Recent status of ν oscillation study and its future

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1. ν oscillation

(i) 2 flavor oscillations in vacuum

$$\left\{\begin{array}{l} i\dfrac{d}{dx}\nu_1(x) = E_1\ \nu_1(x) \\ \\ i\dfrac{d}{dx}\nu_2(x) = E_2\ \nu_2(x) \end{array}\right.$$

mass eigenstates

$$E_j \equiv \sqrt{\vec{p}^2 + m_j^2}$$

$$\begin{pmatrix} \nu_e(x) \\ \nu_\mu(x) \end{pmatrix} = U \begin{pmatrix} \nu_1(x) \\ \nu_2(x) \end{pmatrix}$$

flavor eigenstates

$$U \equiv \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

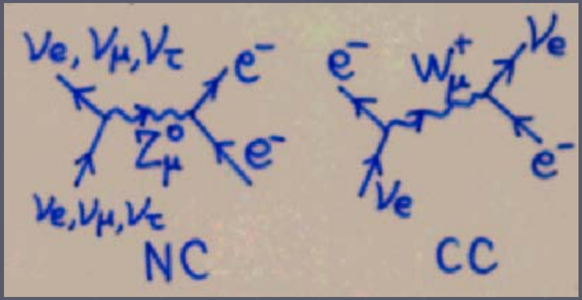
mixing matrix in vacuum

$$\boxed{P(\nu_e \to \nu_\mu; L) \quad = \quad \sin^2 2\theta\ \sin^2\left(\frac{\Delta E L}{2}\right)}$$

$$\Delta E \quad = \quad E_2 - E_1 \simeq \frac{m_2^2 - m_1^2}{2E} \equiv \frac{\Delta m^2}{2E}$$

(ii) 2 flavor oscillations in matter (MSW effect)

$$
\begin{aligned}
\mathcal{L}_{eff} &= \sqrt{2}\,G_F\,\bar{\nu}_e\gamma^\mu\nu_e\,\bar{e}\gamma_\mu e \quad (\langle\bar{e}\gamma_\mu e\rangle \longrightarrow \delta_{\mu 0}N_e(x)) \\
&= A\,\bar{\nu}_e\gamma^0\nu_e \qquad \boxed{(A \equiv \sqrt{2}\,G_F\,N_e(x))}
\end{aligned}
$$



$$
\begin{aligned}
i\frac{d}{dx}\begin{pmatrix}\nu_e(x)\\ \nu_\mu(x)\end{pmatrix} &= \left[U\begin{pmatrix}E_1 & 0\\ 0 & E_2\end{pmatrix}U^{-1} + \begin{pmatrix}A & 0\\ 0 & 0\end{pmatrix}\right]\begin{pmatrix}\nu_e(x)\\ \nu_\mu(x)\end{pmatrix} \\
&= \tilde{U}(x)\begin{pmatrix}\tilde{E}_1 & 0\\ 0 & \tilde{E}_2\end{pmatrix}\tilde{U}^{-1}(x)\begin{pmatrix}\nu_e(x)\\ \nu_\mu(x)\end{pmatrix}
\end{aligned}
$$

If N_e=const.

$$
\boxed{P(\nu_e \rightarrow \nu_\mu; L) = \sin^2 2\tilde{\theta}\,\sin^2\left(\frac{\Delta\tilde{E}L}{2}\right)}
$$

$$
\begin{aligned}
\tan 2\tilde{\theta} &\equiv \frac{\Delta E\sin 2\theta}{\Delta E\cos 2\theta - A} \\
\Delta\tilde{E} &= \left[(\Delta E\cos 2\theta - A)^2 + (\Delta E\sin 2\theta)^2\right]^{1/2}
\end{aligned}
$$

even if θ in vacuum is small $\tilde{\theta}$ in matter could be large (MSW effect)

If N_e varies adiabatically (e.g., in solar ν)

$$\begin{pmatrix} \nu_e(L) \\ \nu_\mu(L) \end{pmatrix} = \tilde{U}(L)\,\exp\left[-i\int_0^L \mathrm{diag}\left(\tilde{E}_1(x),\tilde{E}_2(x)\right)dx\right]\tilde{U}^{-1}(0)\begin{pmatrix} \nu_e(0) \\ \nu_\mu(0) \end{pmatrix}$$

$$A(\nu_\alpha \to \nu_\beta) = \sum_j \tilde{U}(L)_{\beta j}\exp\left(-i\int_0^L \tilde{E}_j(x)dx\right)\tilde{U}(0)^*_{\alpha j}$$

$$P(\nu_\alpha \to \nu_\beta) = \sum_{j,k}\tilde{U}(L)_{\beta j}\tilde{U}(L)^*_{\beta k}\tilde{U}(0)^*_{\alpha j}\tilde{U}(0)_{\alpha k}\exp\left(-i\int_0^L \Delta\tilde{E}_{jk}(x)dx\right)$$

$$(L\to\infty) \to \sum_j \left|\tilde{U}(0)_{\alpha j}\right|^2\left|\tilde{U}(L)_{\beta j}\right|^2$$

$$\boxed{\left(\exp\left(-i\int_0^L \Delta\tilde{E}_{jk}(x)dx\right)\to\delta_{jk}\right)}$$

$$= \sum_j \left|\tilde{U}(0)_{\alpha j}\right|^2 |U_{\beta j}|^2$$

average over rapid oscillations

$$\boxed{P(\nu_e \to \nu_e) = \cos^2\tilde{\theta}(x=0)\cos^2\theta + \sin^2\tilde{\theta}(x=0)\sin^2\theta}$$

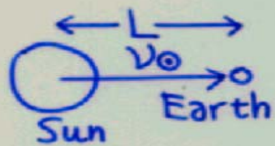
$$\begin{pmatrix} \cos^2\tilde{\theta}(x=0) \\ \sin^2\tilde{\theta}(x=0) \end{pmatrix} = \frac{1}{2}\left[1 \pm \frac{\Delta E\cos 2\theta - A(x=0)}{[(\Delta E\cos 2\theta - A(x=0))^2 + (\Delta E\sin 2\theta)^2]^{1/2}}\right]$$

(iii) 3 flavor ν oscillation

KamLAND(reactor) $\bar{\nu}_e \to \bar{\nu}_e$

- solar ν $\qquad \nu_e \to \nu_e$

① flux of $\nu_\odot$ observed on the Earth is lower than theoretical predictions



GALLEX-GNO, SAGE (Ga), Homestake (Cl), Kamiokande, SK (H_2O), SNO (D_2O)

② data/th depends on exps.

