New plots & parameter degeneracy in ν oscillations

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1. Introduction

Even if we know $P(\nu_\mu \to \nu_e)$ and $P(\overline{\nu_\mu} \to \overline{\nu_e})$ in a long baseline accelerator experiments with approximately monoenergetic neutrino beam, precise determination of θ_{13}, sign(Δm^2_{31}) and δ is difficult because of the 8-fold parameter degeneracy.

- intrinsic (δ, θ_{13}) degeneracy
- $\Delta m^2_{31} \Leftrightarrow -\Delta m^2_{31}$ degeneracy
- $\theta_{23} \Leftrightarrow \pi/2 - \theta_{23}$ degeneracy

Intuitive understanding of 8-fold degeneracy is import ➡ **plots in ($\sin^2 2\theta_{13}$, $1/s^2_{23}$) plane**

Plots in ($\sin^2 2\theta_{13}$, $1/s^2_{23}$) plane

The way curves intersect is easy to see

(P=const, δ=const)

($\bar{P}$=const, δ=const)

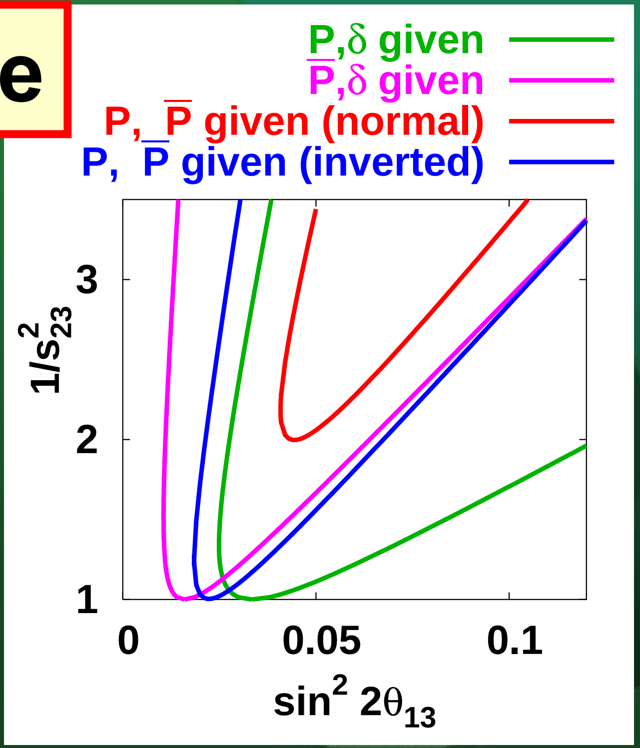
(P=const& $\bar{P}$=const' off OM)

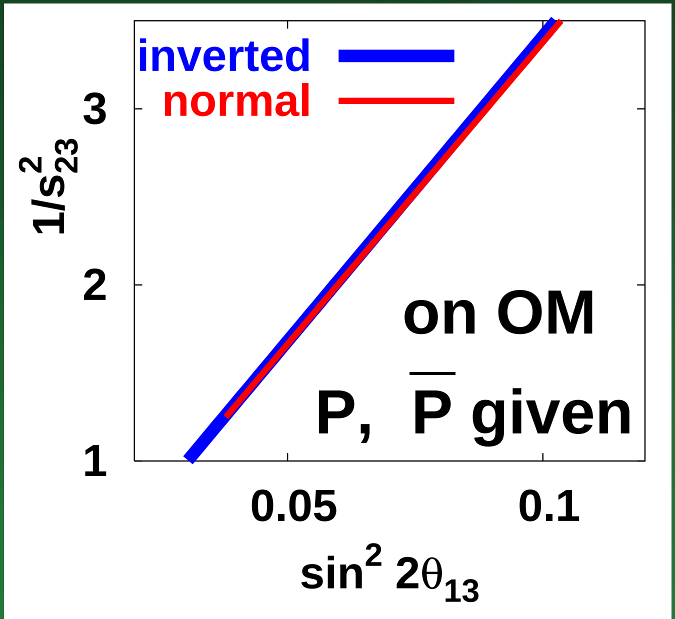
hyperbolas

(or ellipses)

(P=const& $\bar{P}$=const' on OM)

straight lines





Notations:

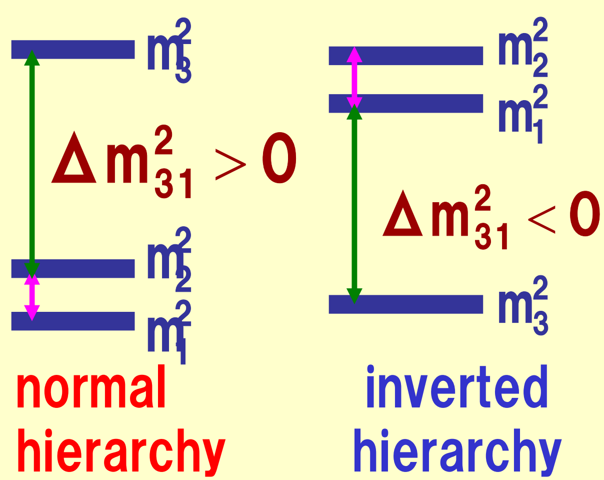
$$P \equiv P(\nu_\mu \to \nu_e)$$

$$\bar{P} \equiv P(\overline{\nu_\mu} \to \overline{\nu_e})$$

$$\Delta \equiv \frac{|\Delta m^2_{31}| L}{4E}$$

Oscillation Maximum (OM)

