Parameter degeneracy and reactor neutrino experiments

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1. Parameter degeneracy in $(s^2_{23}, \sin^2 2\theta_{13})$ plane
2. Resolution of $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$ degeneracy by LBL $\oplus$ reactor
3. Summary

1. Parameter degeneracy in $(s_{23}^2, \sin^2 2\theta_{13})$ plane

Even if $P \equiv P(\nu_\mu \to \nu_e)$ and $\bar{P} \equiv P(\bar{\nu}_\mu \to \bar{\nu}_e)$ are given, there are in general 8 solutions.

3 kinds of degeneracy

- intrinsic (δ, θ_{13}) — Burguet-Castell et al ('01)
- sign (Δm_{31}^2) — Minakata-Nunokawa ('01)
- $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$ — Fogli-Lisi PRD54 ('96) 3667; Barger-Marfatia-Whisnant ('02)

$\boxed{\text{8-fold degeneracy}}$

Here I assume that accelerator beams are approximately monchromatic.

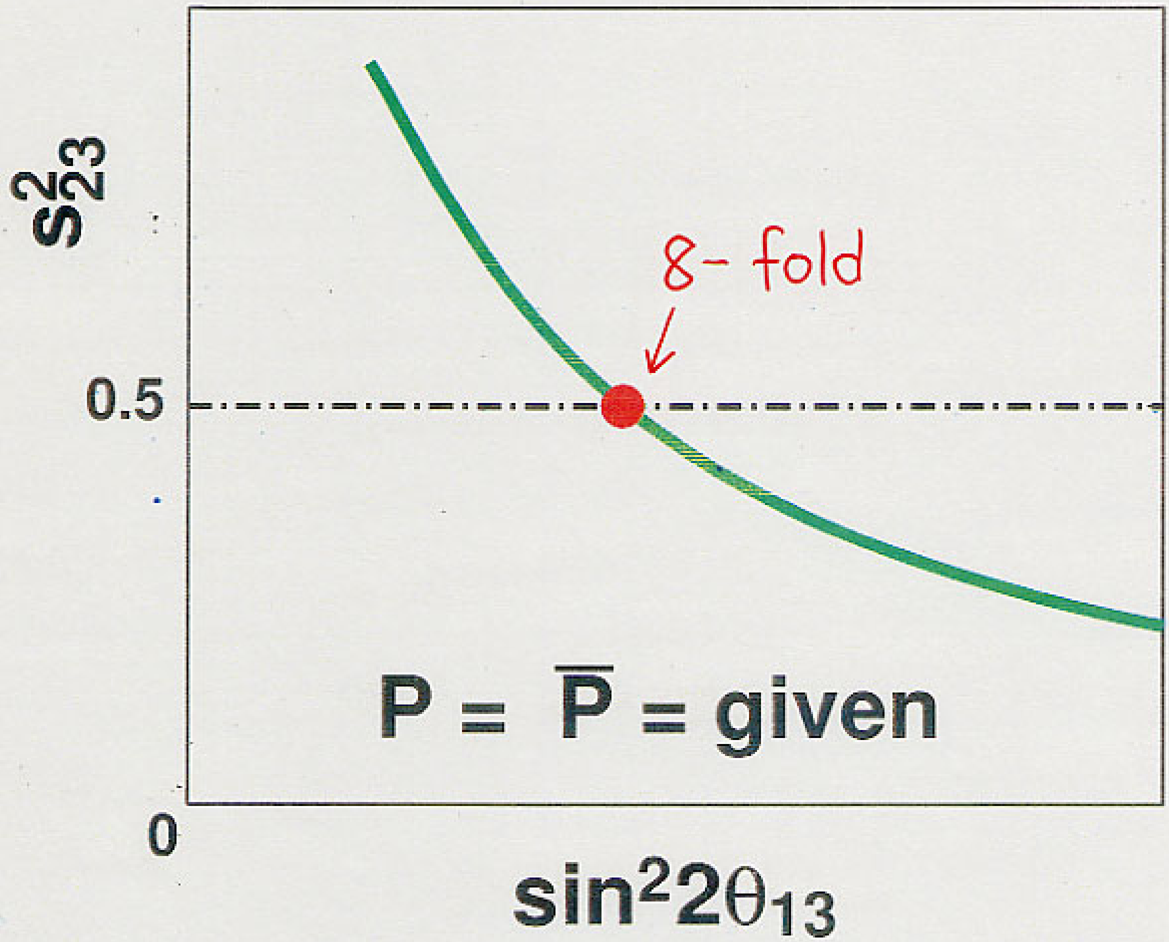
Experimental errors in long baseline experiments are not taken into account.

Here I visualize the 8-fold degeneracy by using the $(s_{23}^2, \sin^2 2\theta_{13})$ plane step by step.

$\epsilon \equiv \dfrac{\Delta m_{21}^2}{\Delta m_{31}^2}$ $\quad \dfrac{AL}{2}$

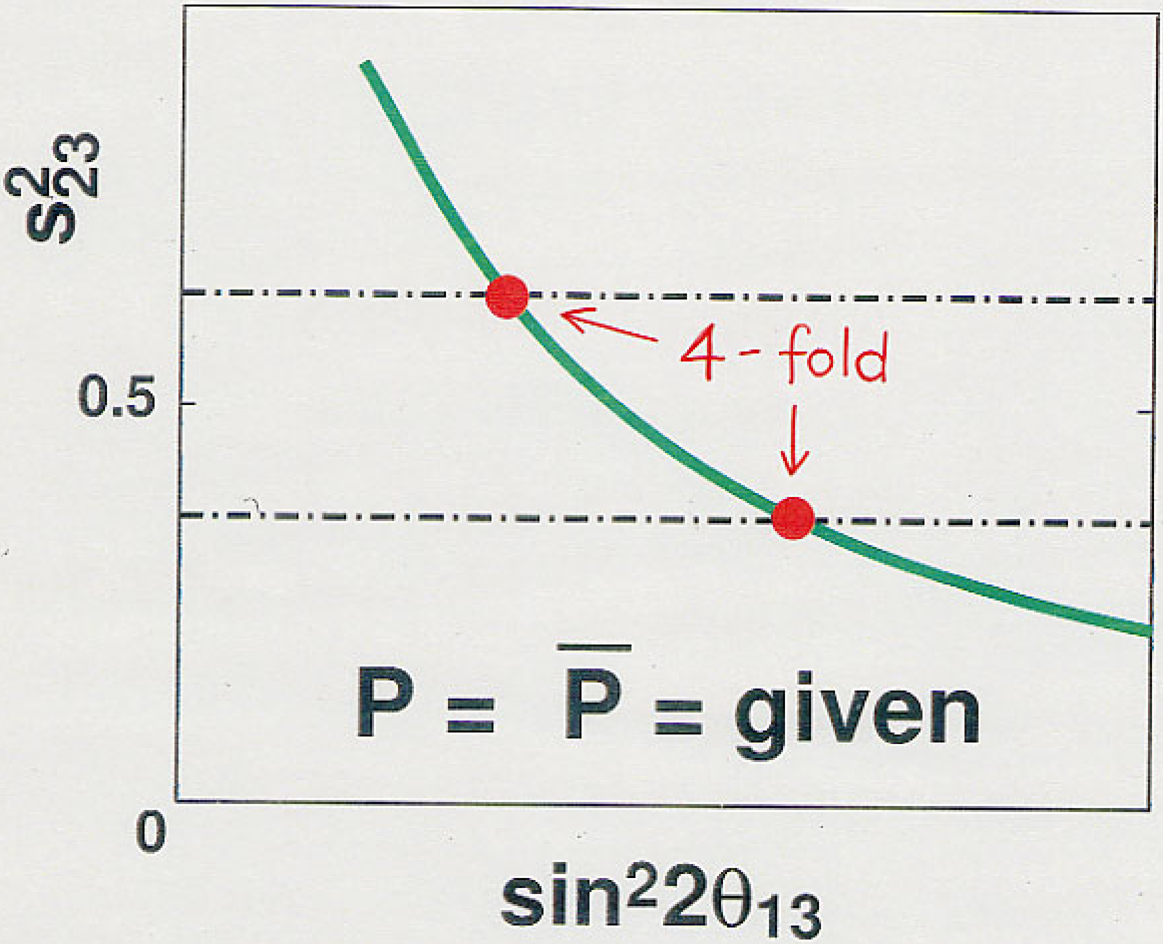
	$\theta_{23}-\frac{\pi}{4}$	Δm_{21}^2	$A \equiv \sqrt{2} G_F N_e$	$\theta_{23} \Leftrightarrow \frac{\pi}{2}-\theta_{23}$	(δ, θ_{13})	$\mathrm{sign}(\Delta m_{31}^2)$
(a)	$=0$	$=0$	$=0$	degen.	degen.	degen.
(b)	$\neq 0$	$=0$	$=0$	lifted	degen.	degen.
(c)	$\neq 0$	$\neq 0$	$=0$	lifted	lifted	degen.
(d) off OM	$\neq 0$	$\neq 0$	$\neq 0$	lifted	lifted	lifted
(e) @OM	$\neq 0$	$\neq 0$	$\neq 0$	lifted	degen.	almost degen.

