

Sensitivity of Future Long Baseline Experiments and Octant Degeneracy

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December 16, 2023
Miami2023 @ Lago Mar Resort

1. Introduction

- Framework of 3 flavor ν oscillation
- Status of 3 ν fit

2. Sensitivity of T2HK & DUNE to $N_\nu=3$ oscillation parameters

Ghosh-OY, NPB 989 ('23) 116142

- Precision of Δm^2_{31} & θ_{23}
- Mass ordering
- Octant degeneracy
- CP

3. Octant parameter degeneracy

Sugama-OY, arXiv:2308.15071

- Situation before and after 2012
- Octant degeneracy in T2HK, DUNE, T2HKK, ESS ν SB

4. Conclusions

1. Introduction

Framework of 3 flavor ν oscillation

Mixing matrix $U_{\alpha j}$ depends on θ_{12} , θ_{23} , θ_{13} , and CP phase δ

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

All 3 mixing angles have been measured:

1998- ν_{atm} + T2K + MINOS + NOvA (accelerators)

$$P(\nu_\mu \rightarrow \nu_\mu)$$

$$\theta_{23} \cong \frac{\pi}{4}, |\Delta m_{32}^2| \cong 2.5 \times 10^{-3} \text{ eV}^2$$

2002 ν_{solar} + KamLAND (reactor)

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$$

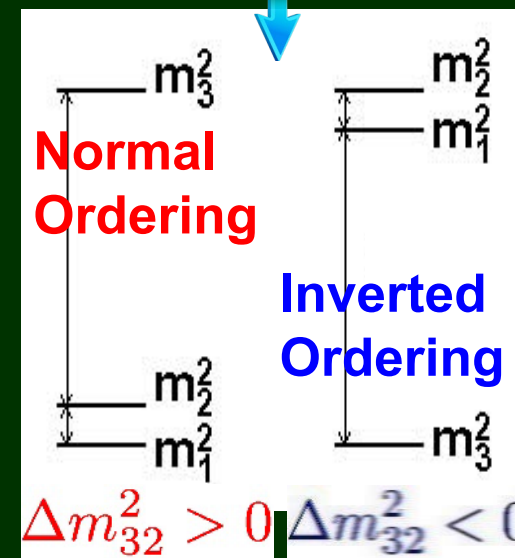
$$\theta_{12} \cong \frac{\pi}{6}, \Delta m_{21}^2 \cong 8 \times 10^{-5} \text{ eV}^2$$

2012 DCHOOZ + Daya Bay + Reno (reactors)

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$$

$$\theta_{13} \cong \pi / 20$$

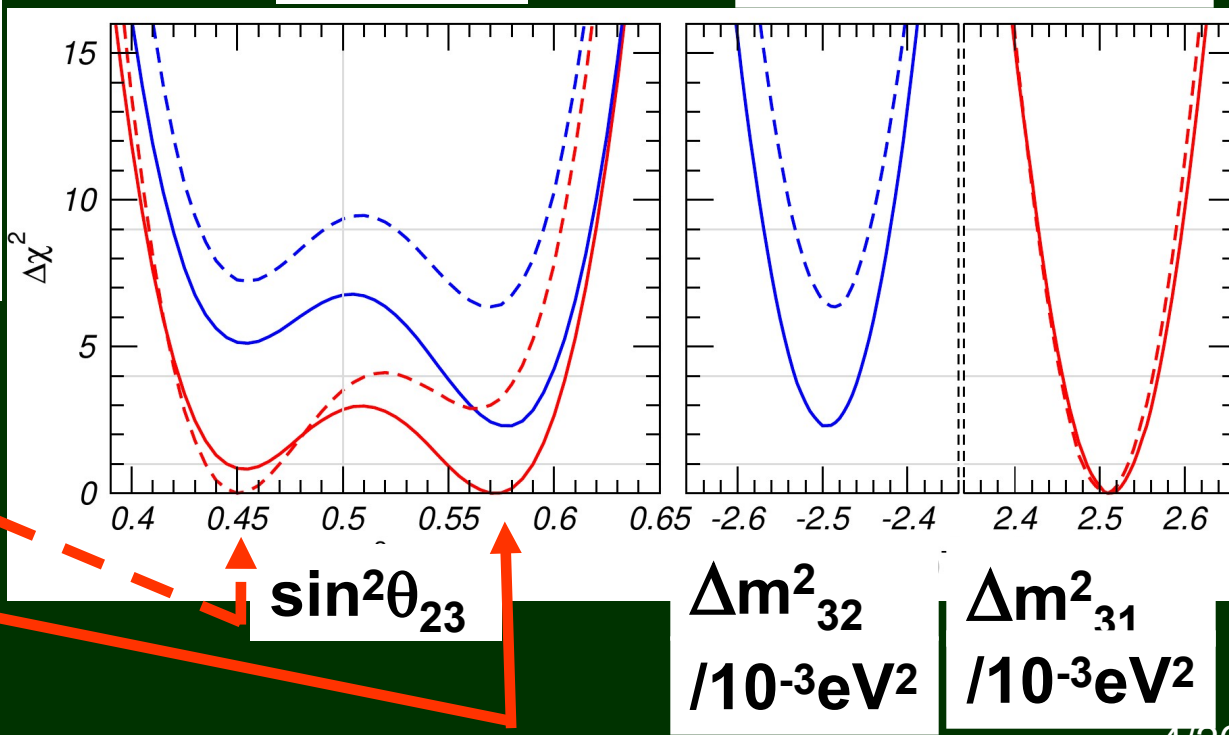
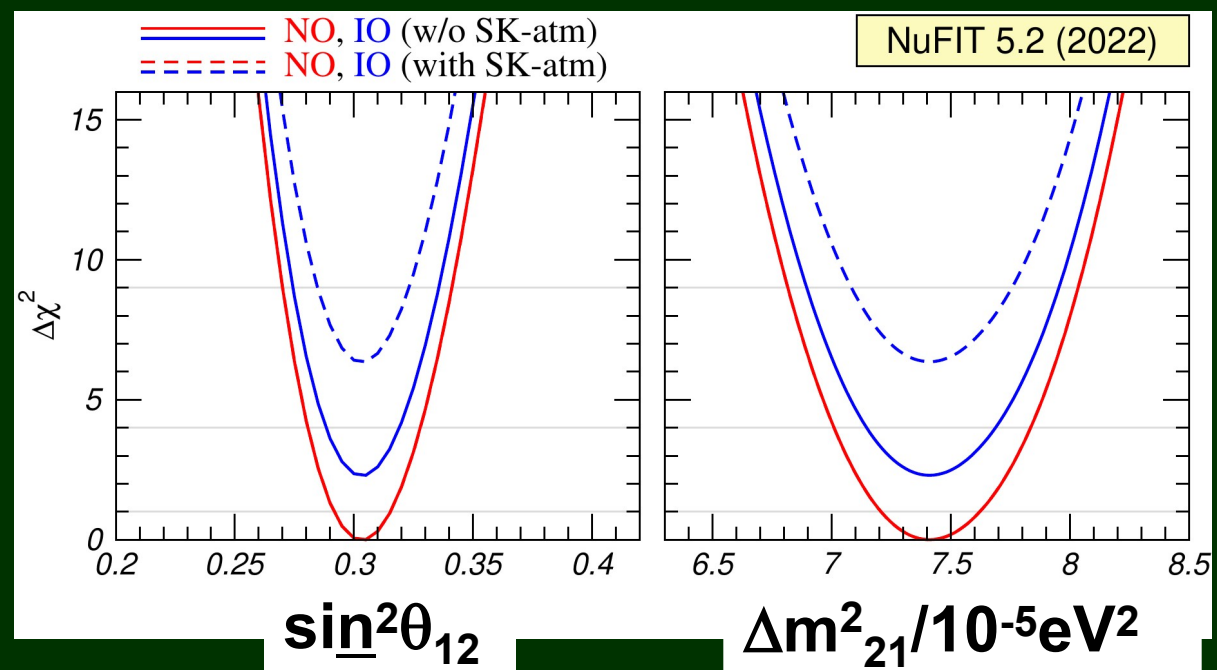
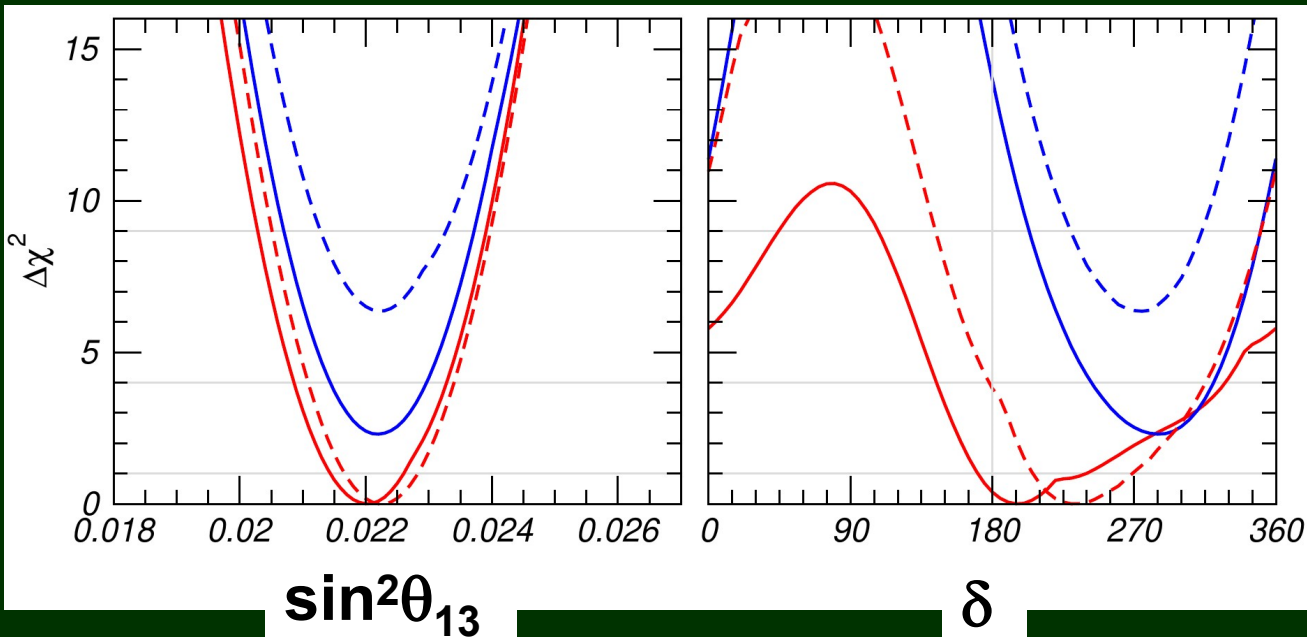
Both **Mass Orderings** are still allowed



Status of 3ν fit (1)

www.nu-fit.org
v5.2 (Nov. 2022)

— NO, IO (w/o SK-atm)
- - NO, IO (with SK-atm)



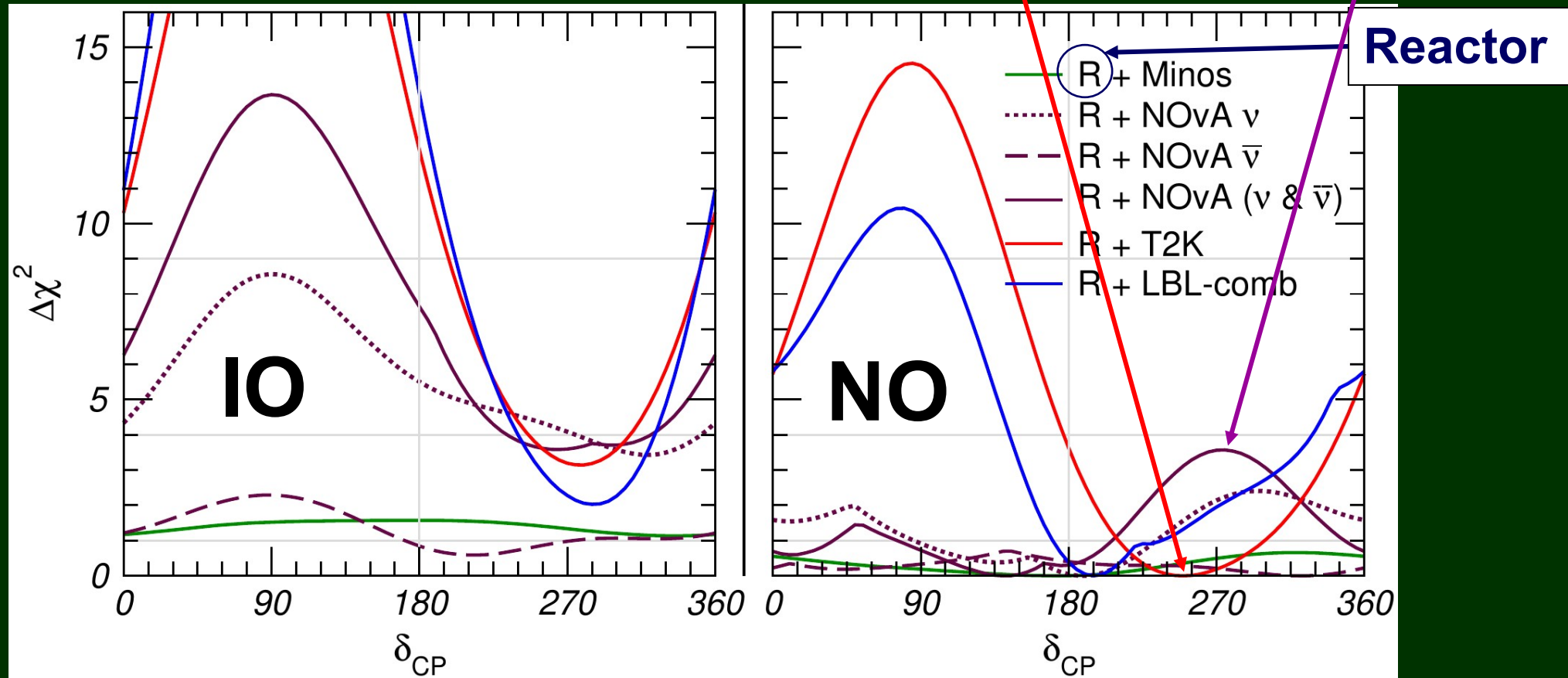
- **NO** seems to be preferred over **IO**
 - **SK-atm** seems to prefer **Lower Octant**, while others prefer **Higher Octant**
- ⇒ Situation of θ_{23} is still confusing.

NuFIT 5.2 (2022)

Status of 3ν fit (2)

● Appearance data of LBL show us potential **tension** for NO, although **T2K** dominates over **NOvA** in statistics. \Rightarrow Situation of δ is still confusing.

www.nu-fit.org v5.2 (Nov. 2022)



Next things to do are to determine the following by long baseline experiments:

● $\text{sign}(\Delta m^2_{31})$

● $\pi/4 - \theta_{23}$

● δ

$$\overline{\nu}_{\mu} \rightarrow \overline{\nu}_{\mu} + \overline{\nu}_{\mu} \rightarrow \overline{\nu}_e$$

Long baseline experiments under construction

• T2HK(JP, JPARC-->HK) L=295km, E~0.6GeV

• DUNE (US, FNAL-->Homestake, SD) , L=1300km, E~2GeV

Explores 1st oscillation maximum

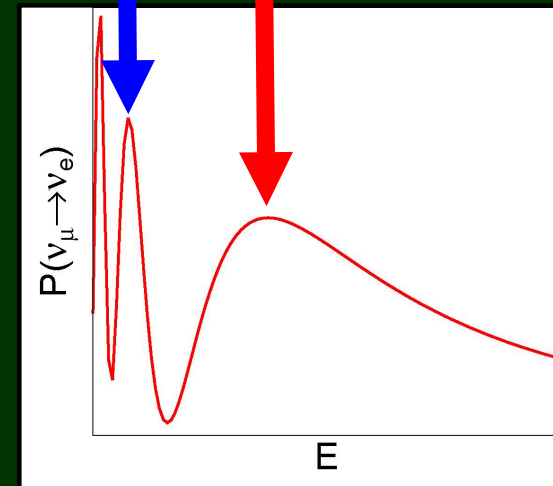
Proposed long baseline experiments

• T2HKK(JP, JPARC-->1st in Japan+2nd detector in Korea) L=1100km, E~0.8GeV

• ESSνSB (Sweden, ESS-->Zinkgruvan mine) , L=360km, E~0.3GeV

Explores 2nd oscillation maximum

These experiments are expected to measure $\text{sign}(\Delta m^2_{31})$, $\pi/4 - \theta_{23}$ and δ



Matter effect in T2HK and DUNE

T2HK: L=295km

DUNE: L=1300km

In a toy 2 flavor case:

To know the sign of $\Delta E = \Delta m^2 / 2E$, large matter effect is necessary.

Matter effect becomes most conspicuous if $\Delta E \cos 2\theta = A$ is satisfied.

$$P(\nu_\mu \rightarrow \nu_e) = \left(\frac{\Delta E \sin 2\theta}{\Delta \tilde{E}} \right)^2 \sin^2 \left(\frac{\Delta \tilde{E} L}{2} \right)$$

$$\tan 2\tilde{\theta} \equiv \frac{\Delta E \sin 2\theta}{\Delta E \cos 2\theta - A}$$

$$\Delta \tilde{E} \equiv \left\{ (\Delta E \cos 2\theta - A)^2 + (\Delta E \sin 2\theta)^2 \right\}^{1/2}$$

$$A \equiv \sqrt{2} G_F N_e \sim \mathbf{1/2000km}$$

In this case, the baseline length L has to be large

$\rightarrow L > \pi/A \sim O(1000km) \rightarrow$ It is satisfied by DUNE but not by T2HK.

