# Phenomenology of neutrino --- Overview ---

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# Introduction Future experiments New Physics & τ detection Summary

### **1. Introduction**

### Framework of 3 flavor v oscillation

#### Mixing matrix

Functions of mixing angles θ<sub>12</sub>, θ<sub>23</sub>, θ<sub>13</sub>, and CP phase δ

$$\begin{pmatrix} \mathbf{v}_{e} \\ \mathbf{v}_{\mu} \\ \mathbf{v}_{\tau} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \mathbf{v}_{1} \\ \mathbf{v}_{2} \\ \mathbf{v}_{3} \end{pmatrix}$$



All 3 mixing angles have been measured (2012):

### **Global analysis**



### Forero, Tortola, Valle arXiv:1205.4018 (nu2012 data included)

• A word on theory: Simple theoretical ansatz to predict  $\theta_{13}$  successfully

Anarchy Hall, Murayama, Weiner, PRL 84 (2000) 2572

sin²2θ<sub>13</sub>∼ 0.1

Quark-lepton complementarity Minakata, Smirnov, PR D70 (2004) 073009

 $\theta_{12} + \theta_{C} = 45 \text{ deg}$   $\theta_{13} = 8.9 \text{ deg}$   $\theta_{12} = 35.4 \text{ deg}$   $\theta_{23} = 42.1 \text{ deg}$ Dighe, Goswami, Roy PR D76 (2007) 096005

sin²2θ<sub>23</sub>∼ 1

### Oscillation vs non-oscillation experiments

Majorana

phases

### neutrino oscillation

neutrinoless double beta decay

$$\mathbf{m}_{ee} = |\Sigma(\mathbf{U}_{ej})^2 \mathbf{m}_j \exp(i\phi_j)|$$

direct measurement

$$\mathbf{m}_{\beta} = (\Sigma | \mathbf{U}_{ej} |^2 \mathbf{m}_{j}^2)^{1/2}$$

cosmology

Σm

### neutrinoless double beta decay

$$m_{ee} = |\Sigma(U_{ej})^2 m_j exp(i\phi_j)|$$

#### Rodejohann@NOW2012

Experiment	Isotope	Status Start of		Sensitivity	
			data-taking	$\langle m_{ u}  angle$ [eV]	
GERDA	<sup>76</sup> Ge	running	$\sim 2011$	0.17-0.42	
		in progress	$\sim$ 2012	0.06-0.16	
		R&D	$\sim 2015$	0.012-0.03	
CUORE	<sup>130</sup> Te	in progress	$\sim 2013$	0.018-0.03	
				0.03-0.066	
MAJORANA	$^{76}$ Ge	in progress	$\sim 2013$	0.06-0.16	
		R&D	$\sim 2015$	0.012-0.03	
EXO	<sup>136</sup> Xe	running	$\sim 2011$	0.073-0.18	
		R&D	$\sim 2015$	0.02-0.05	
SuperNEMO	<sup>82</sup> Se	R&D	$\sim$ 2013-15	0.04-0.096	
KamLAND-Zen	$^{136}Xe$	running	$\sim 2011$	0.03-0.07	
		R&D	$\sim$ 2013-15	0.02-0.046	
SNO+	$^{150}$ Nd	in progress	$\sim$ 2014	0.09-0.18	





### 2. Future LBL (Long BaseLine experiments)

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \cong \begin{pmatrix} C_{12} & S_{12} & E \\ -S_{12}/\sqrt{2} & C_{12}/\sqrt{2} & 1/\sqrt{2} \\ S_{12}/\sqrt{2} & -C_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

 Both mass hierarchies are allowed

# Next task is to measure sign( $\Delta m^2_{31}$ ), $\pi$ /4- $\theta_{23}$ and $\delta$

Most realistic way to measure sign( $\Delta m^2_{31}$ ) and  $\delta$  is Long Base Line experiments by accelerators or reactors



To measure  $\delta$ , we need a lot of numbers of events to improve statistical errors  $\rightarrow$  We need high intensity beam

**Candidates for high intensity beam in the future:** 

- (conventional) superbeam
- neutrino factory
  - $\mu$  in a storage ring
- beta beam

