Discovery Channel at a Neutrino Factory

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1. Introduction
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3. Violation of unitarity w/o light ν_s
4. Summary

1. Introduction

1.1 ν oscillation

Mass eigenstates

$$i\frac{d}{dt}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}=\begin{pmatrix}E_1 & 0\\ 0 & E_2\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

$$E_j\equiv\sqrt{\vec{p}^2+m_j^2}$$

Flavor eigenstates

$$\begin{pmatrix}\nu_\mu\\ \nu_\tau\end{pmatrix}=U\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

$$U\equiv\begin{pmatrix}U_{\mu 1} & U_{\mu 2}\\ U_{\tau 1} & \mathbf{u}_{\tau 2}\end{pmatrix}$$

MNS matrix

Probability of flavor conversion

$$\boxed{P(\nu_\mu\to\nu_\tau)=\sin^2 2\theta\,\sin^2\left(\frac{\Delta E\,L}{2}\right)}$$

$$\Delta E=E_2-E_1\cong\frac{m_2^2-m_1^2}{2E}=\frac{\Delta m^2}{2E}$$

1.2 Framework of 3 flavor ν oscillation

Mixing matrix

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \left(\boxed{\begin{matrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu1} & U_{\mu2} & U_{\mu3}\\ U_{\tau1} & U_{\tau2} & U_{\tau3}\end{matrix}}\right)\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

Functions of mixing angles θ_{12}, θ_{23}, θ_{13}, and CP phase δ

1.3 Information we have obtained so far

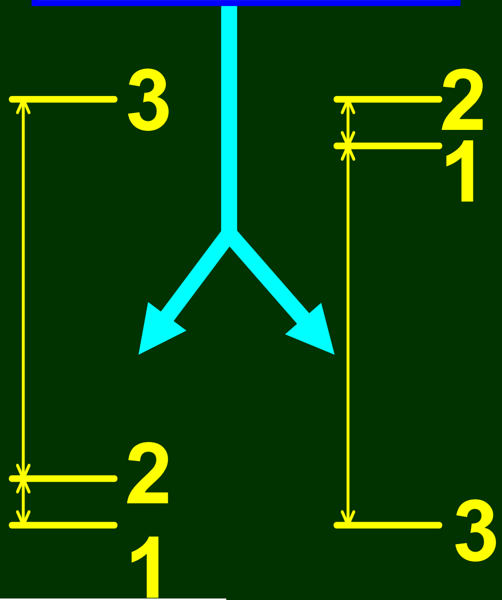
ν_{solar}+KamLAND (reactor) → $\theta_{12} \cong \frac{\pi}{6},\ \Delta m^2_{21} \cong 8\times10^{-5}\,\mathrm{eV}^2$

ν_{atm}+K2K,MINOS(accelerators) → $\theta_{23} \cong \frac{\pi}{4},\ |\Delta m^2_{32}| \cong 2.5\times10^{-3}\,\mathrm{eV}^2$

CHOOZ (reactor) → $|\theta_{13}| \leq \sqrt{0.15}/2$

$$\mathbf{U} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \cong \begin{pmatrix} c_{12} & s_{12} & \varepsilon \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & 1/\sqrt{2} \\ s_{12}/\sqrt{2} & -c_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

- **Both mass hierarchies are allowed**



normal hierarchy

$\Delta m^2_{32} > 0$

inverted hierarchy

$\Delta m^2_{32} < 0$

- θ_{13} :only upper bound is known
- δ :undetermined

Next task is to measure θ_{13}, sign(Δm^2_{31}) and δ.

1.4 Future long baseline experiments

Ongoing & Near future experiments

Accelerator $\Rightarrow \theta_{13}, \mathrm{sgn}(\Delta m^2_{32})?, \delta?$ **→Stanco's talk**

'06～ **MINOS** (FNAL→Soudan) L=730km, E ～10GeV

'08 ～ **OPERA･ICARUS** (CERN→GrandSasso) L=730km, E ～20GeV

'09 ～ **T2K** (JAERI→SK) L=295km, E ～1GeV **phase1** (0.75MW,22.5kt)

↳ **Bravar's talk**

'14 (?) ～ **NOνA** (FNAL→Ash River) L=810km, E ～1GeV (0.7MW,15kt)

Reactor $\Rightarrow \theta_{13}$

'09～ **Double CHOOZ** **→ talks by Novella & Kawasaki**

'10～ **RENO**

'11 (?)～ **Daya Bay**

Far future experiments

Accelerator $\Rightarrow \theta_{13}, \mathrm{sgn}(\Delta m^2_{32}), \delta$

’xx∼ **T2K(K)** (JAERI→HK(+Korea)) L=295km(+1050km), E ∼1GeV
phase2 (4MW,500kt)

’yy∼ ν **factory** (?→?) L ∼ 4000km+7500km, E ∼20GeV
↳**talks by Bross & Mondal**

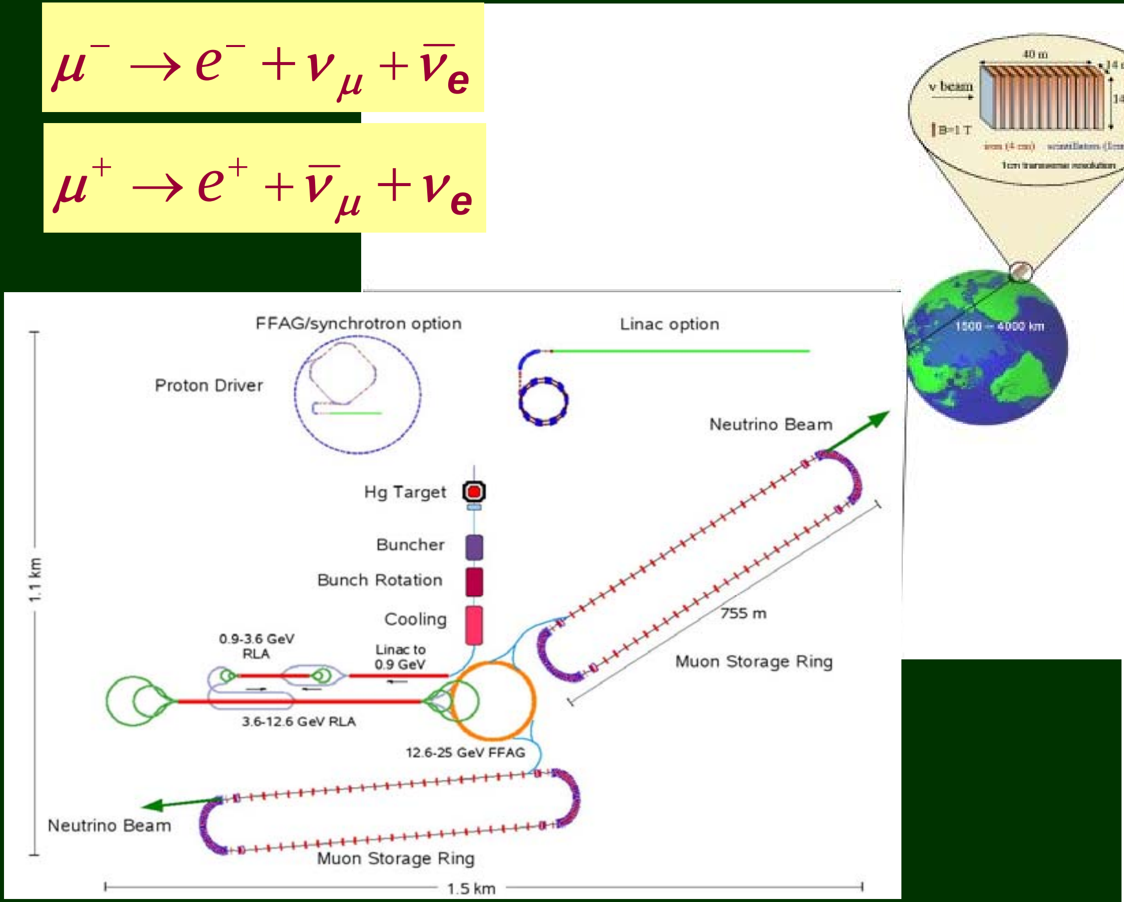
1.5 ν factory: ν from μ decays

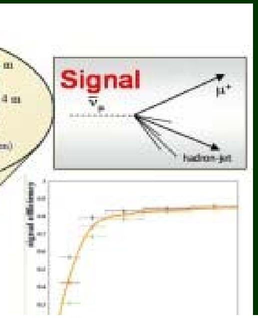
$\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$

