Non oscillation flavor physics at future neutrino oscillation facilities

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(SM+m_ν) + (correction due to New physics) which can be probed through oscillations

1. Motivation for research on New Physics
2. A word on origin of New Physics
3. New Physics in oscillation experiments
4. New Physics at source and detector
5. New Physics in propagation (matter effect)
6. Violation of unitarity
7. Summary

Notations throughout this talk:

$\Delta E_{jk}=\Delta m^2_{jk}/2E$

$A= 2^{1/2}G_F N_e$ (matter effect)

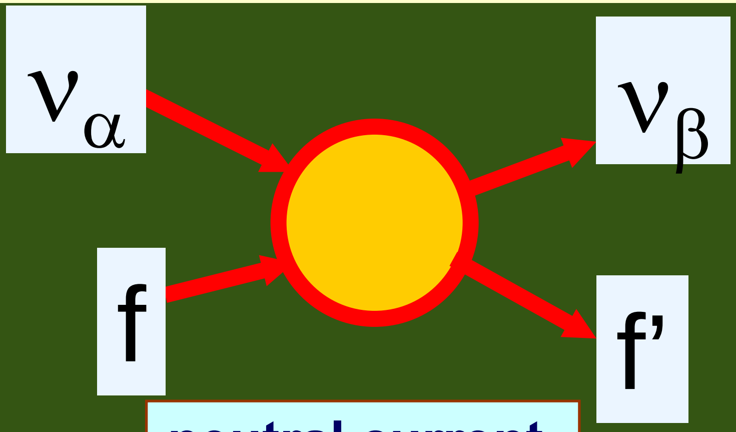
1. Motivation for research on New Physics

Just like at B factories, high precision measurements of ν oscillation in future experiments can be used also to probe physics beyond SM by looking at deviation from SM+massive ν

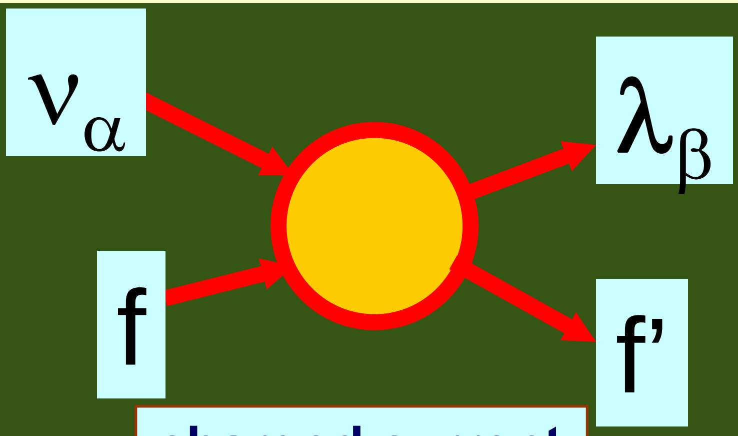
In this talk I will discuss phenomenologically new physics which is described by 4-fermi exotic interactions:

$$\mathcal{L}_{eff} = G_{NP}^{\alpha\beta}\,\bar{\nu}_\alpha\gamma^\mu\nu_\beta\,\bar{f}\gamma_\mu f'$$

$$\mathcal{L}_{NP} = G_N^{\alpha\beta}\bar{\nu}_\alpha\gamma^\mu\ell_\beta\bar{f}\gamma_\mu f'$$



neutral current



charged current

2. A word on the origin of New Physics

If we have New Physics at higher energy scale, there can be higher dimensional operators:

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM}^{d=4} + \frac{1}{\Lambda_{\rm NP}}\mathcal{L}^{d=5} + \frac{1}{\Lambda_{\rm NP}^2}\mathcal{L}^{d=6} + \cdots$$

These operators are supposed to be SU(2) invariant before symmetry breaking of SM. The lower dimensional operators relevant to neutrino oscillation experiments are of dim 6 and dim 8.

→dim 6 operators such as $(H^+\overline{L}_\alpha)\gamma^\mu iD_\mu(HL_\beta)$ **turn out to be strongly constrained by charged lepton processes.**

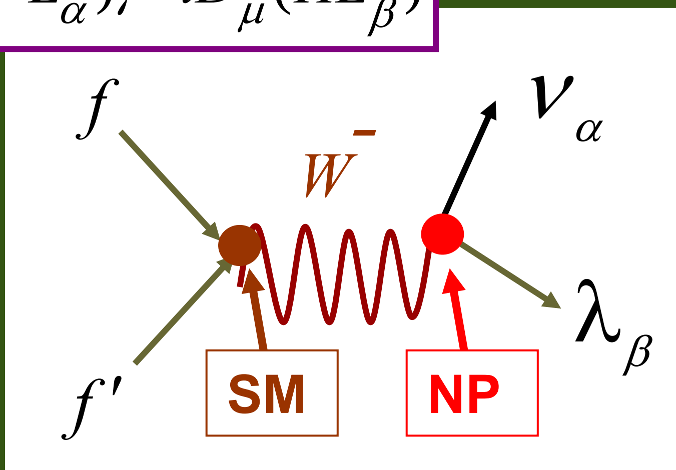
→dim 8 operators such as $\overline{f}\left(H^\dagger\gamma^\rho P_L L_\beta\right)\left(\overline{L}_\alpha\gamma_\rho P_L H\right)f$ **do not have strong constraints.**

→ **To justify the discussions below, dim 8 operators will be assumed.**

Constraints on dimension-6, 8 operators

dim-6 operators

$$(H^{+}\overline{L}_{\alpha})\gamma^{\mu} i D_{\mu}(H L_{\beta})$$



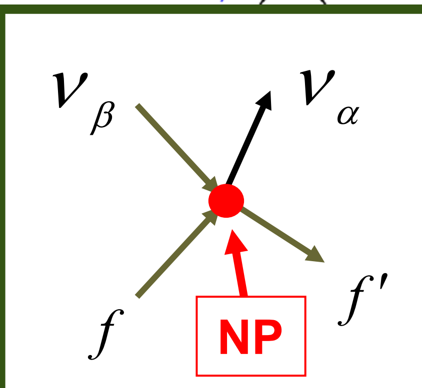
$$\overline{\lambda}_{\alpha}\gamma^{\mu}\lambda_{\beta}\,\bar{f}\gamma_{\mu}f'$$

$$\overline{\nu}_{\alpha}\gamma^{\mu}\lambda_{\beta}\,\bar{f}\gamma_{\mu}f'$$

strong constraints from charged lepton processes, e.g., $\mu^{+} \to e^{+}e^{+}e^{-}$

dim-8 operators

$$\overline{f}\left(H^{\dagger}\gamma^{\rho}P_{L}L_{\beta}\right)\left(\overline{L}_{\alpha}\gamma_{\rho}P_{L}H\right)f$$



$$-v^{2}\left(\overline{\nu}_{\alpha}\gamma_{\rho}P_{L}\nu_{\beta}\right)\left(\overline{f}\gamma^{\rho}P_{L}f\right)$$

$$\left(\overline{l}_{\alpha}\gamma_{\rho}P_{L}l_{\beta}\right)\left(\overline{f}\gamma^{\rho}P_{L}f\right)$$

little constraints from charged lepton

The coefficient of the operator is usually normalized in terms of G_F:

$$G_{NP}^{\alpha\beta} \equiv G_F \, \epsilon_{\alpha\beta}$$ **where** $$\mathcal{L}_{eff} \;=\; G_{NP}^{\alpha\beta} \, \bar{\nu}_\alpha \gamma^\mu \nu_\beta \, \bar{f} \gamma_\mu f'$$ **etc**

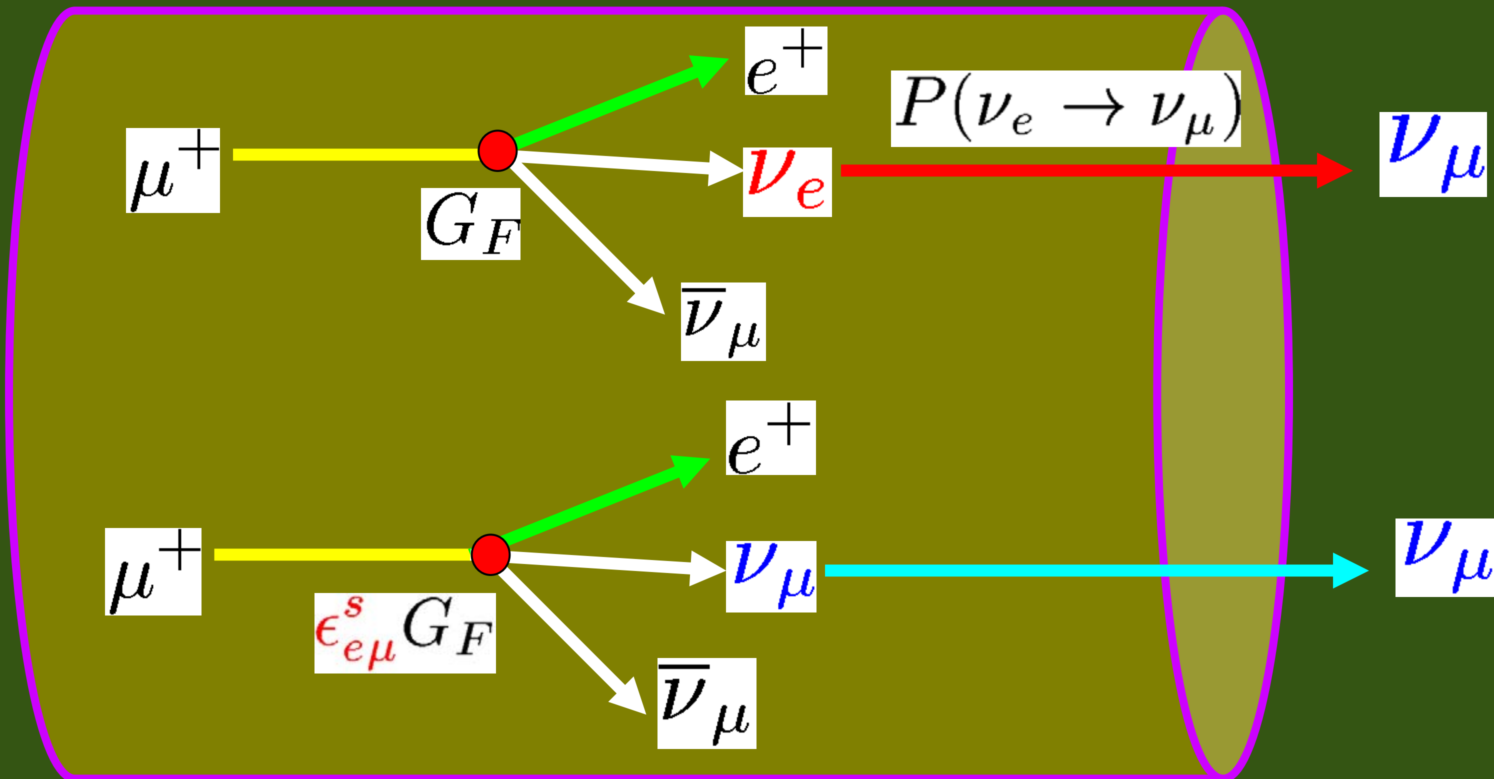
Theoretically the coefficients $\varepsilon_{\alpha\beta}$ of the exotic interactions are expected to be suppressed by ratio $(M_W/\Lambda_{NP})^n << 1$ (n=2 for dim 6, n=4 for dim 8).

In phenomenological analysis, however, we take into account only the constraints from the experiments, and we do not worry about the magnitude of the coefficients $\varepsilon_{\alpha\beta}$ which may be unnaturally large from a theoretical view point.

3. New Physics in oscillation experiments

● NP at source

Grossman, Phys. Lett. B359, 141 (1995)



$$\mu^+ \to e^+ + \overline{\nu}_\mu + \nu^s_\mu$$

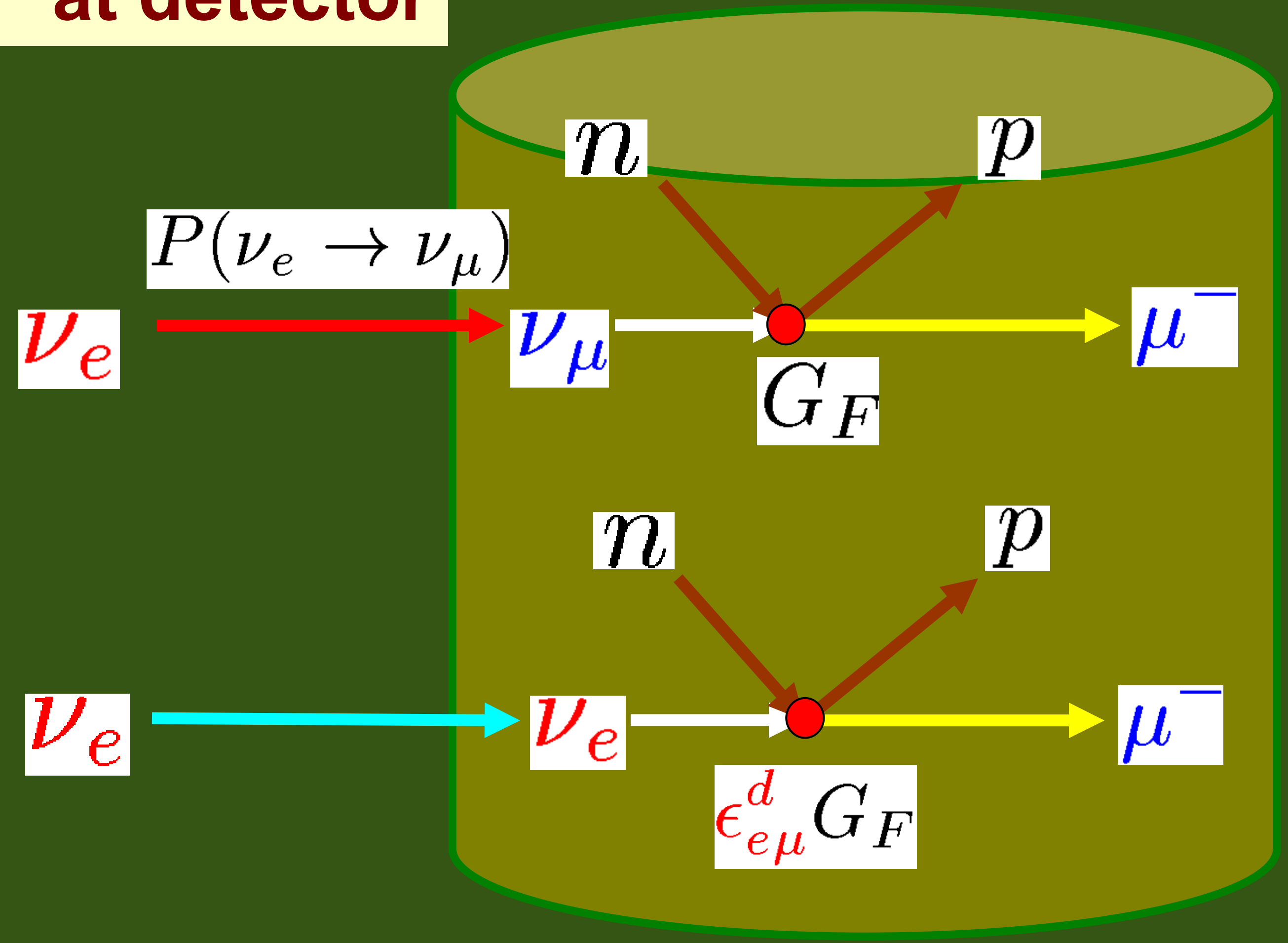
$$\nu^s_e = \nu_e + \epsilon^s_{e\mu} \nu_\mu$$

Effective eigenstate

$$\begin{pmatrix} \nu^s_e \\ \nu^s_\mu \end{pmatrix} = \begin{pmatrix} 1 & \epsilon^s_{e\mu} \\ -\epsilon^s_{e\mu} & 1 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

● NP at detector

Grossman, Phys. Lett. B359, 141 (1995)



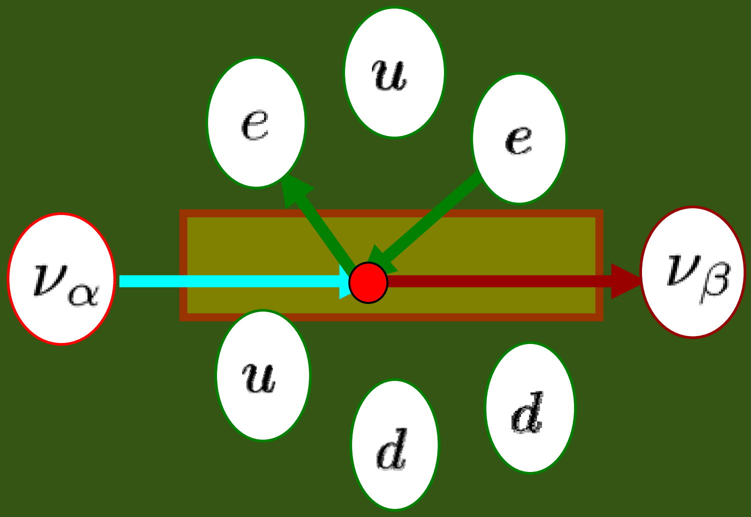
$$\nu_\mu^d + n \to \mu^- + p$$

$$\nu_\mu^d = \nu_\mu - \epsilon^d_{e\mu} \nu_e$$

Effective eigenstate

$$\begin{pmatrix} \nu_e^d \\ \nu_\mu^d \end{pmatrix} = \begin{pmatrix} 1 & \epsilon^d_{e\mu} \\ -\epsilon^d_{e\mu} & 1 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

● NP in propagation (NP matter effect)



SM potential due to W exchange is modified by NP

SM

$$\mathcal{A}_0 \equiv A \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow$$

NP

$$\mathcal{A} \equiv A \begin{pmatrix} 1+\epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau} \end{pmatrix}$$

$A \equiv \sqrt{2} G_F N_e$ $\quad$ $N_e \equiv$ electron density

Oscillation probability w/o and w/ NP

SM+m_ν

$$\mathcal{A}_0 \equiv A \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$U \,\mathrm{diag}(E_j)\, U^{-1} + \mathcal{A}_0 = \tilde{U}_0 \,\mathrm{diag}(\tilde{E}_j^0)\, \tilde{U}_0^{-1}$$

$$P(\nu_\alpha \to \nu_\beta) = \left| \left[\tilde{U}_0 \exp\left\{ -i \,\mathrm{diag}(\tilde{E}_j^0) L \right\} \tilde{U}_0^{-1} \right]_{\beta\alpha} \right|^2$$

