

# Parameter degeneracy and reactor neutrino experiments

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based on

hep-ph/0211111

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TMU KamLAND

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# 1. Introduction

Oscillation parameters in  $N_\nu = 3$  framework

$$(\underbrace{\Delta m_{21}^2, \theta_{12}}; \underbrace{|\Delta m_{32}^2|, \theta_{23}}; \underbrace{\text{sign}(\Delta m_{32}^2), \theta_{13}, \delta})$$

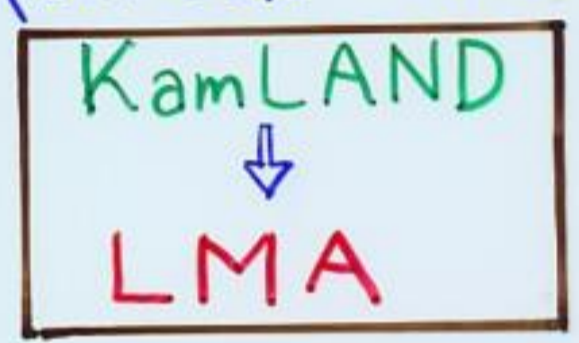
$\uparrow$   
 $\nu_0 + \nu_{\text{KamLAND}}$

$\uparrow$   
 $\nu_{\text{atm}}$

$\uparrow$   
things to do  
in the future

$$\left( \begin{array}{l} \Delta m_{21}^2 \sim 0 \text{ (} 10^{-5} \text{ eV}^2 \text{)} \\ \sin^2 2\theta_{12} \sim 0.8 \end{array} \right)$$

$$\left( \begin{array}{l} |\Delta m_{32}^2| \approx 2.5 \times 10^{-3} \text{ eV}^2 \\ \sin^2 2\theta_{23} \approx 1.0 \end{array} \right)$$



Final goal in  $\nu$  oscillation physics is measurement of  $\mathcal{CP}$  (only possible for **LMA**)

$$\text{Prob}(\mathcal{CP}) \propto J = \frac{c_{13}}{8} \underbrace{\sin 2\theta_{12}}_{\sim \sqrt{0.8}} \underbrace{\sin 2\theta_{13}}_{\sim \sqrt{0.1} \text{ unknown}} \underbrace{\sin 2\theta_{23}}_{\approx 1.0} \underbrace{\sin \delta}_{\text{unknown}}$$

As a first step, we need to know the magnitude of  $\sin 2\theta_{13}$

	parameter degeneracy	sensitivity
Long Base Line exp.	some	$\sin^2 2\theta_{13} (\text{JHF}) \gtrsim 0(10^{-3})$
Reactor exp.	none	$\sin^2 2\theta_{13} \gtrsim 0(10^{-2})$

## 2. Reactor measurement of $\theta_{13}$

F. Suekane  
K. Inoue

) are thinking of the possibility to measure  $\theta_{13}$  by a reactor experiment at Kashiwazaki - Kariwa Nuclear Power Plant.

# Experimental Conditions for $\theta_{13}$

## Optimization of Baseline

SK Result:  $\Delta\bar{m}_{23}^2 \approx 2.5 \times 10^{-3} eV^2$

$$\int f_\nu(E) \sigma(E) \sin^2 \frac{\Delta m^2 L}{4E} dE = \max$$



**L ~ 1.7 km**



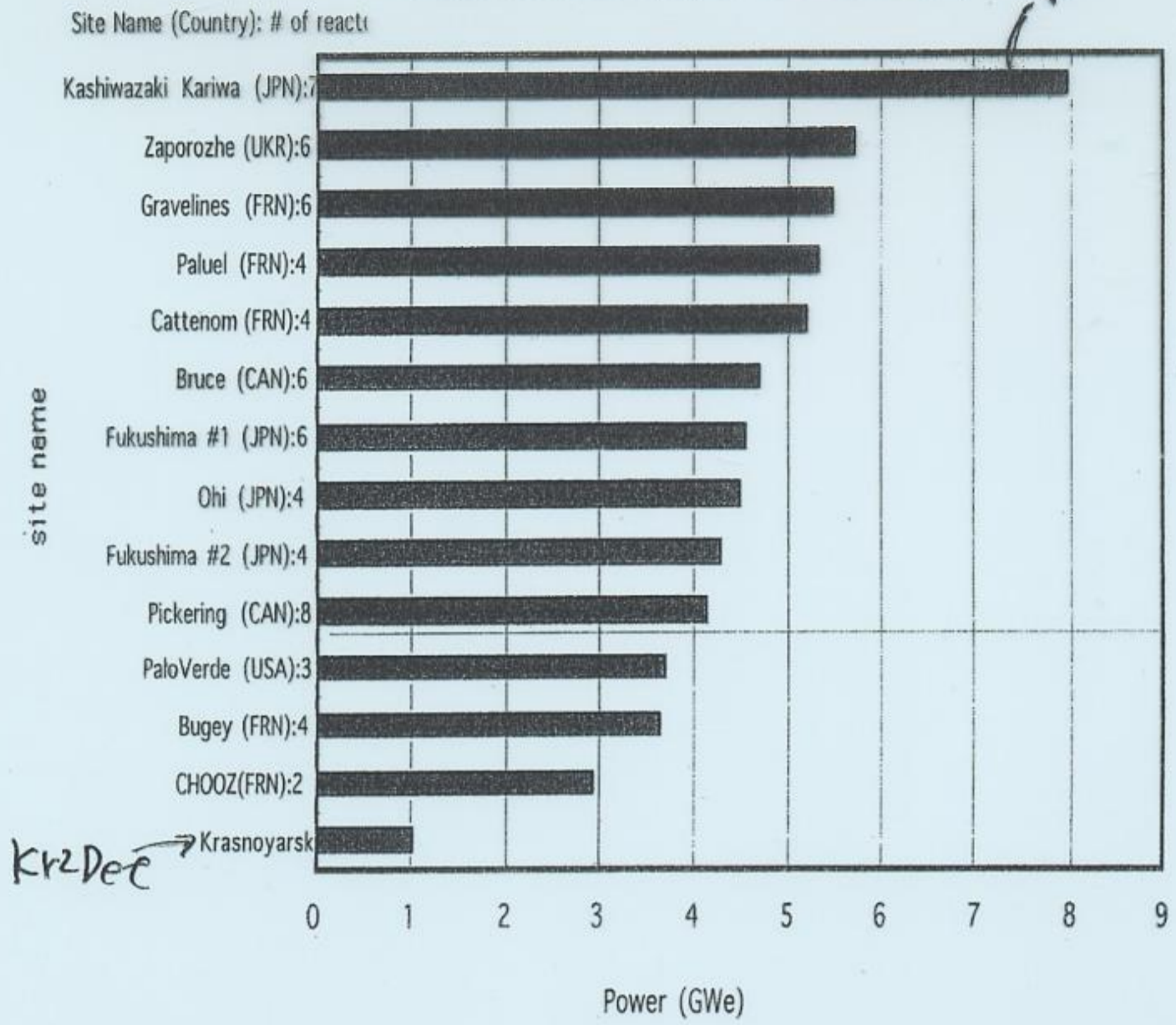
$N_\nu \sim 150/\text{year}/\text{target-ton}/\text{GW}_{th}$

1% stat. error/year



$M_{\text{Target}} * P_{\text{Reactor}} = 70 [\text{ton} * \text{GW}_{th}]$

Reactor Site Powers (Top 10 + neutrino reactors)



(Overviews of the World Nuclear Power, Nuclear Training Centre Jozef Stefan Institute (Slovenia); 17.Sept.2001)

### Kashiwazaki-Kariwa NPP (24.3GW<sub>th</sub>)



Largest Nuclear Reactor Site in the World.

Net  $M_{\text{Traget}} \sim \underline{5\text{tons}}$  for 80% reactor and 70% detection efficiency (=Just CHOOZ size).

## Issues at CHOOZ and solutions

(1) Systematic Error=2.7%

Comes from { rate prediction: 2.3%  
detection efficiency: 1.5%

### Solution:

Identical Front and Far Detectors



most of the systematics cancel out

How good is the cancellation?

Study BUGEY(3 identical detectors) case

Bugey detectors are modular type

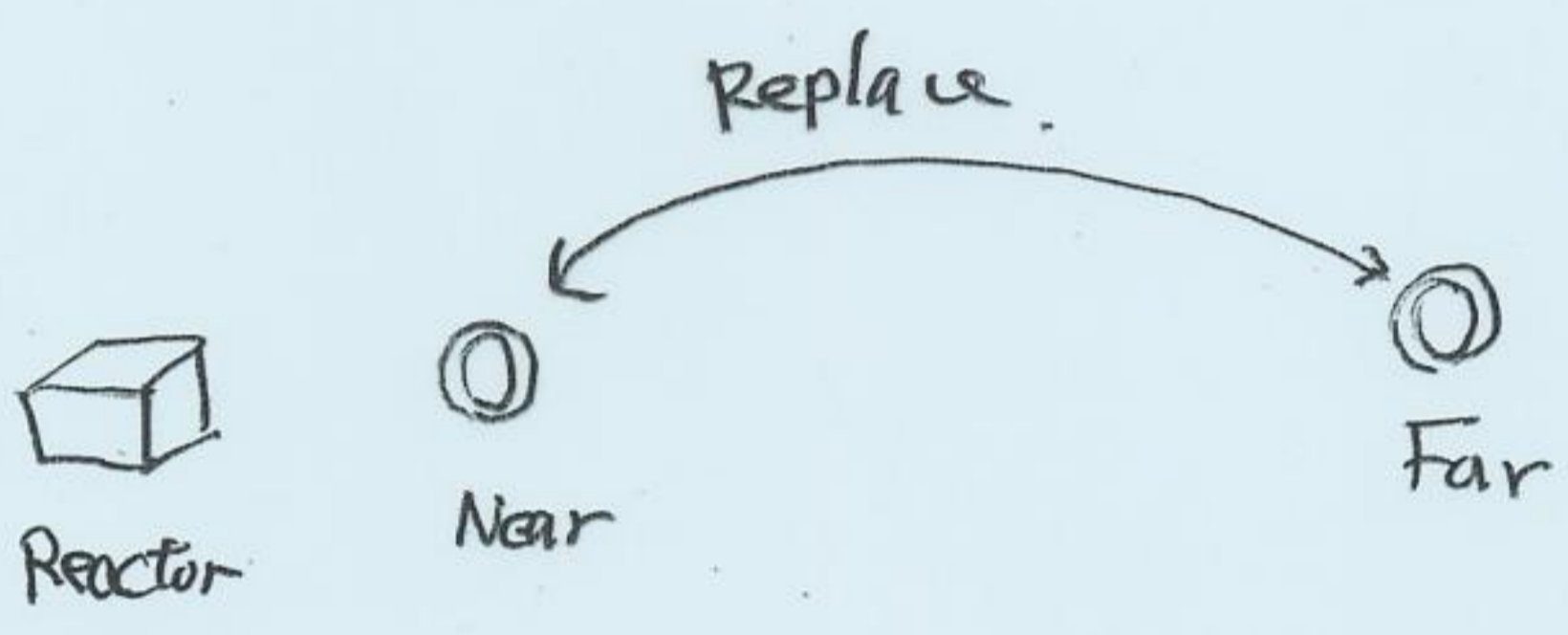
(Intrinsically worse systematics than bulk type)

Example,

	BUGEY Case: (modular detectors)	CHOOZ projection: (same fraction assumed)
$\sigma_{f_\nu}$	2.8% → 0%	2.1% → 0%
$N_p$	1.9% → 0.6%	0.8% → 0.3%
$L^2$	0.5% → 0.5%	---
$\varepsilon$	3.5% → 1.7%	1.5% → 0.7%
=====		
Total	4.9% → 2%	2.7% → 0.8%
	(Kr2Det expects $\sigma_{sys} = 0.5\%$ )	

CHOOZ detector is (in principle) Movable.

If front and far detectors are exchanged during the experiment, the individualities of the detectors are canceled and it is expected that the systematic error is further reduced to  $\sim 0.5\%$ .