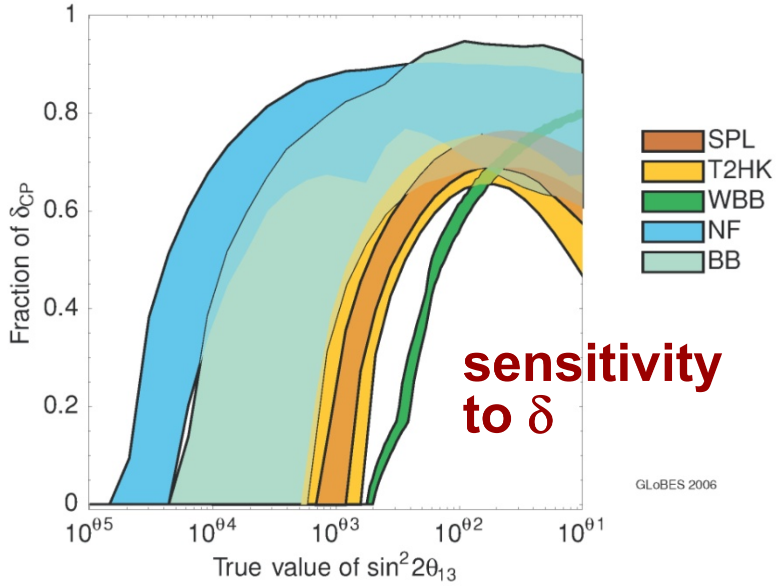
$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$

- **Large No. of events**
- **Low backgrounds**

→ Very good sensitivity







sensitivity to δ

$\nu_e \rightarrow \nu_\mu$

$\bar{\nu}_e \rightarrow \bar{\nu}_\mu$

golden channel

$\nu_\mu \rightarrow \nu_\mu$

$\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$

disappearance channel

$\nu_e \rightarrow \nu_\tau$

$\bar{\nu}_e \rightarrow \bar{\nu}_\tau$

silver channel

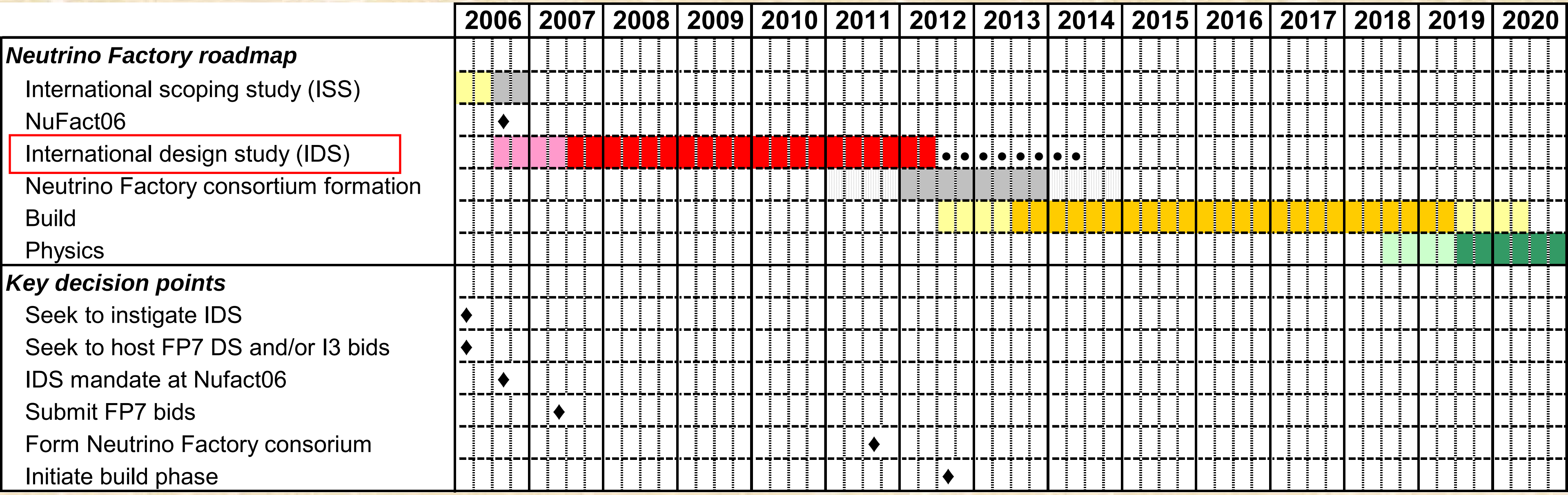
$\nu_\mu \rightarrow \nu_\tau$

$\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$

discovery channel

K.Long

NF roadmap: key decision points



	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Neutrino Factory roadmap															
International scoping study (ISS)	🟨▨														
NuFact06	◆														
International design study (IDS)	🟪	🟪🟥	🟥	🟥	🟥	🟥	🟥 • • •	• • • •	• •						
Neutrino Factory consortium formation						▨	▨	▨	▨						
Build							🟨	🟨🟧	🟧	🟧	🟧	🟧	🟧	🟧🟨	🟨
Physics													🟩	🟩	🟩
Key decision points															
Seek to instigate IDS	◆														
Seek to host FP7 DS and/or I3 bids	◆														
IDS mandate at Nufact06	◆														
Submit FP7 bids		◆													
Form Neutrino Factory consorium						◆									
Initiate build phase							◆								

- Ambitious, science-driven schedule
- Issue now is to establish vibrant R&D programme
- Vision for International Design Study phase:
  - International collaboration; coordinated effort:
    - *Concept development – full system*
    - *Accelerator R&D*
    - *Detector R&D*

Nagashima: ISS 3rd plenary ('06) @ RAL

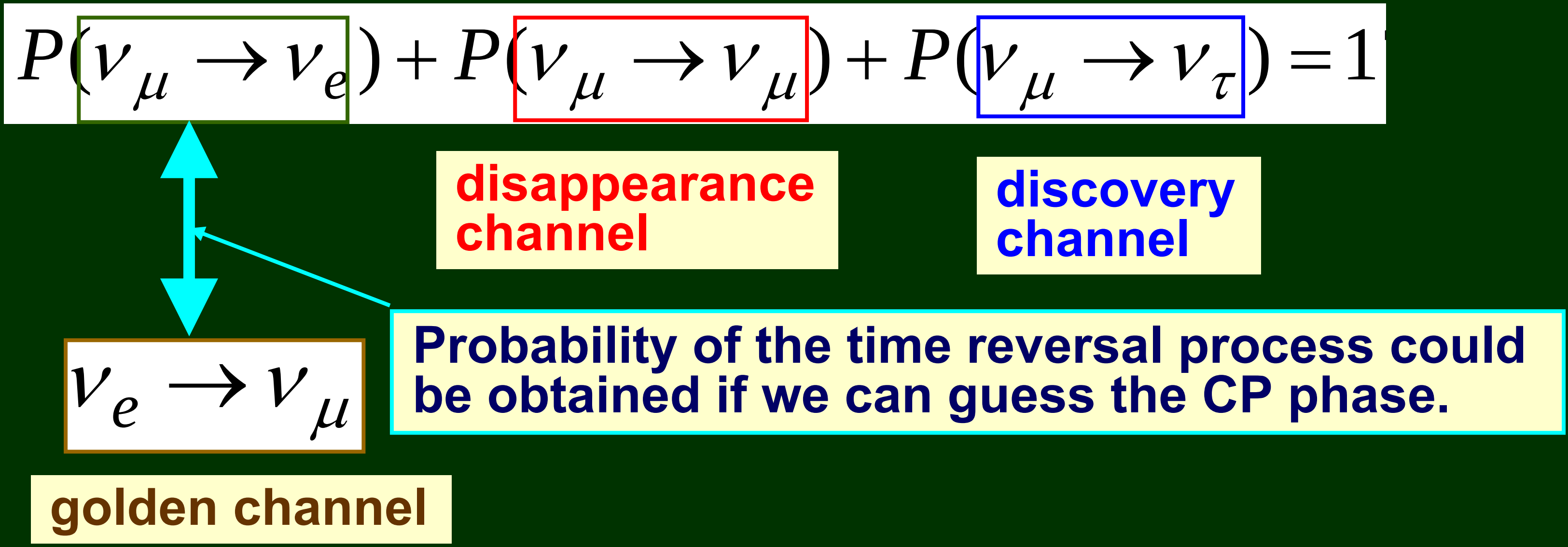
1.6 Motivation for research on New Physics and τ detection at ν factory

- **Just like at B factories, high precision measurements of ν oscillation at ν factory will allow us to probe physics beyond SM by looking at deviation from SM+massive ν.**
- **If θ_{13} turns out to be large, conventional super-beam experiments (T2K etc) may be sufficient.**

→ Search for new physics and test of unitarity would be even more important subjects at ν factory.