2. Sensitivity of T2HK & DUNE to $N_\nu=3$ oscillation parameters

Uncertainty in matter density taken into account

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The parameters assumed here:

T2HK

187 kton fiducial volume

$\nu:\bar{\nu} = 1:1$

Total exposure: 2.7×10^{22} POT

DUNE

40 kt LiAr detector,

$\nu:\bar{\nu} = 1:1$

Total exposure: 1.1×10^{21} POT

Reference value:

$\theta_{23} = 42^\circ$ or 48° ,

$\delta = -90^\circ$,

$\Delta m_{31}^2 = 2.51 \times 10^{-3} \text{eV}^2$,

$\Delta\rho/\rho = 0, 5\%, 10\%$

Recommended by Geller-Hara,
[hep-ph/0111342](https://arxiv.org/abs/hep-ph/0111342)

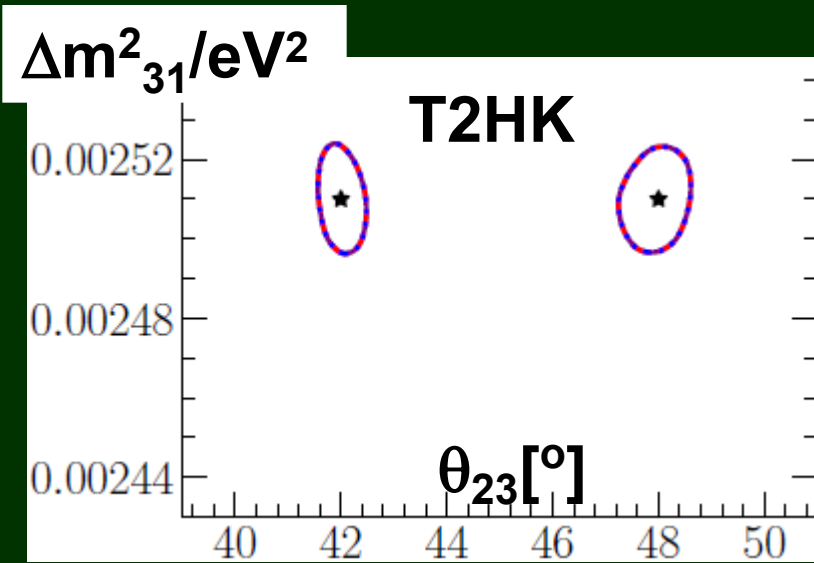
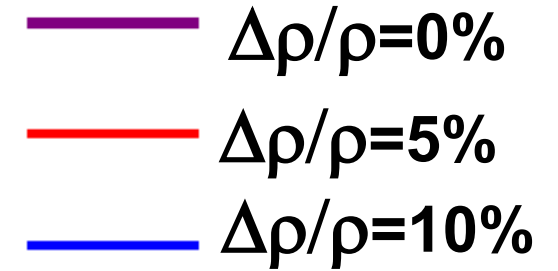
2.1 Precision to oscillation parameters

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Uncertainty in matter density
taken into account

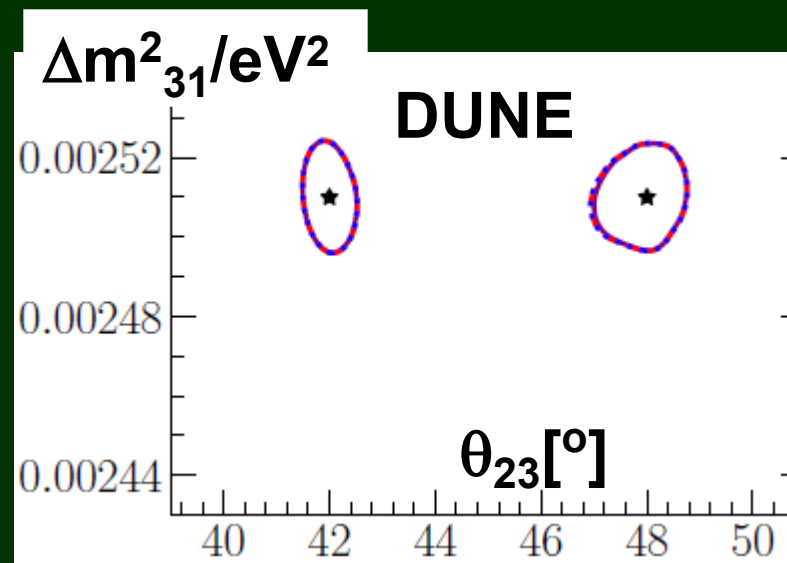
Precision of DUNE is slightly better than that of T2HK.
-> Combined precision is excellent.

Uncertainty in matter density has little effect
-> Major contribution comes from disappearance channel.



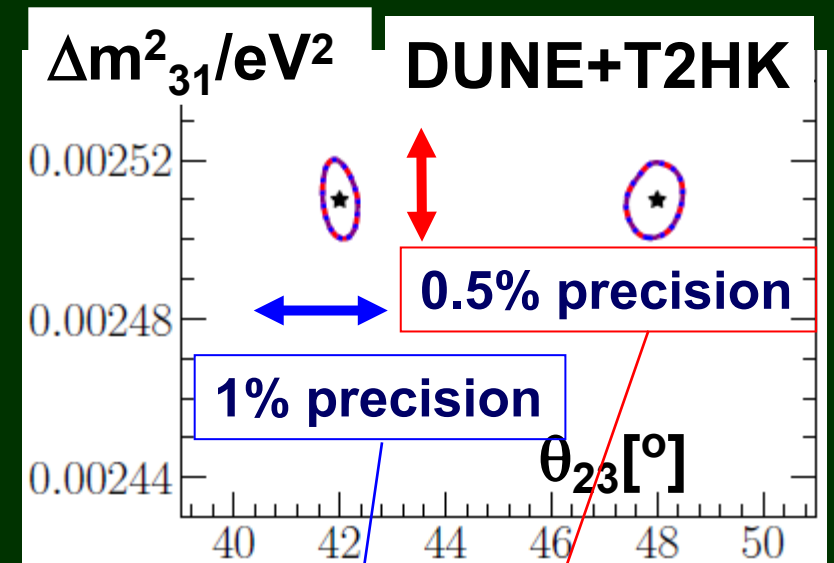
$$\Delta m_{31}^2/10^{-3}eV^2 = (2.51+0.013-0.014)$$

$$\theta_{23} = (42 \pm 0.5)^\circ$$



$$(2.51+0.015-0.014)$$

$$(42 \pm 0.5)^\circ$$



$$(2.51 \pm 0.01)$$

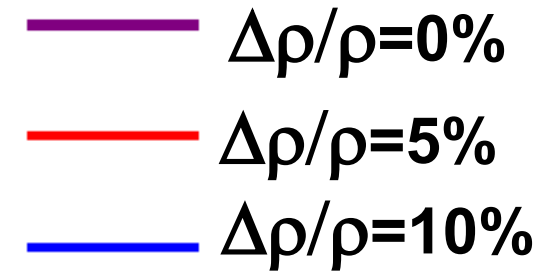
$$(42+0.4-0.3)^\circ$$

DUNE&T2HK vs Present status of global fit

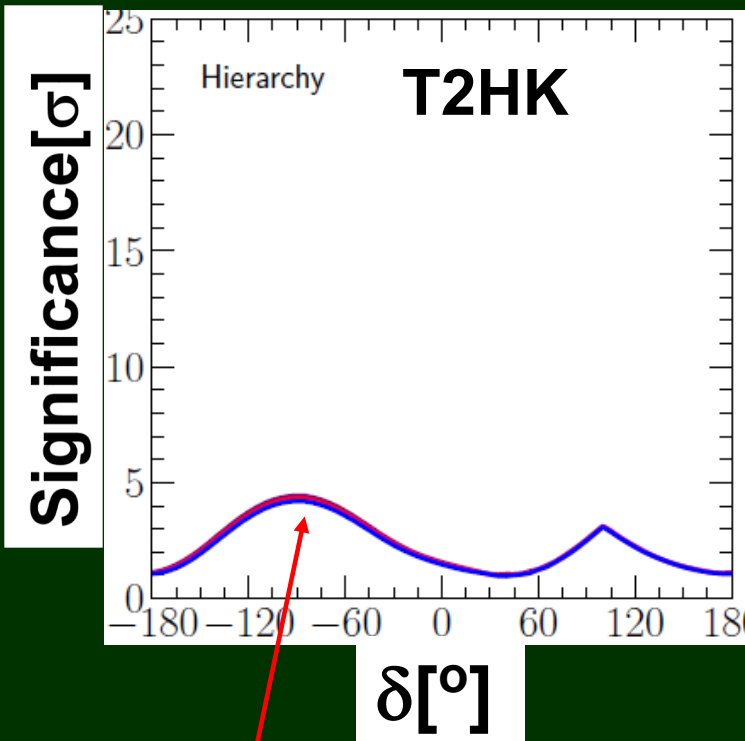
	Ref	$\Delta m^2_{31}/10^{-3}eV^2$	$\theta_{23}[^{\circ}]$
Global fit	www.nu-fit.org v5.2 (Nov. 2022)	2.507+0.026-0.027	42.2+1.1-0.9
Future exp	T2HK Ghosh-OY('23)	2.510+0.013-0.014	42.0\pm0.5
	DUNE Ghosh-OY('23)	2.510+0.015-0.014	42.0\pm0.5
	DUNE+T2HK Ghosh-OY('23)	2.510\pm0.010	42.0+0.4-0.3

2.2 Sensitivity to Mass Ordering

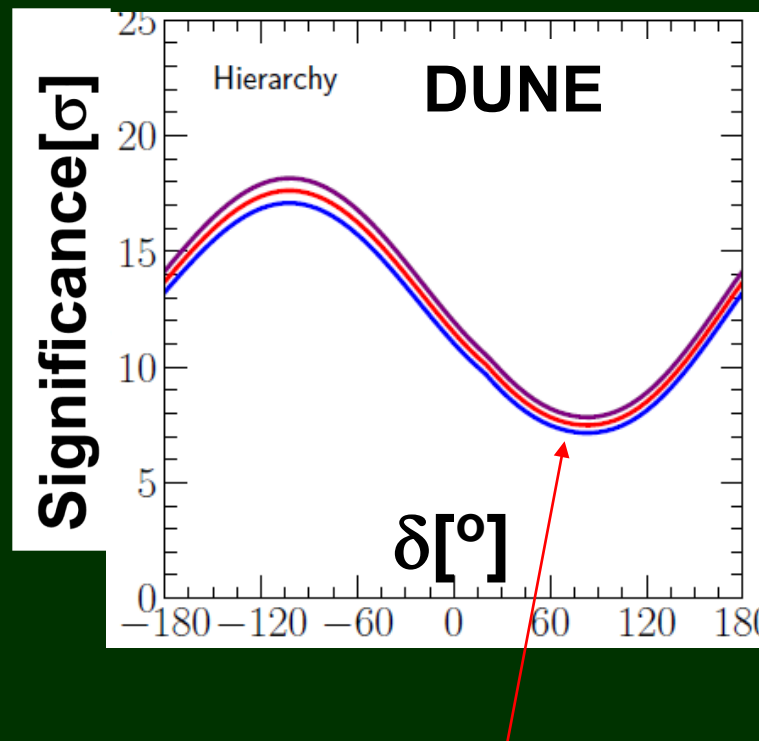
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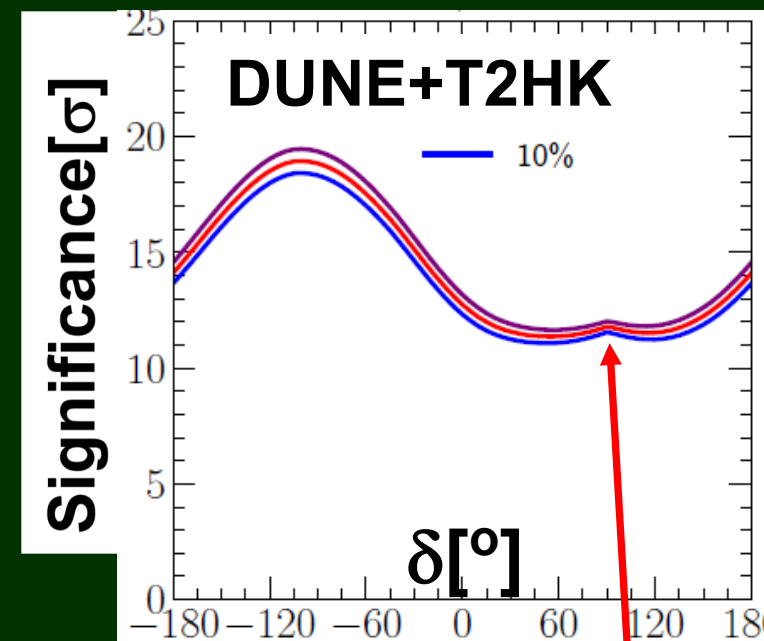
Uncertainty in matter density has some effect on DUNE
← DUNE has longer baseline $L=1300\text{km}$



Sensitivity of T2HK is poor except for $\delta \sim -\pi/2$



Sensitivity of DUNE is excellent for any δ



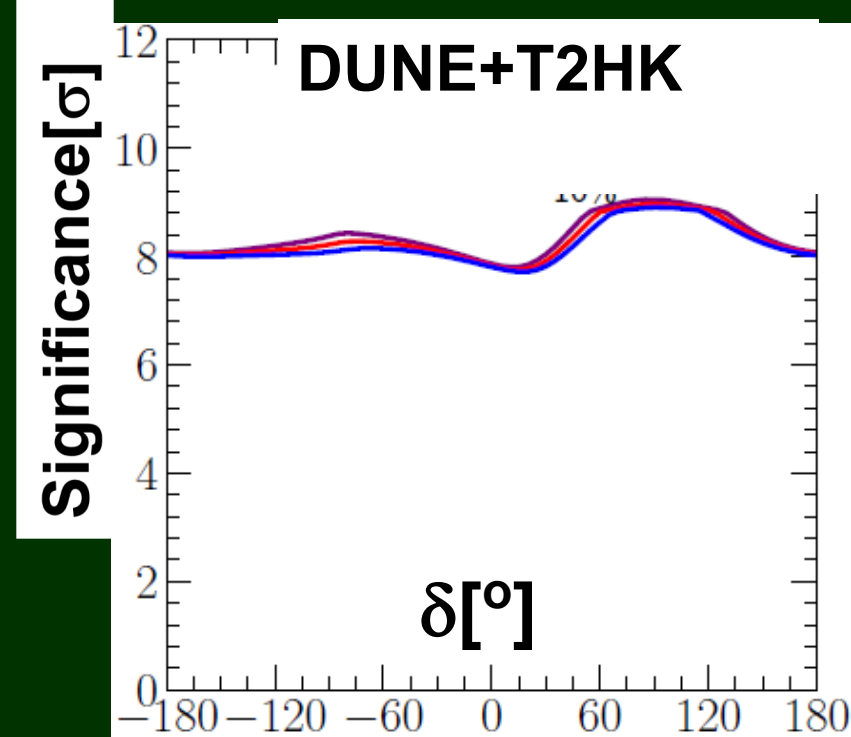
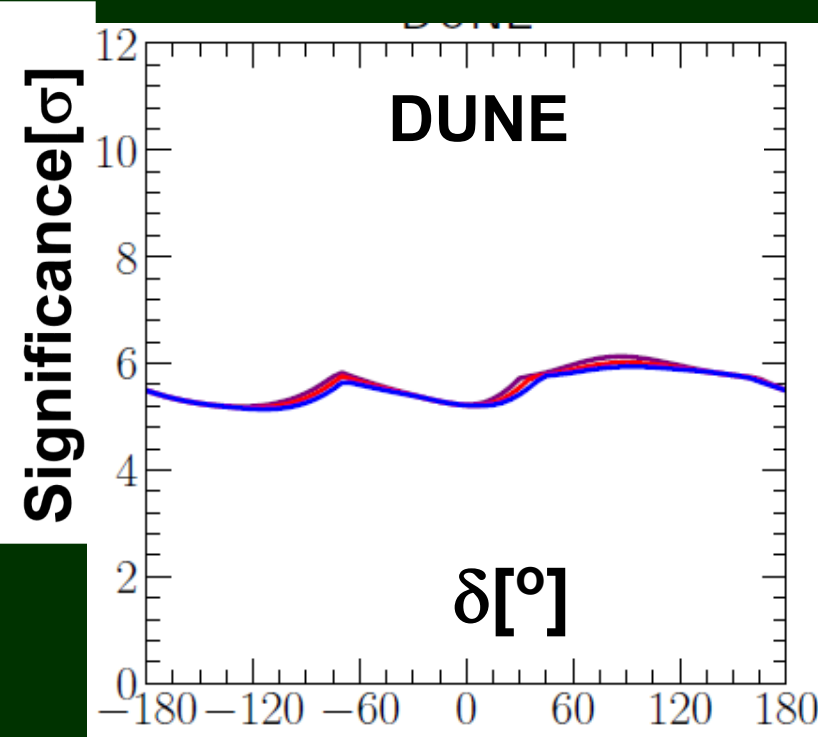
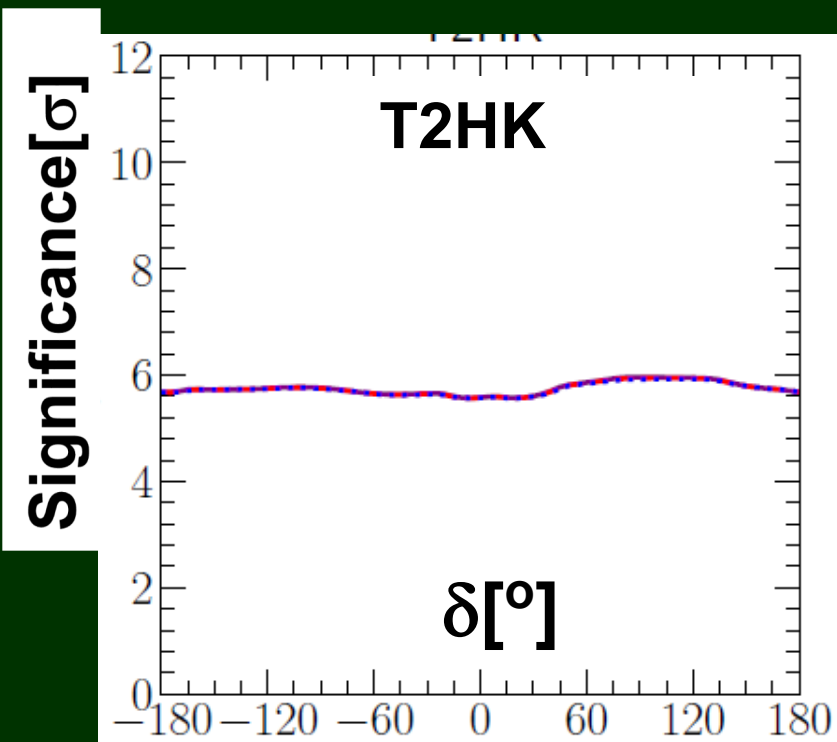
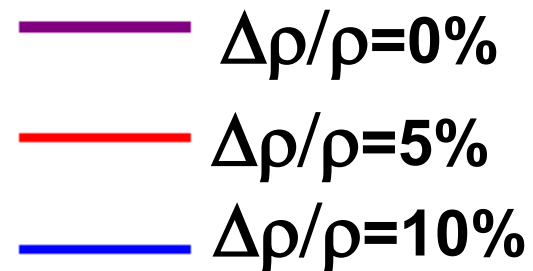
Synergy of T2HK+DUNE at $\delta = \pi/2$: If MO is known from DUNE \rightarrow Sensitivity of T2HK is improved

2.3 Sensitivity to Octant degeneracy

HO-LO Separation is possible for **T2HK** & **DUNE** w/ ν & $\bar{\nu}$ for most of δ

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$$\theta_{23} = 42^\circ$$



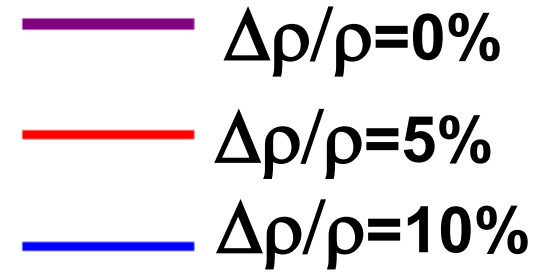
2.4 Sensitivity to CP(1)

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Uncertainty in matter density has some effect on DUNE.