We assume here :

24.3 GWth

80% operation efficiency

70% detection efficiency @  $\begin{cases} L = 1.7 \text{ km} \\ L = 0.3 \text{ km} \end{cases}$

energy spectrum: 14 bins of 0.5 MeV

$$\Delta m_{32}^2 = 2.5 \times 10^{-3} \text{ eV}^2$$

Results :

in the negative case

Excluded region (analysis w/ d.o.f. = 1)

$$\sigma_{\text{sys}} = 2\%, 5 \text{ t}\cdot\text{yr}$$

$$\sin^2 2\theta_{13} \lesssim 0.027$$

$$\sigma_{\text{sys}} = 0.8\%, 20 \text{ t}\cdot\text{yr}$$

$$\sin^2 2\theta_{13} \lesssim 0.013$$

in the affirmative case

The experimental error in  $\sin^2 2\theta_{13}$  is almost independent of the central value

$$\sigma_{\text{sys}} = 2\%, 5 \text{ t}\cdot\text{yr}$$

$$\delta(\sin^2 2\theta_{13}) = 0.034$$

$$\sigma_{\text{sys}} = 0.8\%, 20 \text{ t}\cdot\text{yr}$$

$$\delta(\sin^2 2\theta_{13}) = 0.015$$

d.o.f. = 2

If JHF determines  $\Delta m_{32}^2$  to  $10^{-4} \text{ eV}^2$

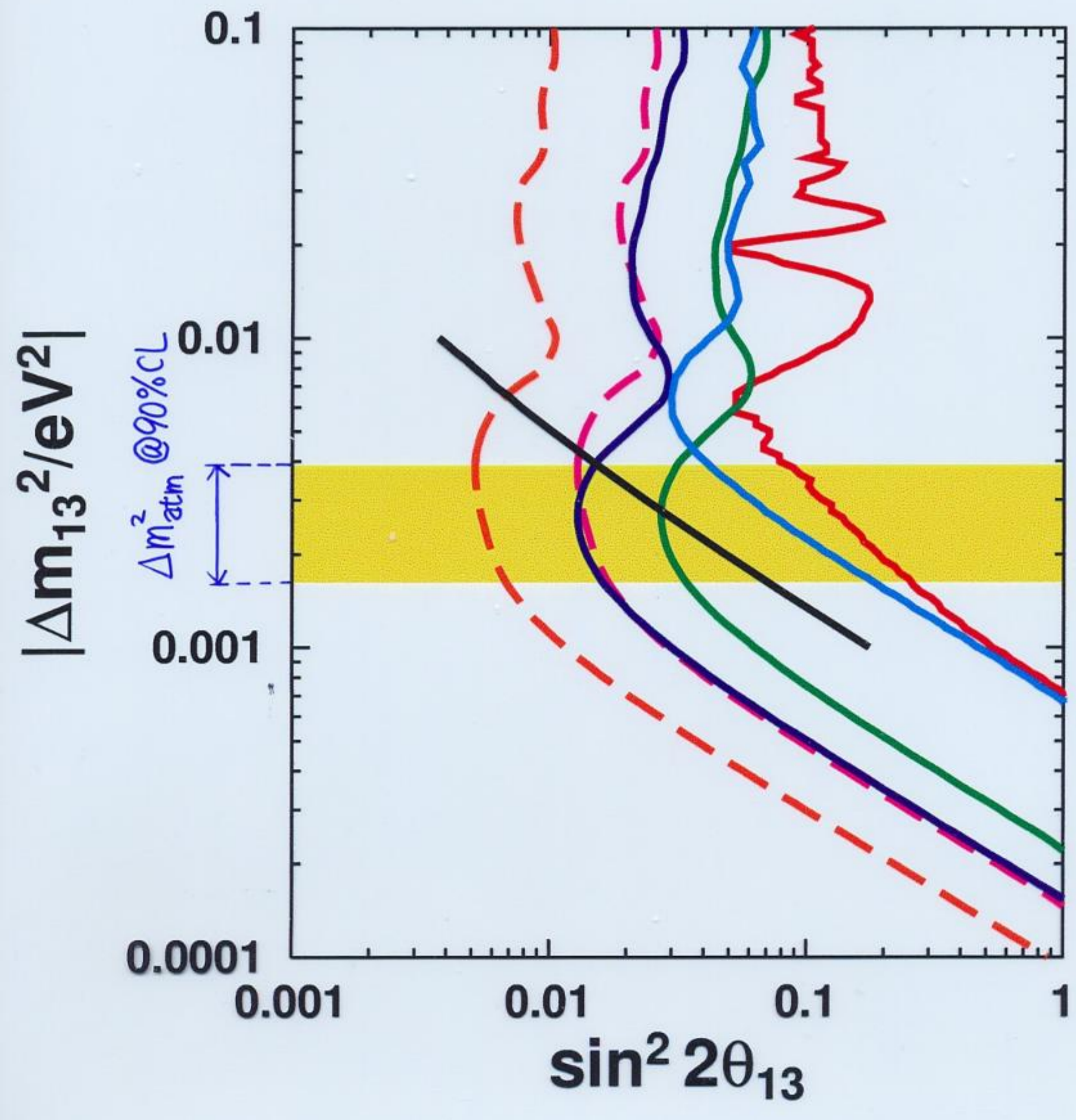
then analysis becomes approximately 1-dimensional (w.r.t.  $\sin^2 2\theta_{13}$  only)

$$\rightarrow \sigma_{\text{sys}} = 0.8\%, 20 \text{ t}\cdot\text{yr}$$

$$\delta(\sin^2 2\theta_{13}) = 0.012 \quad (\text{d.o.f.} = 1)$$

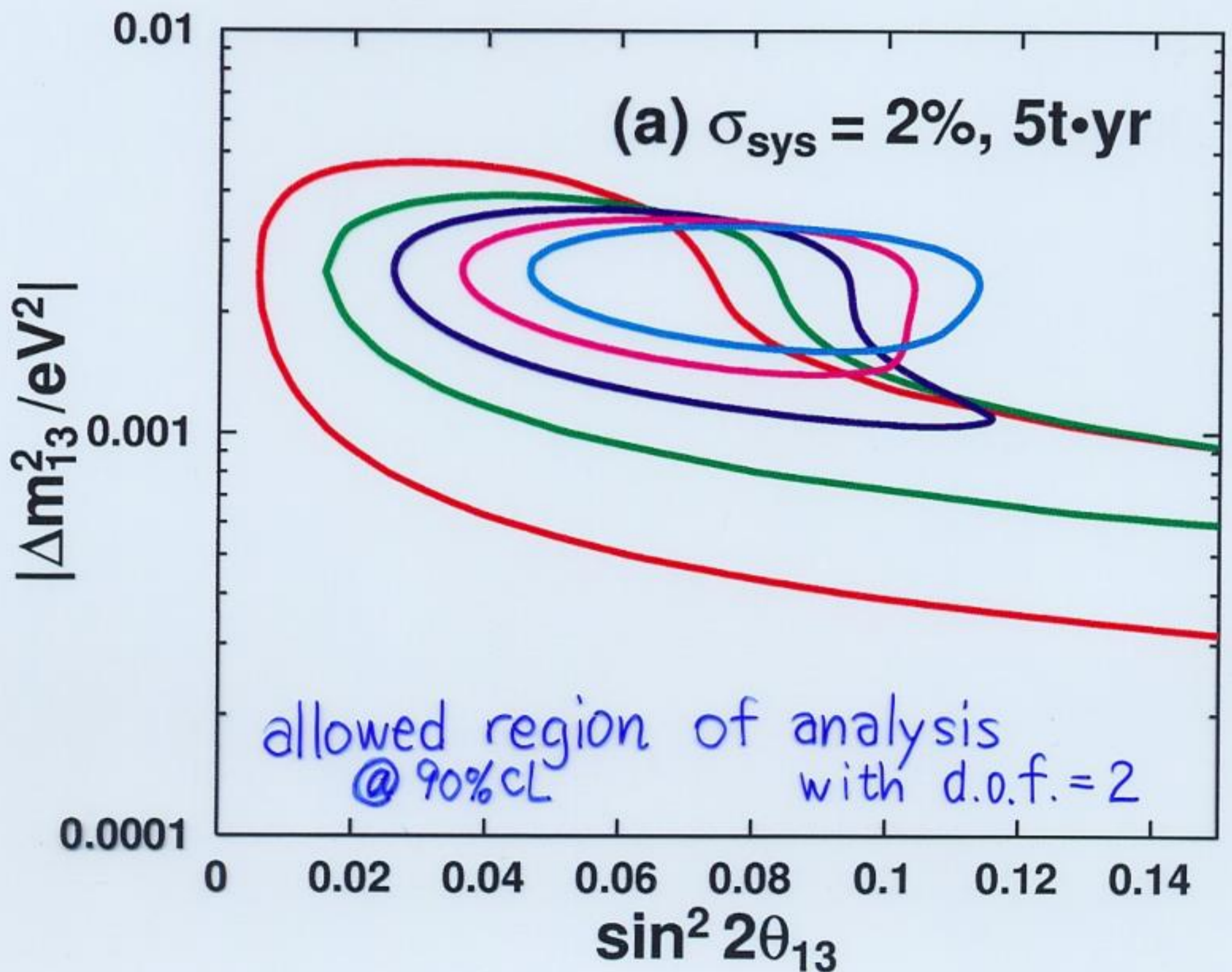
excluded region

- CHOOZ ——— (red)
- $\sigma_{\text{sys}} = 2\%, 5\text{t}\cdot\text{yr}$  ——— (green)
- $\sigma_{\text{sys}} = 0.8\%, 20\text{t}\cdot\text{yr}$  ——— (dark blue)
- $\sigma_{\text{sys}} = 2\%, \infty\text{t}\cdot\text{yr}$  - - - (magenta)
- $\sigma_{\text{sys}} = 0.8\%, \infty\text{t}\cdot\text{yr}$  - - - (orange)
- ICARUS+OPERA ——— (black)
- MINOS ——— (light blue)



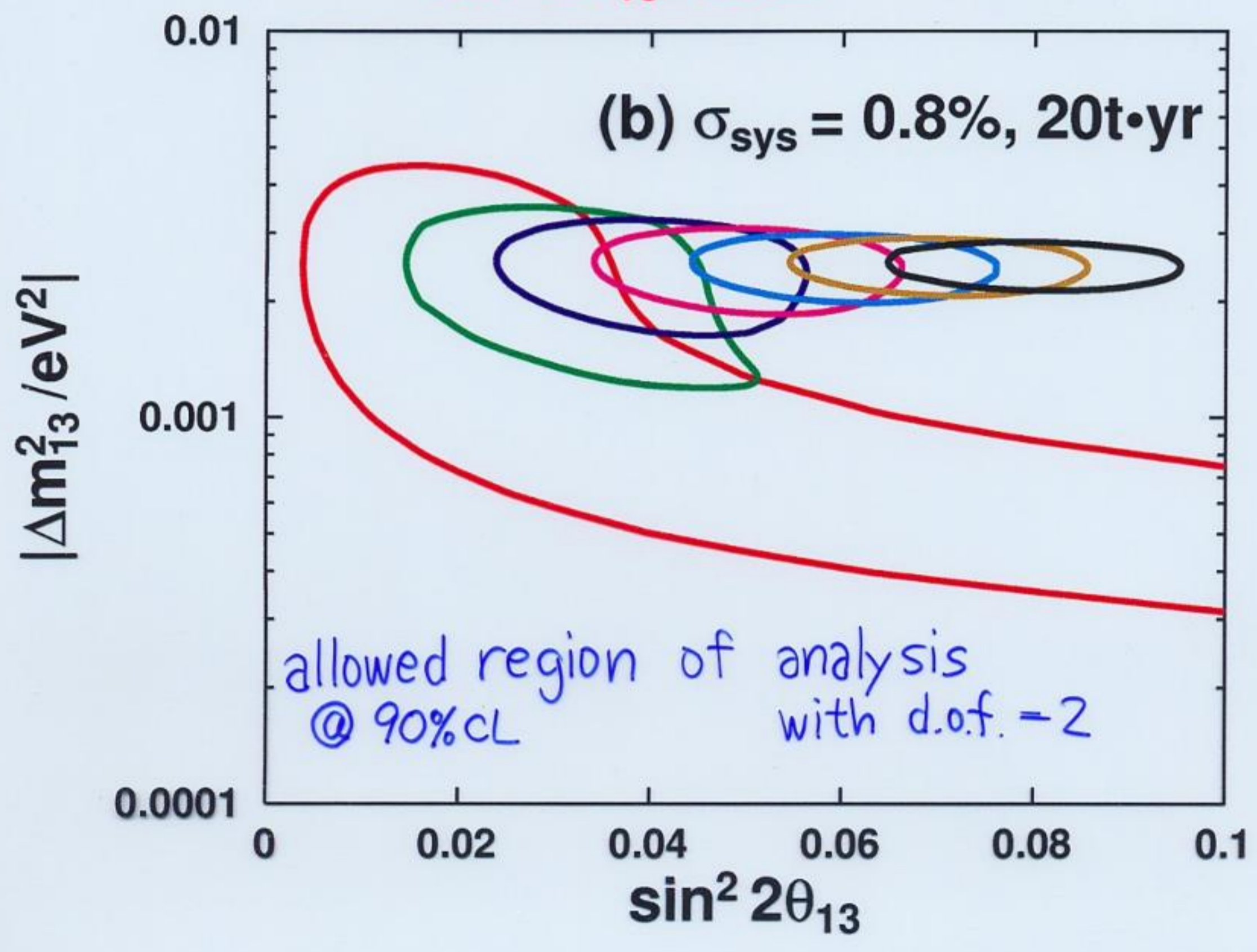
$$\delta(\sin^2 2\theta_{13}) = 0.034$$

- $\sin^2 2\theta_{13} = 0.08$  — (blue line)
- $\sin^2 2\theta_{13} = 0.07$  — (magenta line)
- $\sin^2 2\theta_{13} = 0.06$  — (dark blue line)
- $\sin^2 2\theta_{13} = 0.05$  — (green line)
- $\sin^2 2\theta_{13} = 0.04$  — (red line)



$\delta(\sin^2 2\theta_{13}) = 0.015 \rightarrow 0.012 \text{ (d.o.f. = 1)}$

- $\sin^2 2\theta_{13} = 0.08$  —
- $\sin^2 2\theta_{13} = 0.07$  —
- $\sin^2 2\theta_{13} = 0.06$  —
- $\sin^2 2\theta_{13} = 0.05$  —
- $\sin^2 2\theta_{13} = 0.04$  —
- $\sin^2 2\theta_{13} = 0.03$  —
- $\sin^2 2\theta_{13} = 0.02$  —



3-1. Parameter degeneracy in  $(S_{23}^2, \sin^2 2\theta_{13})$  plane <sup>23</sup>  $\frac{12}{13}$

Even if  $P \equiv P(\nu_\mu \rightarrow \nu_e)$  and  $\bar{P} \equiv P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$  are given, there are in general 8 solutions.

