

Sterile neutrinos and near detectors at a neutrino factory

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- 1. Introduction**
- 2. Light sterile neutrinos**
- 3. Sensitivity to θ_{14} , θ_{24} , θ_{34} at ν factory with a near detector**
- 4. Sensitivity to θ_{14} , θ_{24} , θ_{34} at ν factory with far detectors**
- 5. Summary**

1. Introduction

Motivation for research on **New Physics** and τ detection

- Just like at B factories, **high precision** measurements of ν oscillation in future experiments will allow us to probe **physics beyond SM** by looking at deviation from SM+massive ν .
- If θ_{13} turns out to be large, search for **new physics** and test of **unitarity** will be even more important subjects at ν factory. (cf. $\sin^2\theta_{13}=0.02\pm 0.01@1\sigma$, Fogli et al, arXiv:0905.3549 [hep-ph])

- If 3 flavor unitarity is guaranteed, then roughly speaking, we could guess (**discovery**) from (**golden**) + (**disappearance**) at ν factory from 3 flavor unitarity:

$$P(\nu_{\mu} \rightarrow \nu_e) + P(\nu_{\mu} \rightarrow \nu_{\mu}) + P(\nu_{\mu} \rightarrow \nu_{\tau}) = 1$$

**disappearance
channel**

**discovery
channel**

$$\nu_e \rightarrow \nu_{\mu}$$

Probability of the time reversal process could be obtained if we can guess the CP phase.

golden channel

- Intuitively, therefore, τ detection is supposed to be important to test New Physics which violates unitarity.
→ Quantitative estimate is necessary to draw conclusions.

New physics which can be probed at a neutrino factory includes:

- ◆ Non standard interactions in propagation
- ◆ Non standard interactions at production / detection
- ◆ Violation of unitarity due to heavy particles
- ◆ Schemes with light sterile neutrinos

$$\sum_{\beta=e,\mu,\tau} P(\nu_{\alpha} \rightarrow \nu_{\beta}) = 1$$

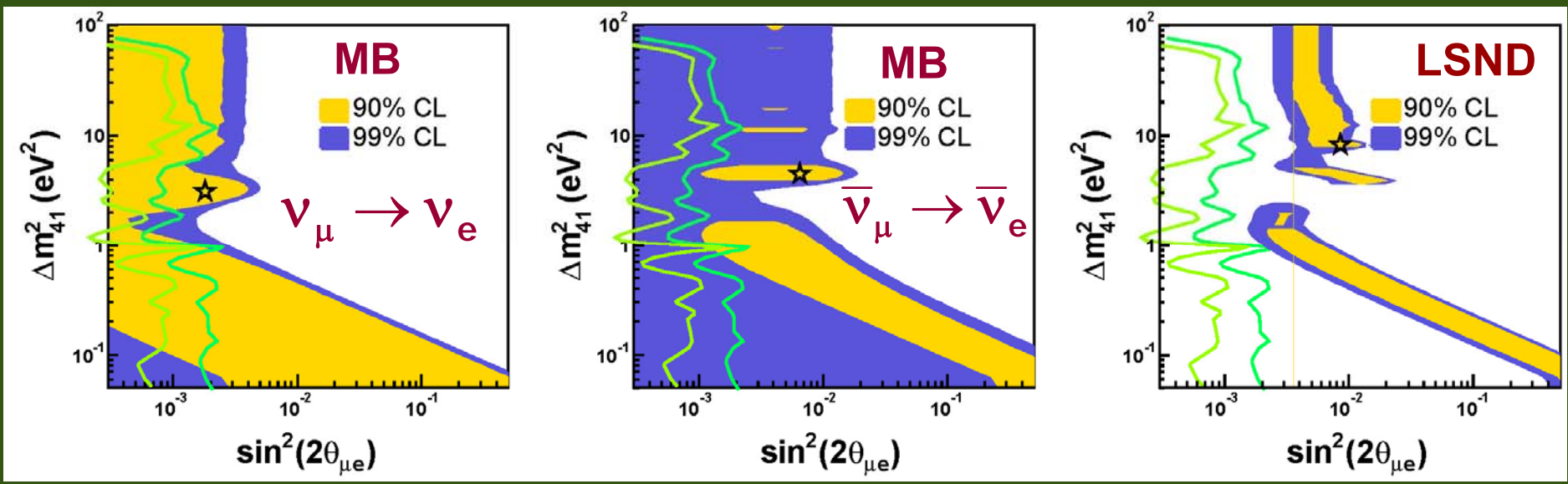
Scenarios	3 flavor unitarity
NSI in propagation	✓
NSI at production / detection	✗
Violation of unitarity due to heavy particles	✗
Light sterile neutrinos	✗

Scenarios	Phenomenological bound on deviation of unitarity
NSI at production / detection	$O(0.1\%)$
Violation of unitarity due to heavy particles	$O(0.1\%)$
Light sterile neutrinos	$O(10\%)$

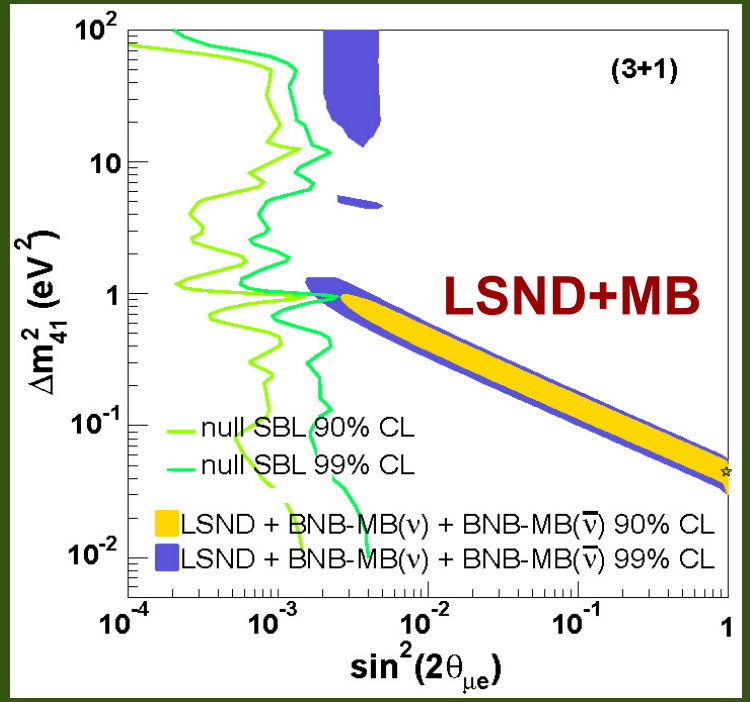
◆ (Except sterile ν) none of these scenarios has ever been supported experimentally.

◆ To encourage experimentalists, one should adopt the most optimistic scenario.

→ Even if LSND anomaly is excluded in the near future, light sterile ν could be phenomenologically even more promising than others!



● Neither MiniBooNE (ν or $\bar{\nu}$) nor disappearance results (CDHSW+Bugey+atm) excludes LSND at 4σ .

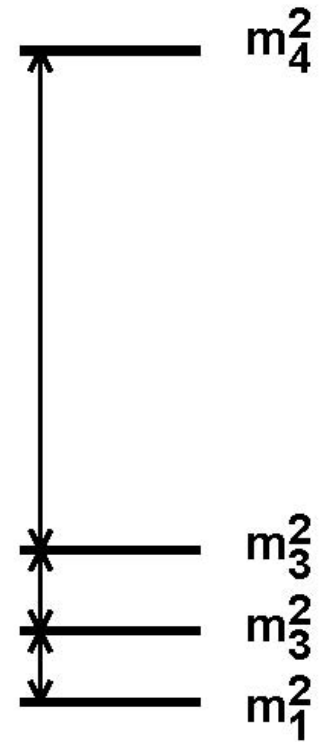


2. Light sterile neutrinos

(3+1)-scheme **w/ LSND**: the situation is unclear, but it's worth checking it

(3+1)-scheme **w/o LSND**: still a possible scenario, provided that the mixing angles satisfy all the constraints of the negative results

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix} \quad U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$



$$U = R_{34}(\theta_{34}, 0) R_{24}(\theta_{24}, 0) R_{23}(\theta_{23}, \delta_3) R_{14}(\theta_{14}, 0) R_{13}(\theta_{13}, \delta_2) R_{12}(\theta_{12}, \delta_1)$$

θ_{34} : ratio of $\nu_\mu \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow \nu_s$ in \mathbf{V}_{atm}

θ_{24} : ratio of $\sin^2\left(\frac{\Delta m_{\text{atm}}^2 L}{4E}\right)$ and $\sin^2\left(\frac{\Delta m_{\text{SBL}}^2 L}{4E}\right)$ in \mathbf{V}_{atm}

