New plots & parameter degeneracy in ν oscillations

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Notations:

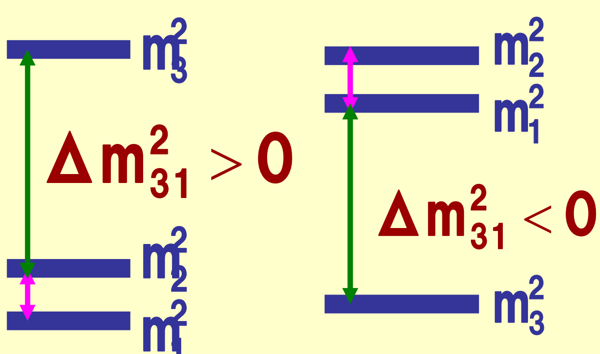
$$P \equiv P(\nu_\mu \to \nu_e)$$

$$\bar{P} \equiv P(\bar{\nu}_\mu \to \bar{\nu}_e)$$

$$\Delta \equiv \frac{|\Delta m^2_{31}|L}{4E}$$

Oscillation Maximum (OM)

$$\Delta \equiv \frac{|\Delta m^2_{31}|L}{4E} = \frac{\pi}{2}$$



normal hierarchy　　inverted hierarchy

normal hierarchy

$$P = f^2x^2 + 2xyfg\cos(\delta + \Delta) + g^2y^2$$

$$\bar{P} = \bar{f}^2x^2 + 2xy\bar{f}g\cos(\delta - \Delta) + g^2y^2$$

inverted hierarchy

$$P = \bar{f}^2x^2 - 2xy\bar{f}g\cos(\delta + \Delta) + g^2y^2$$

$$\bar{P} = f^2x^2 - 2xyfg\cos(\delta - \Delta) + g^2y^2$$

$$x \equiv s_{23}\sin 2\theta_{13},$$

$$y \equiv |\Delta m^2_{21}/\Delta m^2_{31}|\, c_{23}\sin 2\theta_{12},$$

$$f, \bar{f} \equiv \sin(\Delta \mp AL/2)/(1 \mp AL/2\Delta),$$

$$g \equiv \sin(AL/2)/(AL/2\Delta)$$

$$A \equiv \sqrt{2}G_F N_e$$

Plots in ($\sin^2 2\theta_{13}$, $1/s^2_{23}$) plane

The way curves intersect is easy to see

(P=const, δ=const)

($\bar{P}$=const, δ=const)

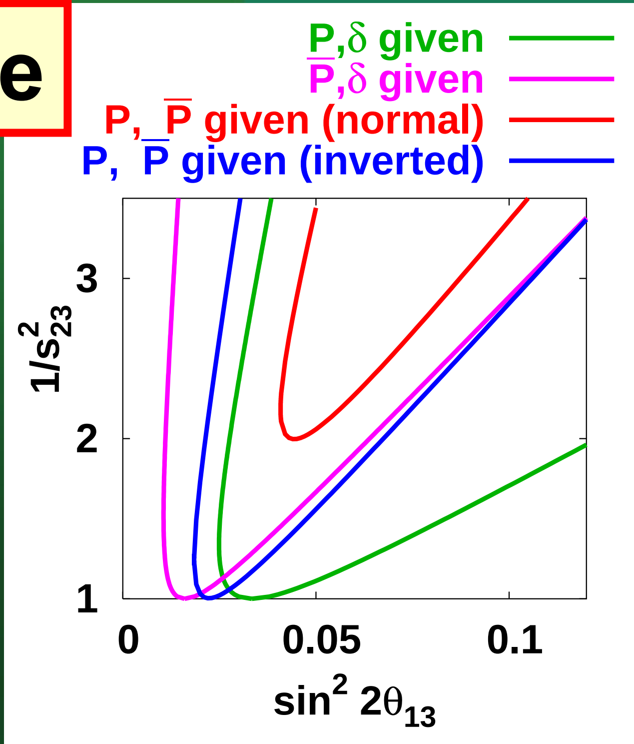
(P=const& $\bar{P}$=const' off OM)

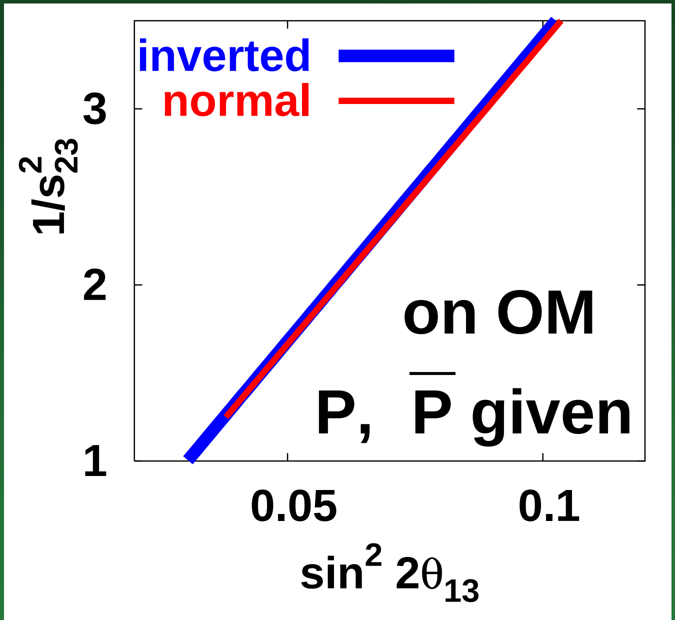
hyperbolas

(or ellipses)

