Parameter degeneracy and reactor neutrino experiments

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1. Parameter degeneracy in $(s^2_{23}, \sin^2 2\theta_{13})$ plane
2. Resolution of $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$ degeneracy by LBL $\oplus$ reactor
3. Summary

Parameter degeneracy in $(s_{23}^2, \sin^2 2\theta_{13})$ plane

Even if $P \equiv P(\nu_\mu \to \nu_e)$ and $\bar{P} \equiv P(\bar{\nu}_\mu \to \bar{\nu}_e)$ are given, there are in general 8 solutions.

3 kinds of degeneracy

- intrinsic (δ, θ_{13}) — Burguet-Castell et al ('01)
- sign (Δm_{31}^2) — Minakata-Nunokawa ('01)
- $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$ — Fogli-Lisi PRD54('96)3667, Barger-Marfatia-Whisnant('02)

$\to$ $\boxed{\text{8-fold degeneracy}}$

Here I assume that accelerator beams are approximately monchromatic.

Experimental errors in long baseline experiments are not taken into account.

I will show how the 8-fold degeneracy is lifted by switching on :

$$\theta_{23} - \frac{\pi}{4}, \qquad \left|\frac{\Delta m_{21}^2}{\Delta m_{31}^2}\right|, \qquad AL$$

$(A \equiv \sqrt{2} G_F N_e)$

(they are all small @ JHF experiment)

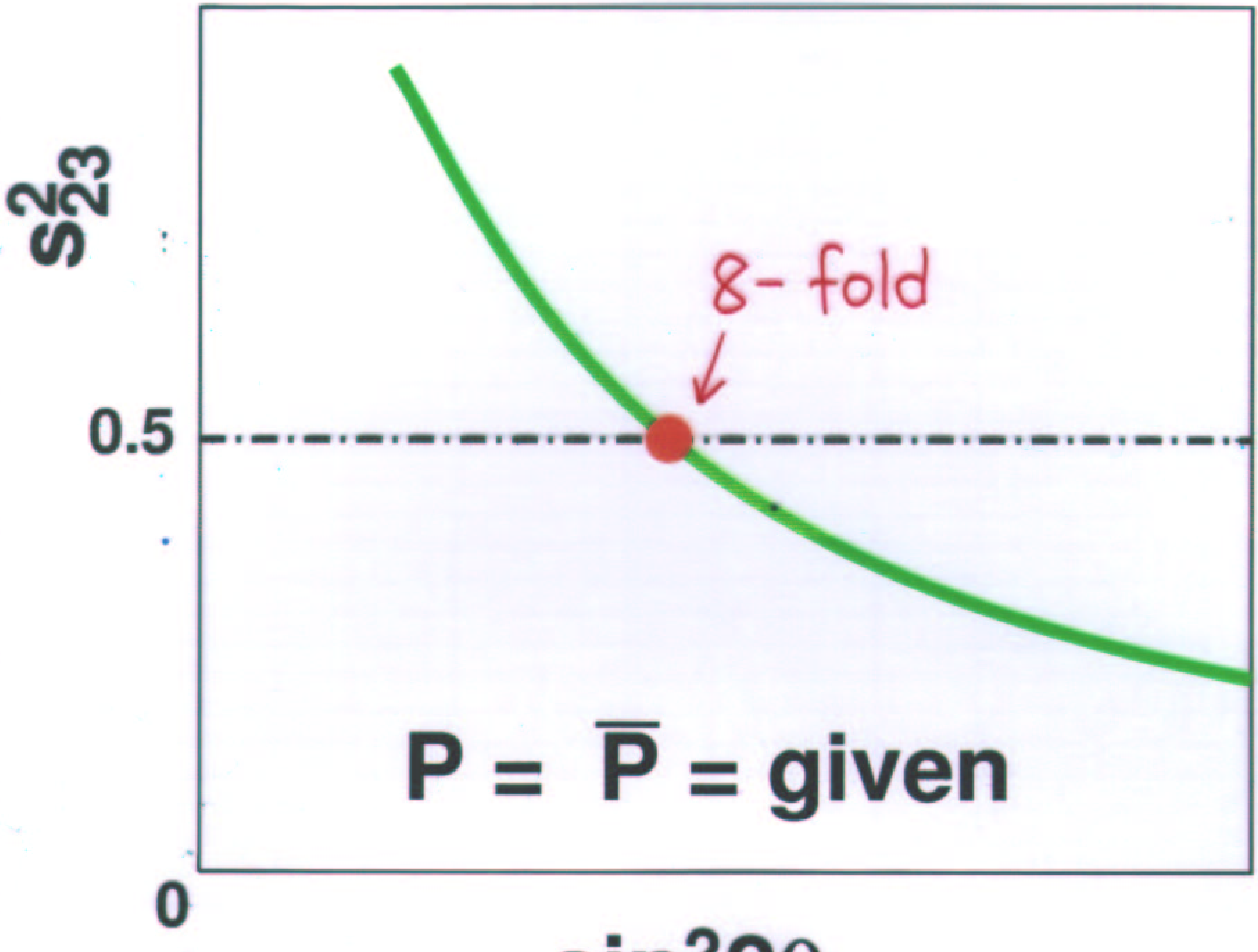
$\sin^2 2\theta_{23} \gtrsim 0.92$ $\quad \left|\frac{\Delta m_{21}^2}{\Delta m_{31}^2}\right| \sim \frac{1}{35}$ $\quad \frac{AL}{2} \sim \frac{1}{13}$

Here I visualize the 8-fold degeneracy by using the $(s_{23}^2, \sin^2 2\theta_{13})$ plane step by step.

$\epsilon \equiv \frac{\Delta m_{21}^2}{\Delta m_{31}^2}$ $\frac{AL}{2}$

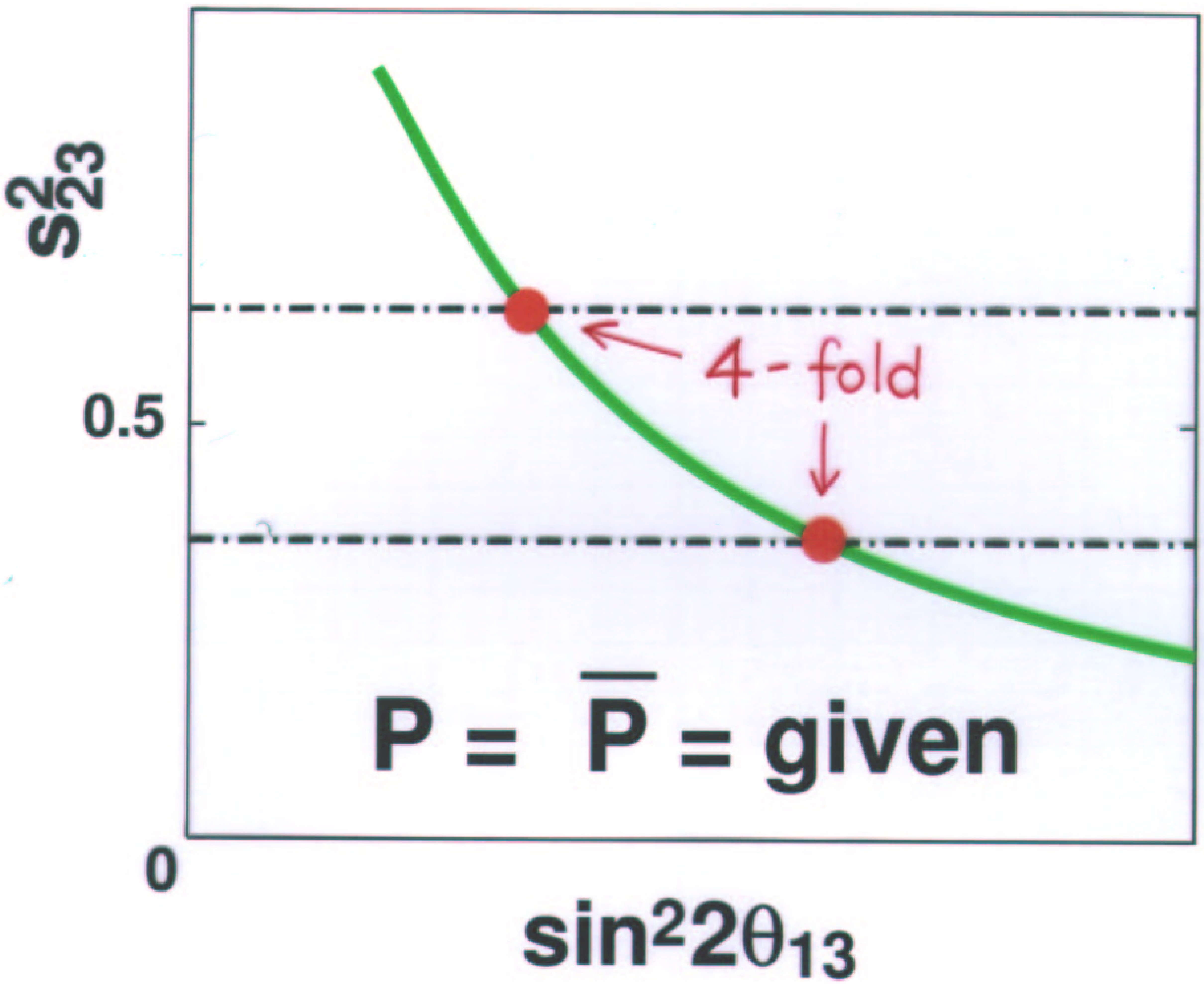
	$\theta_{23} - \frac{\pi}{4}$	Δm_{21}^2	$A \equiv \sqrt{2} G_F N_e$	$\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$	(δ, θ_{13})	$\mathrm{sign}(\Delta m_{31}^2)$
(a)	$=0$	$=0$	$=0$	degen.	degen.	degen.
(b)	$\neq 0$	$=0$	$=0$	lifted	degen.	degen.
(c)	$\neq 0$	$\neq 0$	$=0$	lifted	lifted	degen.
(d) off OM	$\neq 0$	$\neq 0$	$\neq 0$	lifted	lifted	lifted
(e) @OM	$\neq 0$	$\neq 0$	$\neq 0$	lifted	degen.	almost degen.