Large Mixing Angle solution

$\theta_{12} \cong \pi/6$

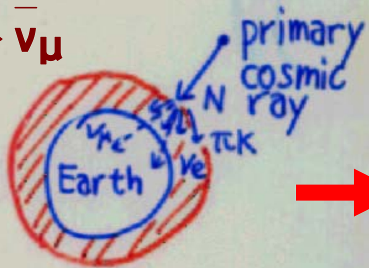
$\Delta m^2_{21} = 8 \times 10^{-5}\,\mathrm{eV}^2$

- atmospheric ν $\qquad \nu_\mu \to \nu_\mu + \bar{\nu}_\mu \to \bar{\nu}_\mu$

naive expectation from

$\pi^+ \to \mu^+ + \nu_\mu$, $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$

$\pi^- \to \mu^- + \bar{\nu}_\mu$, $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$



leads to

$\#(\nu_\mu + \bar{\nu}_\mu)/\#(\nu_e + \bar{\nu}_e) \simeq 2$

but observations show

① $\#(\nu_\mu + \bar{\nu}_\mu)/\#(\nu_e + \bar{\nu}_e) \simeq 1.3$

② data/th depends on zenith angle

Kamiokande
IMB
SK
Soudan2
MACRO

maximal mixing

$\theta_{23} \cong \pi/4$

$|\Delta m^2_{32}| = 2.5 \times 10^{-3}\,\mathrm{eV}^2$

CHOOZ (reactor) $\qquad L \sim 1\,\mathrm{km}$, $E_\nu \sim 3\,\mathrm{MeV}$ $\qquad \bar{\nu}_e \to \bar{\nu}_e$

$$\left|\frac{\Delta m^2_{21} L}{4E}\right| = \left|\frac{\Delta m^2_\odot L}{4E}\right| \ll 1$$

$\Delta m^2|_{N_\nu=2}$ ⫴

$N_\nu = 3$

$$P(\bar{\nu}_e \to \bar{\nu}_e) \simeq 1 - \underbrace{4|U_{e3}|^2(1-|U_{e3}|^2)}_{= \sin^2 2\theta_{13}} \sin^2\left(\frac{\boxed{\Delta m^2_{32}}L}{4E}\right)$$

small mixing

$\sin^2 2\theta_{13} < 0.15$

mixing matrix of 3 flavor ν oscillation

$N_\nu = 3$: $\nu_{atm} + \nu_{solar} + \nu_{reactor}$

Mixing matrix

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \cong \begin{pmatrix} c_{12} & s_{12} & \varepsilon \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & 1/\sqrt{2} \\ s_{12}/\sqrt{2} & -c_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

Mixing angles & mass squared differences

$\theta_{12} \cong \pi/6, \quad \theta_{23} \cong \pi/4$

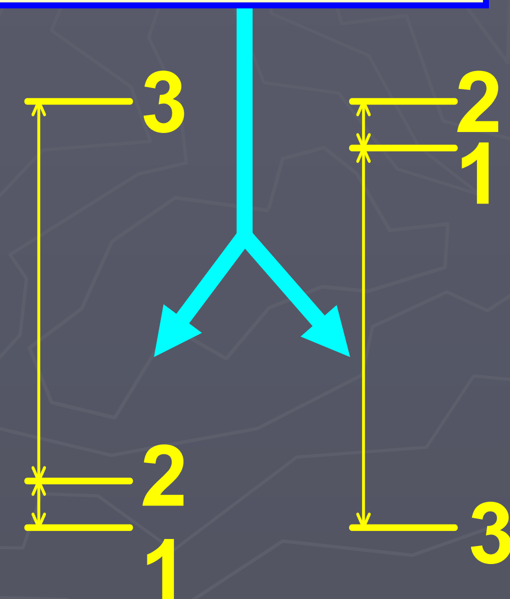
$|\theta_{13}| \cong |\varepsilon| \leq \sqrt{0.15}/2$

$\Delta m^2_{21} = 8 \times 10^{-5}\,\mathrm{eV}^2$

$|\Delta m^2_{32}| = 2.5 \times 10^{-3}\,\mathrm{eV}^2$

- θ_{13} :only upper bound is known
- δ :undetermined

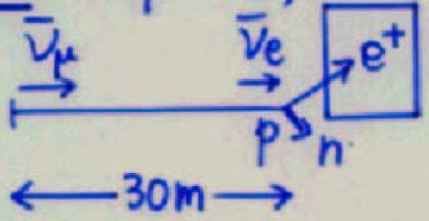
- Both hierarchies are allowed



normal hierarchy	inverted hierarchy
$\Delta m^2_{32} > 0$	$\Delta m^2_{32} < 0$

(iv) Scenario other than 3 flavor ν oscillation

∘LSND ($\bar{\nu}_\mu \to \bar{\nu}_e$)



detector

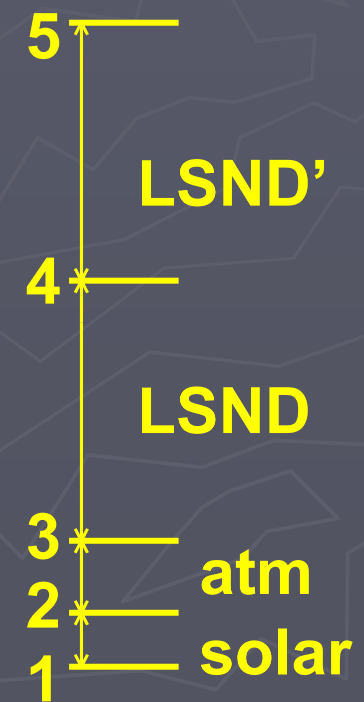
E ~ 50 MeV

$\frac{\Delta m^2 L}{4E} \sim 1 \to \Delta m^2 \sim O(1eV^2)$

??? : MiniBooNE (2001 ~) is expected to (dis)prove LSND.

So far the only promising scenario to explain LSND in terms of ν is (3+2)-scenario with 2 kind of sterile neutrinos.

Until LSND is confirmed by MiniBOONE, sterile neutrino scenarios don't seem to have strong motivations. →In most of the talk, N_ν=3 is assumed.



(v) Theoretical prediction for θ_{13}

All kinds of values of θ_{13} are predicted by theory, and it doesn't look like illuminating.

→ Theory is not yet developed enough to say something from mass & mixing of quarks & leptons.

hep-ex/0402041

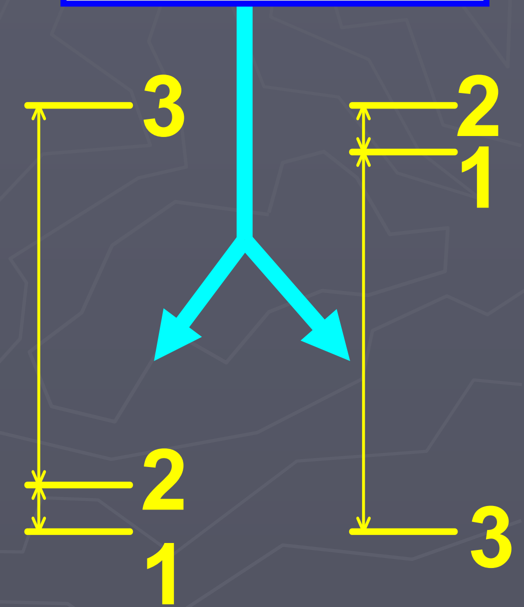
Reference	$\sin\theta_{13}$	$\sin^2 2\theta_{13}$
SO(10)		
Goh, Mohapatra, Ng [40]	0.18	0.13
Orbifold SO(10)		
Asaka, Buchmüller, Covi [41]	0.1	0.04
SO(10) + flavor symmetry		
Babu, Pati, Wilczek [42]	$5.5 \cdot 10^{-4}$	$1.2 \cdot 10^{-6}$
Blazek, Raby, Tobe [43]	0.05	0.01
Kitano, Mimura [44]	0.22	0.18
Albright, Barr [45]	0.014	$7.8 \cdot 10^{-4}$
Maekawa [46]	0.22	0.18
Ross, Velasco-Sevilla [47]	0.07	0.02
Chen, Mahanthappa [48]	0.15	0.09
Raby [49]	0.1	0.04
SO(10) + texture		
Buchmüller, Wyler [50]	0.1	0.04
Bando, Obara [51]	0.01 .. 0.06	$4 \cdot 10^{-4}$ .. 0.01
Flavor symmetries		
Grimus, Lavoura [52, 53]	0	0
Grimus, Lavoura [52]	0.3	0.3
Babu, Ma, Valle [54]	0.14	0.08
Kuchimanchi, Mohapatra [55]	0.08 .. 0.4	0.03 .. 0.5
Ohlsson, Seidl [56]	0.07 .. 0.14	0.02 .. 0.08
King, Ross [57]	0.2	0.15
Textures		
Honda, Kaneko, Tanimoto [58]	0.08 .. 0.20	0.03 .. 0.15
Lebed, Martin [59]	0.1	0.04
Bando, Kaneko, Obara, Tanimoto [60]	0.01 .. 0.05	$4 \cdot 10^{-4}$ .. 0.01
Ibarra, Ross [61]	0.2	0.15
3 × 2 see-saw		
Appelquist, Piai, Shrock [62, 63]	0.05	0.01
Frampton, Glashow, Yanagida [64]	0.1	0.04
Mei, Xing [65] (normal hierarchy)	0.07	0.02
(inverted hierarchy)	> 0.006	$> 1.6 \cdot 10^{-4}$
Anarchy		
de Gouvêa, Murayama [66]	> 0.1	> 0.04
Renormalization group enhancement		
Mohapatra, Parida, Rajasekaran [67]	0.08 .. 0.1	0.03 .. 0.04