(P=const& $\bar{P}$=const' on OM)

straight lines





$$(X \equiv \sin^2 2\theta_{13},\ Y \equiv 1/s^2_{23})$$

All the curves become quadratic or linear.

$$C \equiv \left(\Delta m^2_{21}/\Delta m^2_{31}\right)^2 g^2 \sin^2 2\theta_{12}$$

$$\lambda \equiv C/P$$

ν only (normal hierarchy)

$$\frac{f^2}{P}X = 1 + \left[1 + \frac{2\lambda}{1-\lambda}\cos^2(\delta+\Delta)\right] - \cos(\delta+\Delta)\sqrt{4\lambda(Y-1)\left[\left(1-\lambda+\lambda\cos^2(\delta+\Delta)\right)(Y-1)+1\right]}$$

$\nu + \bar{\nu}$ off OM (normal hierarchy)

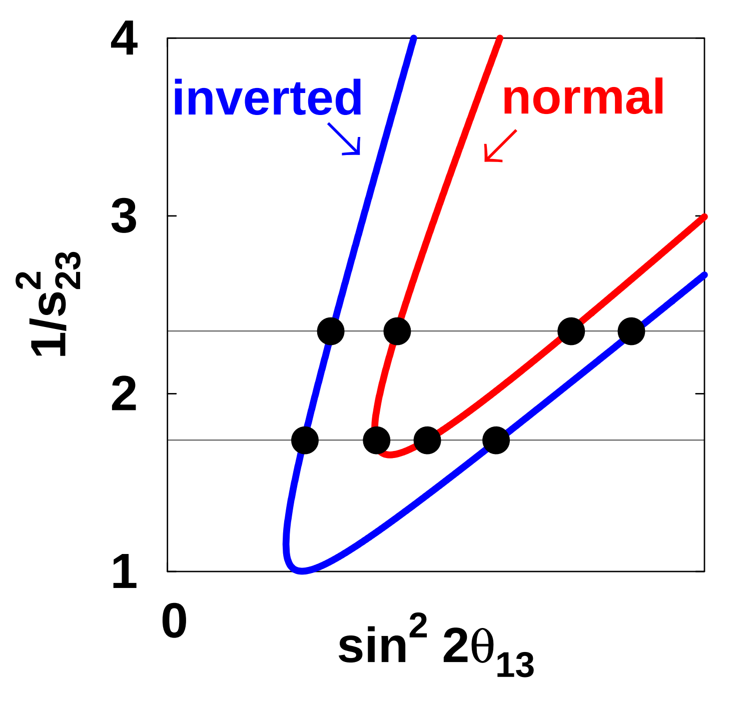
$$16CX(Y-1) = \frac{1}{\cos^2\Delta}\left[\left(\frac{P-C}{f} + \frac{\bar{P}-C}{\bar{f}}\right)(Y-1) - (f+\bar{f})X + \frac{P}{f} + \frac{\bar{P}}{\bar{f}}\right]^2 + \frac{1}{\sin^2\Delta}\left[\left(\frac{P-C}{f} - \frac{\bar{P}-C}{\bar{f}}\right)(Y-1) - (f-\bar{f})X + \frac{P}{f} - \frac{\bar{P}}{\bar{f}}\right]^2$$

$\nu + \bar{\nu}$ @OM (normal hierarchy)

$$Y = \frac{f+\bar{f}}{P/f + \bar{P}/\bar{f} - C(1/f + 1/\bar{f})}\left(X - \frac{C}{f\bar{f}}\right)$$

Ambiguity due to parameter degeneracy

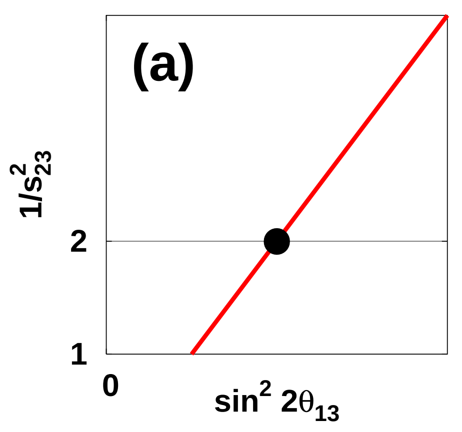
Even if we know $P(\nu_\mu \to \nu_e)$ and $P(\overline{\nu_\mu} \to \overline{\nu_e})$ in a long baseline accelerator experiments with approximately monoenergetic neutrino beam, precise determination of θ_{13}, sign(Δm^2_{31}) and δ is difficult because of the ambiguity due to 8-fold parameter degeneracy.