(a) $\theta_{23}=\frac{\pi}{4}$, $\Delta m^2_{21}=0$, $A=0$



$$P=\overline{P}=\underbrace{s_{23}^2}_{=\frac{1}{2}}\sin^2 2\theta_{13}\underbrace{\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right)}_{=\text{const}}$$

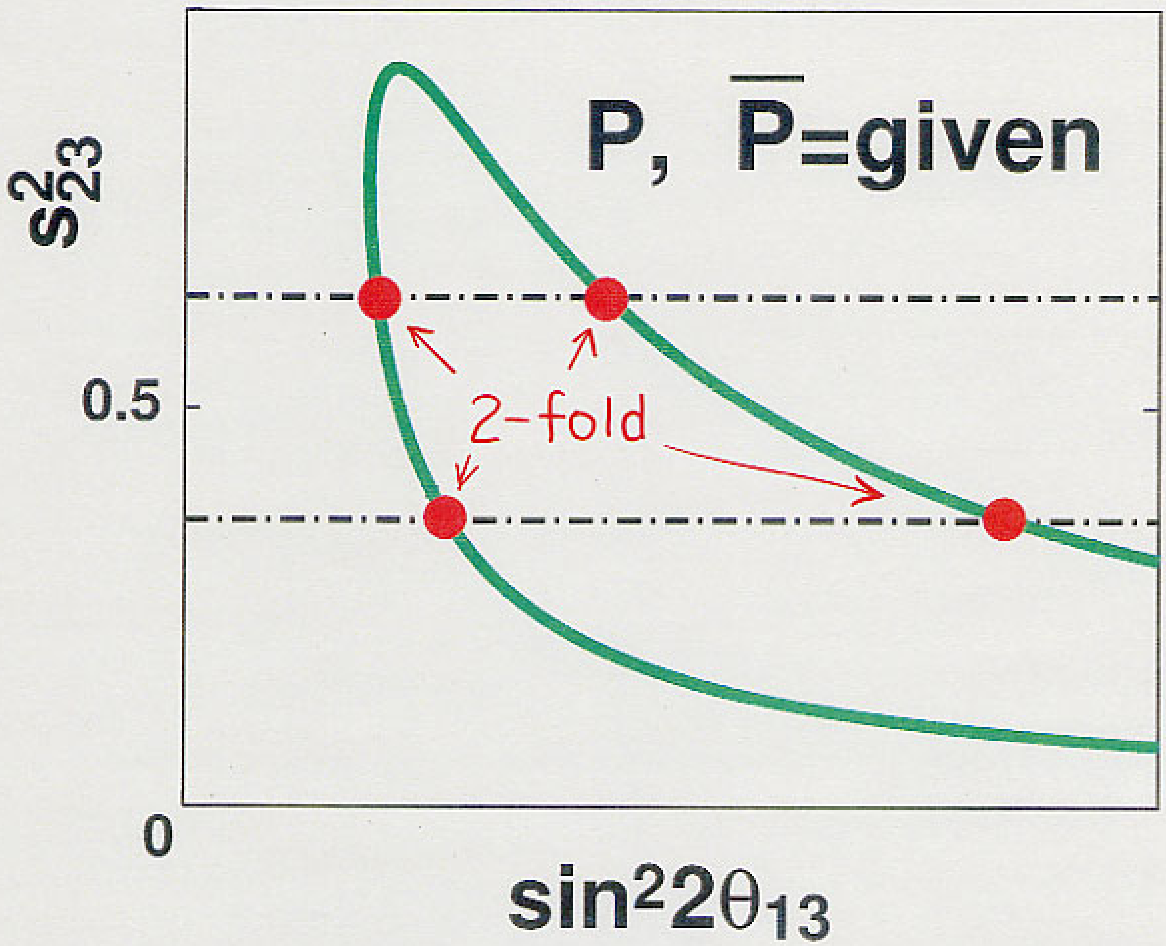
(b) $\theta_{23} \neq \frac{\pi}{4}$, $\Delta m^2_{21}=0$, $A=0$



$$P = \overline{P} = s_{23}^2 \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$s_{23}^2 = \frac{1 \pm \sqrt{1 - \sin^2 2\theta_{23}}}{2}$$ ← known from $\nu_\mu \to \nu_\mu$

(c) $\theta_{23} \neq \frac{\pi}{4}$, $\Delta m^2_{21} \neq 0$, A=0



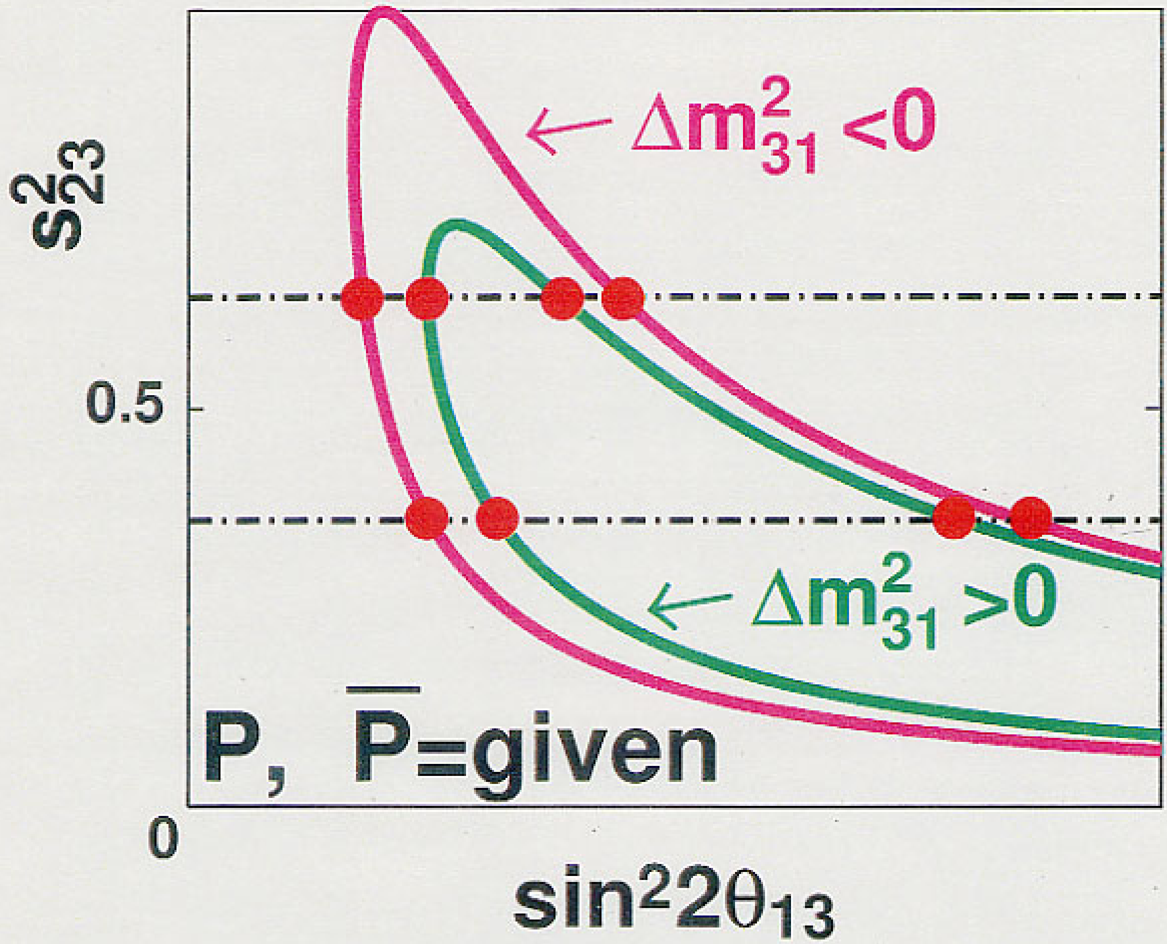
$$\frac{1}{\cos^2\Delta}\left(\frac{P+\bar{P}}{2} - x^2\sin^2\Delta - y^2\Delta^2\right)^2 + \frac{1}{\sin^2\Delta}\left(\frac{P-\bar{P}}{2}\right)^2 = (2xy\Delta\sin\Delta)^2$$