**RI** in a storage ring

n 
$$\begin{cases} \mathbf{\pi}^{+} \rightarrow \mathbf{\mu}^{+} + \mathbf{V}_{\mathbf{\mu}} \\ \mathbf{\pi}^{-} \rightarrow \mathbf{\mu}^{-} + \mathbf{V}_{\mathbf{\mu}} \\ \mathbf{\mu}^{+} \rightarrow \mathbf{e}^{+} + \mathbf{V}_{\mathbf{e}} + \mathbf{V}_{\mathbf{\mu}} \\ \mathbf{\mu}^{-} \rightarrow \mathbf{e}^{-} + \mathbf{V}_{\mathbf{e}} + \mathbf{V}_{\mathbf{\mu}} \\ \end{cases}$$
  
$$\begin{cases} \mathbf{\mu}^{e} \rightarrow \mathbf{e}^{-} + \mathbf{V}_{\mathbf{e}} + \mathbf{V}_{\mathbf{\mu}} \\ \mathbf{\mu}^{e} \rightarrow \mathbf{v}_{\mathbf{e}} \end{pmatrix}$$
  
$$\begin{cases} \mathbf{e}^{e} \mathbf{H} \mathbf{e} \rightarrow \mathbf{e}^{6} \mathbf{L} \mathbf{i} + \mathbf{e}^{-} + \mathbf{V}_{\mathbf{e}} \\ \mathbf{v}_{\mathbf{e}} \rightarrow \mathbf{v}_{\mathbf{\mu}} \\ \mathbf{v}_{\mathbf{$$

### Future LBL exp. (under construction / proposed )

superbeam

T2K phase II (2.2MW+HK(+Okinoshima), E~1GeV, L=295km, 658km)

NOvA (FNAL  $\rightarrow$  Ash River (MN), E~2GeV, L=810km)

LBNE (FNAL→Homestake, E~a few GeV, L=1290km)

CN2PY (CERN→Pyhasalmi, E~several GeV, L=2300km)

• neutrino factory (reference design,  $E_v \sim 10 \text{GeV}$ , L~2000km)

beta beam (E,=0.5-1.5GeV, L~130km)

Mass Hierarchy [sign(∆m<sup>2</sup><sub>31</sub>)] may be determined next

### Parameter degeneracy

Even if we know  $P \equiv P(v_{\mu} \rightarrow v_{e})$  and  $\overline{P} \equiv P(\overline{v_{\mu}} \rightarrow \overline{v_{e}})$ in a long baseline accelerator experiments with approximately monoenergetic neutrino beam, precise determination of  $\theta_{13}$ ,  $\theta_{23}$ , sign( $\Delta m^{2}_{31}$ ) and  $\delta$  is difficult because of the 8-fold parameter degeneracy.



Plots in  $(\sin^2 2 \theta_{13}, 1/s_{23}^2)$  plane are useful to see 8-fold degeneracy

OY, New J.Phys. 6 (2004) 83

Each point has different value of  $\delta$ .  $\rightarrow$  Parameter degeneracy must be resolved for precise measurements of  $\delta$ .

• octant degeneracy  

$$\theta_{23} \leftrightarrow \pi/2 - \theta_{23}$$
  
(a) $\cos 2\theta_{23} = 0 \rightarrow (b)\cos 2\theta_{23} \neq 0$ 



0



# • intrinsic degeneracy $(\delta, \theta_{13})$

(a) 
$$\frac{\Delta m_{21}^2}{|\Delta m_{31}^2|} = 0 \rightarrow (b) \frac{\Delta m_{21}^2}{|\Delta m_{31}^2|} \cong \frac{1}{35} \neq 0$$

• sign degeneracy  

$$\Delta m_{31}^2 \leftrightarrow -\Delta m_{31}^2$$
  
(a)AL/2 = 0  $\rightarrow$  (b)AL/2  $\neq$  0  
 $A \equiv \sqrt{2}G_F N_e \cong 1/2000 \text{ km}$ 



 $\sin^2 2\theta_{13}$ 

0





### Sign degeneracy is more serious than octant one, because $sin\delta(sign)=0 \Rightarrow sin\delta'(sign)=O(1)\neq 0$



### **Current status: T2K+atm+reactors**



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### **Future exp. vs Mass Hierarchy**