$$\Delta \equiv \frac{|\Delta m^2_{31}| L}{4E} = \frac{\pi}{2}$$



normal hierarchy

$$P = f^2 x^2 + 2xyfg\cos(\delta + \Delta) + g^2 y^2$$

$$\bar{P} = \bar{f}^2 x^2 + 2xy\bar{f}g\cos(\delta - \Delta) + g^2 y^2$$

inverted hierarchy

$$P = \bar{f}^2 x^2 - 2xy\bar{f}g\cos(\delta + \Delta) + g^2 y^2$$

$$\bar{P} = f^2 x^2 - 2xyfg\cos(\delta - \Delta) + g^2 y^2$$

$$x \equiv s_{23}\sin 2\theta_{13},$$

$$y \equiv |\Delta m^2_{21} / \Delta m^2_{31}| c_{23} \sin 2\theta_{12},$$

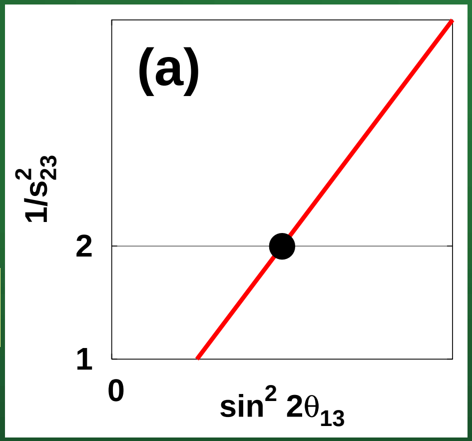
$$f, \bar{f} \equiv \sin(\Delta \mp AL/2) / (1 \mp AL/2\Delta),$$

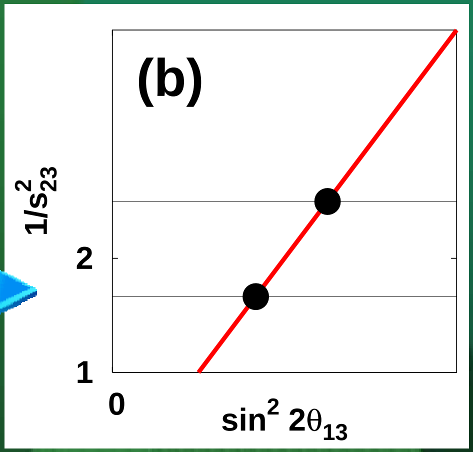
$$g \equiv \sin(AL/2) / (AL/2\Delta)$$

$$A \equiv \sqrt{2} G_F N_e$$

● $\theta_{23} \Leftrightarrow \pi/2 - \theta_{23}$ degeneracy

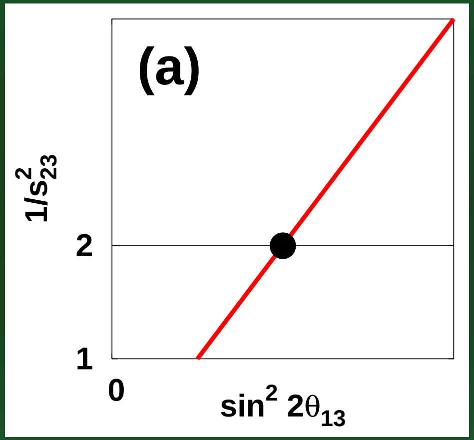
$$(a)\cos 2\theta_{23} = 0 \rightarrow (b)\cos 2\theta_{23} \neq 0$$

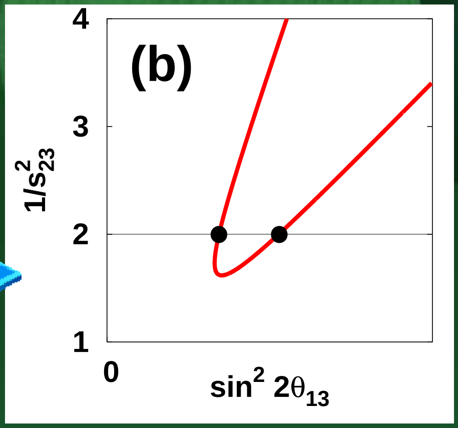




● intrinsic (δ, θ_{13}) degeneracy

$$(a)\frac{\Delta m^2_{21}}{|\Delta m^2_{31}|} = 0 \rightarrow (b)\frac{\Delta m^2_{21}}{|\Delta m^2_{31}|} \cong \frac{1}{35} \neq 0$$