3 kinds of degeneracy

- intrinsic  $(\delta, \theta_{13})$
- sign  $(\Delta m_{31}^2)$
- $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$

Burguet-Castell et al ('01)

Minakata-Nunokawa ('01)

Fogli-Lisi PRD54 ('96) 3667

Barger-Marfatia-Whisnant ('02)

8-fold degeneracy

Here I assume that accelerator beams are approximately monochromatic.

Experimental errors in long baseline experiments are not taken into account.

I will show how the 8-fold degeneracy is lifted by switching on:

$$\theta_{23} = \frac{\pi}{4}, \quad \left| \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right|, \quad AL$$

$(A \equiv \sqrt{2} G_{\nu e})$

(they are all small @ JHF experiment)

$$\sin^2 2\theta_{23} \geq 0.92$$

$$\left| \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right| \sim \frac{1}{35}$$

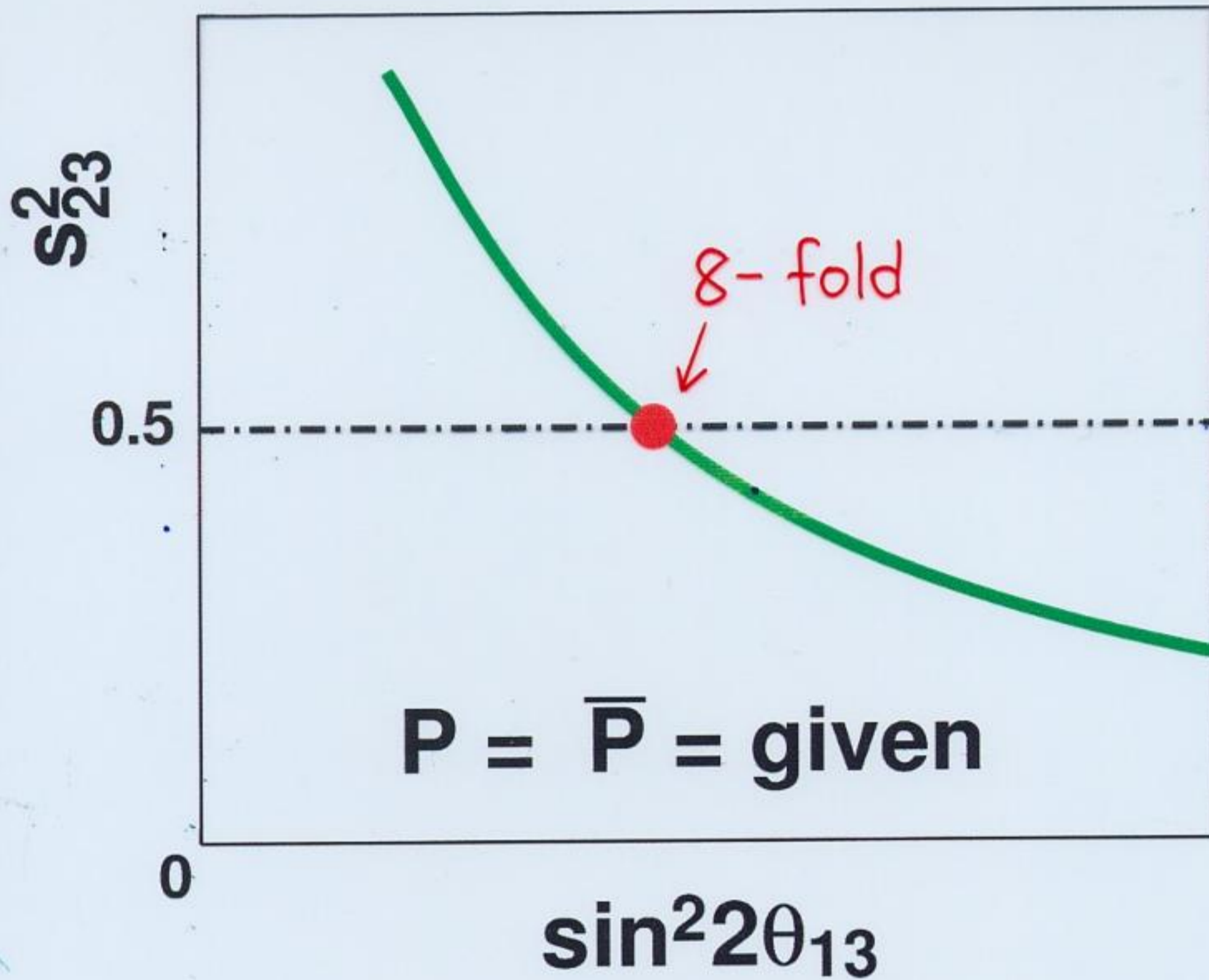
$$\frac{AL}{2} \sim \frac{1}{13}$$

Here I visualize the 8-fold degeneracy by using the  $(S_{23}^2, \sin^2 2\theta_{13})$  plane step by step.

$$\epsilon \equiv \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \quad \frac{AL}{2}$$

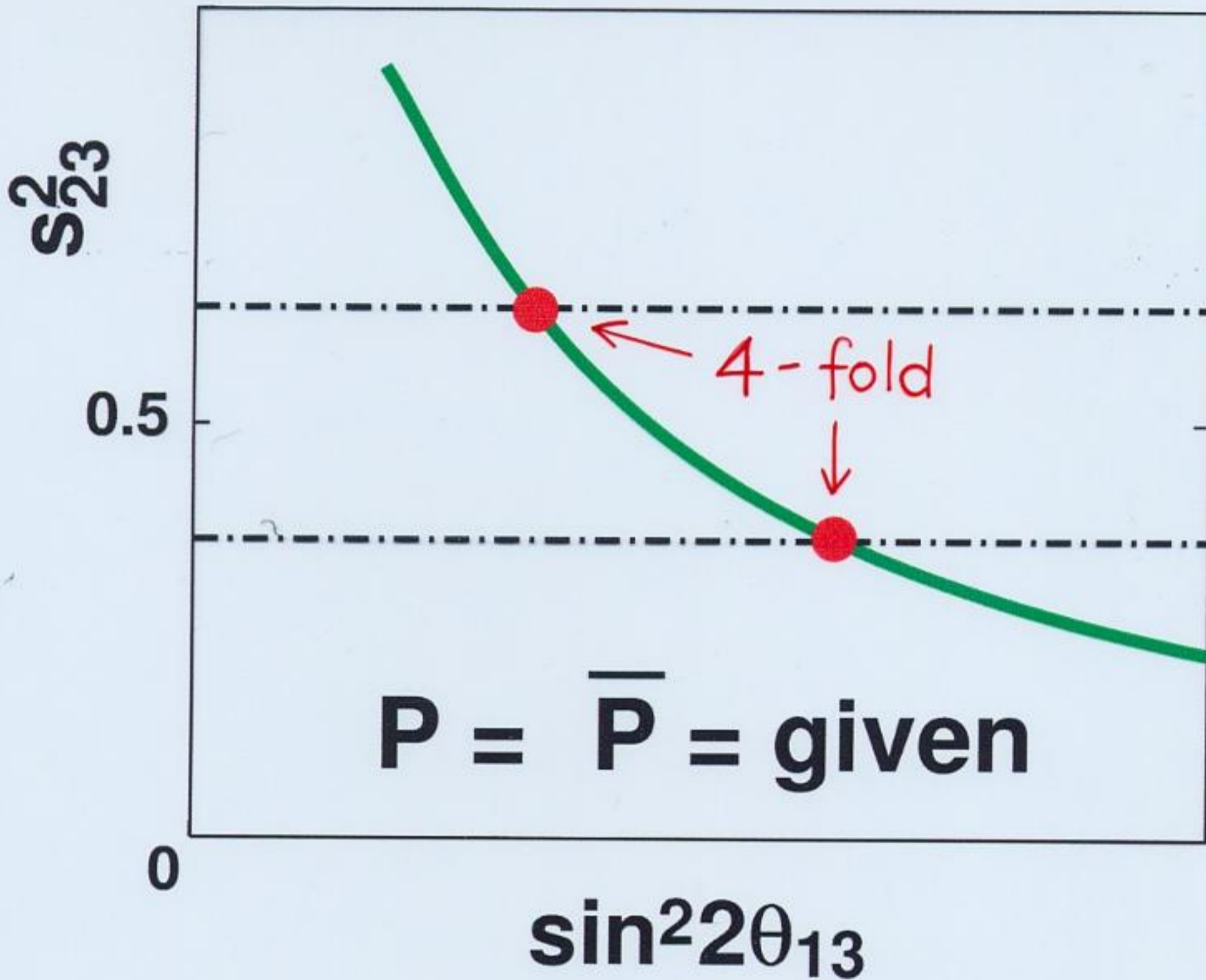
	$\theta_{23} - \frac{\pi}{4}$	$\Delta m_{21}^2$	$A \equiv \sqrt{2} G_F N_e$	$\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$	$(\delta, \theta_{13})$	$\text{sign}(\Delta m_{31}^2)$
(a)	= 0	= 0	= 0	degen.	degen.	degen.
(b)	$\neq 0$	= 0	= 0	lifted	degen.	degen.
(c)	$\neq 0$	$\neq 0$	= 0	lifted	lifted	degen.
(d) off OM	$\neq 0$	$\neq 0$	$\neq 0$	lifted	lifted	lifted
(e) @ OM	$\neq 0$	$\neq 0$	$\neq 0$	lifted	degen.	almost degen.