NP

$$\mathcal{A} \equiv A \begin{pmatrix} 1+\epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau} \end{pmatrix}$$

$$U \,\mathrm{diag}(E_j)\, U^{-1} + \mathcal{A} = \tilde{U} \,\mathrm{diag}(\tilde{E}_j)\, \tilde{U}^{-1}$$

$$P(\nu_\alpha \to \nu_\beta) = \left| \left[U^d \, \tilde{U} \exp\left\{ -i \,\mathrm{diag}(\tilde{E}_j) L \right\} \tilde{U}^{-1} \, (U^s)^{-1} \right]_{\beta\alpha} \right|^2$$

NB

o NP effects at production and at detection becomes important when L is smaller

$$P(\nu_\alpha \to \nu_\beta) \to \left| \left[U^d (U^s)^{-1} \right]_{\beta\alpha} \right|^2$$

i.e., no BG from ν osc. in the limit of L → 0

→ Experiments with a shorter baseline are advantageous

o NP effects in propagation becomes important when L is larger

because AL $\epsilon_{\alpha\beta} \sim \epsilon_{\alpha\beta}$ (L/2000km)

→ Experiments with a longer baseline are advantageous

o Advantage over Direct Detection

Sato: ISS 2nd plenary @ KEK

Transition Probability $\sim |\mathcal{A} + \epsilon|^2$

$\mathcal{A}$: Oscillation Amplitude

$\mathcal{S}$: Systematic Error

Direct Detection ($|\mathcal{A}| \ll |\epsilon|$)

$$\epsilon^2 > \mathcal{S} \longrightarrow \epsilon > \sqrt{\mathcal{S}}$$

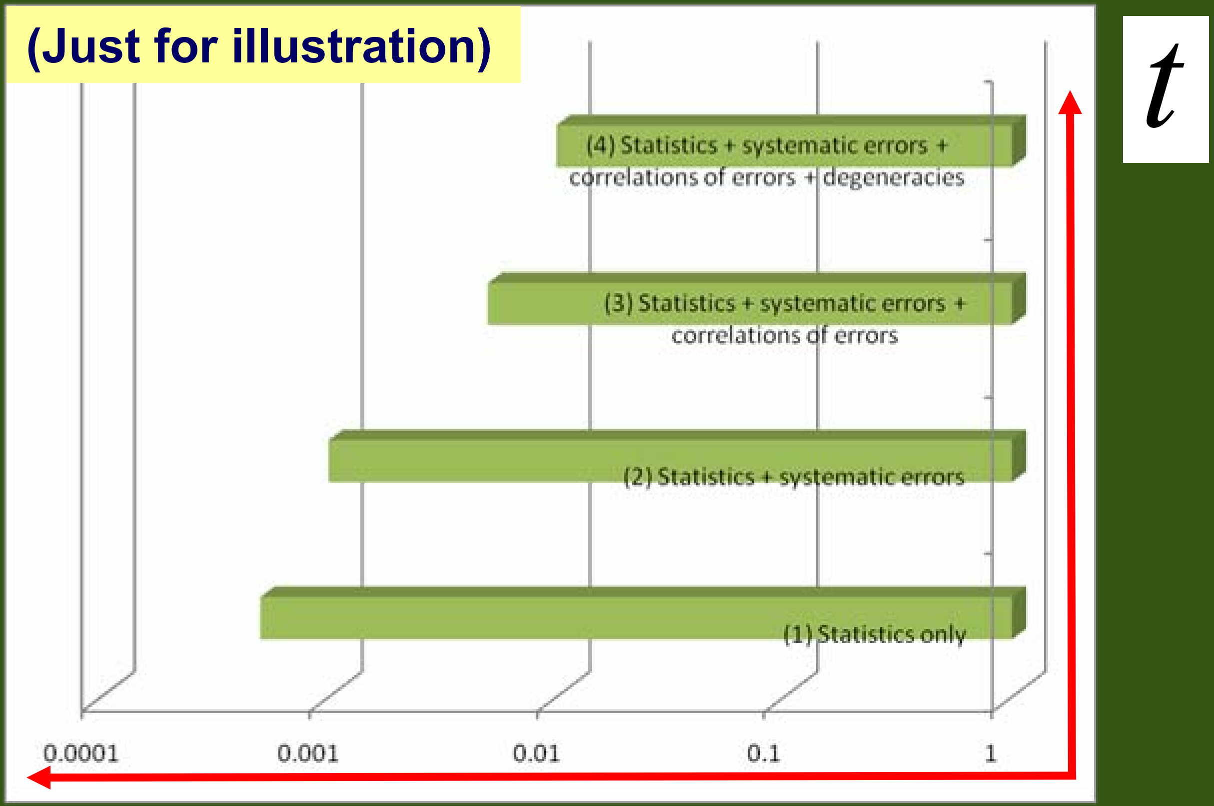
Oscillation Detection

$$\mathcal{A}\epsilon > \mathcal{S} \longrightarrow \epsilon > \frac{\mathcal{S}}{\mathcal{A}} (< \sqrt{\mathcal{S}})$$

$\mathcal{A}^2 > \mathcal{S}$: Always expected

o History of analysis of sensitivity

1. Statistics only
2. Statistics + systematic errors
3. Statistics + systematic errors + correlations of errors
4. Statistics + systematic errors + correlations of errors + degeneracies



$\sin^2 2\theta$ or $|\varepsilon_{\alpha\beta}|$

Unless new ideas appears, sensitivity to unknown parameters is monotonically decreasing as a function of time.

4. New Physics at source and detector

Grossman, Phys. Lett. B359, 141 (1995)

Optimistic bounds on $\varepsilon_{\alpha\beta}$ are obtained by

$$P(\nu_\alpha - \nu_\beta) \cong \left|\varepsilon^s_{\alpha\beta} - \varepsilon^{d*}_{\alpha\beta}\right|^2 \sim \max\left(\left|\varepsilon^s_{\alpha\beta}\right|^2, \left|\varepsilon^d_{\alpha\beta}\right|^2\right) < P(\nu_\alpha \to \nu_\beta)_{\text{upper bound}}$$

and typically are of order 10^{-2}

$$\left|\varepsilon^s_{e\mu}\right|, \left|\varepsilon^d_{e\mu}\right| < 6\times10^{-2}$$ ←MiniBooNE

$$\left|\varepsilon^s_{\mu\tau}\right|, \left|\varepsilon^d_{\mu\tau}\right| < 1\times10^{-2}$$ ←NOMAD

$$\left|\varepsilon^s_{e\tau}\right|, \left|\varepsilon^d_{e\tau}\right| < 0.1$$ ←NOMAD

Sensitivity to $\varepsilon_{e\mu}$ at ν factory

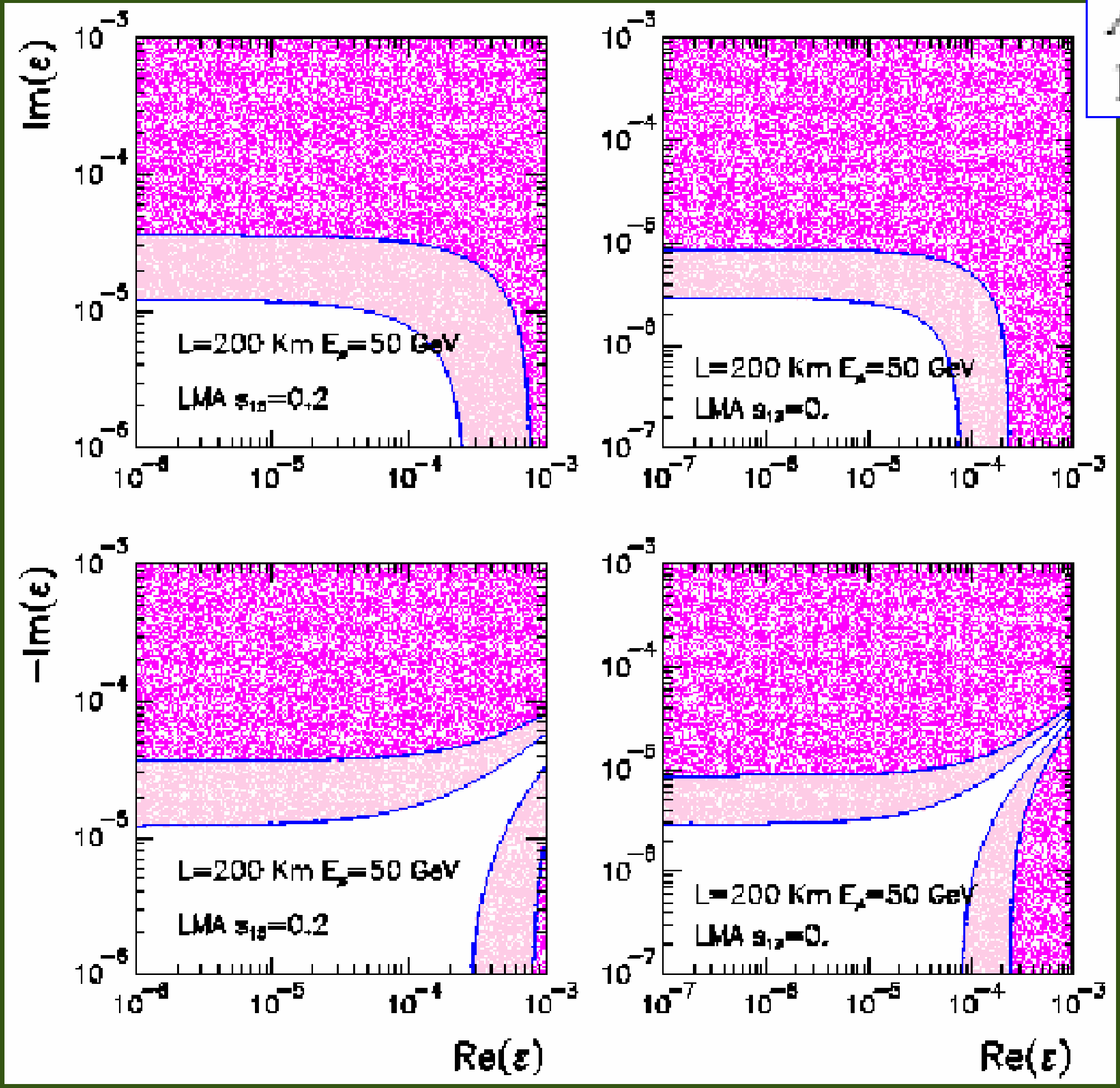


FIG. 6. Regions in the plane of $[\mathrm{Re}(\epsilon^s_{e\mu}), \mathrm{Im}(\epsilon^s_{e\mu})]$ that give $A^{NP}_{CP}/\Delta A = 3$ (darker shadow) and 1 (light shadow).

$$A_{CP} \equiv \frac{P_{e\mu} - P_{\bar{e}\bar{\mu}}}{P_{e\mu} + P_{\bar{e}\bar{\mu}}}$$

Statistics only; naïve arguments

$$\left|\varepsilon^s_{e\mu}\right| < \text{a few} \times 10^{-4}$$

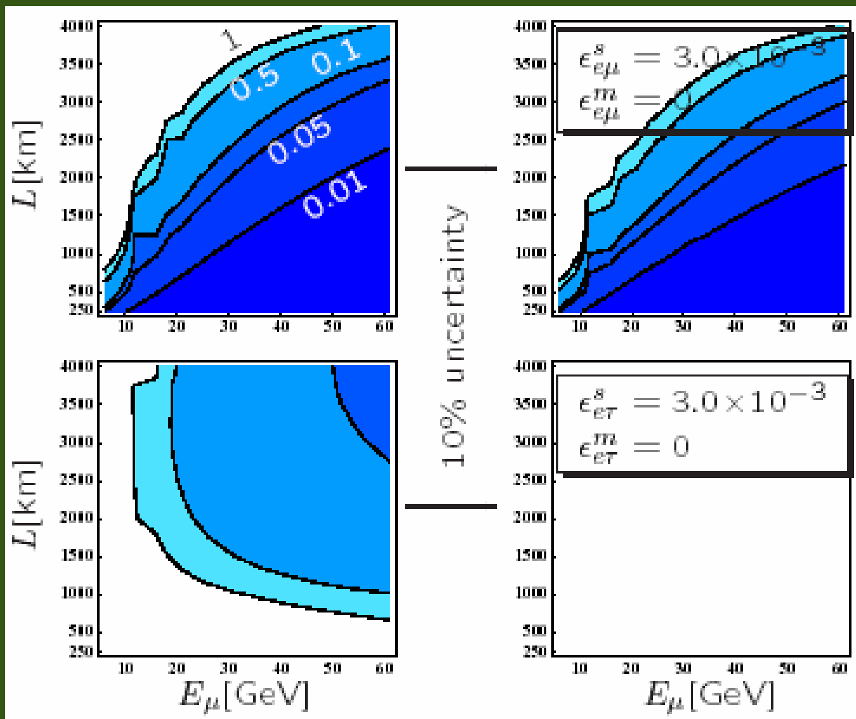
Gonzalez-Garcia, Grossman, Gusso, Nir, Phys. Rev. D64, 096006 (2001)

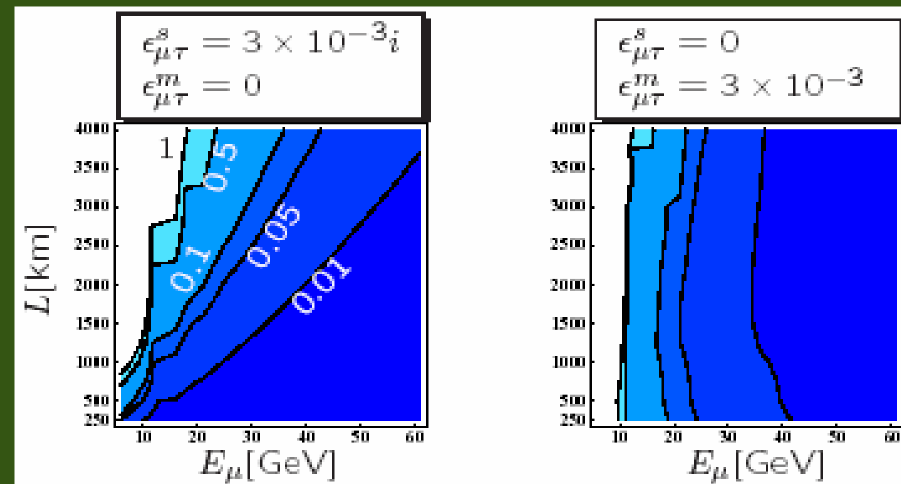
Sensitivity to $\varepsilon_{e\mu}$, $\varepsilon_{e\tau}$, $\varepsilon_{\mu\tau}$ at ν factory

Ota-Sato-Yamashita, PRD65:093015,'02

$\nu_e \to \nu_\mu$ $\quad \left|\varepsilon^s_{e\mu}\right| = 3\times10^{-3}$ $\quad \left|\varepsilon^s_{e\tau}\right| = 3\times10^{-3}$

$\nu_\mu \to \nu_\tau$ $\quad \left|\varepsilon^{s,m}_{\mu\tau}\right| = 3\times10^{-3}$





Sato: ISS 2nd plenary @ KEK

■ The expected sensitivity is $\epsilon \gtrsim \mathcal{O}(10^{-4})$.

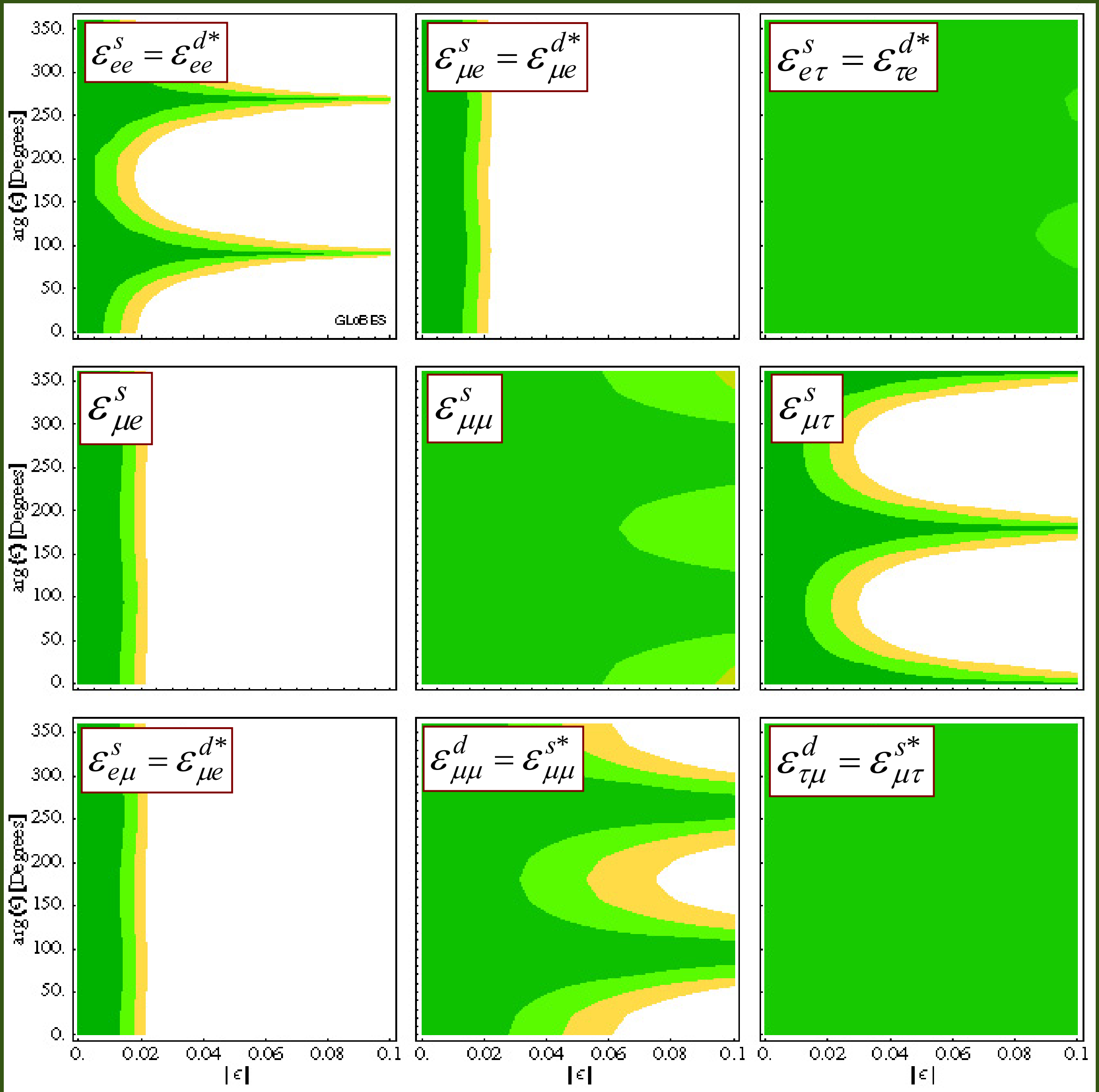
	$\epsilon^{s,m}_{e\mu}\ (\epsilon^s_{\mu e})$	$\epsilon^{s,m}_{e\tau}$	$\epsilon^{s,m}_{\mu\tau}$
$\nu_e \to \nu_\mu$	△	△	×
$\nu_\mu \to \nu_\mu$	×	×	○
$\nu_e \to \nu_\tau$	×	○	△
$\nu_\mu \to \nu_\tau$	×	△	○
$\nu_\mu \to \nu_e$	△	×	×
$\nu_e \to \nu_e$	×	×	×

Statistics + some correlations of errors

Required data size in unit of 10^{21} μ × 100kt

Sensitivity by Noνa + DC-200kt

Kopp, Lindner, Ota, Sato, PRD77:013007,2008



Statistical + systematic errors + correlations of errors + degeneracies

T2K + DCHOOZ

$$|\varepsilon^s_{\mu e}| < 3\times10^{-2}$$

Noνa + DC-200kt

$$|\varepsilon^s_{\mu e}| < 1.5\times10^{-2}$$

5. New Physics in propagation (matter effect)

5.1 Constraints from various ν experiments
(CHARM, LEP, LSND, NuTeV)

Davidson, Pena-Garay, Rius, Santamaria, JHEP 0303:011,2003

See also Berezhiani and A. Rossi, PLB535 ('02) 207; Barranco et al., PRD73 ('06) 113001; Barranco et al., arXiv:0711.0698.