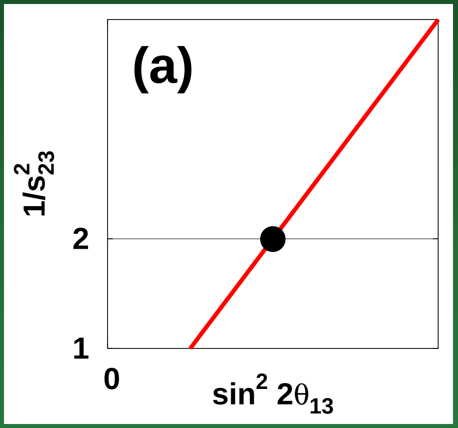


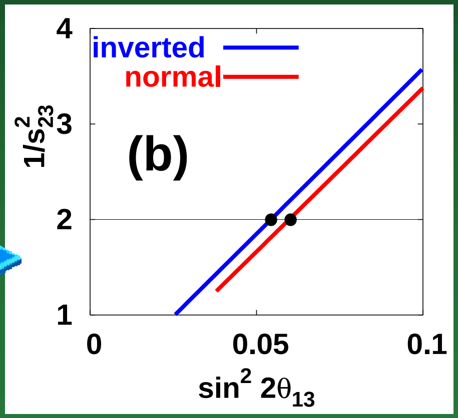


● $\Delta m^2_{31} \Leftrightarrow -\Delta m^2_{31}$ degeneracy

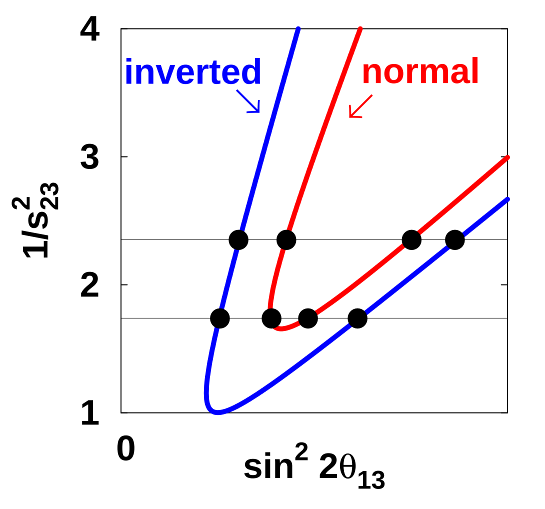
$$(a)AL/2 = 0 \rightarrow (b)AL/2 \neq 0$$

$$A \equiv \sqrt{2}G_F N_e \cong 1/2000\text{km}$$



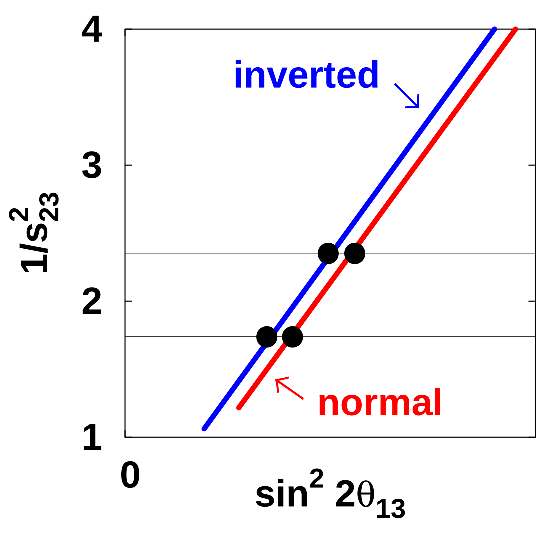


Off OM we have 8-fold parameter degeneracy



Plot of $P(\nu_\mu \to \nu_e), P(\overline{\nu_\mu} \to \overline{\nu_e})$ =const.

On OM we have 4-fold parameter degeneracy



JPARC experiment is expected to be done on OM

→ intrinsic (δ, θ_{13}) degeneracy is not a problem at JPARC

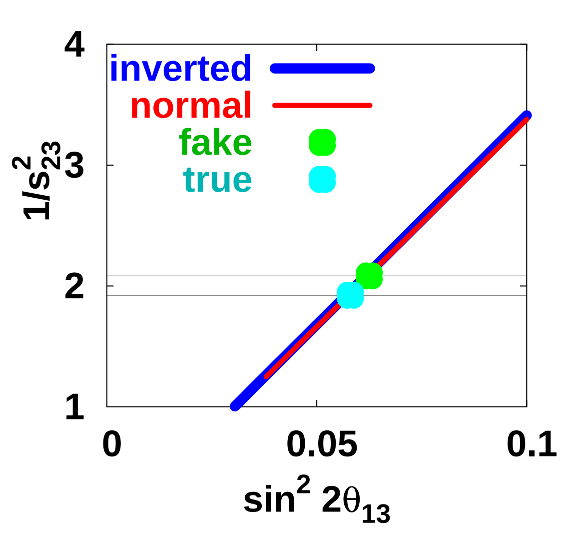
2. Determination of θ_{13}

Assumption: $\nu_\mu \to \nu_e$ and $\overline{\nu_\mu} \to \overline{\nu_e}$ will be measured at JPARC (@OM, 4MW, HK) .