(cf. $\sin^2\theta_{13}=0.02\pm0.01$@$1\sigma$, Fogli et al, arXiv:0905.3549 [hep-ph])

- If 3 flavor unitarity is guaranteed, then roughly speaking, we could guess (discovery) from (golden) + (disappearance) at ν factory from 3 flavor unitarity:



$$P\left(\boxed{\nu_\mu \to \nu_e}\right) + P\left(\boxed{\nu_\mu \to \nu_\mu}\right) + P\left(\boxed{\nu_\mu \to \nu_\tau}\right) = 1$$

disappearance channel

discovery channel

$$\boxed{\nu_e \to \nu_\mu}$$

Probability of the time reversal process could be obtained if we can guess the CP phase.

golden channel

- Intuitively, therefore, τ detection is supposed to be important to test New Physics which violates unitarity.

→ Quantitative estimate is necessary to draw conclusions.

New physics which can be probed at a neutrino factory includes:

- Non standard interactions in propagation
- Non standard interactions at production / detection
- Violation of unitarity due to heavy particles
- Schemes with light sterile neutrinos

$$\sum_{\beta=e,\mu,\tau} P(\nu_\alpha \to \nu_\beta) = 1$$

Scenarios	3 flavor unitarity
NSI in propagation	✓
NSI at production / detection	✗
Violation of unitarity due to heavy particles	✗
Light sterile neutrinos	✗

Scenarios	Phenomenological bound on deviation of unitarity
NSI at production / detection	O(1%)
Violation of unitarity due to heavy particles	O(0.1%)
Light sterile neutrinos	O(10%)

Light sterile neutrinos could be phenomenologically more promising than others!

1.7 Light sterile neutrinos

● accelerator ν anomaly LSND

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$$

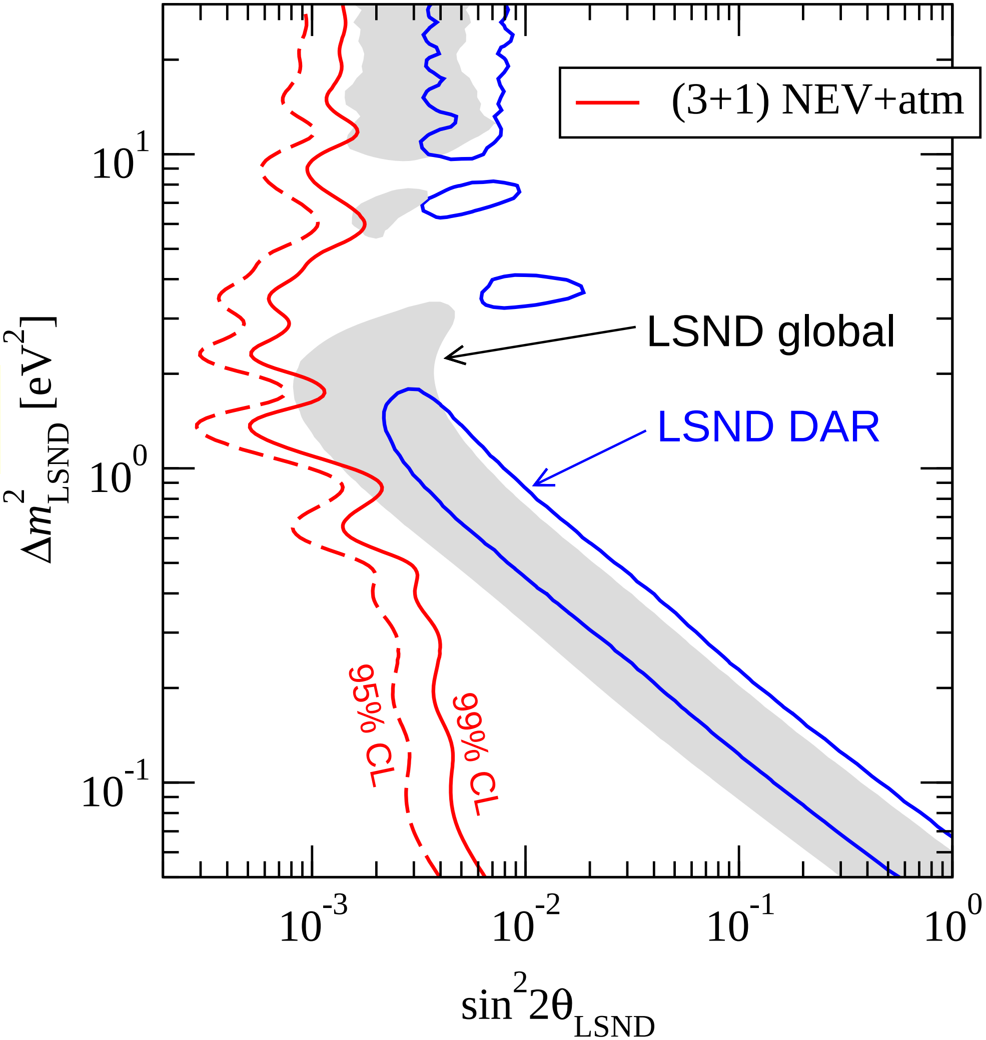
$E_\nu \cong 50\mathrm{MeV}$

$L \cong 30\mathrm{m}$

$\Delta m^2 \cong O(1)\mathrm{eV}^2$

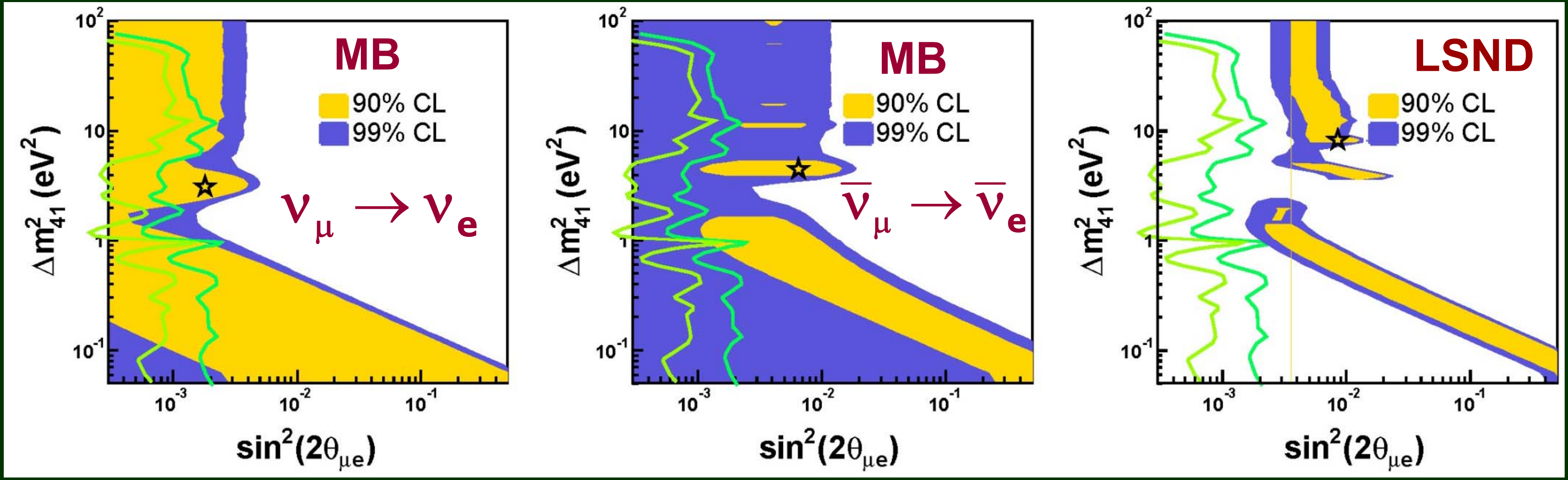
$\sin^2 2\theta \cong O(10^{-2})$??