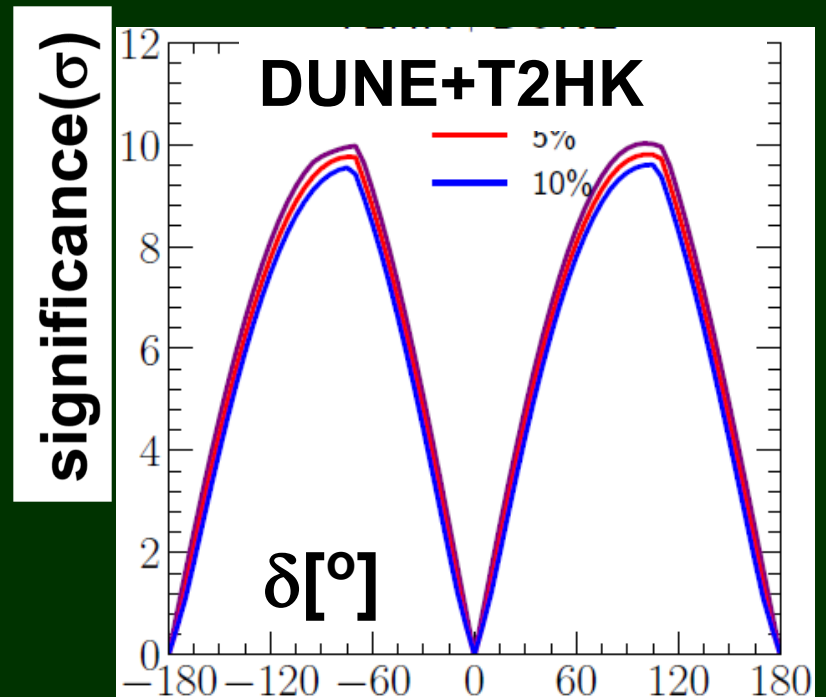
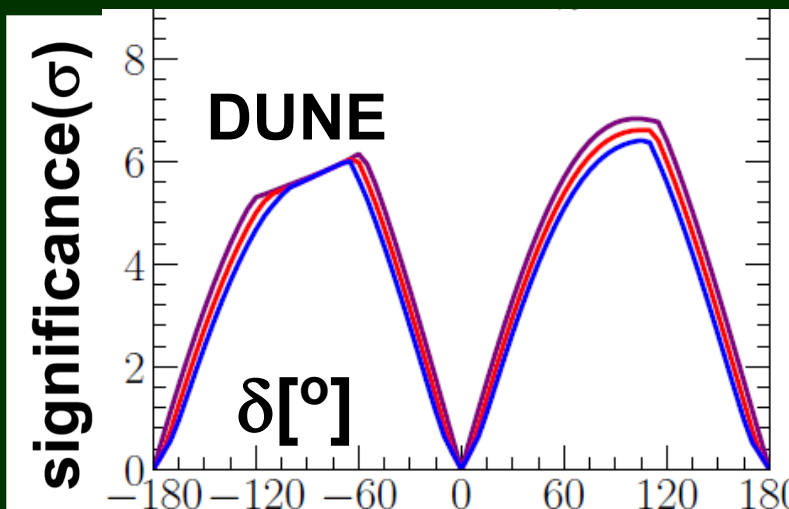
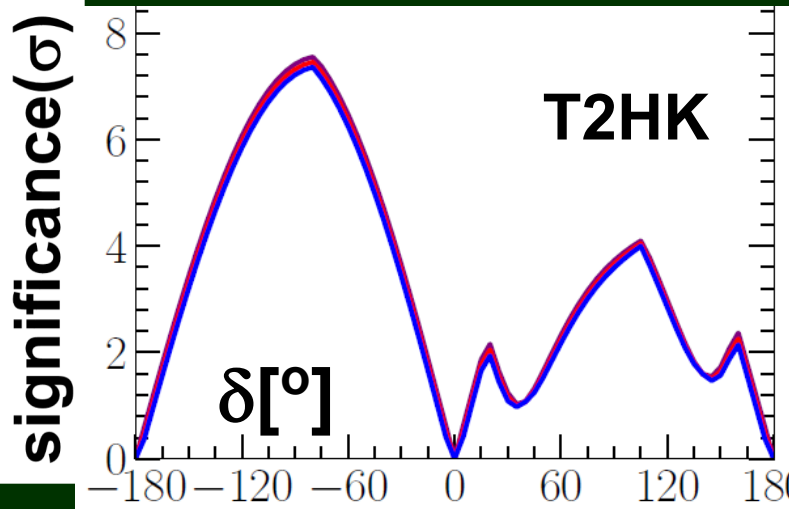
<- DUNE has longer baseline $L=1300\text{km}$

However even with $\Delta\rho/\rho=10\%$, the sensitivity is excellent.



Sensitivity of **T2HK** is poor for (NO, $\delta = +\pi/2$) & (IO, $\delta = -\pi/2$)

Sensitivity of **DUNE** is excellent for $|\sin\delta|=1$ for both mass hierarchy

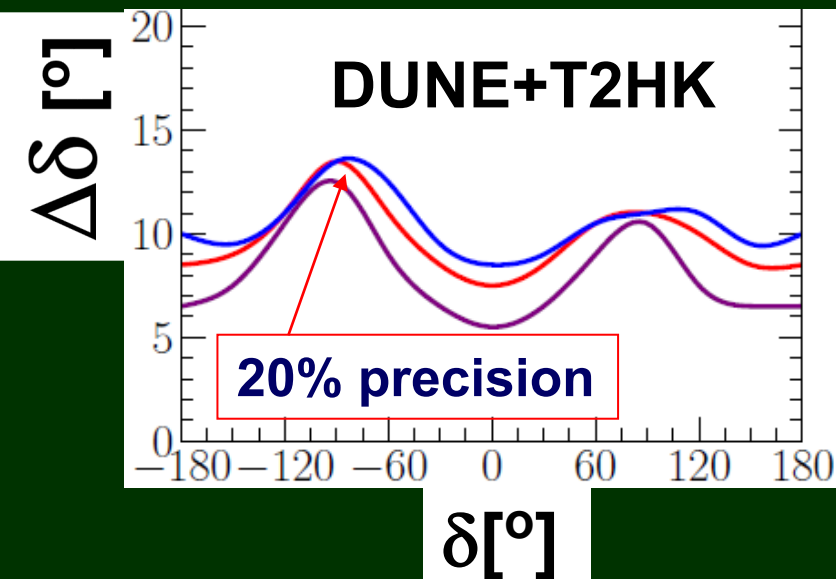
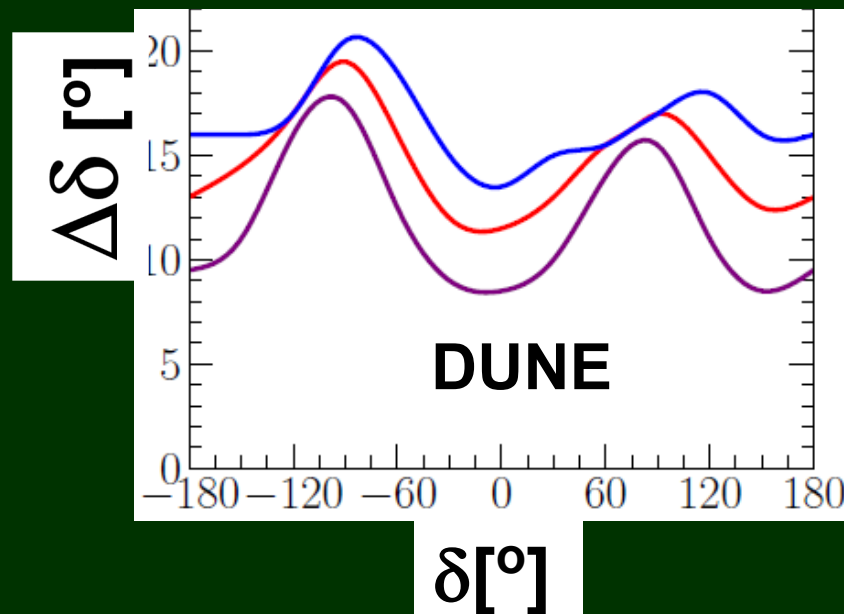
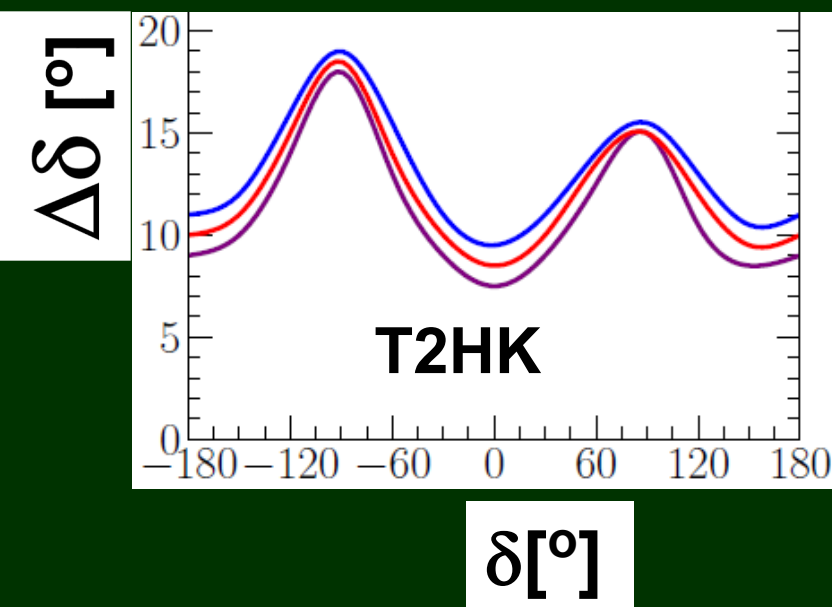
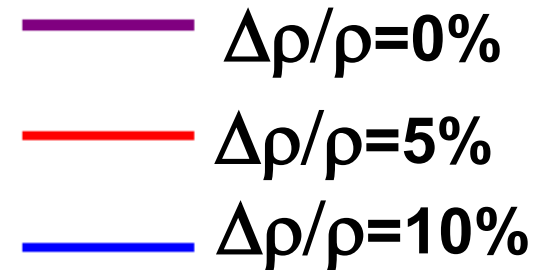


2.4 Sensitivity to CP(2)

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Uncertainty in matter density has some effect on the precision $\Delta\delta$ both for T2HK & DUNE.

$\Delta\delta/\delta$ has mild dependence on δ but not much.



3. Octant parameter degeneracy

Sugama - OY, arXiv:2308.15071

● Parameter degeneracy

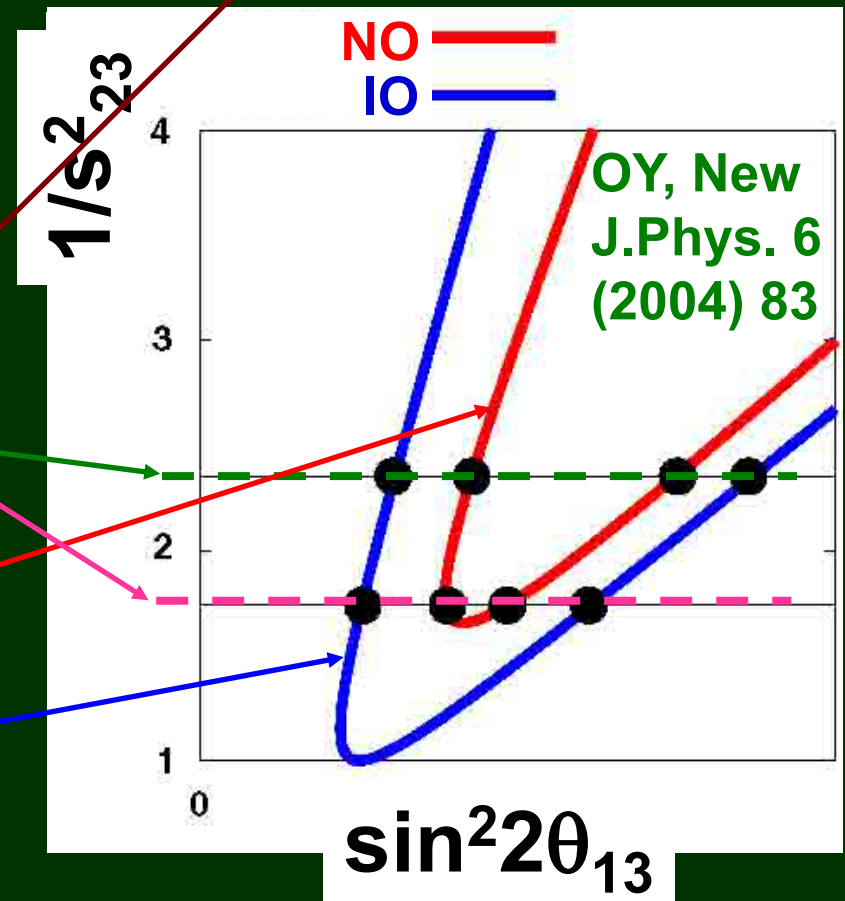
Even if we know $P \equiv P(\nu_\mu \rightarrow \nu_e)$ and $\bar{P} \equiv P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ in LBL experiments with energy E and baseline L , δ cannot be uniquely determined because of the 8-fold **parameter degeneracy**.

● octant degeneracy $\theta_{23} \leftrightarrow \pi/2 - \theta_{23}$
(Fogli-Lisi, '96)

● intrinsic degeneracy (δ, θ_{13})
(Burguet-Castell et al, '01)

● sign degeneracy $\Delta m^2_{31} \leftrightarrow -\Delta m^2_{31}$
(Minakata-Nunokawa, '01)

$(\sin^2 2\theta_{13}, 1/s^2_{23})$ plane
($P=\text{const}$ & $\bar{P}=\text{const}$ gives a **quadratic curve**)



● Appearance oscillation probability

$$A \equiv \sqrt{2}G_F N_e$$

$$P(\nu_\mu \rightarrow \nu_e, E) = x^2 F^2 + 2 \operatorname{sign}(\Delta m_{31}^2) xy F g \cos [\delta + \operatorname{sign}(\Delta m_{31}^2) \Delta] + y^2 g^2$$

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e, E) = x^2 \bar{F}^2 + 2 \operatorname{sign}(\Delta m_{31}^2) xy \bar{F} g \cos [\delta - \operatorname{sign}(\Delta m_{31}^2) \Delta] + y^2 g^2$$

$$x \equiv s_{23} \sin 2\theta_{13}$$

$$y \equiv \left| \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right| \cos \theta_{23} \sin 2\theta_{12}$$

$$(F, \bar{F}) \equiv \begin{cases} (f, f) & \text{for NO} \\ (\bar{f}, f) & \text{for IO} \end{cases}$$

$$\Delta \equiv \frac{|\Delta m_{31}^2| L}{4E}$$

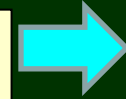
$$g \equiv \frac{\sin(AL/2)}{AL/2\Delta}$$

$$\begin{cases} f \\ \bar{f} \end{cases} \equiv \frac{\sin(\Delta \mp AL/2)}{(1 \mp AL/2\Delta)}$$

● Change of variables

$$X \equiv \sin^2 2\theta_{13}$$

Eliminate δ



For $\sin \Delta \neq 0$: a quadratic curve in (X,Y)-plane

$$Y \equiv \frac{1}{s_{23}^2}$$

$$16CX(Y-1)$$

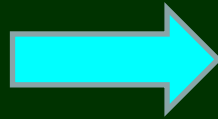
$$C \equiv \left(\frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right)^2 \left[\frac{\sin(AL/2)}{AL/2\Delta} \right]^2 \sin^2 2\theta_{12}$$

$$= \frac{1}{\cos^2 \Delta} \left[\left(\frac{P(E) - C}{F} + \frac{\bar{P}(E) - C}{\bar{F}} \right) (Y - 1) - (F + \bar{F})X + \frac{P(E)}{F} + \frac{\bar{P}(E)}{\bar{F}} \right]^2$$

$$+ \frac{1}{\sin^2 \Delta} \left[\left(\frac{P(E) - C}{F} - \frac{\bar{P}(E) - C}{\bar{F}} \right) (Y - 1) - (F - \bar{F})X + \frac{P(E)}{F} - \frac{\bar{P}(E)}{\bar{F}} \right]^2$$

$$X \equiv \sin^2 2\theta_{13} \quad Y \equiv \frac{1}{s_{23}^2} \quad C \equiv \left(\frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right)^2 \left[\frac{\sin(AL/2)}{AL/2\Delta} \right]^2 \sin^2 2\theta_{12}$$

For $\sin\Delta \neq 0$



A quadratic curve in (X, Y)-plane

$$16CX(Y-1) = \frac{1}{\cos^2 \Delta} \left[\left(\frac{P(E)-C}{F} + \frac{\bar{P}(E)-C}{\bar{F}} \right) (Y-1) - (F+\bar{F})X + \frac{P(E)}{F} + \frac{\bar{P}(E)}{\bar{F}} \right]^2 + \frac{1}{\sin^2 \Delta} \left[\left(\frac{P(E)-C}{F} - \frac{\bar{P}(E)-C}{\bar{F}} \right) (Y-1) - (F-\bar{F})X + \frac{P(E)}{F} - \frac{\bar{P}(E)}{\bar{F}} \right]^2$$

For $\sin\Delta = 0$ (Oscillation Maximum

$$\Delta \equiv \frac{|\Delta m_{31}^2|L}{4E} = \frac{\pi}{2})$$



A straight line in (X, Y)-plane

$$\left(\frac{P(E)-C}{F} + \frac{\bar{P}(E)-C}{\bar{F}} \right) (Y-1) - (F+\bar{F})X + \frac{P(E)}{F} + \frac{\bar{P}(E)}{\bar{F}} = 0$$

- Fit of **test** oscillation parameters to **true** ones

From the values of P and \bar{P} given by the **true** oscillation parameters, can we determine uniquely the **test** oscillation parameters?

$$P(\nu_\mu \rightarrow \nu_e, E; \theta_{jk}^{\text{test}}, \Delta m_{jk}^{2 \text{ test}}, \delta^{\text{test}}) = P(\nu_\mu \rightarrow \nu_e, E; \theta_{jk}^{\text{true}}, \Delta m_{jk}^{2 \text{ true}}, \delta^{\text{true}}) \equiv P(E)$$

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e, E; \theta_{jk}^{\text{test}}, \Delta m_{jk}^{2 \text{ test}}, \delta^{\text{test}}) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e, E; \theta_{jk}^{\text{true}}, \Delta m_{jk}^{2 \text{ true}}, \delta^{\text{true}}) \equiv \bar{P}(E)$$

Test oscillation parameters ($\theta_{13}, \theta_{23}, \delta$): varied (δ is expressed by θ_{13} and θ_{23})
 → 2 independent parameters

are: $X \equiv \sin^2 2\theta_{13}$ $Y \equiv \frac{1}{s_{23}^2}$

True oscillation parameters: fixed

NB Other **test** oscillation parameters ($\Delta m_{31}^2, \theta_{12}, \Delta m_{21}^2$) are fixed

3.1 Situation before 2012

2012 Reactor ν : $P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \rightarrow \sin^2 2\theta_{13}$

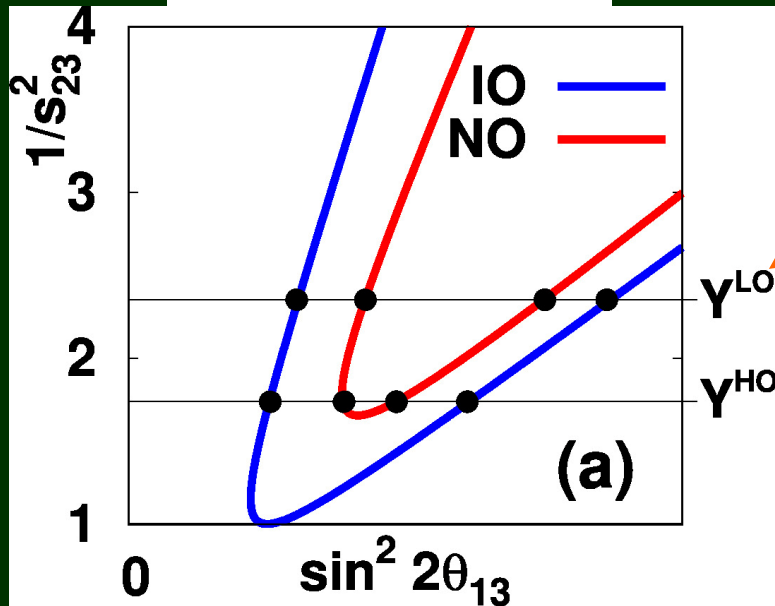
(1) No information on $\sin^2 2\theta_{13}$ was available