(a)  $\theta_{23} = \frac{\pi}{4}$ ,  $\Delta m_{21}^2 = 0$ ,  $A = 0$



$$P = \bar{P} = \underbrace{S_{23}^2}_{\frac{1}{2}} \sin^2 2\theta_{13} \underbrace{\sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right)}_{\text{const}}$$

(b)  $\theta_{23} \neq \frac{\pi}{4}$ ,  $\Delta m_{21}^2 = 0$ ,  $A = 0$

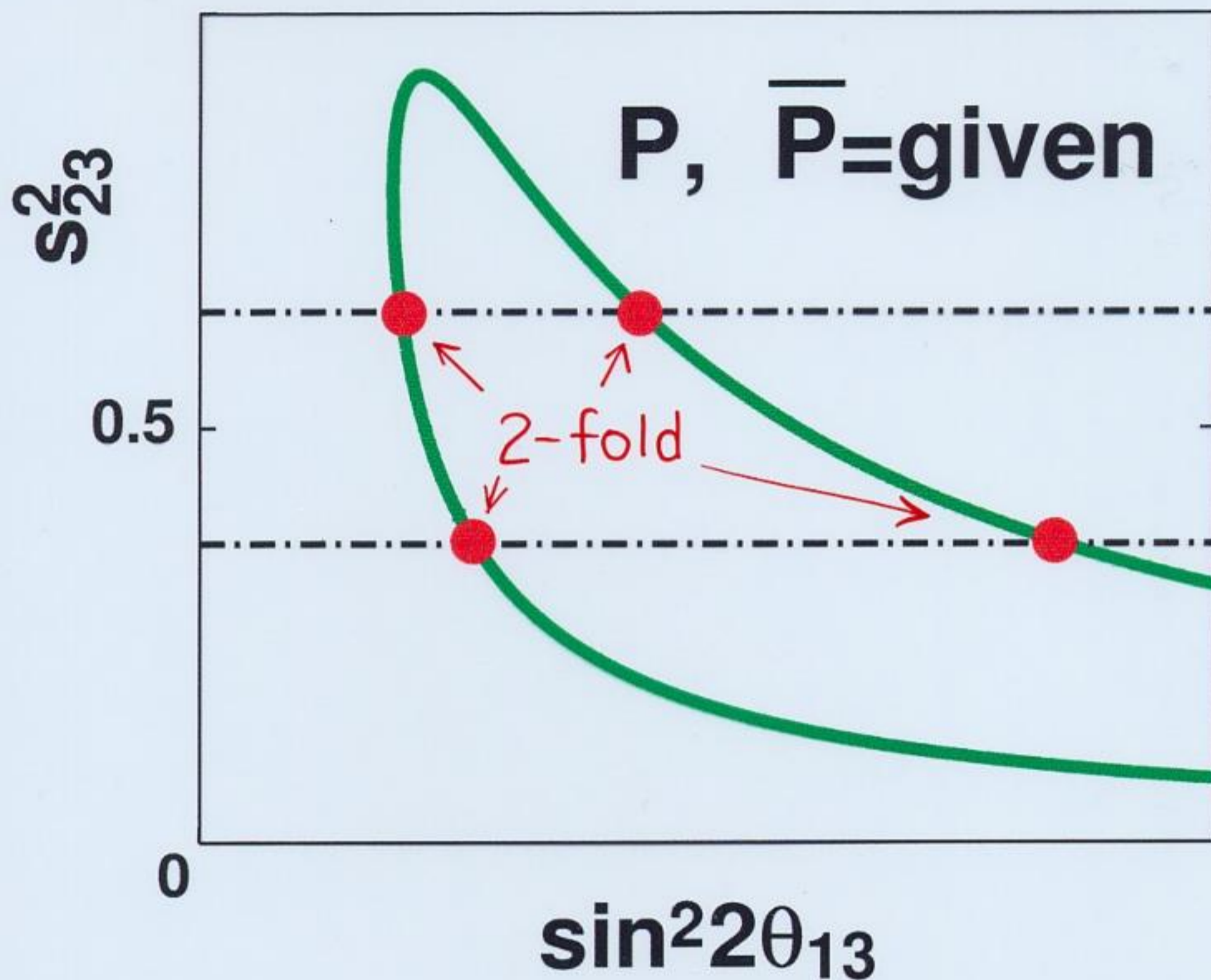


$$P = \bar{P} = S_{23}^2 \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right)$$

$$S_{23}^2 = \frac{1 \pm \sqrt{1 - \sin^2 2\theta_{23}}}{2} \leftarrow \text{known from } \nu_{\mu} \rightarrow \nu_{\mu}$$



(c)  $\theta_{23} \neq \frac{\pi}{4}$ ,  $\Delta m_{21}^2 \neq 0$ ,  $A=0$



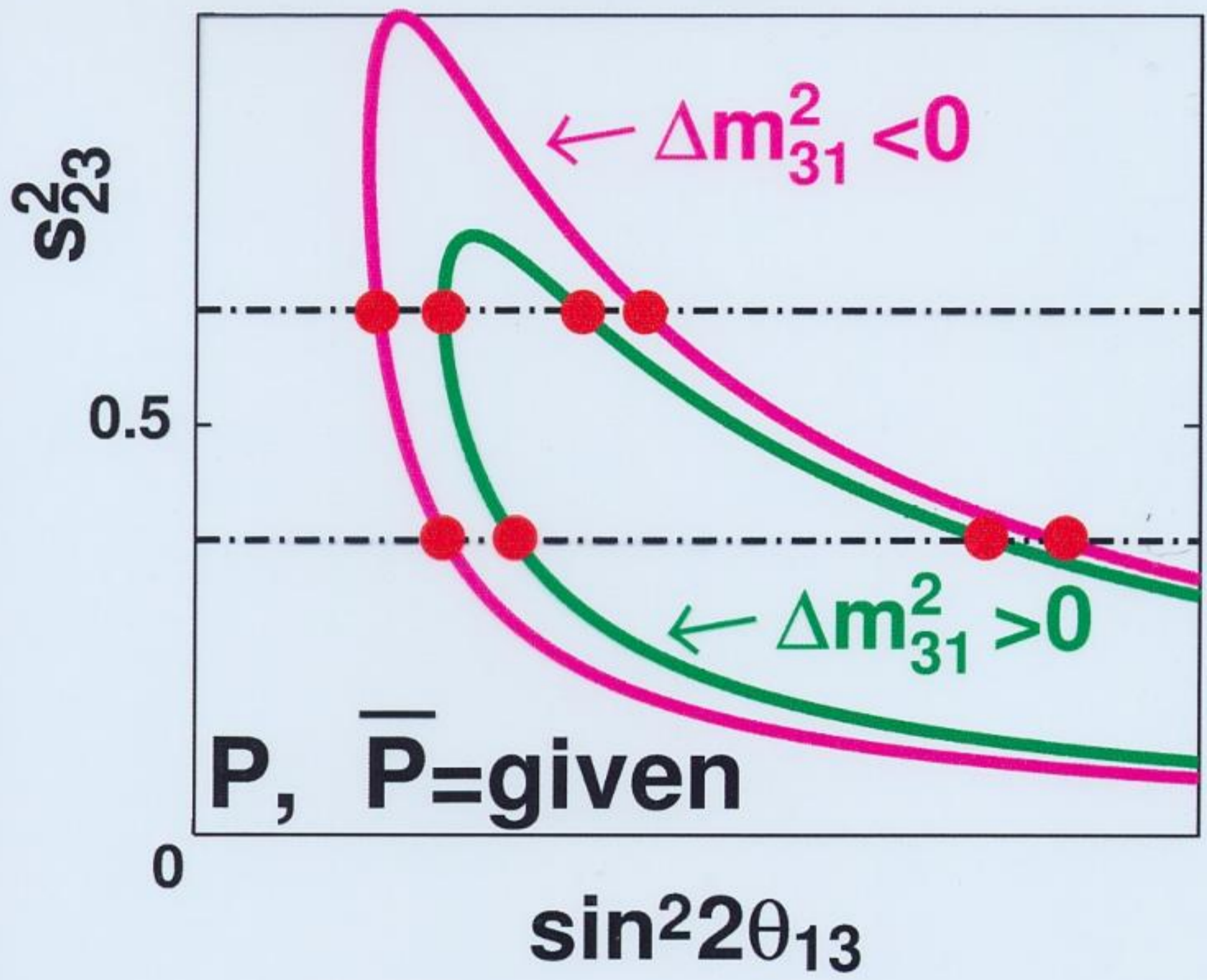
$$\frac{1}{\cos^2 \Delta} \left( \frac{P + \bar{P}}{2} - x^2 \sin^2 \Delta - y^2 \Delta^2 \right)^2 + \frac{1}{\sin^2 \Delta} \left( \frac{P - \bar{P}}{2} \right)^2 = (2xy \Delta \sin \Delta)^2$$

quadratic eq. in  $x^2$

$$\left( \begin{array}{l} x \equiv S_{23} \sin 2\theta_{13} \\ y \equiv \left| \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right| c_{23} \sin 2\theta_{12} \\ \Delta \equiv \frac{\Delta m_{31}^2 L}{4E} \end{array} \right)$$

$$S_{23} = \sqrt{\frac{1 \pm \sqrt{1 - \sin^2 2\theta_{23}}}{2}}$$

(d)  $\theta_{23} \neq \frac{\pi}{4}$ ,  $\Delta m_{21}^2 \neq 0$ ,  $A \neq 0$   
off OM



$$\frac{1}{4 \cos^2 \Delta} \left( \frac{P - x^2 f^{(\mp)^2} - y^2 g^2}{f^{(\mp)}} + \frac{\bar{P} - x^2 f^{(\pm)^2} - y^2 g^2}{f^{(\pm)}} \right)^2 + \frac{1}{4 \sin^2 \Delta} \left( \frac{P - x^2 f^{(\mp)^2} - y^2 g^2}{f^{(\mp)}} - \frac{\bar{P} - x^2 f^{(\pm)^2} - y^2 g^2}{f^{(\pm)}} \right)^2 = (2xyg)^2$$

for  $\Delta m_{31}^2 \begin{cases} > 0 \\ < 0 \end{cases}$   
quadratic in  $x^2$

$$S_{23} = \sqrt{\frac{1 \pm \sqrt{1 - \sin^2 2\theta_{23}}}{2}}$$

$$x \equiv S_{23} \sin 2\theta_{13}$$

$$y \equiv \left| \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right| C_{23} \sin 2\theta_{12}$$

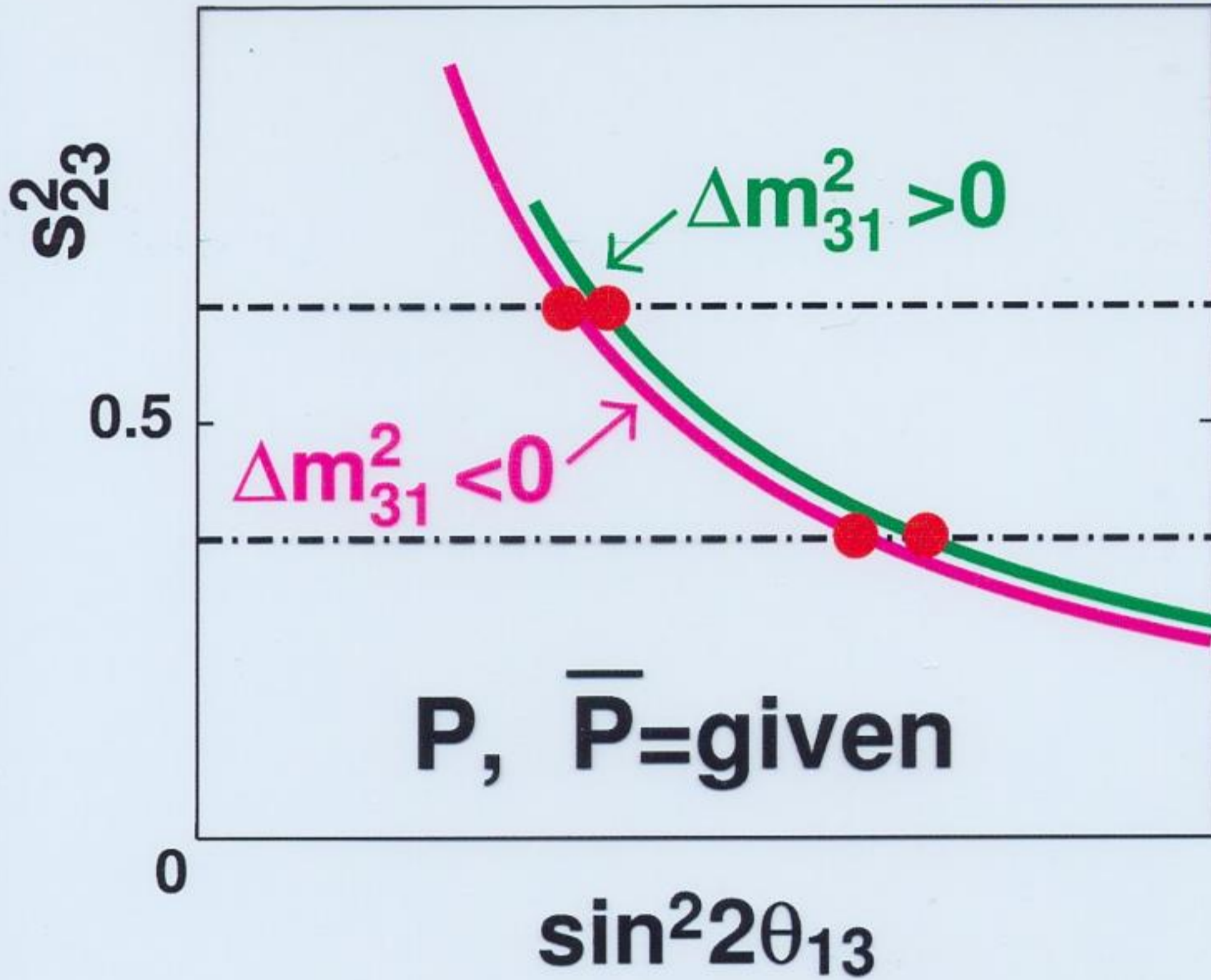
$$\Delta \equiv \frac{\Delta m_{31}^2 L}{4E}$$

