Systematic limits of multi reactors and detectors

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I. One detector + reactors

II. Multi detectors + reactors

Work in collaboration with
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III. Spectrum analysis

Work in collaboration with
H. Sugiyama

Assumptions throughout this talk:

- Systematic errors only (=in the ∞ statistical limit)

$(\sin^2 2\theta)^{\text{sys only}}_{\text{limit}}$: limit on $\sin^2 2\theta_{13}$ w/o stat error

- Background not considered (no difference in calibration for #(reactors) >1)
- Identical systematic errors (in Kashiwazaki)

from detectors $\begin{cases} \sigma_c & \text{correlated} \sim 1.6\% \\ \sigma_u & \text{uncorrelated} \sim 0.6\% \end{cases}$

from reactors $\begin{cases} \sigma_c^{(r)} & \text{correlated} \sim 2.5\% \\ \sigma_u^{(r)} & \text{uncorrelated} \sim 2.3\% \end{cases}$

I. One detector + multi reactors

(1) 1 detector + 1 reactor

$$\chi^2 = \min_{\alpha's}\left\{\left[\frac{M - T\left(1+\alpha_c+\alpha_c^{(r)}+\alpha_u^{(r)}\right)}{T\sigma_u}\right]^2 + \left(\frac{\alpha_c}{\sigma_c}\right)^2 + \left(\frac{\alpha_c^{(r)}}{\sigma_c^{(r)}}\right)^2 + \left(\frac{\alpha_u^{(r)}}{\sigma_u^{(r)}}\right)^2\right\}$$

$$= \frac{(M/T-1)^2}{\sigma_u^2+\sigma_c^2+\left(\sigma_c^{(r)}\right)^2+\left(\sigma_u^{(r)}\right)^2} \equiv \frac{(M/T-1)^2}{\sigma_{eff}^2}$$

M: measured # (events) , T: theoretical # (events)

Limit on $\sin^2 2\theta$

$$T = \int \varepsilon(E)\sigma(E)f(E)dE$$

$$M = \int \varepsilon(E)\sigma(E)f(E)P(E)dE$$

$$T - M = \sin^2 2\theta \int \varepsilon(E)\sigma(E)f(E)\sin^2\left(\frac{\Delta m^2 L}{4E}\right)dE$$

$$\frac{T-M}{T} = \sin^2 2\theta \left\langle \sin^2\left(\frac{\Delta m^2 L}{4E}\right)\right\rangle \cong 0.8 \sin^2 2\theta$$

For L=1.7km,
$\Delta = 2.5 \times 10^{-3}\,\mathrm{eV}^2$

$$\chi^2 = \frac{\left(0.8 \sin^2 2\theta\right)^2}{\sigma_{\mathrm{eff}}^2}, \quad \chi^2\Big|_{90\%\mathrm{CL}} = 2.7$$

$$\left(\sin^2 2\theta\right)_{\mathrm{limit}}^{\mathrm{sys-only}} = \frac{\sqrt{2.7}}{0.8}\sigma_{\mathrm{eff}}$$

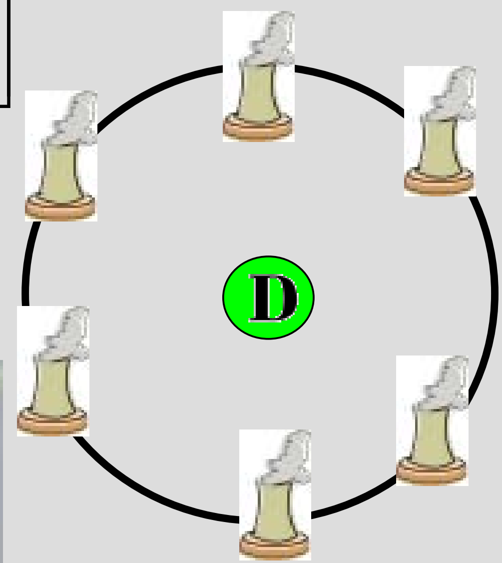
(2) 1 detector +N reactors

$$T=\sum_{a=1}^{N}T_a \rightarrow \sum_{a=1}^{N}T_a\left(1+\alpha_c+\alpha_c^{(r)}+\alpha_{ua}^{(r)}\right)$$

$$=T\left[1+\alpha_c+\alpha_c^{(r)}+\sum_{a=1}^{N}\left(\frac{T_a}{T}\right)\alpha_{ua}^{(r)}\right]$$

$$\chi^2=\min_{\alpha's}\left\{\left[\frac{M-T\left(1+\alpha_c+\alpha_c^{(r)}+\sum_{a=1}^{N}\left(\frac{T_a}{T}\right)\alpha_{ua}^{(r)}\right)}{T\sigma_u}\right]^2+\left(\frac{\alpha_c}{\sigma_c}\right)^2+\left(\frac{\alpha_c^{(r)}}{\sigma_c^{(r)}}\right)^2+\sum_{a=1}^{N}\left(\frac{\alpha_{ua}^{(r)}}{\sigma_u^{(r)}}\right)^2\right\}$$