3. New Physics &  $v_{\tau}$  detection NP:= Deviation from SM+massive v Most of discussions to date are phenomenological

Motivation: High precision measurements of v oscillation in future experiments can be used also to probe physics beyond SM by looking at deviation from SM+massive v

# $v_{\tau}$ channels: not studied much $\rightarrow$ There is a plenty of room to probe New Physic

 MINSIS (Main Injector Non Standard Interactions Search): Near detectors may enable us to study τ channels [Para, Madrid ν NSI Workshop (Dec. 2009)]

### • v factory

- Higher energy option (E\_{\mu}=50GeV, L=3000km) w/  $\tau$  detectors has better sensitivity to NP
- Unfortunately higher energy option has poor sensitivity to  $\delta$  for sin^22 $\theta_{13}$  =0.1 [Pinney-OY, 2001]
- current baseline (E<sub>µ</sub>=10GeV, L=2000km) does not have a  $\tau$  detector [NF Reference Design], but  $\tau$  detector may be included in the future (if physics case is strong)

$$V_e \rightarrow V_\mu$$
 golden  
channel  $V_\mu \rightarrow V_\mu$  disappearance  
channel  $V_e \rightarrow V_\tau$  silver  
channel  $V_\mu \rightarrow V_\tau$  discovery channel

**New physics** which can be probed at a future long baseline neutrino experiments includes:

- NSI at production / detection
- Unitarity Violation due to heavy particles
- Schemes with light sterile neutrinos



Scenarios	Phenomenological bound on deviation of 3 flavor unitarity			
NSI at production / detection	0(1%)			
Violation of unitarity due to heavy particles	0(0.1%)			
Light sterile neutrinos	0(10%)			
<ul> <li>(Except sterile v) none of these scenarios has ever been supported experimentally.</li> <li>Even if LSND anomaly is excluded in the near future,</li> </ul>				

light sterile v could be phenomenologically even more promising than others.

### **3-1. NSI at source and detector**

Grossman, Phys. Lett. B359, 141 (1995)

#### **Possible processes with**

$$\mathcal{L}_{\text{eff}} = G_F \,\epsilon^{ff'}_{\alpha\beta} \,\bar{\nu}_{\alpha} \gamma^{\rho} \ell_{\beta} \bar{f} \gamma_{\rho} f'$$

$$\mu^+ \to e^+ + \overline{\nu}_{\mu} + \nu^s_{\mu}$$

$$\nu^s_e = \nu_e + \epsilon^s_{e\mu} \nu_{\mu}$$

• NP at detector

$$\nu^d_\mu + n \to \mu^- + p$$

$$\nu_{\mu}^{d} = \nu_{\mu} - \epsilon_{e\mu}^{d} \nu_{e\mu}$$



From  $\mu$ ,  $\beta$ ,  $\pi$  decays and zero distance oscillations

$$2\sqrt{2}G_{F}\varepsilon_{\alpha\beta}^{\mu d}\left(\bar{l}_{\beta}\gamma^{\mu}P_{L}\nu_{\alpha}\right)\left(\bar{u}\gamma_{\mu}P_{L,R}d\right) \qquad 2\sqrt{2}G_{F}\varepsilon_{\alpha\beta}^{\mu e}\left(\bar{\mu}\gamma^{\mu}P_{L}\nu_{\beta}\right)\left(\bar{\nu}_{\alpha}\gamma_{\mu}P_{L}e\right)$$

		0.042	0.025	0.042		(0.025)	0.03	0.03
$\boldsymbol{\mathcal{E}}^{ud}$	<	$2.6 \cdot 10^{-5}$	0.1	0.013	$\left  \mathcal{E}^{\mu e} \right  <$	0.025	0.03	0.03
		0.087	0.013	0.13		0.025	0.03	0.03

**Bounds** ~**O(10<sup>-2</sup>)** 

C. Biggio, M. Blennow and EFM 0907.0097

E. Fernandez-Martinez @ NSI workshop at UAM 2009-12-10

### Sensitivity to $\epsilon_{u\tau}$ at $\nu$ factory

#### Ota-Sato-Yamashita, PRD65:093015,2002



### Near τ detector improves sensitivity



ND5: mass 2 kton, L=1 km

# **3-2. Violation of unitarity w/o light** $v_s$ (Minimal Unitarity Violation)

Antusch, Biggio, Fernandez-Martinez, Gavela, Lopez-Pavon, JHEP0610,084, '06

In generic see-saw models, after integrating out  $v_R$ , the kinetic term gets modified, and unitarity is expected to be violated.