Maltoni et al., hep-ph/0405172

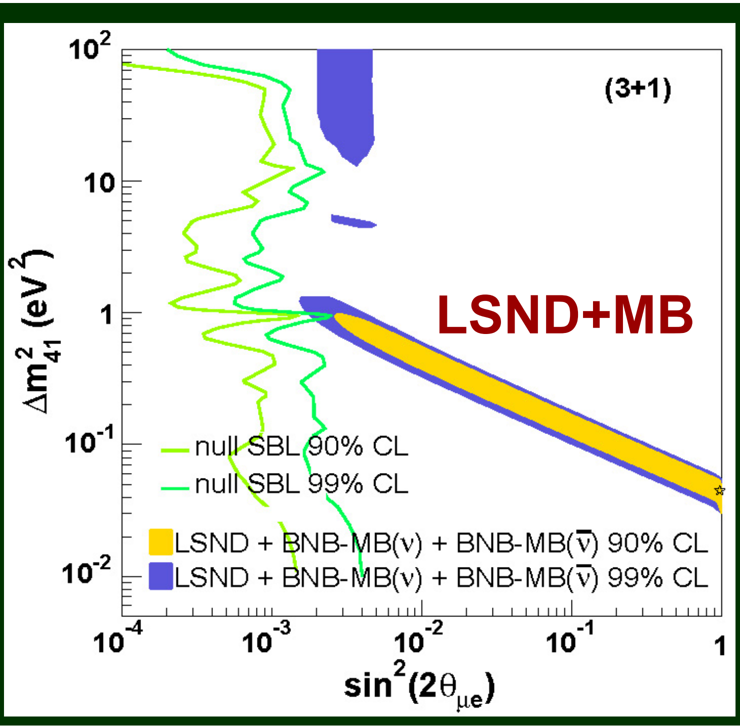


Recent status of LSND: Check by MiniBooNE

Karagiorgi et al,
Phys.Rev.D80:073001,2009



- Neither MiniBooNE (ν or $\bar{\nu}$) nor disappearance results (CDHSW+Bugey+atm) excludes LSND at 4σ.



N_ν=4 schemes

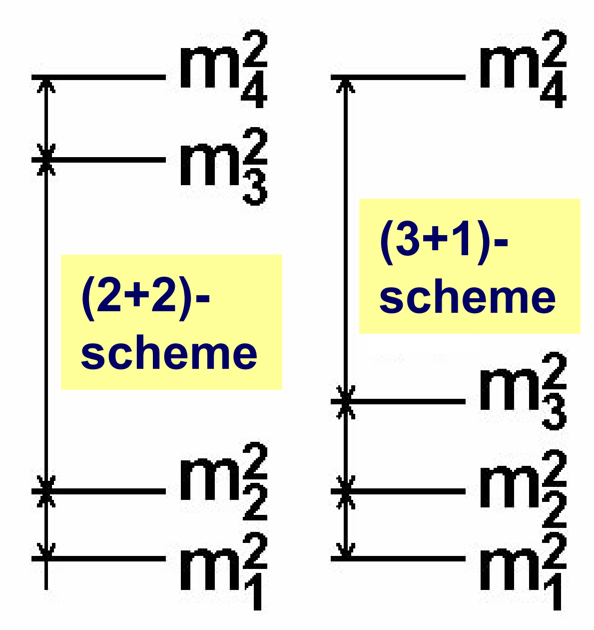
Because of the hierarchy: $\Delta m^2_{sol} \ll \Delta m^2_{atm} \ll \Delta m^2_{LSND}$

N_ν=3 schemes can't explain LSND.

N_ν=4 schemes may be able to explain all.

$\Delta m^2_{21} = \Delta m^2_{sol}, \Delta m^2_{32} = \Delta m^2_{atm}, \Delta m^2_{43} = \Delta m^2_{LSND}$

LEP → 4th ν has to be sterile



(2+2)-scheme is excluded by solar + atmospheric ν

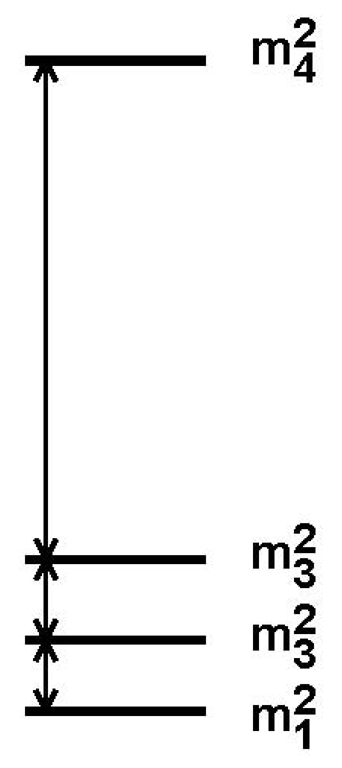
→ (3+1)-scheme will be discussed

2. Sterile neutrinos at ν factory

There are two view points:

(3+1)-scheme w/ LSND: the situation is unclear, so it’s worth checking it

(3+1)-scheme w/o LSND: still a possible scenario, provided that the mixing angles satisfy all the constraints of the negative results



$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

ν factory at L=4000km+7500km has sensitivity to sterile neutrino mixings through various channels:

golden

$$P(\nu_e \to \nu_\mu) = 4\mathrm{Re}\left[U_{e3}U^*_{\mu3}(U^*_{e3}U_{\mu3} + \boxed{U^*_{e4}U_{\mu4}})\right]\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right) + \cdots$$

silver

$$P(\nu_e \to \nu_\tau) = 4\mathrm{Re}\left[U_{e3}U^*_{\tau3}(U^*_{e3}U_{\tau3} + \boxed{U^*_{e4}U_{\tau4}})\right]\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right) + \cdots$$

disappearance

$$P(\nu_\mu \to \nu_\mu) = 1 - 4|U_{\mu3}|^2(1 - |U_{\mu3}|^2 - \boxed{|U_{\mu4}|^2})\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right) + \cdots$$

discovery

$$P(\nu_\mu \to \nu_\tau) = 4\mathrm{Re}\left[U_{\mu3}U^*_{\tau3}(U^*_{\mu3}U_{\tau3} + \boxed{U^*_{\mu4}U_{\tau4}})\right]\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right) + \cdots$$

setup

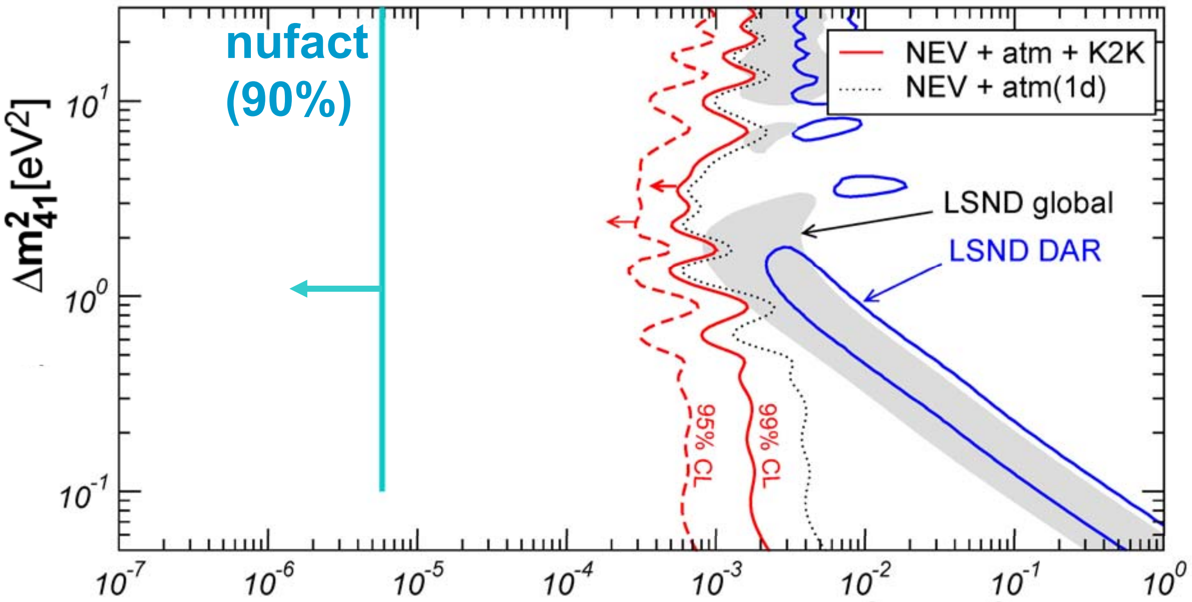
5×10^{20} μ^-+μ^+'s/yr × 4 yrs, E_μ =20GeV,
L= 4000+7500km,
50kton Magnetized Iron ν Detector +
4kton Magnetized Emulsion Cloud Chamber