(a) $\theta_{23}=\frac{\pi}{4}$, $\Delta m^2_{21}=0$, $A=0$



$$P=\overline{P} = \underbrace{s_{23}^2}_{=\frac{1}{2}} \sin^2 2\theta_{13} \underbrace{\sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)}_{=\text{const}}$$

(b) $\theta_{23} \neq \frac{\pi}{4}$, $\Delta m^2_{21} = 0$, $A = 0$

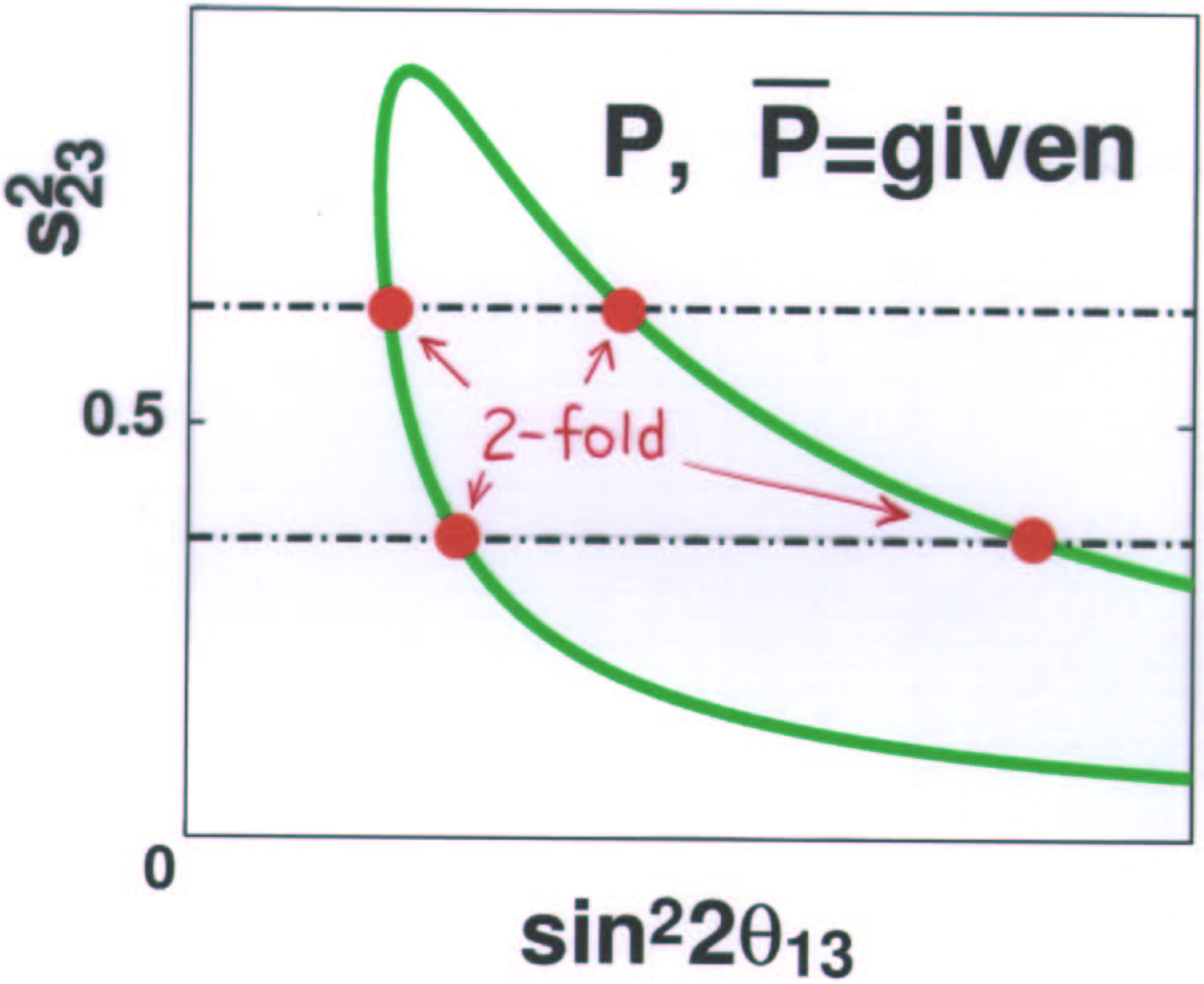


$$P = \overline{P} = s_{23}^2 \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$s_{23}^2 = \frac{1 \pm \sqrt{1 - \sin^2 2\theta_{23}}}{2}$$

