

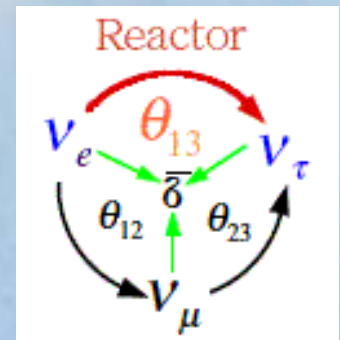
Sensitivity to $\sin^2 2\theta_{13}$ at KASKA

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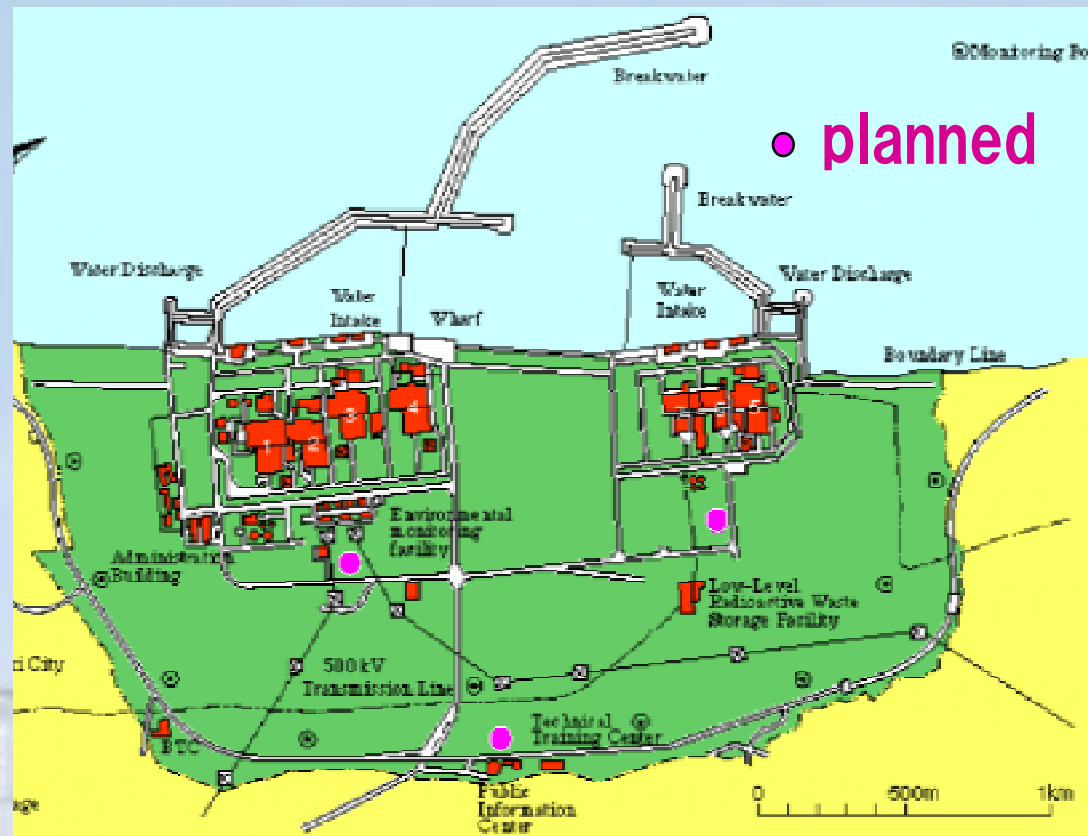
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1. Introduction

The locations of the detectors in the KASKA project are planned as indicated in the figure.



One has to make sure that these locations are close to the optimum.

Assumptions throughout this talk:

- **Identical** systematic errors in KASKA

from detectors $\left\{ \begin{array}{ll} \sigma_c & \text{correlated} \quad \sim 1.6\% \\ \sigma_u & \text{uncorrelated} \quad \sim 0.6\% \end{array} \right.$

from reactors $\left\{ \begin{array}{ll} \sigma_c^{(r)} & \text{correlated} \quad \sim 2.5\% \\ \sigma_u^{(r)} & \text{uncorrelated} \quad \sim 2.3\% \end{array} \right.$

The sensitivity to $\sin^2 2\theta_{13}$ only σ_u depends on to a good approximation.

- The present analysis is **rate** only.

- The sensitivity is defined **@90%CL**.

How sensitivity to $\sin^2 2 \theta_{13}$ is obtained @90%CL

$$\chi^2 = \min_{\alpha\text{'s}} \left\{ \sum_{i=1}^3 \left[\frac{M^i - T^i (1 + \alpha_c + \alpha_c^{(r)} + \sum_{a=1}^7 \left(\frac{T_a^i}{T^i} \right) \alpha_{ua}^{(r)})}{T^i \sigma_u} \right]^2 + \left(\frac{\alpha_c}{\sigma_c} \right)^2 + \left(\frac{\alpha_c^{(r)}}{\sigma_c^{(r)}} \right)^2 + \sum_{a=1}^7 \left(\frac{\alpha_{ua}^{(r)}}{\sigma_u^{(r)}} \right)^2 \right\}$$

M^i : measured # (events), T^i : theoretical # (events)

$$T^i = \sum_{a=1}^7 T_a^i, \quad M^i = \sum_{a=1}^7 M_a^i$$

$i=1,2,3$ (detectors); $a=1,\dots,7$ (reactors)

$$M_a^i = \int \epsilon(E) \sigma(E) f_a(E) P(E; L_{ia}) dE$$

$$T_a^i = \int \epsilon(E) \sigma(E) f_a(E) dE$$

contribution from a -th reactor to yield at i -th detector w/ and w/o osc.

$$2.7 = \chi^2 \Big|_{90\%CL} = \text{const} \times \sin^4 2 \theta_{13} \Rightarrow \sin^2 2 \theta_{13} = \sqrt{\frac{2.7}{\text{const}}}$$