Results

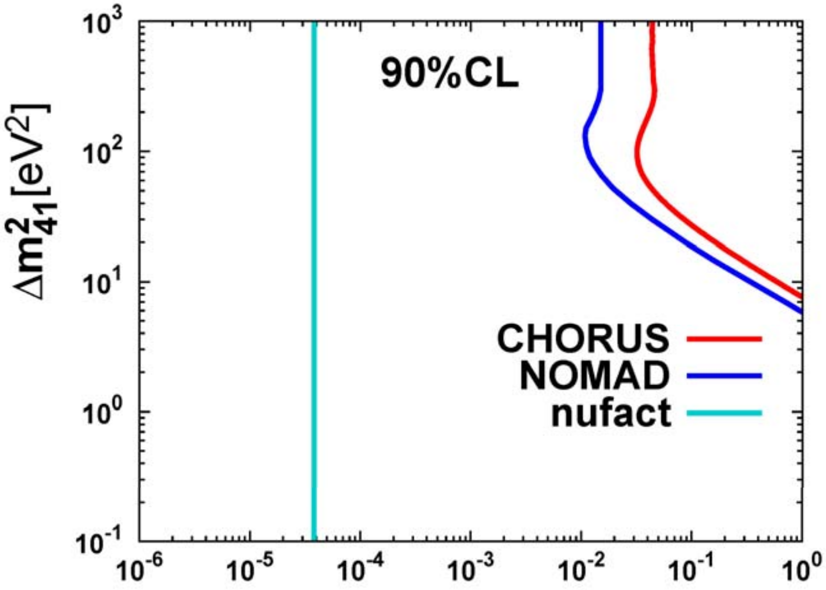
Donini, Fuki, Lopez-Pavon, Meloni, OY, JHEP 0908:041,2009

(1) Sensitivity to mixing angles

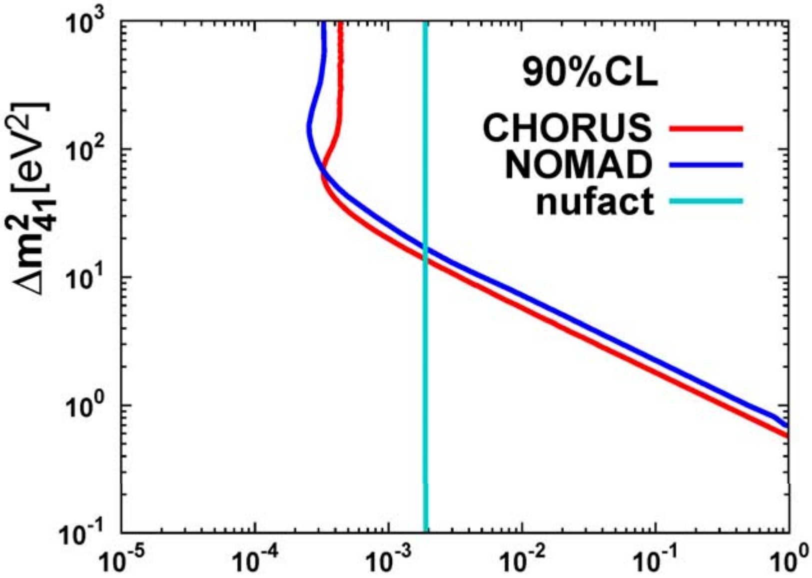
Sensitivity to the 4ν mixings with ν_e is very good compared to the present bound.
→ It could serve as a severe test of LSND/MiniBooNE.



golden $4|U_{e4}|^2|U_{\mu4}|^2$



silver $4|U_{e4}|^2|U_{\tau4}|^2$



discovery $4|U_{\mu4}|^2|U_{\tau4}|^2$

(2) Sensitivity to CP violation

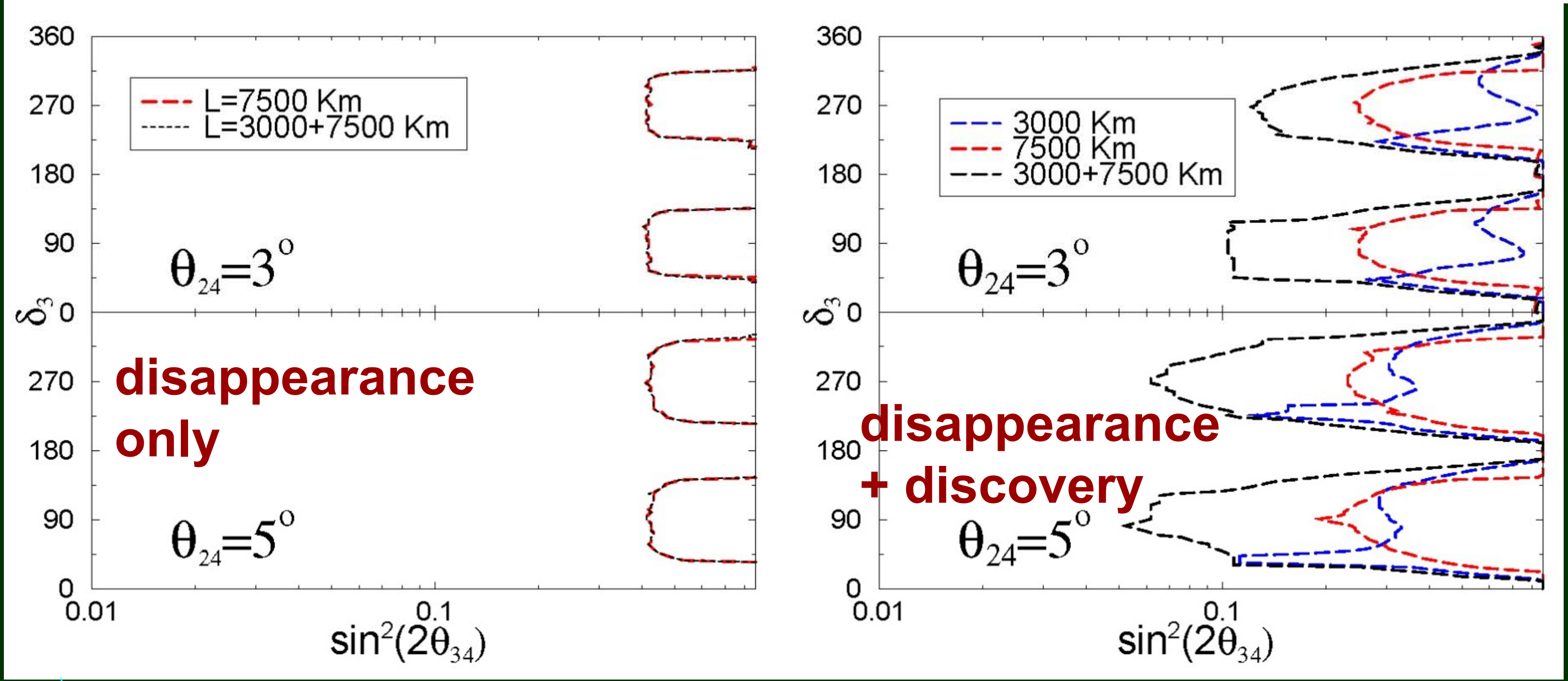
Donini, Fuki, Lopez-Pavon, Meloni, OY, JHEP 0908:041,2009

Potentially the largest CP violation occurs in μ-τ channel:
CP violation due to the new CP phase

$$P(\nu_\mu \to \nu_\tau) - P(\bar{\nu}_\mu \to \bar{\nu}_\tau) = 2 s_{24}\, s_{34} \boxed{\sin\delta_3} \sin(\Delta m^2_{31} L/4E) + \cdots$$

θ_{34} : ratio of $\nu_\mu \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow \nu_s$ in ν_{atm}

θ_{24} : ratio of $\sin^2(\frac{\Delta m^2_{\rm atm} L}{4E})$ and $\sin^2(\frac{\Delta m^2_{\rm SBL} L}{4E})$ in ν_{atm}



disappearance only

disappearance + discovery

Discovery channel is crucial to measure the new CP phase

3. Violation of unitarity w/o light ν_s

Antusch, Biggio, Fernandez-Martinez, Gavela, Lopez-Pavon, JHEP0610,084, ‘06

In generic see-saw models, after integrating out ν_R, the kinetic term gets modified, and unitarity is expected to be violated.

$$L = \frac{1}{2}\left(i\overline{\nu}_\alpha \partial K_{\alpha\beta}\nu_\beta - \overline{\nu}^c{}_\alpha M_{\alpha\beta}\nu_\beta\right) - \frac{g}{\sqrt{2}}\left(W^+_\mu \overline{l}_\alpha \gamma^\mu P_L \nu_\alpha + h.c.\right) + \ldots$$

rescaling ν →

$$L = \frac{1}{2}\left(i\overline{\nu}_i \partial \nu_i - \overline{\nu}^c{}_i m_{ii}\nu_i\right) - \frac{g}{\sqrt{2}}\left(W^+_\mu \overline{l}_\alpha \gamma^\mu P_L N_{\alpha i}\nu_i\right) + \ldots$$

N: non-unitary

Some see-saw models (e.g., inverse see-saw) do have two scales, one to produce small neutrino mass and another which may not be extremely different from M_W .