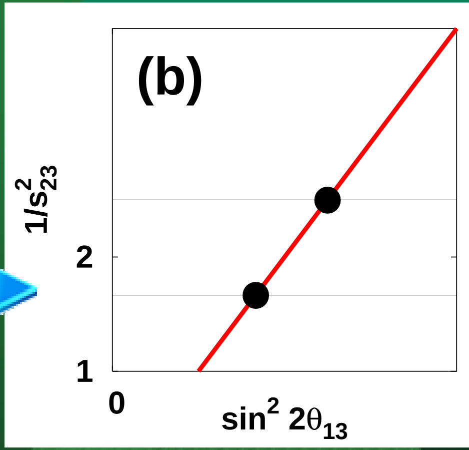


Plot of $P(\nu_\mu \to \nu_e)$, $P(\overline{\nu_\mu} \to \overline{\nu_e})$ =const.

● $\theta_{23} \Leftrightarrow \pi/2 - \theta_{23}$ degeneracy

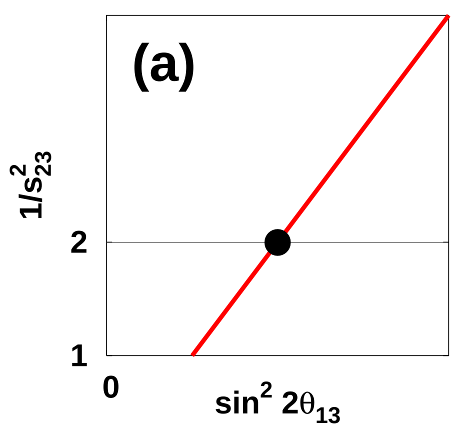
$$(a)\cos 2\theta_{23} = 0 \rightarrow (b)\cos 2\theta_{23} \neq 0$$

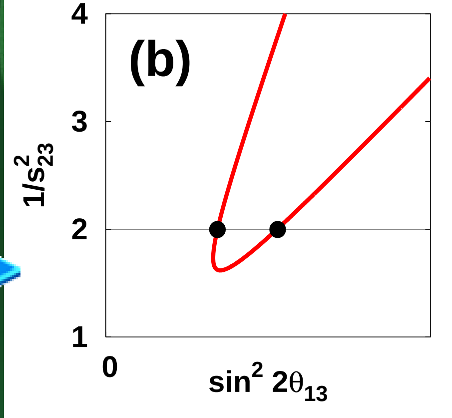




● intrinsic (δ, θ_{13}) degeneracy

$$(a)\frac{\Delta m^2_{21}}{|\Delta m^2_{31}|} = 0 \rightarrow (b)\frac{\Delta m^2_{21}}{|\Delta m^2_{31}|} \cong \frac{1}{35} \neq 0$$

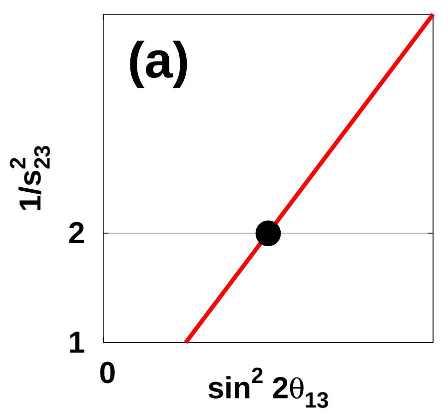


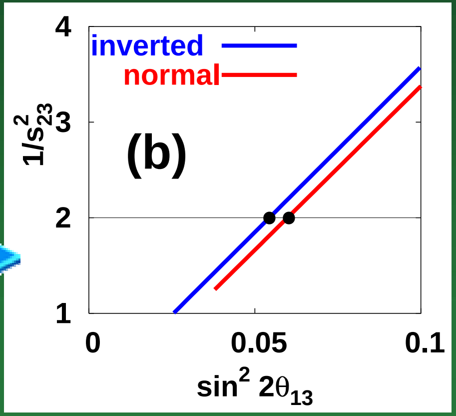


● $\Delta m^2_{31} \Leftrightarrow -\Delta m^2_{31}$ degeneracy

$$(a)AL/2 = 0 \rightarrow (b)AL/2 \neq 0$$

$$A \equiv \sqrt{2} G_F N_e \cong 1/2000\text{km}$$





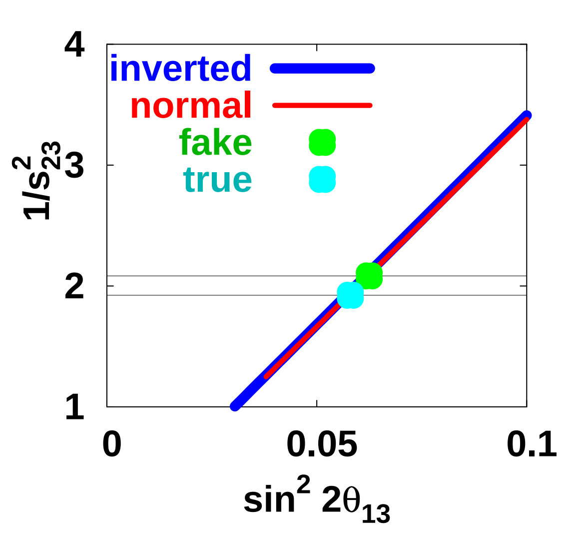
Determination of θ_{13}

Assumption: $\nu_\mu \to \nu_e$ and $\overline{\nu_\mu} \to \overline{\nu_e}$ will be measured at JPARC (@OM, 4MW, HK) .