$$L = \frac{1}{2} \left( i \overline{v_{\alpha}} \partial K_{\alpha\beta} v_{\beta} - \overline{v}^{c}{}_{\alpha} M_{\alpha\beta} v_{\beta} \right) - \frac{g}{\sqrt{2}} \left( W_{\mu}^{+} \overline{l}_{\alpha} \gamma^{\mu} P_{L} v_{\alpha} + h.c. \right) + \dots$$

rescaling v

$$L = \frac{1}{2} \left( i \overline{\nu_i} \partial \nu_i - \overline{\nu}^c {}_i m_{ii} \nu_i \right) - \frac{g}{\sqrt{2}} \left( W^+_\mu \overline{l}_\alpha \gamma^\mu P_L N_\alpha \nu_i \right) + \dots$$

### **N:** non-unitary lepton mixing matrix





![](_page_28_Figure_0.jpeg)

### Sensitivity of MINSIS to $\nu_{\text{s}}$

### Li@Madrid ~ v~ NSI~ Workshop

With  $\mu$  disappearance, have similar sensitivity to  $\sin^2 2\theta_s^{\mu\mu}$  as NOMAD and CHORUS have to  $\sin^2 2\theta_s^{\mu\tau}$ .

au appearance is  $\sim$  100 times more sensitive than  $\mu$  disappearance.

![](_page_29_Figure_5.jpeg)

#### Sensitivity of v factory to CP violation

Potentially the largest CP violation occurs in  $\mu$ - $\tau$  channel: CP violation due to the new CP phase

![](_page_30_Figure_2.jpeg)

**Discovery channel** is crucial to measure the new CP phase 31

### 4. Summary (1)

- Three mixing angles have been determined :  $\theta_{12} \simeq \pi/6, \theta_{23} \simeq \pi/4, \theta_{13} \simeq \pi/20.$
- The remaining parameters to be measured are sign( $\Delta m_{31}^2$ ), sign( $\theta_{23}$ - $\pi/4$ ) and  $\delta$ .
- To determine δ, parameter degeneracy (particularly of mass hierarchy) must be resolved.
- Accelerator and reactor experiments are expected to determine sign( $\Delta m_{31}^2$ ) and  $\delta$  in 10-20 years.

### **4. Summary (2)**

 High precision measurements of v oscillation in the future experiments will enable us to probe New Physics such as (NSI at production / detection, Violation of unitarity, Schemes with light sterile neutrinos).

τ detectors may be placed at NUMI ν beams or at ν factory in the future option.
 In absence of 3 flavor unitarity, τ detectors in principle give us important information on New Physics→ ν<sub>τ</sub> detection deserves further study.

## **Backup slides**

![](_page_34_Picture_0.jpeg)

### Global Fits:

Global Fit Forero, Tortola, Valle arXiv:1205.4018 Fogli, Lisi, Marrone, Montanino , Palazzo, Rotunno Phys.Rev. D86 (2012) 013012 arXiv:1205.5254

Rotunno 12

parameter	best fit $\pm 1\sigma$	best fit $\pm 1\sigma$
$\Delta m_{21}^2  [10^{-5} \mathrm{eV}^2]$	$7.62\pm0.19$	$7.54^{+0.26}_{-0.22}$
$\Delta m_{31}^2  [10^{-3} \text{eV}^2]$	$2.53^{+0.08}_{-0.10} \\ -(2.40^{+0.10}_{-0.07})$	$2.43^{+0.07}_{-0.09}_{-(2.42^{+0.07}_{-0.10})}$
$\sin^2  heta_{12}$	$0.320\substack{+0.015\\-0.017}$	$0.307\substack{+0.018 \\ -0.016}$
$\sin^2 heta_{23}$	$\begin{array}{c} 0.49\substack{+0.08\\-0.05}\\ 0.53\substack{+0.05\\-0.07}\end{array}$	$\begin{array}{c} 0.398\substack{+0.030\\-0.026}\\ 0.408\substack{+0.035\\-0.030}\end{array}$
$\sin^2 heta_{13}$	$\begin{array}{c} 0.026\substack{+0.003\\-0.004}\\ 0.027\substack{+0.003\\-0.004} \end{array}$	$\begin{array}{c} 0.0245^{+0.0034}_{-0.0031} \\ 0.0246^{+0.0034}_{-0.0031} \end{array}$
δ	$ig( 0.83^{+0.54}_{-0.64} ig) \pi \ 0.07 \pi^{-a}$	$(0.89^{+0.29}_{-0.44})\pi$ $(0.90^{+0.32}_{-0.43})\pi$