known from $\nu_\mu \to \nu_\mu$

(c) $\theta_{23} \neq \frac{\pi}{4}$, $\Delta m^2_{21} \neq 0$, $A=0$



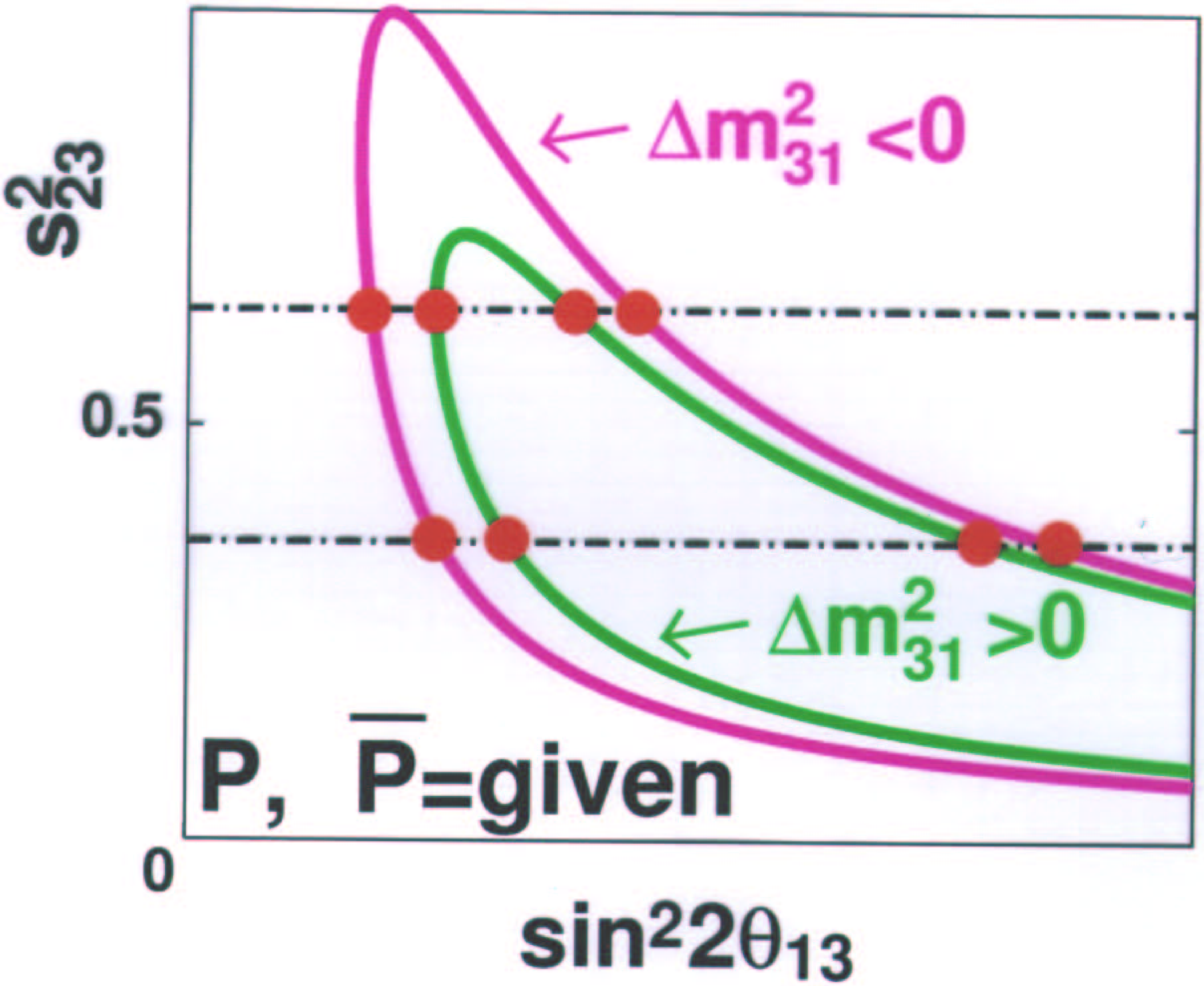
$$\frac{1}{\cos^2\Delta}\left(\frac{P+\bar{P}}{2} - x^2\sin^2\Delta - y^2\Delta^2\right)^2 + \frac{1}{\sin^2\Delta}\left(\frac{P-\bar{P}}{2}\right)^2 = (2xy\Delta\sin\Delta)^2$$

quadratic eq. in x^2

$$\left(\begin{array}{l} x \equiv s_{23}\sin 2\theta_{13} \\ y \equiv \left|\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right| c_{23}\sin 2\theta_{12} \\ \Delta \equiv \frac{\Delta m^2_{31} L}{4E} \end{array}\right)$$

$$s_{23} = \sqrt{\frac{1 \pm \sqrt{1 - \sin^2 2\theta_{23}}}{2}}$$

(d) $\theta_{23}\neq\frac{\pi}{4}$, $\Delta m^2_{21}\neq 0$, $A\neq 0$ off OM



$$\frac{1}{4\cos^2\Delta}\left(\frac{P-x^2f^{(\mp)2}-y^2g^2}{f^{(\mp)}}+\frac{\bar{P}-x^2f^{(\pm)2}-y^2g^2}{f^{(\pm)}}\right)^2$$

$$+\frac{1}{4\sin^2\Delta}\left(\frac{P-x^2f^{(\mp)2}-y^2g^2}{f^{(\mp)}}-\frac{\bar{P}-x^2f^{(\pm)2}-y^2g^2}{f^{(\pm)}}\right)^2=\left(2xyg\right)^2$$

for $\Delta m^2_{31}\left\{\begin{matrix}>0\\<0\end{matrix}\right\}$

quadratic in x^2

$$x\equiv s_{23}\sin 2\theta_{13}$$

$$y\equiv\left|\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right|c_{23}\sin 2\theta_{12}$$

$$\Delta\equiv\frac{\Delta m^2_{31}L}{4E}$$

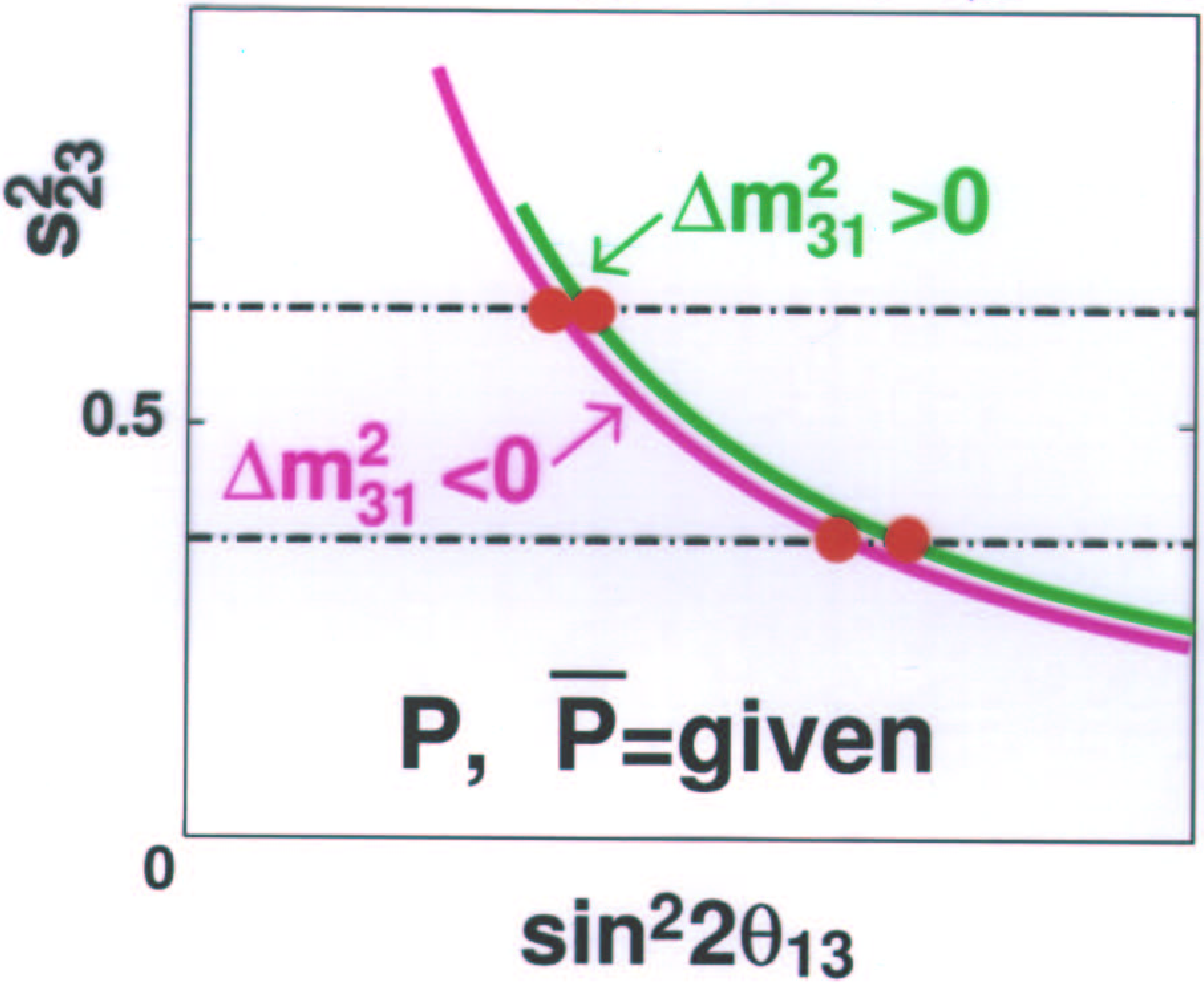
$$s_{23}=\sqrt{\frac{1\pm\sqrt{1-\sin^2 2\theta_{23}}}{2}}$$