Question: **Will that be enough to determine $|U_{e3}|$?**

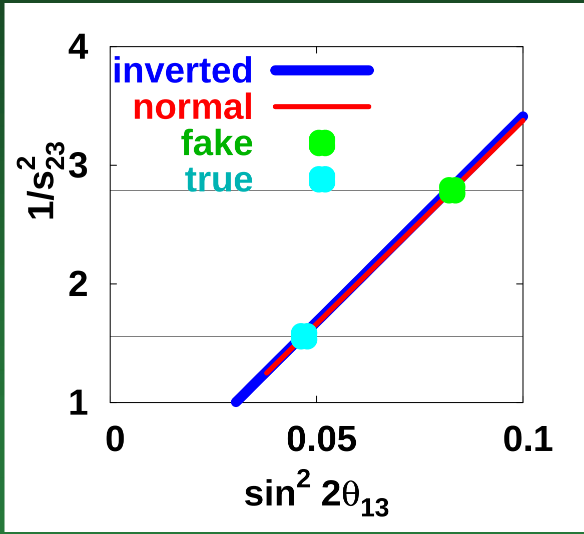
(1) $\sin^2 2\theta_{23} \simeq 1$

JPARC $\nu + \bar{\nu}$ is almost enough, since (a) there is no intrinsic (δ, θ_{13}) degeneracy, and (b) sign(Δm^2_{31}) degeneracy is small.



(2) $\sin^2 2\theta_{23} < 1$

Ambiguity due to $\theta_{23} \Leftrightarrow \pi/2 - \theta_{23}$ degeneracy is significant.



To resolve θ_{23} ambiguity, possible ways are:

(A) reactor measurement of θ_{13}

(B) LBL measurement of $\nu_\mu \to \nu_e$ (or $\nu_e \to \nu_\mu$)

(C) measurement of $\nu_e \to \nu_\tau$

Here we consider (B) and (C).

The reference values used here are:

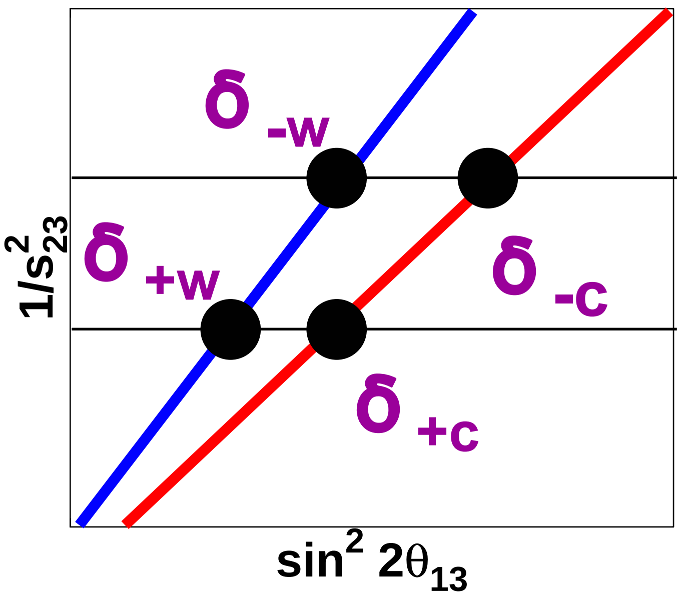
$\sin^2 2\theta_{23}=0.96$, $\sin^2 2\theta_{13}=0.05$, $\delta=\pi/4$, $\Delta m^2_{31}>0$

(B) LBL measurement of $\nu_\mu \to \nu_e$ (or $\nu_e \to \nu_\mu$)

Consider 3rd measurement of $\nu_\mu \to \nu_e$ (or $\nu_e \to \nu_\mu$)

in addition to JPARC $\nu + \bar{\nu}$.

↓ (exaggerated figure)



correct assumption
wrong assumption
on mass hierarchy

The value of δ for each point can be deduced (up to $\delta \Leftrightarrow \pi - \delta$) from

$$\sin\delta = -\frac{P - f^2x^2 - g^2y^2}{2fgxy}$$

Then from the equation for the probability of $\nu_\mu \to \nu_e$ (or $\nu_e \to \nu_\mu$) in the 3rd experiment