→ Magnitude of violation may not be extremely small.

→ Unitarity of the lepton sector is worth checking.

Oscillation probability

$$P(\nu_\alpha \to \beta) = \left| \left[H\tilde{U} \exp\left\{ -i\mathrm{diag}(\tilde{E}_j)L \right\} \tilde{U}^{-1} H \right]_{\beta\alpha} \right|^2$$

$$U\,\mathrm{diag}(E_j)\,U^{-1} + H\mathcal{A}_0 H = \tilde{U}\,\mathrm{diag}(\tilde{E}_j)\,\tilde{U}^{-1}$$

$$N \equiv HU$$

***H*: close to identity**

$$NN^\dagger - 1 = H^2 - 1:\ \text{deviation from unitarity}$$

Constraints from weak decays are more stringent than ν oscillation:

$$|(NN^\dagger)_{\alpha\beta} - \delta_{\alpha\beta}| < \begin{pmatrix} 6\times10^{-3} & 7.1\times10^{-5} & 1.6\times10^{-2} \\ 7.1\times10^{-5} & 5\times10^{-3} & 1.0\times10^{-2} \\ 1.6\times10^{-2} & 1.0\times10^{-2} & 5\times10^{-3} \end{pmatrix}$$

Antusch et al,
JHEP0610,084, '06

Assuming the origin comes from d=6 operator, constraints become even more stringent:

$$|(NN^\dagger)_{\alpha\beta} - \delta_{\alpha\beta}| < \begin{pmatrix} 4.0\times10^{-3} & 1.8\times10^{-3} & 3.2\times10^{-3} \\ 1.8\times10^{-3} & 1.6\times10^{-3} & 2.1\times10^{-3} \\ 3.2\times10^{-3} & 2.1\times10^{-3} & 5.3\times10^{-3} \end{pmatrix}$$

Antusch, Baumann, Fernandez-Martinez, Nucl.Phys.B810:369, 2009

Sensitivity at ν factory

Antusch et al, JHEP0610,084, 2006

- **4kt OPERA-like near detector @100 m**

$\nu_e \to \nu_\tau$ $$\left|\sum_i N_{ei} N^*_{\tau i}\right| < 2.9 \times 10^{-3} \ (\text{present} : 0.016)$$

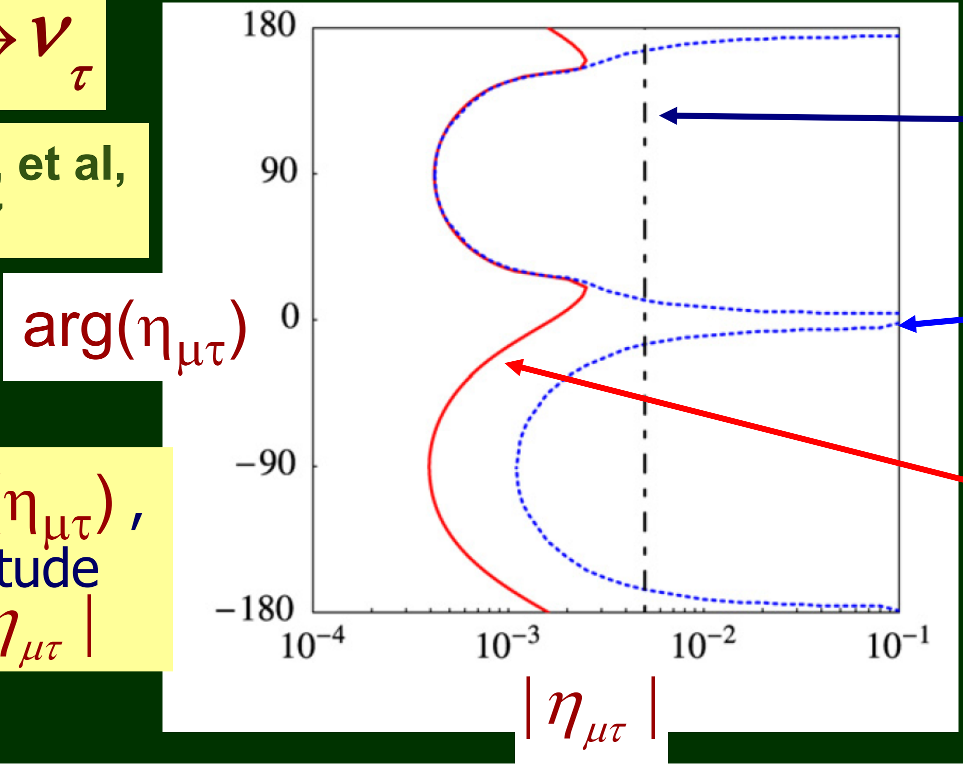
$\nu_\mu \to \nu_\tau$ $$\left|\sum_i N_{\mu i} N^*_{\tau i}\right| < 2.6 \times 10^{-3} \ (\text{present} : 0.013)$$

- **5kt OPERA-like far detector @130 km**

$\nu_\mu \to \nu_\tau$

Fernandez-Martinez, et al, PLB649:427,2007

$H \equiv 1 + \eta$



Present bound from $\tau \to \mu\gamma$

Sensitivity to arg($\eta_{\mu\tau}$)

Sensitivity to $|\eta_{\mu\tau}|$

For non-trivial arg($\eta_{\mu\tau}$), one order of magnitude improvement for $|\eta_{\mu\tau}|$

4. Summary

- ν factory can search for new physics which violates 3 flavor unitarity , such as sterile ν mixings and unitarity violation due to heavy fields, etc.
- In absence of 3 flavor unitarity, τ detectors in principle give us important information on New Physics.
- ν factory can offer a powerful test of LSND/MiniBooNE.
- To measure the new CP phase due to sterile ν mixings or unitarity violation due to heavy fields, discovery channel is important.

Backup slides

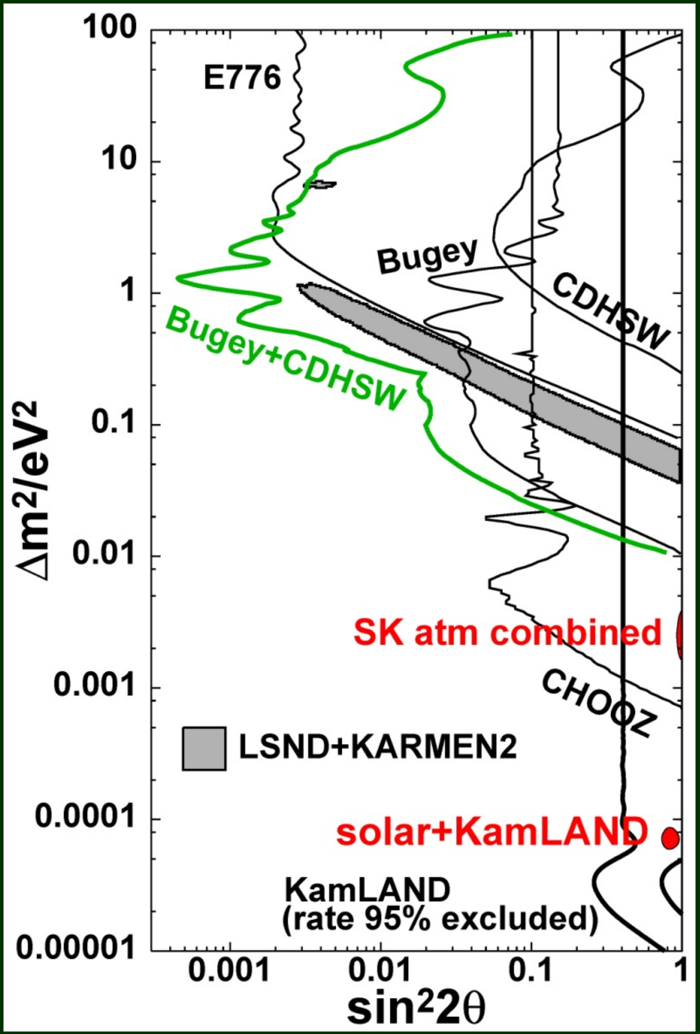
(3+1)-scheme

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - 4|U_{e4}|^2(1 - |U_{e4}|^2)\sin^2(\Delta m^2_{41}L/4E)$$
$$P(\nu_\mu \to \nu_\mu) = 1 - 4|U_{\mu 4}|^2(1 - |U_{\mu 4}|^2)\sin^2(\Delta m^2_{41}L/4E)$$
$$P(\bar{\nu}_\mu \to \bar{\nu}_e) = 4|U_{e4}|^2|U_{\mu 4}|^2\sin^2(\Delta m^2_{41}L/4E)$$