θ_{14} : mixing angle in $\mathbf{V}_{\text{reactor}}$ at $L=O(10\text{m})$

Constraints from ν_{atm} and SBL

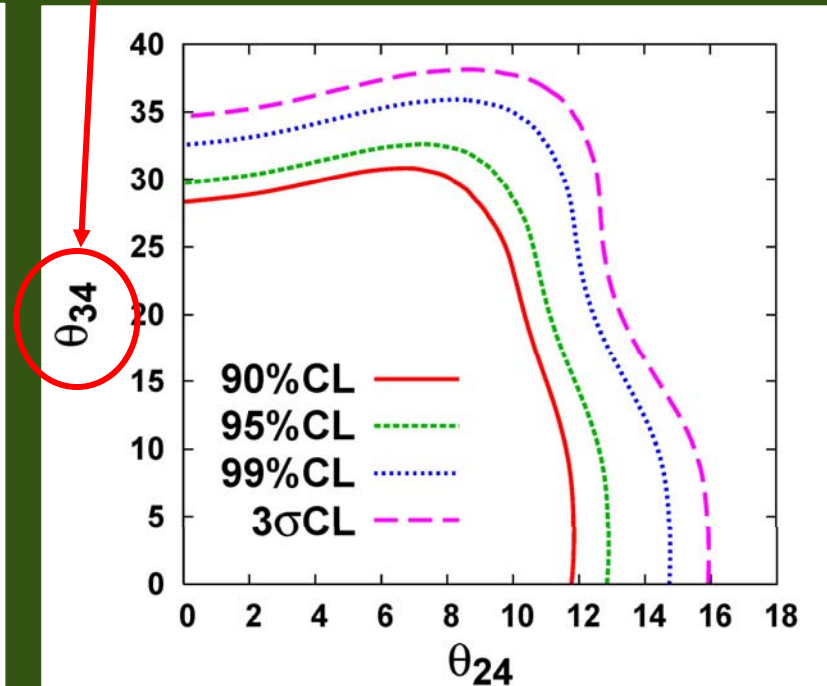
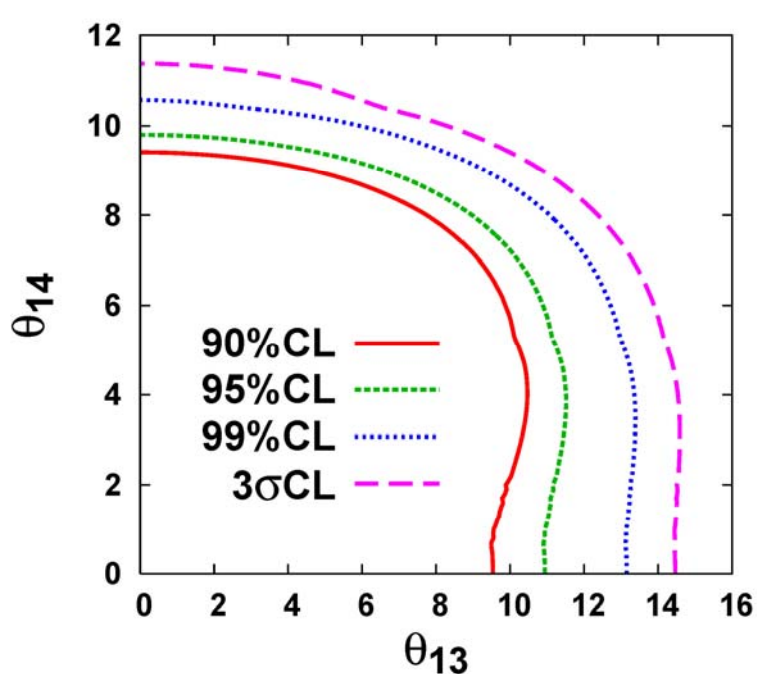
Donini-Maltoni-Meloni-Migliozzi-Terranova, JHEP 0712:013,'07

$$U = R_{34}(\theta_{34}) R_{24}(\theta_{24}) R_{23}(\theta_{23}, \delta_3) R_{14}(\theta_{14}) R_{13}(\theta_{13}, \delta_2) R_{12}(\theta_{12}, \delta_1)$$

Assumption on rapid oscillations in ν_{atm} :

$$\Delta m^2_{41} > 0.1 \text{ eV}^2$$

θ_{34} : could be relatively large



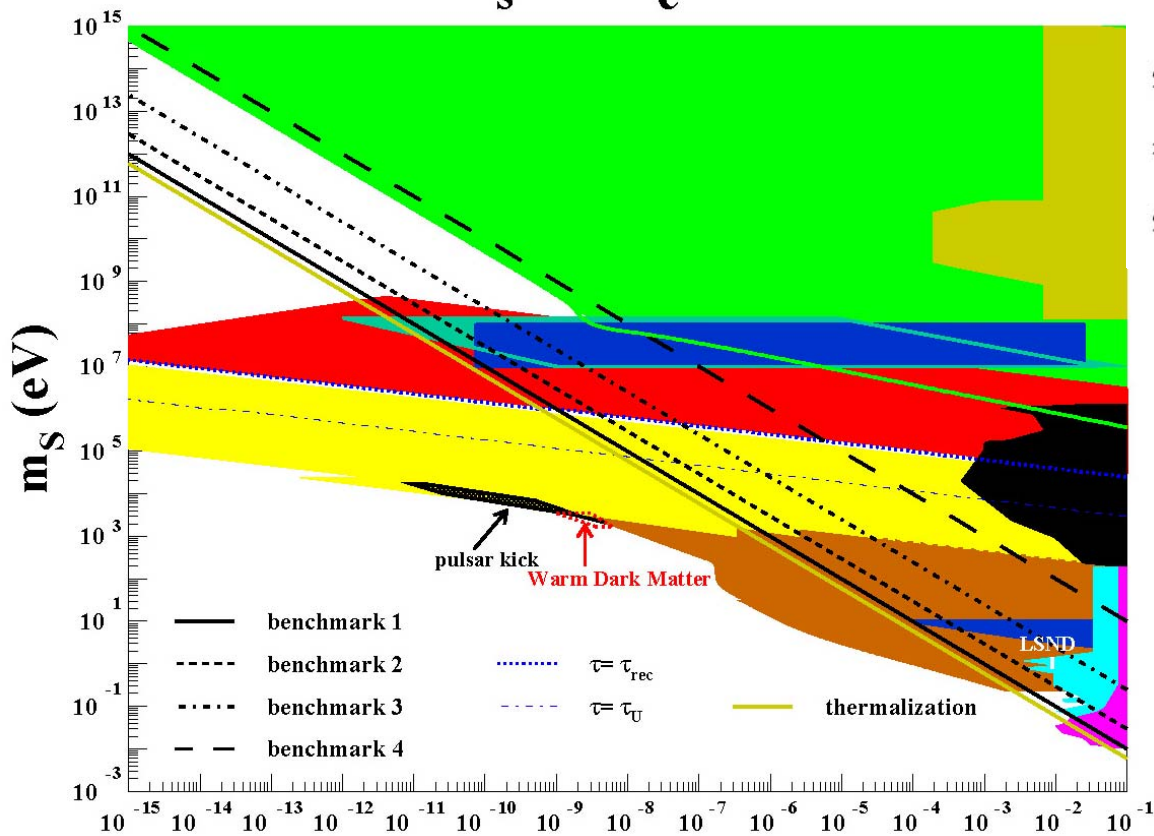
Cosmological constraints on light sterile neutrinos ($s \leftrightarrow e$)

Smirnov & Zukanovich -Funchal, Phys.Rev.D74:013001,2006

$$\nu_s \leftrightarrow \nu_e$$

$$P(\nu_e \rightarrow \nu_s)$$

$$\begin{aligned} &\simeq 4|U_{e4}U_{s4}|^2 \sin^2(\Delta m_{41}^2 L/4E) \\ &= c_{24}^2 c_{34}^2 \sin^2 2\theta_{14} \sin^2(\Delta m_{41}^2 L/4E) \\ &\simeq \sin^2 2\theta_{14} \sin^2(\Delta m_{41}^2 L/4E) \end{aligned}$$