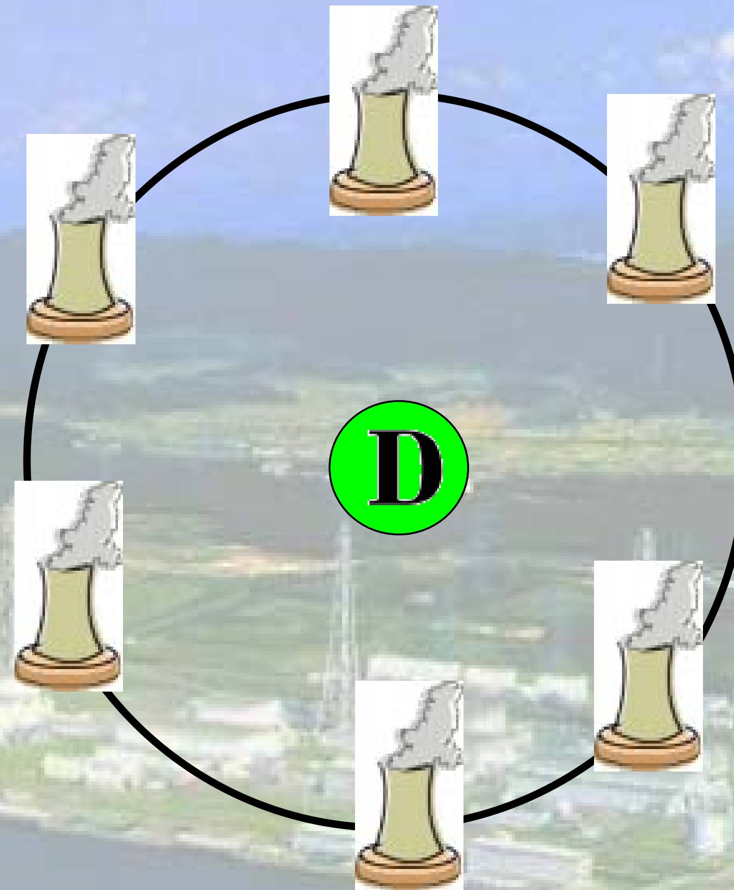
$$\sigma_{eff}^2=\sigma_u^2+\sigma_c^2+\left(\sigma_c^{(r)}\right)^2+\left(\sigma_u^{(r)}\right)^2\sum_{a=1}^{N}\left(\frac{T_a}{T}\right)^2$$



Assume equal yield from each reactor: Then

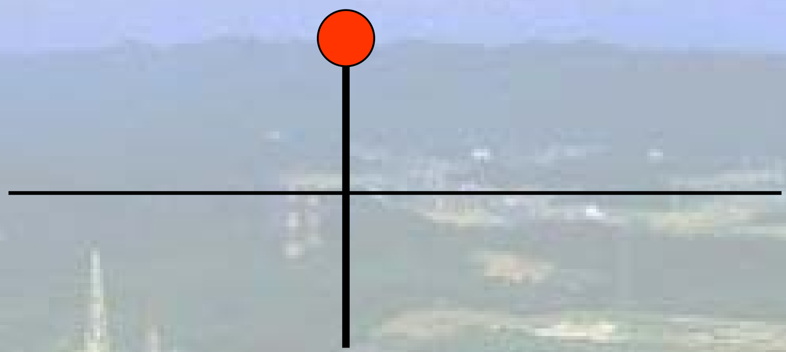
$$\frac{T_a}{T} = \frac{T_a}{\sum_{a=1}^{N} T_a} = \frac{1}{N}$$

$$\sum_{a=1}^{N} \left(\frac{T_a}{T} \right)^2 = \sum_{a=1}^{N} \frac{1}{N^2} = \frac{1}{N}$$

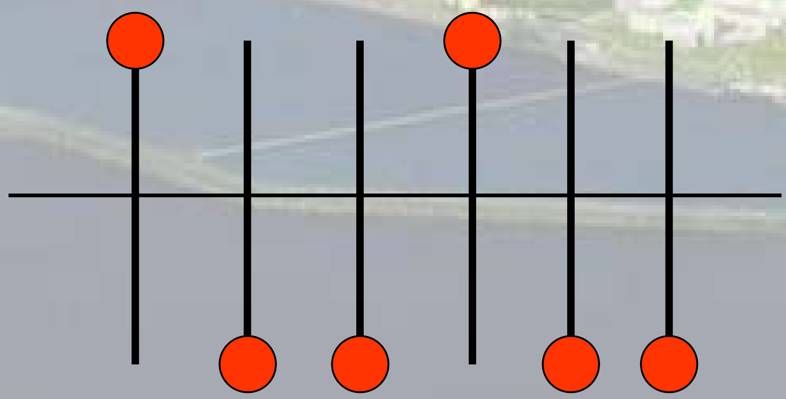


$$\sigma_{eff}^2 = \sigma_u^2 + \sigma_c^2 + \left(\sigma_c^{(r)}\right)^2 + \frac{1}{N}\left(\sigma_u^{(r)}\right)^2$$