Because of the relation $\epsilon_{\alpha\beta} \sim \epsilon^{e}_{\alpha\beta} + 3\epsilon^{u}_{\alpha\beta} + 3\epsilon^{d}_{\alpha\beta}$, the dominant contribution comes from the quantities with largest errors.

$$\begin{pmatrix} -4 \lesssim \epsilon_{ee} \lesssim 2.6 & |\epsilon_{e\mu}| \lesssim 2.8\times 10^{-4} & |\epsilon_{e\tau}| \lesssim 1.9 \\ |\epsilon_{e\mu}| \lesssim 2.8\times 10^{-4} & -0.05 \lesssim \epsilon_{\mu\mu} \lesssim 0.08 & |\epsilon_{\mu\tau}| \lesssim 0.25 \\ |\epsilon_{e\tau}| \lesssim 1.9 & |\epsilon_{\mu\tau}| \lesssim 0.25 & |\epsilon_{\tau\tau}| \lesssim 19 \end{pmatrix}$$

ε_{ee} , $\varepsilon_{e\tau}$, $\varepsilon_{\tau\tau}$ ~O(1) are consistent with accelerator experiments data

update of JHEP 0303:011,2003 on July 1, 2008 courtesy by Sacha Davidson

Only the bound on | $\varepsilon_{e\mu}$ | is modified:

$$\begin{pmatrix} -4 \lesssim \epsilon_{ee} \lesssim 2.6 & |\epsilon_{e\mu}| \lesssim 2.8 \times 10^{-4} & |\epsilon_{e\tau}| \lesssim 1.9 \\ |\epsilon_{e\mu}| \lesssim 2.8 \times 10^{-4} & -0.05 \lesssim \epsilon_{\mu\mu} \lesssim 0.08 & |\epsilon_{\mu\tau}| \lesssim 0.25 \\ |\epsilon_{e\tau}| \lesssim 1.9 & |\epsilon_{\mu\tau}| \lesssim 0.25 & |\epsilon_{\tau\tau}| \lesssim 19 \end{pmatrix}$$

$$\begin{pmatrix} -4 \lesssim \epsilon_{ee} \lesssim 2.6 & |\epsilon_{e\mu}| \lesssim 1.4 \times 10^{-4} & |\epsilon_{e\tau}| \lesssim 1.9 \\ |\epsilon_{e\mu}| \lesssim 1.4 \times 10^{-4} & -0.05 \lesssim \epsilon_{\mu\mu} \lesssim 0.08 & |\epsilon_{\mu\tau}| \lesssim 0.25 \\ |\epsilon_{e\tau}| \lesssim 1.9 & |\epsilon_{\mu\tau}| \lesssim 0.25 & |\epsilon_{\tau\tau}| \lesssim 19 \end{pmatrix}$$

current limits	current limits
$|\varepsilon^{eP}_{\tau\mu}| < 1.2$ $(\tau \to \mu\bar{e}e)^{*)}$ $|\varepsilon^{eP}_{\tau\mu}| < 0.1$ CHARM II	$|\varepsilon^{eP}_{\tau\mu}| < .31$ $(\tau \to \mu\bar{e}e)^{*)}$ $|\varepsilon^{eP}_{\tau\mu}| < 0.1$ CHARM II
$|\varepsilon^{uP}_{\tau\mu}| < 2.8$ $(\tau \to \mu\rho)^{*)}$ $|\varepsilon^{uP}_{\tau\mu}| < 0.05$ NuTeV	$|\varepsilon^{uP}_{\tau\mu}| < .50$ $(\tau \to \mu\rho)^{*)}$ $|\varepsilon^{uP}_{\tau\mu}| < 0.05$ NuTeV
$|\varepsilon^{dP}_{\tau\mu}| < 2.8$ $(\tau \to \mu\rho)^{*)}$ $|\varepsilon^{dP}_{\tau\mu}| < 0.05$ NuTeV	$|\varepsilon^{dP}_{\tau\mu}| < .50$ $(\tau \to \mu\rho)^{*)}$ $|\varepsilon^{dP}_{\tau\mu}| < 0.05$ NuTeV
$|\varepsilon^{eP}_{\mu e}| < 5 \times 10^{-4}$ $(\mu \to 3e)^{*)}$	$|\varepsilon^{eP}_{\mu e}| < 5 \times 10^{-4}$ $(\mu \to 3e)^{*)}$
$|\varepsilon^{uP}_{\mu e}| < 7.7 \times 10^{-4}$ $(\mathrm{Ti}\mu \to \mathrm{Ti}e)^{*)}$	$|\varepsilon^{uP}_{\mu e}| < 3.1 \times 10^{-4}$ $(\mathrm{Ti}\mu \to \mathrm{Ti}e)^{*)}$
$|\varepsilon^{dP}_{\mu e}| < 7.7 \times 10^{-4}$ $(\mathrm{Ti}\mu \to \mathrm{Ti}e)^{*)}$	$|\varepsilon^{dP}_{\mu e}| < 3.1 \times 10^{-4}$ $(\mathrm{Ti}\mu \to \mathrm{Ti}e)^{*)}$
$|\varepsilon^{eP}_{\tau e}| < 2.9$ $(\tau \to e\bar{e}e)^{*)}$ $|\varepsilon^{eL}_{\tau e}| < 0.4, |\varepsilon^{eR}_{\tau e}| < 0.7$ LSND	$|\varepsilon^{eP}_{\tau e}| < .76$ $(\tau \to e\bar{e}e)^{*)}$ $|\varepsilon^{eL}_{\tau e}| < 0.4, |\varepsilon^{eR}_{\tau e}| < 0.7$ LSND
$|\varepsilon^{uP}_{\tau e}| < 1.6$ $(\tau \to e\rho)^{*)}$ $|\varepsilon^{uP}_{\tau e}| < 0.5$ CHARM	$|\varepsilon^{uP}_{\tau e}| < .9$ $(\tau \to e\rho)^{*)}$ $|\varepsilon^{uP}_{\tau e}| < 0.5$ CHARM
$|\varepsilon^{dP}_{\tau e}| < 1.6$ $(\tau \to e\rho)^{*)}$ $|\varepsilon^{dP}_{\tau e}| < 0.5$ CHARM	$|\varepsilon^{dP}_{\tau e}| < .9$ $(\tau \to e\rho)^{*)}$ $|\varepsilon^{dP}_{\tau e}| < 0.5$ CHARM

5.2 Phenomenology with ε_{ee} , $\varepsilon_{e\tau}$, $\varepsilon_{\tau\tau}$ ~O(1)

● Constraints from ν_{atm}

Friedland-Lunardini-Maltoni, PRD70:111301,'04; Friedland-Lunardini, PRD72:053009,'05

ε_{ee} , $\varepsilon_{e\tau}$, $\varepsilon_{\tau\tau}$ ~O(1) are consistent with atmospheric neutrino data, provided that

$$\epsilon_{\tau\tau} \simeq \frac{|\epsilon_{e\tau}|^2}{1+\epsilon_{ee}} \qquad -1 \lesssim \epsilon_{ee} \lesssim 1.5 \qquad 0 \leq |\epsilon_{e\tau}| \lesssim 1+\epsilon_{ee}$$

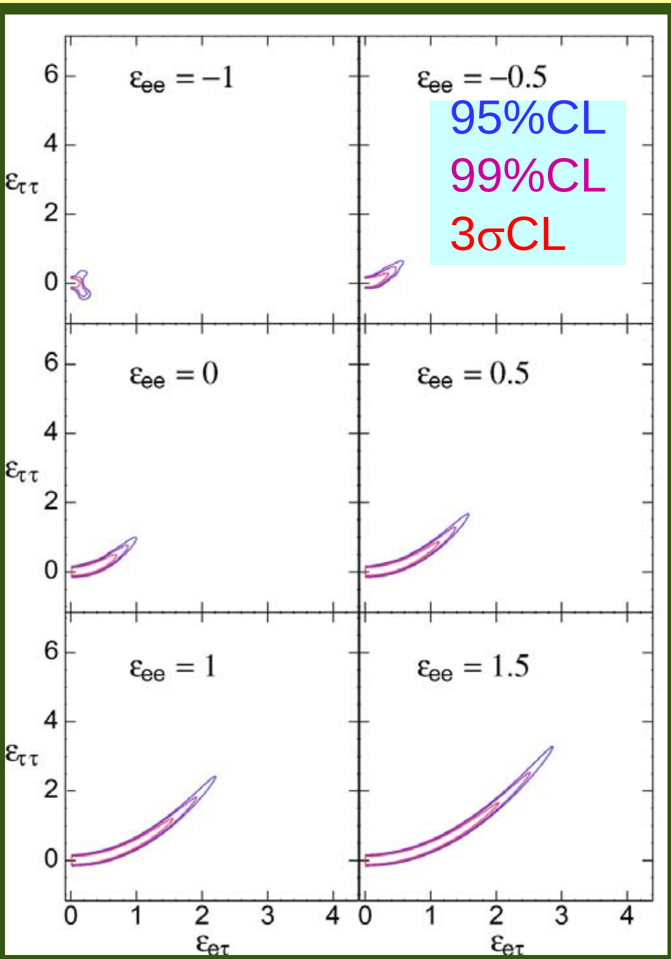
$$2.5\times10^{-3}\mathrm{eV}^2 = \Delta m^2_{\rm atm} \simeq \frac{2\cos^2\beta}{1+\cos^2\beta}|\Delta m^2_{31}|$$

$$1.0 = \sin^2 2\theta_{\rm atm} \simeq \frac{4\cos^2\beta}{(1+\cos^2\beta)^2}$$

$$\tan\beta \equiv \frac{|\epsilon_{e\tau}|}{1+\epsilon_{ee}} \qquad \gamma \equiv \frac{1}{2}\arg(\epsilon_{e\tau})$$

$$A\begin{pmatrix} 1+\epsilon_{ee} & 0 & \epsilon_{e\tau} \\ 0 & 0 & 0 \\ \epsilon^*_{e\tau} & 0 & \epsilon_{\tau\tau} \end{pmatrix}$$

$$\equiv\; e^{i\gamma\,\mathrm{diag}(1,0-1)}e^{-i\beta\lambda_5}\mathrm{diag}\left(\lambda_{e'},0,0\right)e^{i\beta\lambda_5}e^{-i\gamma\,\mathrm{diag}(1,0-1)}$$



- **Exact analytical formula for the oscillation probability with ε_{ee} , $\varepsilon_{e\tau}$, $\varepsilon_{\tau\tau}$ in the limit $\Delta m^2_{21} \to 0$**

OY arXiv:0704.1531 [hep-ph]

$$U\,\mathrm{diag}(0,0,\Delta E_{31})\,U^{-1} + A\begin{pmatrix} 1+\epsilon_{ee} & 0 & \epsilon_{e\tau} \\ 0 & 0 & 0 \\ \epsilon_{e\tau}^* & 0 & \epsilon_{\tau\tau} \end{pmatrix} = \tilde{U}\,\mathrm{diag}(\Lambda_-,0,\Lambda_+)\,\tilde{U}^{-1}$$

Eigenvalues can be exactly obtained in the limit $\Delta m^2_{21} \to 0$:

$$\Lambda_\pm = \frac{1}{2}\left[\Delta E_{31} + \frac{A(1+\epsilon_{ee})}{\cos^2\beta}\right] \pm \frac{1}{2}\sqrt{\left[\Delta E_{31}\cos 2\theta''_{13} - \frac{A(1+\epsilon_{ee})}{\cos^2\beta}\right]^2 + (\Delta E_{31}\sin 2\theta''_{13})^2}$$

$$\tan\beta = \frac{|\epsilon_{e\tau}|}{1+\epsilon_{ee}} \qquad \epsilon_{\tau\tau} = \frac{|\epsilon_{e\tau}|^2}{1+\epsilon_{ee}} \qquad \theta''_{13} = \sin^{-1}\left|e^{-i\arg(\epsilon_{e\mu})}U_{e3}\cos\beta + U_{\tau 3}\sin\beta\right|$$

This includes corrections to all orders in θ_{13}, ε_{ee} , $\varepsilon_{e\tau}$, $\varepsilon_{\tau\tau}$.
The assumptions are Δm^2_{21} =0 and $\varepsilon_{\tau\tau} = |\varepsilon_{e\tau}|^2/(1+\varepsilon_{ee})$

Corrections in Δm^2_{21} and other $\varepsilon_{\alpha\beta}$ can be also obtained to first order.

Once the eigenvalues are known, we can easily get the analytical formula for the oscillation probability by KTY's method

Kimura, Takamura, Yokomakura (PLB537:86,2002)



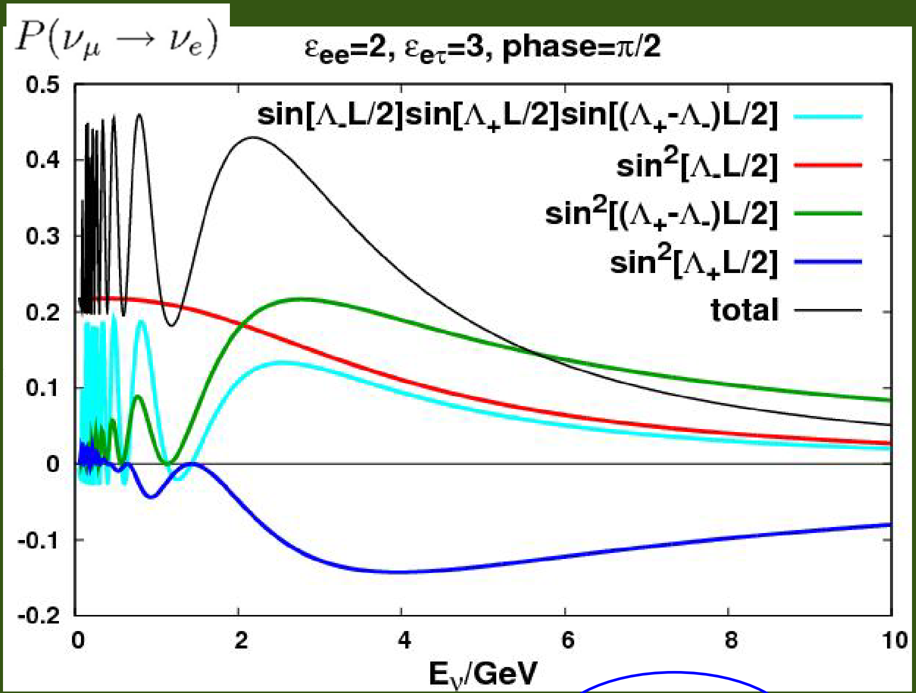
$$U\mathcal{E}U^{-1} + \mathcal{A} \equiv \tilde{U} \begin{pmatrix} \tilde{E}_1 & 0 & 0 \\ 0 & \tilde{E}_2 & 0 \\ 0 & 0 & \tilde{E}_3 \end{pmatrix} \tilde{U}^{-1}$$

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) \; &= \; \delta_{\alpha\beta} - 4 \sum_{j<k} \mathrm{Re}\left(\tilde{X}_j^{\alpha\beta} \tilde{X}_k^{\alpha\beta *}\right) \sin^2\left(\frac{\Delta \tilde{E}_{jk} L}{2}\right) \\ &\quad -2 \sum_{j<k} \mathrm{Im}\left(\tilde{X}_j^{\alpha\beta} \tilde{X}_k^{\alpha\beta *}\right) \sin\left(\Delta \tilde{E}_{jk} L\right), \end{aligned}$$

$$\begin{pmatrix} \tilde{X}_1^{\alpha\beta} \\ \tilde{X}_2^{\alpha\beta} \\ \tilde{X}_3^{\alpha\beta} \end{pmatrix} = \begin{pmatrix} \frac{1}{\Delta \tilde{E}_{21} \Delta \tilde{E}_{31}} (\tilde{E}_2 \tilde{E}_3, \; -(\tilde{E}_2 + \tilde{E}_3), \; 1) \\ \frac{-1}{\Delta \tilde{E}_{21} \Delta \tilde{E}_{32}} (\tilde{E}_3 \tilde{E}_1, \; -(\tilde{E}_3 + \tilde{E}_1), \; 1) \\ \frac{1}{\Delta \tilde{E}_{31} \Delta \tilde{E}_{32}} (\tilde{E}_1 \tilde{E}_2, \; -(\tilde{E}_1 + \tilde{E}_2), \; 1) \end{pmatrix} \begin{pmatrix} \delta_{\alpha\beta} \\ \left[U\mathcal{E}U^{-1} + \mathcal{A}\right]_{\alpha\beta} \\ \left[(U\mathcal{E}U^{-1} + \mathcal{A})^2\right]_{\alpha\beta} \end{pmatrix}$$

The problem of obtaining the exact analytical oscillation probability is reduced to obtaining only the eigenvalues!