quadratic eq. in x^2

$$\left(\begin{array}{l} x \equiv s_{23}\sin 2\theta_{13} \\ y \equiv \left|\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right| c_{23}\sin 2\theta_{12} \\ \Delta \equiv \frac{\Delta m^2_{31} L}{4E} \end{array}\right)$$

$$s_{23} = \sqrt{\frac{1 \pm \sqrt{1-\sin^2 2\theta_{23}}}{2}}$$

(d) $\theta_{23} \neq \frac{\pi}{4}$, $\Delta m^2_{21} \neq 0$, $A \neq 0$ off OM



$$\frac{1}{4\cos^2\Delta}\left(\frac{P - x^2 f^{(\mp)2} - y^2 g^2}{f^{(\mp)}} + \frac{\bar{P} - x^2 f^{(\pm)2} - y^2 g^2}{f^{(\pm)}}\right)^2$$

$$+\frac{1}{4\sin^2\Delta}\left(\frac{P - x^2 f^{(\mp)2} - y^2 g^2}{f^{(\mp)}} - \frac{\bar{P} - x^2 f^{(\pm)2} - y^2 g^2}{f^{(\pm)}}\right)^2 = (2xyg)^2$$

for $\Delta m^2_{31} \begin{Bmatrix} >0 \\ <0 \end{Bmatrix}$

quadratic in x^2

$x \equiv s_{23} \sin 2\theta_{13}$

$y \equiv \left|\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right| c_{23} \sin 2\theta_{12}$

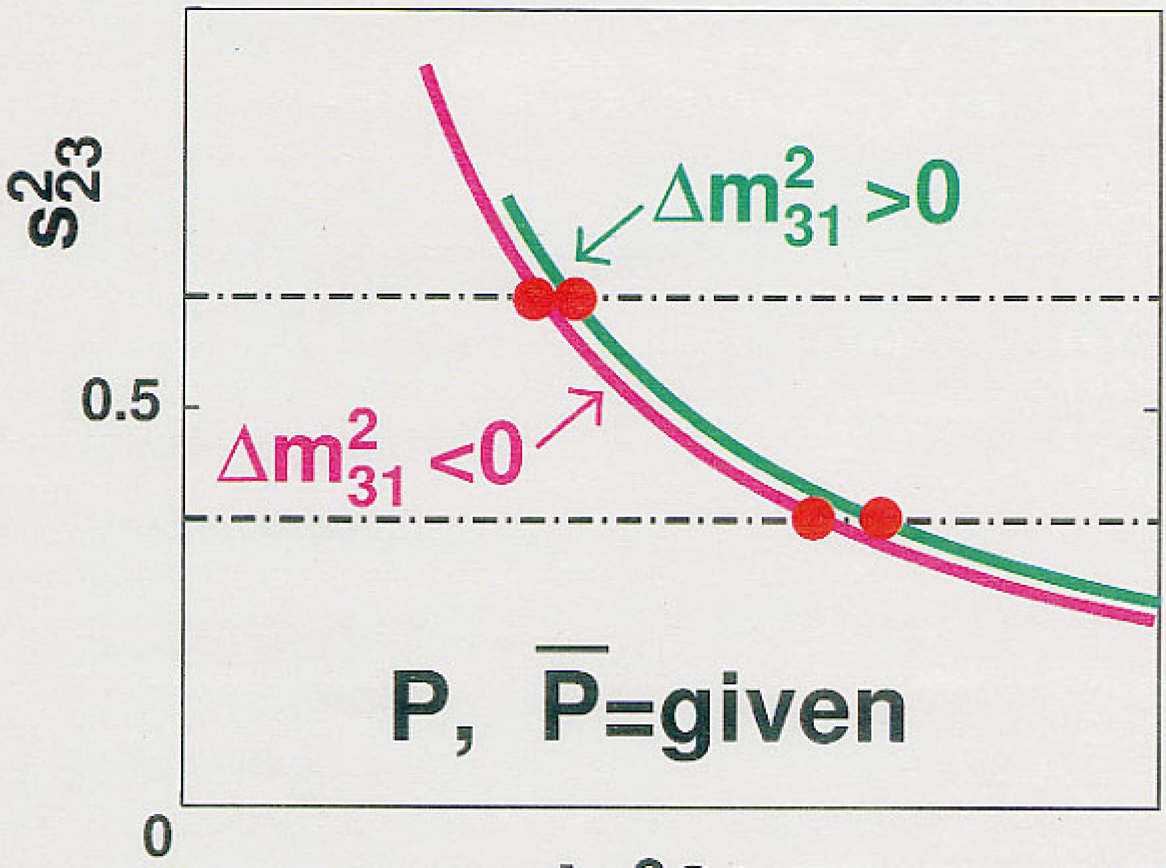
$\Delta \equiv \frac{\Delta m^2_{31} L}{4E}$

$f^{(\pm)} \equiv \frac{\sin(\Delta \pm AL/2)}{1 \mp AL/2\Delta}$

$g \equiv \frac{\sin(AL/2)}{AL/2\Delta}$

$$s_{23} = \sqrt{\frac{1 \pm \sqrt{1 - \sin^2 2\theta_{23}}}{2}}$$

(e) $\theta_{23} \neq \frac{\pi}{4}$, $\Delta m^2_{21} \neq 0$, $A \neq 0$ @OM $\left(\frac{\Delta m^2_{31} L}{4E} = \frac{\pi}{2}\right)$



$$\frac{P - x^2 f^{(\mp)2} - y^2 g^2}{f^{(\mp)}} = -\frac{\bar{P} - x^2 f^{(\pm)2} - y^2 g^2}{f^{(\pm)}} \quad \text{for } \Delta m^2_{31} \begin{Bmatrix} >0 \\ <0 \end{Bmatrix}$$