$$f^{(\pm)}\equiv\frac{\sin(\Delta\pm AL/2)}{1\mp AL/2\Delta}$$

$$g\equiv\frac{\sin(AL/2)}{AL/2\Delta}$$

(e) $\theta_{23} \neq \frac{\pi}{4}$, $\Delta m^2_{21} \neq 0$, $A \neq 0$

@OM $\left(\frac{\Delta m^2_{31} L}{4E} = \frac{\pi}{2} \right)$



$$\frac{P - x^2 f^{(\mp)2} - y^2 g^2}{f^{(\mp)}} = - \frac{\bar{P} - x^2 f^{(\pm)2} - y^2 g^2}{f^{(\pm)}} \quad \text{for } \Delta m^2_{31} \begin{Bmatrix} >0 \\ <0 \end{Bmatrix}$$

linear in x^2

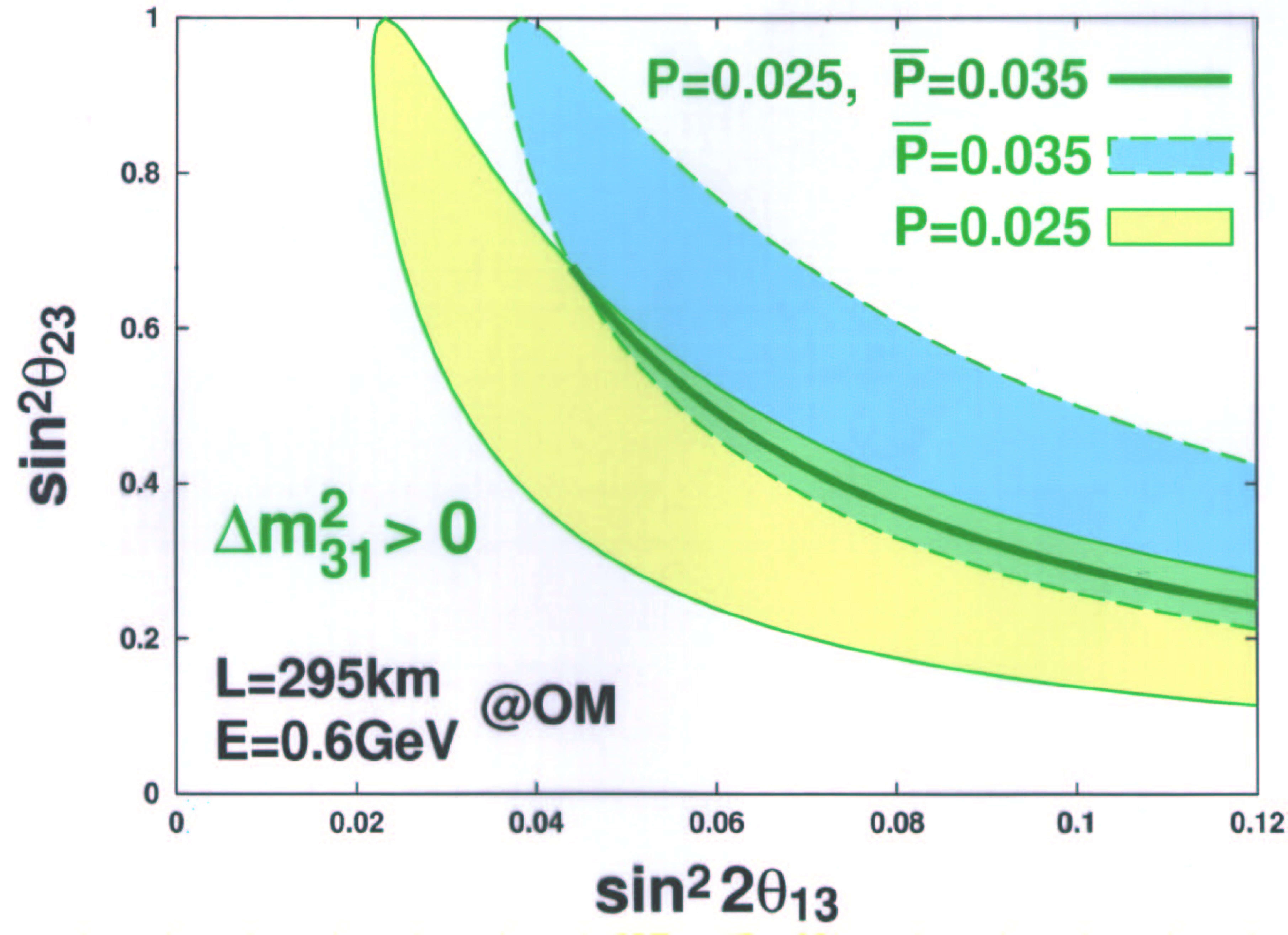
$$x \equiv s_{23} \sin 2\theta_{13}$$

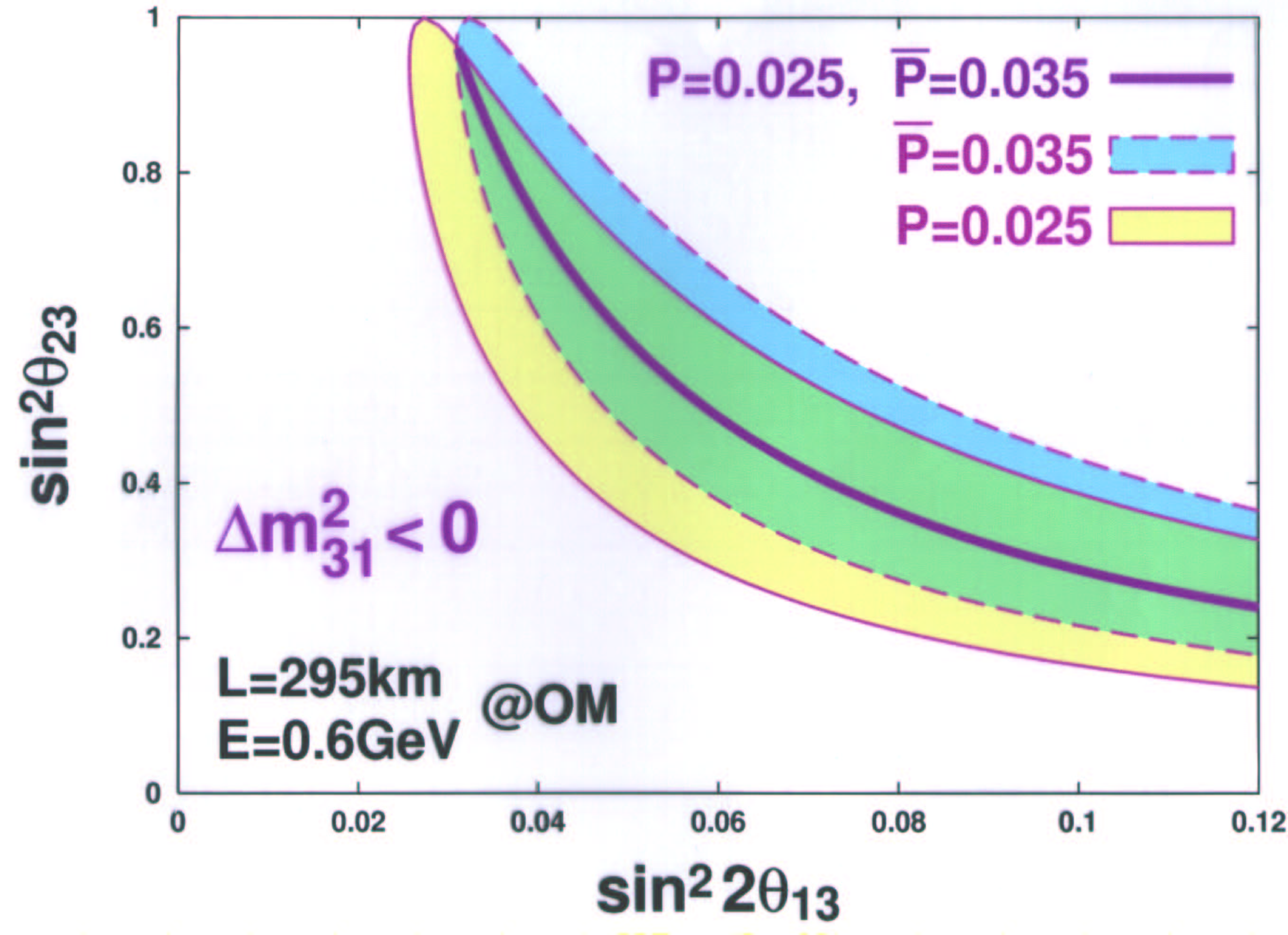
$$y \equiv \left| \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \right| c_{23} \sin 2\theta_{12}$$