(2) From $P(\nu_\mu \rightarrow \nu_\mu)$ only $\sin^2 2\theta_{23}$ is known

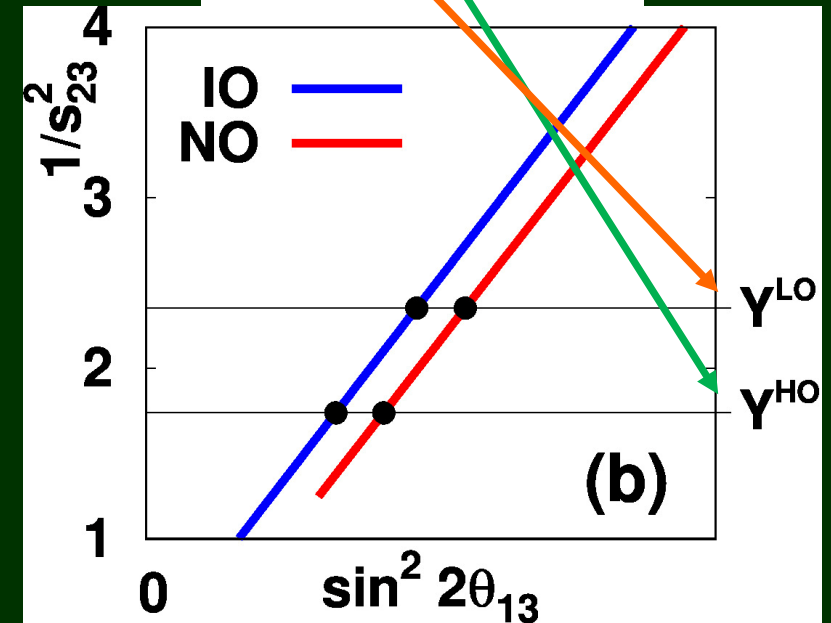
2 possibilities for $Y=1/s^2_{23}$ remain:

$$Y = \begin{cases} Y^{\text{HO}} \\ Y^{\text{LO}} \end{cases} \equiv \frac{2}{1 \pm \sqrt{1 - \sin^2 2\theta_{23}^{\text{true}}}}$$

For $\sin\Delta \neq 0$



For $\sin\Delta = 0$



3.2 Situation after 2012

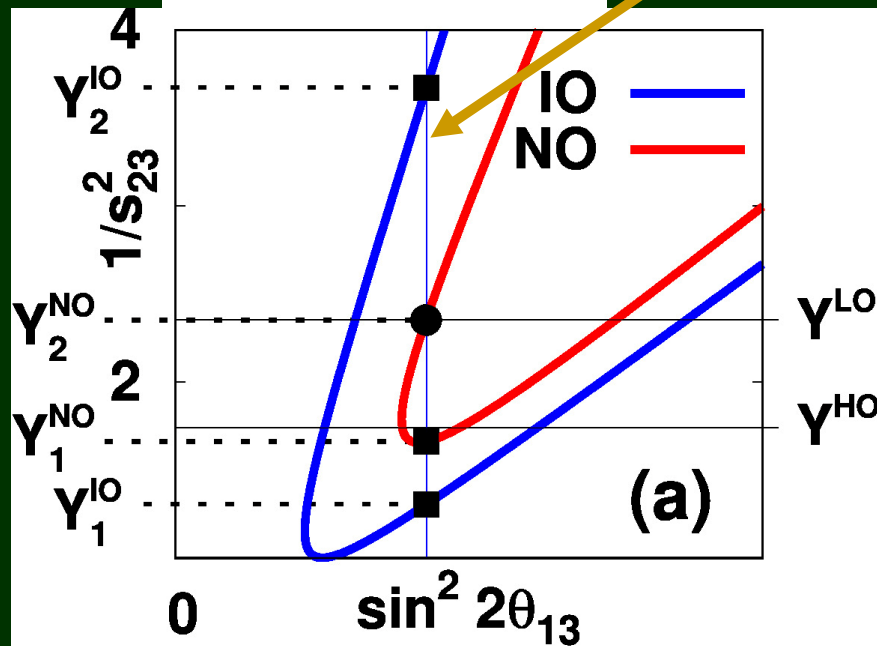
2012 Reactor ν : $P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \rightarrow \sin^2 2\theta_{13}$

(1) From $P(\nu_\mu \rightarrow \nu_\mu)$ only $\sin^2 2\theta_{23}$ is known

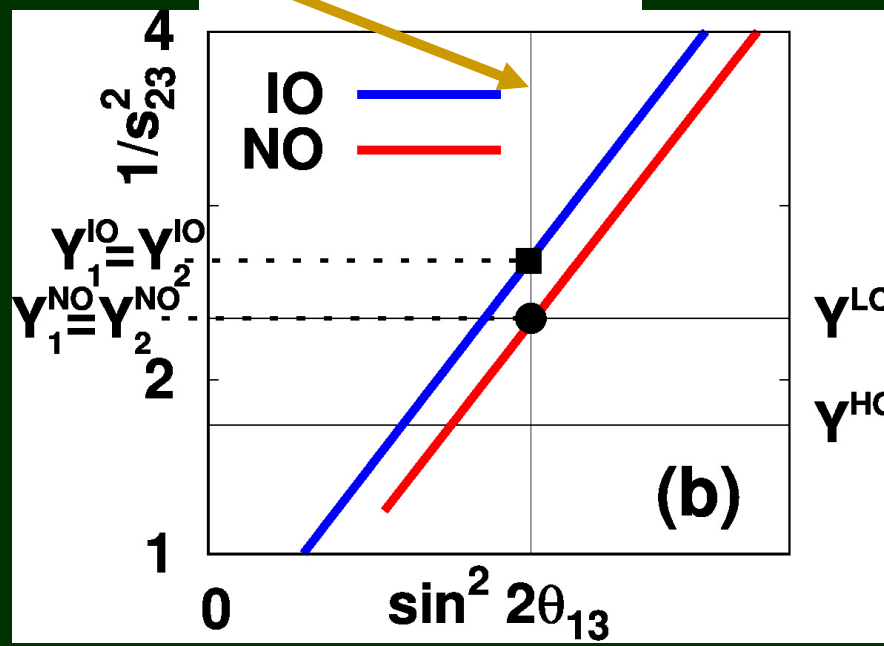
(2) Information on $\sin^2 2\theta_{13}$ is available

From these two, θ_{23} should be determined!?

For $\sin\Delta \neq 0$



For $\sin\Delta = 0$



These figures are for a fixed energy. \rightarrow Later we will see the behavior of Y_j^{MO} over the whole energy spectrum.

3.3 T2HK: (X,Y) plot

$L=295\text{km}$

$E\sim 0.6\text{GeV}$

If true Mass Ordering is NO and true δ_{CP} is $-\pi/2$, then T2HK can exclude IO.

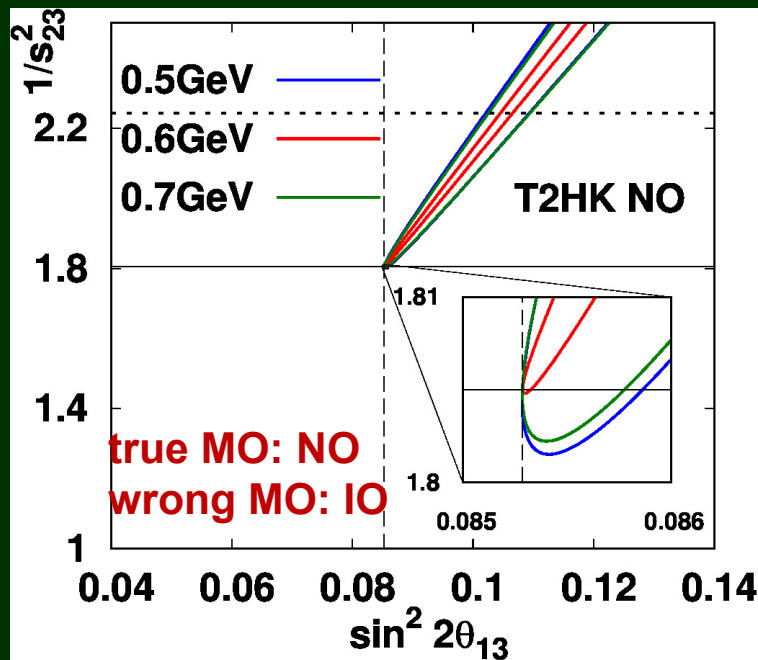
Assumption:
 $\delta_{CP}(\text{true})=-\pi/2$

$Y_2^{\text{NO}}(E=0.7\text{GeV})$

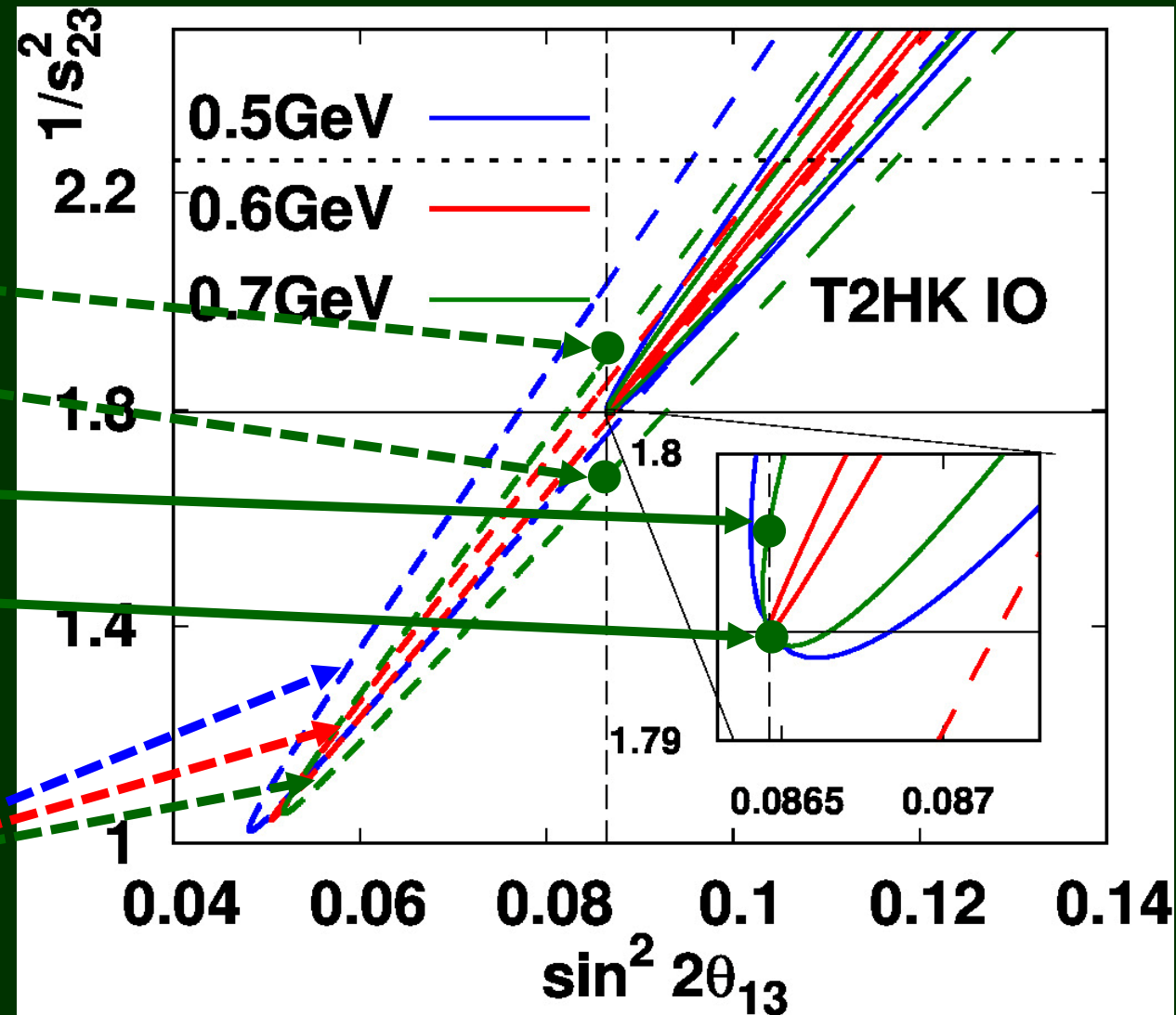
$Y_1^{\text{NO}}(E=0.7\text{GeV})$

$Y_2^{\text{IO}}(E=0.7\text{GeV})$

$Y_1^{\text{IO}}(E=0.7\text{GeV})$



Dashed lines:
Trajectory assuming wrong Mass Ordering



true MO: IO
wrong MO: NO

Sugama - OY, arXiv:2308.15071

3.3 T2HK: $(E, Y_j^{MO}(E))$ plot

Assumption: True Octant=Higher Octant

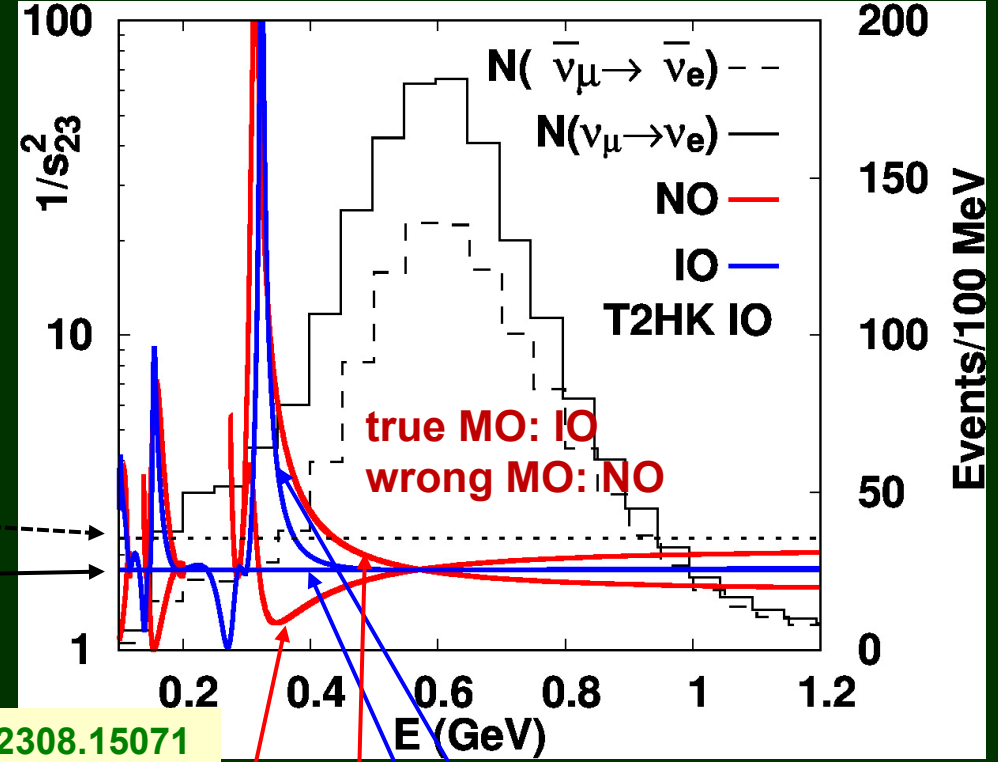
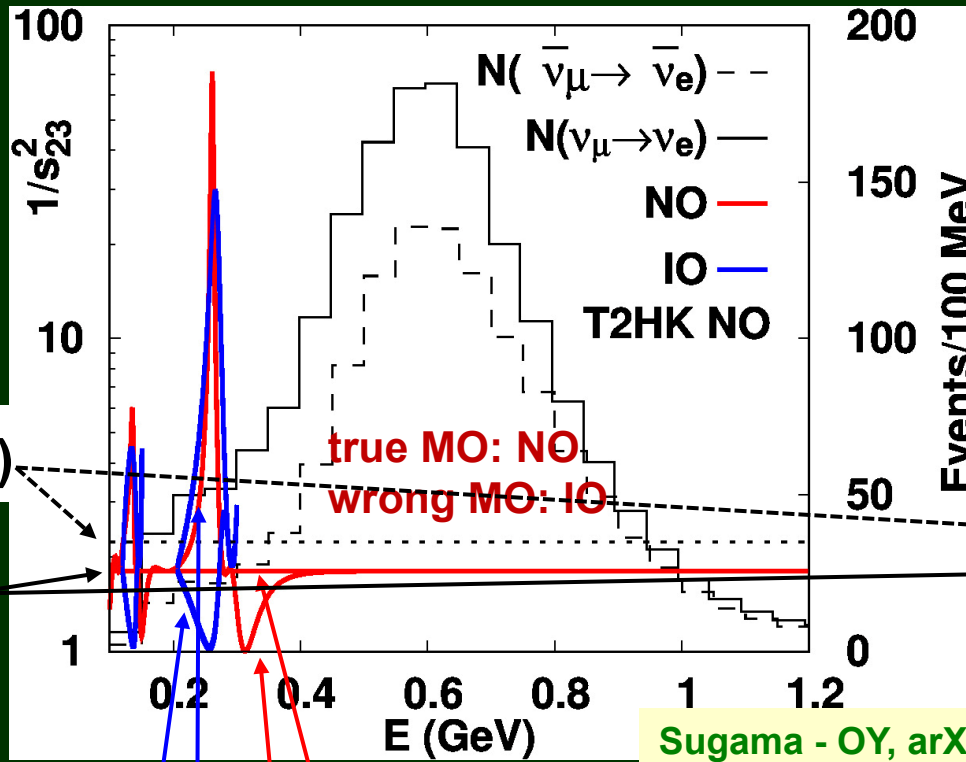
$L=295\text{km}$
 $E \sim 0.6\text{GeV}$

Sugama - OY,
 arXiv:2308.15071

Y^{LO} (wrong octant)

Y^{HO} (true octant)

For dominant energy bins, Y_j^{MO} agree with Y^{HO} → Octant degeneracy can be resolved.