• ν_e appearance probability



$$P(\nu_\mu \to \nu_e) =$$

$$-4\mathrm{Re}\left(\tilde{X}_1^{\mu e}\tilde{X}_2^{\mu e*}\right)\sin^2\left(\frac{\Lambda_- L}{2}\right)$$

$$-4\mathrm{Re}\left(\tilde{X}_2^{\mu e}\tilde{X}_3^{\mu e*}\right)\sin^2\left(\frac{\Lambda_+ L}{2}\right)$$

$$-4\mathrm{Re}\left(\tilde{X}_1^{\mu e}\tilde{X}_3^{\mu e*}\right)\sin^2\left[\frac{(\Lambda_+ - \Lambda_-)L}{2}\right]$$

$$-8\mathrm{Im}\left(\tilde{X}_1^{\mu e}\tilde{X}_2^{\mu e*}\right)\sin\left(\frac{\Lambda_- L}{2}\right)\sin\left(\frac{\Lambda_+ L}{2}\right)\sin\left[\frac{(\Lambda_+ - \Lambda_-)L}{2}\right]$$

$$\tilde{X}_1^{\mu e} = \frac{-1}{\Lambda_-(\Lambda_+ - \Lambda_-)}[\xi + \eta e^{-i(\arg(\epsilon_{e\mu})+\delta)} - \Lambda_+\zeta]$$

$$\tilde{X}_2^{\mu e} = \frac{1}{\Lambda_+\Lambda_-}[\xi + \eta e^{-i(\arg(\epsilon_{e\mu})+\delta)} - (\Lambda_+ + \Lambda_-)\zeta]$$

$$\tilde{X}_3^{\mu e} = \frac{1}{\Lambda_+(\Lambda_+ - \Lambda_-)}[\xi + \eta e^{-i(\arg(\epsilon_{e\mu})+\delta)} - \Lambda_-\zeta]$$

$$\xi \equiv [(\Delta E_{31})^2 + A(1+\epsilon_{ee})\Delta E_{31}]U_{\mu 3}|U_{e3}|$$

$$\eta \equiv A\Delta E_{31}|\epsilon_{e\tau}|U_{\mu 3}U_{\tau 3}$$

$$\zeta \equiv \Delta E_{31}U_{\mu 3}|U_{e3}|,$$

• ν_μ disappearance probability

$$1 - P(\nu_\mu \to \nu_\mu) \simeq 4\left(\frac{U_{\mu 3}^2}{c_{13}''^2}\right)\left(1 - \frac{U_{\mu 3}^2}{c_{13}''^2}\right)\sin^2\left(\frac{\Delta E_{31} c_{13}''^2 L}{2}\right)$$

$$(E \to \infty)$$

At first sight NP with ε_{ee} , $\varepsilon_{e\tau}$, $\varepsilon_{\tau\tau}$ ~O(1) seems to be inconsistent with ν_{atm}, but it is not the case:

- **At high energy limit, matter effect becomes dominant, but because the two eigenvalues of matter potential matrix turn out to be zero, the scenario is reduced to vacuum oscillation which is consistent with the high energy atmospheric neutrino data.**

A>> |ΔE$_{jk}$|

$$1 - P(\nu_\mu \to \nu_\mu) \simeq 4\left(\frac{U_{\mu 3}^2}{c_{13}''^2}\right)\left(1 - \frac{U_{\mu 3}^2}{c_{13}''^2}\right)\sin^2\left(\frac{\Delta E_{31} c_{13}''^2 L}{2}\right)$$

$$c_{13}''^2 \equiv 1 - \left|c_\beta e^{-i\gamma} U_{e3} + s_\beta e^{i\gamma} U_{\tau 3}\right|^2 \simeq 1 - s_\beta^2 c_{23}^2$$

$$(\theta_{13} \to 0)$$

In the limit $\theta_{13} \to 0$

$$4\left(\frac{U_{\mu 3}^2}{c_{13}''^2}\right)\left(1 - \frac{U_{\mu 3}^2}{c_{13}''^2}\right) = 1 \quad \to \quad s_{23}^2 = \frac{c_\beta^2}{1 + c_\beta^2}$$

$$2.5 \times 10^{-3}\mathrm{eV}^2 = |\Delta m_{31}^2| c_{13}''^2 \quad \to \quad |\Delta m_{31}^2| = \frac{1 + c_\beta^2}{2 c_\beta^2} \times 2.5 \times 10^{-3}\mathrm{eV}^2$$

Friedland-Lunardini, PRD72:053009,'05

- At low energy or at shorter baseline (such as K2K & MINOS), matter effect is negligible compared to ΔE_{jk}, so the sub-GeV atmospheric and K2K + MINOS data are supposed to give us the true value for Δm^2_{23} and $\sin^2 2\theta_{23} \rightarrow c^2_\beta > 0.45$.

Friedland-Lunardini, PRD72:053009,'05

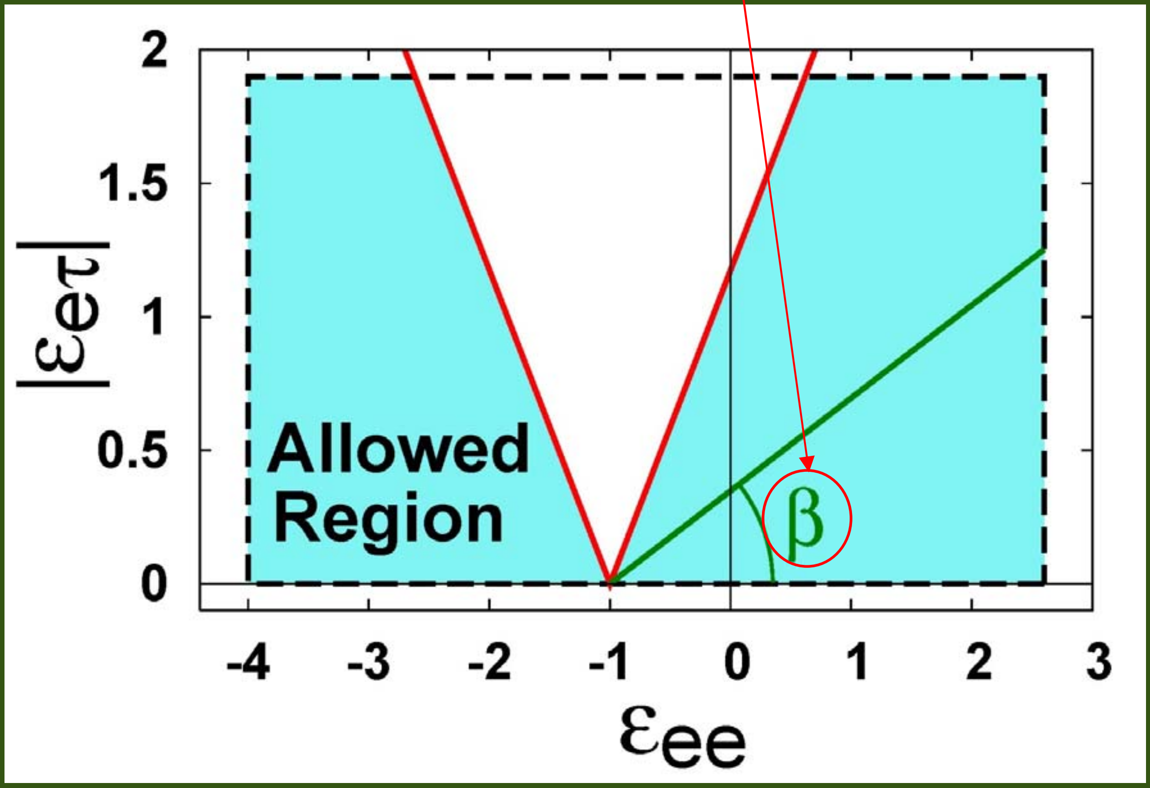
- The multi-GeV atmospheric data should in principle give us a signature for NP, but statistics is so low that we can't say anything conclusive.

→ Will HK improve the situation?

Thus NP with ε_{ee} , $\varepsilon_{e\tau}$, $\varepsilon_{\tau\tau}$ ~O(1) is consistent with ν_{atm} for the moment.

So, as approximation, we can eliminate $\varepsilon_{\tau\tau}$ by $\varepsilon_{\tau\tau} = |\varepsilon_{e\tau}|^2/(1+\varepsilon_{ee})$ and we can reduce the problem in $(\varepsilon_{ee}, |\varepsilon_{e\tau}|, \varepsilon_{\tau\tau})$ to the allowed region in $(\varepsilon_{ee}, |\varepsilon_{e\tau}|)$.

$|\varepsilon_{e\tau}|=\tan\beta(1+\varepsilon_{ee})$ (β stands for the gradient of the line),
$c^2_\beta>0.45 \rightarrow \tan^2\beta<1.2$

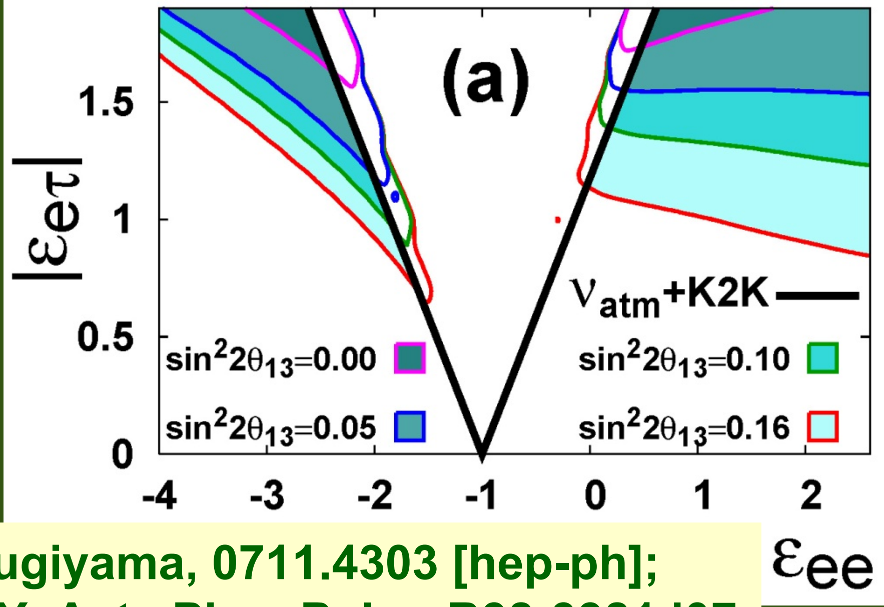


degeneracy:
$(1+\varepsilon_{ee}) \Leftrightarrow -(1+\varepsilon_{ee})$
$\Delta m^2_{31} \Leftrightarrow -\Delta m^2_{31}$

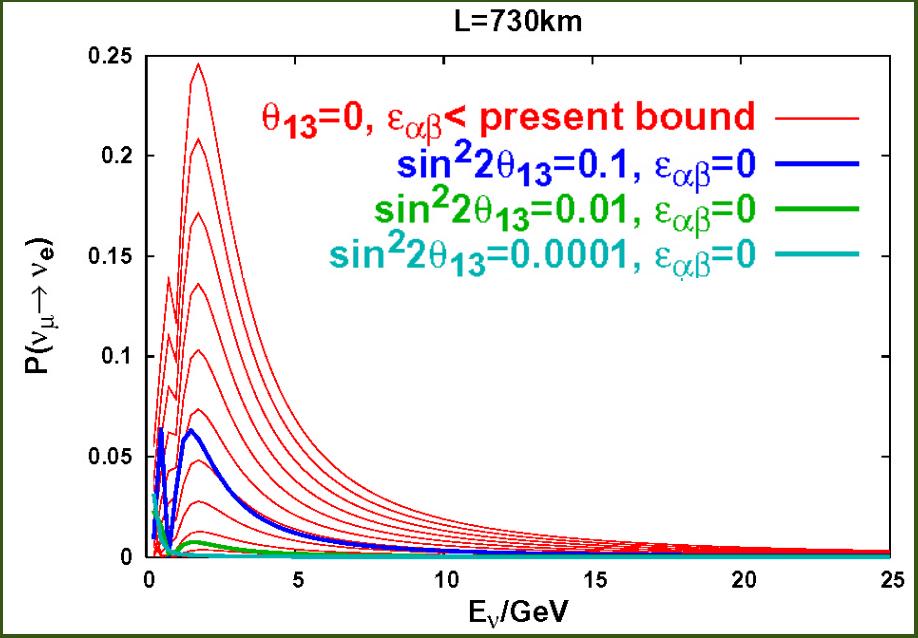
● ν_e appearance

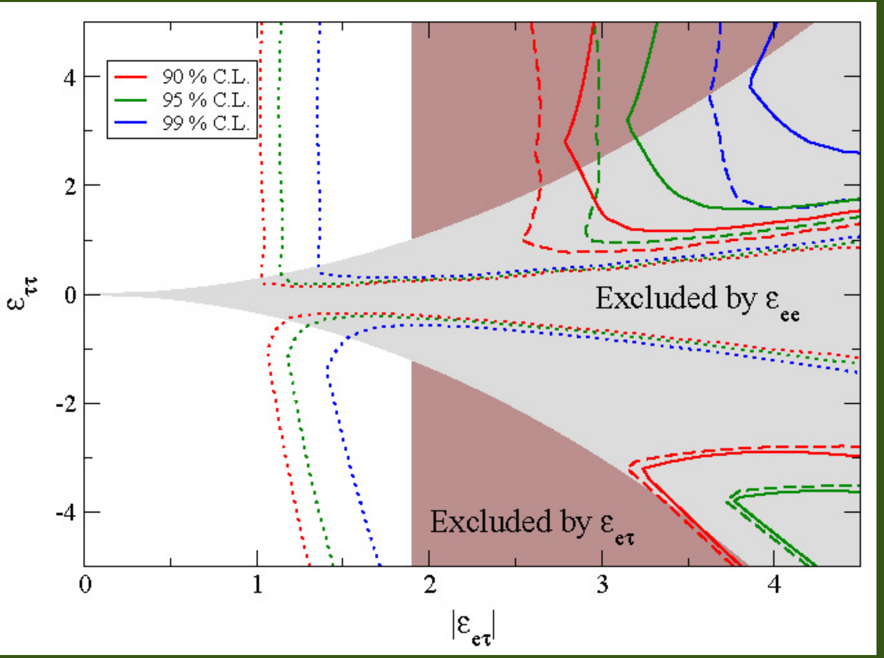
Region testable by ν_e appearance at MINOS

If $|\varepsilon_{e\tau}|$ is very large then we may be able to prove the existence of NP from ν_e appearance at MINOS



Sugiyama, 0711.4303 [hep-ph]; OY, Acta Phys.Polon.B38:3381,'07

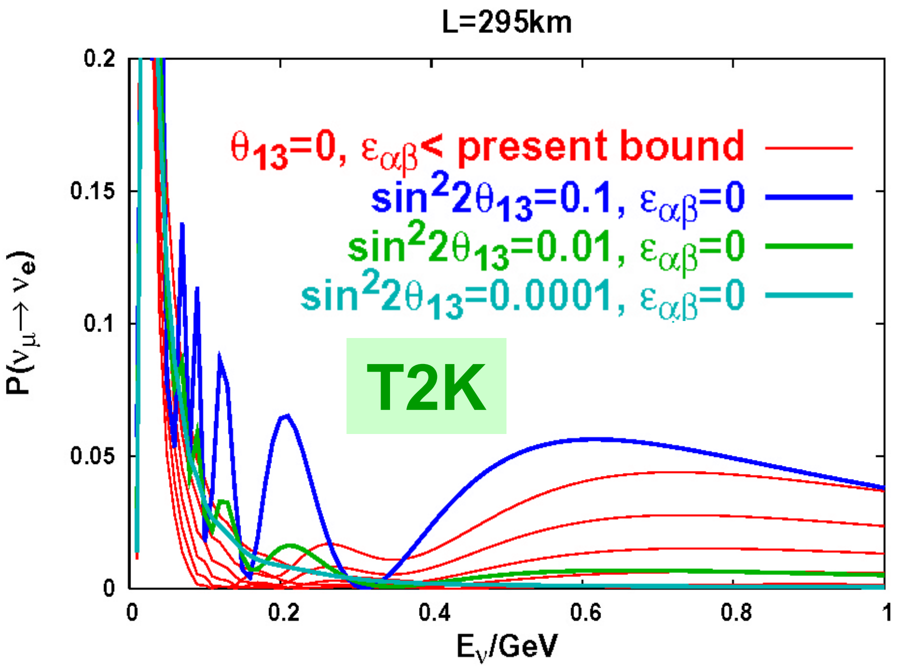


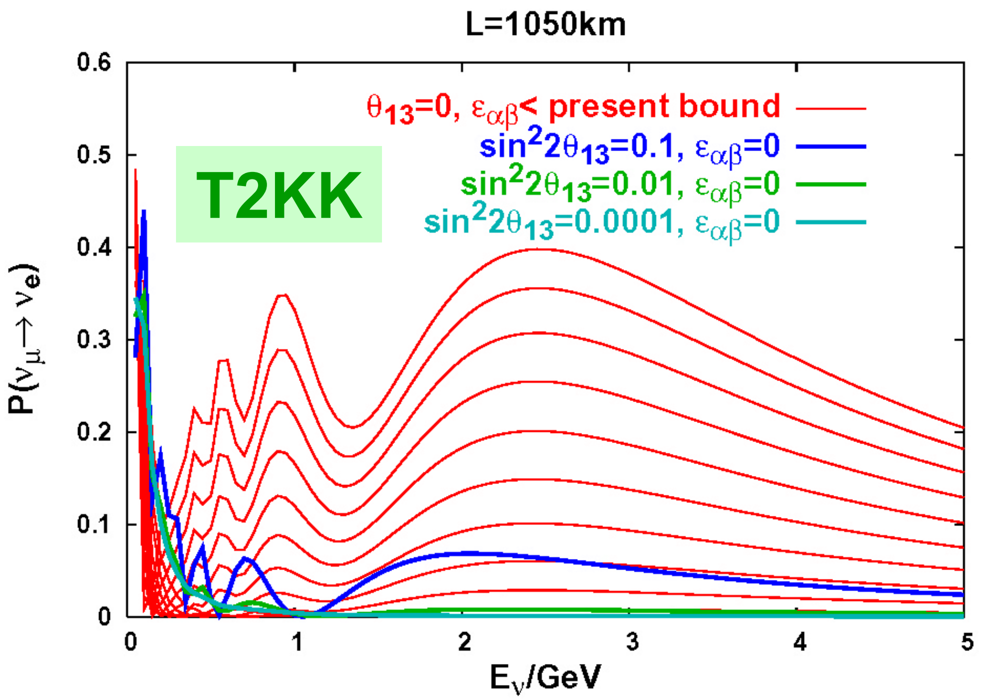


Blennow-Ohlsson-Skrotzki, PLB660:522,'08

ν_e appearance at T2K(K)

T2K(T2KK) is insensitive (sensitive) to $|\varepsilon_{e\tau}|$, so if $|\varepsilon_{e\tau}|$ is very large then we may be able to prove the existence of NP from ν_e appearance by T2K-T2KK complex.