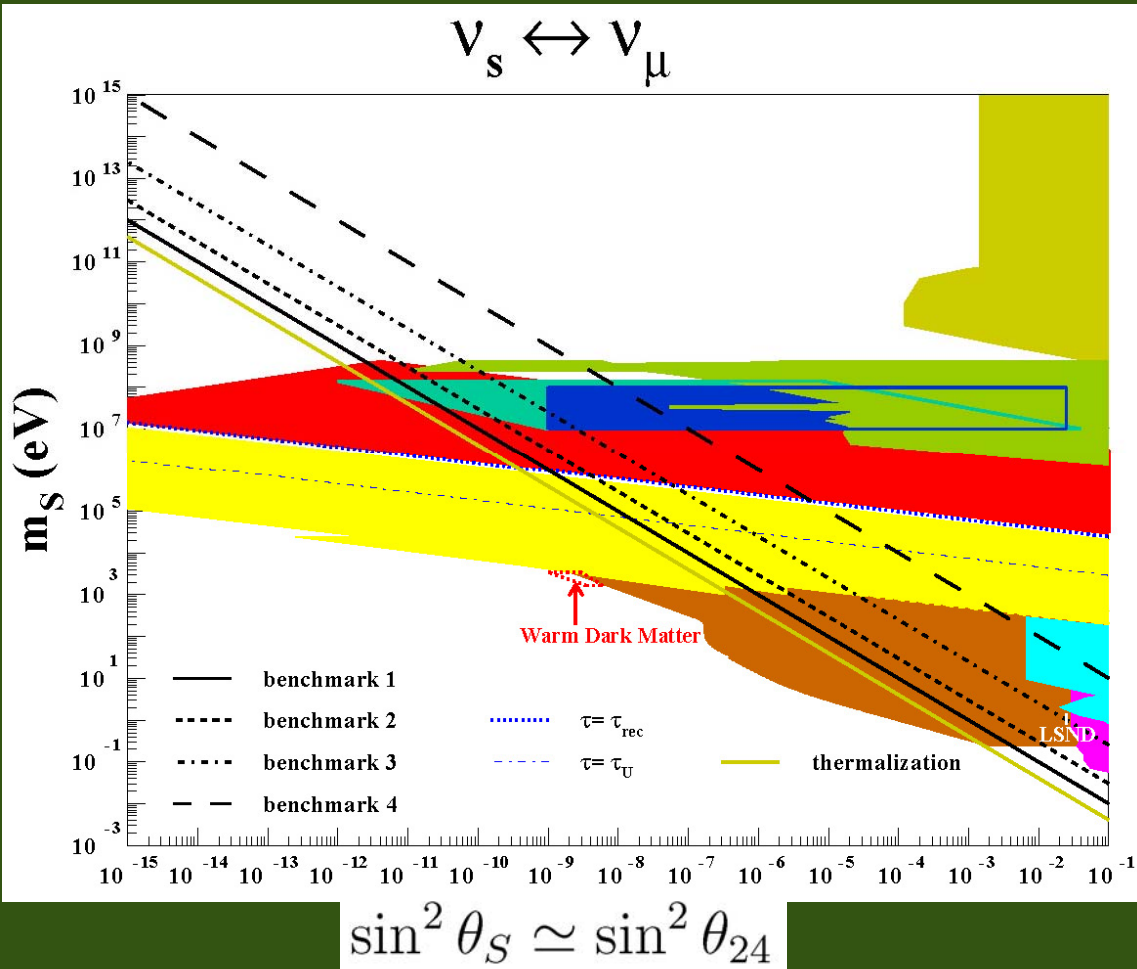
$$\sin^2 \theta_s \simeq \sin^2 \theta_{14}$$

- $0\nu\beta\beta$
- LSS
- X-ray
- BBN
- CMB

- SN1987A
- β -decay
- Accelerator
- Atmospheric
- Reac.+Beam

Cosmological constraints on light sterile neutrinos ($s \leftrightarrow \mu$)

Smirnov & Zukanovich -Funchal, Phys.Rev.D74:013001,2006

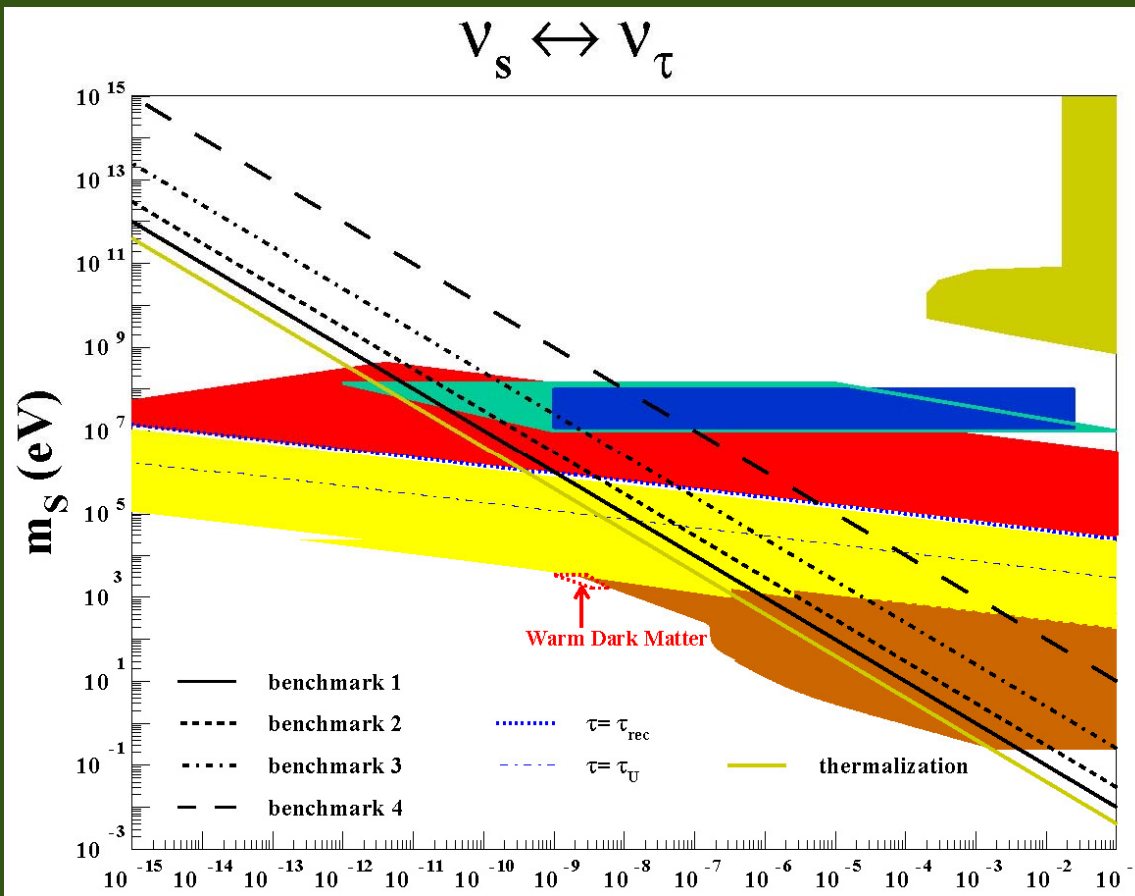


$$\begin{aligned}
 P(\nu_\mu \rightarrow \nu_s) & \\
 &\simeq 4|U_{\mu 4}U_{s 4}|^2 \sin^2(\Delta m_{41}^2 L/4E) \\
 &= c_{14}^4 c_{34}^2 \sin^2 2\theta_{24} \sin^2(\Delta m_{41}^2 L/4E) \\
 &\simeq \sin^2 2\theta_{24} \sin^2(\Delta m_{41}^2 L/4E)
 \end{aligned}$$

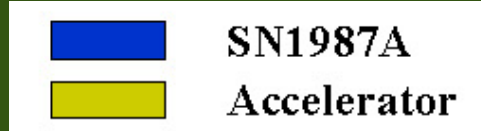
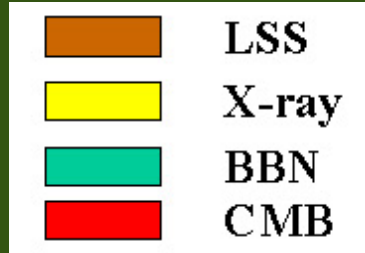
- | | |
|--|--------------------|
| | LSS |
| | X-ray |
| | BBN |
| | CMB |
| | |
| | SN1987A |
| | Accelerator |
| | Decays |
| | Atmospheric |
| | Beam |

Cosmological constraints on light sterile neutrinos ($s \leftrightarrow \tau$)

Smirnov & Zukanovich -Funchal, Phys.Rev.D74:013001,2006



$$\begin{aligned}
 P(\nu_\tau \rightarrow \nu_s) &\simeq 4|U_{\tau 4}U_{s4}|^2 \sin^2(\Delta m_{41}^2 L/4E) \\
 &= c_{24}^4 \sin^2 2\theta_{34} \sin^2(\Delta m_{41}^2 L/4E) \\
 &\simeq \sin^2 2\theta_{34} \sin^2(\Delta m_{41}^2 L/4E)
 \end{aligned}$$



NB: Constraints from LSS, X-ray, BBN, CMB may be avoided if some suppression mechanism (e.g., lepton asymmetry) exists

**In
that
case**



θ_{14} : major constraints from ν_{atm} & SBL

for $\Delta m^2_{41} : < 10^5 \text{ eV}^2$

θ_{24}, θ_{34} : major constraints from ν_{atm} & SBL

for $\Delta m^2_{41} : < 10^{14} \text{ eV}^2$

Hence there may be some room for sterile neutrino mixings for these values of Δm^2_{41} .