linear in x^2

$$x \equiv s_{23} \sin 2\theta_{13}$$

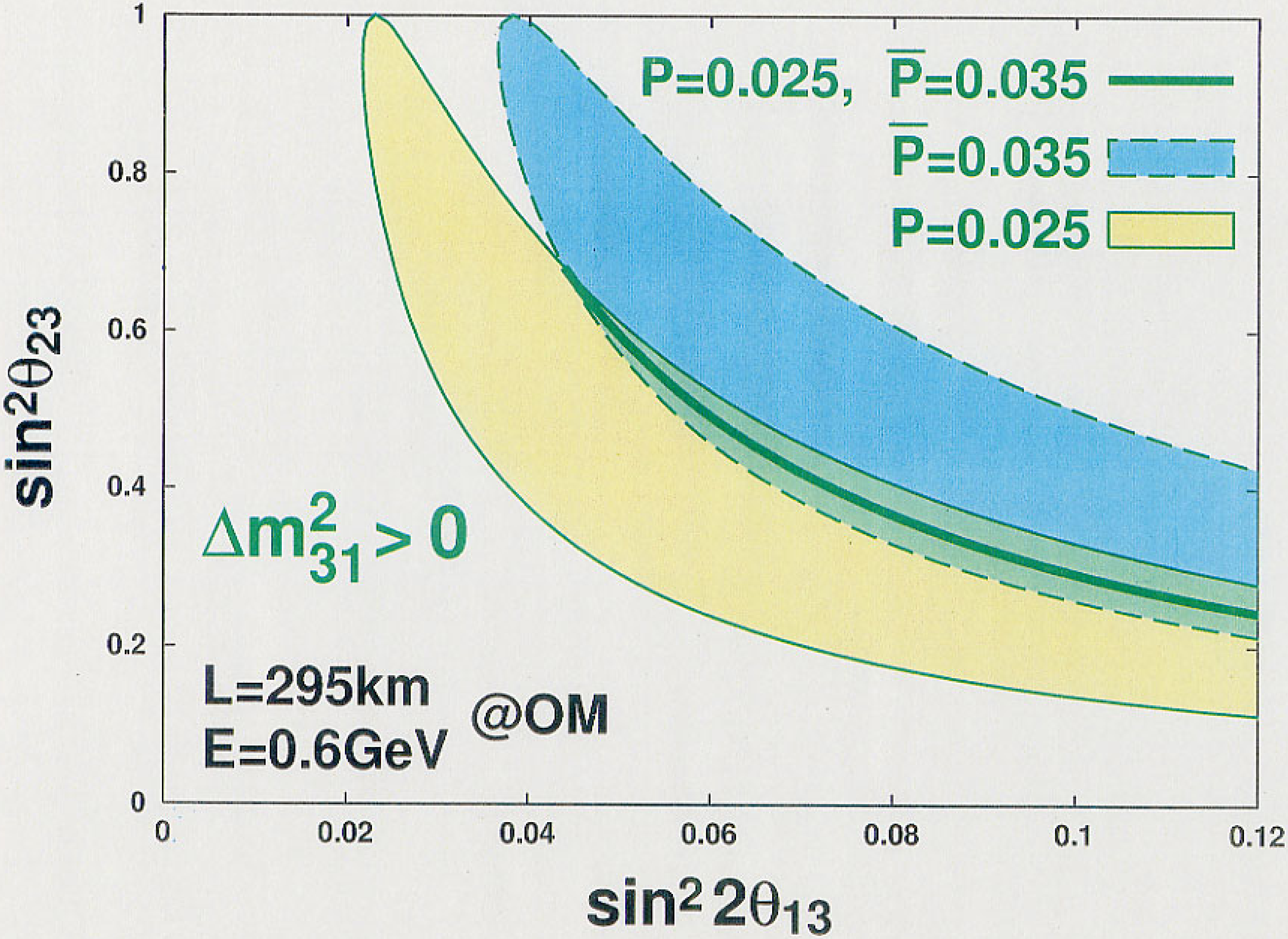
$$y \equiv \left|\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right| c_{23} \sin 2\theta_{12}$$

$$s_{23} = \sqrt{\frac{1 \pm \sqrt{1 - \sin^2 2\theta_{23}}}{2}}$$

$$f^{(\pm)} \equiv \frac{\cos(AL/2)}{1 \mp AL/\pi}$$

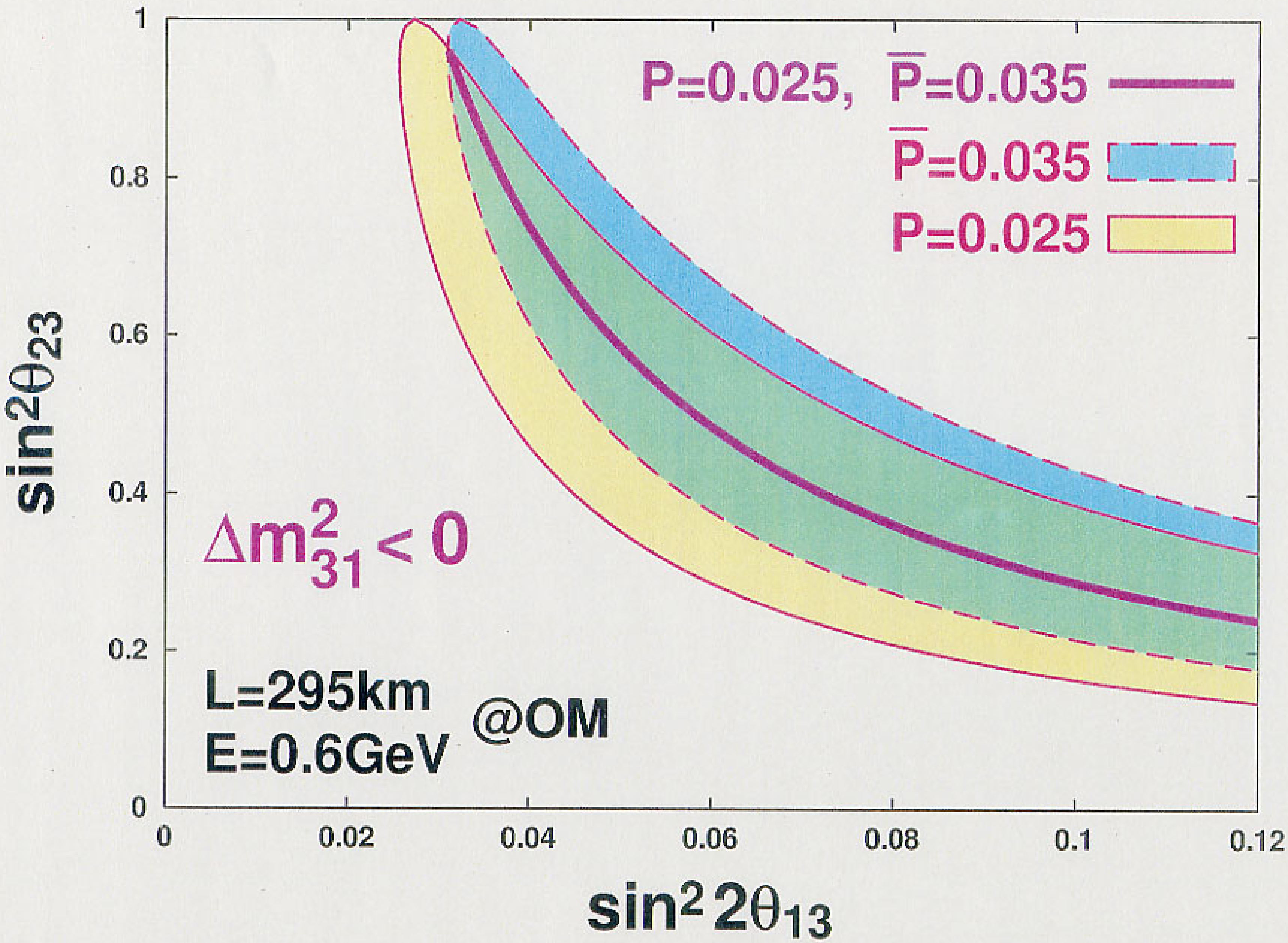
$$g \equiv \frac{\sin(AL/2)}{AL/\pi}$$

L9



$P=0.025,\ \bar{P}=0.035$
$\bar{P}=0.035$
$P=0.025$
$\Delta m^2_{31} > 0$
L=295km
E=0.6GeV
@OM
$\sin^2\theta_{23}$
$\sin^2 2\theta_{13}$
0
0.2
0.4
0.6
0.8
1
0.02
0.04
0.06
0.08
0.1
0.12