2. Future LBL (Long BaseLine experiments)

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \cong \begin{pmatrix} c_{12} & s_{12} & \varepsilon \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & 1/\sqrt{2} \\ s_{12}/\sqrt{2} & -c_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

- θ_{13} :only upper bound is known
- δ :undetermined

- Both hierarchies are allowed



normal hierarchy

$\Delta m^2_{32} > 0$

inverted hierarchy

$\Delta m^2_{32} < 0$

Next task is to measure θ_{13} , sign(Δm^2_{31}) and δ.

Most realistic way to measure θ_{13} , sign(Δm^2_{31}) and δ is long base line experiments by accelerators or reactors.

→Matter effect contributes in LBL in most cases

Measurement of θ_{13} and sign of Δm^2_{31}

$$P(\nu_\mu \to \nu_e) = s^2_{23} \sin^2 2\theta_{13} \left(\frac{\Delta E_{31}}{\Delta \tilde{E}^{(-)}_{31}} \right)^2 \sin^2 \left(\frac{\Delta \tilde{E}^{(-)}_{31} L}{2} \right)$$

$$P(\bar{\nu}_\mu \to \bar{\nu}_e) = s^2_{23} \sin^2 2\theta_{13} \left(\frac{\Delta E_{31}}{\Delta \tilde{E}^{(+)}_{31}} \right)^2 \sin^2 \left(\frac{\Delta \tilde{E}^{(+)}_{31} L}{2} \right)$$

$$\Delta \tilde{E}^{(\pm)}_{31} \equiv \sqrt{(\Delta E_{31} \cos 2\theta_{13} \pm A)^2 + (\Delta E_{31} \sin 2\theta_{13})^2}$$

to leading order in $\Delta m^2_{21} / |\Delta m^2_{32}|$

$$\text{If } \Delta m^2_{31} > 0 \text{ then } \left(\Delta E_{31}/\Delta \tilde{E}^{(-)}_{31} > 1 > \Delta E_{31}/\Delta \tilde{E}^{(+)}_{31} \right)^2$$

$$\text{If } \Delta m^2_{31} < 0 \text{ then } \left(\Delta E_{31}/\Delta \tilde{E}^{(-)}_{31} < 1 < \Delta E_{31}/\Delta \tilde{E}^{(+)}_{31} \right)^2$$

For large L, difference between $P(\nu_\mu \to \nu_e)$ and $P(\bar{\nu}_\mu \to \bar{\nu}_e)$ due to matter effect becomes significant

Measurement of δ

$\sin^2 2\theta_{13} < 0.15$

All the contributions of δ appear with the factor of $\sin\theta_{13}$
→ Unless there is enhancement due to matter effect, effects of CP phase δ are expected to be small, or may be ignored in the zero-th approximation

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & \boxed{s_{13}e^{-i\delta}} \\ 0 & 1 & 0 \\ \boxed{-s_{13}e^{i\delta}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Particle physicists are interested in CP violation, so no matter how small θ_{13} may be we make efforts to measure δ → Attitude by astrophysicists may be different

Theoretical argument on measurement of δ

$$P(\nu_\alpha \to \nu_\beta; L) = \delta_{\alpha\beta} - 4\sum_{j<k} \mathrm{Re}\left(U_{\alpha j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k}\right)\sin^2\left(\frac{\Delta E_{jk}L}{2}\right)$$
$$+2\sum_{j<k} \mathrm{Im}\left(U_{\alpha j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k}\right)\sin\left(\Delta E_{jk}L\right),$$

the CP violation in vacuum is given by

$$\begin{aligned} & P(\nu_\alpha \to \nu_\beta) - P(\bar\nu_\alpha \to \bar\nu_\beta) \\ = \; & 4\sum_{j<k} \mathrm{Im}\left(U_{\alpha j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k}\right)\sin\left(\Delta E_{jk}L\right) \\ = \; & 4\; J\left[\sin\left(\Delta E_{12}L\right) + \sin\left(\Delta E_{23}L\right) + \sin\left(\Delta E_{31}L\right)\right], \end{aligned}$$

$$J \equiv \mathrm{Im}\left(U_{\alpha 1}U^*_{\beta 1}U^*_{\alpha 2}U_{\beta 2}\right)$$

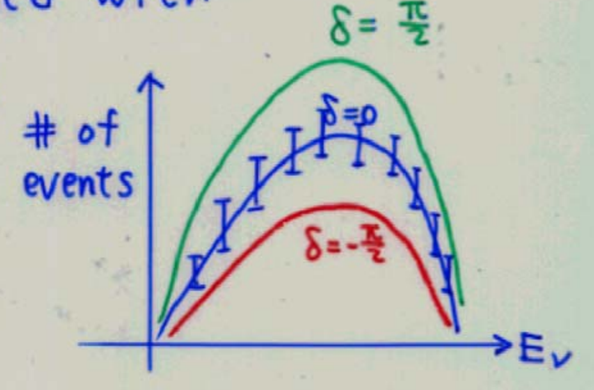
$$J = c_{13}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}\sin\delta$$

Jarlskog factor

Practical measurement of δ

Measure
$P(\nu_\mu \to \nu_e)$
and
$P(\bar{\nu}_\mu \to \bar{\nu}_e)$
and then

Assume three flavor mixing
and compare the data with
prediction for $\delta = 0$:



$$\Delta\chi^2 = \sum_j \frac{[N_j(\delta) - N_j(\delta=0)]^2}{\sigma_j^2}$$

To reject a hypothesis "$\delta = 0$" at 3σ CL

$$\Delta\chi^2 \geq \Delta\chi^2(3\sigma)$$

→ We can estimate detector size
to reject "$\delta = 0$" at 3σ CL

Enhancement of CP violating term

It is illuminating to consider T-violating term in matter with constant density

$$P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$$

$$= 4 \sum_{j<k} \mathrm{Im}(U^M_{\beta j} U^{M*}_{\alpha j} U^{M*}_{\beta k} U^M_{\alpha k}) \sin(\Delta E^M_{jk} L)$$

$$= \pm 16 J^M \sin\left(\frac{\Delta E^M_{21} L}{2}\right) \sin\left(\frac{\Delta E^M_{32} L}{2}\right) \sin\left(\frac{\Delta E^M_{31} L}{2}\right)$$

$$\boxed{J^M = \mathrm{Im}(U^M_{e1} U^{M*}_{\mu 1} U^{M*}_{e2} U^M_{\mu 2})}$$

modified Jarlskog factor in matter

(identity: Naumov, Sov.Phys.JETP 74('92)1

$$J^M \prod_{j<k} \Delta E^M_{jk} = J \prod_{j<k} \Delta E_{jk} \Big)$$

$$= \boxed{\pm 16 J \frac{\Delta E_{21} \Delta E_{32} \Delta E_{31}}{\Delta E^M_{21} \Delta E^M_{32} \Delta E^M_{31}} \sin\left(\frac{\Delta E^M_{21} L}{2}\right) \sin\left(\frac{\Delta E^M_{32} L}{2}\right) \sin\left(\frac{\Delta E^M_{31} L}{2}\right)}$$