3. Sensitivity to θ_{14} , θ_{24} , θ_{34} at ν factory with a near detector

Donini, Meloni, Eur.Phys.J.C22:179-186,2001

$2 \times 10^{20} \mu^-$'s/yr \times 5 yrs, $E_\mu = 20\text{GeV}$

10kton MIND @ L = 40 Km + 1ton ECC @ L = 1 Km

efficiency=0.5 for μ , 0.35 for τ

statistical errors + BG (w/o systematic errors)

golden

$$P(\nu_e \rightarrow \nu_\mu) = 4c_{24}^2 c_{34}^4 s_{14}^2 s_{24}^2 \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right)$$

sensitivity to $s_{14}^2 s_{24}^2$

silver

$$P(\nu_e \rightarrow \nu_\tau) = 4c_{24}^2 c_{34}^2 s_{14}^2 s_{34}^2 \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right)$$

sensitivity to $s_{14}^2 s_{34}^2$

discovery

$$P(\nu_\mu \rightarrow \nu_\tau) = 4c_{34}^2 s_{24}^2 s_{34}^2 \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right)$$

sensitivity to $s_{24}^2 s_{34}^2$

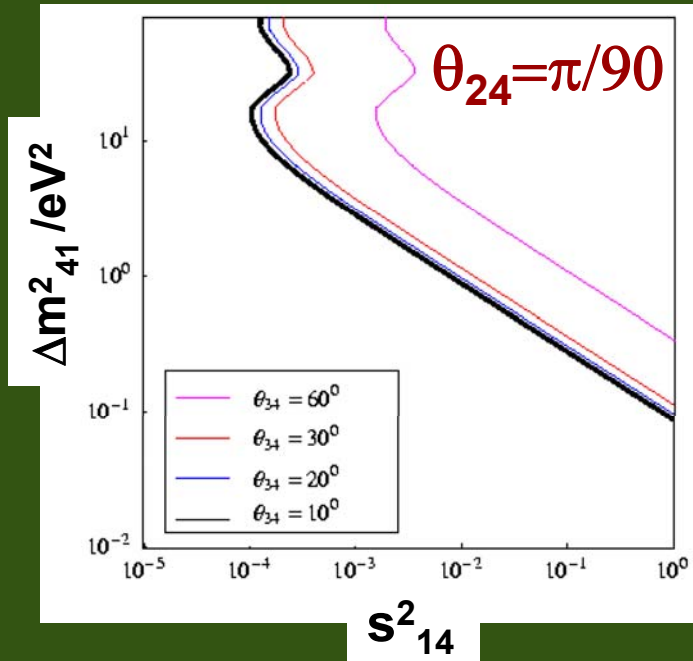
disappearance

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - 4c_{34}^2 s_{24}^2 (1 - c_{34}^2 s_{24}^2) \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right)$$

sensitivity to s_{24}^2

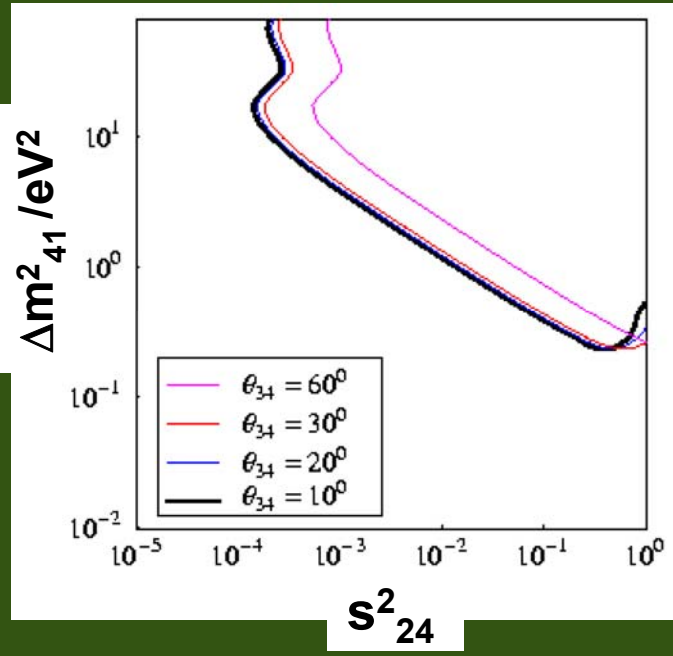
golden

$$P(\nu_e \rightarrow \nu_\mu) = 4c_{24}^2 c_{34}^4 s_{14}^2 s_{24}^2 \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right)$$



disappearance

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - 4c_{34}^2 s_{24}^2 (1 - c_{34}^2 s_{24}^2) \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right)$$



$$4|U_{e4}U_{\mu4}|^2 > 1.4 \times 10^{-7}$$

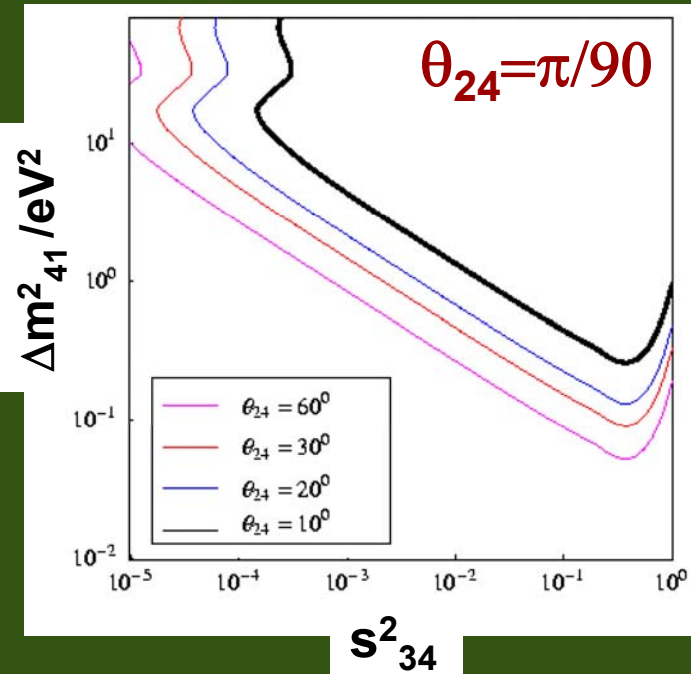
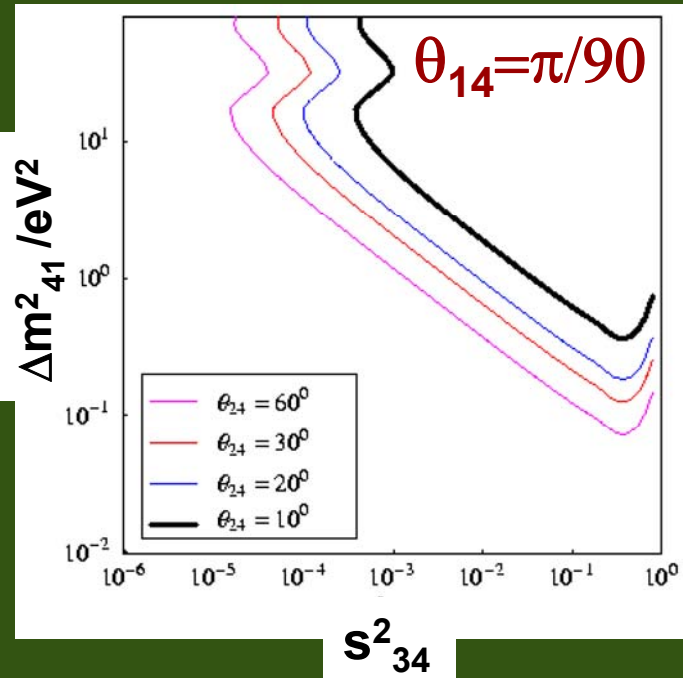
$$4|U_{\mu4}|^2 > 5.3 \times 10^{-4}$$

silver

discovery

$$P(\nu_e \rightarrow \nu_\tau) = 4c_{24}^2 c_{34}^2 s_{14}^2 s_{34}^2 \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right)$$

$$P(\nu_\mu \rightarrow \nu_\tau) = 4c_{34}^2 s_{24}^2 s_{34}^2 \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right)$$



$$4|U_{e4} U_{\tau 4}|^2 > 4.8 \times 10^{-5}$$

$$4|U_{\mu 4} U_{\tau 4}|^2 > 1.8 \times 10^{-5}$$

4. Sensitivity to θ_{14} , θ_{24} , θ_{34} at ν factory with far detectors

Donini et al, JHEP 0908:041,2009

$5 \times 10^{20} \mu^- + \mu^+$ s/yr \times 4 yrs
(E_μ/GeV , L/km) = (50, 3000+7500) or (20, 4000+7500)
50kton MIND + 4kton MECC