$$P_{true} = P\left(\sin^2 2\theta_{13}, \delta_{\pm[cw]}, s^2_{23}\right)$$

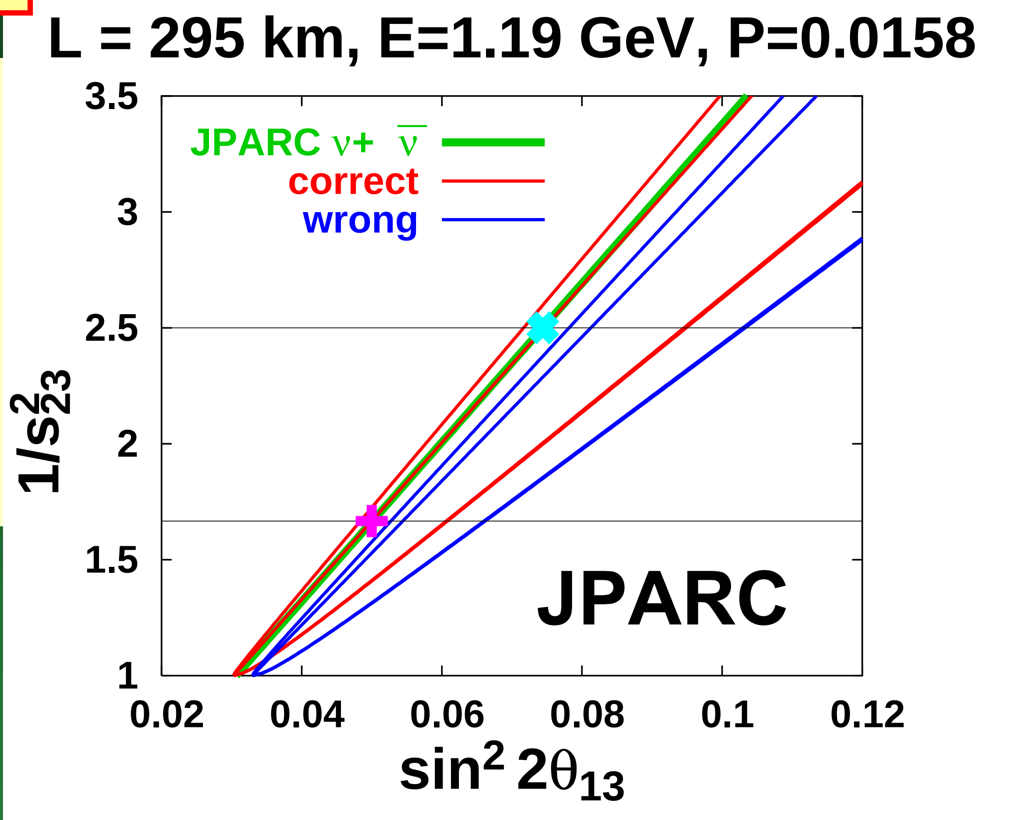
or

$$P_{true} = P\left(\sin^2 2\theta_{13}, \pi - \delta_{\pm[cw]}, s^2_{23}\right)$$

where

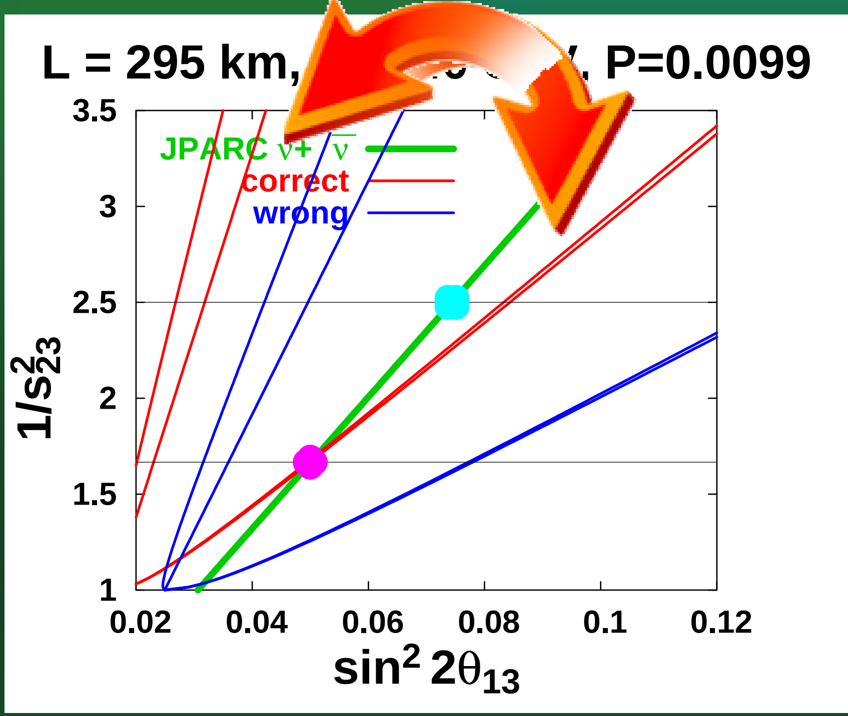
$$P_{true} \equiv P\left((\sin^2 2\theta_{13})_{true}, \delta_{true}, (s^2_{23})_{true}\right)$$

we can get a unique line (a hyperbola or an ellipse) in $(\sin^2 2\theta_{13}, 1/s^2_{23})$ plane for $\delta_{\pm[cw]}$ or $\pi - \delta_{\pm[cw]}$.