$\propto \sin\delta$

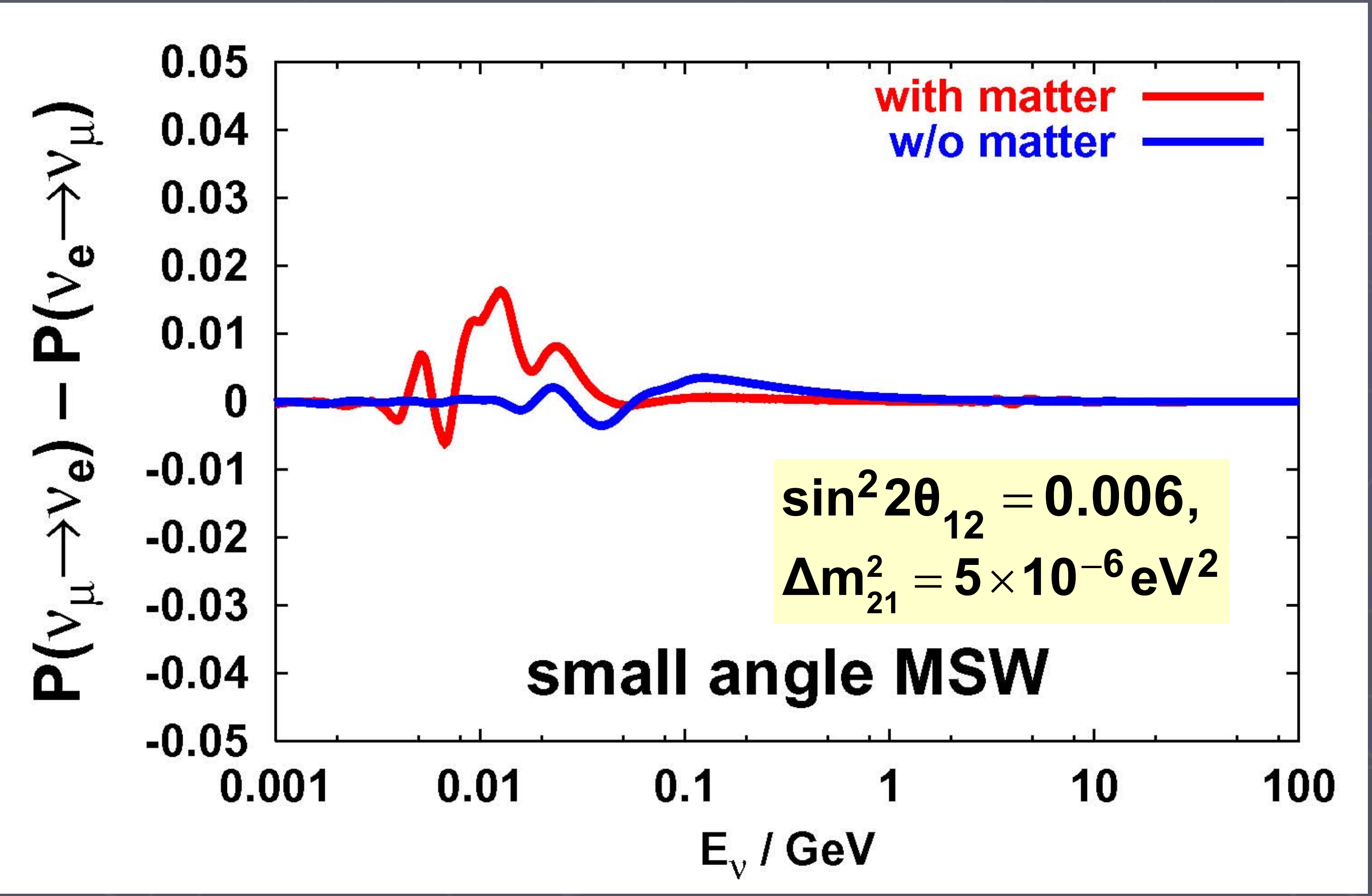
$\cancel{T} \neq 0 \iff \sin\delta \neq 0$

cf. Complete treatment of oscillation probability in matter with constant density

Kimura, Takamura, Yokomakura '02

Example: Enhancement of T-violation after passing through the Earth

OY (Ustron'99)



Future LBL

To perform precise measurements of θ_{13} and δ, one has to have a lot of numbers of events to improve statistical errors.

→We need **high intensity** beam

Candidates for high intensity beam in the future:

- (conventional) superbeam $\pi^+ \to \mu^+ + \nu_\mu$

T2K phase I (0.75MW) , II (4MW)