Results for $E_\mu=20\text{GeV}$ case are shown below for a fair comparison

statistical errors + systematic errors + BG

efficiency ~ 0.7 for μ , ~ 0.65 for τ

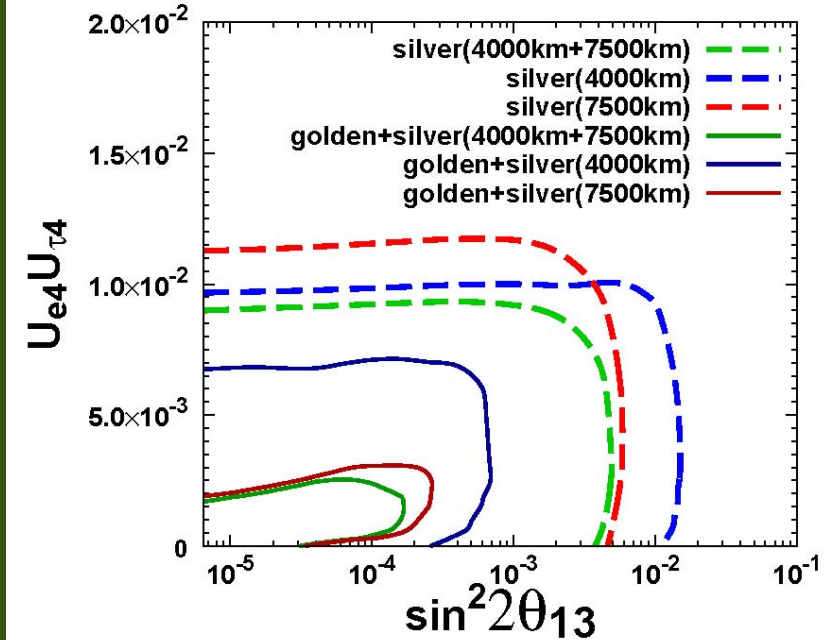
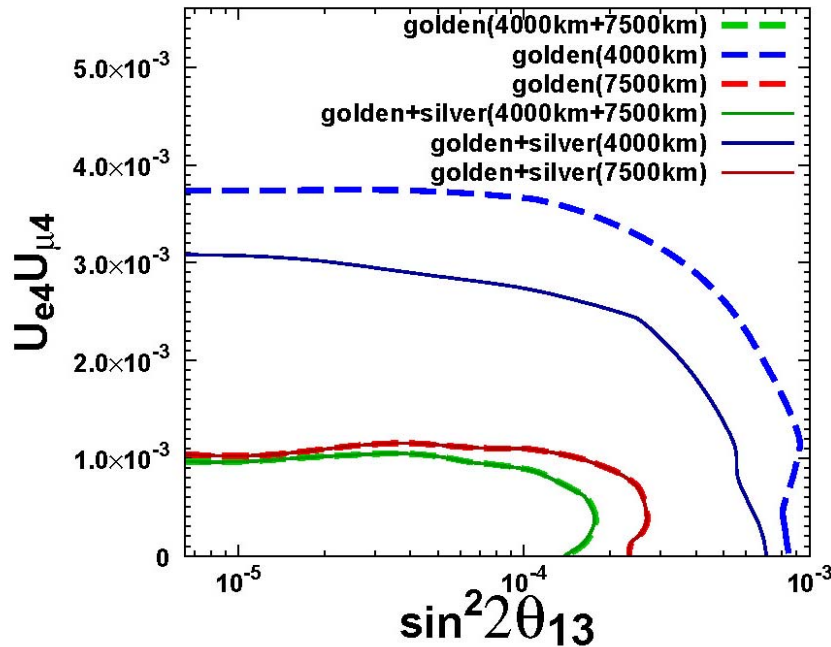
NB. Magnetized Emulsion Cloud Chamber (MECC)

active target: iron

$\tau \rightarrow \mu$ decay + $\tau \rightarrow e$ decay + $\tau \rightarrow$ hadron decay are used

$$P(\nu_e \rightarrow \nu_\mu) = 4\text{Re} [U_{e3}U_{\mu 3}^*(U_{e3}^*U_{\mu 3} + U_{e4}^*U_{\mu 4})] \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right) + \dots$$

$$P(\nu_e \rightarrow \nu_\tau) = 4\text{Re} [U_{e3}U_{\tau 3}^*(U_{e3}^*U_{\tau 3} + U_{e4}^*U_{\tau 4})] \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right) + \dots$$



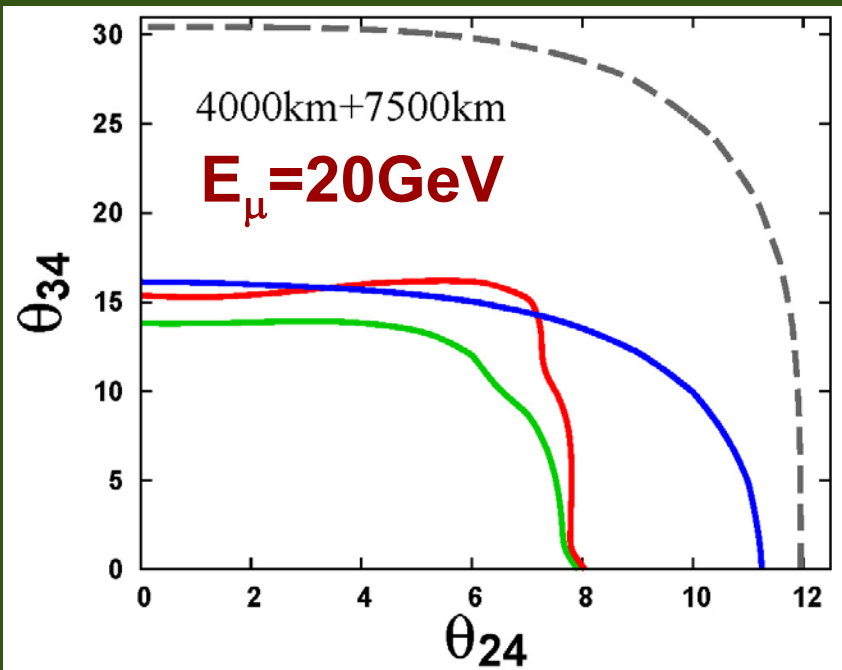
$$4|U_{e4}U_{\mu 4}|^2 > 5.8 \times 10^{-6}$$

$$4|U_{e4}U_{\tau 4}|^2 > 3.8 \times 10^{-5}$$

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - 4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2 - |U_{\mu 4}|^2) \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right) + \dots$$

$$P(\nu_\mu \rightarrow \nu_\tau) = 4\text{Re} \left[U_{\mu 3} U_{\tau 3}^* (U_{\mu 3}^* U_{\tau 3} + U_{\mu 4}^* U_{\tau 4}) \right] \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right) + \dots$$

--- current **— disappearance** **— discovery** **— combined**



$$4|U_{\mu 4}|^2 > 7.6 \times 10^{-2}$$

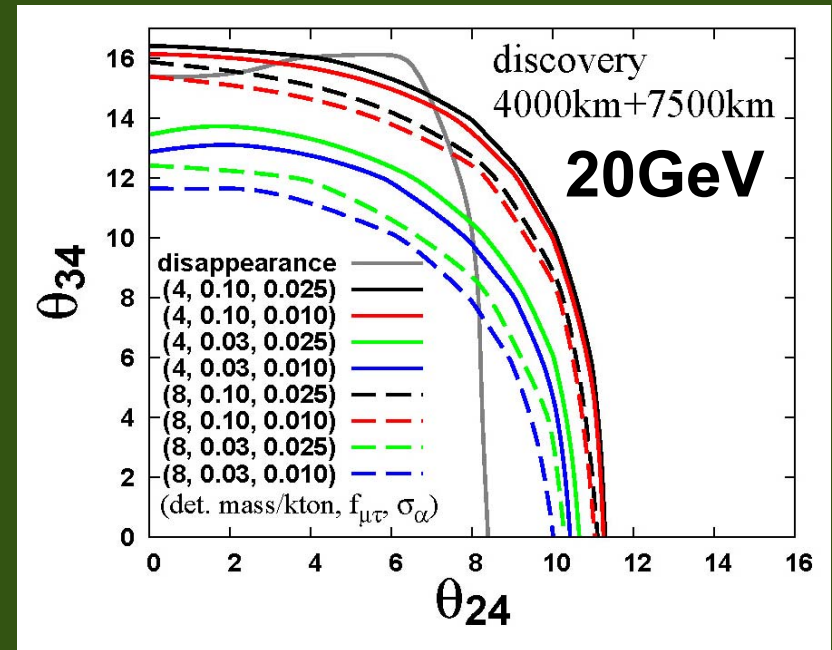
$$4|U_{\mu 4} U_{\tau 4}|^2 > 1.9 \times 10^{-3}$$

- Byproduct of the near detectors: improvement of systematic errors at far detectors

Dependence of sensitivity on systematic errors

Donini et al, JHEP 0908:041,2009

In previous page, $f_{\mu\tau}=10\%$, $\sigma_\alpha=2.5\%$ (black solid lines above) was assumed



$f_{\mu\tau}$: uncorrelated bin-to-bin systematic error (error in detection efficiency in each bin etc.)

σ_α : correlated systematic error (error in detector volume etc.)