$$f^{(\pm)} \equiv \frac{\cos(AL/2)}{1 \mp AL/\pi}$$

$$s_{23} = \sqrt{\frac{1 \pm \sqrt{1 - \sin^2 2\theta_{23}}}{2}}$$

$$g \equiv \frac{\sin(AL/2)}{AL/\pi}$$


P=0.025, $\overline{P}$=0.035
$\overline{P}$=0.035
P=0.025
$\Delta m^2_{31} > 0$
L=295km
E=0.6GeV
@OM
$\sin^2\theta_{23}$
$\sin^2 2\theta_{13}$
0
0.2
0.4
0.6
0.8
1
0.02
0.04
0.06
0.08
0.1
0.12



P=0.025, $\overline{P}$=0.035
$\overline{P}$=0.035
P=0.025
$\Delta m^2_{31} < 0$
L=295km
E=0.6GeV
@OM
$\sin^2\theta_{23}$
$\sin^2 2\theta_{13}$
0
0.2
0.4
0.6
0.8
1
0.02
0.04
0.06
0.08
0.1
0.12

2. Resolution of $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$ degeneracy by LBL $\oplus$ reactor

cf. Fogli-Lisi PRD54('96) 3667
Barenboim-de Gouvea ('02)

Our scenario

- JHF $\nu \oplus \bar{\nu}$ @ Oscillation Maximum

$\oplus$

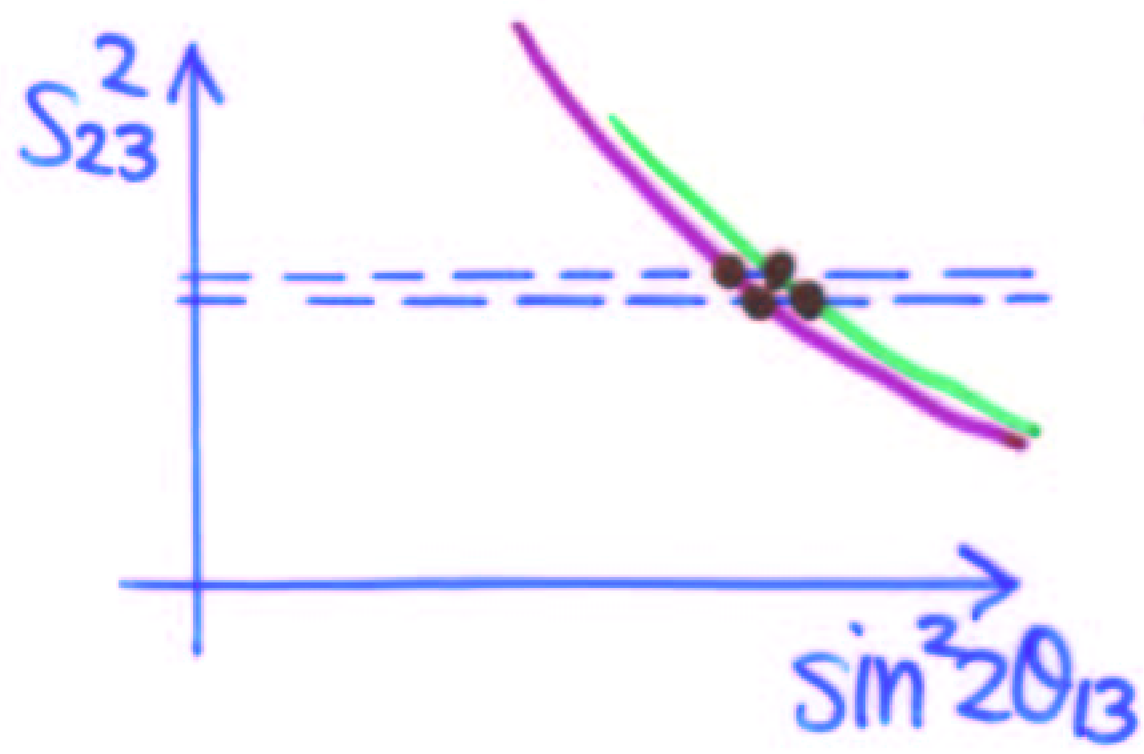
- reactor experiment (@ Kashiwazaki ?)

From $\nu_\mu \leftrightarrow \nu_\mu$ @JHF we will know that θ_{23} satisfies either of the followings:

(A) $|1 - \sin^2 2\theta_{23}| <$ a few $\times 10^{-2}$

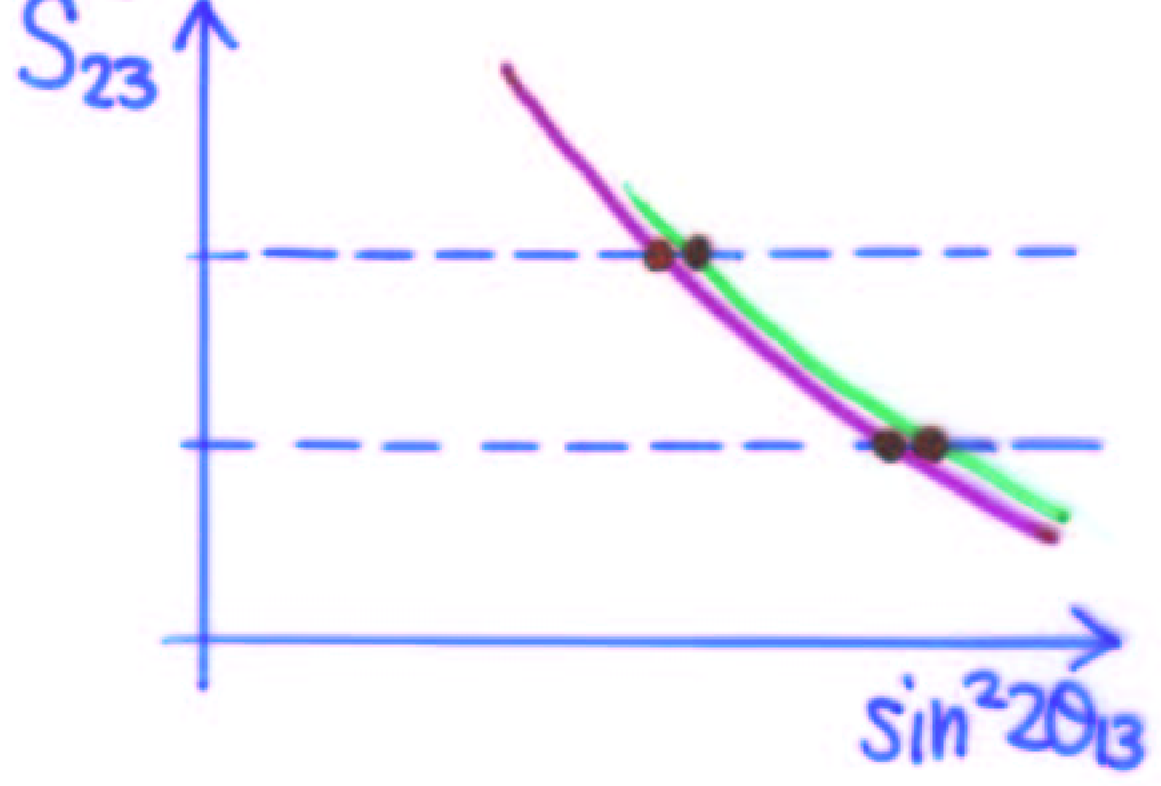
(B) $|1 - \sin^2 2\theta_{23}| \geq$ a few $\times 10^{-2}$

(A) with JHF $\nu \oplus \bar{\nu}$ @OM



The situation looks like the upper figure.
The precise determination of true $\sin^2 2\theta_{13}$ is difficult, but the values of $\sin^2 2\theta_{13}$ for the 4 solutions are approximately the same.

