathbf{3}$ Flavor Mixing Matrix

$$\begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\phi} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\phi} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\phi} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\phi} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\phi} & c_{23}c_{13} \end{pmatrix}$$

3 Textures in Three Neutrino Scheme

Atmospheric + Solar Neutrinos

Many Models predict Flavor Mixings:

$$V^{\mathrm{CKM}} \equiv U_\ell^\dagger U_\nu$$

$$\simeq \begin{pmatrix} 1 & u^\ell_{12} & u^\ell_{13} \\ -u^\ell_{21}\,{\scriptstyle 12} & 1 & u^\ell_{23} \\ u^\ell_{31} & -u^\ell_{32} & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix}$$

$$\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} e \\ \mu \\ \tau \end{array} \simeq & \begin{pmatrix} 1 & \underline{c}\, u^\ell_{12} & \underline{s}\, u^\ell_{12} \\ -u^\ell_{21}\,{\scriptstyle 12} & c & s \\ u^\ell_{31} & -s & c \end{pmatrix} \end{array}$$

$$s \equiv \sin\theta \simeq \frac{1}{\sqrt{2}} \sim \frac{2}{\sqrt{6}}$$

$$u^\ell_{12} = \sqrt{\frac{m_e}{m_\mu}}, \quad u^\ell_{23} = u^\ell_{32} \ = \frac{1}{\sqrt{2}}\frac{m_\mu}{m_\tau}$$

$$|u^\ell_{13}| \simeq |u^\ell_{31}| \simeq \frac{\sqrt{2 m_e m_\mu}}{m_\tau}$$

$\simeq c\sqrt{\frac{m_e}{m_\mu}}$ $\sqrt{\frac{m_e}{m_\mu}} \simeq 0.07$

$$V_{e2}^{\rm CKM} = c\; u_{12}^{\ell} \simeq 0.04 \sim 0.05$$

$$V_{e3}^{\rm CKM} = s\; u_{12}^{\ell} \simeq 0.057 \sim 0.05$$

Oscillation Probability in LBL Experiments

$$\nu_\mu \to \nu_e, \quad \nu_\mu \to \nu_\tau, \quad \nu_e \to \nu_\tau$$

$$P(\nu_\mu \to \nu_e) \simeq 4|V_{\mu 3}^* V_{e3}|^2 \sin^2 \frac{\Delta m_{31}^2 L}{4E} - 2J_{CP}\, S_{CP}$$

$$P(\nu_\mu \to \nu_\tau) \simeq 4|V_{\mu 3}^* V_{\tau 3}|^2 \sin^2 \frac{\Delta m_{31}^2 L}{4E} - 2J_{CP}\, S_{CP}$$

$$P(\nu_e \to \nu_\tau) \simeq 4|V_{e3}^* V_{\tau 3}|^2 \sin^2 \frac{\Delta m_{31}^2 L}{4E} - 2J_{CP}\, S_{CP}$$

$$J_{CP} = c_{12} c_{23} c_{13}^2 s_{12} s_{23} s_{13} \sin\phi \sim \frac{1}{4}\frac{m_e}{m_\mu} \sim 10^{-3}$$

$$S_{CP} = \sin\frac{\Delta m_{12}^2 L}{2E} + \sin\frac{\Delta m_{23}^2 L}{2E} + \sin\frac{\Delta m_{31}^2 L}{2E} \sim 10^{-2}$$

$2J_{CP}S_{CP} \sim 10^{-5}$ terms can be neglected.

$$\sin^2 2\theta_{\rm Atm} \simeq 4c^2 s^2 \qquad \sin^2 2\theta_\odot \simeq 4c^2 \frac{m_e}{m_\mu}$$

$$P(\nu_\mu \to \nu_e) \simeq 4s^4 \frac{m_e}{m_\mu} \sin^2 \frac{\Delta m^2_{31} L}{4E}$$

$$P(\nu_e \to \nu_\tau) \simeq 4c^2 s^2 \frac{m_e}{m_\mu} \sin^2 \frac{\Delta m^2_{31} L}{4E}$$

Once $\sin^2 2\theta_{\rm Atm}$ and $\Delta m^2_{\rm Atm}$ will be determined precisely, we have three predictions:

$$\boxed{\sin^2 2\theta_\odot \quad P(\nu_\mu \to \nu_e) \quad P(\nu_e \to \nu_\tau)}$$