$$\sin^2 2\theta_{\mathrm{Bugey}} > 4|U_{e4}|^2(1 - |U_{e4}|^2) \cong 4|U_{e4}|^2$$

$$\sin^2 2\theta_{\mathrm{CDHSW}} > 4|U_{\mu 4}|^2(1 - |U_{\mu 4}|^2) \cong 4|U_{\mu 4}|^2$$

$$\sin^2 2\theta_{\mathrm{LSND}} = 4|U_{e4}|^2|U_{\mu 4}|^2$$

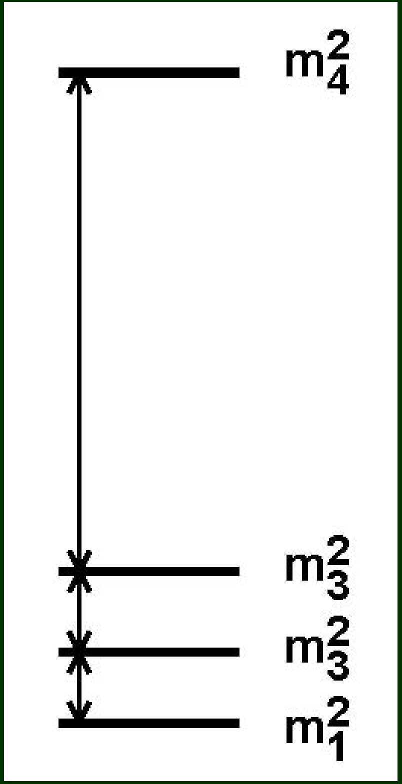


$$\sin^2 2\theta_{\mathrm{LSND}}(\Delta m^2) < \frac{1}{4}\sin^2 2\theta_{\mathrm{Bugey}}(\Delta m^2)\sin^2 2\theta_{\mathrm{CDHSW}}(\Delta m^2)$$

must be satisfied (Okada-OY Int.J.Mod.Phys.A12:3669,1997)

But there is no overlap between LSND and left side of Bugey+CDHSW

Sterile neutrinos at ν factory



(3+1)-scheme w/ LSND: the situation is unclear, so it's worth checking it

(3+1)-scheme w/o LSND: still a possible scenario, provided that the mixing angles satisfy all the constraints of the negative results

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix} \qquad U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

$$U = R_{34}(\theta_{34},\, 0)\; R_{24}(\theta_{24},\, 0)\; R_{23}(\theta_{23},\, \delta_3)\; R_{14}(\theta_{14},\, 0)\; R_{13}(\theta_{13},\, \delta_2)\; R_{12}(\theta_{12},\, \delta_1)$$

θ_{34} : ratio of $\nu_\mu \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow \nu_s$ in ν_{atm}

θ_{24} : ratio of $\sin^2(\frac{\Delta m^2_{\rm atm} L}{4E})$ and $\sin^2(\frac{\Delta m^2_{\rm SBL} L}{4E})$ in ν_{atm}

θ_{14} : mixing angle in $\nu_{reactor}$ at L=O(10m)

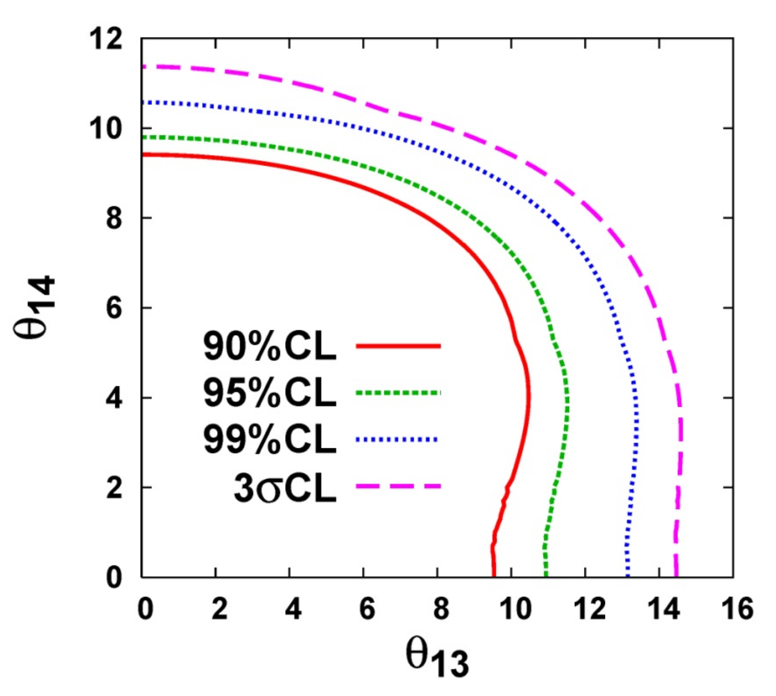
Constraints on (3+1)-scheme from ν_{atm} and SBL

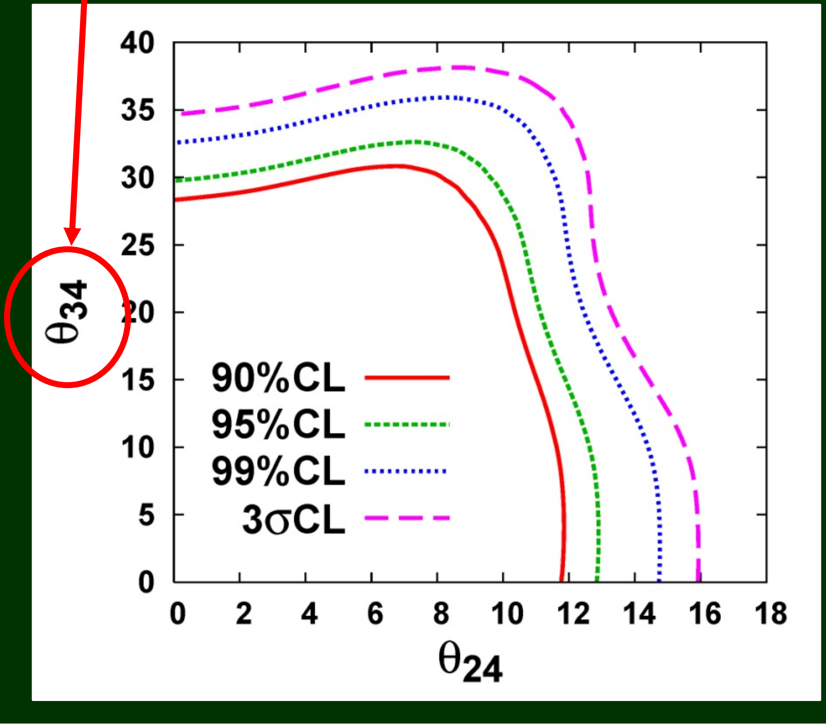
Donini-Maltoni-Meloni-Migliozzi-Terranova, JHEP 0712:013,'07

$$U = R_{34}(\theta_{34})\; R_{24}(\theta_{24})\; R_{23}(\theta_{23},\, \delta_3)\; R_{14}(\theta_{14})\; R_{13}(\theta_{13},\, \delta_2)\; R_{12}(\theta_{12},\, \delta_1)$$

Assumption on rapid oscillations in ν_{atm}: $\Delta m^2_{41} > 0.1\ \mathrm{eV}^2$