10


P=0.025, $\bar{P}$=0.035
$\bar{P}$=0.035
P=0.025
$\Delta m^2_{31} < 0$
L=295km
E=0.6GeV
@OM
$\sin^2\theta_{23}$
$\sin^2 2\theta_{13}$
0
0.2
0.4
0.6
0.8
1
0.02
0.04
0.06
0.08
0.1
0.12

@ Oscillation Maximum $\left(\frac{\Delta m^2_{31} L}{4E} = \frac{\pi}{2}\right)$

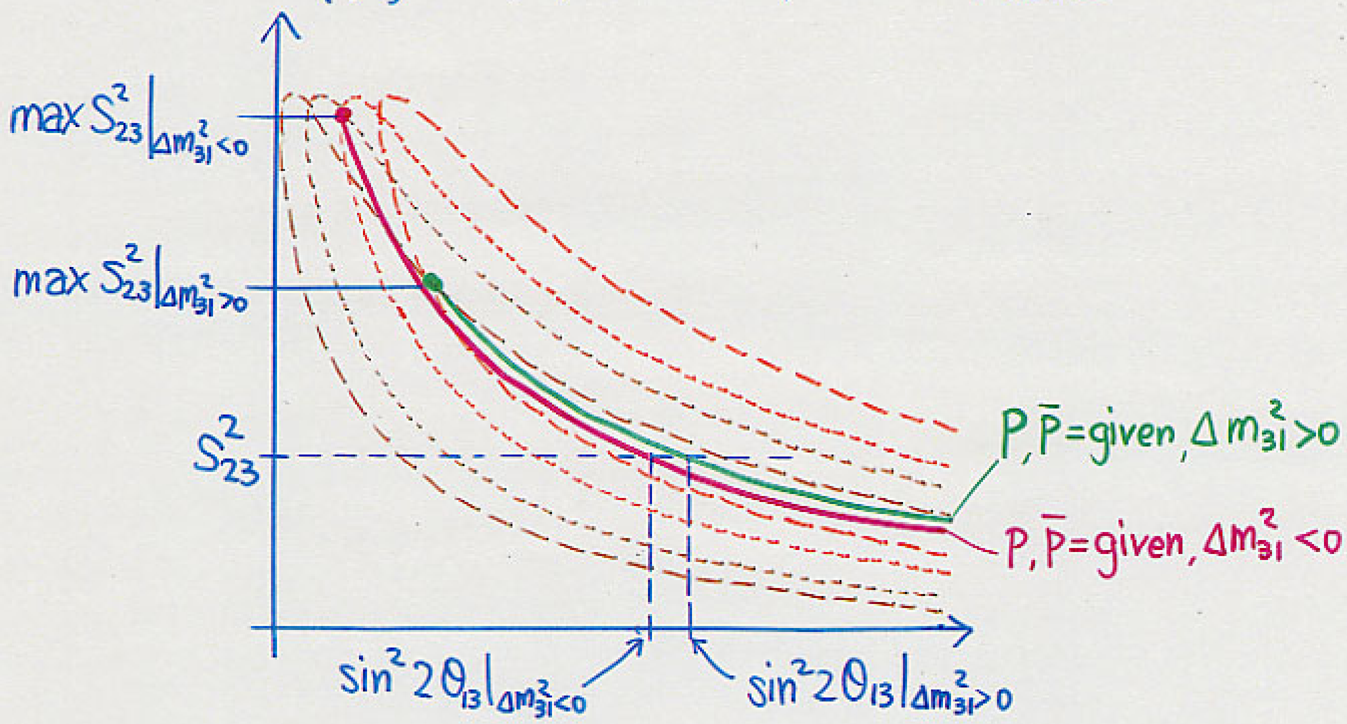
$$P \equiv P(\nu_\mu \to \nu_e) = x^2 f^2 - 2xy\, fg \sin\delta + y^2 g^2$$

$$\bar{P} \equiv P(\bar{\nu}_\mu \to \bar{\nu}_e) = x^2 \bar{f}^2 + 2xy\, \bar{f} g \sin\delta + y^2 g^2$$

where $x \equiv S_{23} \sin 2\theta_{13}$

$y \equiv \epsilon C_{23} \sin 2\theta_{12}$, $\epsilon \equiv \left|\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right|$

$\left\{\begin{matrix} f \\ \bar{f} \end{matrix}\right\} \equiv \frac{\cos(AL/2)}{1 \mp AL/\pi}$, $g \equiv \frac{\sin(AL/2)}{AL/\pi}$, $A \equiv \sqrt{2}\, G_F N_e$



When P & $\bar{P}$ are given, one can show

- $\sin^2 2\theta_{13}|_{\Delta m^2_{31}>0} - \sin^2 2\theta_{13}|_{\Delta m^2_{31}<0} = \frac{1}{S^2_{23}} \cdot \frac{1}{f\bar{f}} \cdot \frac{f-\bar{f}}{f+\bar{f}}(\bar{P}-P) \simeq \frac{AL}{S^2_{23}\pi}(\bar{P}-P) \sim 5\times10^{-4}$

  for any θ_{23} — if $|AL/2| \ll 1$

- $\max S^2_{23}|_{\Delta m^2_{31}<0} - \max S^2_{23}|_{\Delta m^2_{31}>0} = \frac{1}{\epsilon^2} \cdot \frac{1}{\sin^2 2\theta_{12}} \cdot \frac{1}{g^2} \frac{f-\bar{f}}{f+\bar{f}}(\bar{P}-P) \simeq \frac{1}{\epsilon^2} \frac{(2/\pi)^2}{\sin^2 2\theta_{12}} \cdot \frac{AL}{\pi}(\bar{P}-P)$

  ~ 0.3

if $P=0.025$, $\bar{P}=0.035$, $L=295$ km, $\epsilon = \frac{7\times10^{-5}\,\mathrm{eV}^2}{2.5\times10^{-3}\,\mathrm{eV}^2} \simeq \frac{1}{35}$

$A \simeq \frac{1}{1900\,\mathrm{km}}$

2. Resolution of $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$ degeneracy by LBL ⊕ reactor

Our scenario

- JHF $\nu \oplus \bar{\nu}$ @ Oscillation Maximum

⊕

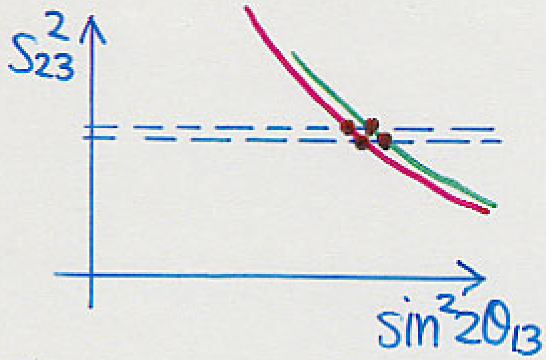
- reactor experiment (@ Kashiwazaki ?)

From $\nu_\mu \leftrightarrow \nu_\mu$ @JHF we will know that θ_{23} satisfies either of the followings:

(A) $|1 - \sin^2 2\theta_{23}| <$ a few $\times 10^{-2}$

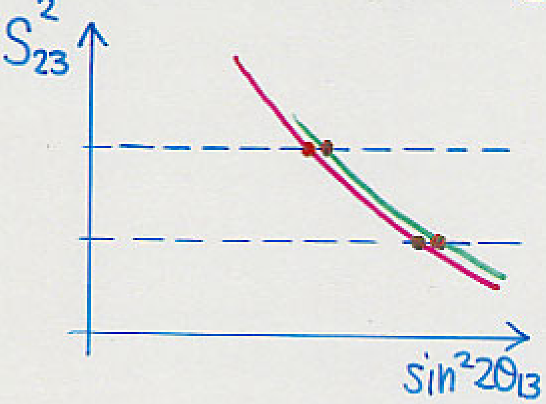
(B) $|1 - \sin^2 2\theta_{23}| \geq$ a few $\times 10^{-2}$

(A) with JHF $\nu \oplus \bar{\nu}$ @OM



The situation looks like the upper figure.
The precise determination of true $\sin^2 2\theta_{13}$ is difficult, but the values of $\sin^2 2\theta_{13}$ for the 4 solutions are approximately the same.