Independent fluctuations of N variables



N=1: Fluctuation is large



N>1: Fluctuation is small in average

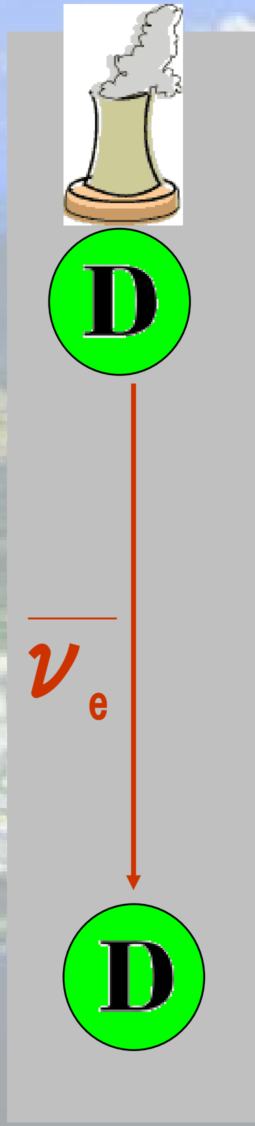
II. Multi detectors + multi reactors

(1) 2 detectors + 1 reactor

$$\chi^2 = \min_{\alpha's} \left\{ \left[\frac{M^n - T^n(1+\alpha_c+\alpha_c^{(r)}+\alpha_u^{(r)})}{T^n\sigma_u} \right]^2 + \left[\frac{M^f - T^f(1+\alpha_c+\alpha_c^{(r)}+\alpha_u^{(r)})}{T^f\sigma_u} \right]^2 + \left(\frac{\alpha_c}{\sigma_c}\right)^2 + \left(\frac{\alpha_c^{(r)}}{\sigma_c^{(r)}}\right)^2 + \left(\frac{\alpha_u^{(r)}}{\sigma_u^{(r)}}\right)^2 \right\}$$

$$= (y^n, \quad y^f) V^{-1} \begin{pmatrix} y^n \\ y^f \end{pmatrix}, \qquad y^n \equiv \frac{M^n - T^n}{T^n}, \quad y^f \equiv \frac{M^f - T^f}{T^f},$$

$$V \equiv \begin{pmatrix} \sigma_u^2 + \sigma_c^2 + (\sigma_c^{(r)})^2 + (\sigma_u^{(r)})^2 & \sigma_c^2 + (\sigma_c^{(r)})^2 + (\sigma_u^{(r)})^2 \\ \sigma_c^2 + (\sigma_c^{(r)})^2 + (\sigma_u^{(r)})^2 & \sigma_u^2 + \sigma_c^2 + (\sigma_c^{(r)})^2 + (\sigma_u^{(r)})^2 \end{pmatrix}$$



After diagonalization of V

$$\chi^2 = \frac{\left(y^f - y^n\right)^2}{2\,\sigma_u^2} + \frac{\left(y^f + y^n\right)^2}{2\,\sigma_u^2 + 4\left[\sigma_c^2 + (\sigma_c^{(r)})^2 + (\sigma_u^{(r)})^2\right]}$$

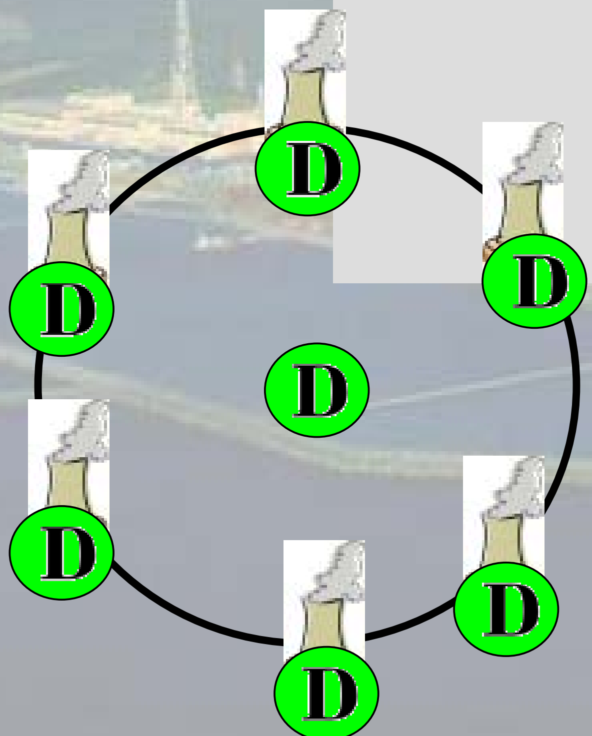
No correlated error

$$\sigma_{eff} \cong \sqrt{2}\sigma_u \left[1 + \frac{1}{1 + 2\left(\sigma_c^2 + \left(\sigma_c^{(r)}\right)^2 + \left(\sigma_u^{(r)}\right)^2\right) / \sigma_u^2}\right]^{-1/2}$$

σ_{eff} is dominated by the uncorrelated error

(2) (N+1) detectors+N reactors

$$\chi^2 = \min_{\alpha' s} \left\{ \sum_{i=1}^{N+1} \left[\frac{M^i - T^i\left(1+\alpha_c + \alpha_c^{(r)} + \sum_{a=1}^{N} \left(\frac{T_a^i}{T^i}\right)\alpha_{ua}^{(r)}\right)}{T^i \sigma_u} \right]^2 + \left(\frac{\alpha_c}{\sigma_c}\right)^2 + \left(\frac{\alpha_c^{(r)}}{\sigma_c^{(r)}}\right)^2 + \sum_{a=1}^{N}\left(\frac{\alpha_{ua}^{(r)}}{\sigma_u^{(r)}}\right)^2 \right\}$$



$$\sigma_{eff} = \sqrt{1+\frac{1}{N}}\,\sigma_u \left[1 + \frac{1/N}{\underbrace{1+(N+1)\left(\sigma_c^2 + \left(\sigma_c^{(r)}\right)^2 + \left(\sigma_u^{(r)}\right)^2/N\right)/\,\sigma_u^2}_{\text{small}}} \right]^{-1/2}$$