- neutrino factory $\mu^+ \to e^+ + \nu_e + \overline{\nu}_\mu$

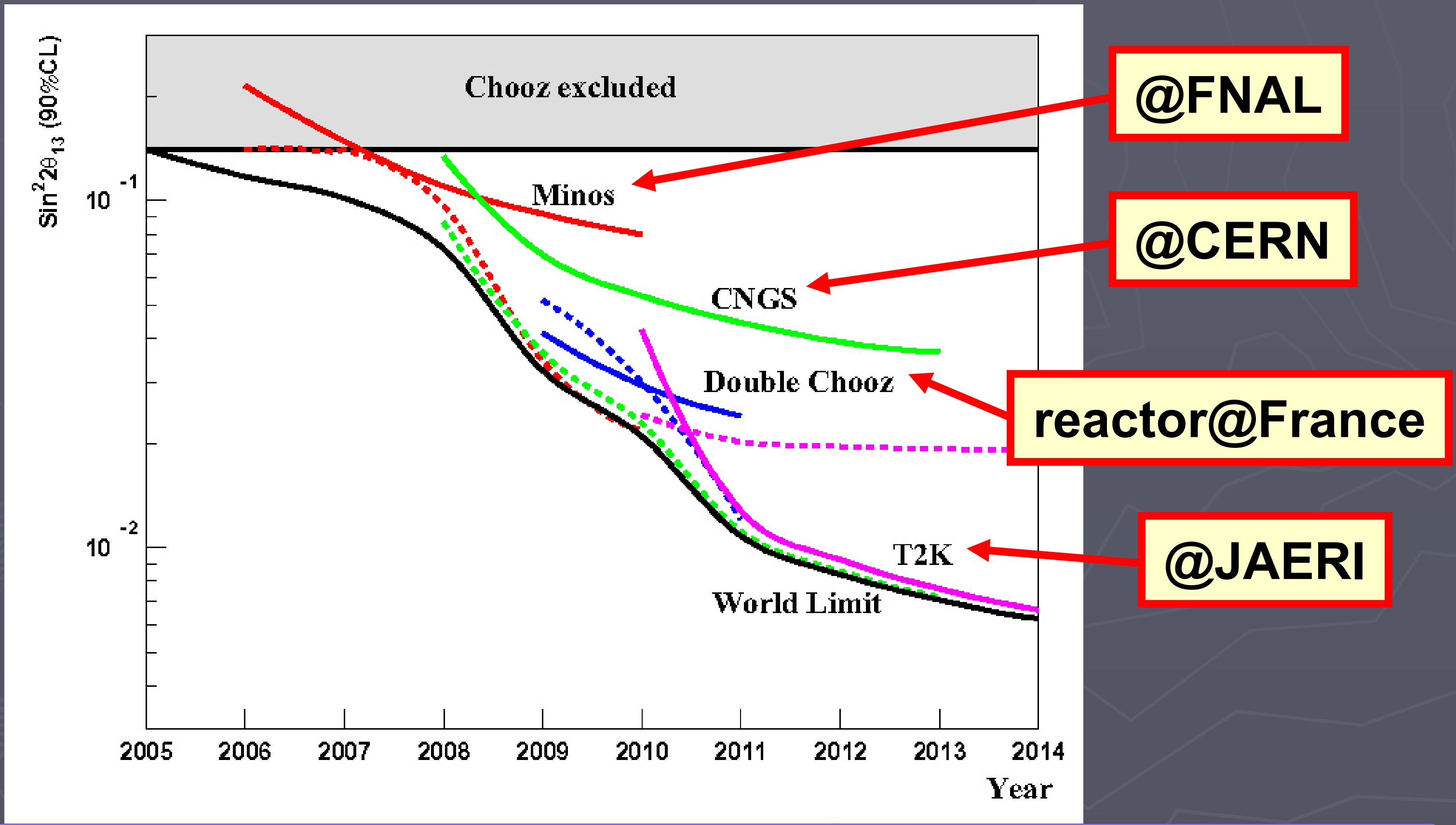
  μ in a storage ring

- beta beam $n \to p + e^- + \overline{\nu}_e$

  RI in a storage ring

Expected sensitivity to $\sin^2 2\theta_{13}$ of ongoing and near future experiments

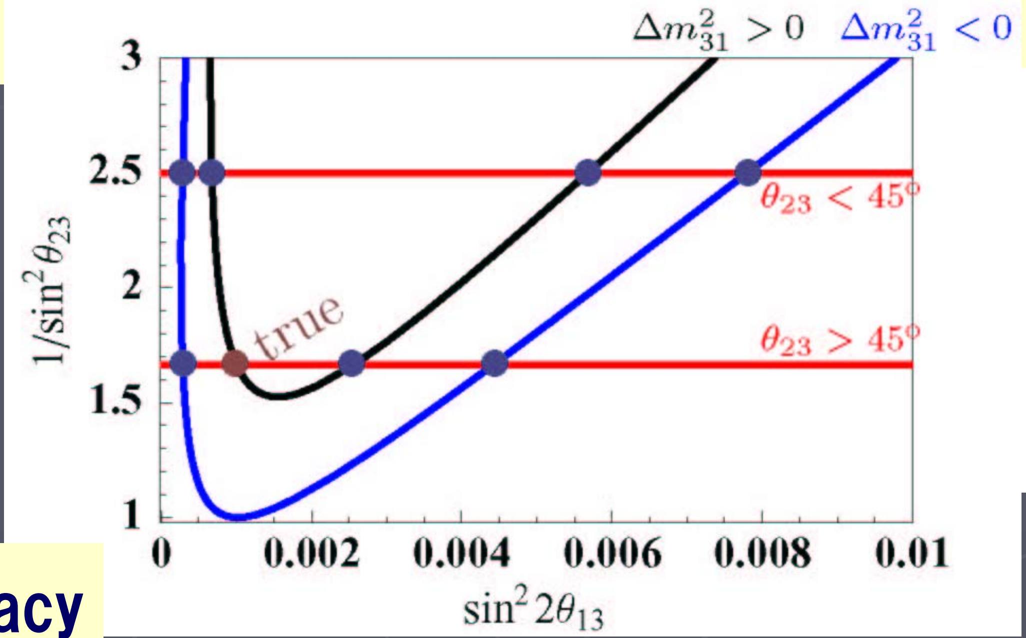
Guglielmi, Mezzetto, Migliozzi , Terranova, hep-ph/0508034



Planned reactor experiments to measure $\sin^2 2\theta_{13}$: Double CHOOZ (FR), KASKA (JP), Braidwood (US), Daya Bay (CN), Angra (BR), RENO (KR), …

Parameter degeneracy

Even if we know $P(\nu_\mu \to \nu_e)$ and $P(\overline{\nu_\mu} \to \overline{\nu_e})$ in a long baseline accelerator experiments with approximately monoenergetic neutrino beam, precise determination of θ_{13}, sign(Δm^2_{31}) and δ is difficult because of the 8-fold parameter degeneracy.



- intrinsic (δ, θ_{13}) degeneracy
- $\Delta m^2_{31} \Leftrightarrow -\Delta m^2_{31}$ degeneracy
- $\theta_{23} \Leftrightarrow \pi/2 - \theta_{23}$ degeneracy

To solve parameter degeneracy, combine the following:

(A) LBL measurement at $|\Delta m^2_{31}|L/4E = \pi/2$

→ hyperbola shrinks to a straight line

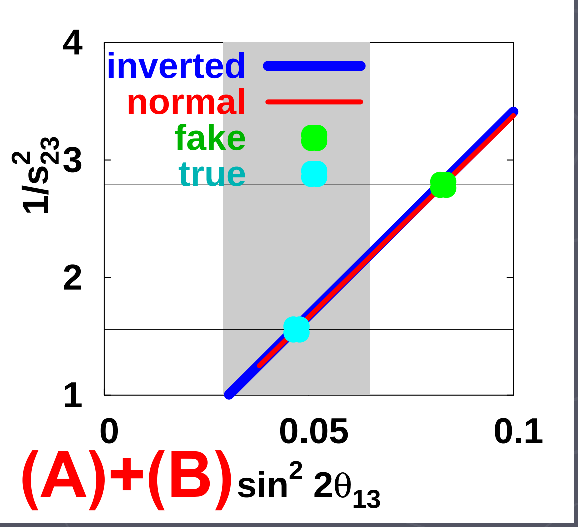
(B) reactor measurement of θ_{13} $\bar{\nu}_e \to \bar{\nu}_e$

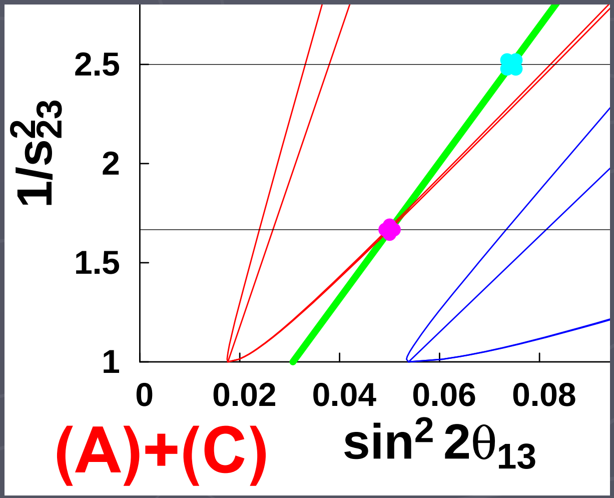
→ depends only on θ_{13}

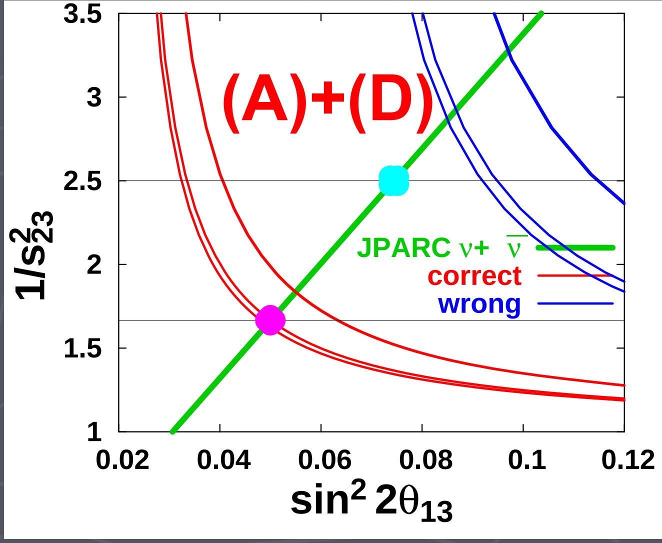
(C) LBL measurement of $\nu_\mu \to \nu_e$ **(or** $\nu_e \to \nu_\mu$ **)**