Question: **Will that be enough to determine $|U_{e3}|$?**

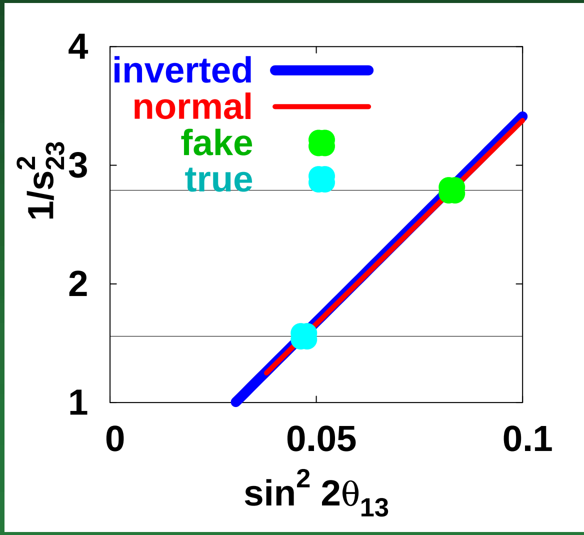
(1) $\sin^2 2\theta_{23} \simeq 1$

JPARC $\nu + \bar{\nu}$ is almost enough, since (a) there is no intrinsic (δ, θ_{13}) degeneracy, and (b) sign(Δm^2_{31}) degeneracy is small.



(2) $\sin^2 2\theta_{23} < 1$

Ambiguity due to $\theta_{23} \Leftrightarrow \pi/2 - \theta_{23}$ degeneracy is significant.



In the case of (1):

JPARC experiment is enough to determine θ_{13}.

In the case of (2):

To resolve θ_{23} ambiguity, possible ways are:

(A) reactor measurement of θ_{13}

(B) LBL measurement of $\nu_\mu \to \nu_e$ (or $\nu_e \to \nu_\mu$)

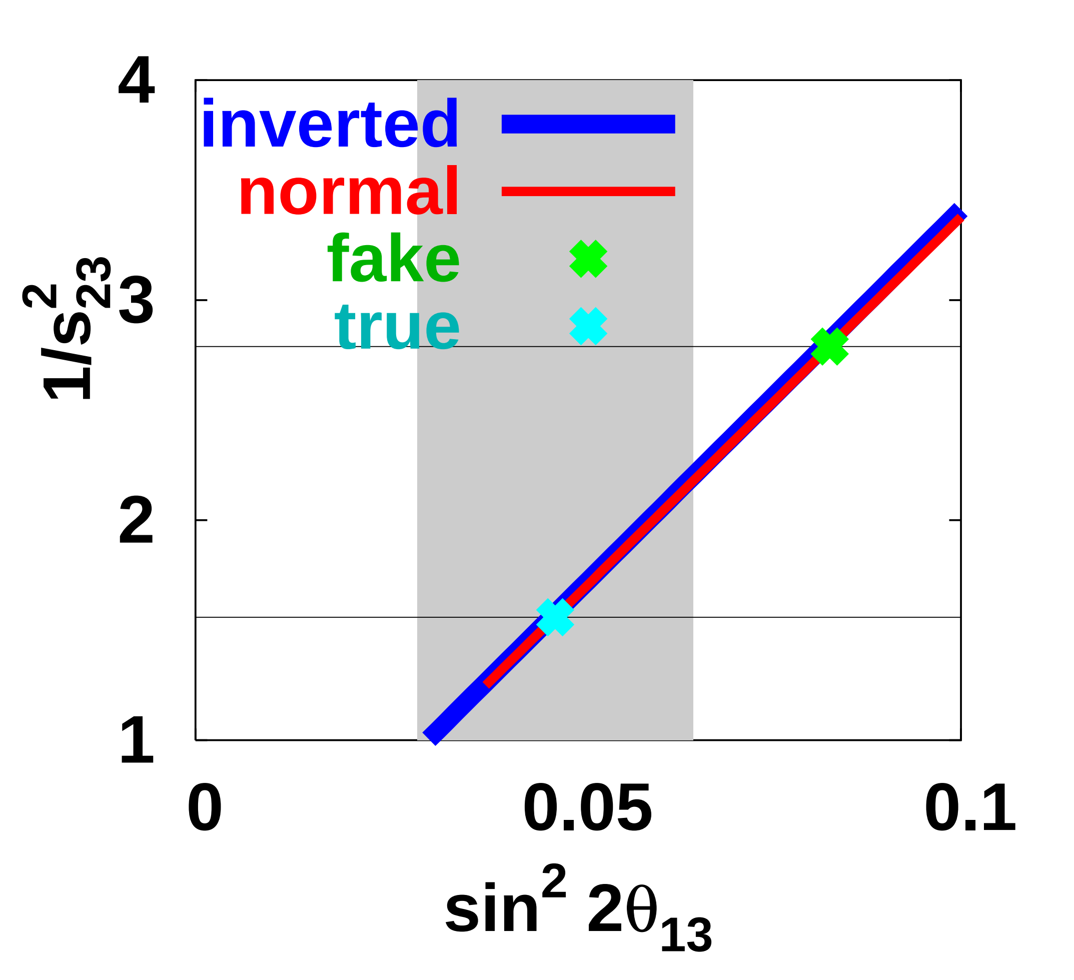
(C) measurement of $\nu_e \to \nu_\tau$

The reference values used here are:

$\sin^2 2\theta_{23}=0.96$, $\sin^2 2\theta_{13}=0.05$, $\delta=\pi/4$, $\Delta m^2_{31}>0$

(A) reactor measurement of θ 13 $\bar{\nu}_e \to \bar{\nu}_e$

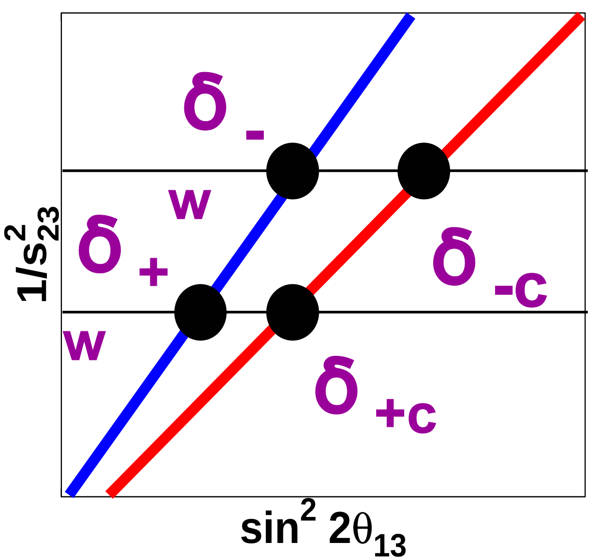
One can resolve θ 23 ambiguity at 90%CL.



(B) LBL measurement of $\nu_\mu \to \nu_e$ (or $\nu_e \to \nu_\mu$)

Consider 3rd measurement of $\nu_\mu \to \nu_e$ (or $\nu_e \to \nu_\mu$) in addition to JPARC $\nu + \bar{\nu}$.

↓ (exaggerated figure)



correct assumption
wrong assumption
on mass hierarchy

The value of δ for each point can be deduced (up to δ⇔π - δ) from

$$\sin\delta = -\frac{P - f^2x^2 - g^2y^2}{2fgxy}$$

Then from the equation for the probability of $\nu_\mu \rightarrow \nu_e$ (or $\nu_e \rightarrow \nu_\mu$) in the 3rd experiment