$$f^{(\pm)} \equiv \frac{\sin(\Delta \pm AL/2)}{1 \mp AL/2\Delta}$$

$$g \equiv \frac{\sin(AL/2)}{AL/2\Delta}$$

(e)  $\theta_{23} \neq \frac{\pi}{4}$ ,  $\Delta m_{21}^2 \neq 0$ ,  $A \neq 0$

@OM  $\left( \frac{\Delta m_{31}^2 L}{4E} = \frac{\pi}{2} \right)$



$$\frac{P - x^2 f^{(\mp)^2} - y^2 g^2}{f^{(\mp)}} = - \frac{\bar{P} - x^2 f^{(\pm)^2} - y^2 g^2}{f^{(\pm)}} \quad \text{for } \Delta m_{31}^2 \begin{cases} > 0 \\ < 0 \end{cases}$$

linear in  $x^2$

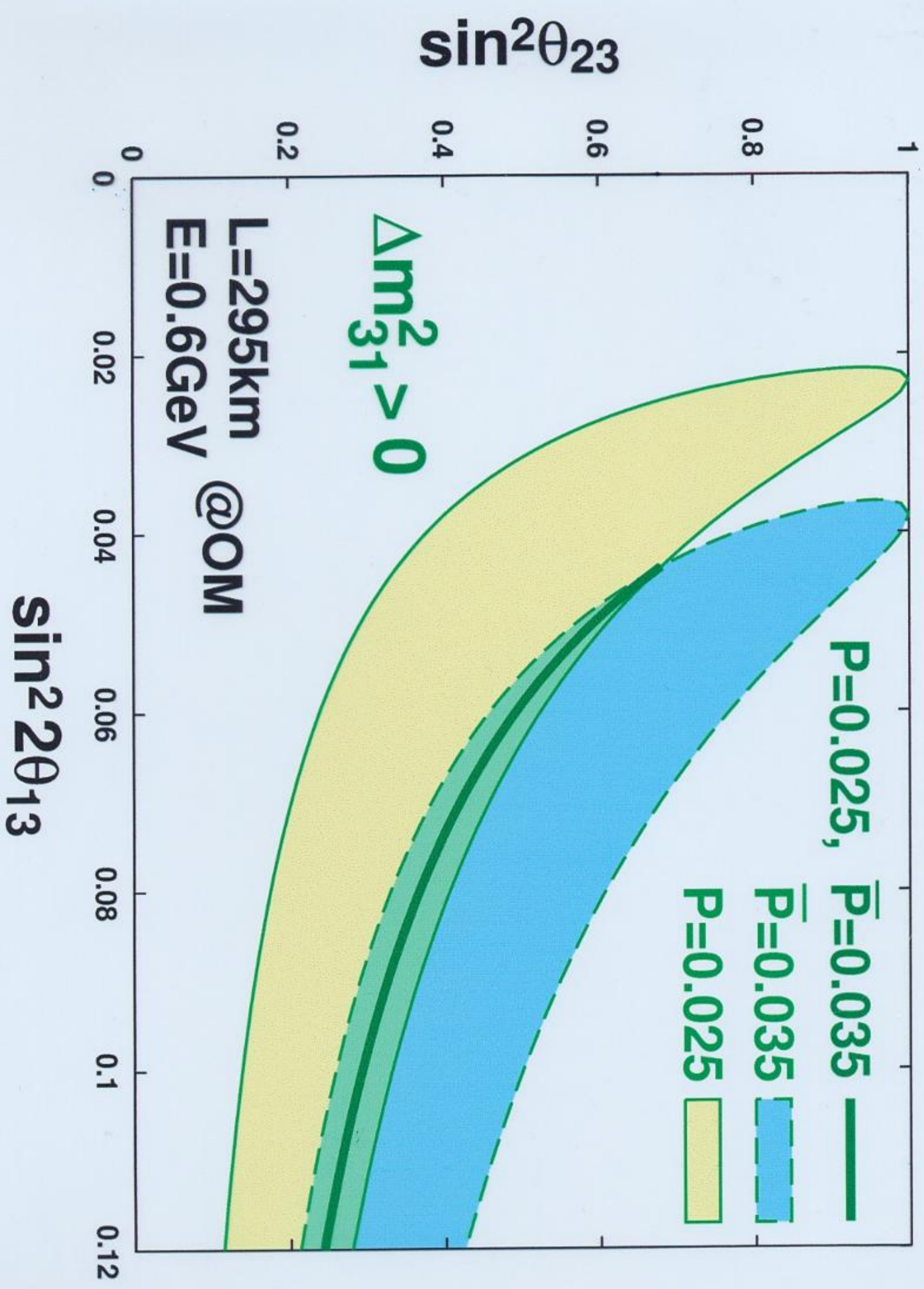
$$S_{23} = \sqrt{\frac{1 \pm \sqrt{1 - \sin^2 2\theta_{23}}}{2}}$$

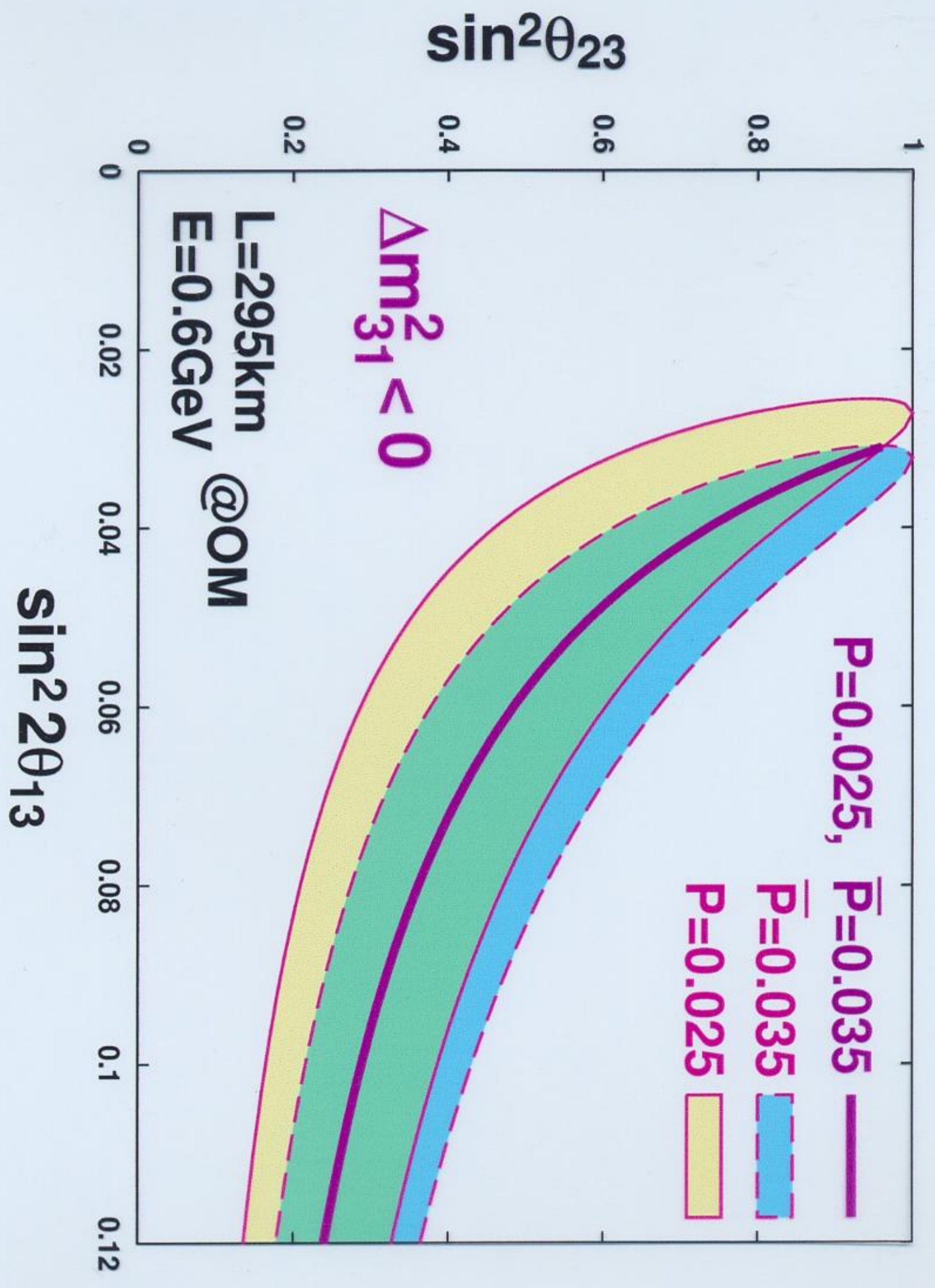
$$x \equiv S_{23} \sin 2\theta_{13}$$

$$y \equiv \left| \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right| c_{23} \sin 2\theta_{12}$$

$$f^{(\pm)} \equiv \frac{\cos(AL/2)}{1 \mp AL/\pi}$$

$$g \equiv \frac{\sin(AL/2)}{AL/\pi}$$





@ Oscillation Maximum  $\left(\frac{\Delta m_{31}^2 L}{4E} = \frac{\pi}{2}\right)$

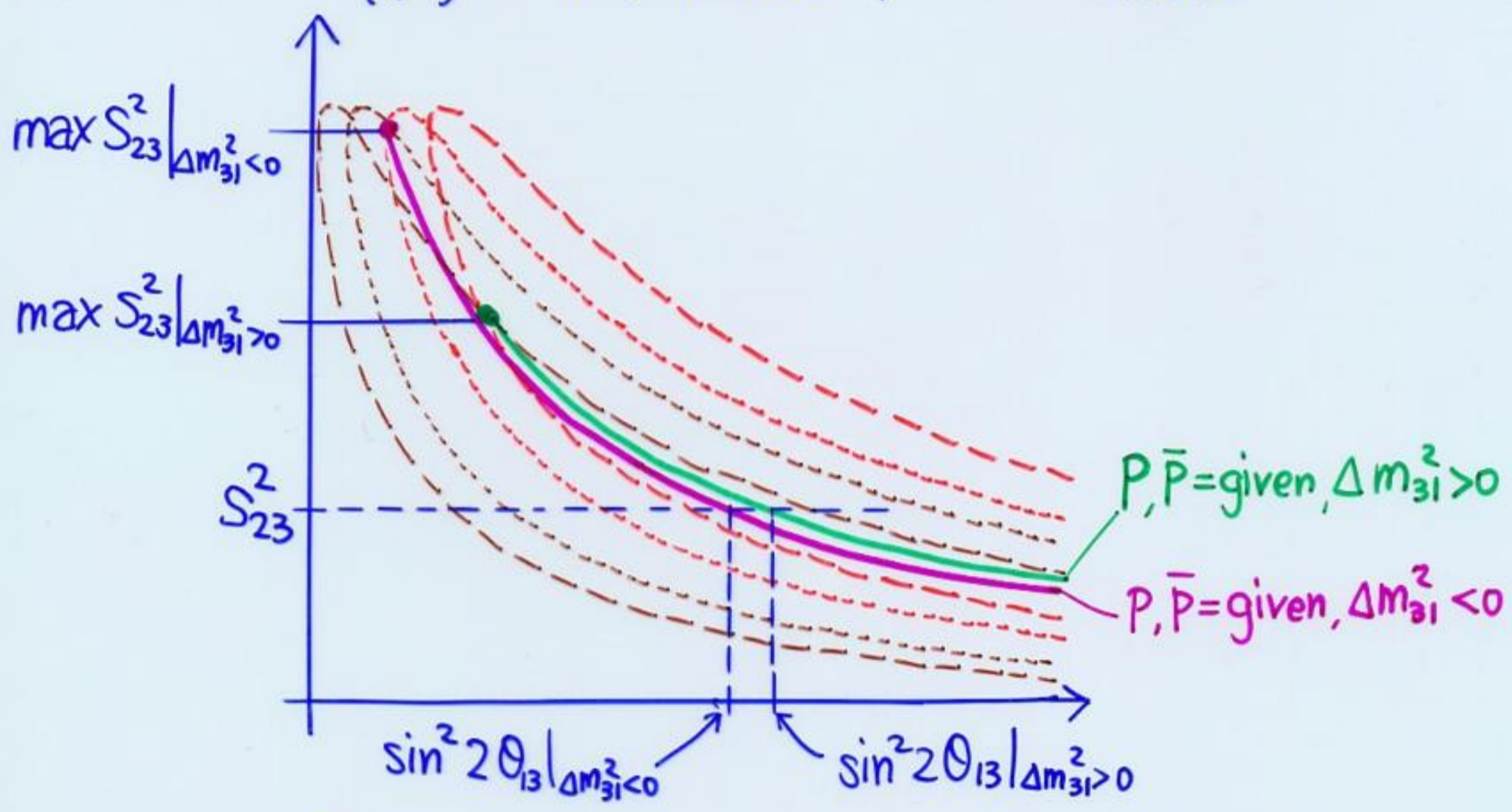
$$P \equiv P(\nu_\mu \rightarrow \nu_e) = x^2 f^2 - 2xyfg \sin \delta + y^2 g^2$$

$$\bar{P} \equiv P(\bar{\nu}_\mu \rightarrow \nu_e) = x^2 \bar{f}^2 + 2xy\bar{f}g \sin \delta + y^2 g^2$$

where  $x \equiv S_{23} \sin 2\theta_{13}$

$$y \equiv \epsilon C_{23} \sin 2\theta_{12}, \quad \epsilon \equiv \left| \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right|$$

$$\left\{ \begin{matrix} f \\ \bar{f} \end{matrix} \right\} \equiv \frac{\cos(AL/2)}{1 \mp AL/\pi}, \quad g \equiv \frac{\sin(AL/2)}{AL/\pi}, \quad A \equiv \sqrt{2} G_F N_e$$