2. Sensitivity to $\sin^2 2\theta_{13}$ at KASKA

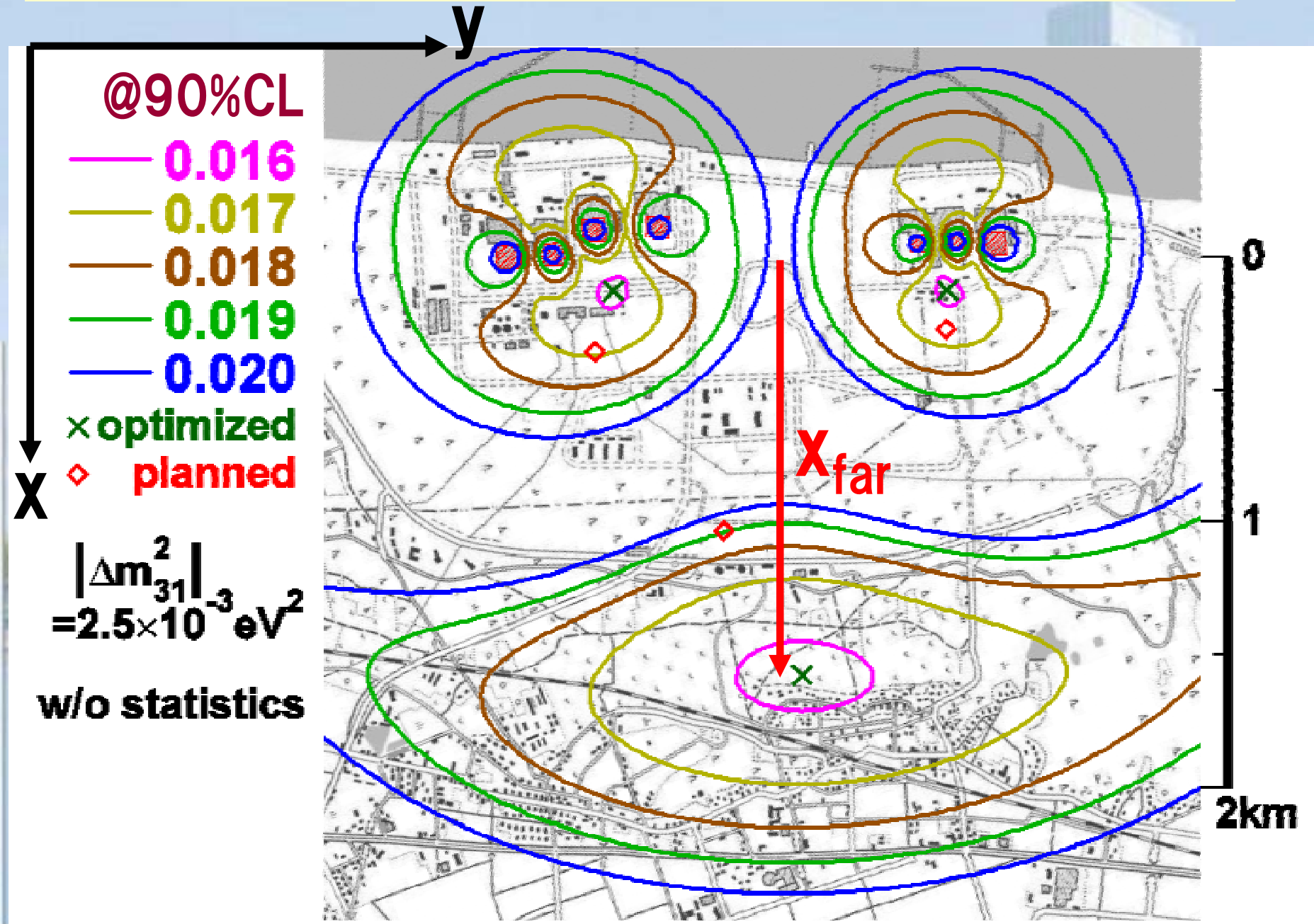
(1) Dependence on positions of detectors

1. Obtain the optimized positions of the three detectors.
2. Vary the position of one detector while keeping the other two in the optimized locations.
3. Combine each contour of the sensitivity to $\sin^2 2\theta_{13}$ in one figure for $\infty \text{ton} \cdot \text{yr}$ and $20 \text{ton} \cdot \text{yr}$.

(2) Dependence on data size

Obtain sensitivity to $\sin^2 2\theta_{13}$ for various data size assuming the optimized positions of the detectors.