with different L/E

(D) measurement of $\nu_e \to \nu_\tau$







International scoping study of a future

Neutrino Factory and super-beam facility

Mandate

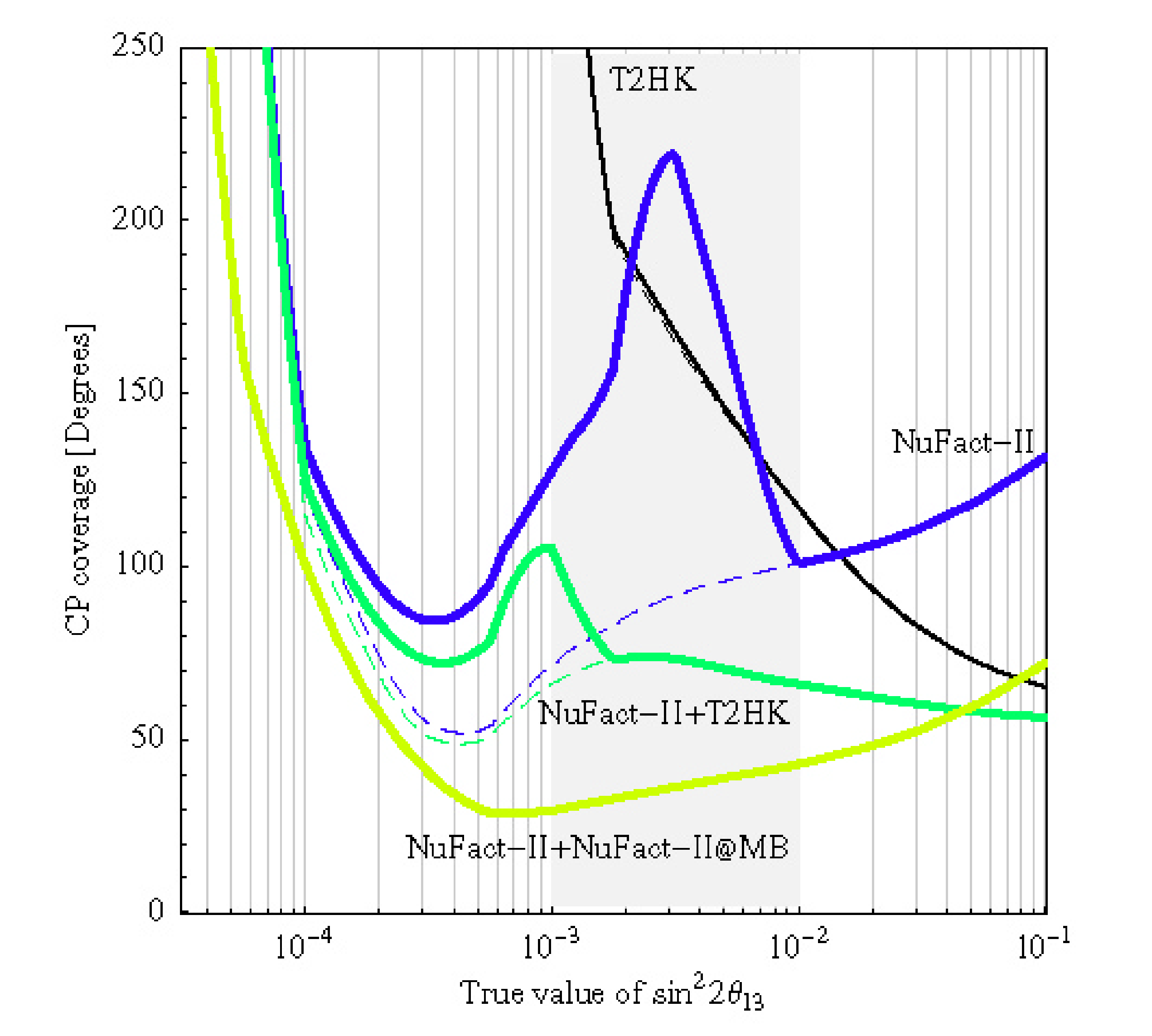
The international scoping study of a future accelerator neutrino complex will be carried by the international community between NuFact05, Frascati, 21-26 June 2005, and NuFact06. The plan for the scoping study is summarised below. The physics case for the facility will be evaluated and options for the accelerator complex and neutrino detection systems will be studied. The principal objective of the study will be to lay the foundations for a full conceptual-design study of the facility. The plan for the scoping study has been prepared in collaboration by the international community that wishes to carry it out; the ECFA/BENE network in Europe, the Japanese NuFact-J collaboration, the US Muon Collider and Neutrino Factory Collaboration and the UK Neutrino Factory collaboration. CCLRC's Rutherford Appleton Laboratory will be the 'host laboratory' for the study.

June 2005 ～ Sept. 2006

http://www.hep.ph.ic.ac.uk/iss/

- **Evaluate the physics case for the facility**
- **Study options for the accelerator complex and neutrino detection systems**

Error (or sensitivity) of the CP phase δ of far future experiments



T2HK = Tokai to HyperKamiokande

Huber-Lindner-Winter '05

Physics which could be done for a ν factory

- Check of unitarity (as at a B factory)
- Study of new physics
  - exotic interactions

$$\mathcal{L}_{eff} = \sqrt{2}\,\epsilon_{\alpha\beta\rho\sigma}\, G_F\, \bar{\nu}_\alpha \gamma^\mu \nu_\beta\, \bar{\ell}_\rho \gamma_\mu \ell_\sigma$$