When  $P$  &  $\bar{P}$  are given, one can show

$$\bullet \sin^2 2\theta_{13} \Big|_{\Delta m_{31}^2 > 0} - \sin^2 2\theta_{13} \Big|_{\Delta m_{31}^2 < 0} = \frac{1}{S_{23}^2} \cdot \frac{1}{f\bar{f}} \cdot \frac{f-\bar{f}}{f+\bar{f}} (\bar{P}-P) \approx \frac{AL}{S_{23}^2 \pi} (\bar{P}-P) \sim 5 \times 10^{-4}$$

for any  $\theta_{23}$  if  $|AL/2| \ll 1$

$$\bullet \max S_{23}^2 \Big|_{\Delta m_{31}^2 < 0} - \max S_{23}^2 \Big|_{\Delta m_{31}^2 > 0} = \frac{1}{\epsilon^2} \cdot \frac{1}{\sin^2 2\theta_{12}} \cdot \frac{1}{g^2} \cdot \frac{f-\bar{f}}{f+\bar{f}} (\bar{P}-P) \approx \frac{1}{\epsilon^2} \frac{(2/\pi)^2}{\sin^2 2\theta_{12}} \frac{AL}{\pi} (\bar{P}-P)$$

$\sim 0.3$

if  $P=0.025, \bar{P}=0.035, L=295 \text{ km}, \epsilon = \frac{7 \times 10^{-5} \text{ eV}^2}{2.5 \times 10^{-3} \text{ eV}^2} \approx \frac{1}{35}$

$A \approx \frac{1}{1900 \text{ km}}$

### 3-2. Resolution of $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$ degeneracy

by LBL  $\oplus$  reactor

cf. Fogli-Lisi PRD54('96)3667  
Barenboim-de Gouvea ('02)

29 112  
23

Our scenario

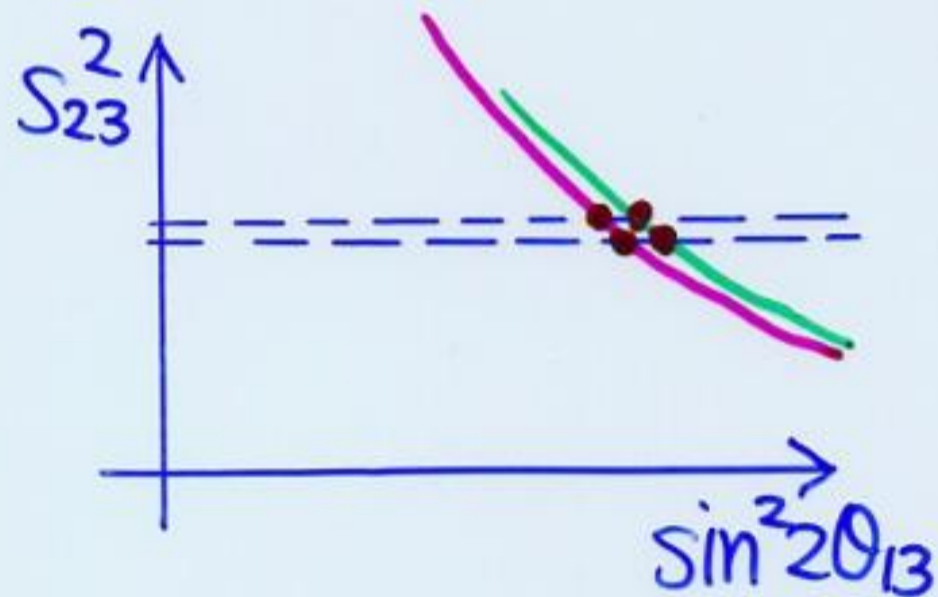
- JHF  $\nu \oplus \bar{\nu}$  @ Oscillation Maximum  $\oplus$
- reactor experiment (@ Kashiwazaki?)

From  $\nu_{\mu} \leftrightarrow \nu_{\mu}$  @ JHF we will know that  $\theta_{23}$  satisfies either of the followings:

(A)  $|1 - \sin^2 2\theta_{23}| < \text{a few} \times 10^{-2}$

(B)  $|1 - \sin^2 2\theta_{23}| \geq \text{a few} \times 10^{-2}$

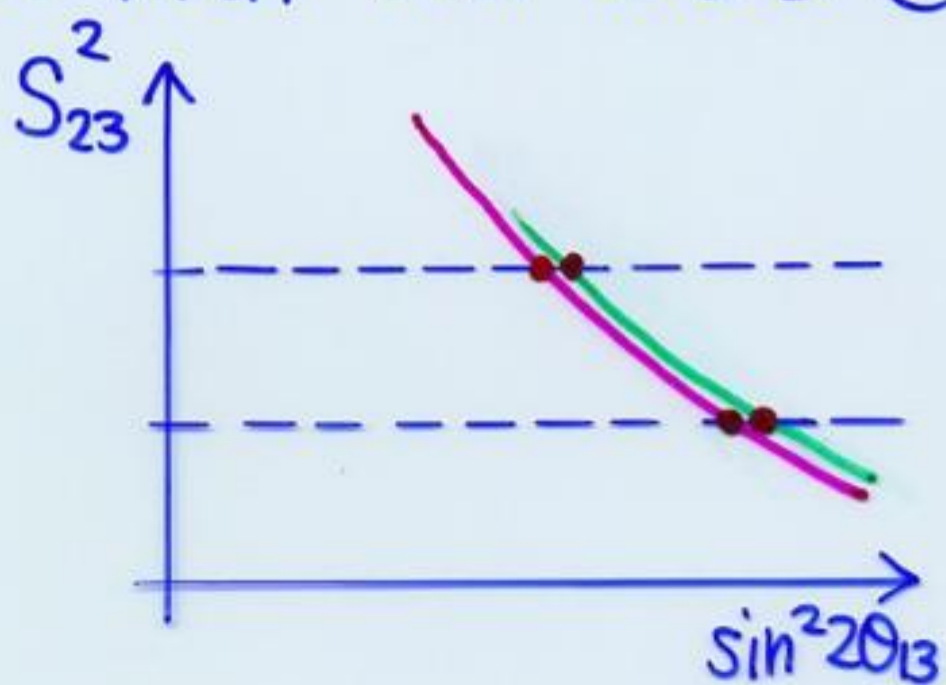
(A) with JHF  $\nu \oplus \bar{\nu}$  @ OM



The situation looks like the upper figure.

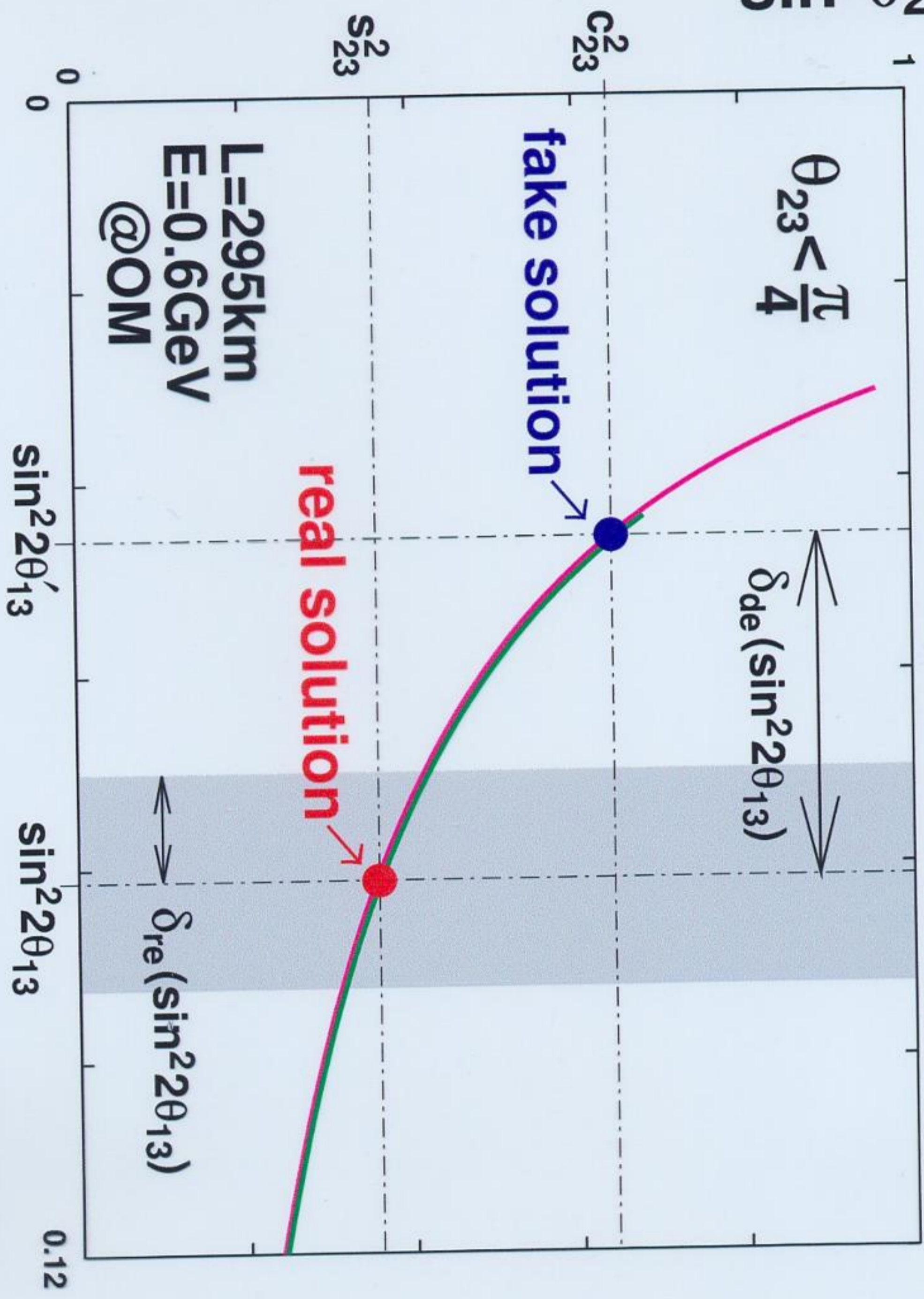
The precise determination of true  $\sin^2 2\theta_{13}$  is difficult, but the values of  $\sin^2 2\theta_{13}$  for the 4 solutions are approximately the same.

(B) with JHF  $\nu \oplus \bar{\nu}$  @ OM



The values of  $\sin^2 2\theta_{13}$  for  $\theta_{23} < \frac{\pi}{4}$  and  $\theta_{23} > \frac{\pi}{4}$  are quite different and it may be possible to determine the true value of  $\sin^2 2\theta_{13}$  if the error  $\delta_{re}(\sin^2 2\theta_{13})$  of the reactor exp. is smaller than the ambiguity  $\delta_{de}(\sin^2 2\theta_{13})$  due to the degeneracy.