● $\delta \Leftrightarrow \pi - \delta$ ambiguity

$\propto \cos\delta \cos\Delta$



Assuming for simplicity P>>C

$$C \equiv \left(\Delta m^2_{21} / \Delta m^2_{31}\right)^2 g^2 \sin^2 2\theta_{12}$$
$$g \equiv \sin(AL/2)/(AL/2\Delta)$$

$$\left.\frac{dX}{dY}\right|_{\delta} \cong -\frac{2\sqrt{PC}}{f^2}\cos(\delta+\Delta)$$

$$\left.\frac{dX}{dY}\right|_{\pi-\delta} \cong \frac{2\sqrt{PC}}{f^2}\cos(\delta-\Delta)$$

$$\left.\frac{dX}{dY}\right|_{\delta} - \left.\frac{dX}{dY}\right|_{\pi-\delta} \cong -\frac{4\sqrt{PC}}{f^2}\cos\delta\cos\Delta$$

Difference in the gradients is large for $\Delta = 0$ or π

sign(Δm^2_{31}) ambiguity

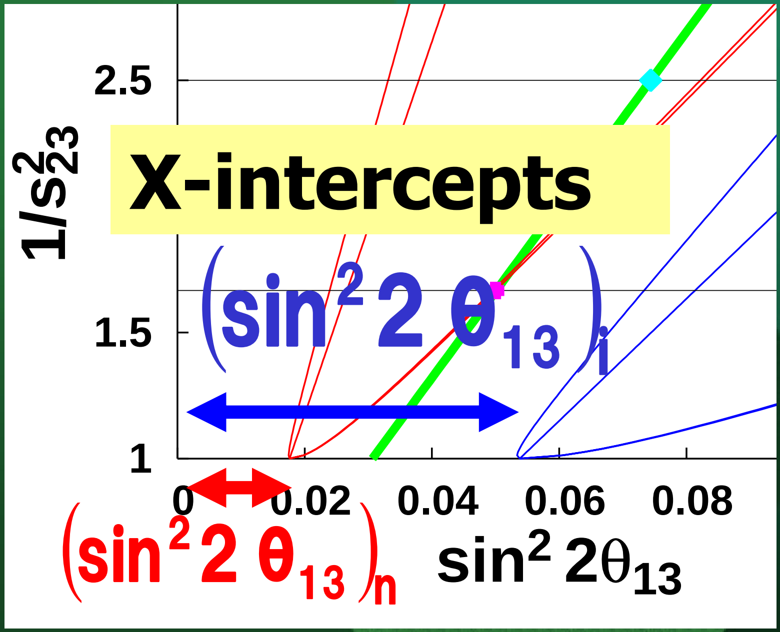
(a) L: AL～L/1900km

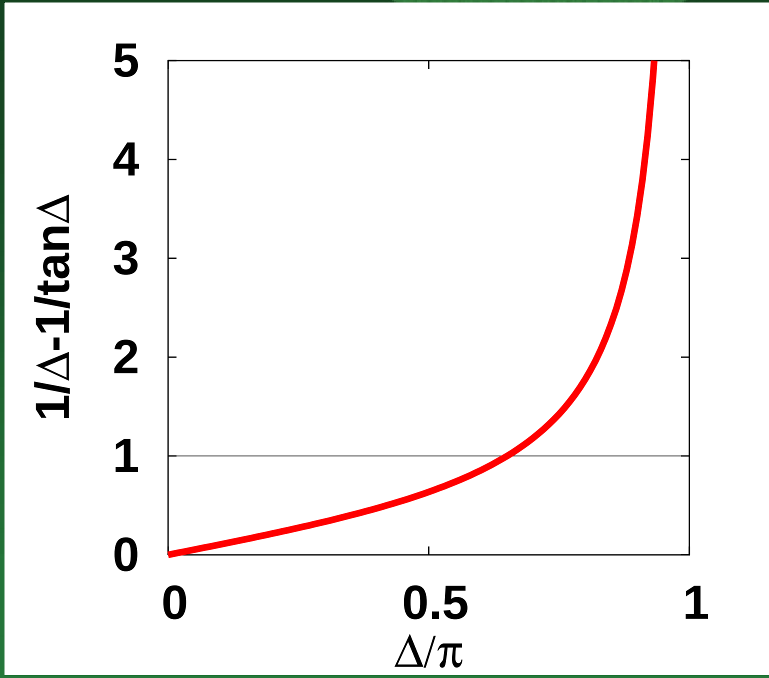
→ L>2000km is good to distinguish Δm^2_{31} from $-\Delta m^2_{31}$

(b) E: → low energy is advantageous

$$\frac{(\sin^2 2\theta_{13})_i}{(\sin^2 2\theta_{13})_n} \cong 1 + 2AL\left(\frac{1}{\Delta} - \frac{1}{\tan\Delta}\right)$$