● ν_μ disappearance

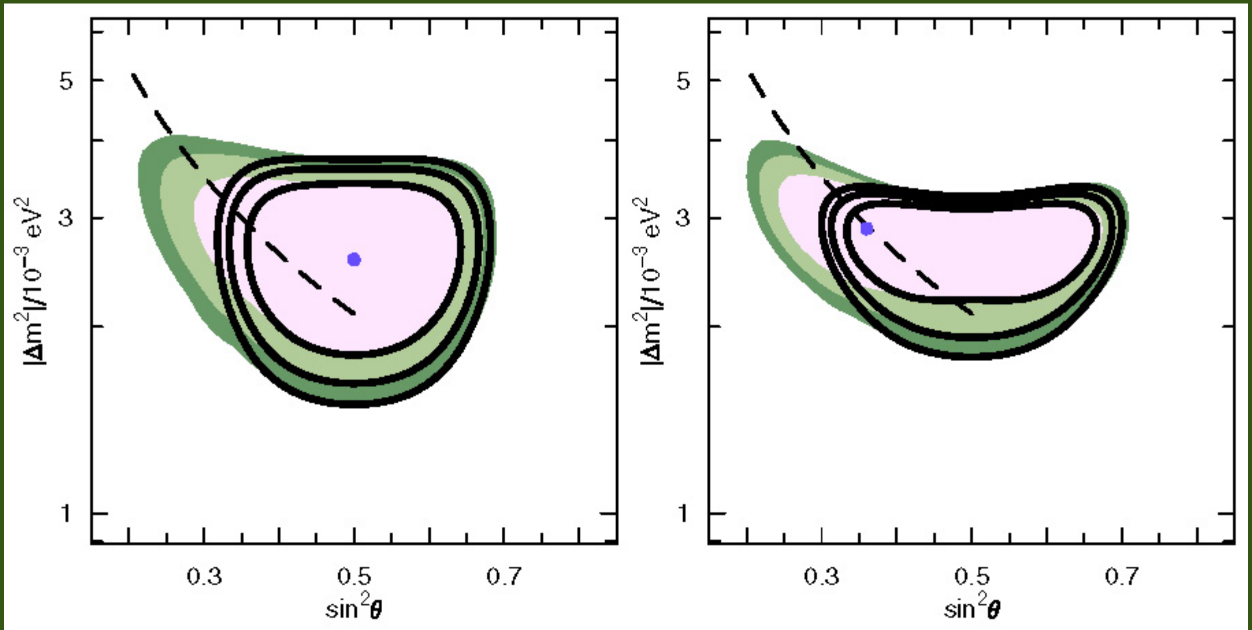
Region testable by ν_μ disappearance at MINOS

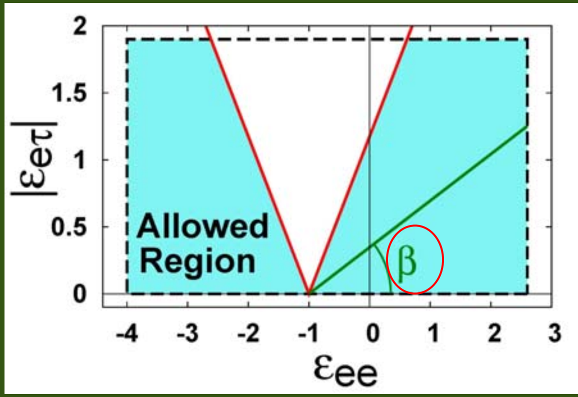
Friedland, Lunardini, PRD74:033012,'06

Because of the two constraints due to ν_{atm}, $\sin^2 2\theta_{23}$ and Δm^2_{23} are correlated:

$$\frac{\sin^2 2\theta_{\rm atm}}{\sin^2 2\theta_{23}} = \frac{(1+c_\beta^2)^2}{4c_\beta^2}$$

$$\frac{\Delta m^2_{\rm atm}}{|\Delta m^2_{31}|} = \frac{2c_\beta^2}{1+c_\beta^2}$$





The allowed region before (left) and after (right) the first MINOS results

Speculation on ν_μ disappearance at T2K

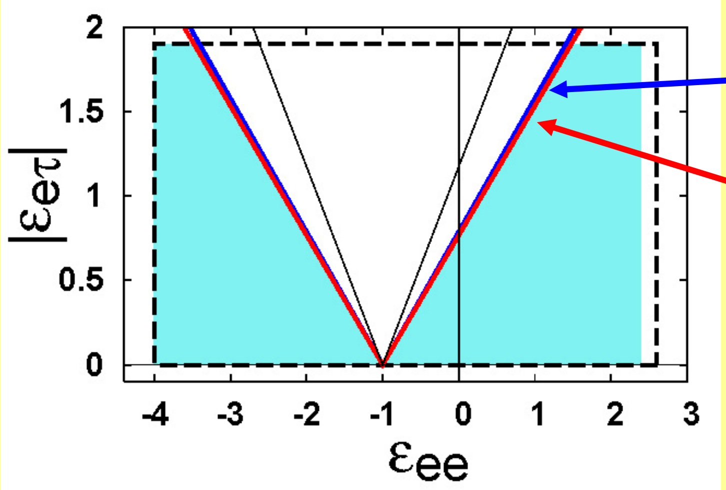
Determination of ε_{ee} , $\varepsilon_{e\tau}$, $\varepsilon_{\tau\tau}$ from future long baseline experiments with $P_{\mu\mu}$:

$$\frac{\sin^2 2\theta_{\rm atm}}{\sin^2 2\theta_{23}} = \frac{(1+c_\beta^2)^2}{4c_\beta^2} \qquad \frac{2c_\beta^2}{1+c_\beta^2} = \frac{\Delta m^2_{\rm atm}}{|\Delta m^2_{31}|}$$

- $0.94 < \sin^2 2\theta_{atm} <= 1$ (Raaf@ν2008)
- $\Delta m^2_{atm} = (2.2 + 0.45 - 0.55) \times 10^{-3}\,eV^2$ (Raaf@ν2008)

Even if $|\Delta m^2_{31}|$ is determined by T2K precisely in the future, if the central values are the same as in ν_{atm}, then the errors in $\sin^2 2\theta_{atm}$ (+-6%) and Δm^2_{atm} (+-20%) will remain and will be dominant.

In the case where the central values are the same as in ν_{atm}



from $\sin^2 2\theta_{23}$

$\tan^2\beta < 0.79$

from Δm^2_{23}

$\tan^2\beta < 0.77$

The constraint may not improve very much.

Analysis of solar neutrinos with NP

Friedland, Lunardini, Pena-Garay
Phys.Lett.B594:347,2004

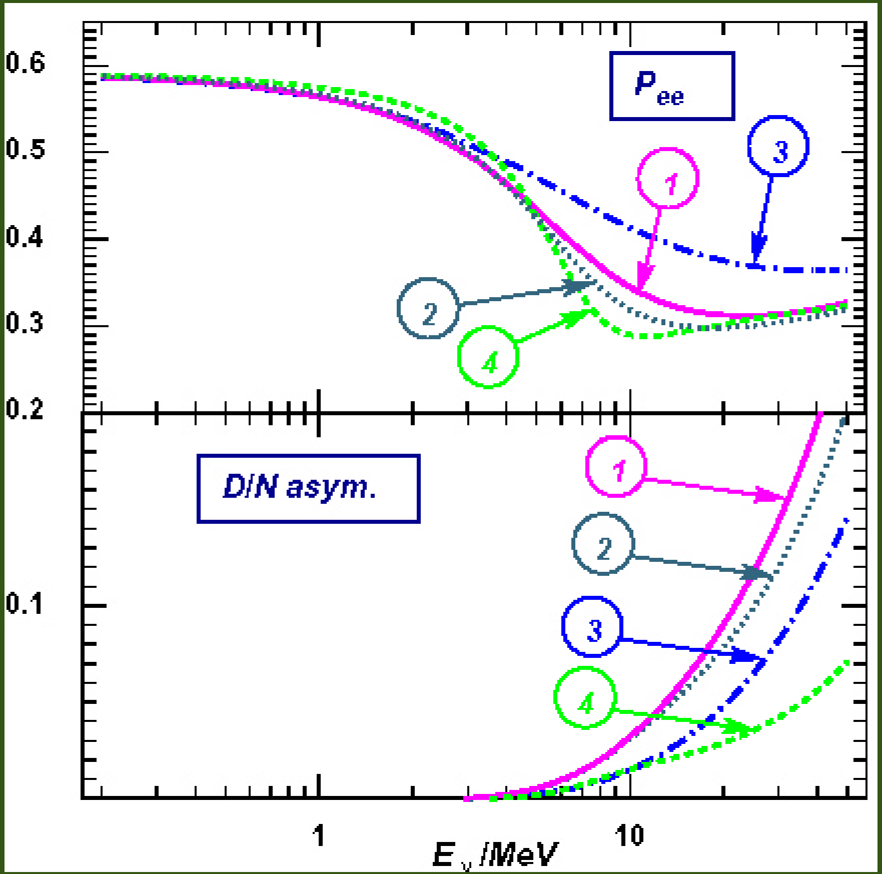


FIG. 1: The electron neutrino survival probability and the day/night asymmetry as a function of energy for $\Delta m^2 = 7\times 10^{-5}$ eV^2, $\tan^2\theta = 0.4$ and several representative values of the NSI parameters: (1) $\epsilon^u_{11} = \epsilon^d_{11} = \epsilon^u_{12} = \epsilon^d_{12} = 0$; (2) $\epsilon^u_{11} = \epsilon^d_{11} = -0.008$, $\epsilon^u_{12} = \epsilon^d_{12} = -0.06$; (3) $\epsilon^u_{11} = \epsilon^d_{11} = -0.044$, $\epsilon^u_{12} = \epsilon^d_{12} = 0.14$; (4) $\epsilon^u_{11} = \epsilon^d_{11} = -0.044$, $\epsilon^u_{12} = \epsilon^d_{12} = -0.14$. Recall that the parameters in Eq. (5) equal $\epsilon_{ij} = \epsilon^u_{ij} n_u/n_e + \epsilon^d_{ij} n_d/n_e$.

degeneracy:

$(1+\varepsilon_{ee}) \Leftrightarrow -(1+\varepsilon_{ee})$

$\theta_{12} \Leftrightarrow \pi/2-\theta_{12}$

The survival probability for solar neutrino can be obtained by KTY formalism

SM+m$_\nu$

$$P_{ee} = \frac{1}{2}\left(1 + \cos 2\theta_{12} \cos 2\tilde{\theta}_{12}\right)$$

$$\cos 2\tilde{\theta}_{12} \equiv \frac{\Delta E_{21} \cos 2\theta_{12} - A}{\Delta \tilde{E}_{21}}$$

$$\Delta \tilde{E}_{21} \equiv \sqrt{(\Delta E_{21} \cos 2\theta_{12} - A)^2 + (\Delta E_{21} \sin 2\theta_{12})^2}$$

For simplicity the non-adiabatic corrections are neglected

NP

$$P_{ee}^{NP} = \frac{1}{2}\left(1 + \cos 2\theta_{12} \cos 2\tilde{\theta}'_{12}\right)$$

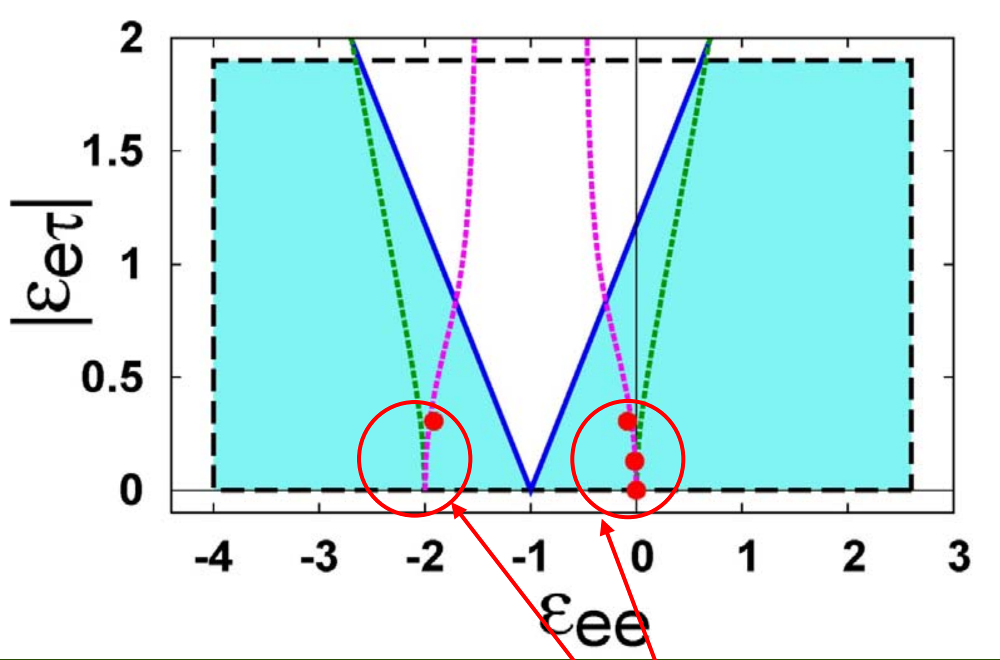
$$\cos 2\tilde{\theta}'_{12} \equiv \frac{\Delta E_{21} \cos 2\theta'_{12} - \dfrac{2c_\beta^2(1+\epsilon_{ee})}{1+c_\beta^2} A}{\Delta \tilde{E}'_{21}}$$

$$\Delta \tilde{E}'_{21} \equiv \sqrt{\left(\Delta E_{21} \cos 2\theta'_{12} - \frac{2(1+\epsilon_{ee})}{1+c_\beta^2} A\right)^2 + (\Delta E_{21} \sin 2\theta'_{12})^2}$$

$$\tan \theta'_{12} \equiv \frac{|c_\beta s_{12} - s_\beta c_{12} s_{23} e^{i\arg(\epsilon_{e\tau})}|}{|c_\beta c_{12} + s_\beta s_{12} s_{23} e^{i\arg(\epsilon_{e\tau})}|}$$

This agrees with the analytical formula in Friedland, Lunardini, Pena-Garay Phys.Lett.B594:347,'04

Solar ν may give a strong constraint on the allowed region in (ε_{ee} , $|\varepsilon_{e\tau}|$)



● : 4 points studied in detail by Friedland, Lunardini, Pena-Garay

green dotted line : $\frac{2(1+\epsilon_{ee})}{1+c_\beta^2}$ =1

magenta dotted line : $\frac{2c_\beta^2(1+\epsilon_{ee})}{1+c_\beta^2}$ =1

If $2c_\beta^2(1+\epsilon_{ee})/(1+c_\beta^2) \simeq 1$ and $2(1+\epsilon_{ee})/(1+c_\beta^2) \simeq 1$, then by adjusting the free parameter arg ($\varepsilon_{e\tau}$) in such a way that $\theta'_{12}=\theta_{12}$, we have $P_{ee}^{NP} \simeq P_{ee}$

Thus on the region in the red circles fit is expected to be good.

$$\cos 2\tilde{\theta}'_{12} \equiv \frac{\Delta E_{21}\cos 2\theta'_{12} - \frac{2c_\beta^2(1+\epsilon_{ee})}{1+c_\beta^2}A}{\Delta\tilde{E}'_{21}}$$

$$\Delta\tilde{E}'_{21} \equiv \sqrt{\left(\Delta E_{21}\cos 2\theta'_{12} - \frac{2(1+\epsilon_{ee})}{1+c_\beta^2}A\right)^2 + (\Delta E_{21}\sin 2\theta'_{12})^2}$$

However, off these lines fit may not be good. → Scanning the whole region may give a new constraint.

- **Summary of possibility with ε_{ee} , $\varepsilon_{e\tau}$, $\varepsilon_{\tau\tau}$ ~O(1)**

> **ν_e appearance at present/near future LBL:**
> **With longer baseline length (L> ~ 1000km), it may be possible to see the effect of NP if $|\varepsilon_{e\tau}|$ is very large.**

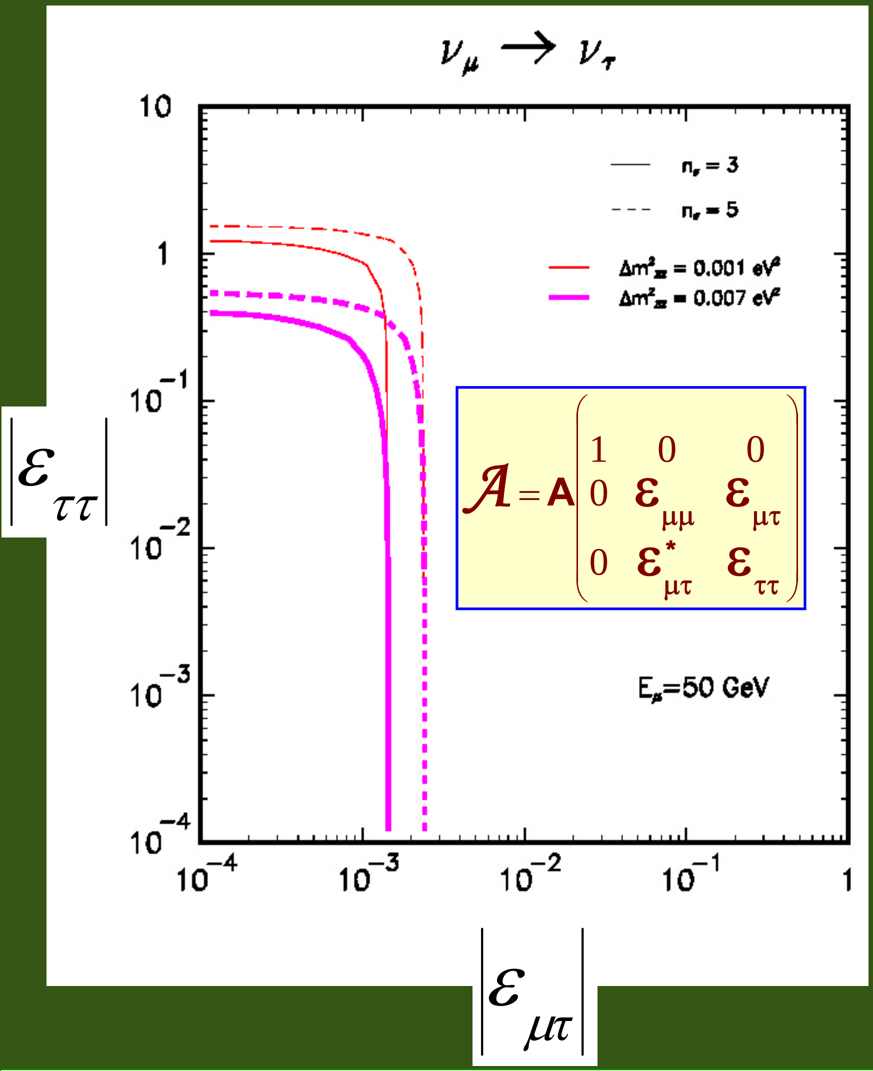
> **ν_μ disappearance at present/near future LBL :**
> **Because of dominant errors of atmospheric neutrino data, even if T2K measures $\sin^2 2\theta_{23}$ and $|\Delta m^2_{32}|$ precisely, it is difficult to give a strong constraint on ε_{ee} , $\varepsilon_{e\tau}$, $\varepsilon_{\tau\tau}$.**