$$P_{true} = P\left(\sin^2 2\theta_{13}, \delta_{\pm[cw]}, s^2_{23}\right)$$

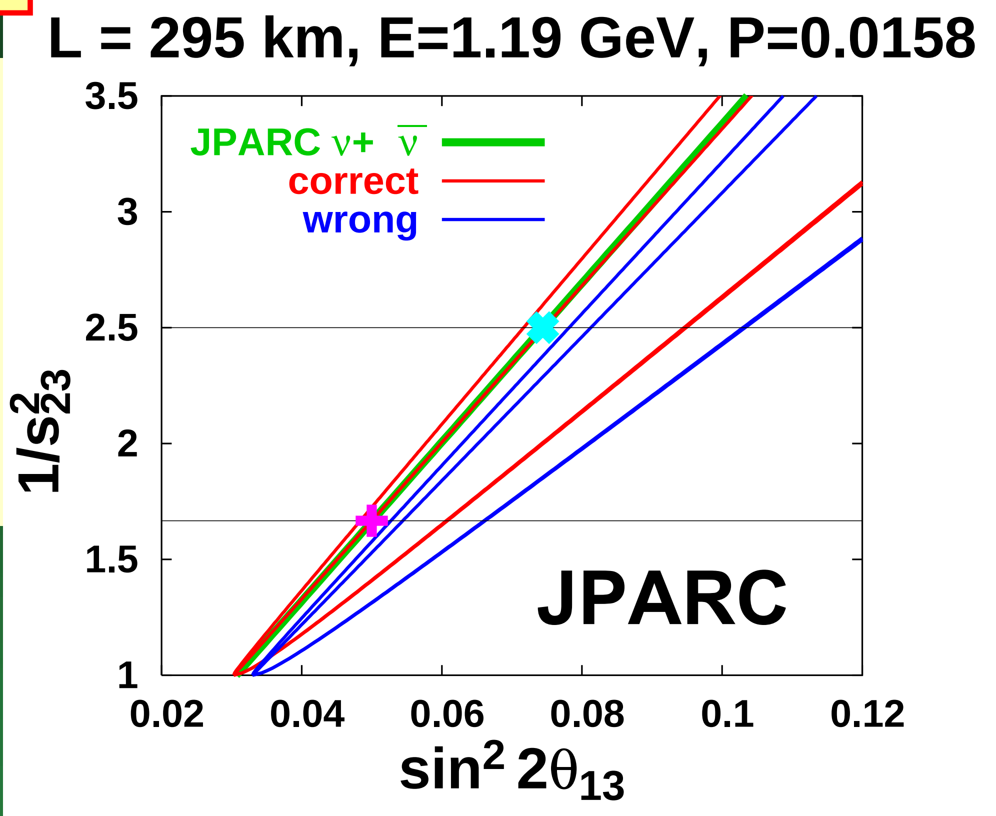
or

$$P_{true} = P\left(\sin^2 2\theta_{13}, \pi - \delta_{\pm[cw]}, s^2_{23}\right)$$

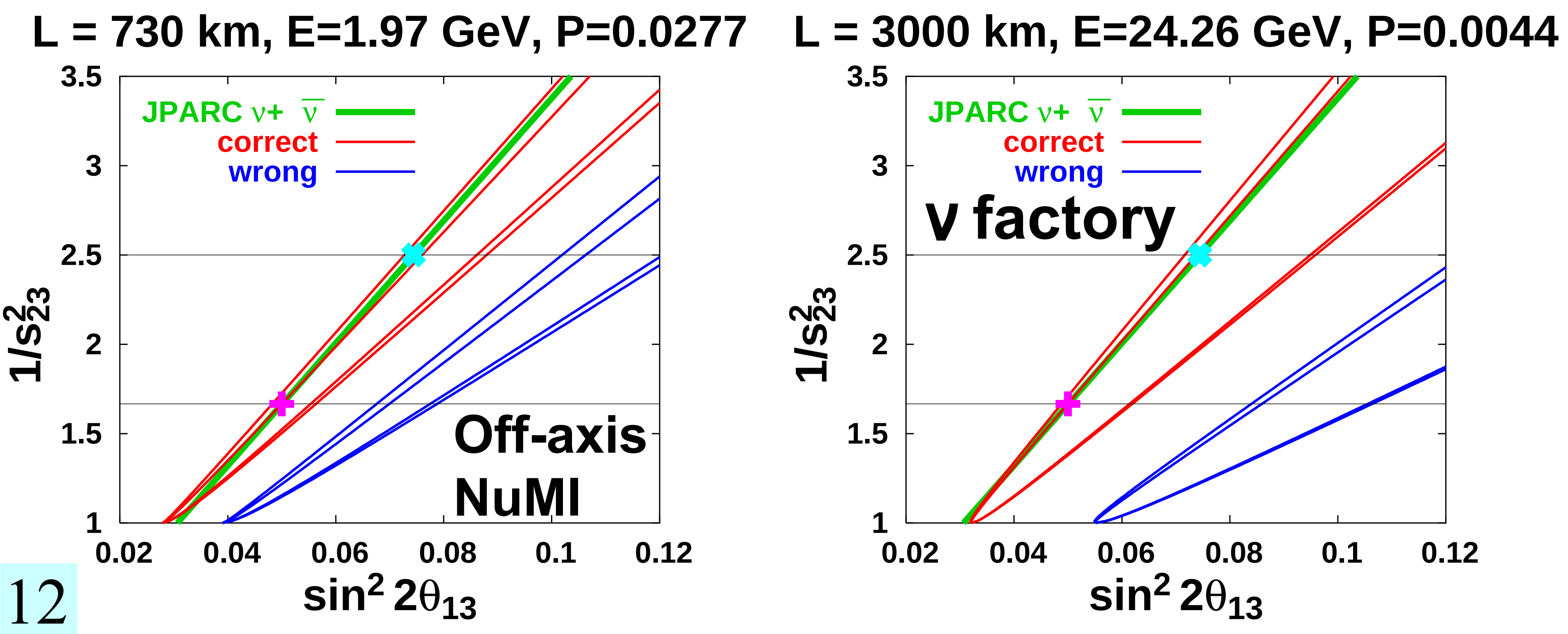
where

$$P_{true} \equiv P\left((\sin^2 2\theta_{13})_{true}, \delta_{true}, (s^2_{23})_{true}\right)$$

we can get a unique line (a hyperbola or an ellipse) in $(\sin^2 2\theta_{13}, 1/s^2_{23})$ plane for $\delta_{\pm[cw]}$ or $\pi - \delta_{\pm[cw]}$.