(1) Dependence on positions of detectors



@90%CL

— 0.020

— 0.021

— 0.022

— 0.023

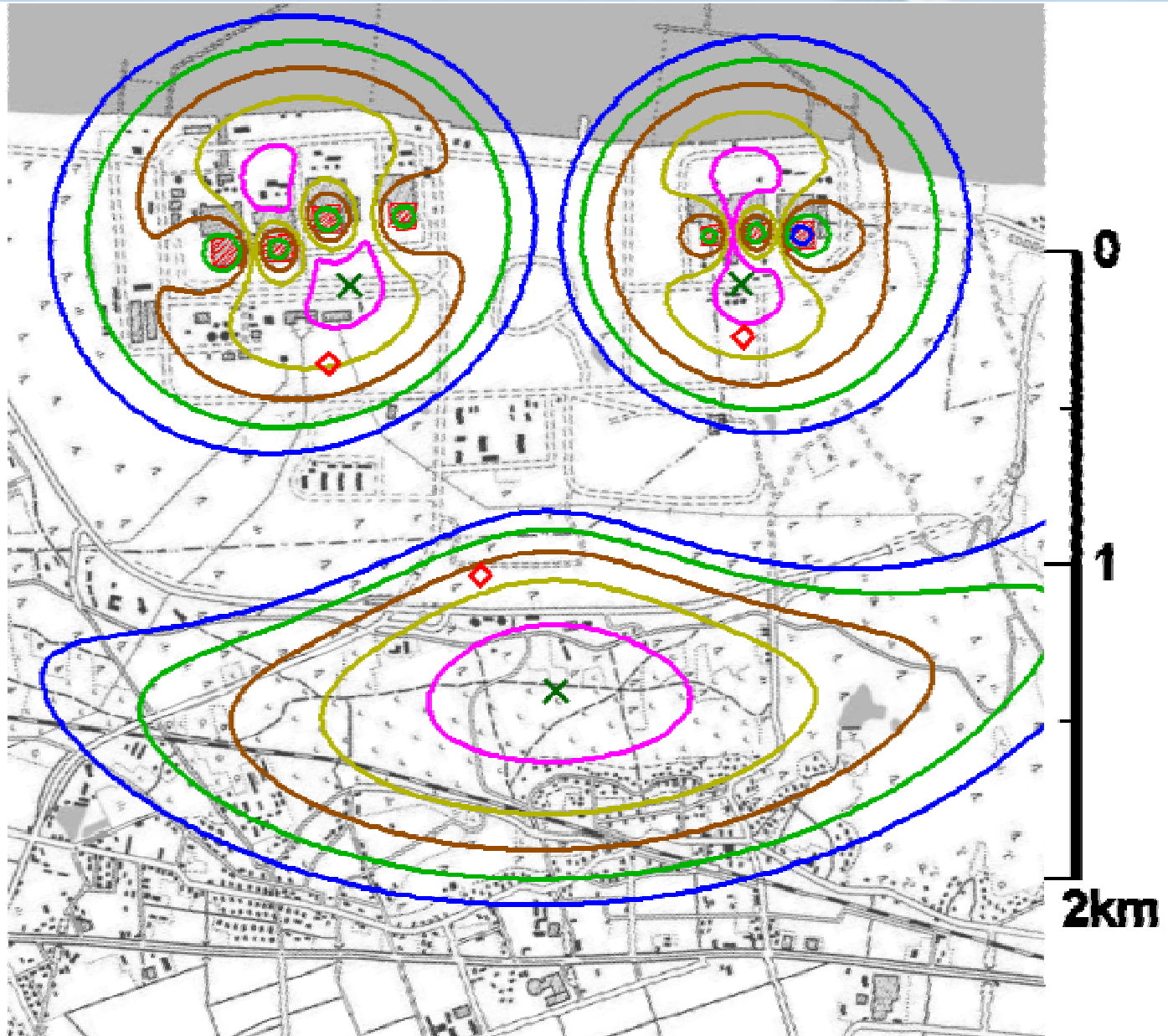
— 0.024

× optimized

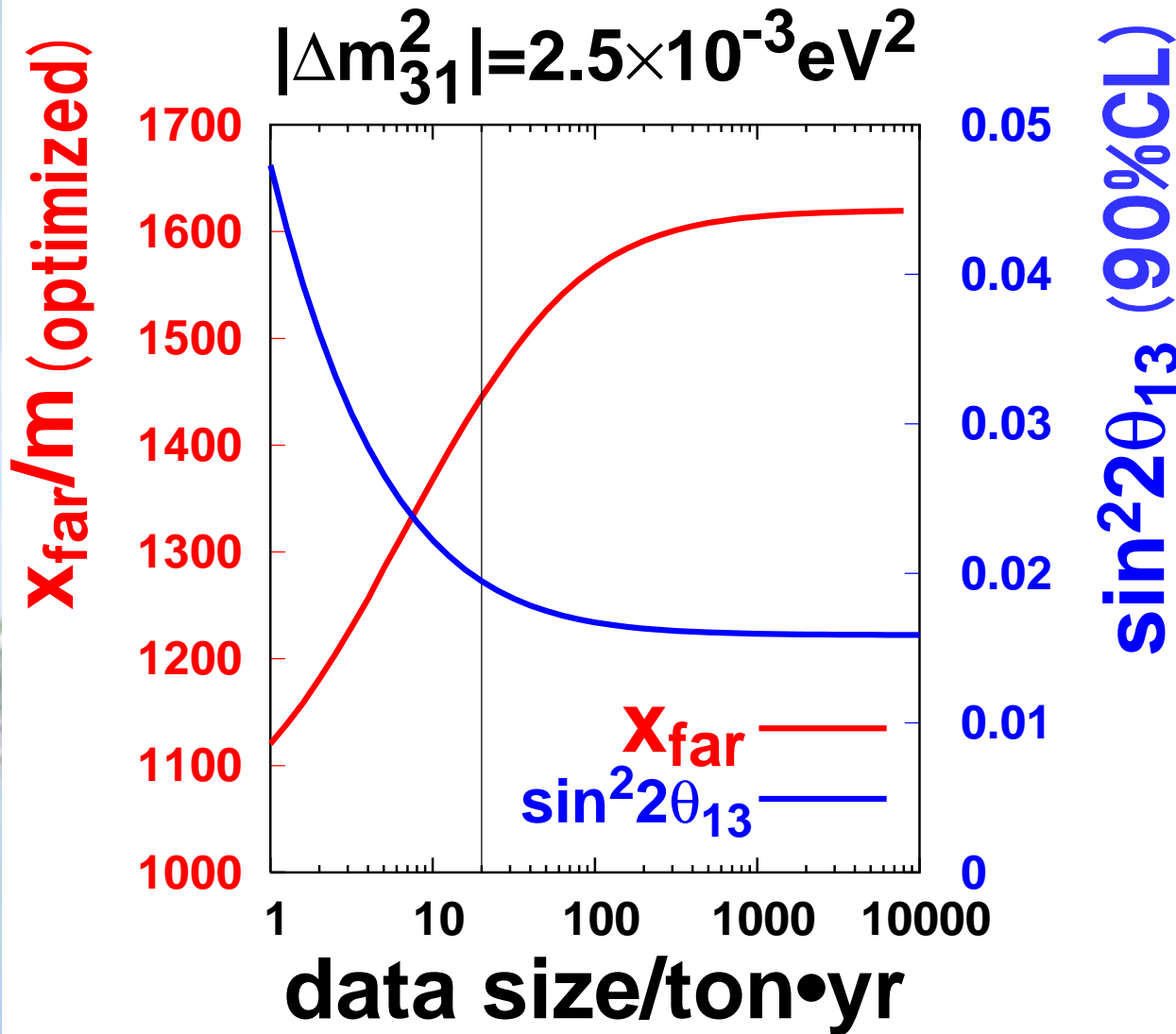
◇ planned

$$|\Delta m_{31}^2| = 2.5 \times 10^{-3} \text{ eV}^2$$

20 ton·yr

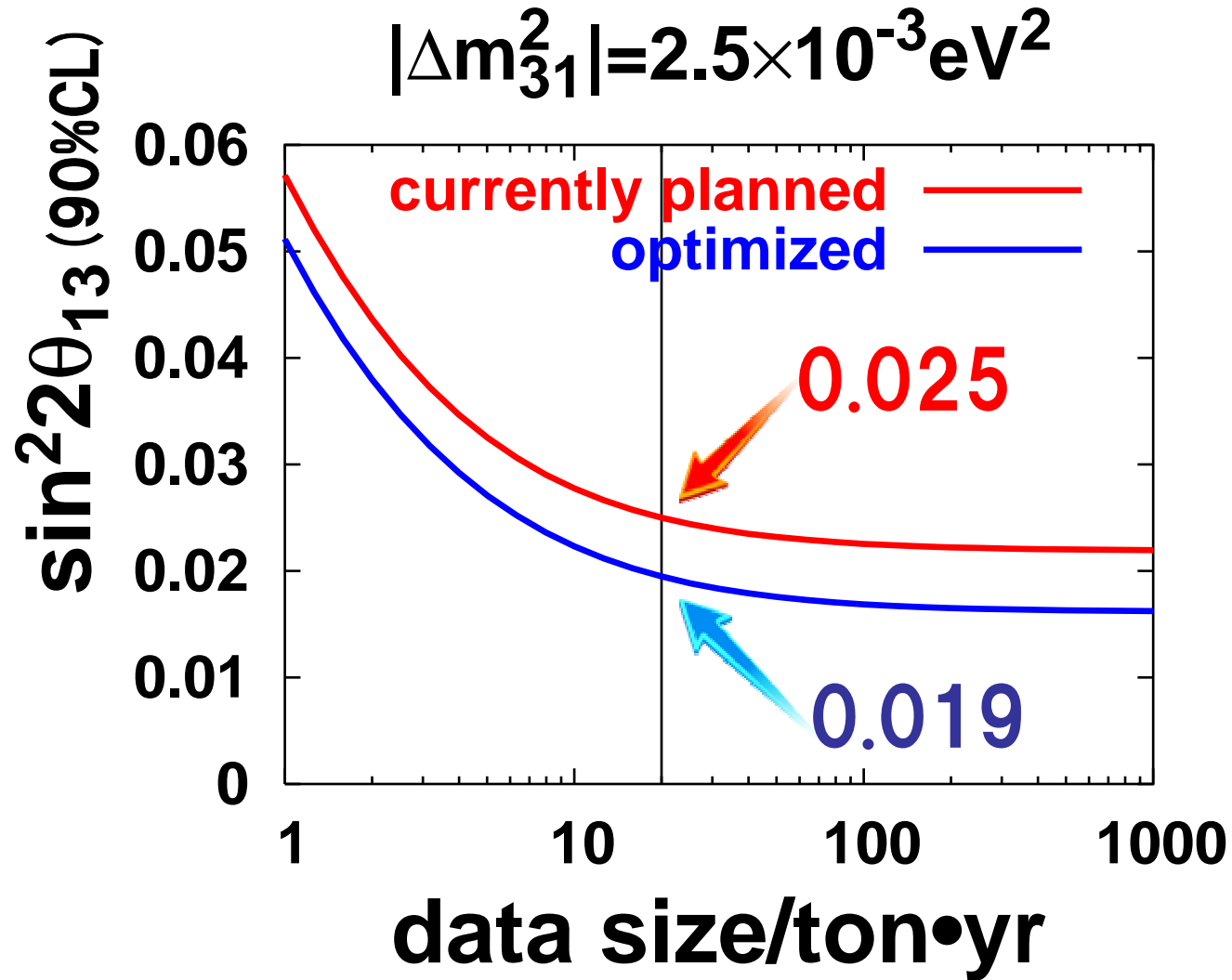


(2) Dependence on data size



Sensitivity is obtained assuming the optimized positions of the three detectors for **each** value of data size.

Sensitivity to $\sin^2 2\theta_{13}$ with the **planned** and **optimized** locations of the detectors



If $\Delta m^2_{31} = 2.0 \times 10^{-3} \text{eV}^2$, the situation changes:

@90%CL

— 0.021

— 0.022

— 0.023

— 0.024

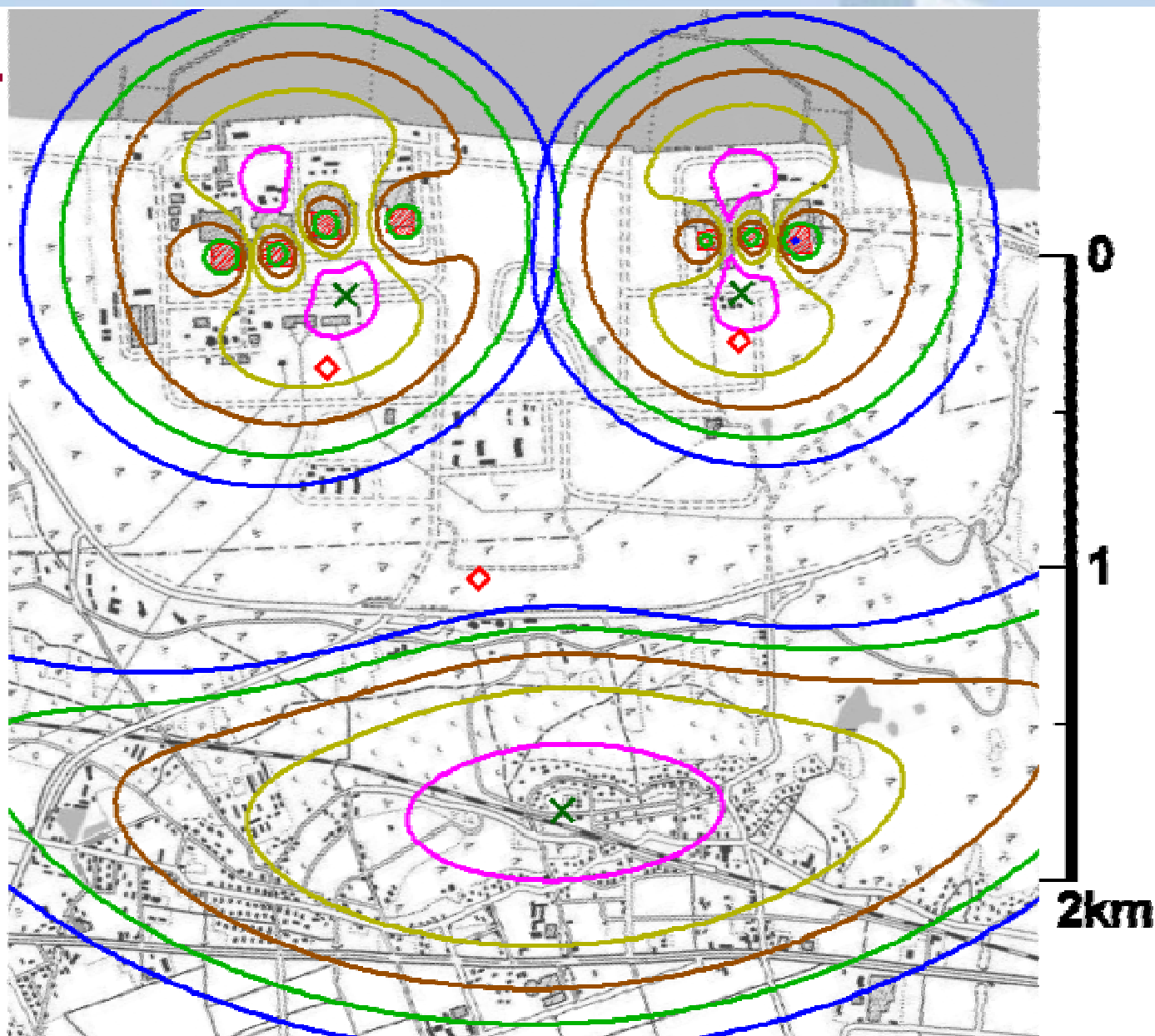
— 0.025

× optimized

◇ planned

$$|\Delta m^2_{31}| = 2.0 \times 10^{-3} \text{eV}^2$$

20 ton·yr



@90%CL

— 0.016

— 0.017

— 0.018

— 0.019

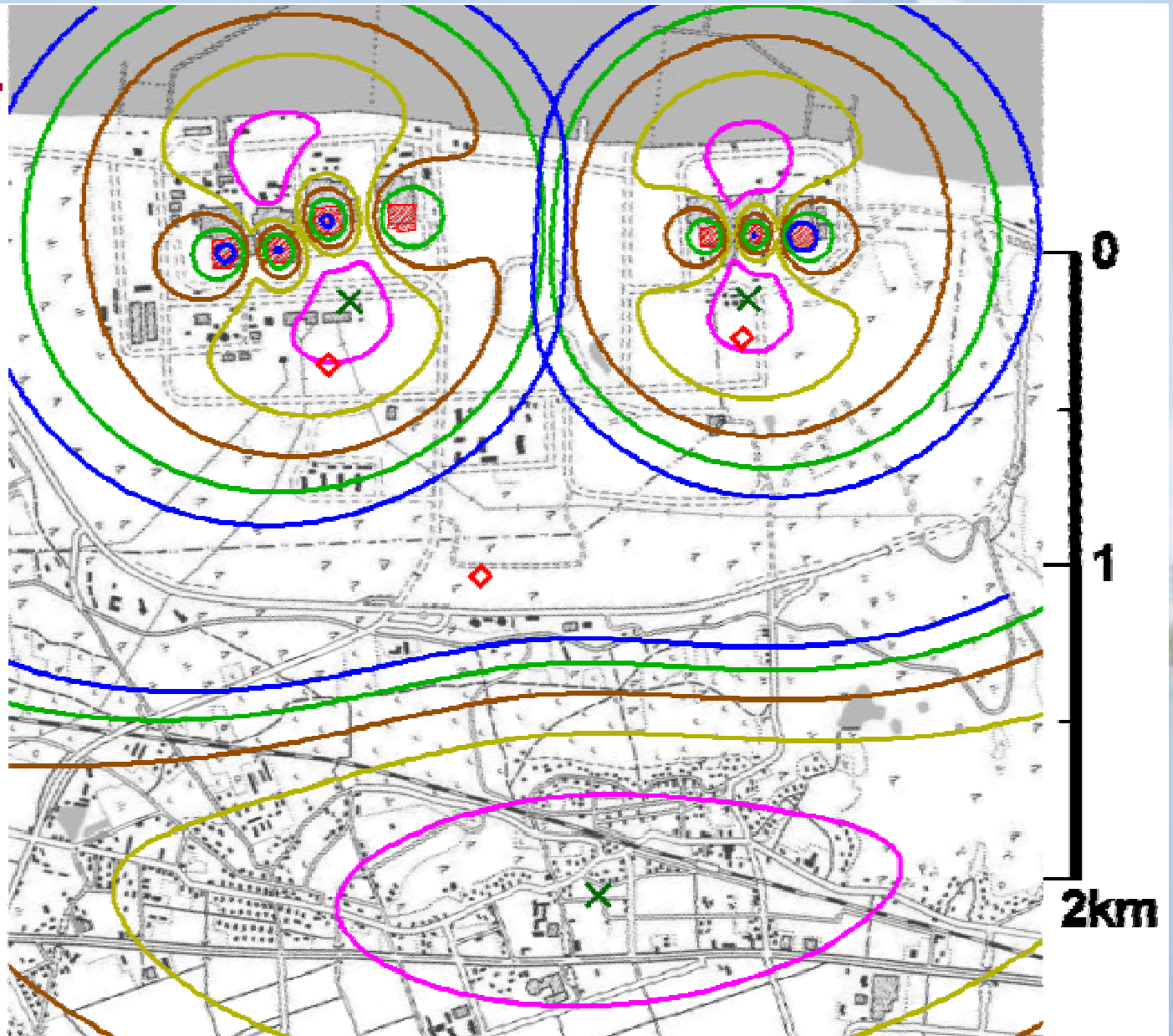
— 0.020

× optimized

◇ planned

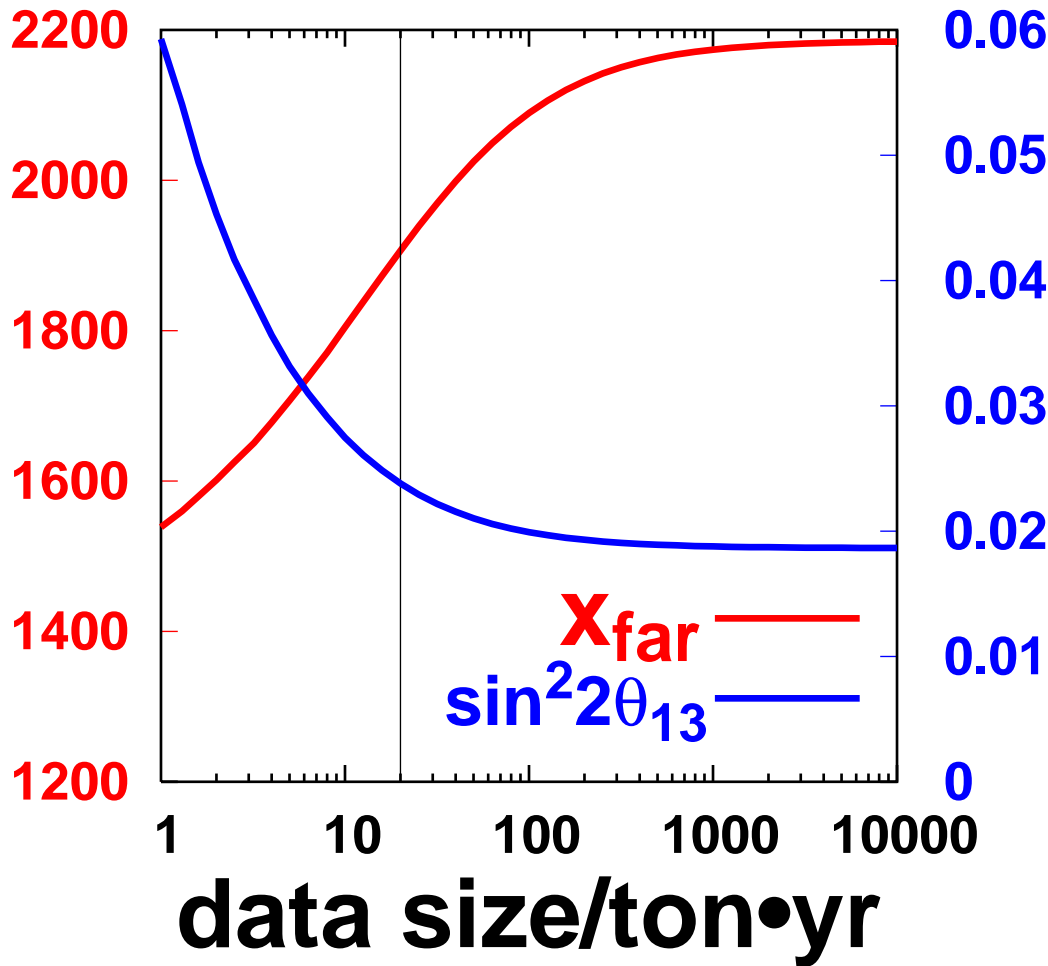
$$|\Delta m_{31}^2| = 2.0 \times 10^{-3} \text{ eV}^2$$

w/o statistics



X_{far}/m (optimized)