(B) with JHF $\nu \oplus \bar{\nu}$ @OM



The values of $\sin^2 2\theta_{13}$ for $\theta_{23} < \frac{\pi}{4}$ and $\theta_{23} > \frac{\pi}{4}$ are quite different and it may be possible to determine the true value of $\sin^2 2\theta_{13}$ if the error $\delta_{re}(\sin^2 2\theta_{13})$ of the reactor exp. is smaller than the ambiguity $\delta_{de}(\sin^2 2\theta_{13})$ due to the degeneracy.

Measurement of θ_{13} by reactors

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$\sin^2 2\theta_{13}$ is measured

with an experimental error

$\delta_{re}(\sin^2 2\theta_{13})$

without any ambiguity from θ_{23} & δ

reator measurement @Kashiwazaki-Kariwa

with $L = 1.7$ km, $\sigma_{sys} = 0.8\%$, 20t·yr

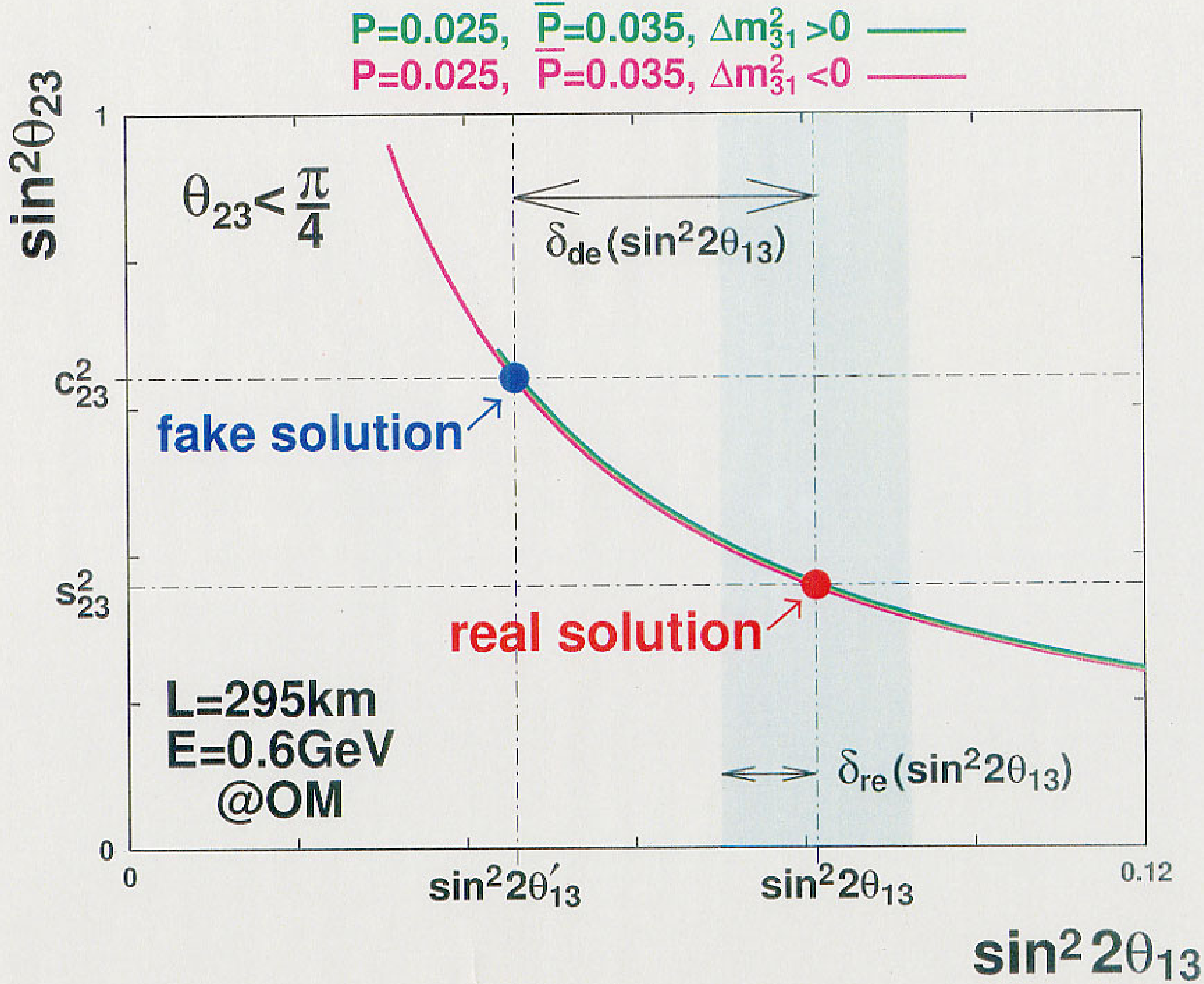
cf. Suekan-san's talk

$\delta_{re}(\sin^2 2\theta_{13}) \simeq 0.012$

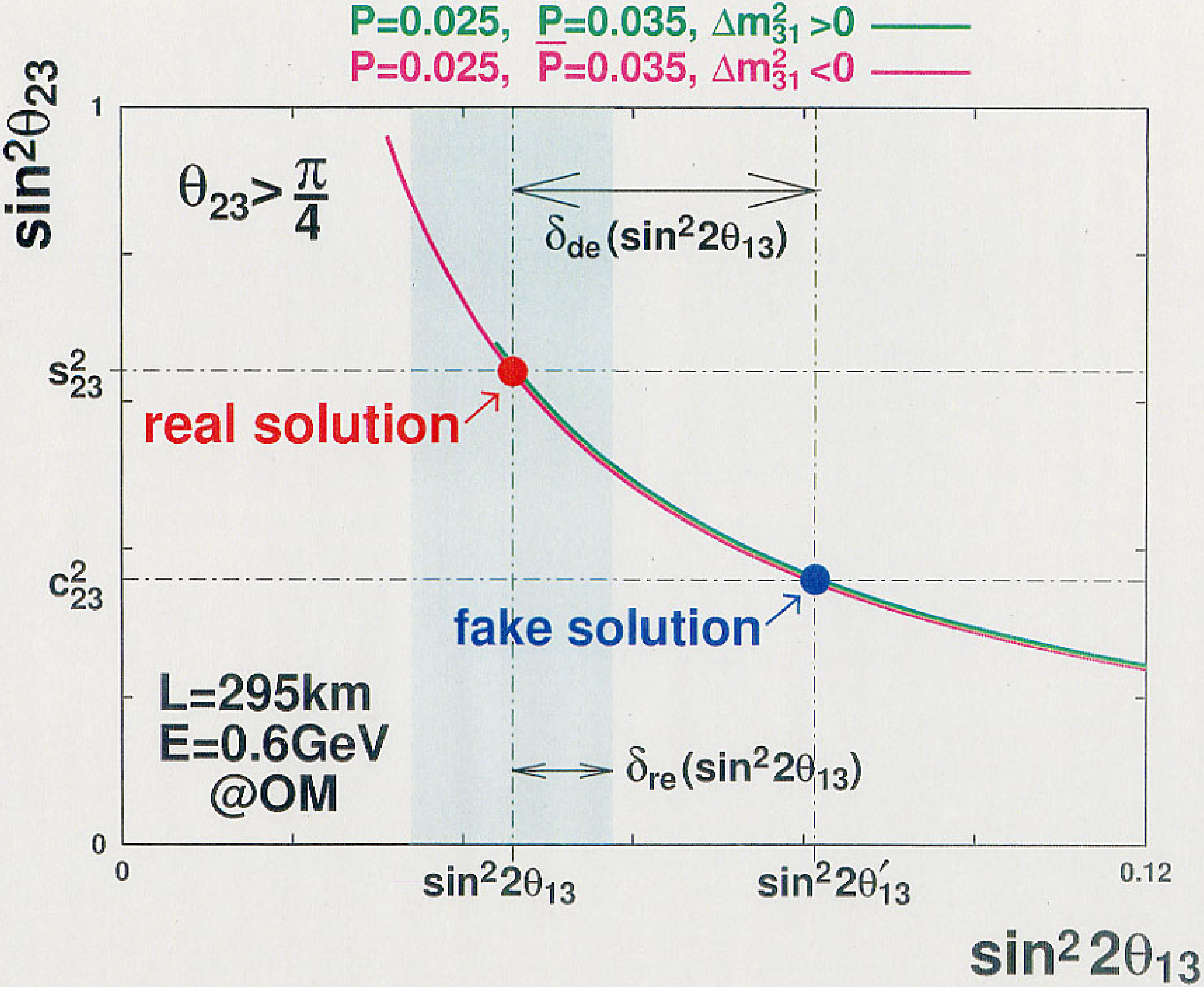
Minakata et al. hep-ph/0211111

d.o.f. = 1 ($\sin^2 2\theta_{13}$ only, assuming that $|\Delta m^2_{31}|$ is known from JHF)