By placing a near τ detector, systematic errors could be reduced

cf. Reduction of systematic errors in
the 2 detector complex at **reactor** experiments

Minakata et al.,
Phys.Rev.D68:033017,2003

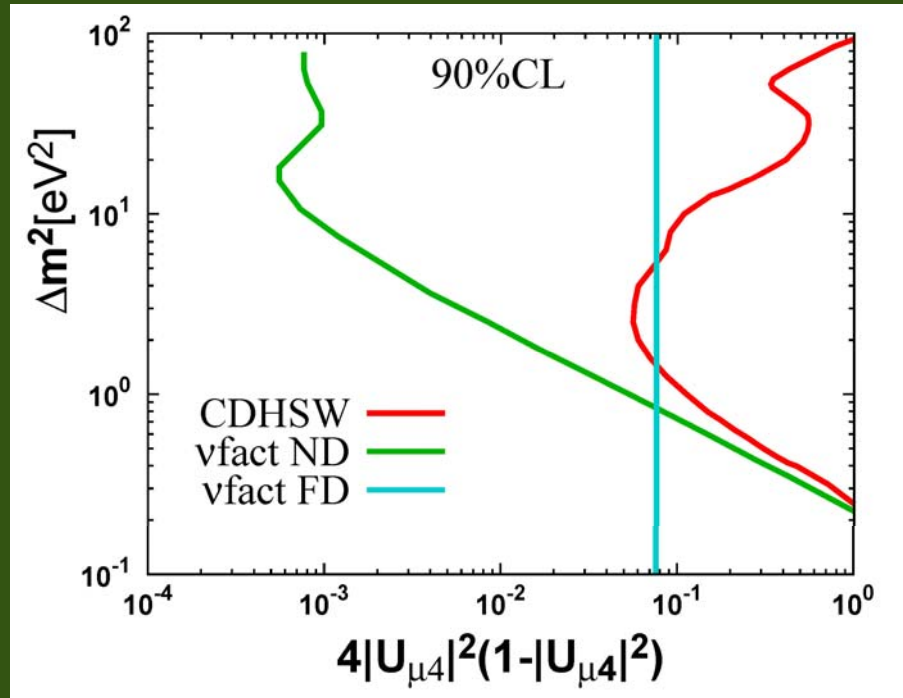
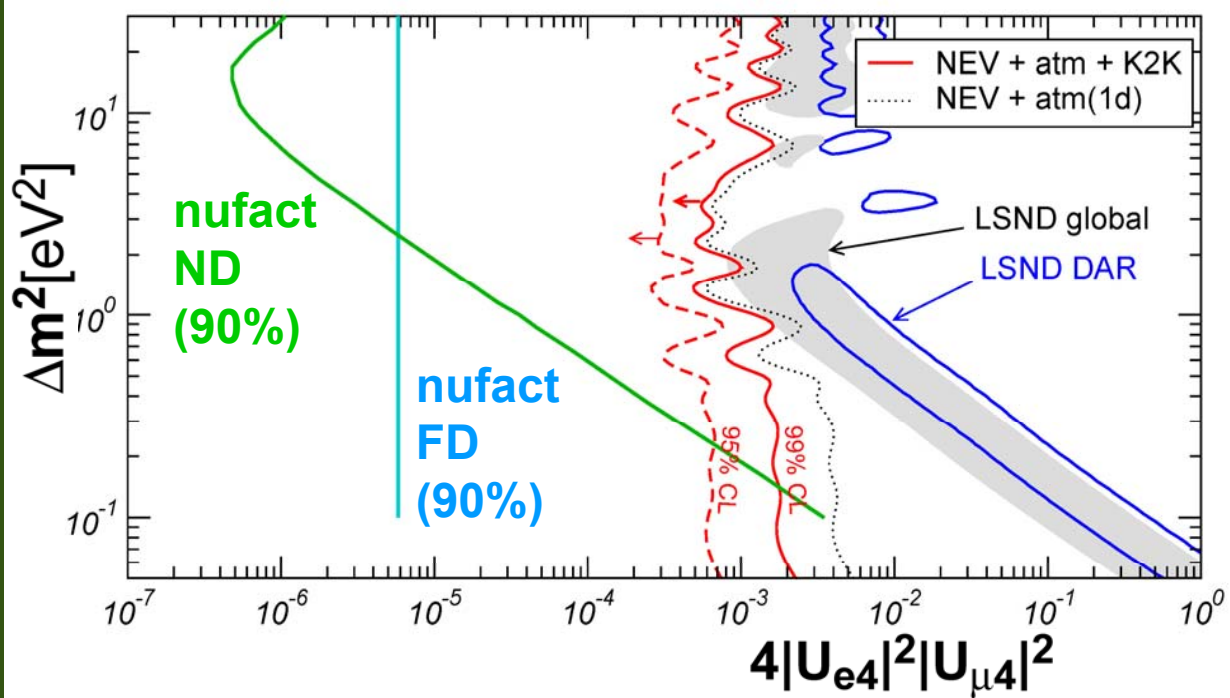
CHOOZ-like	absolute normalization	relative normalization (expected)	relative/absolute
flux	2.1%	0.0%	0
number of protons	0.8%	0.3%	0.38
detection efficiency	1.5%	0.7%	0.47
total	2.7%	0.8%	
for bins	8.1%	2.4%	

2.6 Summary

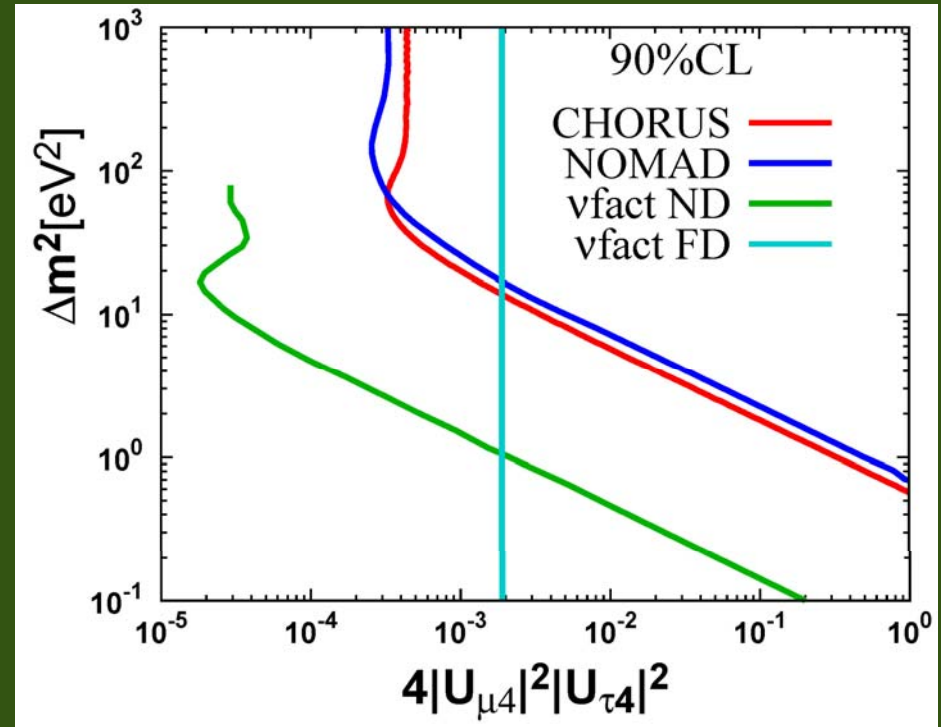
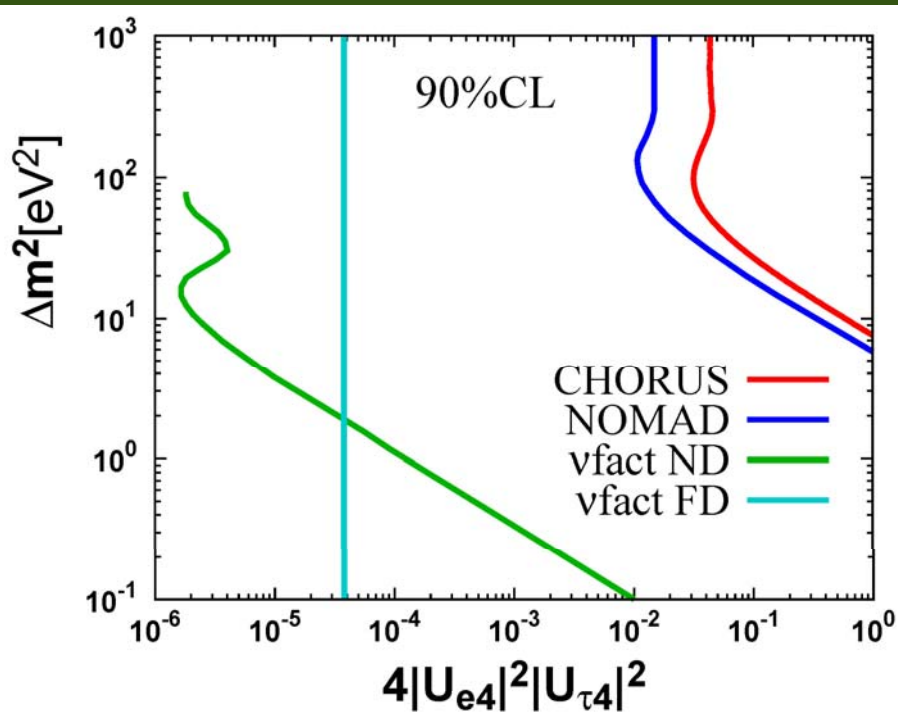
- In most cases sensitivity to the sterile mixings is better at a near detector (@Oscillation Maximum) than at far detectors. (Notice the absence of the systematic errors in the analysis of ND. → Further study is necessary.)

	near@Osc.Max	far
$4 U_{e4}U_{\mu4} ^2$	1.4×10^{-7}	5.8×10^{-6}
$4 U_{e4}U_{\tau4} ^2$	4.8×10^{-5}	3.8×10^{-5}
$4 U_{\mu4} ^2$	5.3×10^{-4}	7.6×10^{-2}
$4 U_{\mu4}U_{\tau4} ^2$	1.8×10^{-5}	1.9×10^{-3}

Sensitivity to the sterile mixings at ND & FD is very good compared to the present bound. → It could serve as a severe test of LSND/MiniBooNE.