  - mass varying neutrino scenario

$$M_i^{eff} = \frac{\lambda_{\nu_i}\lambda_N}{m_\phi^2} n_N$$

Yukava couplings

Nucleon number density

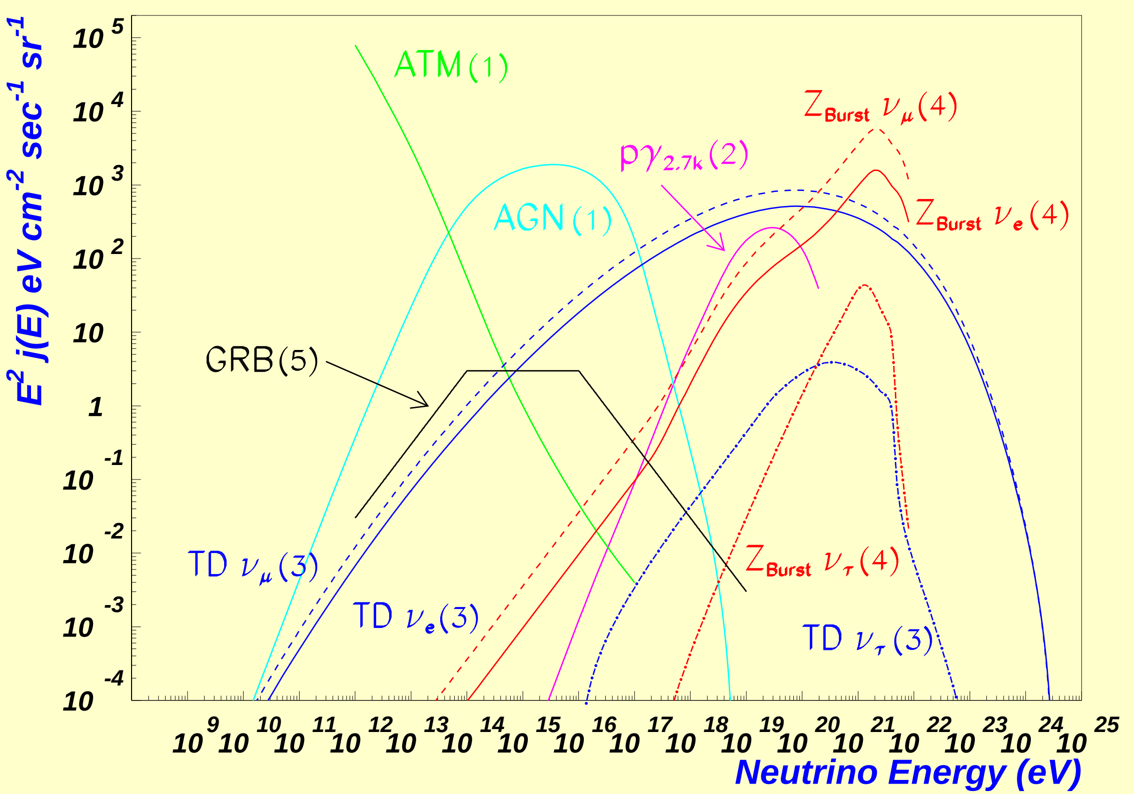
  - existence of sterile neutrinos

scenarios with or w/o LSND

Future problems

3. High energy astrophysical ν

Flux of high energy cosmic ν from Active Galactic Nuclei or Gamma Ray Burst etc.



S/N ratio is expected to be large due to little background of atmospheric ν

- **Precise normalization of flux is not known**

$\rightarrow$ **The ratio of different flavors is important quantity to observe**

- **Initial flux:**

Just like in ν_{atm}, the source of ν is π decay

$$\pi^+ \rightarrow \mu^+ + \nu_\mu, \quad \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu, \quad \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

$\Rightarrow$ $F^0(\nu_e):F^0(\nu_\mu):F^0(\nu_\tau) \cong 1:2:0$

- **Observed flux on Earth:**

Due to ν oscillations

$|\theta_{13}| \ll 1$, $|\pi/4-\theta_{23}| \ll 1$ $\Rightarrow$ $\boxed{F(\nu_e):F(\nu_\mu):F(\nu_\tau) \cong 1:1:1}$

In standard N_ν=3, when L→∞ oscillation probability in vacuum

Learned-Pakvasa '95

$$P_{\alpha\beta} = \sum_j \left|U_{\alpha j}\right|^2 \left|U_{\beta j}\right|^2$$

$$\left|U_{\alpha j}\right|^2 \cong \begin{pmatrix} c_{12}^2 & s_{12}^2 & 0 \\ s_{12}^2/2 & c_{12}^2/2 & 1/2 \\ s_{12}^2/2 & c_{12}^2/2 & 1/2 \end{pmatrix}$$

$$\boxed{\begin{aligned} F(v_e) &= F^0(v_e)(P_{ee} + 2P_{\mu e}) = F^0(v_e)(1\text{-}P_{\tau e} + P_{\mu e}) = 1 \\ F(v_\mu) &= F^0(v_e)(P_{e\mu} + 2P_{\mu\mu}) = F^0(v_e)(1\text{-}P_{\tau\mu} + P_{\mu\mu}) = 1 \\ F(v_\tau) &= F^0(v_e)(P_{e\tau} + 2P_{\mu\tau}) = F^0(v_e)(1\text{-}P_{\tau\tau} + P_{\mu\tau}) = 1 \end{aligned}}$$

$$F(v_\alpha) = F^0(v_e)P_{e\alpha} + F^0(v_\mu)P_{\mu\alpha} = F^0(v_e)(P_{e\alpha} + 2P_{\mu\alpha})$$

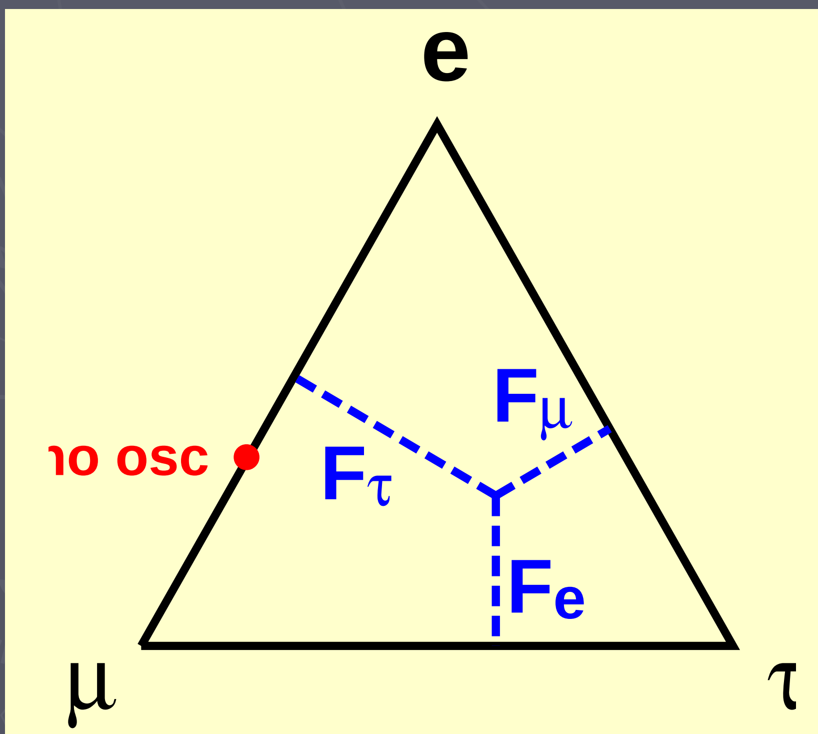
$$P_{e\alpha} + 2P_{\mu\alpha} = (P_{e\alpha} + P_{\mu\alpha}) + P_{\mu\alpha} = 1\text{-}P_{\tau\alpha} + P_{\mu\alpha} = 1$$