In the following discussions, this value for $\delta_{re}(\sin^2 2\theta_{13})$ is used, but qualitative features do not change.



P=0.025, $\overline{P}$=0.035, $\Delta m^2_{31}>0$
P=0.025, $\overline{P}$=0.035, $\Delta m^2_{31}<0$
$\sin^2\theta_{23}$
1
$\theta_{23}<\frac{\pi}{4}$
$\delta_{de}(\sin^2 2\theta_{13})$
c^2_{23}
fake solution
s^2_{23}
real solution
L=295km
E=0.6GeV
@OM
$\delta_{re}(\sin^2 2\theta_{13})$
0
0
$\sin^2 2\theta'_{13}$
$\sin^2 2\theta_{13}$
0.12
$\sin^2 2\theta_{13}$



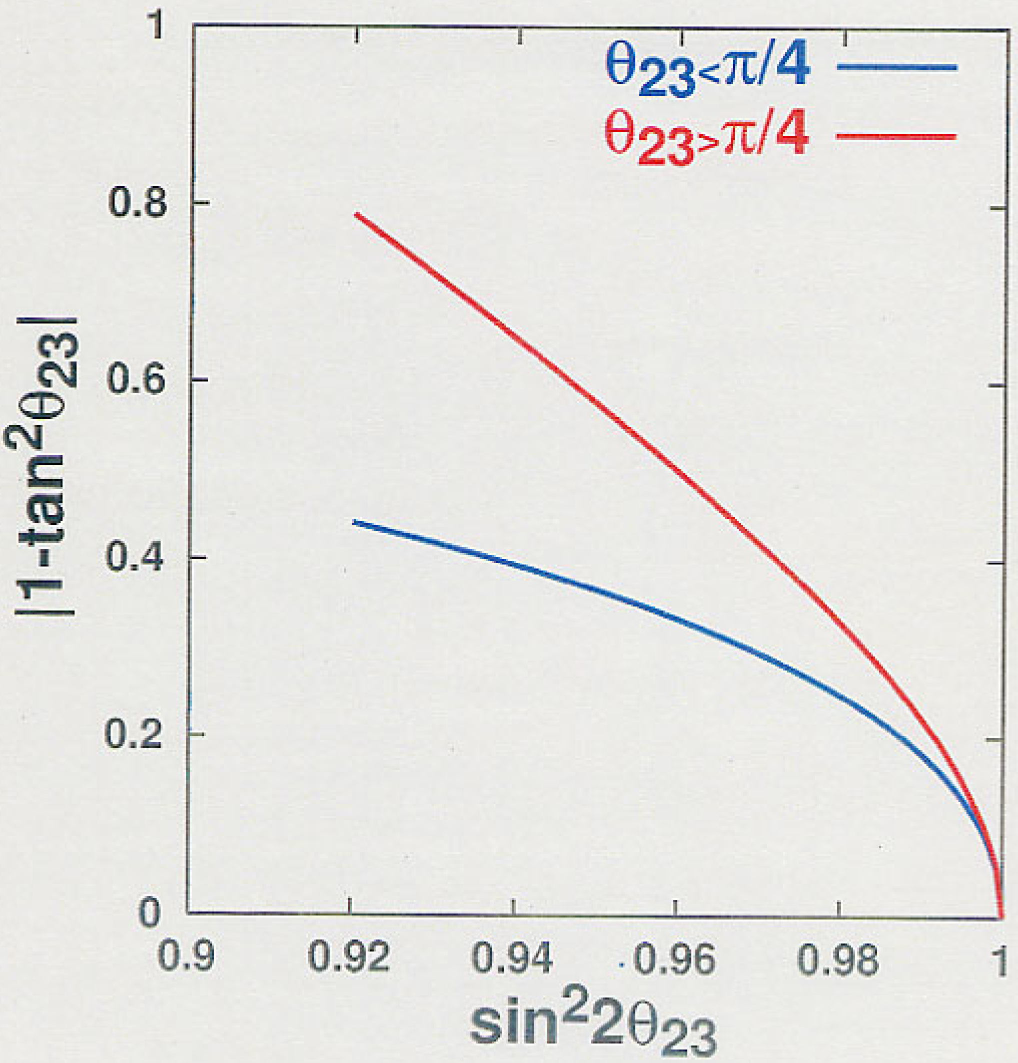
P=0.025, P̄=0.035, Δm²₃₁ >0
P=0.025, P̄=0.035, Δm²₃₁ <0
sin²θ23
θ23 > π/4
δde (sin²2θ13)
s²23
c²23
real solution
fake solution
L=295km
E=0.6GeV
@OM
δre (sin²2θ13)
1
0
0
0.12
sin²2θ13
sin²2θ′13
sin²2θ13

$$\frac{\delta_{de}(\sin^2 2\theta_{13})}{\sin^2 2\theta_{13}} \equiv \frac{|\sin^2 2\theta'_{13} - \sin^2 2\theta_{13}|}{\sin^2 2\theta_{13}}$$

$$= |1-\tan^2\theta_{23}| \cdot \left\{ 1 + \left(\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right)^2 \frac{\tan^2(AL/2)}{AL/\pi} \left[1-\left(\frac{AL}{\pi}\right)^2\right] \sin^2 2\theta_{12} \right\}$$