$\sin^2\theta_{23}$

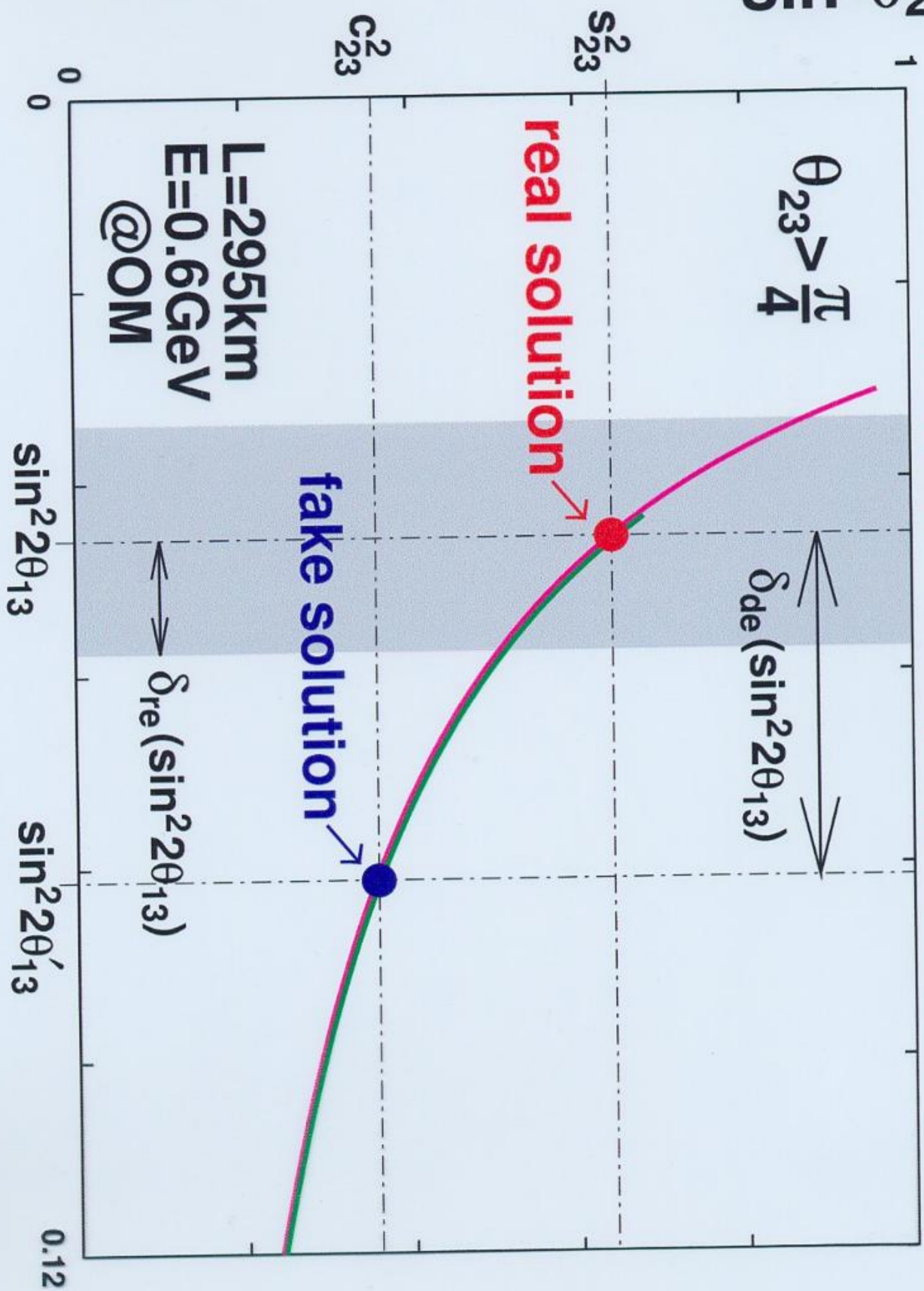


$P=0.025, \bar{P}=0.035, \Delta m_{31}^2 > 0$  — green line  
 $P=0.025, \bar{P}=0.035, \Delta m_{31}^2 < 0$  — pink line

$\sin^2 2\theta_{13}$



$\sin^2\theta_{23}$



$P=0.025, \bar{P}=0.035, \Delta m_{31}^2 > 0$  — green line  
 $P=0.025, \bar{P}=0.035, \Delta m_{31}^2 < 0$  — pink line

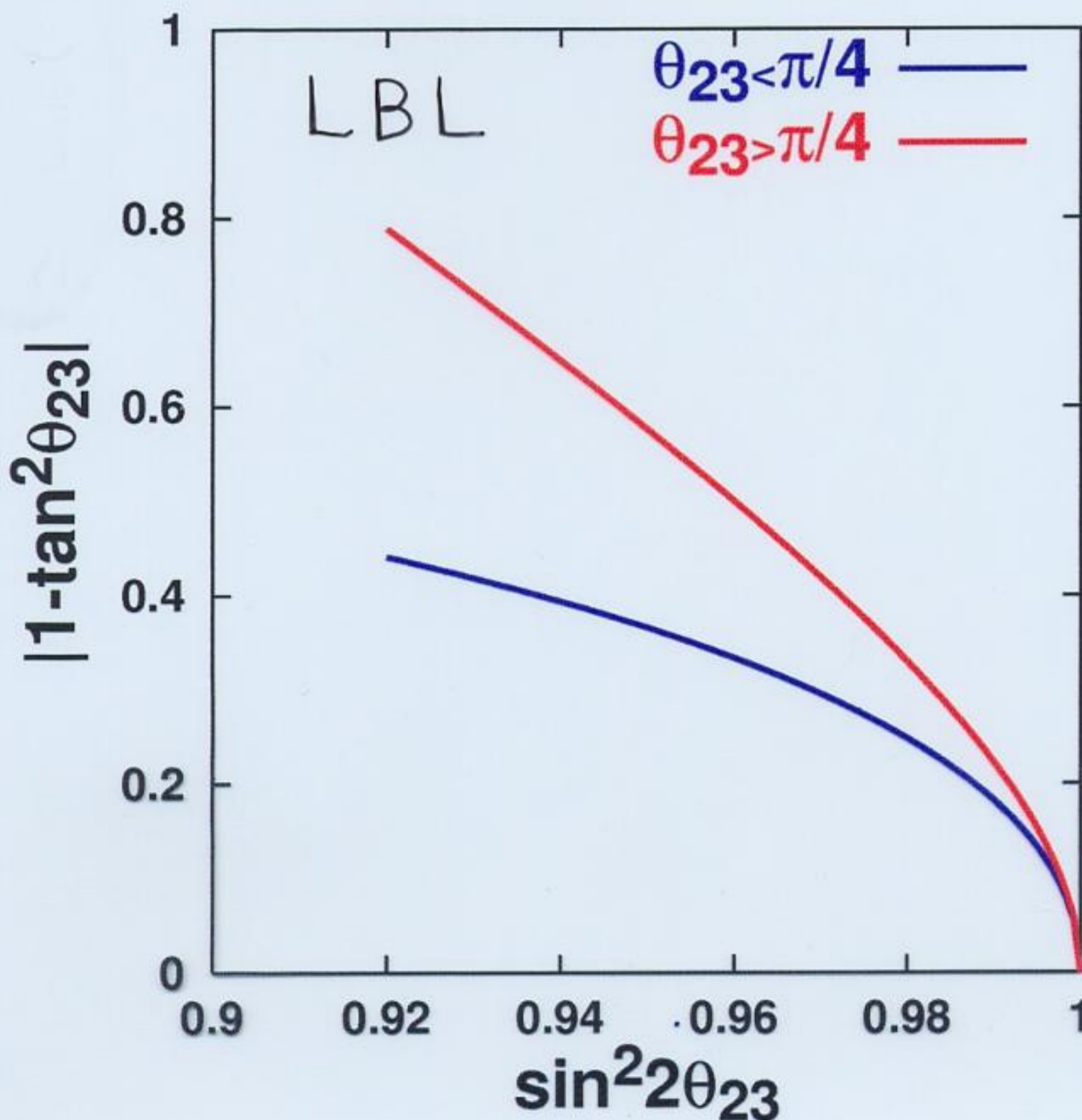
$\sin^2 2\theta_{13}$

$$\frac{\delta_{de}(\sin^2 2\theta_{13})}{\sin^2 2\theta_{13}} \equiv \frac{|\sin^2 2\theta'_{13} - \sin^2 2\theta_{13}|}{\sin^2 2\theta_{13}}$$

$$= |1 - \tan^2 \theta_{23}| \cdot \left\{ 1 + \left( \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right)^2 \frac{\tan^2(AL/2)}{AL/\pi} \left[ 1 - \left( \frac{AL}{\pi} \right)^2 \right] \sin^2 2\theta_{12} \right\}$$

$$\simeq |1 - \tan^2 \theta_{23}|$$

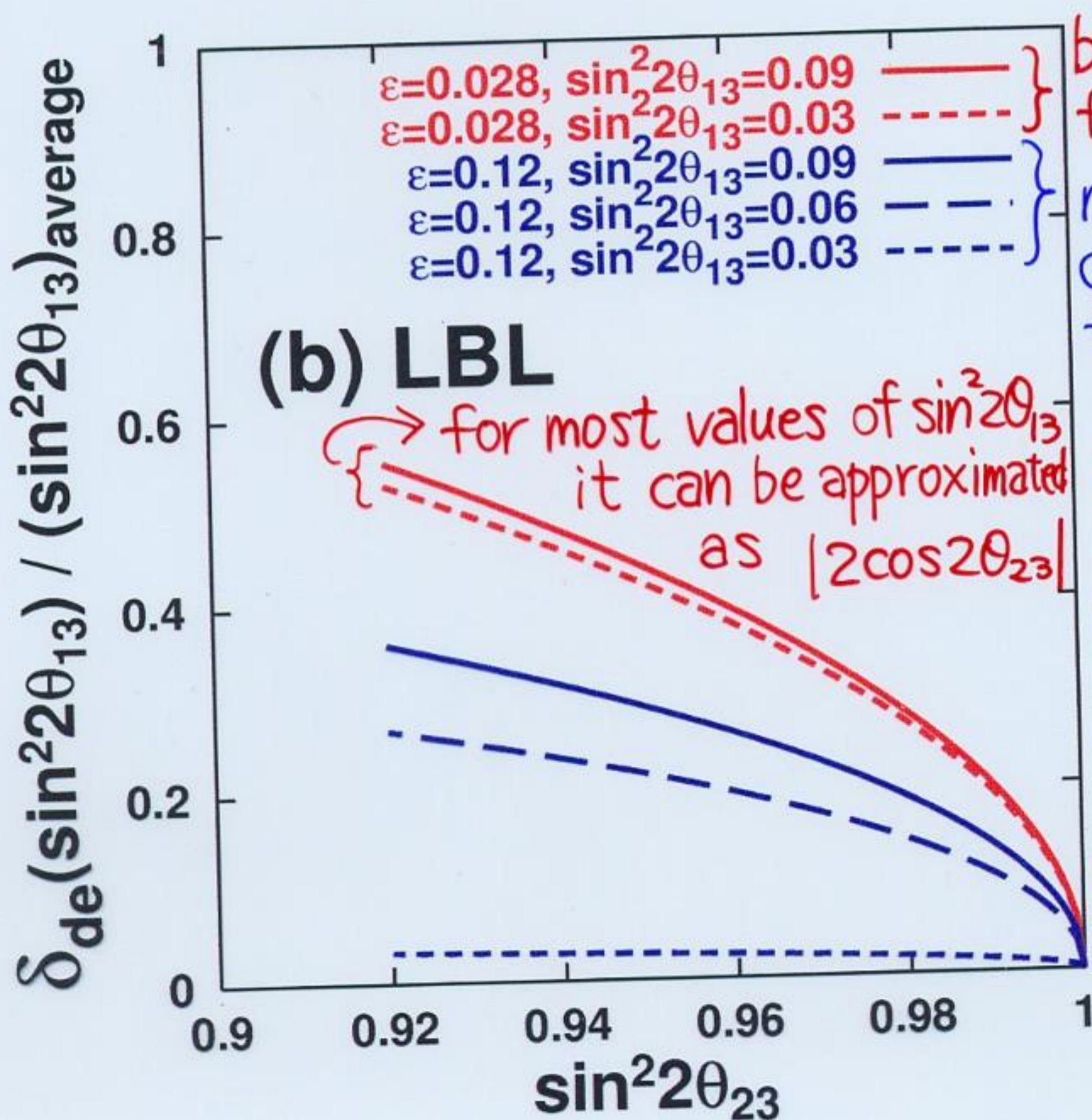
$\delta_{de}(\sin^2 2\theta_{13})$ : ambiguity due to the  
 $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$  degeneracy



ambiguity due to the degeneracy

$$\delta_{de}(\sin^2 2\theta_{13}) \equiv |\sin^2 2\theta_{13} - \sin^2 2\theta'_{13}|$$

$$(\sin^2 2\theta_{13})_{\text{average}} \equiv \frac{1}{2}(\sin^2 2\theta_{13} + \sin^2 2\theta'_{13})$$