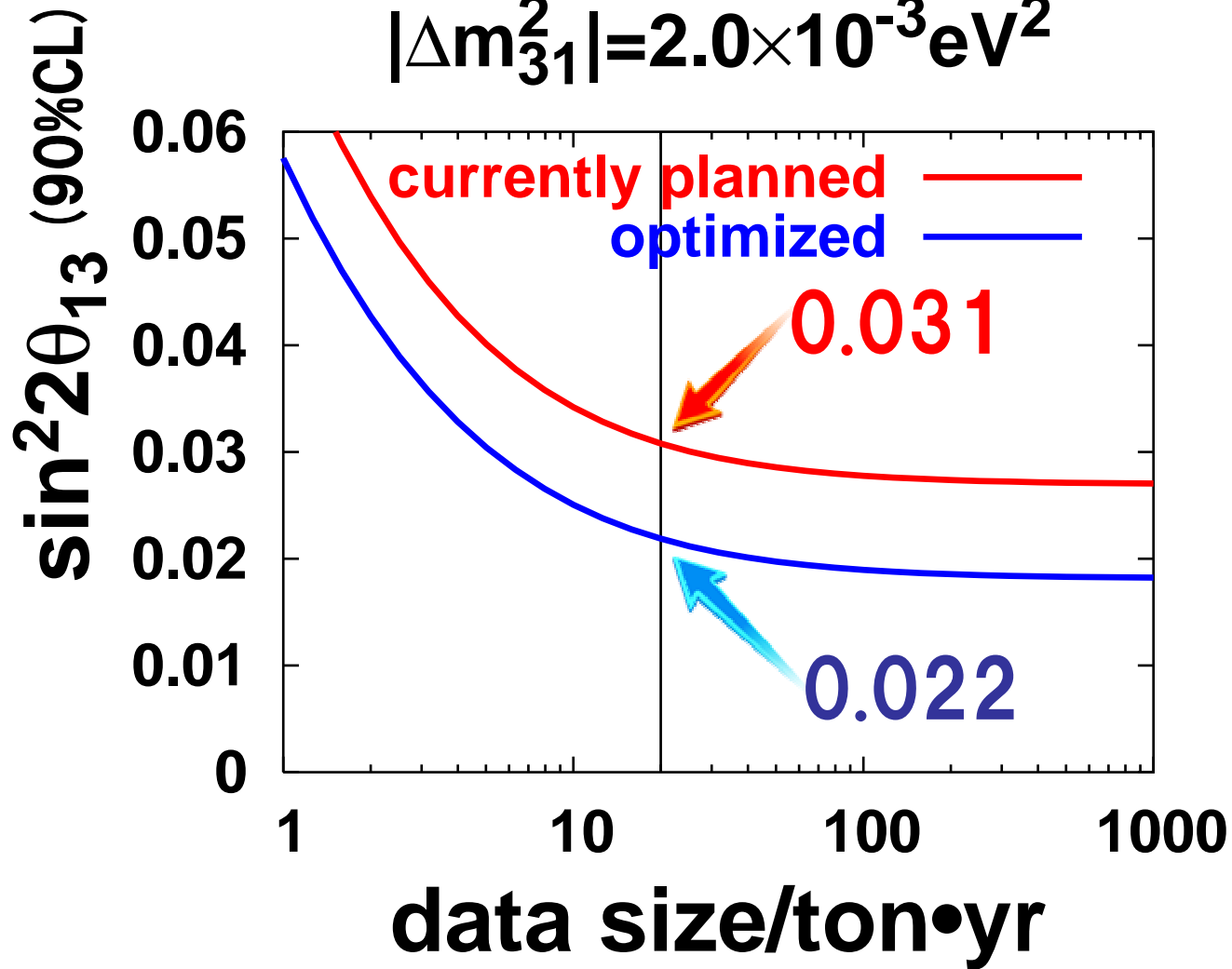
$$|\Delta m_{31}^2| = 2.0 \times 10^{-3} \text{ eV}^2$$



$\sin^2 2\theta_{13}$ (90%CL)

Sensitivity is obtained assuming the optimized positions of the three detectors.

$$|\Delta m_{31}^2| = 2.0 \times 10^{-3} \text{eV}^2$$



3. Comparison between KASKA and lower bound on sensitivity to $\sin^2 2\theta_{13}$ in reactor experiments

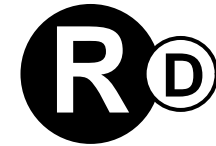
(1) lower bound on sensitivity to $\sin^2 2\theta_{13}$