Sugama - OY, arXiv:2308.15071

correct Mass Ordering

Larger: $Y_2^{NO}(E)$

Smaller: $Y_1^{NO}(E)$

Larger: $Y_2^{IO}(E)$

Smaller: $Y_1^{IO}(E)$

wrong Mass Ordering

correct Mass Ordering

Larger: $Y_2^{IO}(E)$

Smaller: $Y_1^{IO}(E)$

Larger: $Y_2^{NO}(E)$

Smaller: $Y_1^{NO}(E)$

wrong Mass Ordering

3.4 DUNE

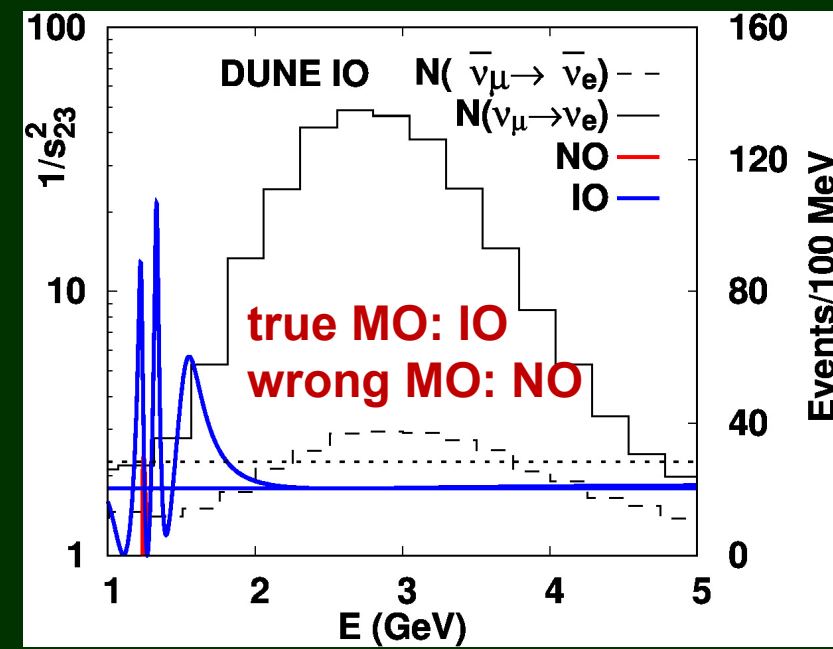
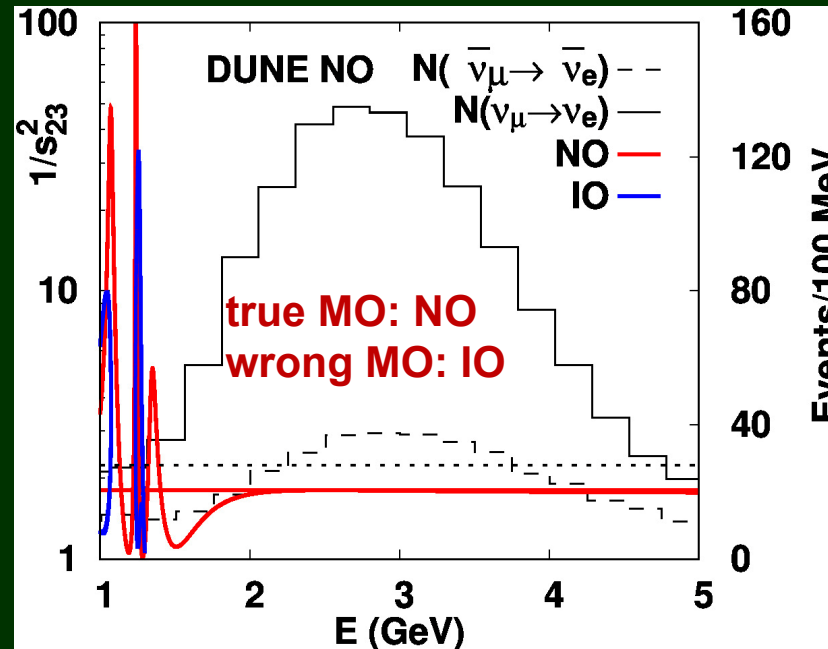
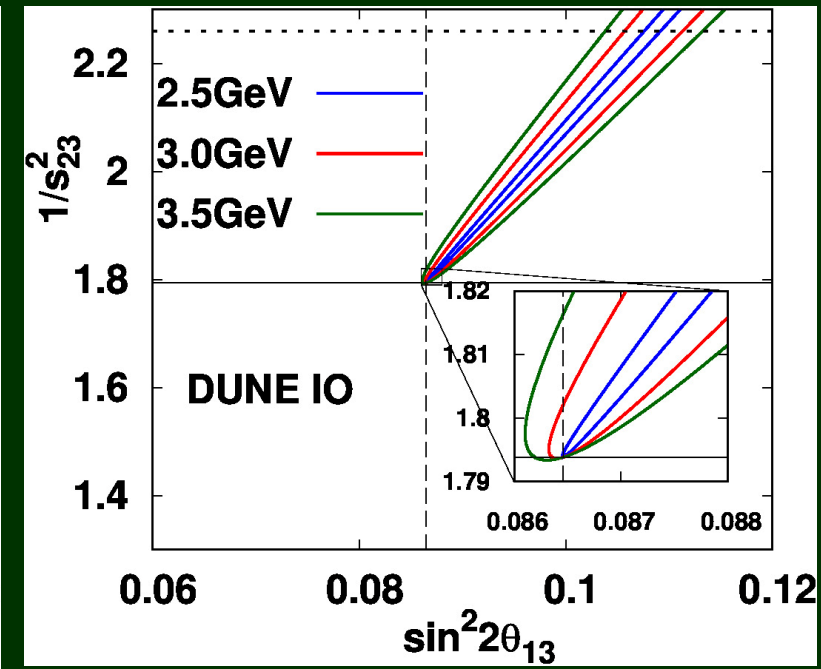
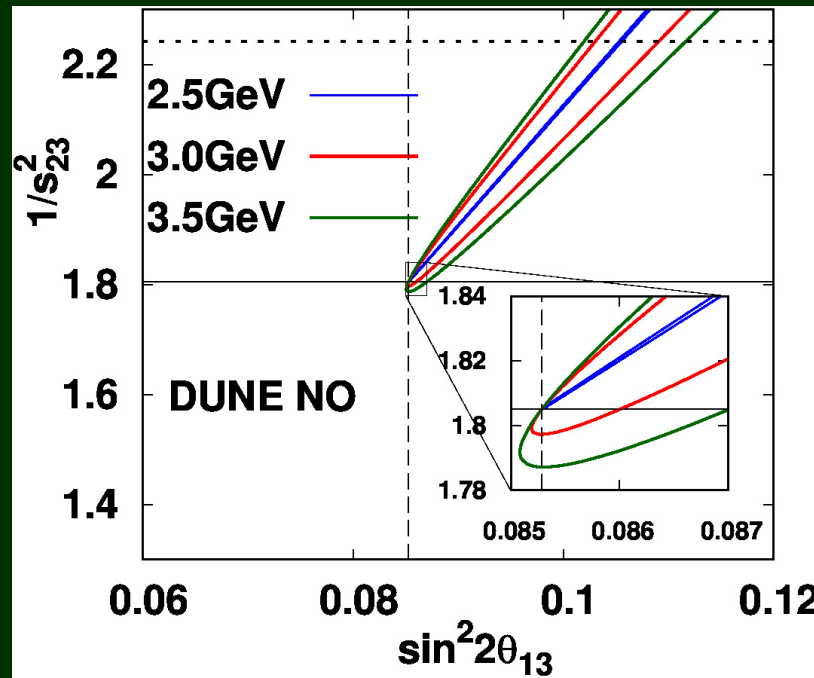
Sugama - OY,
arXiv:2308.15071

L=1300km

E=2GeV - 4GeV

Because of the long baseline, wrong Mass Ordering can be always excluded.

→ Octant degeneracy can be resolved, as in T2HK.



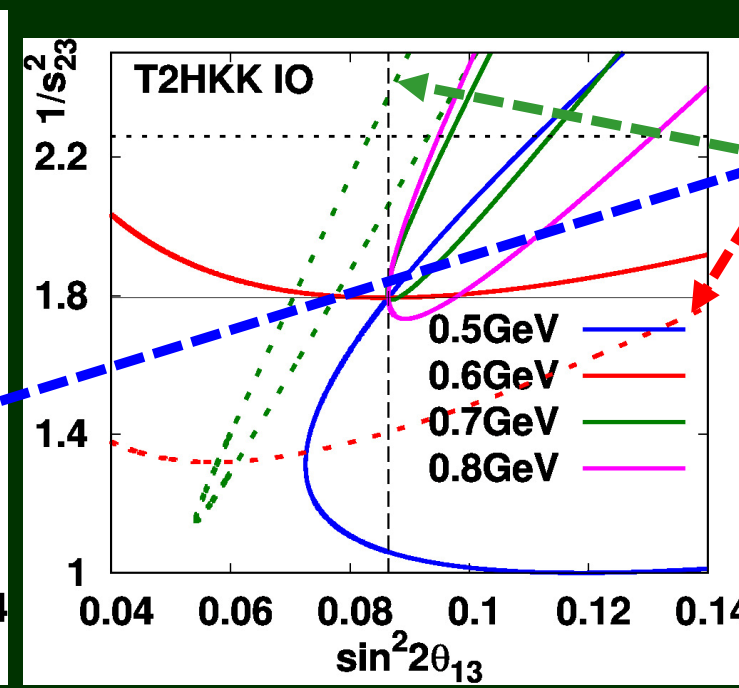
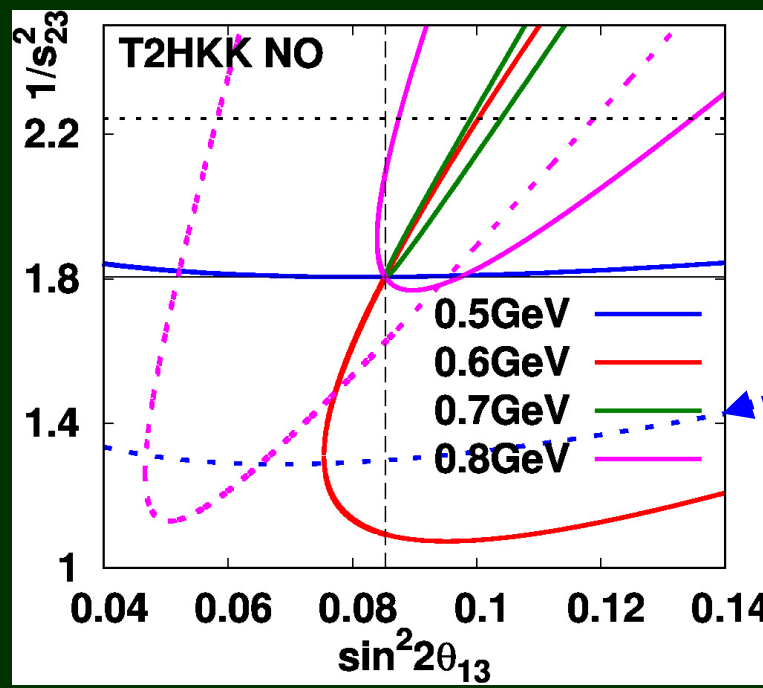
3.5 T2HKK

L=1100km

E=0.5GeV – 0.8GeV

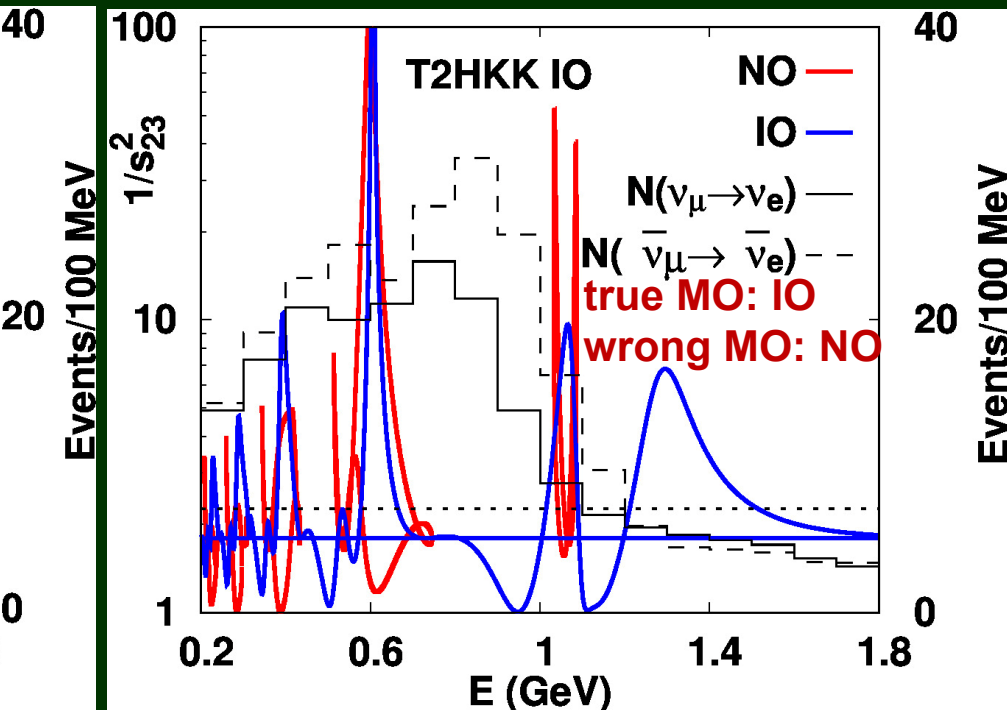
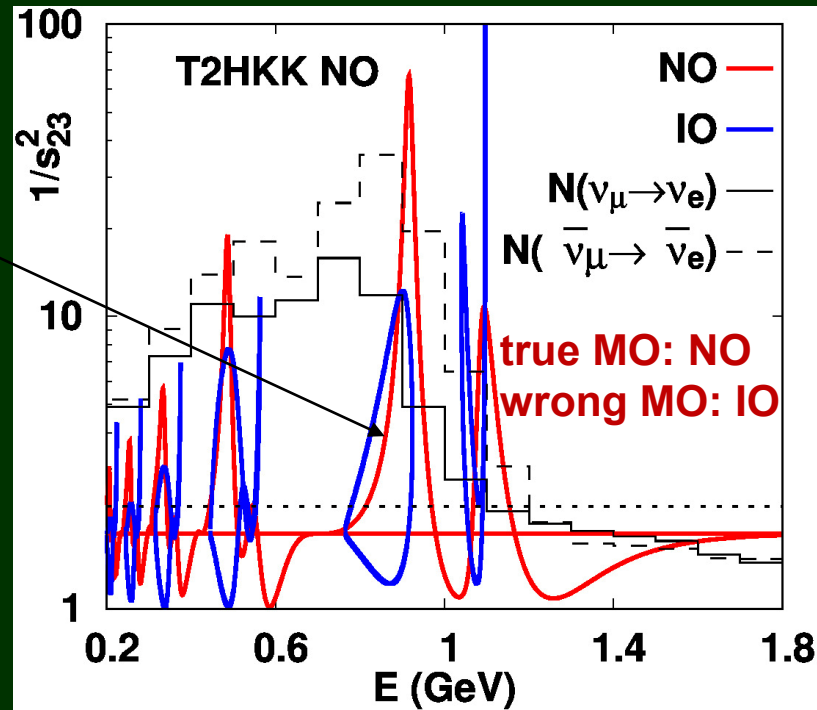
1st Oscillation Maximum ($\Delta=\pi/2$, E=2.2GeV) is missed, but 2nd Oscillation Maximum ($\Delta=3\pi/2$, E=0.75GeV) is covered.

→ Because of large deviation and rapid oscillations near 2nd oscillation maximum, it is difficult to resolve octant degeneracy. [← already known by numerical simulations in P. Panda et al. (arXiv:2206.10320)]



Dashed lines:
Trajectory assuming wrong Mass Ordering

Sugama - OY,
arXiv:2308.15071



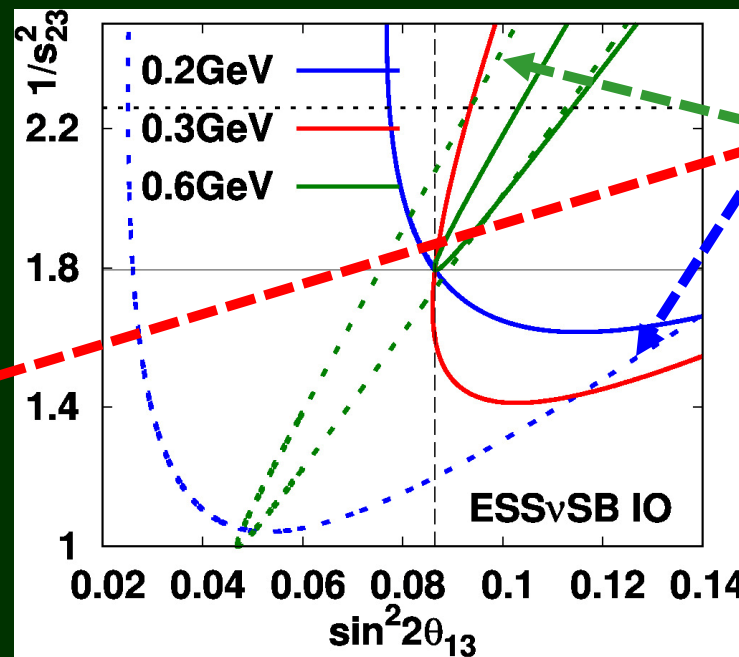
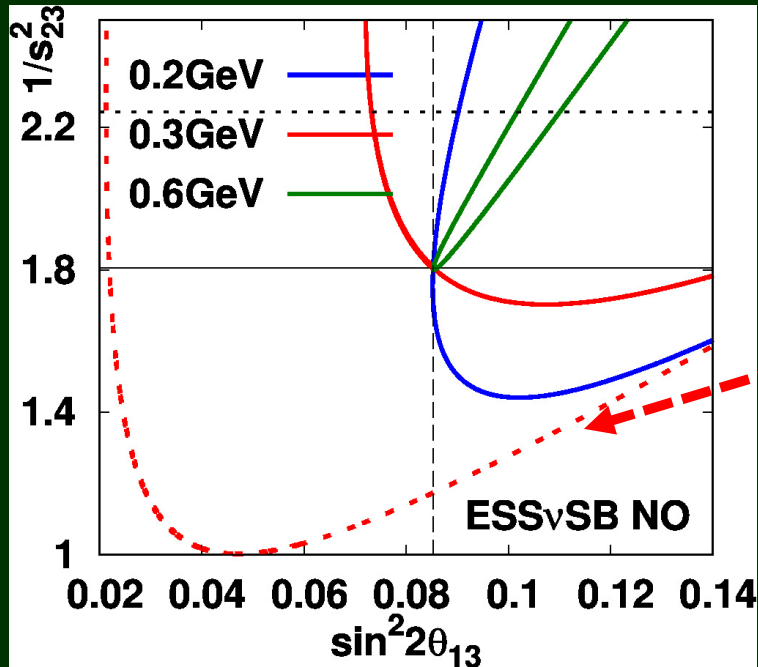
3.6 ESSνSB

L=530km

E=0.2GeV – 0.4GeV

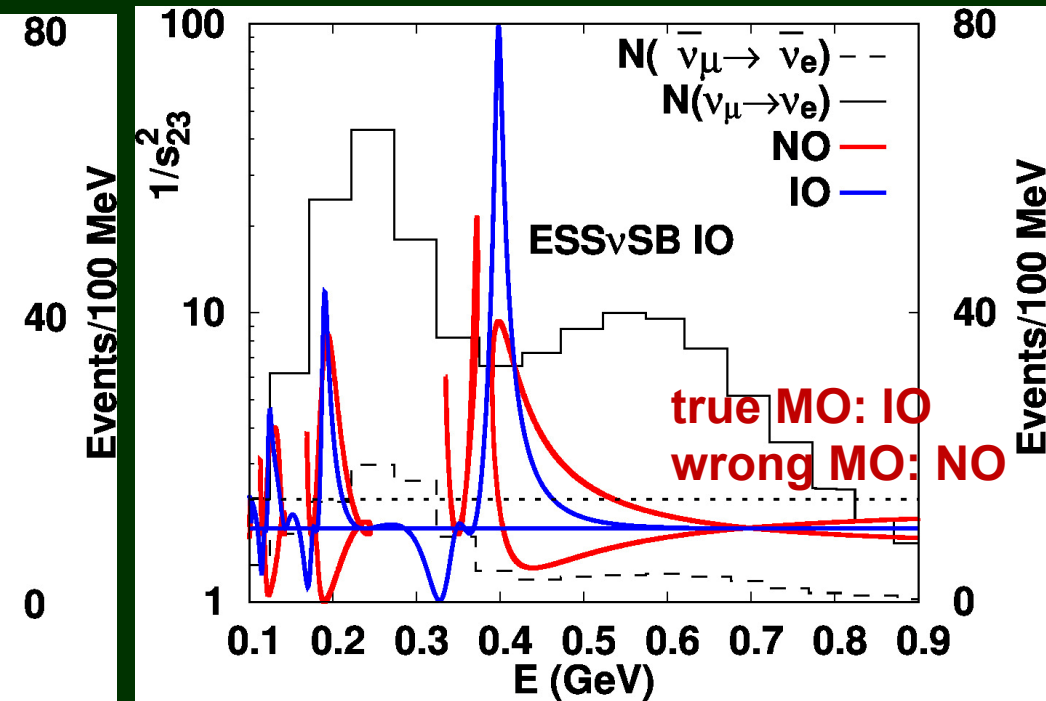
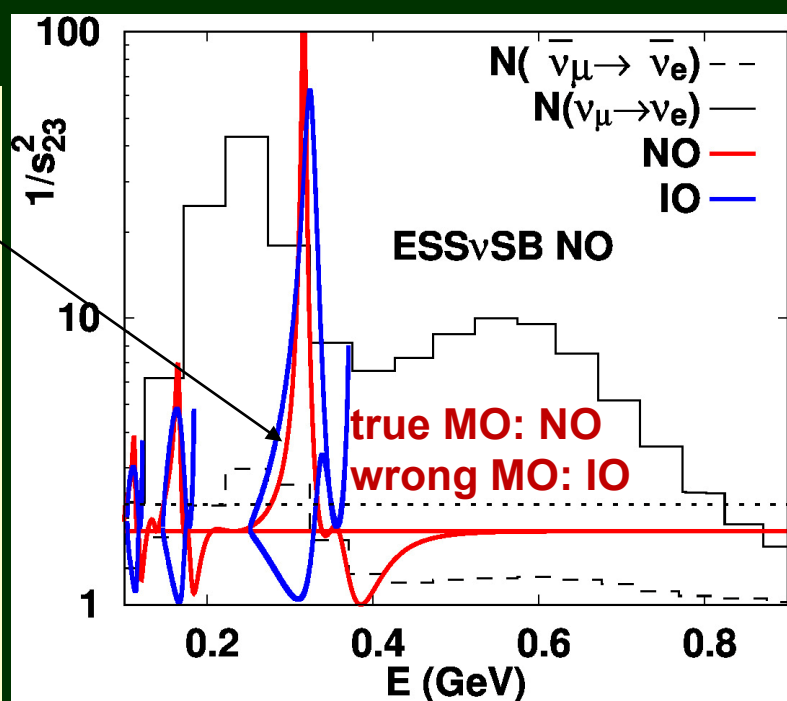
1st Oscillation Maximum ($\Delta=\pi/2$, E=0.73GeV) is missed for $\bar{\nu}$ mode, but 2nd Oscillation Maximum ($\Delta=3\pi/2$, E=0.24GeV) is covered.