- Sensitivity to $4|U_{e4}U_{\tau4}|^2$ & $4|U_{\mu4}U_{\tau4}|^2$:
improvement over the present bound by
one order of magnitude



- **Sensitivity to Δm_{41}^2 :**
 - FD: insensitive**
 - ND: sensitive but some fine tuning required**
 - ND & very near detector are necessary.**
 - To cover the region $\Delta m_{41}^2 \sim O(1\text{eV}^2)$, $L \sim O(10\text{km})$**

- **To measure the new CP phase due to sterile neutrinos, discovery channel at far detectors is crucial. → ND & FD are complementary in the study of sterile neutrinos at a ν factory.**

- **Near τ detectors are useful not only to improve sensitivity to sterile neutrino mixings by themselves, but also to reduce the systematic errors of the far τ detectors.**

In answering Belen's questions on τ -ND (1)

- Is it worth the trouble to develop such a "tau-sensitive" detector?

Yes, since it doesn't cost much, it's worth putting a tau ND.

- Is it doable?

The analysis on ND has to be done again, but presumably it is.

- How good results one can get?

We can improve sensitivity to $4|U_{e4}U_{\mu4}|^2$ ($4|U_{e4}U_{\tau4}|^2$, $4|U_{\mu4}U_{\tau4}|^2$) by 2 (4, 1) orders of magnitude.

- If we only improve a limit - will anybody care?

At least I do. The neutrino factory would give a limit independent of cosmology.

As for $4|U_{e4}U_{\mu4}|^2$, by putting a MIND, we can exclude LSND by 4 orders of magnitude. Presumably it is possible only with a neutrino factory.

In answering Belen's questions on τ -ND (2)

- For instance, for your chosen physics goal or subject, what sensitivity would be required to be worth physics-wise? $e\mu$: Both ND & FD w/ assumed σ_{sys} will test LSND. $e\tau$ or $\mu\tau$: W/o particular physics model, typically improvement by one order of magnitude is one goal.
- And what are the optimal characteristics of the detector? MECC seem to be the best so far as a τ detector to have better efficiency.
- How does it compare with respect to physics reach with other future planned detectors? Complementarity? Liquid argon TPC could be alternative. \rightarrow Further study is required.
- For what other beams is it appropriate or possible? Superbeam: ν_τ contamination
 β beam: Energy is too low to produce τ .

Backup slides

- τ detection is potentially advantage of ν factory:
- Detection of large number of τ 's is possible at ν factory
- No ν_τ contamination at a neutrino factory (cf. super-beam, [Van de Vyver-Zucchelli, NIM A385:91,1997])
- τ channels in 3 family model are not so useful:
- (golden) @4000km+7500km is better than (silver)+(golden) @4000km to solve intrinsic degeneracy [ISS report]
- (disappearance) is better than (discovery) to measure atmospheric parameters [Donini, 0th IDS mtg@CERN]

$$\nu_e \rightarrow \nu_\mu$$

golden channel

$$\nu_\mu \rightarrow \nu_\mu$$

disappearance channel

$$\nu_e \rightarrow \nu_\tau$$

silver channel

$$\nu_\mu \rightarrow \nu_\tau$$

discovery channel

τ detectors

- **Emulsion Cloud Chamber (ECC)**

Prototype: the OPERA detector at the CNGS

active target: lead

spectrometers to measure the charge

only $\tau \rightarrow \mu$ decay is used: detection efficiency $\sim O(5\%)$

→ **Proposal of Magnetized Emulsion Cloud Chamber (MECC)**

active target: iron

$\tau \rightarrow \mu$ decay + $\tau \rightarrow e$ decay + $\tau \rightarrow$ hadron decay are used:

detection efficiency $\sim O(25\%)$

- **Liquid Argon TPC (LAr-TPC)**

Prototype: the ICARUS T600 at the CNGS

$$P_{\mu\mu} = 1 - 2\theta_{24}^2 - \left[1 - 4(\delta\theta_{23})^2 - 2\theta_{24}^2 + \theta_{34}^2 \frac{A_n}{\Delta_{31}} \left(4\delta\theta_{23} - \theta_{34}^2 \frac{A_n}{\Delta_{31}} \right) \right] \sin^2 \frac{\Delta_{31}L}{2} - (A_n L) \left\{ 2\theta_{24} \theta_{34} \cos \delta_3 - \frac{\theta_{34}^2}{2} \left(4\delta\theta_{23} - \theta_{34}^2 \frac{A_n}{2\Delta_{31}} \right) \right\} \sin \Delta_{31}L + O(\epsilon^5), \quad (16)$$

$$P_{\mu\tau} = \left\{ 1 - 4(\delta\theta_{23})^2 - \theta_{24}^2 - \theta_{34}^2 \left[1 - \frac{\theta_{34}^2}{3} - \frac{A_n}{\Delta_{31}} \left(4\delta\theta_{23} - \theta_{34}^2 \frac{A_n}{\Delta_{31}} \right) \right] \right\} \sin^2 \frac{\Delta_{31}L}{2} + \left\{ \theta_{24} \theta_{34} \sin \delta_3 + (A_n L) \left[2\theta_{24} \theta_{34} \cos \delta_3 - \frac{\theta_{34}^2}{2} \left(4\delta\theta_{23} - \theta_{34}^2 \frac{A_n}{2\Delta_{31}} \right) \right] \right\} \sin \Delta_{31}L + O(\epsilon^5), \quad (17)$$

Numbers of events in (3+1)-scheme

Donini, Fuki, Lopez-Pavon, Meloni, Yasuda, JHEP 0908:041,2009

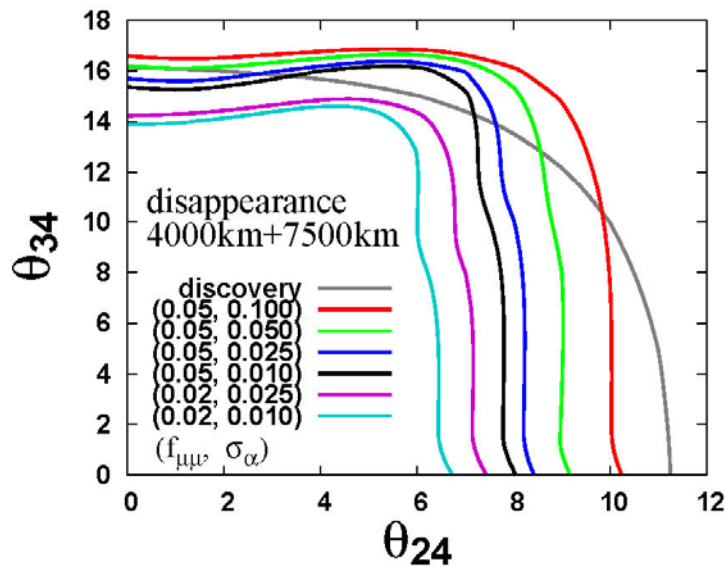
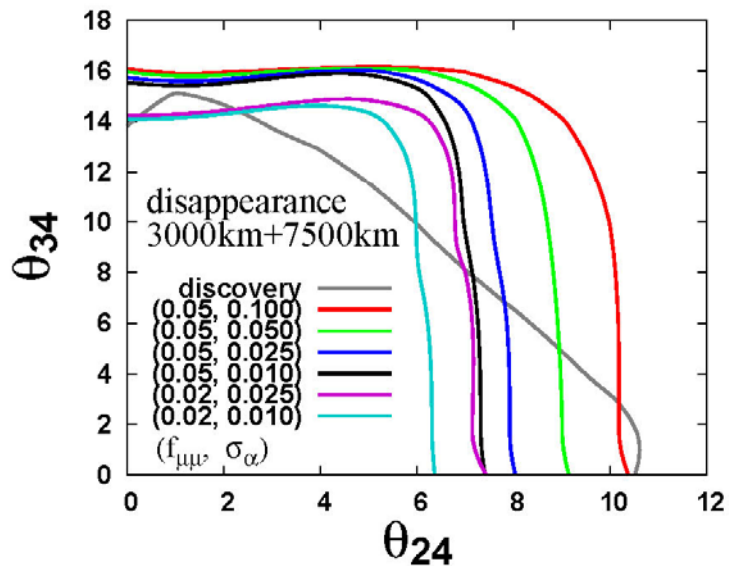
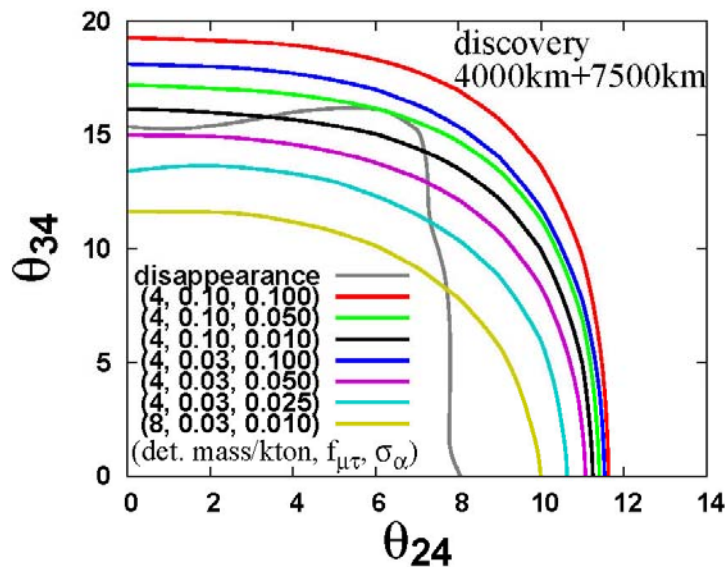
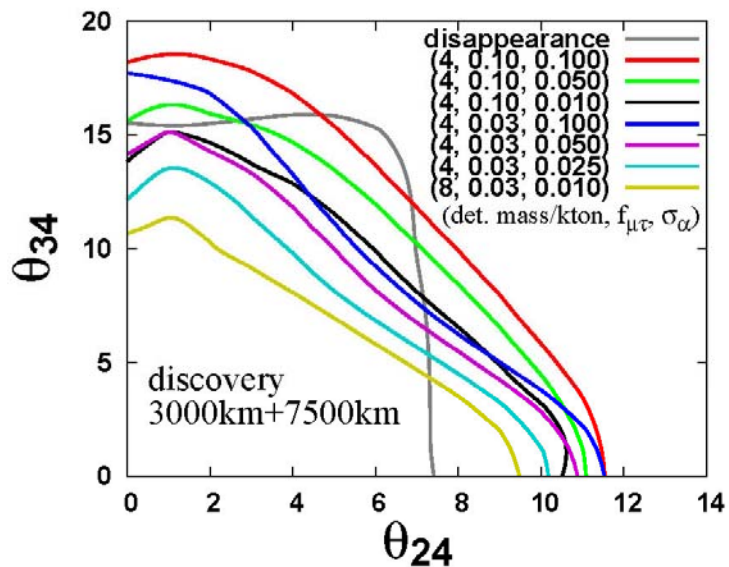
$$\nu_\mu \rightarrow \nu_\tau \quad \bar{\nu}_e \rightarrow \bar{\nu}_\tau \quad \nu_\mu \rightarrow \nu_\tau \quad \bar{\nu}_e \rightarrow \bar{\nu}_\tau$$