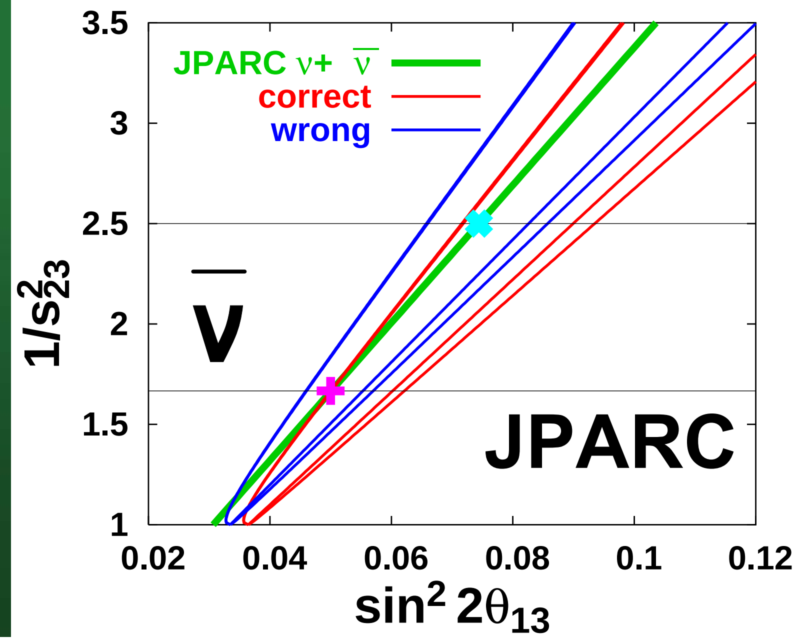


In general, the gradient of the hyperbola is almost equal to that of the JPARC line, and this additional curve does not help to resolve θ_{23} ambiguity if $\Delta \leqq \pi/2$.

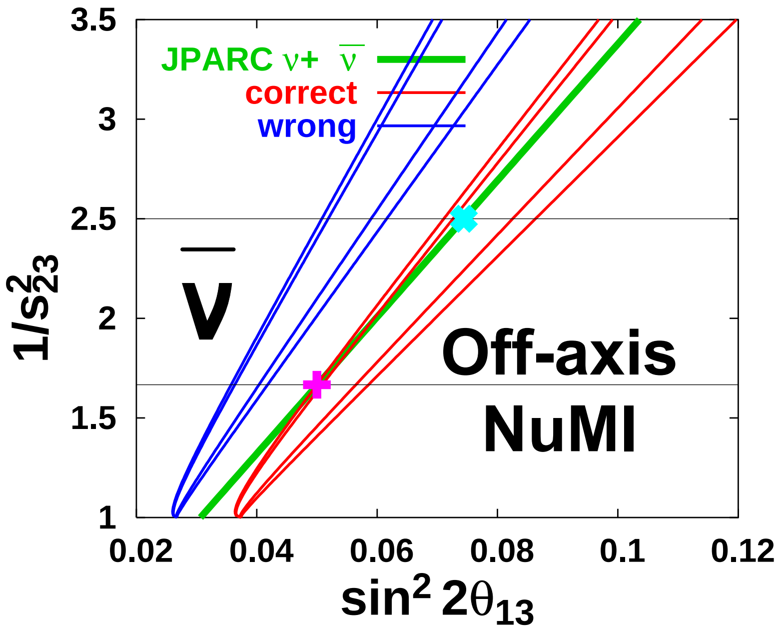


The situation doesn't change much for $\overline{\nu}_\mu \to \overline{\nu}_e$ if $\Delta \leqq \pi/2$.