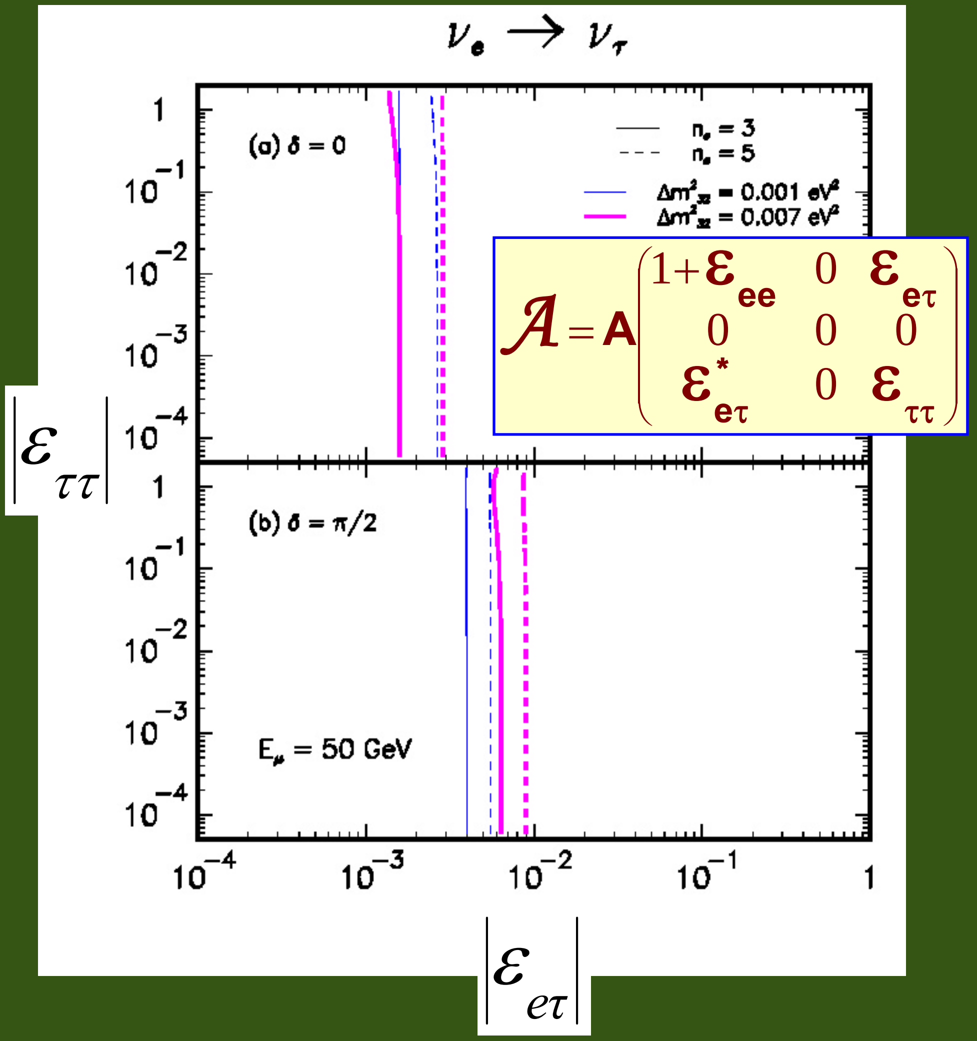
⇒ **ν factory will be necessary to pin down $|\varepsilon_{e\tau}|$ to <<1.**

5.3 Sensitivity to $\varepsilon_{\alpha\beta}$ (<<1) in future experiments

Sensitivity at ν factory

Statistics only

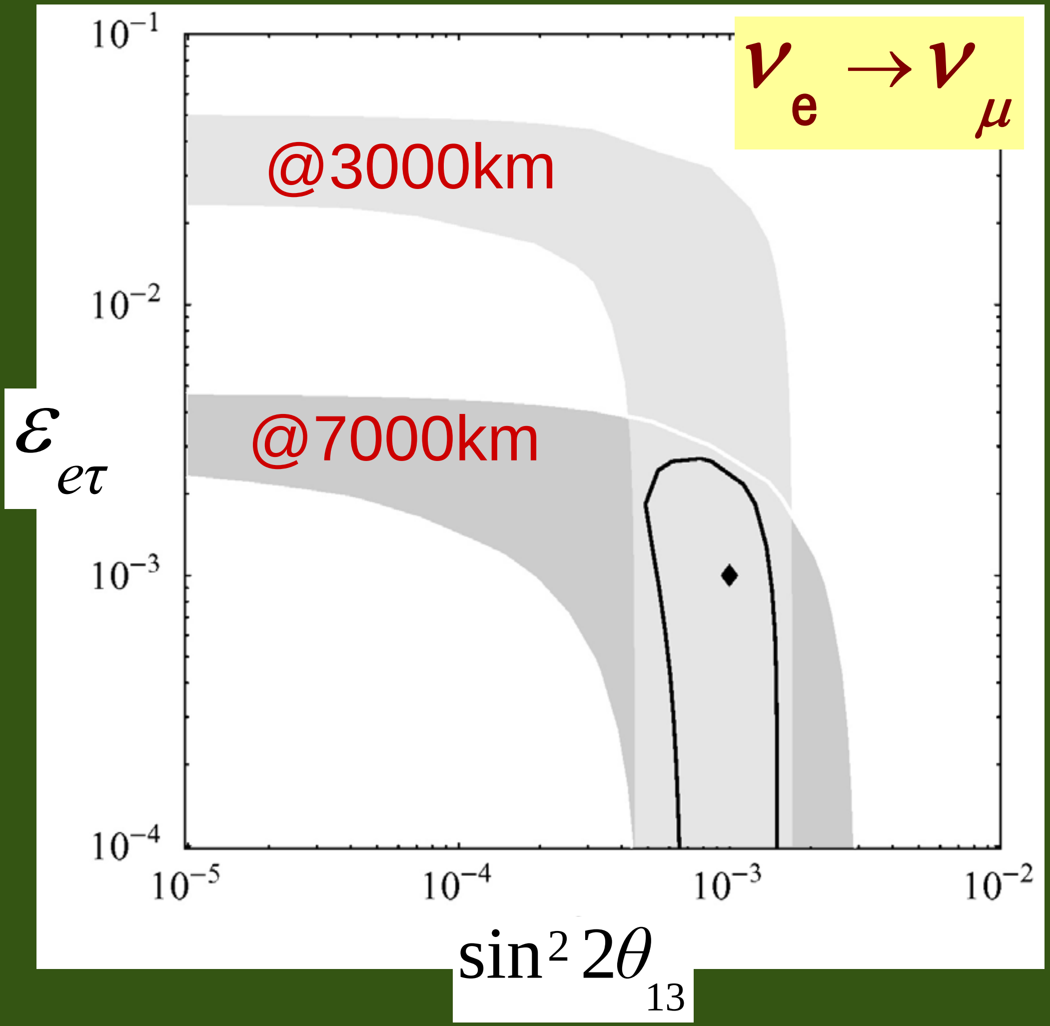




Gago-Guzzo-Nunokawa-Teves-Zukanovich Funchal, Phys.Rev.D64:073003,2001

Degeneracy between θ_{13} and $\varepsilon_{e\tau}$

ν factory



Degeneracy between θ_{13} and $\varepsilon_{e\tau}$ may be removed by combining two baselines

Huber, Schwetz, Valle, Phys. Rev. Lett. 88, 101804 (2002)

Sensitivity at ν factory

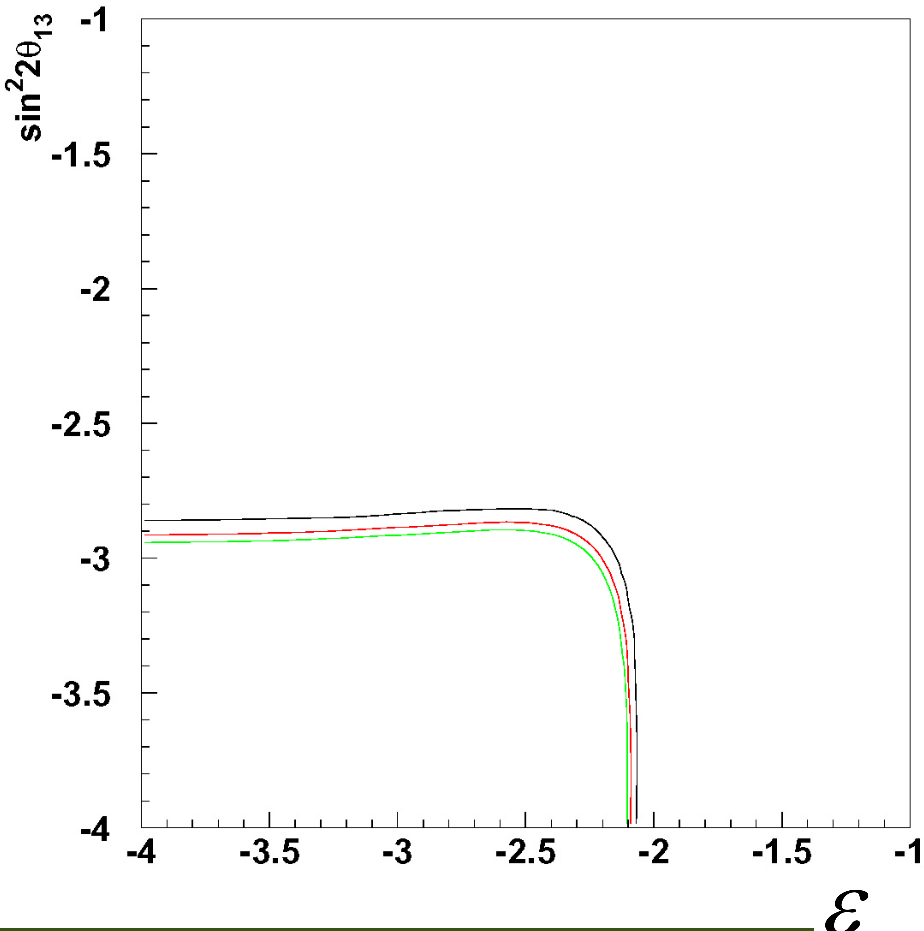
Campanelli-Romanino, PRD66:113001,'02

$\nu_e \rightarrow \nu_\tau$

Statistics only

high energy behavior

$$P(\nu_e \rightarrow \nu_\tau) \sim 4 \left| \epsilon_{\tau e} + \frac{E_{\rm res}}{E} c_{23} s_{13} \right|^2 \sin^2 \frac{LV}{2}$$



$\varepsilon_{e\tau}$

Davidson, Pena-Garay, Rius, Santamaria, JHEP 0303:011,2003

$$\epsilon_{\alpha\beta} \sim \epsilon^{e}_{\alpha\beta} + 3\epsilon^{u}_{\alpha\beta} + 3\epsilon^{d}_{\alpha\beta}$$

- Sensitivity at **near detectors** of ν factory

leptonic s^2_W at ν factory

s^2_W in DIS at ν factory

$f = e, u, d$

$$\left|\varepsilon^{f}_{ee}\right| < O(10^{-3}) \qquad \left|\varepsilon^{f}_{\mu\mu}\right| < O(10^{-3})$$

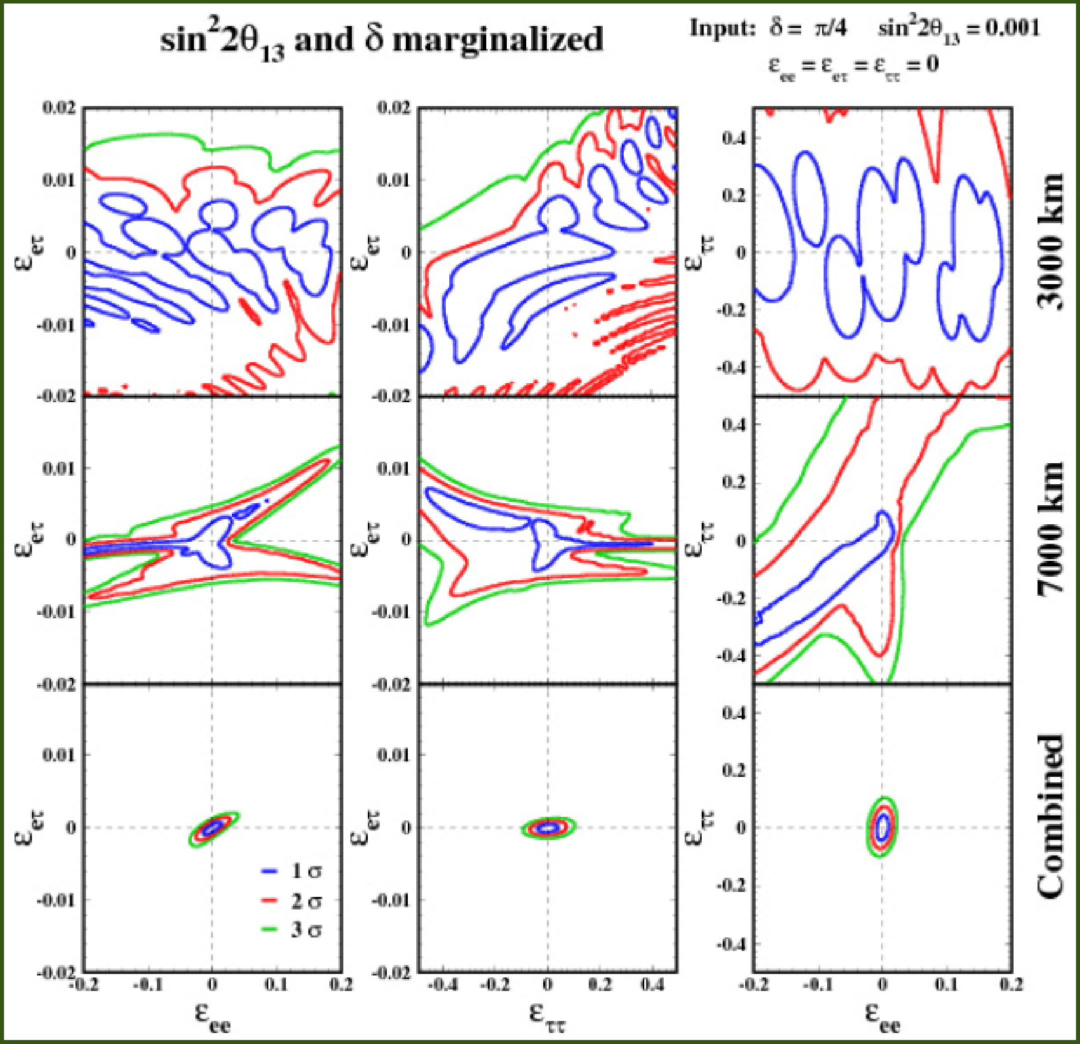
$$\left|\varepsilon^{f}_{\mu\tau}\right| < O(10^{-2}) \qquad \left|\varepsilon^{f}_{e\tau}\right| < O(10^{-2})$$

- Sensitivity of KamLAND and SNO/SK

$$\left|\varepsilon^{f}_{\tau\tau}\right| < 0.3$$

Sensitivity at ν factory

Ribeiro-Minakata-Nunokawa-Uchinami-Zukanovich-Funchal, JHEP 0712:002,2007.



Statistics + some correlations of errors

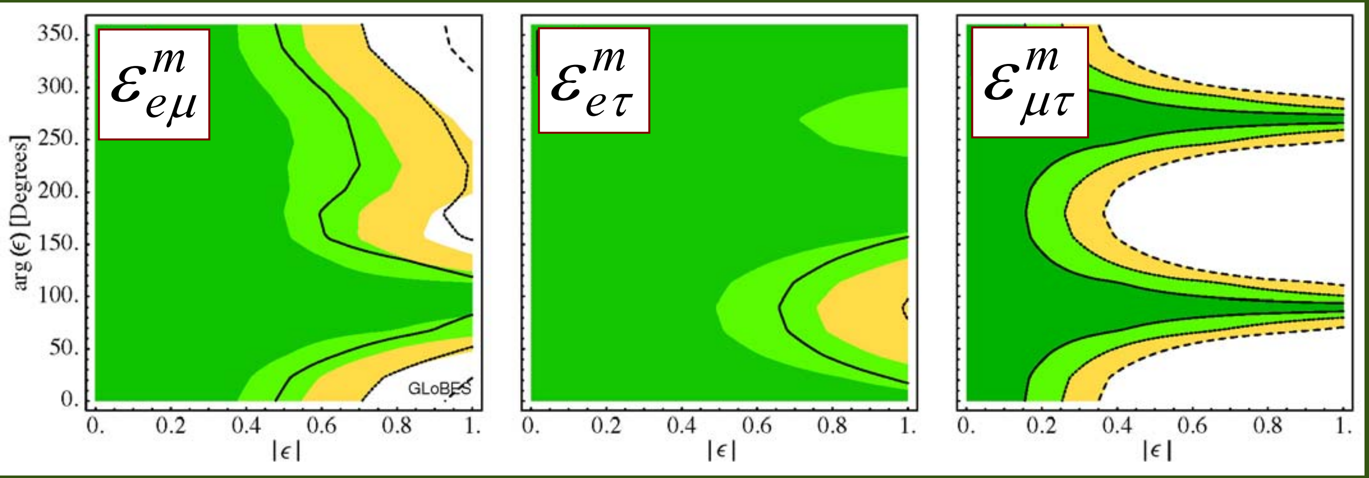
$$\mathcal{A} = \mathbf{A}\begin{pmatrix} 1+\varepsilon_{ee} & 0 & \varepsilon_{e\tau} \\ 0 & 0 & 0 \\ \varepsilon^*_{e\tau} & 0 & \varepsilon_{\tau\tau} \end{pmatrix}$$

$$|\varepsilon_{e\tau}| < \text{a few} \times 10^{-3}$$

Sensitivity by superbeam + reactor

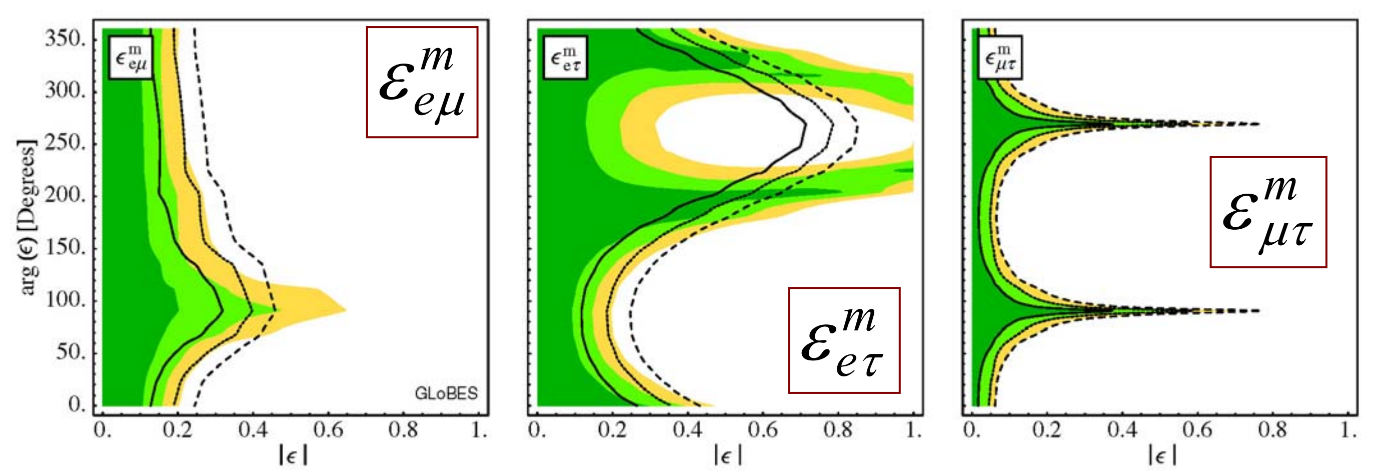
Kopp, Lindner, Ota, Sato, PRD77:013007,2008