![](_page_35_Picture_0.jpeg)

#### MINOS+T2K+sol+ KL+DC+DB+RENO

#### MINOS+T2K+sol+ KL+DC+DB+RENO+atm

![](_page_35_Figure_3.jpeg)

π/4- θ<sub>23</sub> <0 is preferred

Forero, Tortola, Valle arXiv:1205.4018

![](_page_35_Picture_6.jpeg)

### Octant of $\theta_{23}$ ( $\pi/4-\theta_{23}$ >0?) appears to be subtle

![](_page_36_Figure_1.jpeg)

Fogli, Lisi, Marrone, Montanino, Palazzo, Rotunno Phys.Rev. D86 (2012) 013012 arXiv:1205.5254

### Info on MH seems necessary

### ltow@v2012

![](_page_37_Figure_2.jpeg)

![](_page_37_Figure_3.jpeg)

### 3 flavor atmospheric v oscillations

$$\frac{\Phi(\nu_e)}{\Phi_0(\nu_e)} - 1 \approx P_2 \cdot (r \cdot \cos^2 \theta_{23} - 1)$$

$$-r \cdot \sin \tilde{\theta}_{13} \cdot \cos^2 \tilde{\theta}_{13} \cdot \sin 2\theta_{23} \cdot (\cos \delta \cdot R_2 - \sin \delta \cdot I_2)$$

$$+2 \sin^2 \tilde{\theta}_{13} \cdot (r \cdot \sin^2 \theta_{23} - 1)$$
Normal Hierarchy Hierarchy stands, inverted hierarchy stands, inve

![](_page_39_Figure_0.jpeg)

### Differences in values of CP phases

$$\theta_{13} := \theta_{13} \text{(true)}, \quad \theta_{13} := \theta_{13} \text{(false)}$$

$$\delta := \delta$$
(true), δ':=  $\delta$ (false)

### sign degeneracy

$$\sin^2 2\theta'_{13} = \sin^2 2\theta_{13} \tan^2 \theta_{23} + \frac{\alpha^2 g^2 \sin^2 2\theta_{12}}{f\bar{f}} (1 - \tan^2 \theta_{23}),$$
  
$$\sin 2\theta'_{13} \sin \delta' = \sin 2\theta_{13} \sin \delta + \frac{\alpha g (f - \bar{f}) \sin 2\theta_{12}}{f\bar{f}} \frac{\cot 2\theta_{23}}{\sin \Delta},$$

### octant degeneracy

$$x'^{2} = \frac{x^{2}(f^{2} + \bar{f}^{2} - f\bar{f}) - 2yg(f - \bar{f})x\sin\delta\sin\Delta}{f\bar{f}},$$
$$x'\sin\delta' = x\sin\delta\frac{f^{2} + \bar{f}^{2} - f\bar{f}}{f\bar{f}} - \frac{x^{2}}{\sin\Delta}\frac{f^{2} + \bar{f}^{2}}{f\bar{f}}\frac{f - \bar{f}}{2yg}.$$

### Barger Marfatia Whisnant Phys.Rev.D65:073023,2002

# To solve parameter degeneracy, various combinations have been proposed:

- (A) LBL measurement at  $|\Delta m_{31}^2|L/4E = \pi/2$
- (B) reactor measurement of  $\theta_{13}$   $\mathbf{v_e} \rightarrow \mathbf{v_e}$
- (C) LBL measurement of  $v_{\mu} \rightarrow v_{e}$  (or  $v_{e} \rightarrow v_{\mu}$ )

with different L/E (larger L separates NH&IH more)