$$\simeq |1-\tan^2\theta_{23}|$$

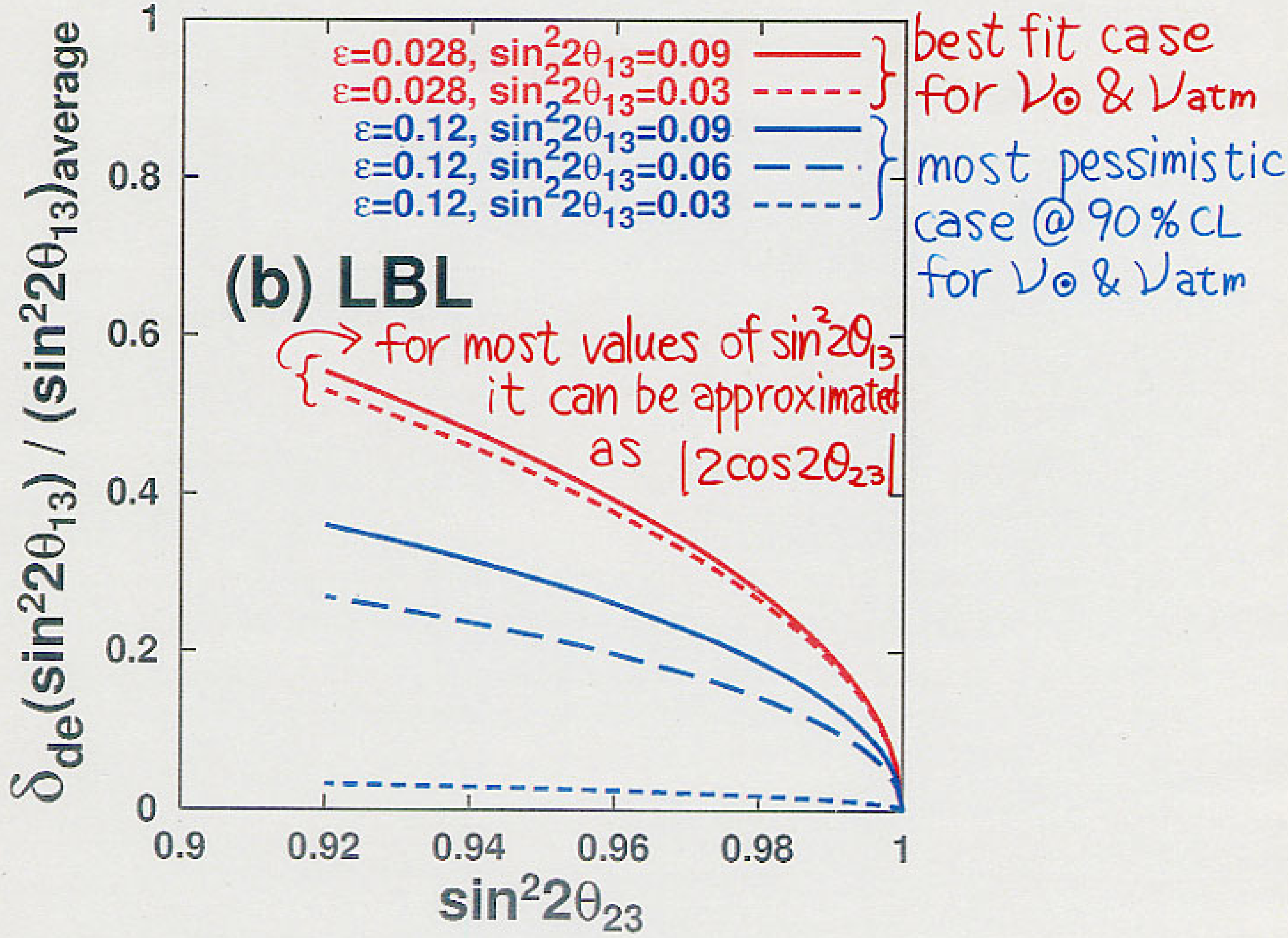
$\delta_{de}(\sin^2 2\theta_{13})$: ambiguity due to the $\theta_{23} \Leftrightarrow \frac{\pi}{2} - \theta_{23}$ degeneracy



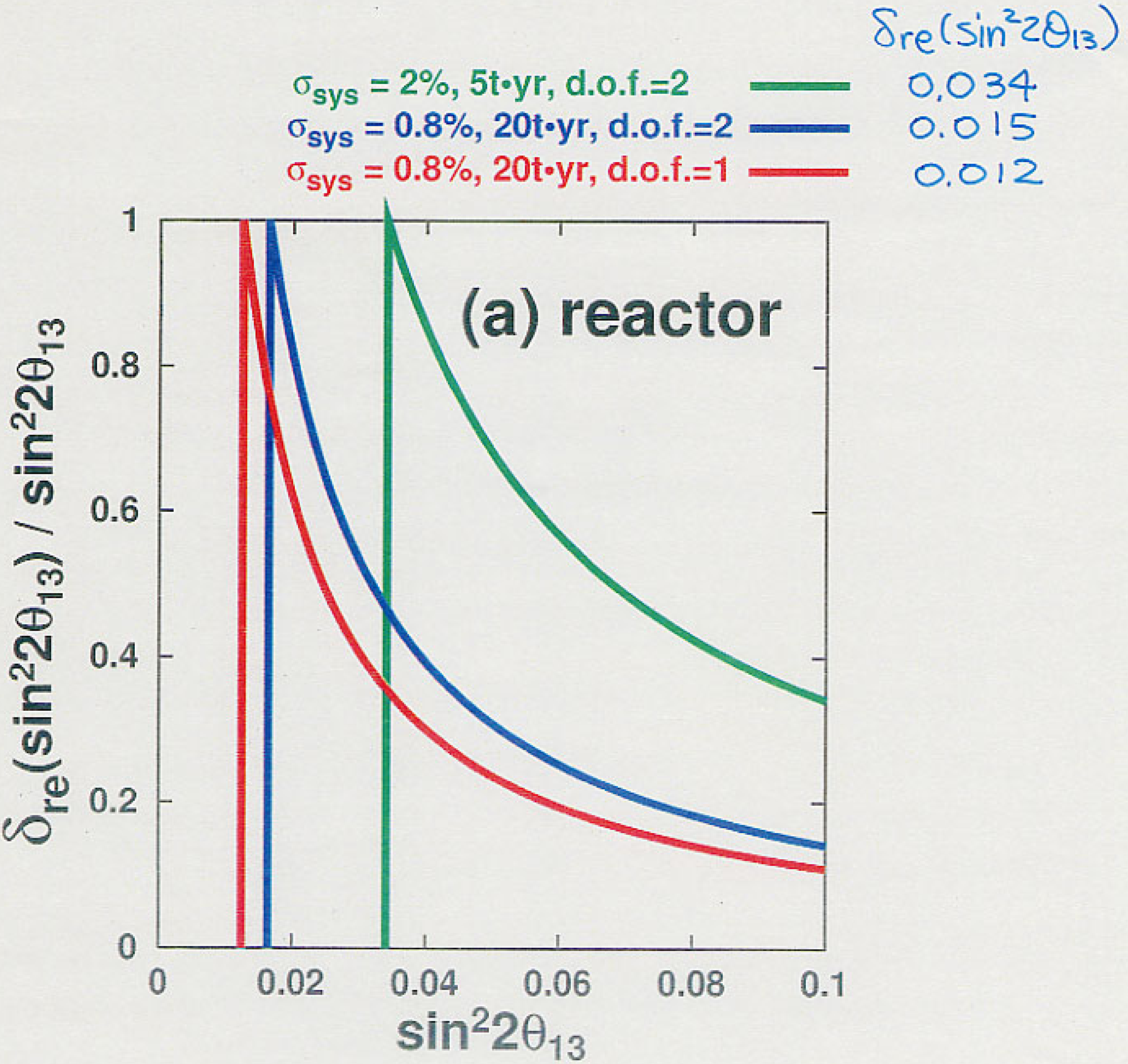
ambiguity due to the degeneracy

$$\delta_{de}(\sin^2 2\theta_{13}) \equiv |\sin^2 2\theta_{13} - \sin^2 2\theta'_{13}|$$

$$(\sin^2 2\theta_{13})_{average} \equiv \frac{1}{2}(\sin^2 2\theta_{13} + \sin^2 2\theta'_{13})$$



error in the reactor experiment
$\delta_{re}(\sin^2 2\theta_{13})$



3. Summary

* 8-fold degeneracy can be visualized using the $(s^2_{23}, \sin^2 2\theta_{13})$ plane.

* $\left\{\begin{array}{l}\text{JHF } \nu \oplus \bar{\nu} \text{ @ OM} \\ \quad\oplus \\ \text{reactor}\end{array}\right\}$ → may solve $\theta_{23} \leftrightarrow \frac{\pi}{2} - \theta_{23}$ degeneracy if $|1-\sin^2 2\theta_{23}|$ and $\sin^2 2\theta_{13}$ are relatively large.

cf. other ways to resolve the degeneracy

Parke-san's talk $\left\{\begin{array}{l}\text{JHF } \nu \oplus \bar{\nu} \text{ @ } \frac{4}{3}\text{OM} \\ \quad\oplus \\ \text{NuMI } \nu \oplus \bar{\nu} \text{ @ OM}\end{array}\right\}$

Signor Donini's talk $\left\{\begin{array}{l}\nu \text{ factory } \overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_\mu \text{ (golden)} \\ \qquad\qquad\qquad \oplus \\ \qquad\qquad\quad \overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_\tau \text{ (silver)}\end{array}\right\}$