→ Because of large deviation and rapid oscillations near 2nd oscillation maximum, it is difficult to resolve octant degeneracy. [← already known by numerical simulations in S. K. Agarwalla et al., (arXiv:1406.2219)]



**Dashed lines:
Trajectory assuming wrong Mass Ordering**

**Sugama - OY,
arXiv:2308.15071**



4. Conclusions

- **T2HK+DUNE gives us excellent precision in θ_{23} (1%), Δm_{32}^2 (0.5%), δ (20%), although DUNE suffers from uncertainty in the density (20%).**
- **T2HK and DUNE are expected to resolve octant degeneracy, while it seems difficult to resolve octant degeneracy for far future long baseline experiments, T2HKK and ESS ν SB, which focus on 2nd oscillation maximum.**

Backup slides

Historical background of ν oscillation studies:

1998- Atmospheric ν / Long baseline ν : $P(\nu_\mu \rightarrow \nu_\mu) \rightarrow \sin^2 2\theta_{23}$

2000- Phenomenology of Long baseline ν :

How to determine δ_{CP} from $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$

2001- Parameter degeneracy was pointed out.

2004 Plot of 8-fold parameter degeneracy was proposed.

2012 Reactor ν : $P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \rightarrow \sin^2 2\theta_{13}$

2023 Plot of 8-fold parameter degeneracy is revisited, taking into account the measured values of $\sin^2 2\theta_{13}$ and $\sin^2 2\theta_{23}$ (this talk).

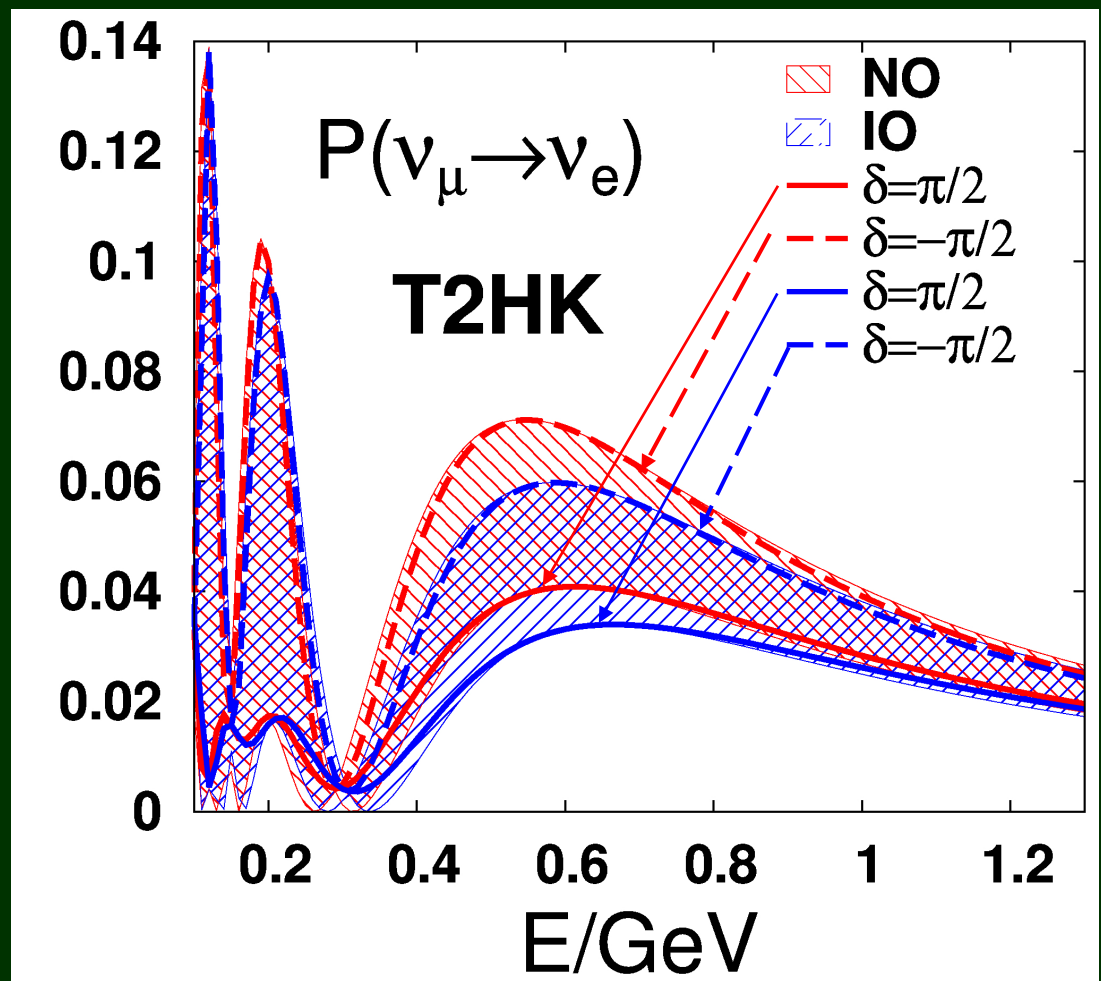
● Understanding degeneracy by appearance probabilities

hierarchy - δ

Prakash, Raut, Sankar, PRD 86, 033012 (2012)

octant - δ

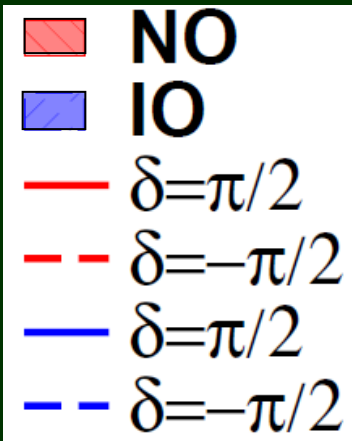
Agarwalla, Prakash, Sankar, JHEP 1307, 131 (2013)



Due to uncertainty in δ , the appearance probabilities has finite width.
-> Each border is approximately realized for $\delta = +\pi/2$ or $-\pi/2$

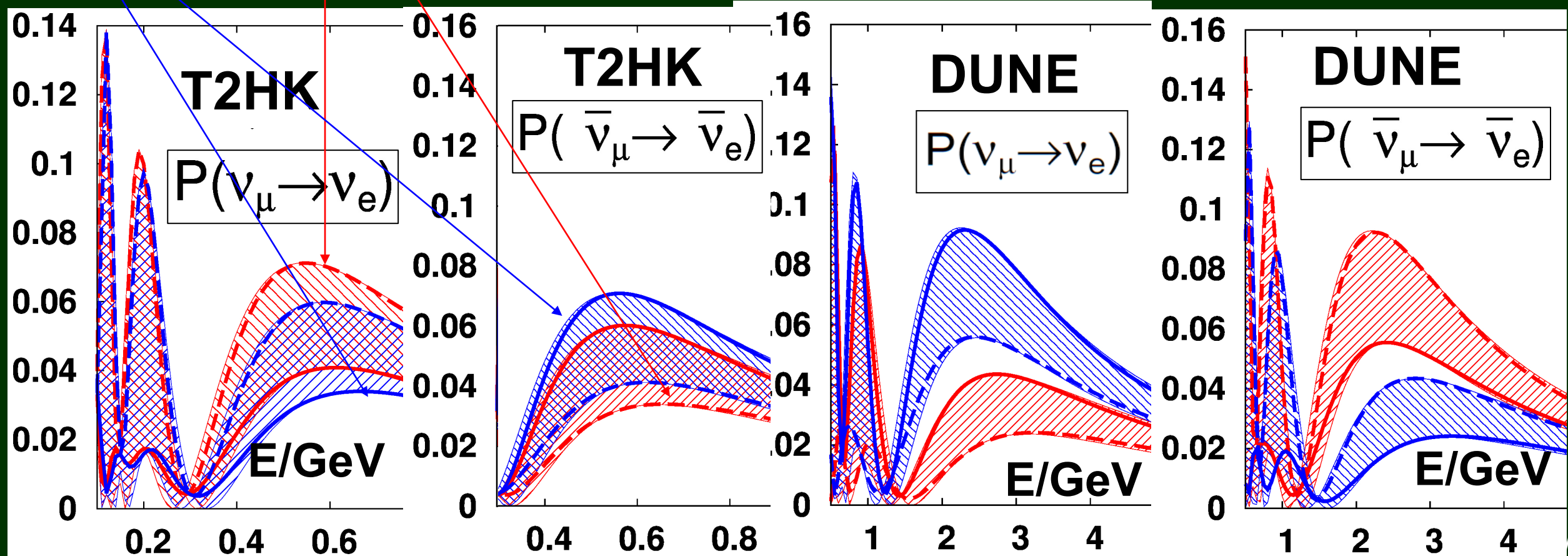
Mass Ordering

At T2HK, MO separation is good only for $\delta \sim -\pi/2$ (NO), $\delta \sim +\pi/2$ (IO)



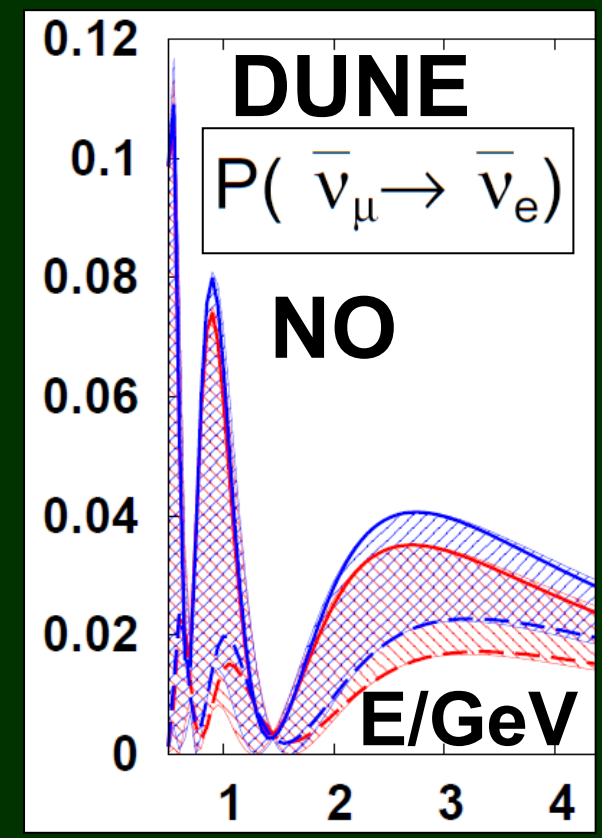
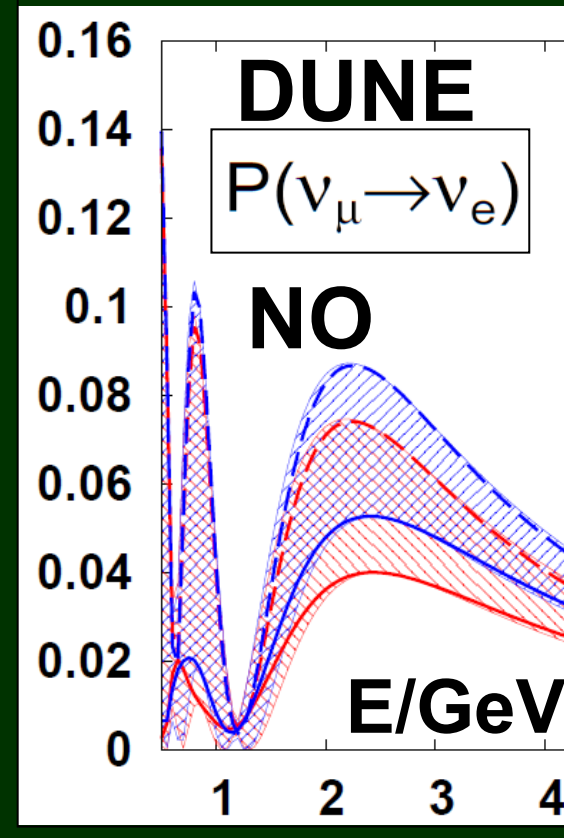
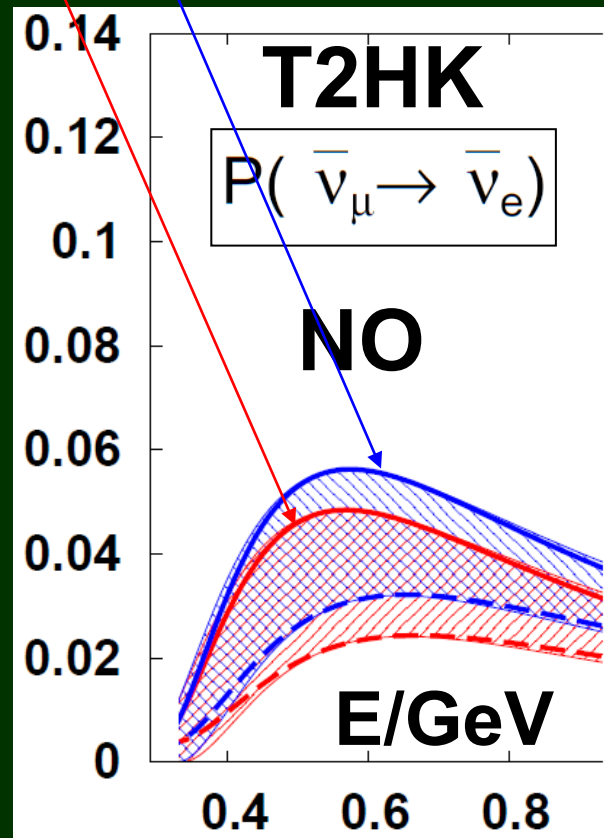
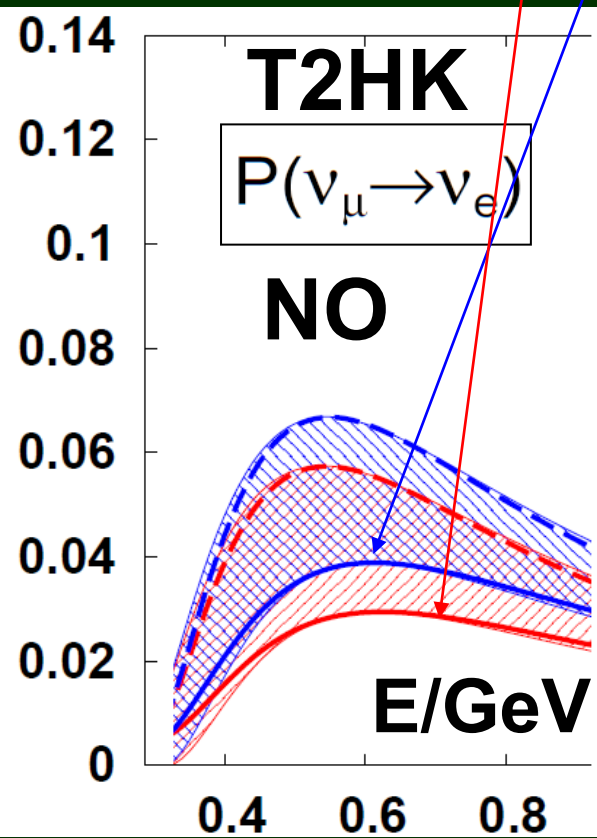
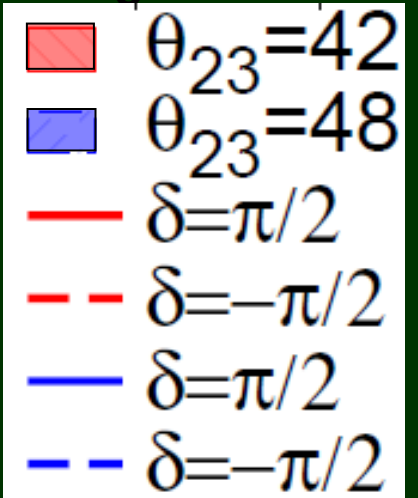
Fukasawa, Ghosh, OY, NPB 918 ('17) 337