I will discuss
“systematic limit”:

$$\left(\sin^2 2\theta_{13}\right)_{\text{sensitivity}} \geq \left(\sin^2 2\theta_{13}\right)_{\text{limit}}^{\text{sys only}}$$

(equality when $\#(\text{events}) \rightarrow \infty$)

● case with 1 reactor + 2 detectors



$$\left(\sin^2 2\theta_{13}\right)_{\text{limit}}^{\text{sys only}} \cong \frac{\sqrt{2.7} \sqrt{2} \sigma_u}{D(L_{\text{far}}) - D(L_{\text{near}})}$$

@90%CL
rate only

where σ_u : uncorrelated systematic error

$$D(L) \equiv \left\langle \sin^2 \left(\frac{\Delta m^2 L}{4E} \right) \right\rangle \equiv \frac{\int dE \sigma(E) f(E) \varepsilon(E) \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)}{\int dE \sigma(E) f(E) \varepsilon(E)}$$

$$\left(\sin^2 2\theta_{13} \right)_{\text{limit}}^{\text{sys only}} \quad \text{@90\%CL}$$

rate only

$$\approx \frac{\sqrt{2.7} \sqrt{2} \sigma_u}{D(L_{\text{far}}) - D(L_{\text{near}})}$$

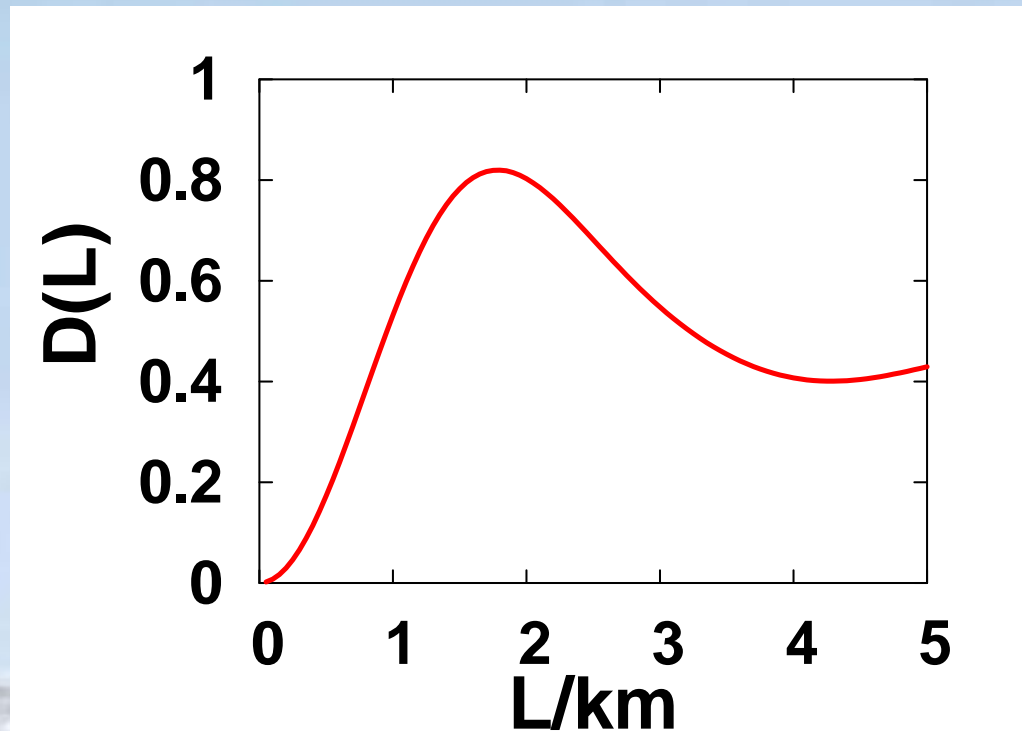
$$\geq \frac{\sqrt{2.7} \sqrt{2} \sigma_u}{0.8}$$



equality : $\left\{ \begin{array}{l} L_{\text{far}} = 1.8\text{km} \\ L_{\text{near}} = 0\text{km} \end{array} \right\}$

$$= 2.8 \sigma_u$$

$$= 0.017 \quad (\text{if } \sigma_u = 0.6\%)$$



→ With 1 reactor + 2 detectors sensitivity @90%CL cannot be better than 0.017!

KASKA (~0.016) is not so bad!

$\sigma_u = 0.6\%$: extrapolation from Bugey+CHOOZ

($\sigma_u < 0.6\%$ seems to be hard to achieve.)

(2) Possible way to improve sensitivity (**theorist's personal speculation**)

If one puts **N** near detectors and **N** far detectors with the same σ_u , then **theoretically** sensitivity becomes:

$$\min_{L_f, L_n} \left(\sin^2 2 \theta_{13} \right)_{\text{limit}}^{\text{sys only}} = 2.8 \sigma_u \text{ @90\%CL}$$