(B) with JHF $\nu \oplus \bar{\nu}$ @OM



The values of $\sin^2 2\theta_{13}$ for $\theta_{23} < \frac{\pi}{4}$ and $\theta_{23} > \frac{\pi}{4}$ are quite different and it may be possible to determine the true value of $\sin^2 2\theta_{13}$ if the error $\delta_{re}(\sin^2 2\theta_{13})$ of the reactor exp. is smaller than the ambiguity $\delta_{de}(\sin^2 2\theta_{13})$ due to the degeneracy.

Measurement of θ_{13} by reactors

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \underbrace{\sin^2 2\theta_{13}} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$\sin^2 2\theta_{13}$ is measured
with an experimental error
$\delta_{re}(\sin^2 2\theta_{13})$
without any ambiguity from θ_{23} & δ

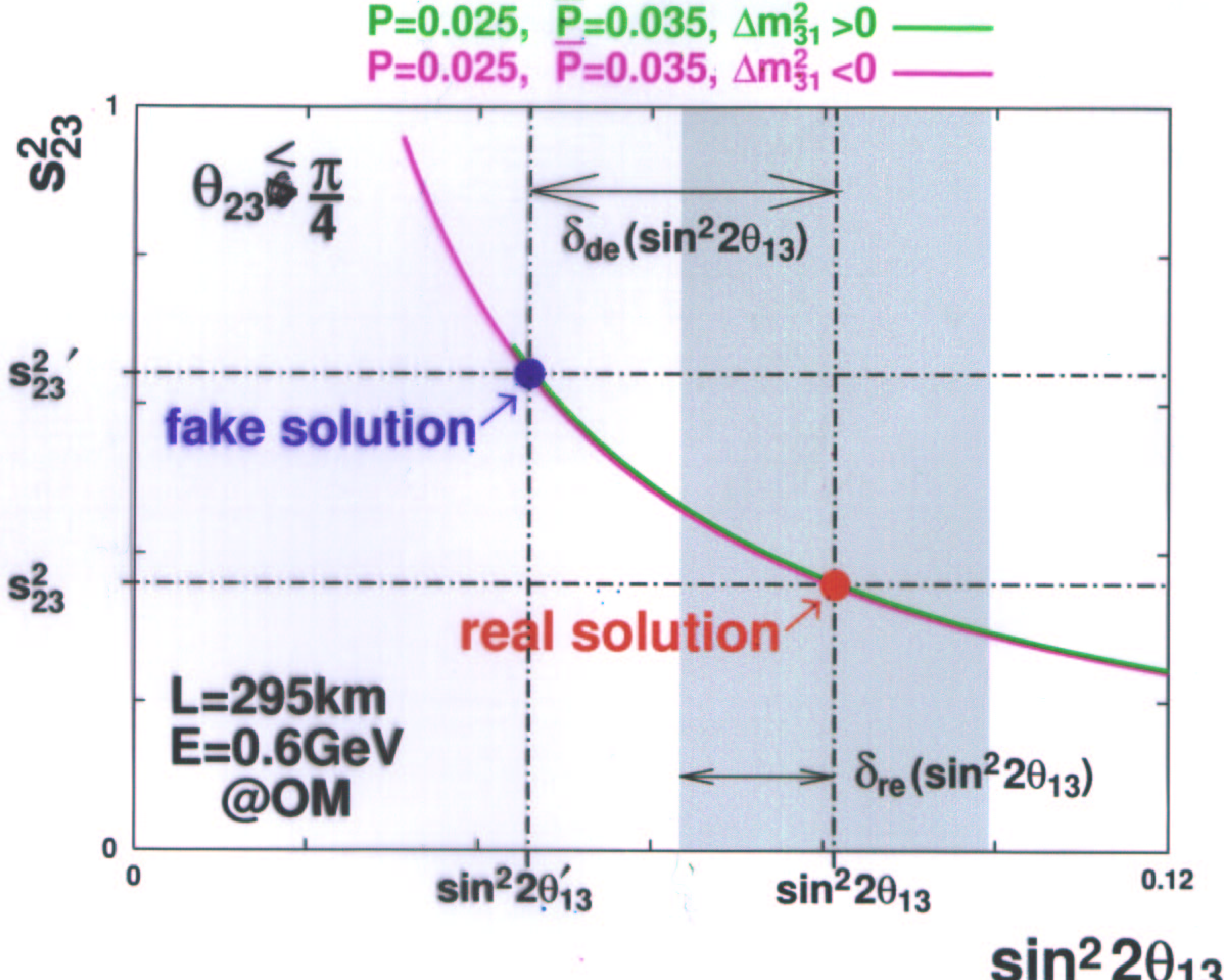
reactor measurement @Kashiwazaki-Kariwa
with $L = 1.7$ km, $\sigma_{sys} = 0.8\%$, 40t·yr
$\delta_{re}(\sin^2 2\theta_{13}) \simeq 0.018$

Minakata et al. hep-ph/0211111
revised version (to appear)

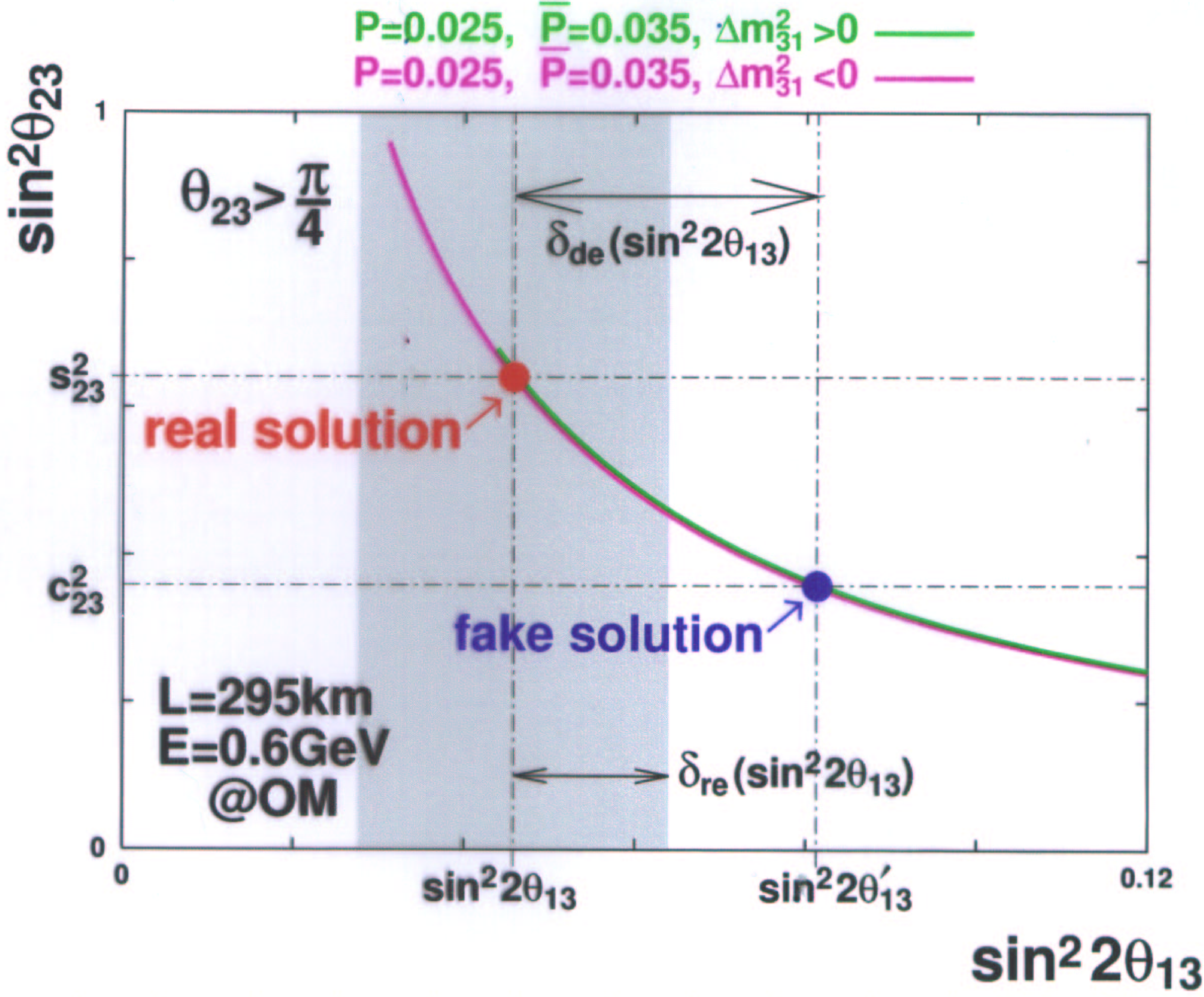
d.o.f = 1 ($\sin^2 2\theta_{13}$ only, assuming that $|\Delta m^2_{31}|$ is known from JHF)