Numerical Example:

Input: $c = \frac{1}{\sqrt{3}}$ $\qquad$ $(\Delta m^2_{\rm Atm} = 5 \times 10^{-3} {\rm eV}^2)$

Predictions: $\sin^2 2\theta_\odot = 8 \times 10^{-3}$

$$P(\nu_\mu \to \nu_e) = 6 \times 10^{-3} \sin^2 \frac{\Delta m^2_{31} L}{4E}$$

$$P(\nu_e \to \nu_\tau) = 4 \times 10^{-3} \sin^2 \frac{\Delta m^2_{31} L}{4E}$$

Mass Matrix Model

S_3 Flavor Symmetry

$\mathbf{2}\ (\mathbf{1st}, \mathbf{2nd\ family}), \mathbf{1}_{\mathrm{A}}\ (\mathbf{3rd\ family})$

$S_{3L} \times S_{3R}$ Invariant Dirac Mass Matrix

changed lepton

$$M_\ell = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

S_{3L} Invariant Majorana Mass Matrix

$$M_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + r \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$\Downarrow$$

$$M_\nu = \begin{pmatrix} 1+2r & 0 & 0 \\ 0 & 1-r & 0 \\ 0 & 0 & 1-r \end{pmatrix}$$

Degenerate Two Light Neutrinos

$r = 0$ or $r = -2 \Rightarrow$

Degenerate Three Light Neutrinos

S_{3L} Breaking Terms give

Quasi Degenerate Three Neutrino Masses

$r = 0$ Case: PR D57(1998)4429

M. Fukugita, M. Tanimoto and T. Yanagida

$$M_\nu = c_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & \epsilon_\nu & 0 \\ \epsilon_\nu & 0 & 0 \\ 0 & 0 & \delta_\nu \end{pmatrix}$$

$$\Delta m^2_{31} \simeq 2\overset{\delta}{\cancel{\epsilon}}_\nu c_\nu \qquad \Delta m^2_{21} \simeq 4\epsilon_\nu c_\nu$$

$r = -2$ Case: hep-ph/9802328

K. Kang and S. Kang

$$M_\nu = c_\nu \begin{pmatrix} 1+2r & 0 & 0 \\ 0 & 1-r & \epsilon_\nu \\ 0 & \epsilon_\nu & 1-r \end{pmatrix}$$

$$c_\nu \simeq 0.3345, \ \ r \simeq -1.9925, \quad \epsilon_\nu \simeq 0.0075$$

Alternative Representation of S_{3L} based on

USY(Universal Strength for Yukawa Couplings)

G.C. Branco, M.N. Rebelo and J.I. Silva-Marcos

hep-ph/9802340

$$M_\nu = c_\nu \begin{pmatrix} e^{i\alpha} & 1 & 1 \\ 1 & e^{i\alpha} & 1 \\ 1 & 1 & e^{i\alpha} \end{pmatrix} \qquad \mathrm{S_{3L}}\ \text{Invariant}$$

$\alpha = \frac{2}{3}\pi \Rightarrow$ Degenerate Three Light Neutrinos

$$M_\nu = c_\nu \begin{pmatrix} e^{i\alpha} & 1 & 1 \\ 1 & e^{i\alpha} & 1 \\ 1 & 1 & e^{i\beta} \end{pmatrix} M_\ell = c_\ell \begin{pmatrix} e^{-ia} & 1 & 1 \\ 1 & e^{ia} & 1 \\ 1 & 1 & e^{ib} \end{pmatrix}$$

$$\alpha = \frac{2}{3}\pi - \frac{1}{\sqrt{3}}\frac{\Delta m_{32}^2}{m_3^2} - \frac{3\sqrt{3}}{4}\frac{\Delta m_{21}^2}{m_3^2}$$

$$\beta = \frac{2}{3}\pi + \frac{2}{\sqrt{3}}\frac{\Delta m_{32}^2}{m_3^2}$$

$$a = 3\sqrt{3}\frac{\sqrt{m_e m_\mu}}{m_\tau}, \qquad b = \frac{9 m_\mu}{2 m_\tau}$$