At DUNE, NO-IO separation is good for any δ



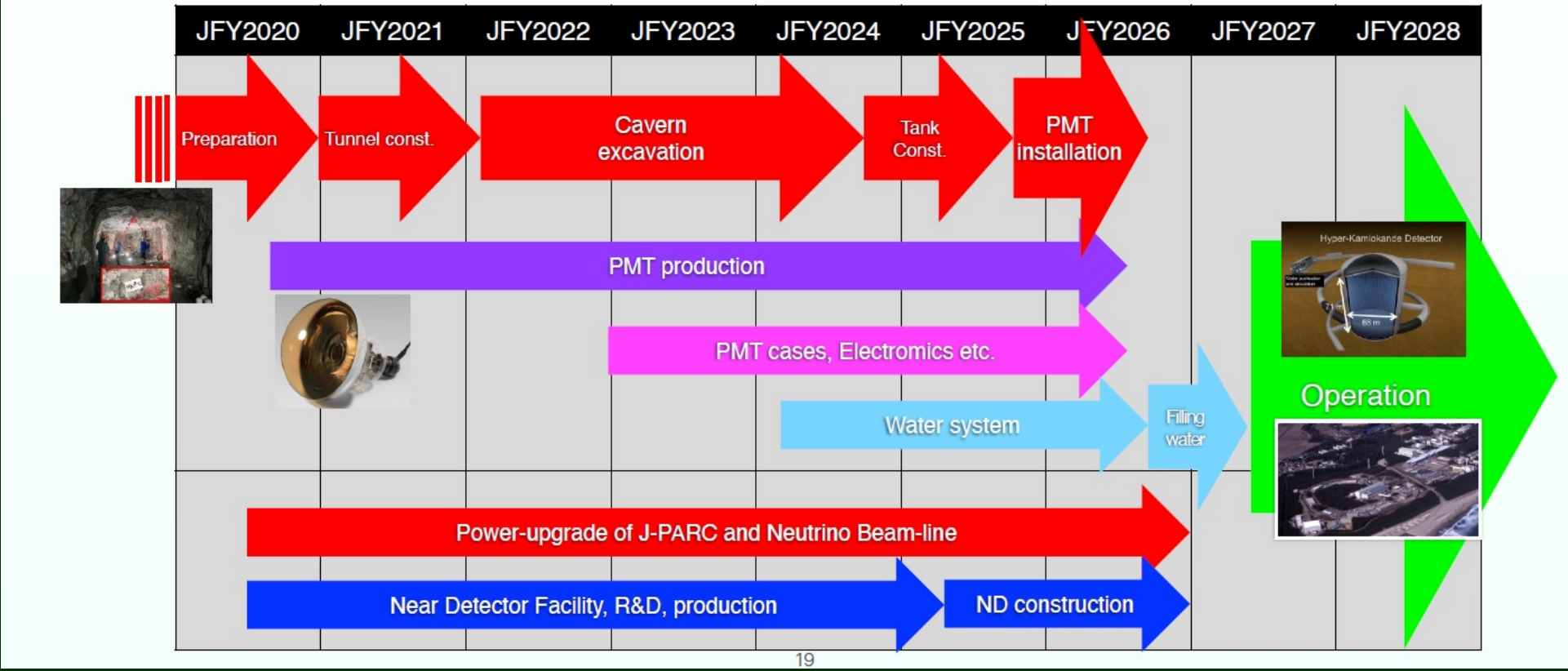
At both T2HK & DUNE, HO-LO separation is possible w/ ν & $\bar{\nu}$ for most of δ

Unlike hierarchy degeneracy, $\delta = -\pi/2$ lies on the same side for ν & $\bar{\nu}$

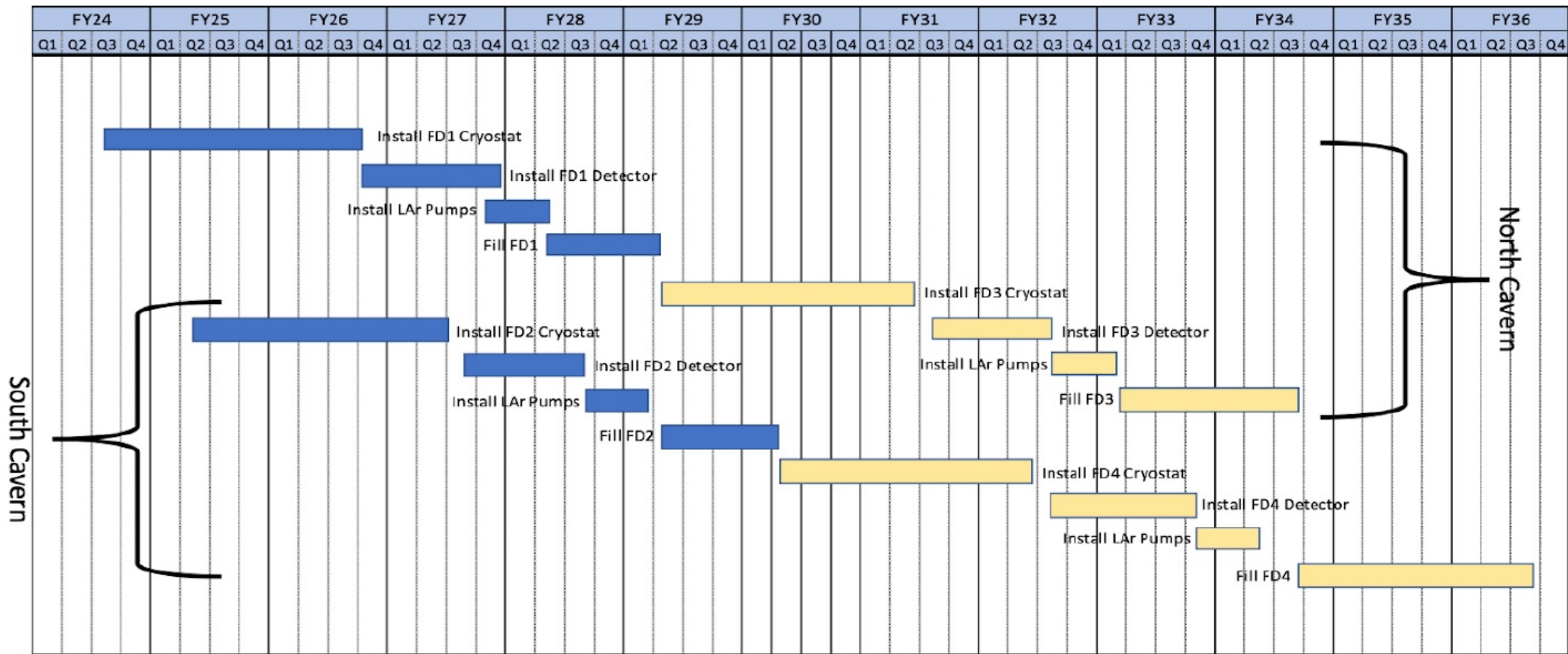


Timeline of Hyperkamiokande

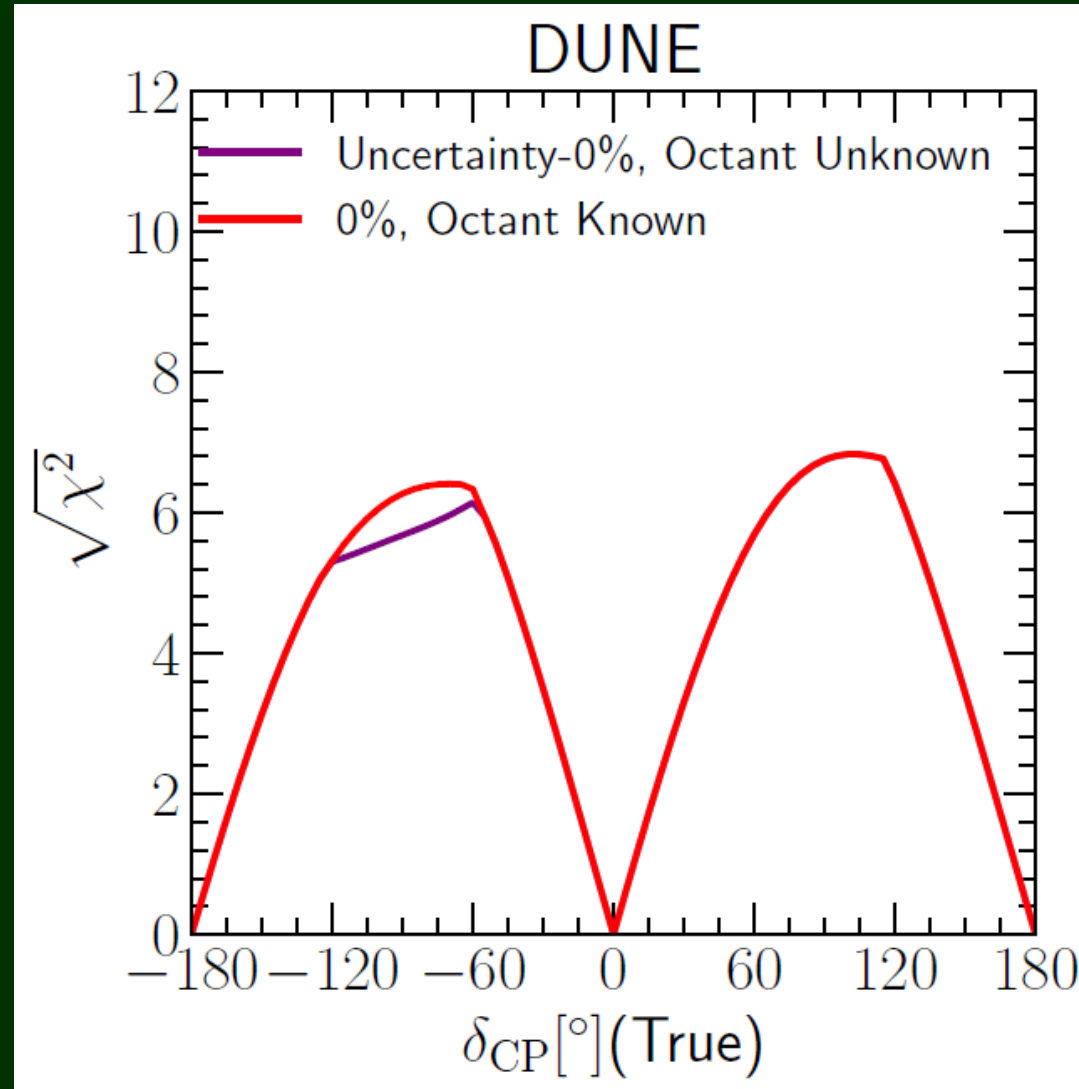
- 2022-2027: Construction, 2027- : Operation
- No change of schedule since the approval of project in 2020



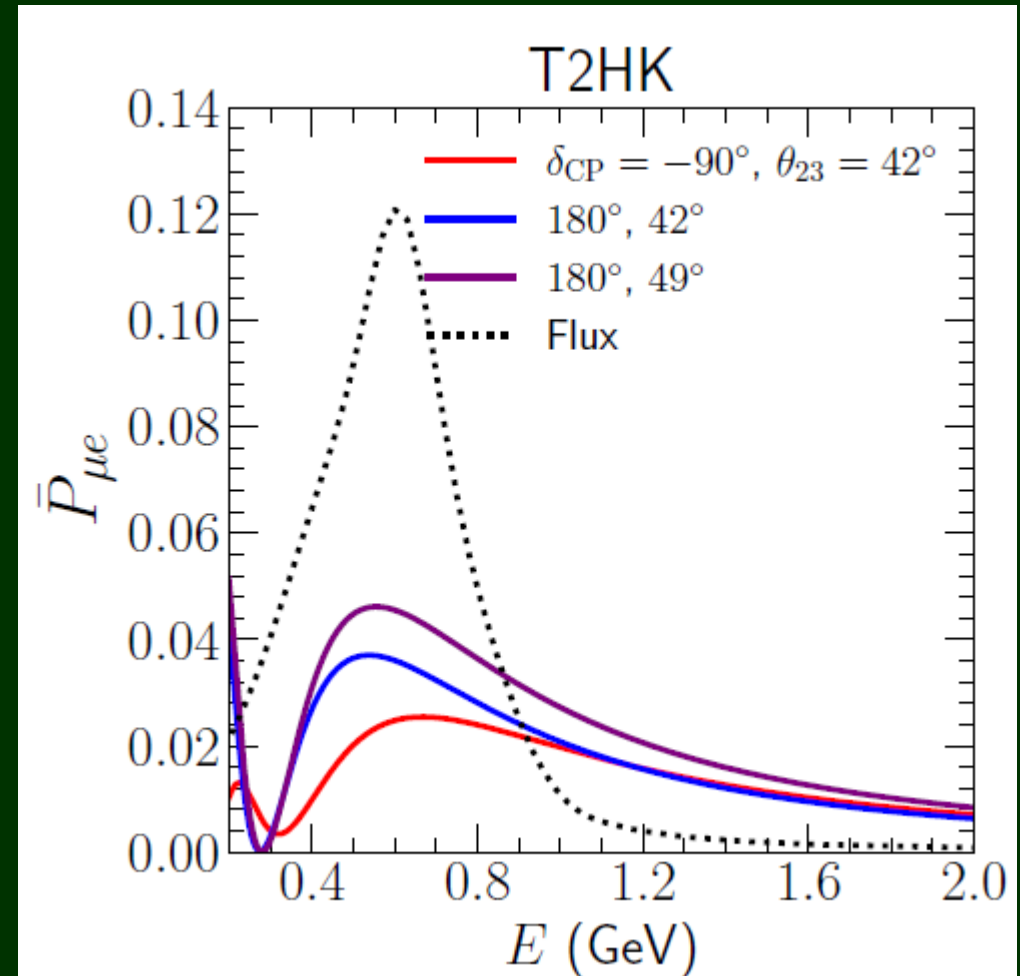
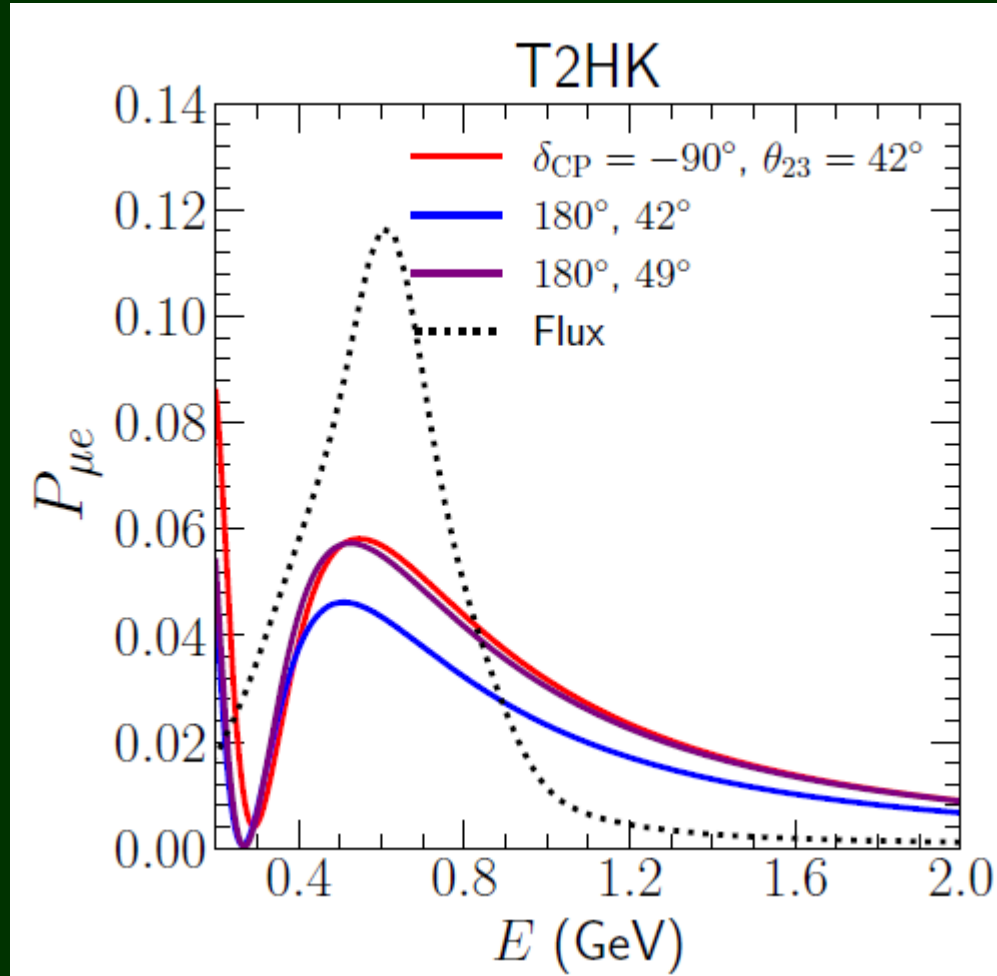
Timeline of DUNE (2029(?)-)



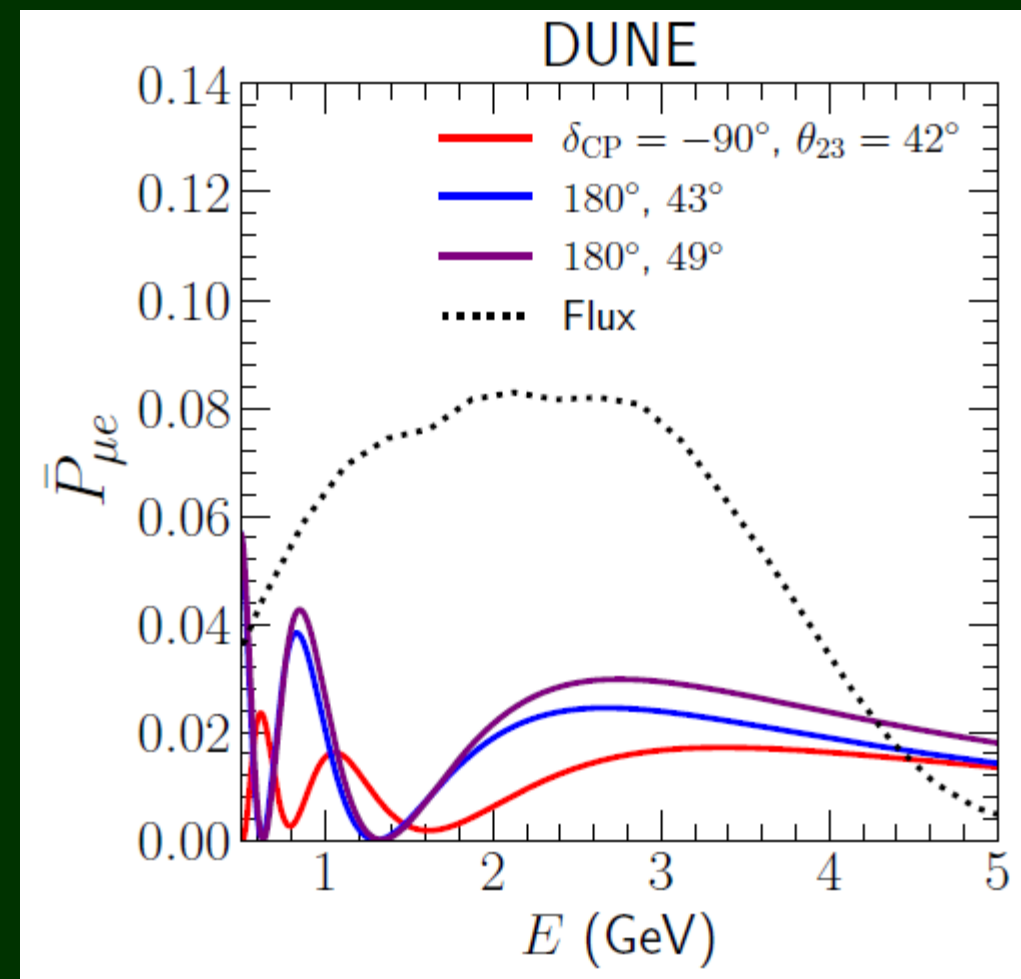
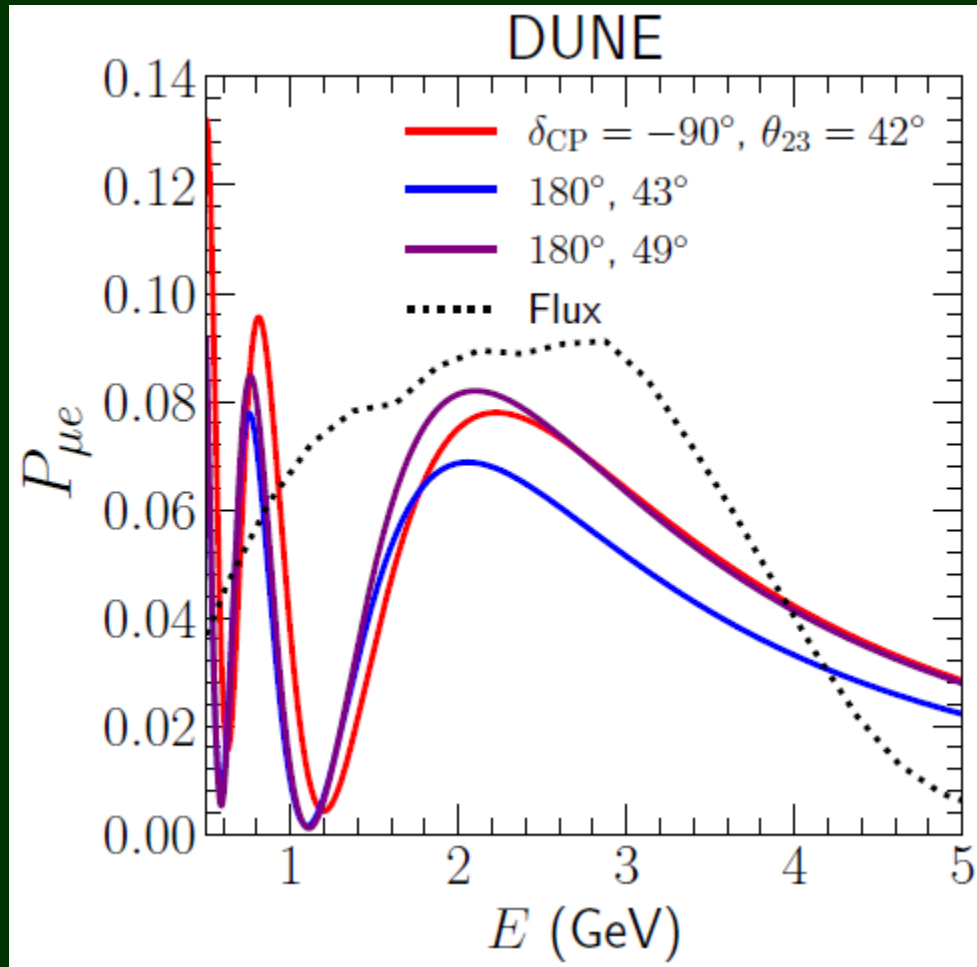
Earliest installation start in 2029 with FD3 completed in Q4, 2034 and FD4 in Q4, 2036