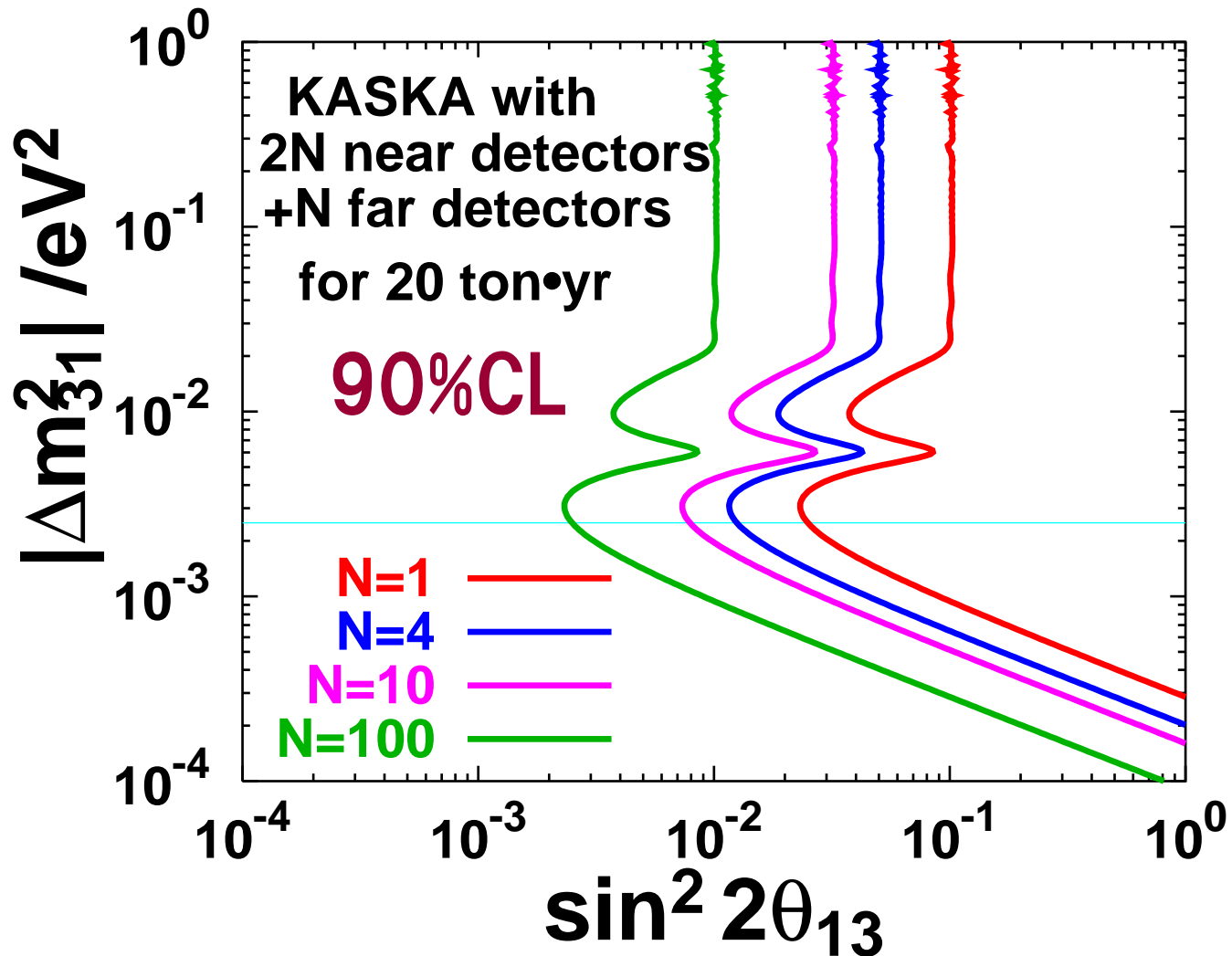


$$\chi^2 \Rightarrow N \chi^2$$

$$\min_{L_f, L_n} \left(\sin^2 2 \theta_{13} \right)_{\text{limit}}^{\text{sys only}} = 2.8 \sqrt{\frac{1}{N}} \sigma_u \text{ @90\%CL}$$

Assumption: σ_u is independent of **N**. (Is it correct?)

Here is what **a theorist** would get by putting **3N** detectors at KASKA:



4. Conclusion

Part I

● At **KASKA** the following sensitivity is obtained with 20 t·yr, $\sigma_u = 0.6\%$ @90%CL (**rate only**) :

$\sin^2 2\theta_{13} \sim 0.025$, $x_{\text{far}} = 1.1\text{km}$ (in the campus)

$\sin^2 2\theta_{13} \sim 0.019$, $x_{\text{far}} = 1.4\text{km}$ (outside the campus)

Part II

● If σ_u is fixed, then sensitivity to $\sin^2 2\theta_{13}$ has a lower bound **@90%CL (rate only)**:

$$\begin{aligned}\min(\sin^2 2\theta_{13}) &\cong 2.8 \sigma_u \quad (\text{one reactor case}) \\ &\geq 0.017 \quad (\sigma_u = 0.6\%) \end{aligned}$$

The sensitivity of KASKA (~ 0.02) is not far from this bound.

● Sensitivity may be improved by increasing the numbers of near and far detectors, or **by combining the reactor experiments all over the world.**

→ Dependence of σ_u on the numbers has to be carefully studied.

Appendix

Proof
of
lower
bound

$$\begin{aligned} \mathbf{x}^2 &= \frac{(\mathbf{y}^f - \mathbf{y}^n)^2}{2 \sigma_u^2} + \frac{(\mathbf{y}^f + \mathbf{y}^n)^2}{2 \sigma_u^2 + 4 [\sigma_c^2 + (\sigma_c^{(r)})^2 + (\sigma_u^{(r)})^2]} \\ &\cong \frac{(\mathbf{y}^f - \mathbf{y}^n)^2}{2 \sigma_u^2} = \frac{(D(L_f) - D(L_n))^2}{2 \sigma_u^2} \sin^2 2 \theta_{13} \end{aligned}$$

$$\mathbf{y}^n \equiv \frac{\mathbf{M}^n - \mathbf{T}^n}{\mathbf{T}^n} = -\sin^2 2 \theta_{13} D(L_n),$$

$$\mathbf{y}^f \equiv \frac{\mathbf{M}^f - \mathbf{T}^f}{\mathbf{T}^f} = -\sin^2 2 \theta_{13} D(L_f)$$

M: measured

T: theoretical

$$D(L) \equiv \left\langle \sin^2 \left(\frac{\Delta m^2 L}{4E} \right) \right\rangle \equiv \frac{\int dE \sigma(E) f(E) \varepsilon(E) \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)}{\int dE \sigma(E) f(E) \varepsilon(E)}$$

$$\mathbf{x}^2 \Big|_{90\%CL} = 2.7 \Rightarrow \sin^2 2 \theta_{13} \cong \frac{\sqrt{2.7} \sqrt{2} \sigma_u}{D(L_f) - D(L_n)}$$

● case with M reactors + (M+1) detectors

$$\left(\sin^2 2 \theta_{13} \right)_{\text{sys only limit}}$$

@90%CL (rate only)

$$\approx \frac{\sqrt{2.7} \sqrt{1+1/M} \sigma_u}{D(L_{\text{far}}) - D(L_{\text{near}})}$$

$$\geq \frac{\sqrt{2.7} \sqrt{1+1/M} \sigma_u}{0.8} \quad (\text{equality : } L_{\text{far}} = 1.8\text{km}, L_{\text{near}} = 0\text{km})$$

$$= 2.0 \sigma_u \quad (\text{if } M \gg 1)$$

$$= 0.012 \quad (\text{if } \sigma_u = 0.6\%)$$

→ With M reactor
+ (M+1) detectors,
sensitivity cannot be
better than 0.012!