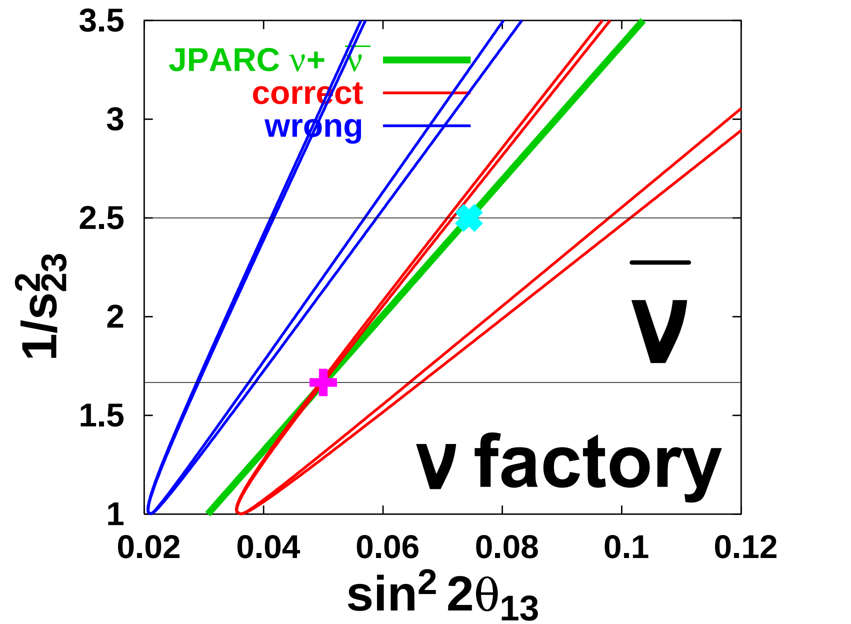
L = 295 km, E=1.19 GeV, P=0.0174



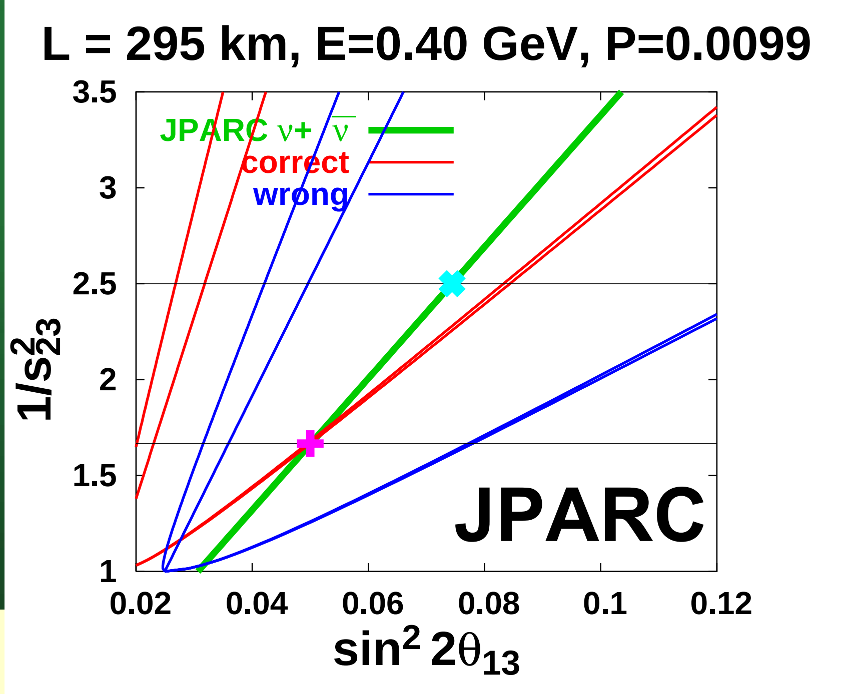
L = 730 km, E=1.97 GeV, P=0.0265



L = 3000 km, E=24.26 GeV, P=0.0029



On the other hand, for $\pi/2<\Delta<\pi$, the situation is different.



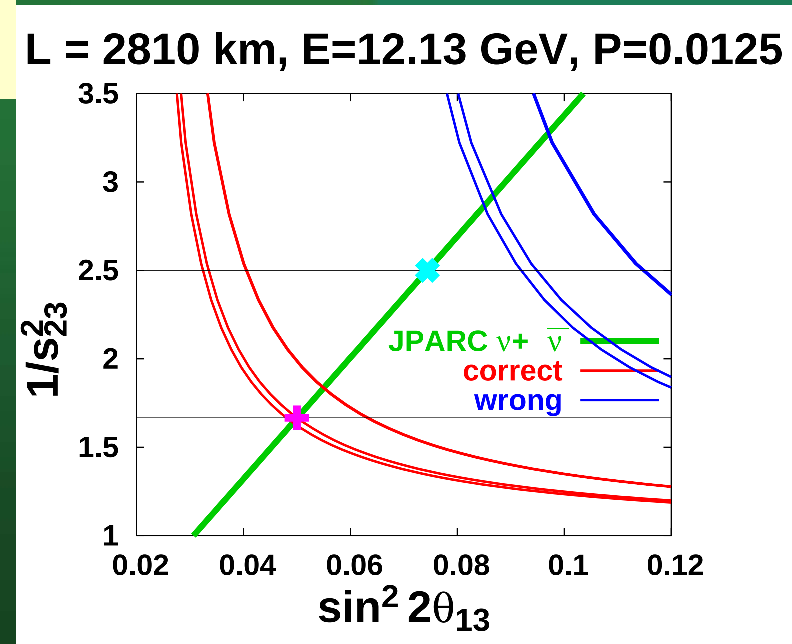
Good news is

- θ_{23} ambiguity may be resolved.
- $\delta \Leftrightarrow \pi - \delta$ ambiguity may be resolved.

Bad news is

- E is so low that statistics is low.
- osc. prob. Is small (～solar ν osc. prob.).

(C) measurement of $\nu_e \rightarrow \nu_\tau$



Curves intersect with the JPARC line almost orthogonally.

- θ_{23} ambiguity may be resolved.
- $\delta \Leftrightarrow \pi - \delta$ ambiguity may be resolved.
- sign(Δm^2_{31}) ambiguity may be resolved .

This channel may be interesting to be combined with JPARC in the future.

3. Summary

- Plots in ($\sin^2 2\theta_{13}$, $1/s^2_{23}$) are useful to see 8-fold degeneracy.

- It is important for determination of θ_{13} to resolve θ_{23} ambiguity if $\sin^2 2\theta_{23} < 1$.

- After $\nu_\mu \to \nu_e$ + $\overline{\nu_\mu} \to \overline{\nu_e}$ by JPARC, another $\nu_\mu \to \nu_e$ or $\overline{\nu_\mu} \to \overline{\nu_e}$ is not useful (but $\nu_e \to \nu_\tau$ is useful) to resolve θ_{23} ambiguity.