Alternative Flavor Mixing Matrix

H. Fritzsch and Z. Xing, PL 372B(1996)265
M. Fukugita, M. Tanimoto and T. Yanagida

$$M_\nu = c_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} -\epsilon_\nu & 0 & 0 \\ 0 & \epsilon_\nu & 0 \\ 0 & 0 & \delta_\nu \end{pmatrix}$$

$$M_\ell = c_\ell \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} -\epsilon_\ell & 0 & 0 \\ 0 & \epsilon_\ell & 0 \\ 0 & 0 & \delta_\ell \end{pmatrix}$$

$\sqrt{\frac{m_e}{m_\mu}} = V_{12}$

$$V^{\mathrm{CKM}} \simeq \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} e \\ \mu \\ \tau \end{array} & \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{2}{\sqrt{6}}\sqrt{\frac{m_e}{m_\mu}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \end{array}$$

$$V_{e2}^{\mathrm{CKM}} \simeq \frac{1}{\sqrt{2}}\,, \quad V_{e3}^{\mathrm{CKM}} \simeq 0.057$$

Just so Solution for Solar Neutrinos

$\sin^2 2\theta_\odot \simeq 1$

Other Models with See-Saw Mechanism

M. Bando, T. Kugo and K. Yoshioka
PRL80(1998)3004

$$M_\nu^{LR} = m \begin{pmatrix} 0 & 0 & x \\ 0 & x & 0 \\ x & 0 & \underset{1}{\cancel{x}} \end{pmatrix} \quad M_\nu^{RR} = \begin{pmatrix} \alpha M & \beta M & 0 \\ \beta M & \gamma M & 0 \\ 0 & 0 & M' \end{pmatrix}$$

K.S. Babu and Q. Shafi: PL294B(1992)235

Based on $SO(10)$

$$M_{\rm up}^{LR} = \begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{pmatrix} \quad M_\nu^{LR} = \begin{pmatrix} 0 & A & 0 \\ A & 0 & -3B \\ 0 & -3B & C \end{pmatrix}$$

$$M_\nu^{RR} = \begin{pmatrix} M_1 e^{i\alpha} & 0 & 0 \\ 0 & M_2 e^{i\beta} & M_3 e^{i\gamma} \\ 0 & M_3 r^{i\gamma} & 0 \end{pmatrix}$$

A, C: Higgs **10**, B: Higgs **126**, M_i: Higgs **126**

4 CP Violating Phase

LSND + Atmospheric + Solar Neutrinos

3 Active Neutrinos + 1 Sterile Neutrino

Minimal Mass Matrix needed to describe data

Small Mixing in $\nu_e - \nu_s$
Negligible Mixings in $\nu_\mu - \nu_s$ and $\nu_\tau - \nu_s$

For active neutrinos, 3×3 Unitary Mixing Matrix is still available approximately

$$
\begin{array}{cc}
 & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\
\begin{array}{c} e \\ \mu \\ \tau \end{array} &
\begin{pmatrix}
c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\phi} \\
-c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\phi} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\phi} & s_{23}c_{13} \\
s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\phi} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\phi} & c_{23}c_{13}
\end{pmatrix}
\end{array}
$$

Remark: $s_{23} \simeq \frac{1}{\sqrt{2}} \Longrightarrow$ No More $V_{e2} \simeq V_{\mu 1}$

Depending on s_{13} and ϕ

$V_{e2} \simeq s_{12} \Longrightarrow$ **LBL** $\nu_\mu \to \nu_e$

$V_{\mu 1} \simeq \frac{1}{\sqrt{2}}(s_{12} + s_{13}e^{i\phi}) \Longrightarrow$ **LSND** $\nu_\mu \to \nu_e$

$V_{e2} \neq V_{\mu 1}$

5 Summary

There are many models to lead

Large Flavor Mixing (maximal)

Models should predict V_{e3},

which will be tested in LBL experiments.

$$\nu_\mu \to \nu_e \qquad \nu_e \to \nu_\tau$$

If LSND data is included in the model,

~~CP~~ violating phase may be important

to predict LBL oscillations.

$$\left.\begin{array}{l} P(\nu_\mu \to \nu_e) \\ P(\nu_e \to \nu_\tau) \end{array}\right\} O(10^{-3})$$

out of sensitivity K2K

Wait future LBL experiments

as well as precise predictions

of Models