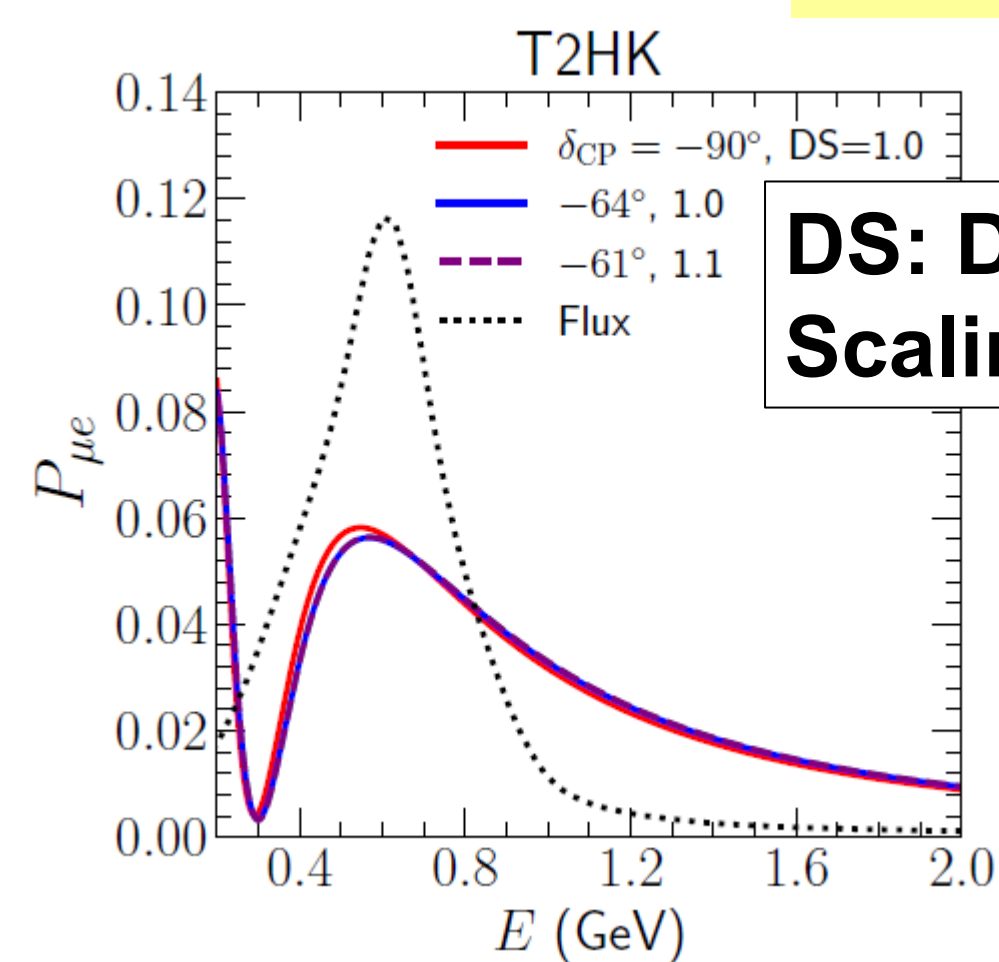
octant degeneracy at DUNE



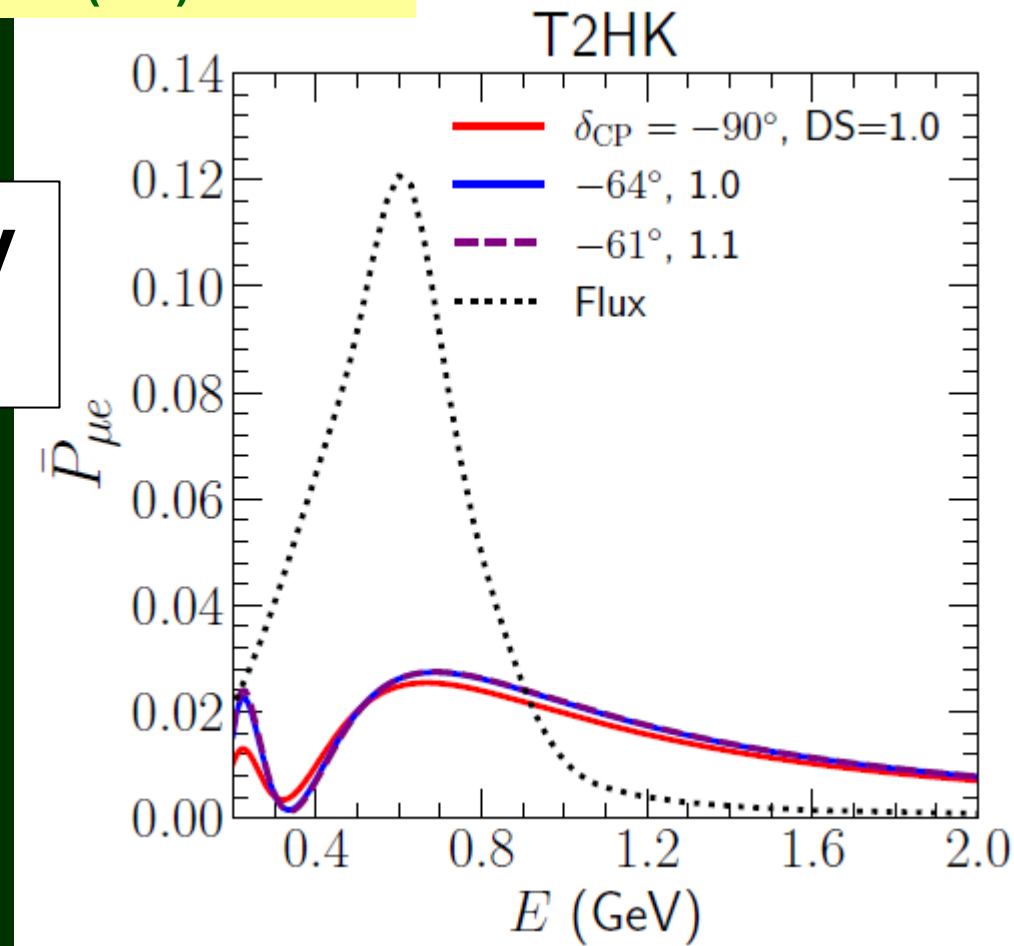
Probability vs octant degeneracy at T2HK



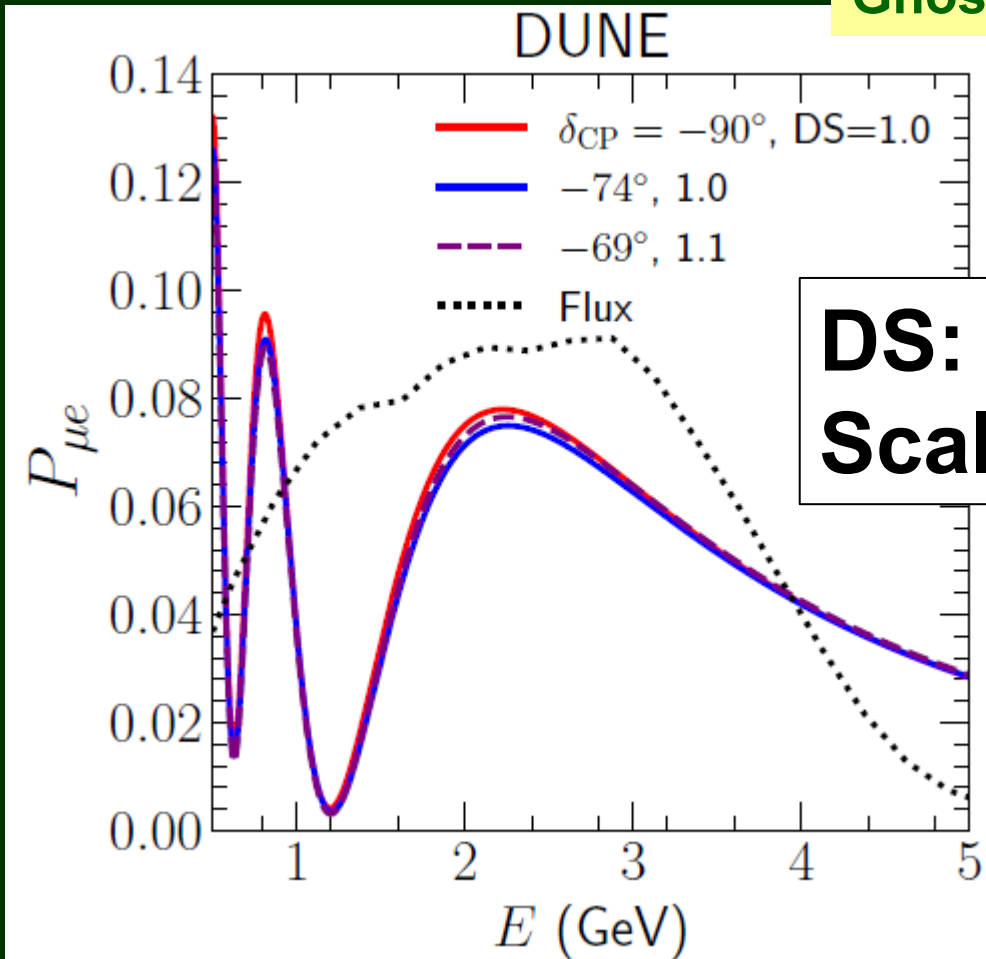
Probability vs octant degeneracy at DUNE



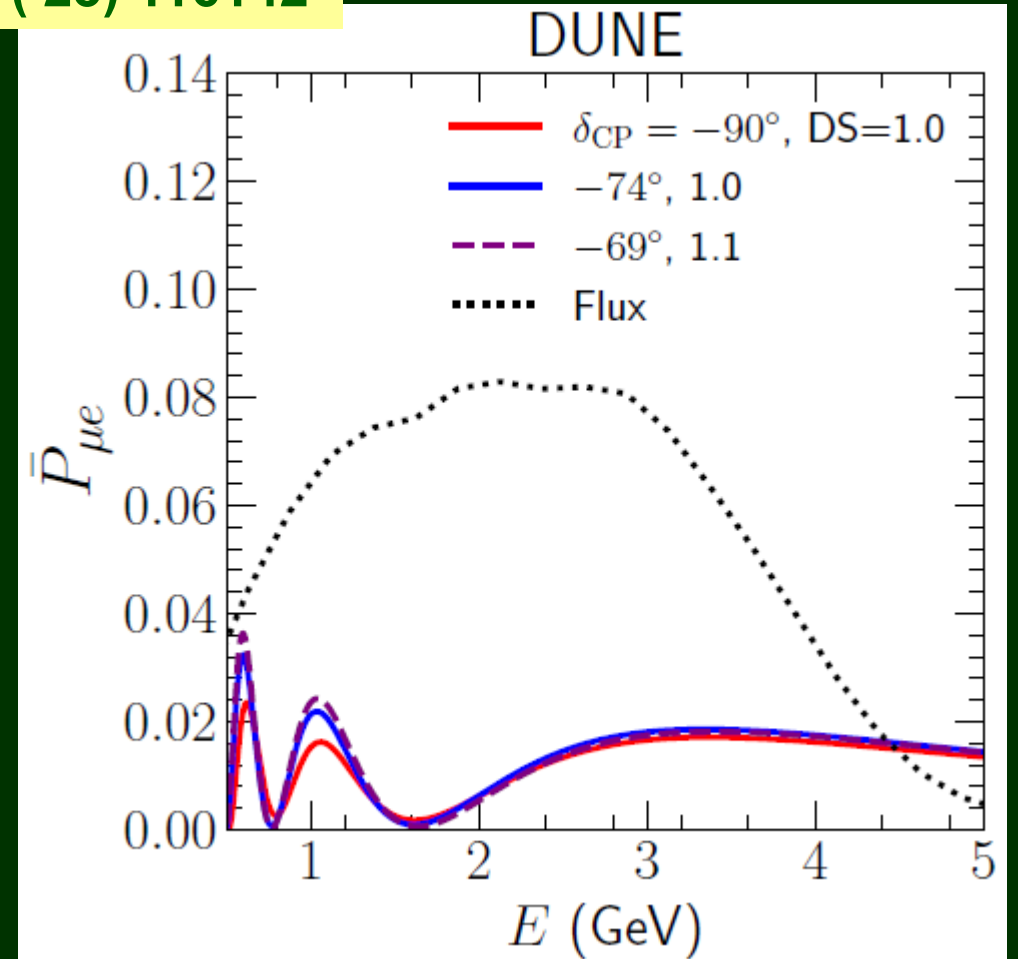
**DS: Density
Scaling**



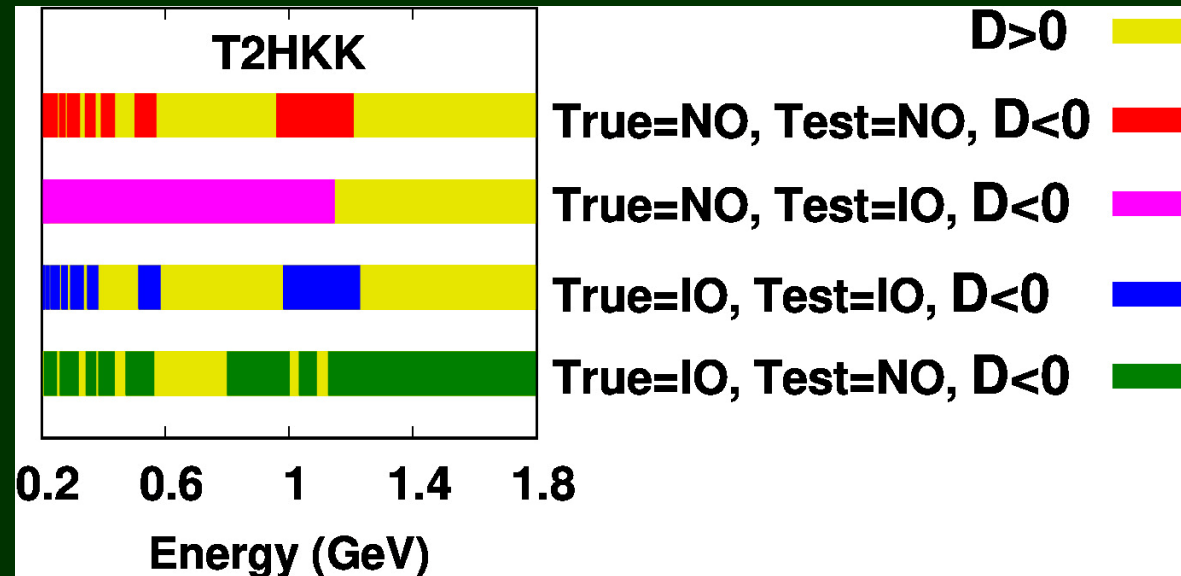
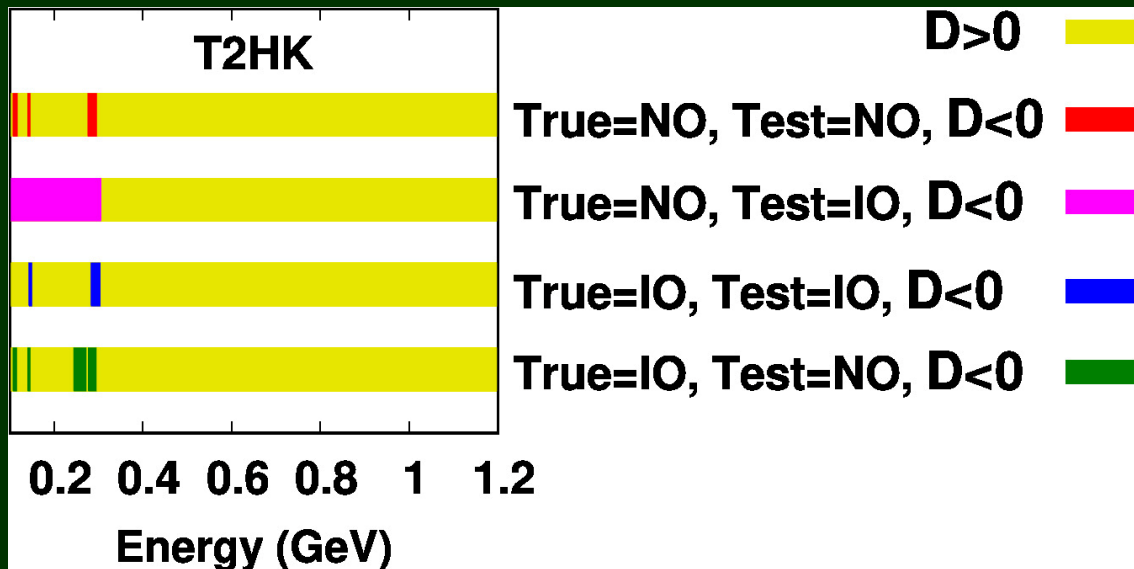
For T2HK, $(\delta, DS)=(-64^\circ, 1.0)$ is degenerate with $(-61^\circ, 1.1)$.
 For $\delta(\text{true}) = -90^\circ$, $(\delta(\text{test}), DS(\text{test})) = (-61^\circ, 1.0)$ is excluded but $(\delta(\text{test}), DS(\text{test})) = (-61^\circ, 1.1)$ is allowed.



**DS: Density
Scaling**



For DUNE, $(\delta, \text{DS})=(-74^\circ, 1.0)$ is degenerate with $(-69^\circ, 1.1)$.
 For $\delta(\text{true}) = -90^\circ$, $(\delta(\text{test}), \text{DS}(\text{test})) = (-69^\circ, 1.0)$ is excluded but $(\delta(\text{test}), \text{DS}(\text{test})) = (-69^\circ, 1.1)$ is allowed.



Sugama-OY, arXiv:2308.15071