$(1+1/N)^{1/2}$: σ_{eff} improves as $N \rightarrow \infty$
(more info w/ more detectors)

$1/N$: slightly better from cancellation of uncorrelated error due to multi reactors

(3) Ideal limit of Kashiwazaki-Kariwa

(3 detectors+2 reactors)

$P=3P_0$

$$\chi^2 = \min_{\alpha\text{'s}} \left\{ \sum_{i=1}^{3} \left[\frac{M^i - T^i\left(1+\alpha_c + \alpha_c^{(r)} + \sum_{a=1}^{2}\left(\frac{T_a^i}{T^i}\right)\alpha_{ua}^{(r)}\right)}{T^i \sigma_u} \right]^2 + \left(\frac{\alpha_c}{\sigma_c}\right)^2 + \left(\frac{\alpha_c^{(r)}}{\sigma_c^{(r)}}\right)^2 + \sum_{a=1}^{2}\left(\frac{\alpha_{ua}^{(r)}}{\sigma_u^{(r)}}\right)^2 \right\}$$

$$\sigma_{eff} \simeq \frac{\sqrt{74}}{7}\sigma_u$$

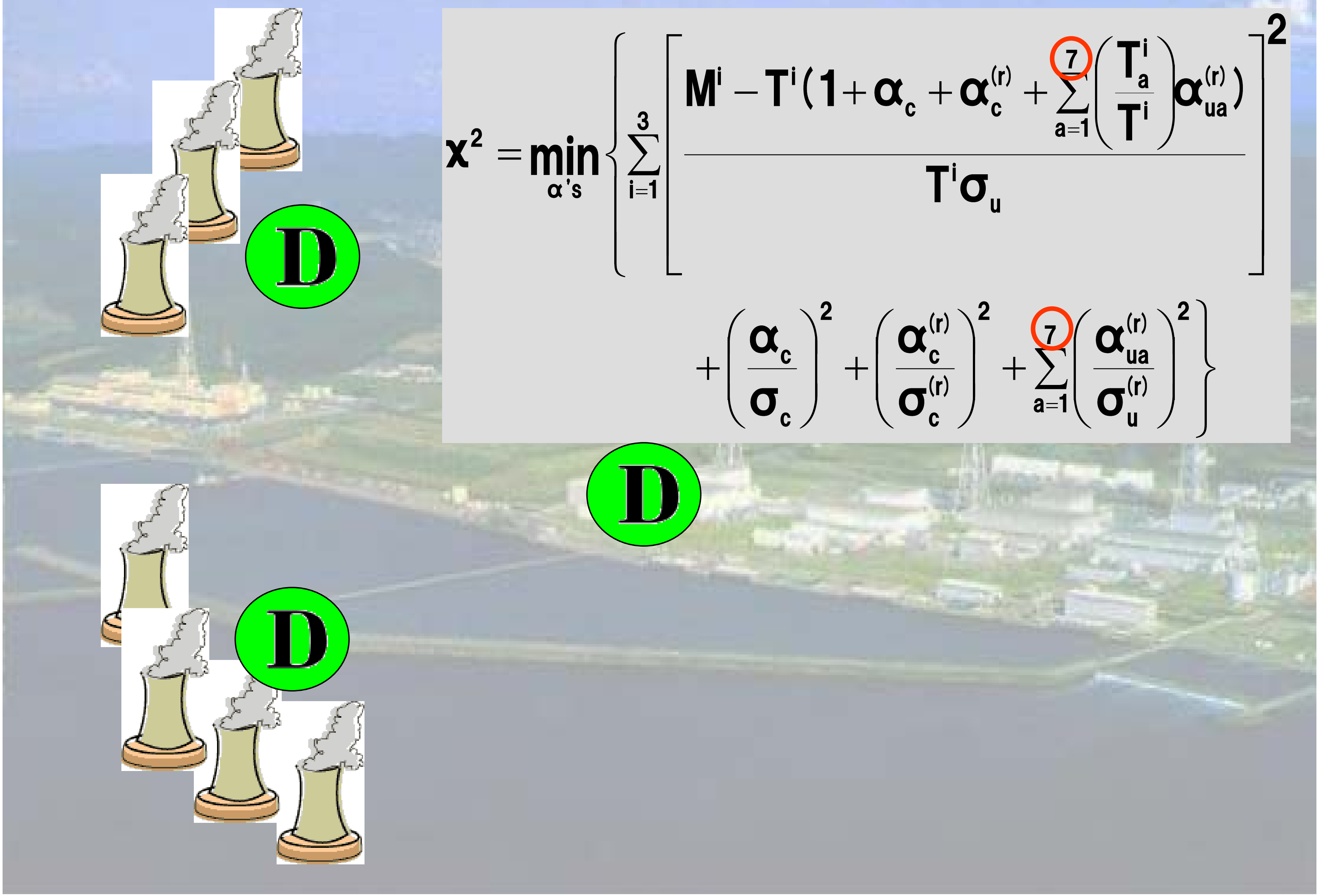
L_f=1.3km
L_n=0.3km

$$\left(\sin^2 2\theta\right)_{limit}^{sys-only} = \frac{\sqrt{2.7}}{0.64}\sigma_{eff} = 0.018$$