$$(\sigma_{sys})^2 (N_{tot})^2 = \sum_j (\sigma^{bin}_{sys})^2 N_j^{\,2} = (\sigma^{bin}_{sys})^2 \sum_j N_j^{\,2}$$

$$\sigma^{bin}_{sys} \simeq 3\,\sigma_{sys} \simeq 2.4\,\%$$



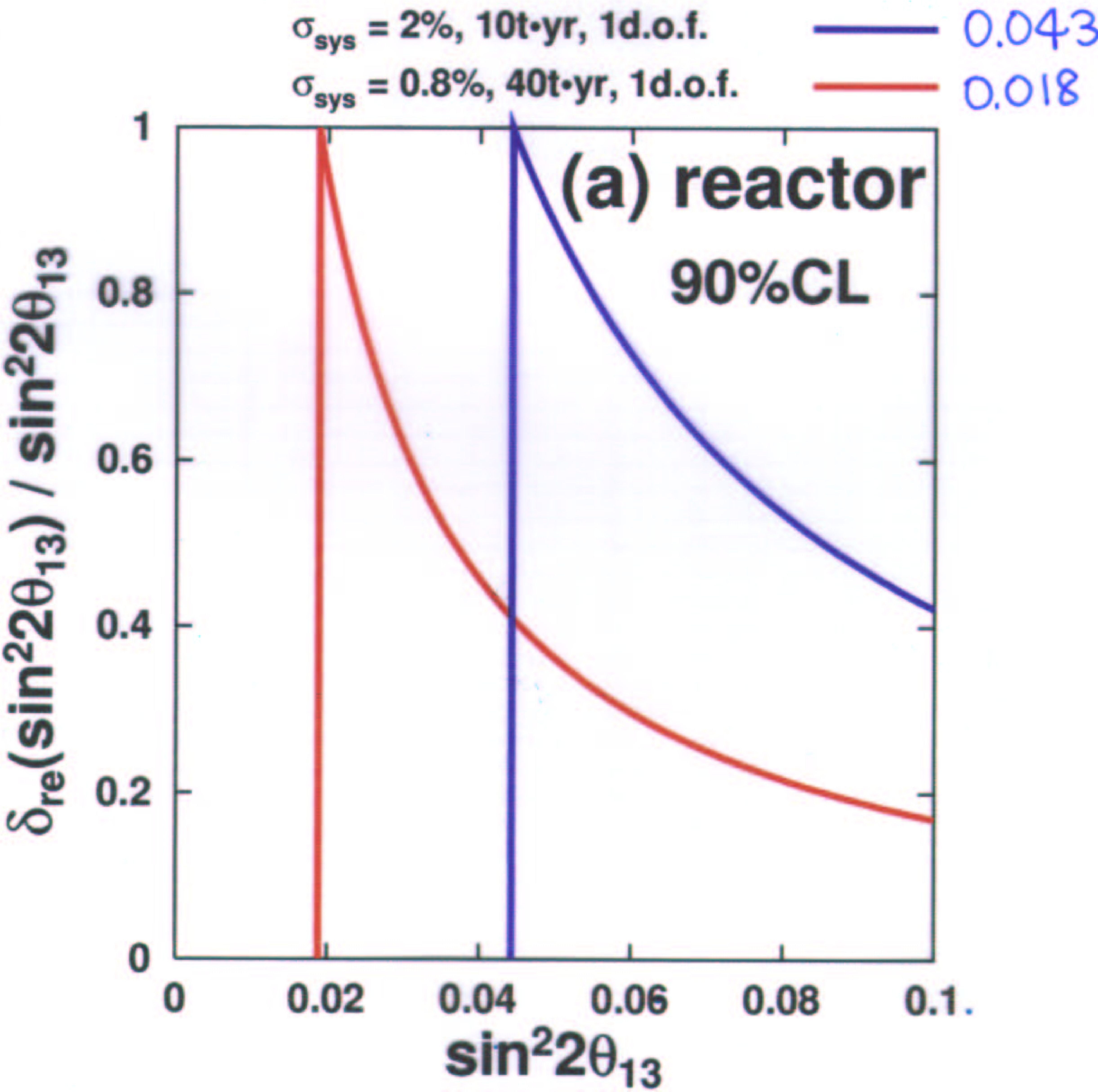
P=0.025, $\overline{P}$=0.035, Δm^2_{31} >0
P=0.025, $\overline{P}$=0.035, Δm^2_{31} <0
s^2_{23}
1
$\theta_{23} \lessgtr \frac{\pi}{4}$
$\delta_{de}(\sin^2 2\theta_{13})$
$s^2_{23}{}'$
fake solution
s^2_{23}
real solution
L=295km
E=0.6GeV
@OM
$\delta_{re}(\sin^2 2\theta_{13})$
0
0
$\sin^2 2\theta'_{13}$
$\sin^2 2\theta_{13}$
0.12
$\sin^2 2\theta_{13}$



P=0.025, $\overline{P}$=0.035, $\Delta m^2_{31}>0$
P=0.025, $\overline{P}$=0.035, $\Delta m^2_{31}<0$
$\sin^2\theta_{23}$
1
$\theta_{23}>\frac{\pi}{4}$
$\delta_{de}(\sin^2 2\theta_{13})$
s^2_{23}
real solution
c^2_{23}
fake solution
L=295km
E=0.6GeV
@OM
$\delta_{re}(\sin^2 2\theta_{13})$
0
0
$\sin^2 2\theta_{13}$
$\sin^2 2\theta'_{13}$
0.12
$\sin^2 2\theta_{13}$