Enhancement of matter effect for $\pi/2 < \Delta < \pi$





● θ_{23} ambiguity

Resolution of θ_{23} ambiguity

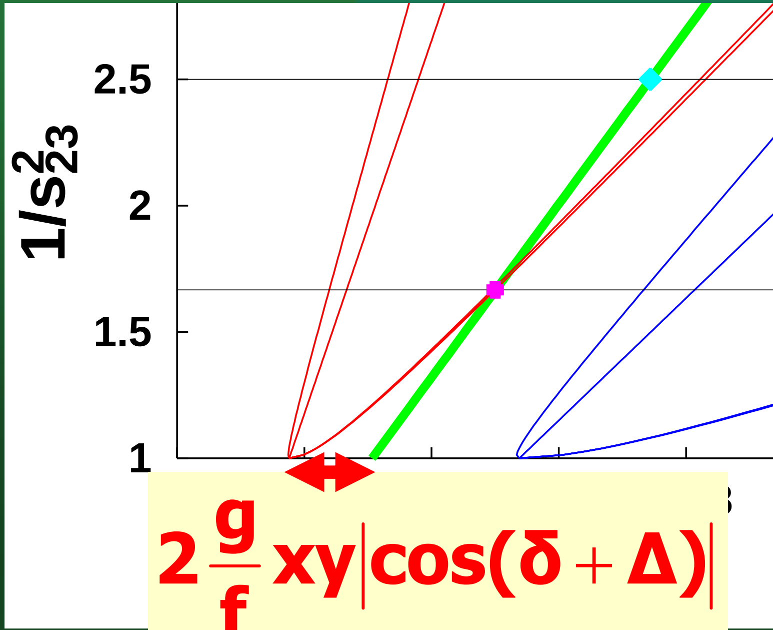
(a) $f \equiv \dfrac{\sin(\Delta - AL/2)}{1 - AL/2\Delta}$ has to be small

(b) $|\cos(\delta + \Delta)|$ has to be large

⇓

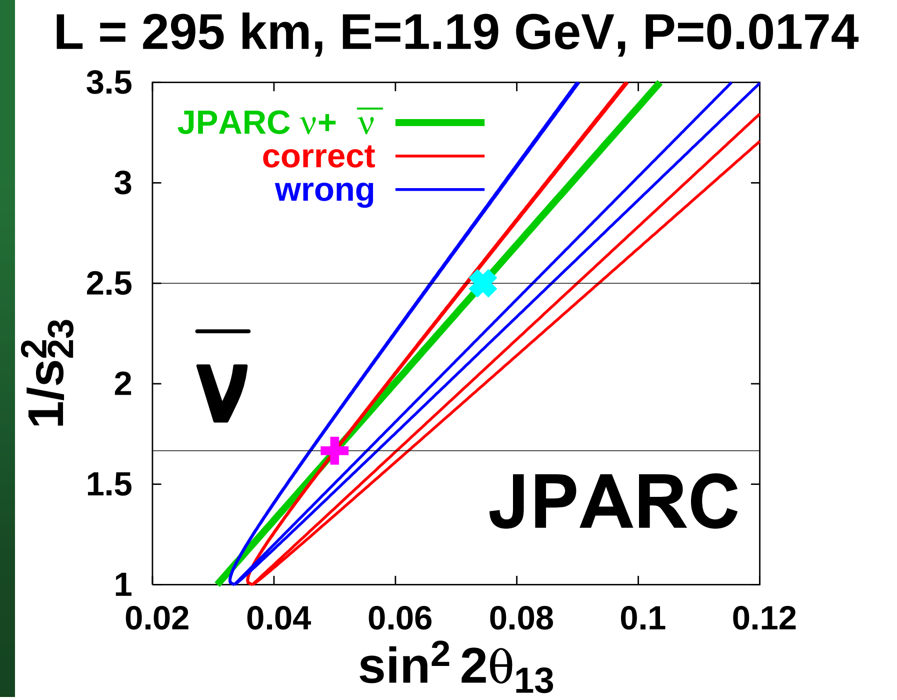
Δ is unknown at first, so it is impossible to design to optimize this resolution.

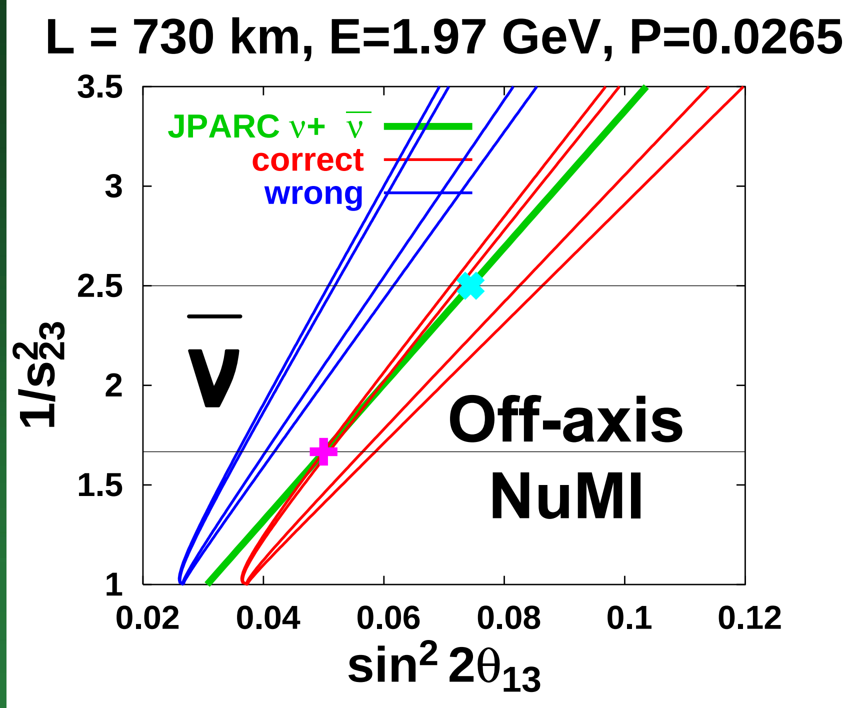
It may happen that this ambiguity can be resolved as a byproduct.