θ_{34} : could be relatively large





Sensitivity to θ_{14}, θ_{24}, θ_{34} at ν factory with far detectors

Donini, Fuki, Lopez-Pavon, Meloni, OY, JHEP 0908:041,2009

5×10^{20} $\mu^{-}+\mu^{+}$'s/yr × 4 yrs
$(E_\mu$/GeV, L/km)= (50,3000+7500) or (20, 4000+7500)
50kton MIND + 4kton MECC

statistical errors + systematic errors + BG

efficiency ~ 0.7 for μ, ~0.65 for τ

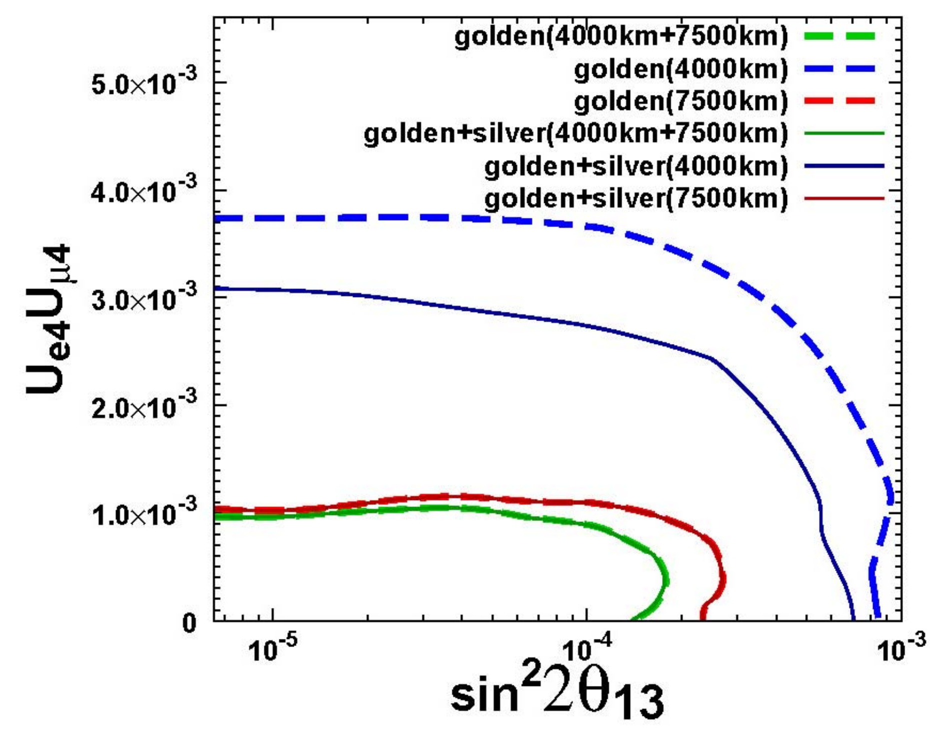
NB. Magnetized Emulsion Cloud Chamber (MECC)
active target: iron
$\tau\rightarrow\mu$ decay + $\tau\rightarrow e$ decay + $\tau\rightarrow$hadron decay are used

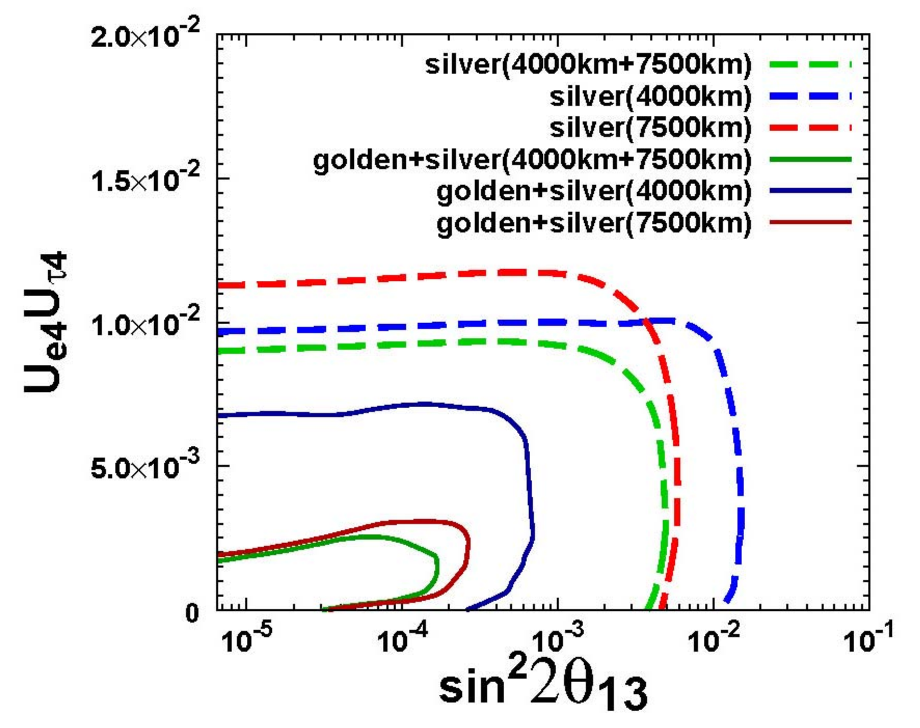
golden + silver

Donini, Fuki, Lopez-Pavon, Meloni, OY,
JHEP 0908:041,2009

$$P(\nu_e \to \nu_\mu) = 4\mathrm{Re}\left[U_{e3}U^*_{\mu 3}(U^*_{e3}U_{\mu 3} + \boxed{U^*_{e4}U_{\mu 4}})\right] \sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right) + \cdots$$

$$P(\nu_e \to \nu_\tau) = 4\mathrm{Re}\left[U_{e3}U^*_{\tau 3}(U^*_{e3}U_{\tau 3} + \boxed{U^*_{e4}U_{\tau 4}})\right] \sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right) + \cdots$$





$4|U_{e4}U_{\mu 4}|^2 > 5.8\times10^{-6}$

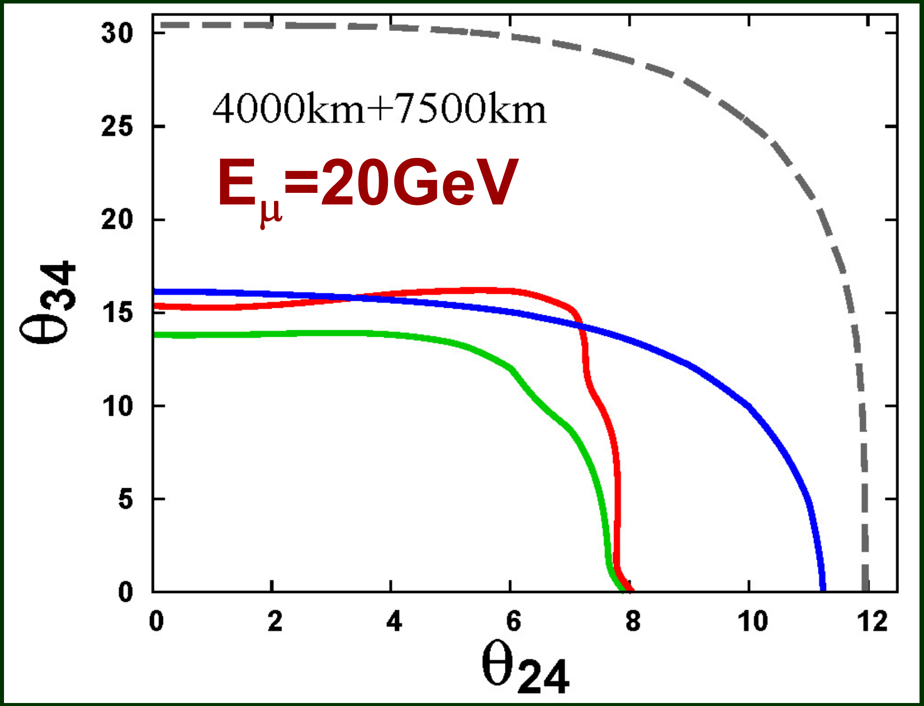
$4|U_{e4}U_{\tau 4}|^2 > 3.8\times10^{-5}$

disappearance + discovery

Donini, Fuki, Lopez-Pavon, Meloni, OY,
JHEP 0908:041,2009

$$P(\nu_\mu \to \nu_\mu) \;=\; 1 - 4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2 - \boxed{|U_{\mu 4}|^2}) \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) + \cdots$$

$$P(\nu_\mu \to \nu_\tau) \;=\; 4\mathrm{Re}\left[U_{\mu 3}U^*_{\tau 3}(U^*_{\mu 3}U_{\tau 3} + \boxed{U^*_{\mu 4}U_{\tau 4}})\right] \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) + \cdots$$



--- current
--- disappearance
--- discovery
--- combined

$4|U_{\mu 4}|^2 > 7.6\text{x}10^{-2}$

$4|U_{\mu 4}U_{\tau 4}|^2 > 1.9\text{x}10^{-3}$