error in the reactor experiment

$\delta_{re}(\sin^2 2\theta_{13})$

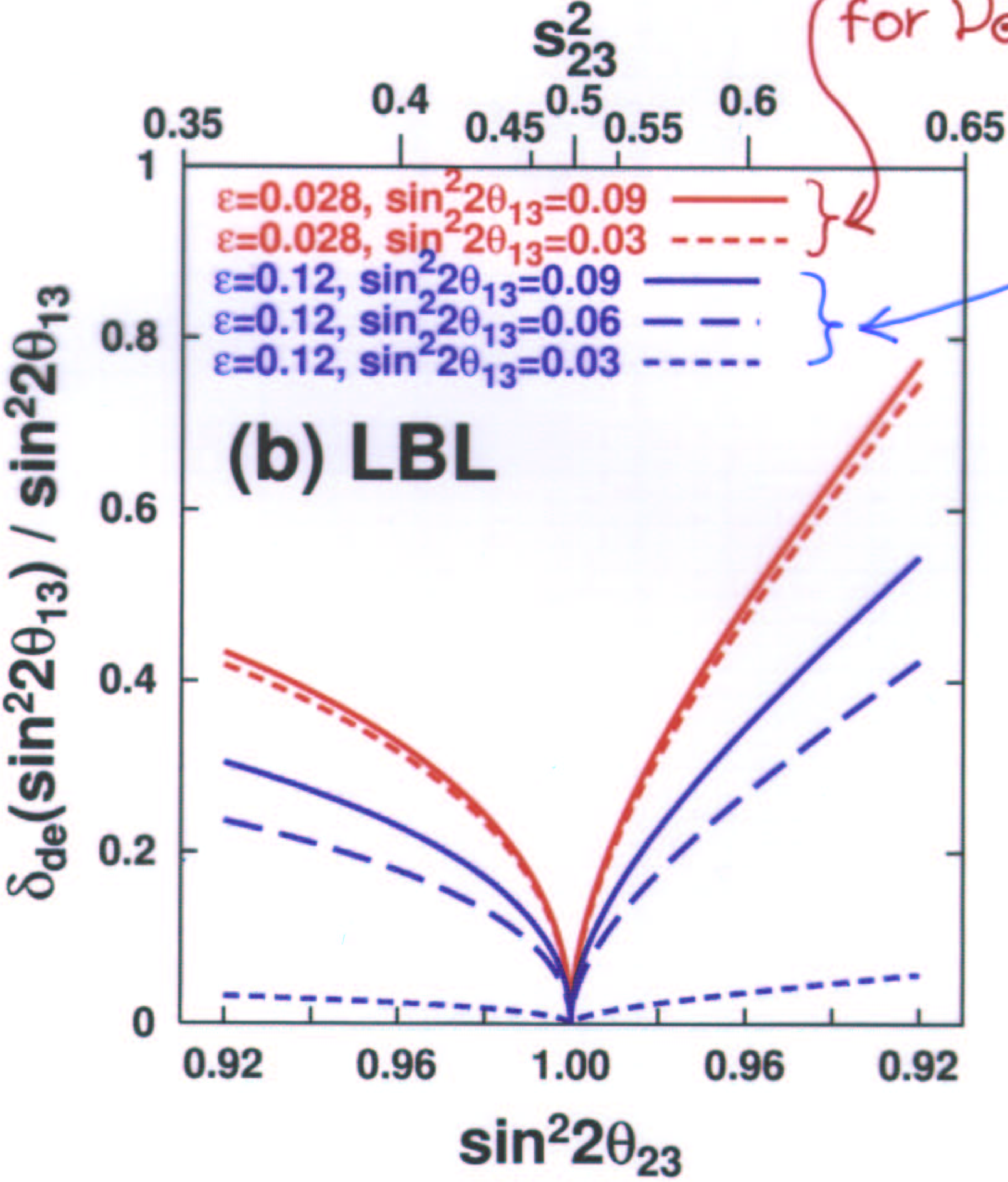


ambiguity due to the degeneracy

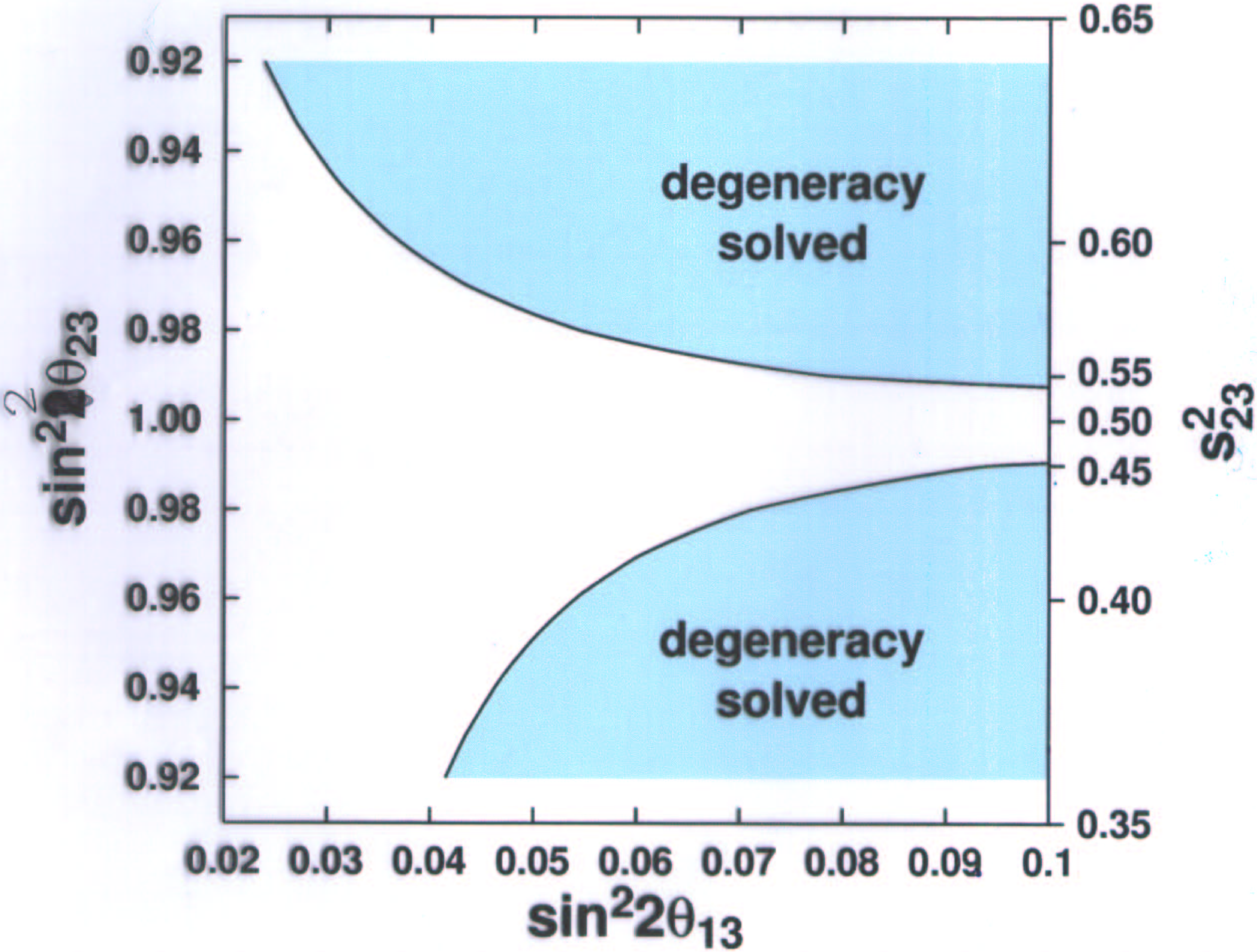
$\delta_{de}(\sin^2 2\theta_{13}) \equiv |\sin^2 2\theta'_{13} - \sin^2 2\theta_{13}|$

most pessimistic case @90%CL for $\nu_\odot$ & ν_{atm}

best fit case for $\nu_\odot$ & ν_{atm}



the region in which the θ_{23}-degeneracy is resolved



3. Summary

* 8-fold degeneracy can be visualized using the $(s^2_{23}, \sin^2 2\theta_{13})$ plane.

* $\left\{ \begin{array}{l} \text{JHF } \nu \oplus \bar{\nu} \text{ @OM} \\ \quad \oplus \\ \text{reactor} \end{array} \right\} \rightarrow$ may solve $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$ degeneracy if $|1-\sin^2 2\theta_{23}|$ and $\sin^2 2\theta_{13}$ are relatively large.