Statistics + systematic errors + correlations of errors + degeneracies



T2K + DCHOOZ

$$\varepsilon^m_{e\mu} < 0.5$$



Nova + DC-200kt

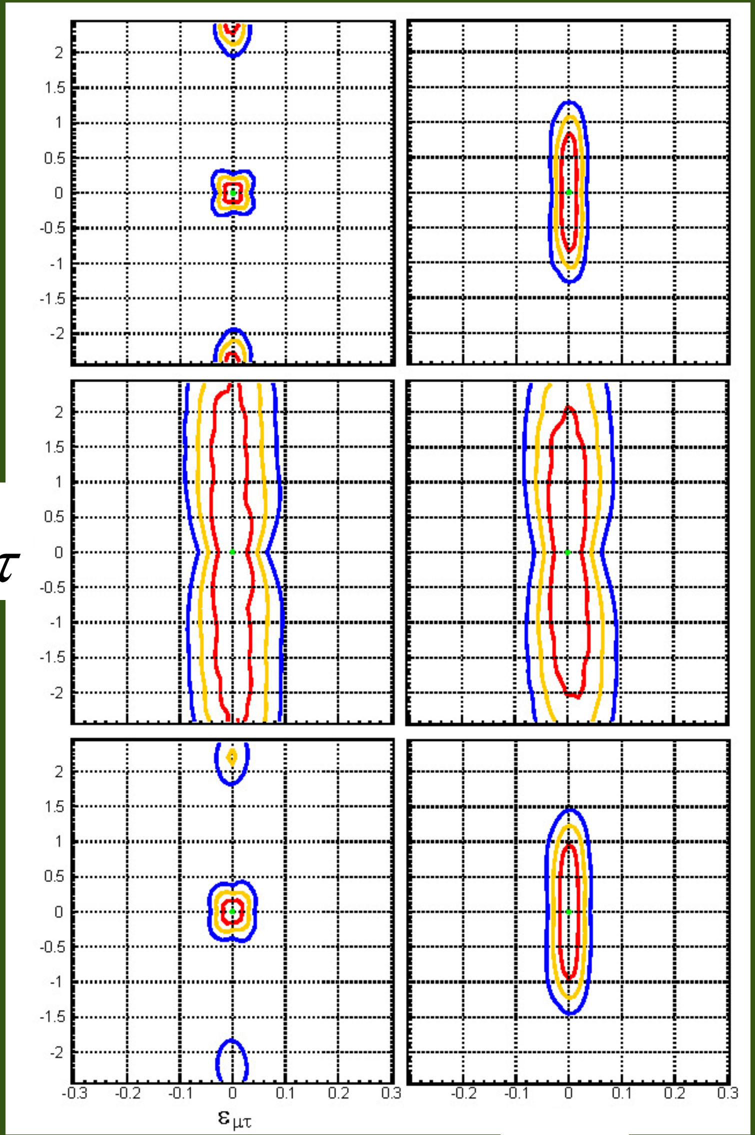
$$\varepsilon^m_{e\mu} < 0.1$$

Sensitivity to $\varepsilon_{\mu\tau}$ and $\varepsilon_{\tau\tau}$ at T2KK

truncation

$$i\frac{d}{dt}\begin{pmatrix}\nu_\mu\\ \nu_\tau\end{pmatrix} = \left[U\begin{pmatrix}0 & 0\\ 0 & \Delta E_{31}\end{pmatrix}U^{-1} + A\begin{pmatrix}\epsilon_{\mu\mu} & \epsilon_{\mu\tau}\\ \epsilon_{\mu\tau} & \epsilon_{\tau\tau}\end{pmatrix}\right]\begin{pmatrix}\nu_\mu\\ \nu_\tau\end{pmatrix}$$

Statistical + systematic errors some correlations of errors



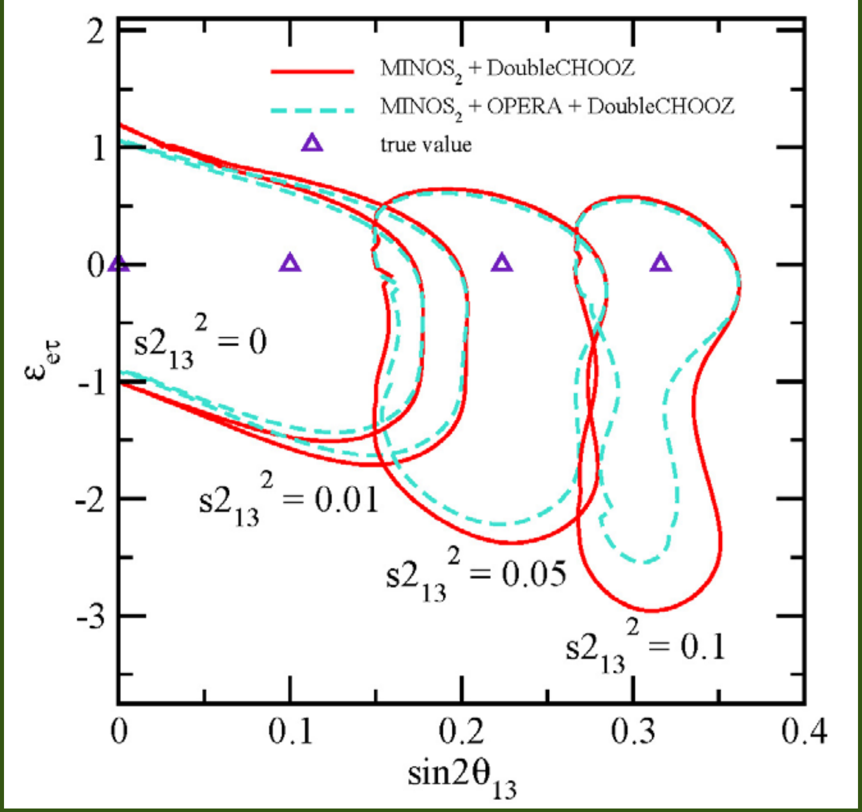
$$\left|\varepsilon_{\mu\tau}\right| < 0.03$$

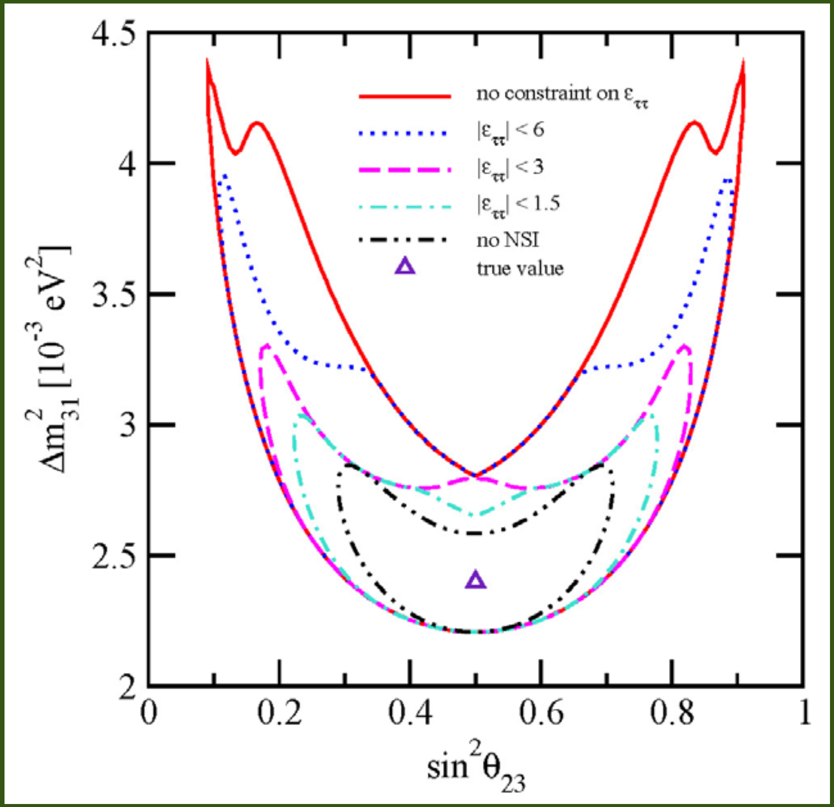
$$\left|\varepsilon_{\tau\tau}\right| < 0.3$$

Ribeiro-Kajita-Ko-Minakata-Nakayama-Nunokawa, PRD77:073007,'08

Sensitivity of MINOS+OPERA+DCHOOZ

Does OPERA help to resolve θ_{13} - $\varepsilon_{e\tau}$ degeneracy?



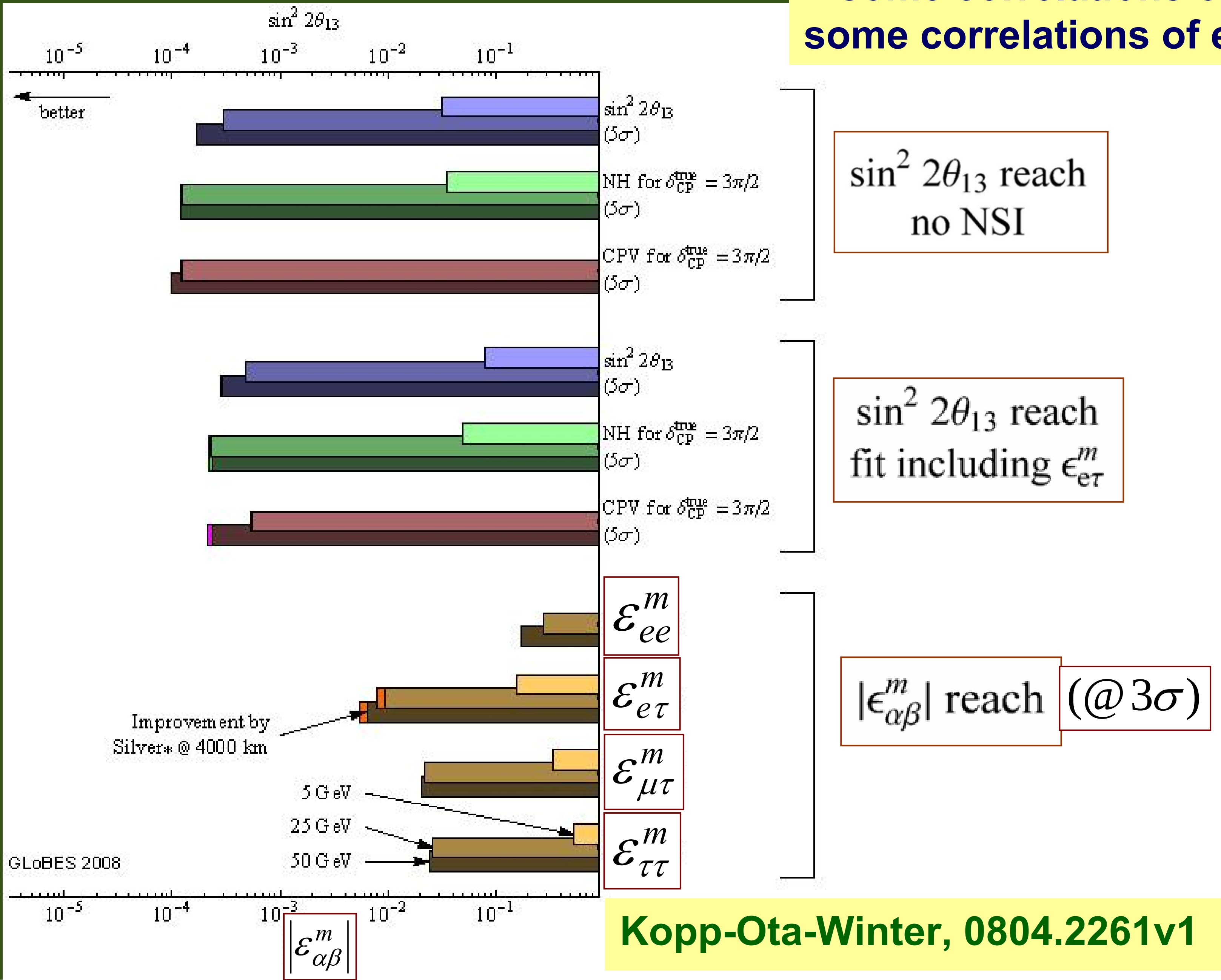


Statistics at OPERA: too small to be significant to constrain $\varepsilon_{e\tau}$

Esteban-Pretel, Valle, Huber, arXiv:0803.1790 [hep-ph]

Sensitivity at ν factory

Statistical + systematic errors + some correlations of errors + some correlations of errors



Kopp-Ota-Winter, 0804.2261v1 [hep-ph]

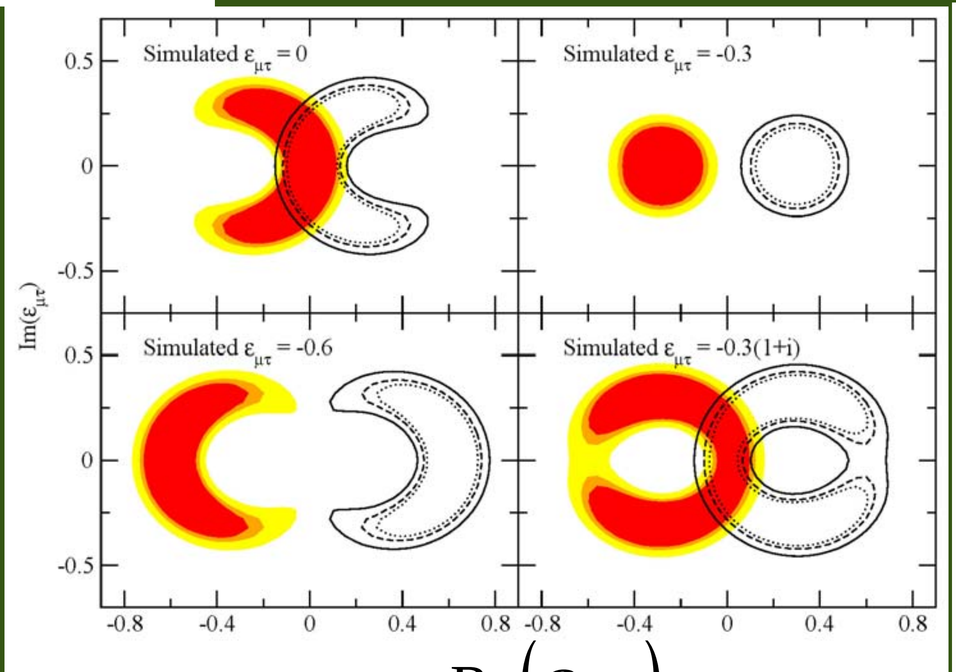
Sensitivity at OPERA

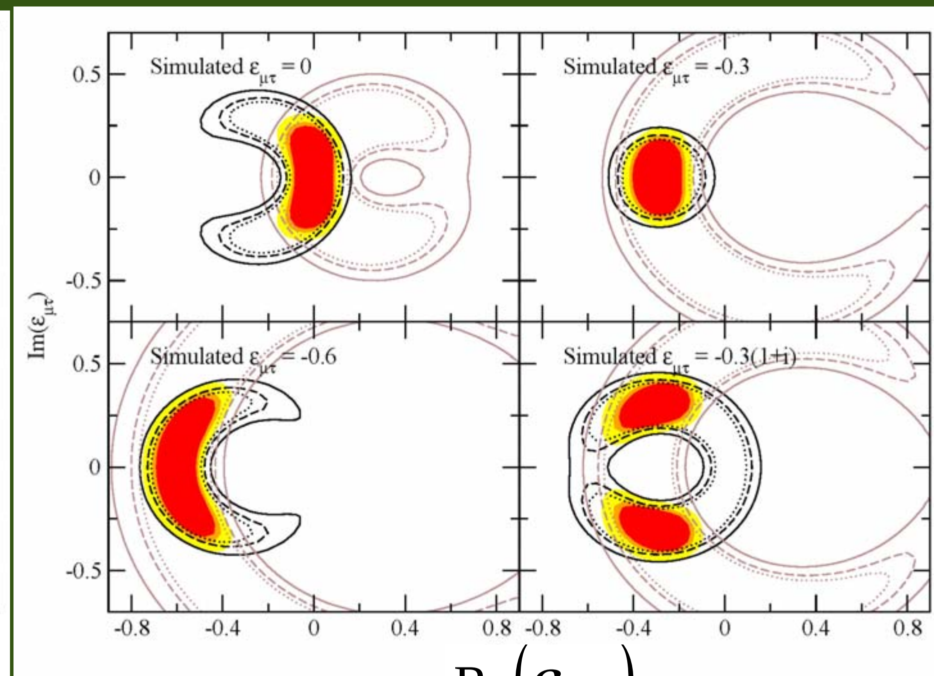
Blennow-Meloni-Ohlsson-Terranova-Westerberg, 0804.2744v1 [hep-ph]

$\varepsilon_{\mu\tau}$: important at shorter baseline length

$$P_{\mu\tau} = |S_{\tau\mu}|^2 = \left| c_{13}^2 \sin(2\theta_{23}) \frac{\Delta m_{31}^2}{4E_\nu} + \varepsilon_{\mu\tau}^* V \right|^2 L^2 + \mathcal{O}(L^3)$$

$\mathrm{Im}(\varepsilon_{\mu\tau})$





$\mathrm{Re}(\varepsilon_{\mu\tau})$ $\mathrm{Re}(\varepsilon_{\mu\tau})$

ν only $\nu + \overline{\nu}$

6. Violation of unitarity w/o light ν_s

Antusch, Biggio, Fernandez-Martinez, Gavela, Lopez-Pavon, JHEP0610,084, '06

In generic see-saw models, after integrating out ν_R, the kinetic term gets modified, and unitarity is expected to be violated.

$$L = \frac{1}{2}\left(i\bar{\nu}_\alpha \partial K_{\alpha\beta}\nu_\beta - \bar{\nu}^c{}_\alpha M_{\alpha\beta}\nu_\beta\right) - \frac{g}{\sqrt{2}}\left(W^+_\mu \bar{l}_\alpha \gamma^\mu P_L \nu_\alpha + h.c.\right) + \ldots$$

rescaling ν →

$$L = \frac{1}{2}\left(i\bar{\nu}_i \partial \nu_i - \bar{\nu}^c{}_i m_{ii}\nu_i\right) - \frac{g}{\sqrt{2}}\left(W^+_\mu \bar{l}_\alpha \gamma^\mu P_L N_{\alpha i}\nu_i\right) + \ldots$$

N: non-unitary

→ The nontrivial issue is the magnitude of violation. Some of see-saw models (e.g., inverse see-saw) do have two scales, one to produce small neutrino mass and another which may not be extremely different from M_W. Then magnitude of violation may not be extremely small.