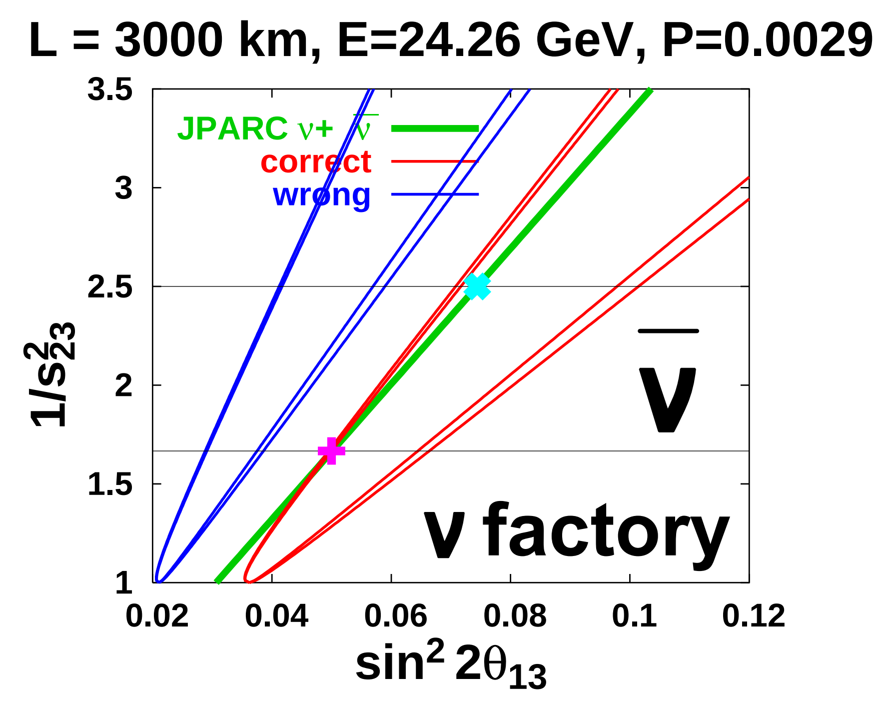


$$2\frac{g}{f}xy|\cos(\delta + \Delta)|$$

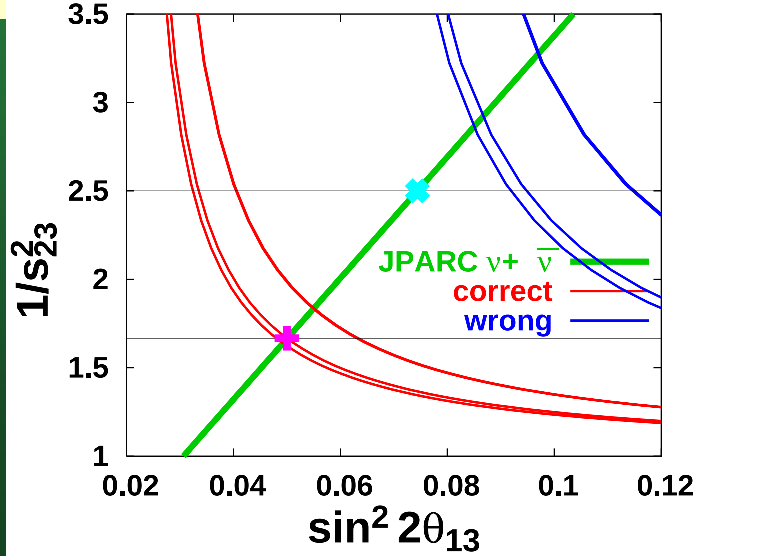
Situation for $\bar{\nu}_\mu \to \bar{\nu}_e$ is similar to that for $\nu_\mu \to \nu_e$







(C) measurement of $\nu_e \to \nu_\tau$



Curves intersect with the JPARC line almost orthogonally.

- **θ_{23} ambiguity may be resolved.**
- **$\delta \Leftrightarrow \pi - \delta$ ambiguity may be resolved.**
- **sign(Δm^2_{31}) ambiguity may be resolved .**

This channel may be interesting to be combined with JPARC in the future.

Summary

$$\Delta \equiv \frac{|\Delta m^2_{31}| L}{4E}$$

△: may be OK

		intrinsic	$\delta \leftrightarrow \pi - \delta$	$\mathrm{sign}(\Delta m^2)$	$\theta_{23}\left(\text{if } \theta_{23} \neq \frac{\pi}{4}\right)$
JPARC $\nu + \bar{\nu}$ @OM		✓	×	×	×
JPARC $\nu + \bar{\nu}$ + reactor (90%CL)		✓	×	×	✓
JPARC $\nu + \bar{\nu}$ + LBL (ν and/or $\bar{\nu}$)	$\Delta < \pi/8$	✓	✓	×	×
	$\Delta < \pi/2$, L<500km	✓	△	×	△
	$\Delta < \pi/2$, L>500km	✓	△	✓	△
	$\Delta > \pi/2$	✓	△	△	△
JPARC $\nu + \bar{\nu}$ + $\nu_e \to \nu_\tau$	L<500km	✓	△	×	✓
	L>500km	✓	△	✓	✓