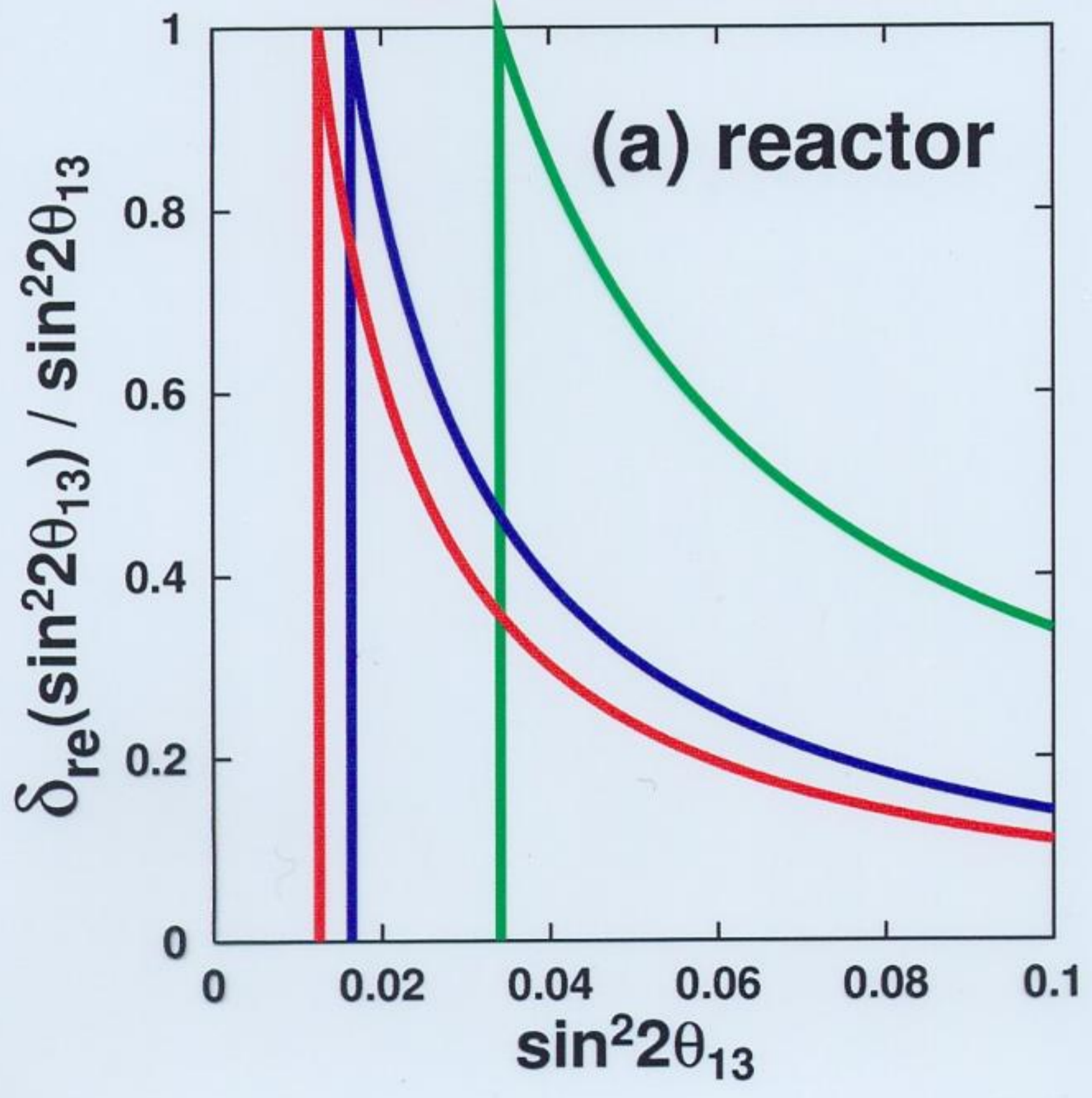
best fit case  
for  $\nu_e$  &  $\nu_{atm}$   
most pessimistic  
case @ 90% CL  
for  $\nu_e$  &  $\nu_{atm}$

for most values of  $\sin^2 2\theta_{13}$   
it can be approximated  
as  $|2\cos 2\theta_{23}|$

error in the reactor experiment  
 $\delta_{re}(\sin^2 2\theta_{13})$

$\delta_{re}(\sin^2 2\theta_{13})$

- $\sigma_{sys} = 2\%, 5t \cdot yr, d.o.f.=2$  ——— 0,034
- $\sigma_{sys} = 0.8\%, 20t \cdot yr, d.o.f.=2$  ——— 0.015
- $\sigma_{sys} = 0.8\%, 20t \cdot yr, d.o.f.=1$  ——— 0.012



#### 4. Summary

Reactor experiment on  $\theta_{13}$

\*  $\left. \begin{array}{l} \text{much cheaper} \\ \text{may be done earlier} \end{array} \right\}$  than JHF

\* sensitivity

$$\sin^2 2\theta_{13} \geq 0.013 \quad \text{for } \sigma_{\text{sys}} = 0.8\%, 20 \text{ ton}\cdot\text{yr (d.o.f.=1)}$$

@ KK - NPP

\* free from degeneracy

$$\text{if } \sin^2 2\theta_{23} \leq 0.96 \quad \& \quad \sin^2 2\theta_{13} \geq 0.06$$

then a reactor experiment may be able to determine  $\sin^2 2\theta_{13}$  &  $S_{23}^2$  for the true solution.

ocean

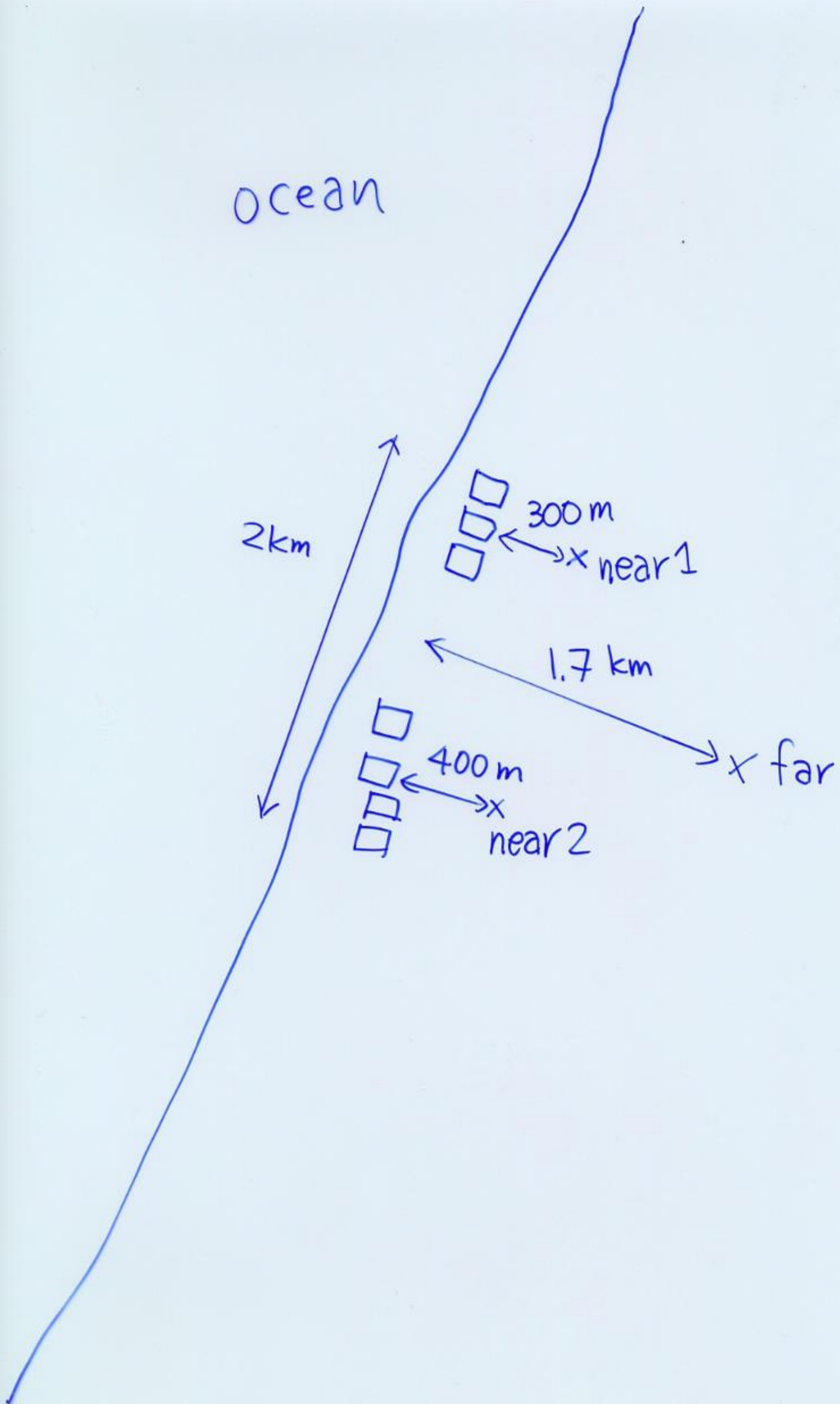
2 km

300 m  
x near 1

1.7 km

400 m  
x near 2

x far

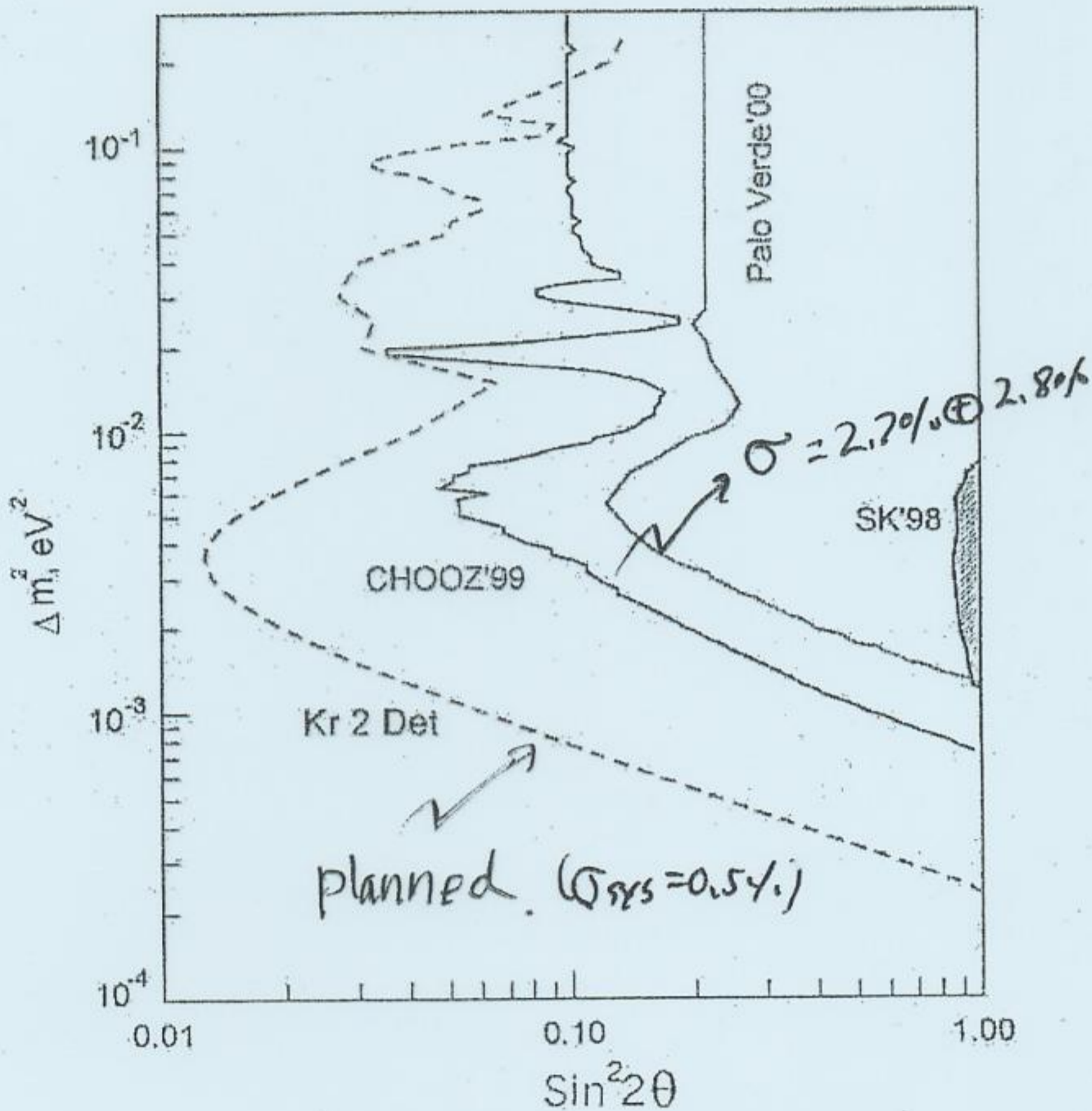


# Reactor $\theta_{13}$ Status

Mikael - 02

## OSCILLATION LIMITS

Mikaelian



For reactor  $\tilde{\nu}_e$  at 1 km:

$$\sin^2 2\theta = 4U_{e3}^2 (1 - U_{e3}^2) \approx 4U_{e3}^2$$

$$\nu_e = U_{e1} \nu_1 + U_{e2} \nu_2 + U_{e3} \nu_3$$

$$U_{e3} \equiv \sin \theta_{13}$$