$P=4P_0$

(4) Actual Kashiwazaki-Kariwa

$$\chi^2 = \min_{\alpha\text{'s}} \left\{ \sum_{i=1}^{3} \left[\frac{M^i - T^i\left(1 + \alpha_c + \alpha_c^{(r)} + \sum_{a=1}^{7} \left(\frac{T_a^i}{T^i}\right) \alpha_{ua}^{(r)}\right)}{T^i \sigma_u} \right]^2 + \left(\frac{\alpha_c}{\sigma_c}\right)^2 + \left(\frac{\alpha_c^{(r)}}{\sigma_c^{(r)}}\right)^2 + \sum_{a=1}^{7} \left(\frac{\alpha_{ua}^{(r)}}{\sigma_u^{(r)}}\right)^2 \right\}$$



$$\frac{\sigma_{eff}\big|_{actual\ \ KK}}{\sigma_{eff}\big|_{ideal\ \ KK}} = 1-7\times10^{-3}$$

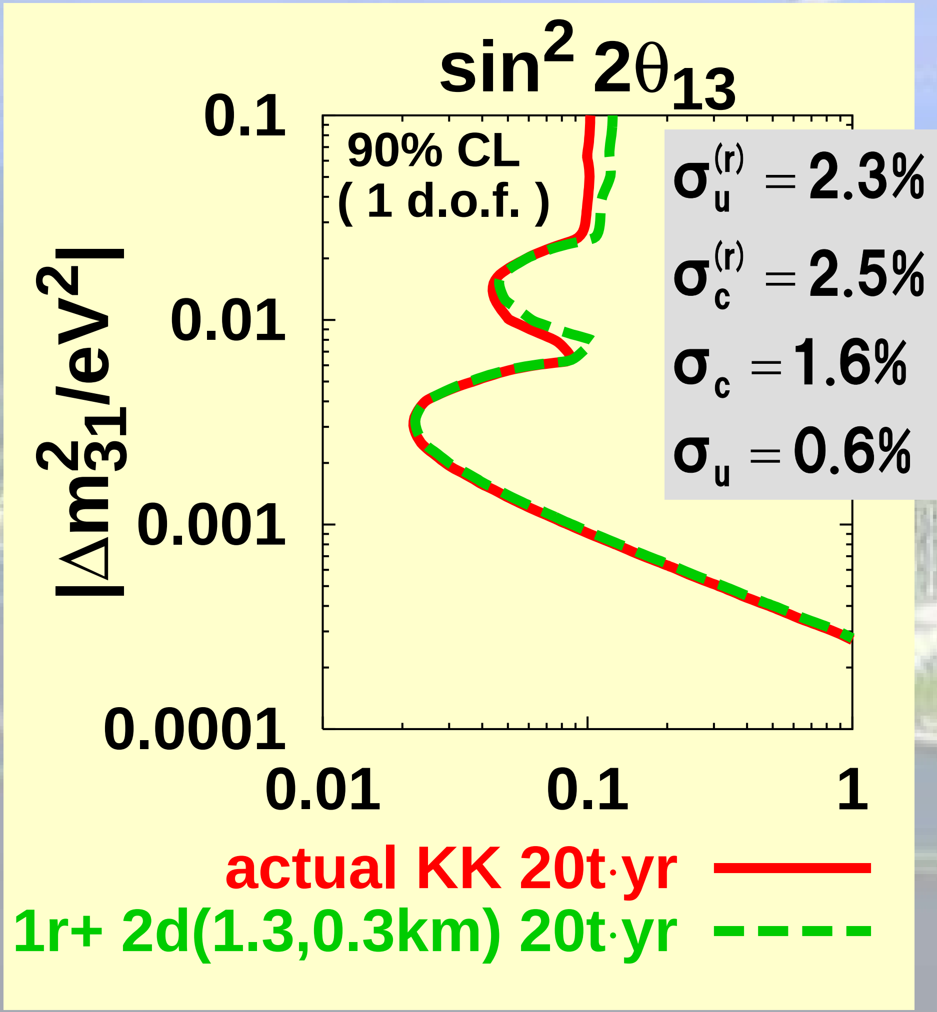
Therefore also for actual KK

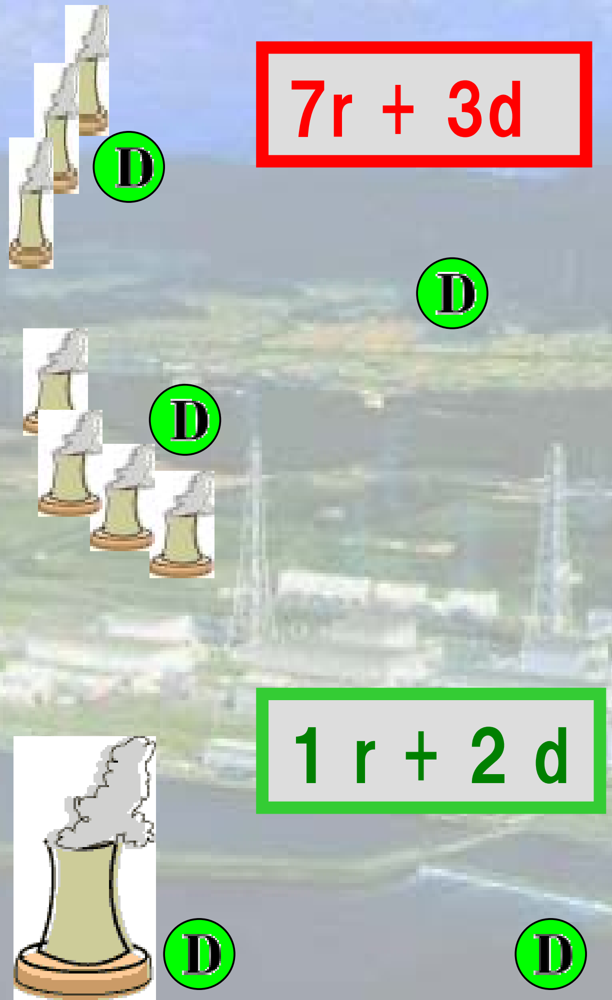
$$\boxed{\left(\sin^2 2\theta\right)^{sys-only}_{limit} = \frac{\sqrt{2.7}}{0.64}\sigma_{eff} = \mathbf{0.018}}$$