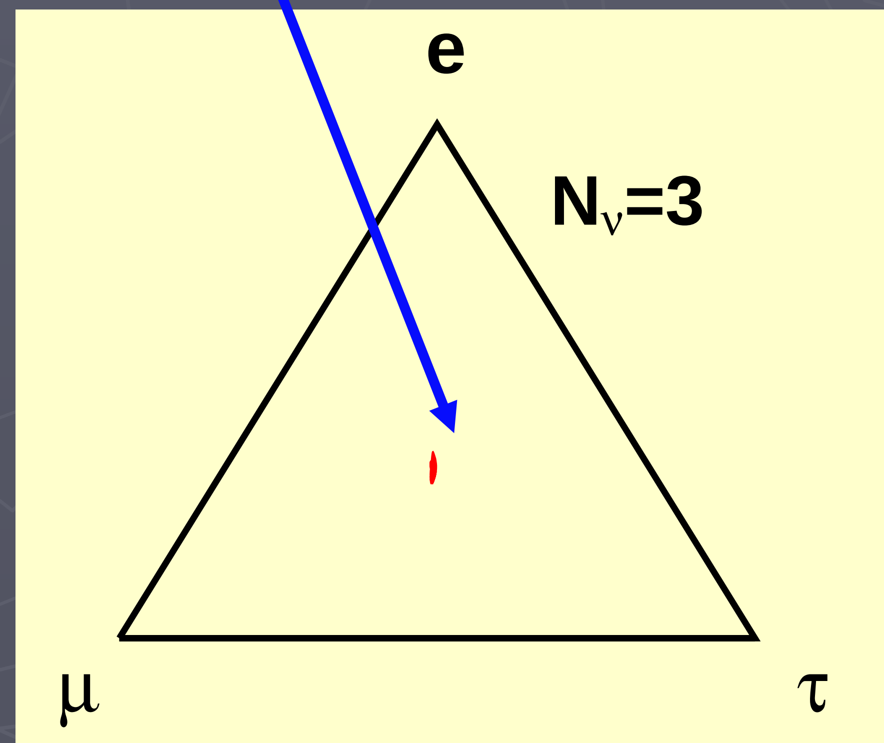
CHOOZ+ν_{atm}: $|\theta_{13}|$<<1

ν_{atm}: $|\pi/4 - \theta_{23}|$<<1

Deviation from 1:1:1 is small

Athar-Jezabek-OY '00





A few scenarios to predict deviation from 1:1:1 have been proposed

- **Standard flux + ν decay**

  Beacom-Bell-Hooper-Pakvasa-Weiler '03

  α:1:1 (α=1.4~6)

- **Standard flux + pseudo-Dirac ν**

  Beacom -Bell-Hooper-Learned-Pakvasa-Weiler'04

  α:1:1 (α=2/3~14/9)

- **Electromagnetic energy losses of π & μ**

  Kashti-Waxman '05

  α:1:1 (α=1/1.8~1)

4. CP phases in ν oscillations

Derivation of propagation of ν oscillations

Grimus-Scharnagl, '93

Dirac eq. → (Tani-Foldy like transf.) → positive energy part → Schroedinger type eq. to order m^2/p^2

For simplicity Majorana case is discussed here

$$\mathcal{A} \equiv \mathrm{diag}\,(A_e + A_n, A_n, A_n)$$
$$A_e \equiv \sqrt{2} G_F N_e$$
$$A_n \equiv \frac{1}{\sqrt{2}} G_F N_n$$

$$i\frac{d}{dx}\begin{pmatrix} \nu_L \\ (\nu_L)^c \end{pmatrix} = \mathcal{M}\begin{pmatrix} \nu_L \\ (\nu_L)^c \end{pmatrix}$$

$$\mathcal{L}_{mag} = \mu_{\alpha\beta}\overline{(\nu_{\alpha L})^c}\sigma_{\rho\sigma}F^{\rho\sigma}\nu_{\beta L} + h.c.$$

$$\mathcal{M} \equiv \begin{pmatrix} |\vec{p}| + \frac{1}{2|\vec{p}|}m^\dagger m + \mathcal{A} & |B_\perp|\mu \\ |B_\perp|\mu & |\vec{p}| + \frac{1}{2|\vec{p}|}m m^\dagger - \mathcal{A} \end{pmatrix}$$

$\mu_{\alpha\beta}$: magnetic transitions

$$\mathcal{M} \equiv \begin{pmatrix} |\vec{p}| + \frac{1}{2|\vec{p}|} m^\dagger m + \mathcal{A} & |B_\perp|\mu \\ |B_\perp|\mu & |\vec{p}| + \frac{1}{2|\vec{p}|} m m^\dagger - \mathcal{A} \end{pmatrix}$$

$$\begin{aligned} \mathcal{A} &\equiv \mathrm{diag}\,(A_e + A_n, A_n, A_n) \\ A_e &\equiv \sqrt{2} G_F N_e \\ A_n &\equiv \frac{1}{\sqrt{2}} G_F N_n \end{aligned}$$

$$\begin{aligned} m &= V^* \,\mathrm{diag}(m_j)\, V^\dagger \\ V &= e^{i\alpha} \boxed{e^{i\beta'\lambda_3} e^{i\gamma'\lambda_8}}\, U \boxed{e^{-i\gamma\lambda_8} e^{-i\beta\lambda_3}} \\ m^\dagger m &= V \,\mathrm{diag}(m_j^2)\, V^\dagger = \boxed{e^{i\beta'\lambda_3} e^{i\gamma'\lambda_8}}\, U \,\mathrm{diag}(m_j^2)\, U^\dagger \boxed{e^{-i\gamma'\lambda_8} e^{-i\beta'\lambda_3}} \\ m m^\dagger &= V^* \,\mathrm{diag}(m_j^2)\, V^T = \boxed{e^{-i\gamma'\lambda_8} e^{-i\beta'\lambda_3}}\, U^* \,\mathrm{diag}(m_j^2)\, U^T \boxed{e^{i\beta'\lambda_3} e^{i\gamma'\lambda_8}} \end{aligned}$$

- **β, γ (Majorana CP phases) don't appear in the eq.**

cf. neutrinoless double β decay

$$\langle m_{\nu e} \rangle = \left| c_{12}^2 c_{13}^2 m_1 e^{-i(\beta+\gamma)} + s_{12}^2 c_{13}^2 m_2 e^{i(\beta-\gamma)} + s_{13}^2 m_3 e^{2i(\gamma-\delta)} \right|$$

- **CP phases β', γ' from the charged lepton sector remain if off-diagonal part of μ or $\mathcal{A}$ ❍0**

In this case analysis of 6x6 matrix is reduced to that of 3x3:

$$\mathcal{M} = \begin{pmatrix} e^{i\Phi} & 0 \\ 0 & e^{-i\Phi} \end{pmatrix} \times \begin{pmatrix} UDU^{-1} + e^{-i\Phi} \mathcal{A} e^{i\Phi} & e^{-i\Phi} |B_\perp| \mu e^{-i\Phi} \\ e^{i\Phi} |B_\perp| \mu e^{i\Phi} & UDU^{-1} - e^{i\Phi} \mathcal{A} e^{-i\Phi} \end{pmatrix} \begin{pmatrix} e^{-i\Phi} & 0 \\ 0 & e^{i\Phi} \end{pmatrix}$$

$$D \equiv \mathrm{diag}(|\vec{p}| + \frac{1}{2|\vec{p}|} m_j^2)$$

$$\Phi \equiv \beta' \lambda_3 + \gamma' \lambda_8$$

$$\boldsymbol{\lambda}_3 = \mathbf{diag}\,(1, -1, 0), \quad \boldsymbol{\lambda}_8 = \mathbf{diag}\,(1, 1, -2)$$

If the effect of magnetic transitions μ or the off-diagonal part of $\mathcal{A}$ due to new physics, then the effect of these CP phase may be important and should be taken into account

5. Summary

- **CP phase δ is always accompanied by a small factor $\sin\theta_{13}$ so unless it is enhanced by matter effect, it may be neglected in the leading order.**
- **There are CP phases from the charged lepton sector and in principle they appear in ν oscillations if off-diagonal elements of magnetic transition or matter term are non-zero. Again unless these terms are large, the CP phases may be ignored.**