(D) measurement of  $V_e \rightarrow V_{\tau}$  at v factory

![](_page_41_Figure_6.jpeg)

### **Current status of appearance experiments**

![](_page_42_Figure_1.jpeg)

![](_page_42_Figure_2.jpeg)

0.4

 $\sin^2 2\theta_{13}$ 

68% C.L

0.4

 $\sin^2 2\theta_{13}$ 

![](_page_43_Figure_0.jpeg)

# Violation of unitarity due to heavy fields (Minimal Unitarity Violation)

![](_page_44_Figure_1.jpeg)

Antusch, Biggio, Fernandez-Martinez, Gavela, Lopez-Pavon, JHEP0610,084, '06

$$\frac{\Gamma(\ell_{\alpha} \to \ell_{\beta} \gamma)}{\Gamma(\ell_{\alpha} \to \nu_{\alpha} \ell_{\beta} \overline{\nu}_{\beta})} = \frac{3\alpha}{32\pi} \, \frac{|\sum_{k} N_{\alpha k} N_{k\beta}^{\dagger} F(x_{k})|^{2}}{(NN^{\dagger})_{\alpha \alpha} (NN^{\dagger})_{\beta \beta}},$$

where  $x_k \equiv m_k^2/M_W^2$  with  $m_k$  being the masses of the light neutrinos and

$$F(x) \equiv \frac{10 - 43x + 78x^2 - 49x^3 + 4x^4 + 18x^3 \ln x}{3(x - 1)^4} \,.$$

# Violation of unitarity due to heavy fields (Minimal Unitarity Violation)

Antusch, Biggio, Fernandez-Martinez, Gavela, Lopez-Pavon, JHEP0610,084, '06

 $F(x) \approx 10/3$ , it follows that

$$\frac{\Gamma(\ell_{\alpha} \to \ell_{\beta} \gamma)}{\Gamma(\ell_{\alpha} \to \nu_{\alpha} \ell_{\beta} \overline{\nu}_{\beta})} = \frac{100\alpha}{96\pi} \frac{|(NN^{\dagger})_{\alpha\beta}|^2}{(NN^{\dagger})_{\alpha\alpha} (NN^{\dagger})_{\beta\beta}}$$

leading to the constraint

$$\frac{|(NN^{\dagger})_{\alpha\beta}|^2}{(NN^{\dagger})_{\alpha\alpha}(NN^{\dagger})_{\beta\beta}} = \frac{\Gamma(\ell_{\alpha} \to \ell_{\beta}\gamma)}{\Gamma(\ell_{\alpha} \to \nu_{\alpha}\ell_{\beta}\overline{\nu}_{\beta})} \frac{96\pi}{100\alpha}$$

# Violation of unitarity due to heavy fields (Minimal Unitarity Violation)

Antusch, Biggio, Fernandez-Martinez, Gavela, Lopez-Pavon, JHEP0610,084, '06

#### **Experimental bound (in 2006)**

$$Br(\tau \to \mu\gamma) < 6.8 \cdot 10^{-8},$$

$$Br(\tau \to e\gamma) < 1.1 \cdot 10^{-7},$$

$$Br(\mu \to e\gamma) < 1.2 \cdot 10^{-11}$$

$$Br(\mu \to \nu_{\mu}e\overline{\nu}_{e}) = 0.1736 \pm 0.0006,$$

$$Br(\tau \to \nu_{\tau}e\overline{\nu}_{e}) = 0.1784 \pm 0.0006,$$

$$Br(\mu \to \nu_{\mu}e\overline{\nu}_{e}) \approx 100\%$$

$$|(NN^{\dagger})_{\mu\tau}|^{2} = (NN^{\dagger})_{\mu\mu}(NN^{\dagger})_{\tau\tau}\frac{\Gamma(\tau \to \mu\gamma)}{\Gamma(\tau \to \nu_{\tau}\mu\overline{\nu}_{\mu})}\frac{96\pi}{100\alpha}$$

$$\approx \frac{6.8 \cdot 10^{-8}}{0.1736} \cdot \frac{96\pi}{100\alpha}$$

$$= 1.6 \times 10^{-4}$$