σ_{eff} of actual KK plan is almost the same as that of the ideal case.

It is slightly better because of 5 more reactors.

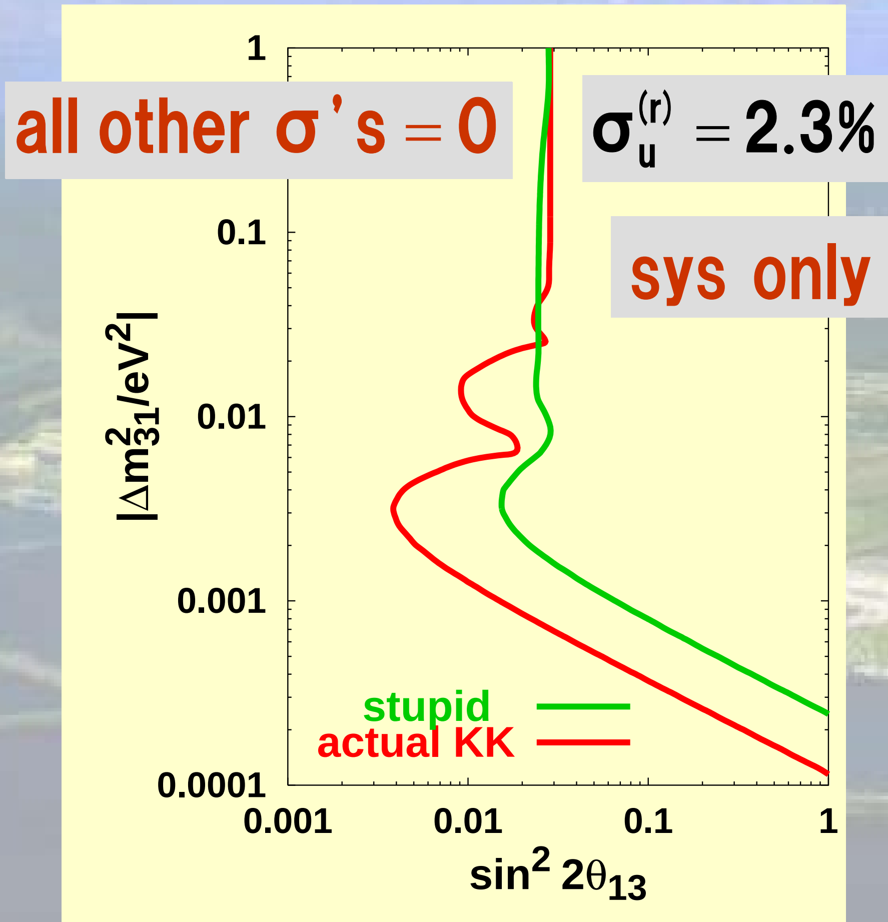
Comparison of (1reactor+2detectors) and KK