→ Unitarity of the lepton sector is worth checking.

Oscillation probability is similar to but different from that of NP:

$$P(\nu_\alpha \to \beta) = \left| \left[H\tilde{U} \exp\left\{ -i\mathrm{diag}(\tilde{E}_j)L \right\} \tilde{U}^{-1} H \right]_{\beta\alpha} \right|^2$$

$$U \,\mathrm{diag}(E_j)\, U^{-1} + H \mathcal{A}_0 H = \tilde{U}\, \mathrm{diag}(\tilde{E}_j)\, \tilde{U}^{-1}$$

$N \equiv HU$

H: close to identity

$NN^\dagger - 1$: deviation from unitarity

Constraints from weak decays turned out to be more stringent than ν oscillation:

mostly from rare decays

$$\left| NN^\dagger \right| \approx \begin{pmatrix} 0.994 \pm 0.005 & < 7.1 \cdot 10^{-5} & < 1.6 \cdot 10^{-2} \\ < 7.1 \cdot 10^{-5} & 0.995 \pm 0.005 & < 1.0 \cdot 10^{-2} \\ < 1.6 \cdot 10^{-2} & < 1.0 \cdot 10^{-2} & 0.995 \pm 0.005 \end{pmatrix}$$

90% C.L.

→ Deviation from unitarity < O(1%)

Sensitivity at ν factory

- **4kt OPERA-like near detector @100 m**

Antusch et al, JHEP0610,084, '06

$\nu_e \rightarrow \nu_\tau$ $\left|\sum_i N_{ei} N^*_{\tau i}\right| < 2.9 \times 10^{-3}$ **(present : 0.016)**

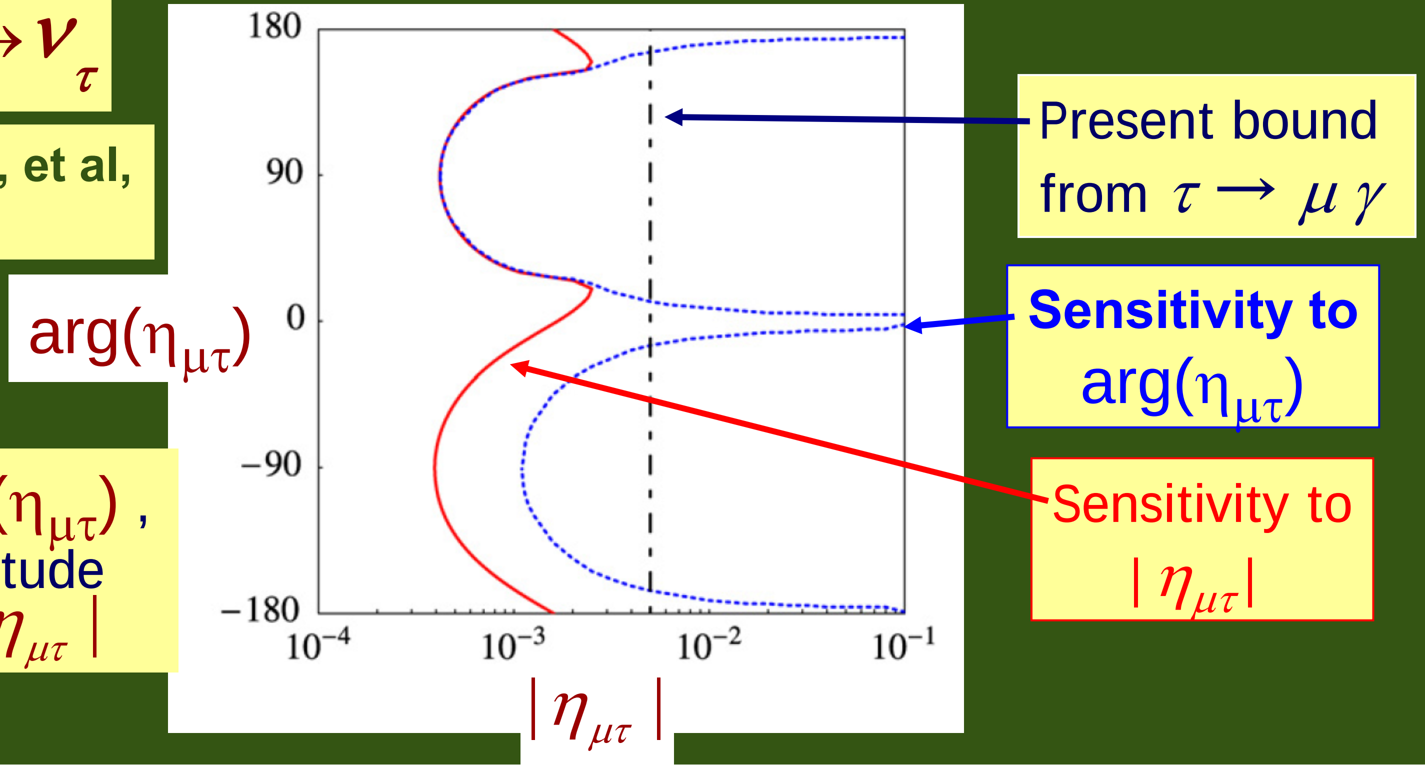
$\nu_\mu \rightarrow \nu_\tau$ $\left|\sum_i N_{\mu i} N^*_{\tau i}\right| < 2.6 \times 10^{-3}$ **(present : 0.013)**

- **5kt OPERA-like far detector @130 km**

$\nu_\mu \rightarrow \nu_\tau$

Fernandez-Martinez, et al, PLB649:427,'07

$H \equiv 1 + \eta$



For non-trivial $\arg(\eta_{\mu\tau})$, one order of magnitude improvement for $|\eta_{\mu\tau}|$

7. Summary

● New physics

- Current bounds on $\varepsilon_{\alpha\beta}$ are typically of order 10^{-3} for production and detection

- $\varepsilon_{\alpha\beta}$ for propagation have bounds typically of order 10^{-2} but ε_{ee} , $|\varepsilon_{e\tau}|$, $\varepsilon_{\tau\tau}$ <~O(1) and it is difficult to give strong constraints on these three from the present data.

- ν factory may be able to improve bounds on $\varepsilon_{\alpha\beta}$ for propagation dramatically.

● Violation of 3 flavor unitarity

Deviation from unitarity is expected in generic models (e.g., see-saw) but phenomenologically its magnitude is less than O(1%). Further studies are necessary.

There are a lot of problems to be worked out:

- **Correlations of errors, degeneracies, etc. in the presence of all new physics parameters $\varepsilon_{\alpha\beta}$**
- **Distinction between the new physics effects (e.g., 4-fermi interactions vs. unitarity violation due to modification in the kinetic term)**

Backup slides

In the present case 1-$P_{\mu\mu}$ can be expressed as

$$1 - P(\nu_\mu \to \mu) = 4\tilde{X}_1^{\mu\mu}\tilde{X}_2^{\mu\mu}\sin^2\left(\frac{\Lambda_- L}{2}\right) + 4\tilde{X}_2^{\mu\mu}\tilde{X}_3^{\mu\mu}\sin^2\left(\frac{\Lambda_+ L}{2}\right)$$

$$+ \; 4\tilde{X}_1^{\mu\mu}\tilde{X}_3^{\mu\mu}\sin^2\left[\frac{(\Lambda_+ - \Lambda_-)L}{2}\right]$$

$$\tilde{X}_1^{\mu\mu} = |U_{\mu 3}|^2\frac{(\Lambda_+ - \Delta E_{31})\Lambda_+}{\lambda_{e'}(\Lambda_+ - \Lambda_-)c_{13}^{\prime\prime 2}} \to \frac{|U_{\mu 3}|^2}{c_{13}^{\prime\prime 2}} \quad (E \to \infty)$$

$$\tilde{X}_2^{\mu\mu} = 1 - \frac{|U_{\mu 3}|^2}{c_{13}^{\prime\prime 2}} \to 1 - \frac{|U_{\mu 3}|^2}{c_{13}^{\prime\prime 2}} \quad (E \to \infty)$$

$$\tilde{X}_3^{\mu\mu} = |U_{\mu 3}|^2\frac{(\Delta E_{31} - \Lambda_-)\Lambda_-}{\lambda_{e'}(\Lambda_+ - \Lambda_-)c_{13}^{\prime\prime 2}} \to 0 \quad (E \to \infty)$$

$$\lambda_{e'} \equiv \frac{A(1+\epsilon_{ee})}{c_\beta^2}$$

$$A\begin{pmatrix} 1+\epsilon_{ee} & 0 & \epsilon_{e\tau} \\ 0 & 0 & 0 \\ \epsilon_{e\tau}^* & 0 & \epsilon_{\tau\tau} \end{pmatrix} \equiv e^{i\gamma\,\mathrm{diag}(1,0-1)}e^{-i\beta\lambda_5}\mathrm{diag}\left(\lambda_{e'},0,0\right)e^{i\beta\lambda_5}e^{-i\gamma\,\mathrm{diag}(1,0-1)}$$

$$\gamma \equiv \frac{1}{2}\arg(\epsilon_{e\tau})$$

$$1 - P(\nu_\mu \to \nu_\mu) \simeq 4\left(\frac{U_{\mu 3}^2}{c_{13}^{\prime\prime 2}}\right)\left(1 - \frac{U_{\mu 3}^2}{c_{13}^{\prime\prime 2}}\right)\sin^2\left(\frac{\Delta E_{31}c_{13}^{\prime\prime 2}L}{2}\right) \quad (E \to \infty)$$

Analytical formula for the oscillation probability in matter with New Physics in the limit $\Delta m^2_{21} \to 0$

OY arXiv:0704.1531 [hep-ph]

$$
\begin{aligned}
P(\nu_\mu \to \nu_e) = & -4\mathrm{Re}\left(\tilde{X}_1^{\mu e}\tilde{X}_2^{\mu e *}\right)\sin^2\left(\frac{\Lambda_- L}{2}\right) - 4\mathrm{Re}\left(\tilde{X}_2^{\mu e}\tilde{X}_3^{\mu e *}\right)\sin^2\left(\frac{\Lambda_+ L}{2}\right) \\
& -4\mathrm{Re}\left(\tilde{X}_1^{\mu e}\tilde{X}_3^{\mu e *}\right)\sin^2\left[\frac{(\Lambda_+ - \Lambda_-)L}{2}\right] \\
& -8\mathrm{Im}\left(\tilde{X}_1^{\mu e}\tilde{X}_2^{\mu e *}\right)\sin\left(\frac{\Lambda_- L}{2}\right)\sin\left(\frac{\Lambda_+ L}{2}\right)\sin\left[\frac{(\Lambda_+ - \Lambda_-)L}{2}\right]
\end{aligned}
$$

$$
\mathcal{A} = A\begin{pmatrix} 1+\varepsilon_{ee} & 0 & \varepsilon_{e\tau} \\ 0 & 0 & 0 \\ \varepsilon^*_{e\tau} & 0 & \varepsilon_{\tau\tau} \end{pmatrix}
$$

$$A \equiv \sqrt{2}G_F N_e$$

$$
\Lambda_\pm = \frac{1}{2}\left[\Delta E_{31} + \frac{A(1+\epsilon_{ee})}{\cos^2\beta}\right] \pm \frac{1}{2}\sqrt{\left[\Delta E_{31}\cos 2\theta''_{13} - \frac{A(1+\epsilon_{ee})}{\cos^2\beta}\right]^2 + (\Delta E_{31}\sin 2\theta''_{13})^2}
$$

$$\tan\beta = \frac{|\epsilon_{e\tau}|}{1+\epsilon_{ee}}$$

$$\epsilon_{\tau\tau} = \frac{|\epsilon_{e\tau}|^2}{1+\epsilon_{ee}}$$

$$\theta''_{13} = \sin^{-1}\left|e^{-i\arg(\epsilon_{e\mu})}U_{e3}\cos\beta + U_{\tau 3}\sin\beta\right|$$

$$
\begin{aligned}
\tilde{X}_1^{\mu e} &= \frac{-1}{\Lambda_-(\Lambda_+ - \Lambda_-)}\left[\xi + \eta e^{-i(\arg(\epsilon_{e\mu})+\delta)} - \Lambda_+\zeta\right] \\
\tilde{X}_2^{\mu e} &= \frac{1}{\Lambda_+\Lambda_-}\left[\xi + \eta e^{-i(\arg(\epsilon_{e\mu})+\delta)} - (\Lambda_+ + \Lambda_-)\zeta\right] \\
\tilde{X}_3^{\mu e} &= \frac{1}{\Lambda_+(\Lambda_+ - \Lambda_-)}\left[\xi + \eta e^{-i(\arg(\epsilon_{e\mu})+\delta)} - \Lambda_-\zeta\right]
\end{aligned}
$$

$$
\begin{aligned}
\xi &\equiv [(\Delta E_{31})^2 + A(1+\epsilon_{ee})\Delta E_{31}]U_{\mu 3}|U_{e3}| \\
\eta &\equiv A\Delta E_{31}|\epsilon_{e\tau}|U_{\mu 3}U_{\tau 3} \\
\zeta &\equiv \Delta E_{31}U_{\mu 3}|U_{e3}|,
\end{aligned}
$$

depends only on $\arg(\varepsilon_{e\tau})+\delta$

Features of the probability (in the limit $\Delta m^2_{21} \to 0$)

A) It depends only on arg($\varepsilon_{e\tau}$)+δ.

→ **This is approximately the case also for** $\Delta m^2_{21} \neq 0$.

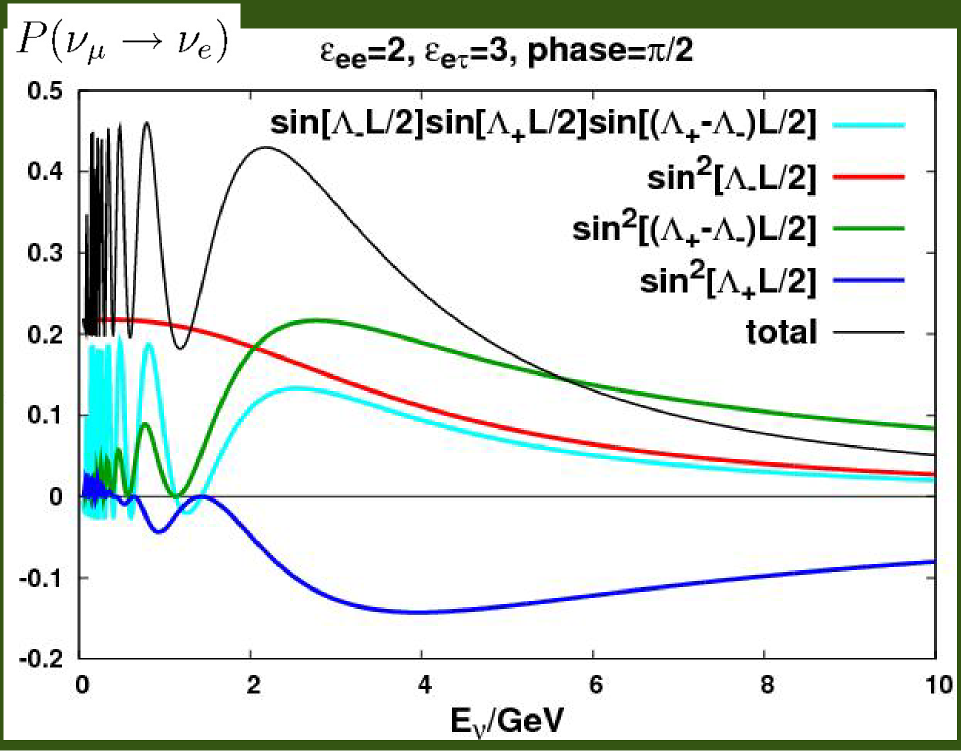
B) Each term gives a large contribution (See Fig. below).

→ **Interpretation of behavior of probability is not obvious.**

cf In standard 3 flavor case, only one term dominates:

$$P(\nu_\mu \to \nu_e) = \sin^2 2\theta_{13} \left(\frac{\Delta E_{31}}{\Delta \tilde{E}_{31}}\right)^2 \sin^2\left(\frac{\Delta \tilde{E}_{31} L}{2}\right)$$

$$\Delta \tilde{E}_{31} \equiv \left[(\Delta E_{31}\cos 2\theta_{13} - A)^2 + (\Delta E_{31}\sin 2\theta_{13})^2\right]^{1/2}$$



$$P(\nu_\mu \to \nu_e) =$$

$$-4\mathrm{Re}\left(\tilde{X}_1^{\mu e}\tilde{X}_2^{\mu e *}\right)\sin^2\left(\frac{\Lambda_- L}{2}\right)$$

$$-4\mathrm{Re}\left(\tilde{X}_2^{\mu e}\tilde{X}_3^{\mu e *}\right)\sin^2\left(\frac{\Lambda_+ L}{2}\right)$$

$$-4\mathrm{Re}\left(\tilde{X}_1^{\mu e}\tilde{X}_3^{\mu e *}\right)\sin^2\left[\frac{(\Lambda_+ - \Lambda_-)L}{2}\right]$$

$$-8\mathrm{Im}\left(\tilde{X}_1^{\mu e}\tilde{X}_2^{\mu e *}\right)\sin\left(\frac{\Lambda_- L}{2}\right)\sin\left(\frac{\Lambda_+ L}{2}\right)\sin\left[\frac{(\Lambda_+ - \Lambda_-)L}{2}\right]$$