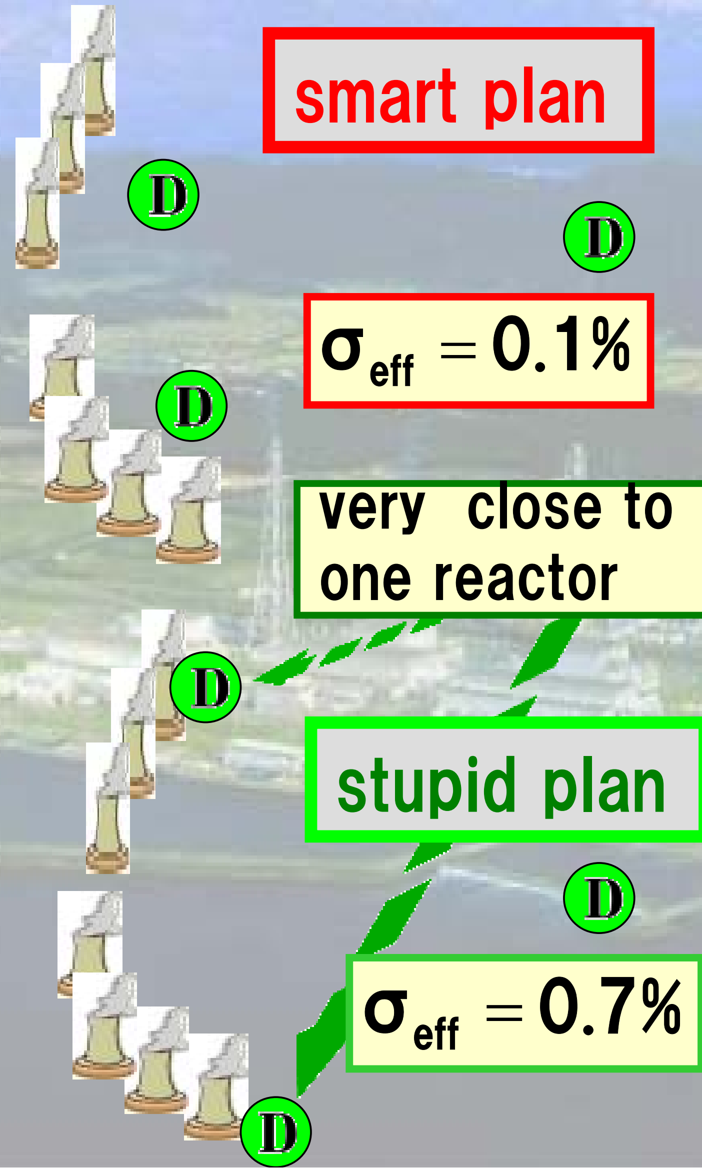




KK is slightly better because of one more detector.

Comparison of locations of near detectors



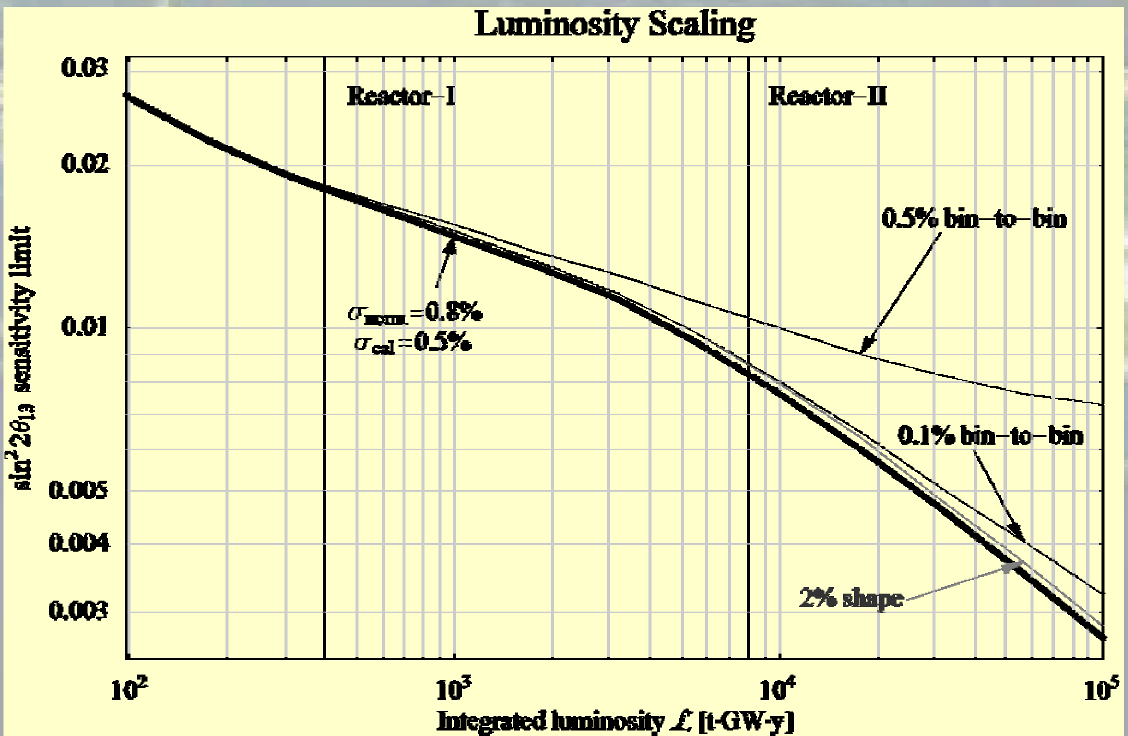


KK plan is almost optimal.

III. Energy spectrum (with H. Sugiyama)

Examination of the result in
Huber-Lindner-Schwetz -Winter, NP B665, 487 ('03)

By building Super KamLAND, is it really possible to reach $\sin^2 2\theta_{13} \sim 0.003$?



2 detectors + 1 reactor

$$\chi^2 = \min_{\alpha's}\left\{\sum_j\left[\frac{M_j^n - T_j^n(1+\alpha^{(r)}+\alpha_j+\alpha_n)}{T_j^n\sigma_u}\right]^2 + \sum_j\left[\frac{M_j^f - T_j^f(1+\alpha^{(r)}+\alpha_j+\alpha_f)}{T_j^f\sigma_u}\right]^2 + \left(\frac{\alpha^{(r)}}{\sigma^{(r)}}\right)^2 + \sum_j\left(\frac{\alpha_j}{\sigma_{shape}}\right)^2 + \left(\frac{\alpha_n}{\sigma_c}\right)^2 + \left(\frac{\alpha_f}{\sigma_c}\right)^2\right\}$$

$$\sigma^{(r)} \leftarrow \sigma_a, \quad \sigma_c \leftarrow \sigma_b$$
$$\sigma_u \leftarrow \sigma_{exp}, \quad \sigma_{shape} \leftarrow \sigma_{shape}$$

※Error of energy calibration gives little contribution and omitted for simplicity.

Assumptions: Uncorrelated bin-to-bin error is independent of bin: $\sigma_{uj} = \sigma_u$

$n = \#(\text{bins})$

$$\sigma_{eff} \cong \sqrt{\frac{a_n}{n}}\,\sigma_u \left[1 + \frac{\sigma_u^2}{\sigma_u^2 + 2\sigma_{shape}^2} + b_n \left(\frac{\sigma_u^2}{\sigma_u^2 + n\sigma_c^2} + \frac{\sigma_u^2}{\sigma_u^2 + 2\sigma_{shape}^2 + n\sigma_c^2 + 2n(\sigma^{(r)})^2} \right) \right]^{-\frac{1}{2}}$$

n	a_n	b_n
2	151	44
4	76	20
6	60	15
8	51	12
16	41	10
≧30	～40	～9

For simplicity, assume: $\sigma_{shape} \to 0, \quad \sigma^{(r)} >> \sigma_c, \sigma_u$

$$\sigma_{eff} \cong \sqrt{\frac{a_n}{2n}}\,\sigma_u \left[1 + \frac{b_n}{2}\frac{\sigma_u^2}{\sigma_u^2 + n\sigma_c^2}\right]^{-\frac{1}{2}}$$

Consider two cases:

(i) $\sigma_c = \sigma_u = 0.6\%$

$$\to \sigma_{eff} \cong \sqrt{\frac{a_n}{2n}}\,\sigma_u \left(1 + \frac{b_n/2}{n+1}\right)^{-\frac{1}{2}}$$

Contribution of σ_c is not negligible for n<10.

n=# (bin)	$(a_n / 2n)^{1/2}$	$b_n / (2n+2)$
2	6.2	7.3
4	3.1	2.0
6	2.2	1.1
8	1.8	0.7
16	1.1	0.3
62	0.6	0.1

(ii) $\sigma_u = 0.1\% \ << \ \sigma_c = 0.6\%$

$$\to \sigma_{eff} \cong \sqrt{\frac{a_n}{2n}}\,\sigma_u \xrightarrow{n=62} \left(\sin^2 2\theta\right)^{\text{sys only}}_{\text{limit}} = \sqrt{2.7}\,\sigma_{eff} \simeq 0.0009 \ !?$$

σ_{eff} is determined by σ_u .

In any case, unless $\sigma_u >> \sigma_c$ is satisfied, σ_u gives dominant contribution to σ_{eff} & $\left(\sin^2 2\theta\right)^{sys\ only}_{limit}$

Realistic σ_u has to be estimated carefully.

IV. Summary

(1) With 1 detector: limit on $\sin^2 2\theta$ is dominated by the correlated error, but the uncorrelated error from reactors decreases with multi reactors. (N=#(reactors))

$$\left(\sin^2 2\theta\right)^{\text{sys-only}}_{\text{limit}} = \frac{\sqrt{2.7}}{0.8}\sqrt{\sigma_u^2 + \sigma_c^2 + (\sigma_u^{(r)})^2/N}$$

(2) With multiple detectors: limit on $\sin^2 2\theta$ is dominated by the uncorrelated error, and #(reactors) is irrelevant. (N=#(detectors))

$$\left(\sin^2 2\theta\right)^{\text{sys-only}}_{\text{limit}} = \frac{\sqrt{2.7}}{0.8}\sqrt{1+\frac{1}{N}}\,\sigma_u\,(1+\text{small correction})$$

(2') Kashiwazaki plan does not have any disadvantage over plans w/ 1 or 2 reactor(s).

$$\left(\sin^2 2\theta\right)^{sys-only}_{limit} = \frac{\sqrt{2.7}}{0.64}\sigma_{eff} = 0.018$$

$$\left(\sin^2 2\theta\right)_{sens.} = \frac{\sqrt{2.7}}{0.64}\sqrt{\sigma^2_{eff} + \sigma^2_{stat}} = 0.020$$

For 20ton•yr: $\sigma^2_{stat} = 1/60000 = (0.4\%)^2$

(3) The energy spectrum analysis:

For larger n≡# (bins), limit on $\sin^2 2\theta$ is dominated by the uncorrelated bin-to-bin error σ_u, and one has to estimate realistic σ_u.

$$\left(\sin^2 2\theta\right)_{\text{limit}}^{\text{sys-only}} = \frac{\sqrt{2.7}}{0.8}\sqrt{\frac{a_n}{n}}\,\sigma_u\left[1+\cdots\right]$$

Appendix

$$V_{ideal} = \sigma_u^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + [\sigma_c^2 + (\sigma_c^{(r)})^2] \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$+ (\sigma_u^{(r)})^2 \begin{pmatrix} 1/4 & 0 & 1/7 \\ 0 & 1/3 & 1/7 \\ 1/7 & 1/7 & 1/7 \end{pmatrix}$$

$$V_{actual} = \sigma_u^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + [\sigma_c^2 + (\sigma_c^{(r)})^2] \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$+ (\sigma_u^{(r)})^2 \begin{pmatrix} 0.221 & 0.041 & 0.149 \\ 0.041 & 0.291 & 0.134 \\ 0.149 & 0.134 & 0.144 \end{pmatrix}$$