$(\theta_{13}; \theta_{14}; \theta_{24}; \theta_{34})$	$N_{\tau^-}^{3000}$	$N_{\tau^+}^{3000}$	$N_{\tau^-}^{7500}$	$N_{\tau^+}^{7500}$
$(5^\circ; 5^\circ; 5^\circ; 20^\circ)$	559	10	544	2
$(5^\circ; 5^\circ; 10^\circ; 20^\circ)$	474	11	529	2
$(5^\circ; 5^\circ; 10^\circ; 30^\circ)$	384	18	454	3
$(5^\circ; 5^\circ; 10^\circ; 30^\circ)$	384	18	454	3
$(10^\circ; 5^\circ; 5^\circ; 20^\circ)$	522	22	512	2
$(10^\circ; 5^\circ; 10^\circ; 20^\circ)$	443	22	498	2
$(10^\circ; 5^\circ; 5^\circ; 30^\circ)$	397	30	413	4
$(10^\circ; 5^\circ; 10^\circ; 30^\circ)$	361	30	428	4
3 families, $\theta_{13} = 5^\circ$	797	3	666	0
3 families, $\theta_{13} = 10^\circ$	755	12	632	1

Number of events
 2×10^{20} flux
1 year
1 Kton MECC
perfect efficiency

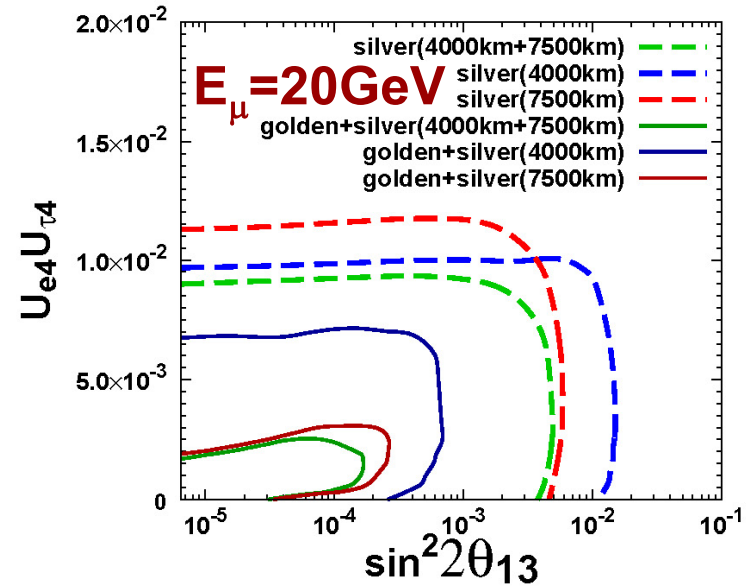
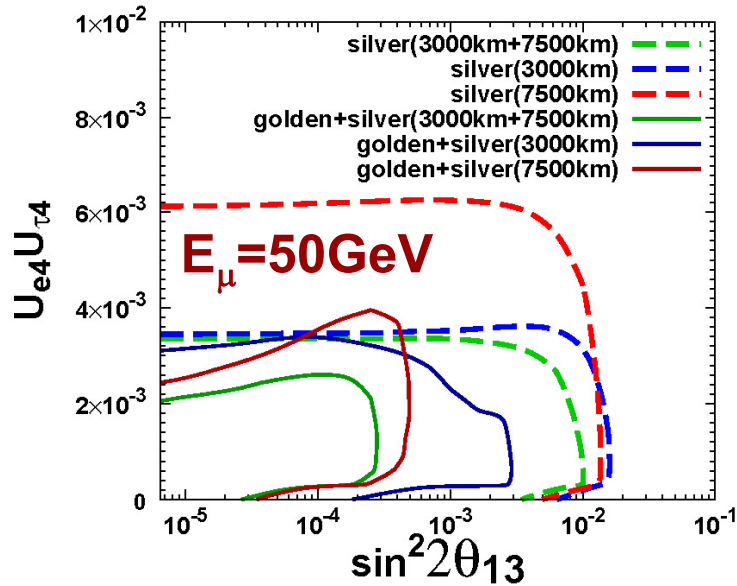
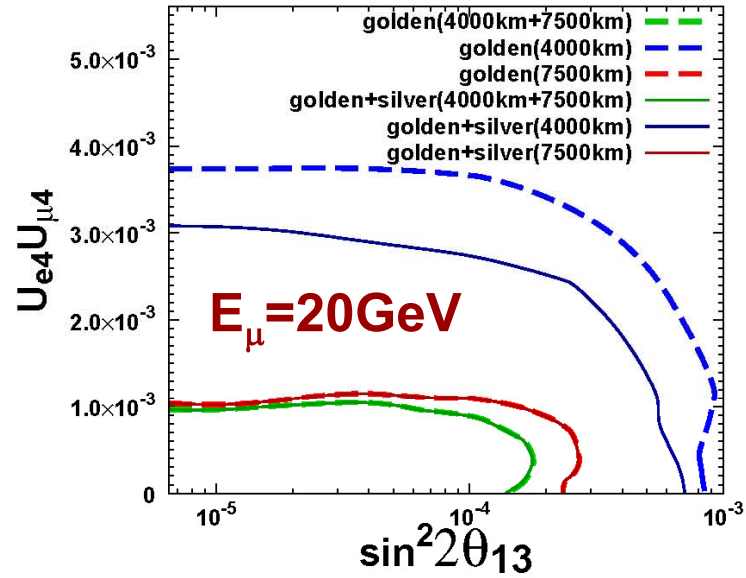
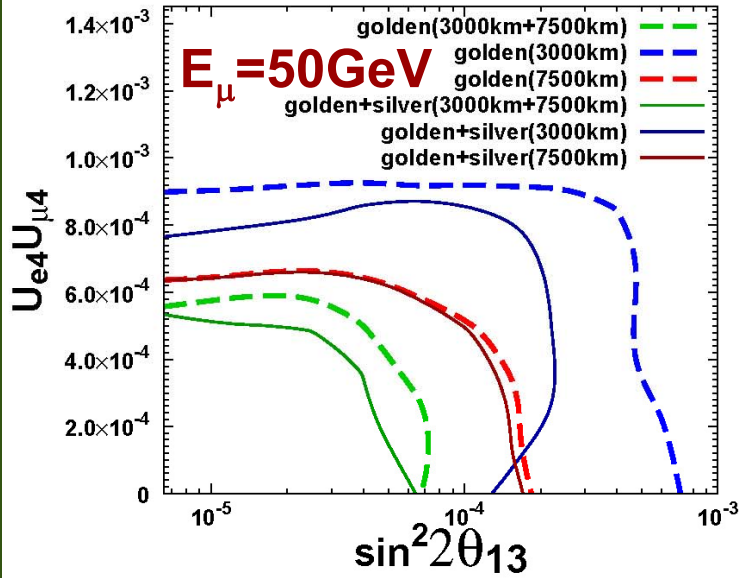
Dependence of sensitivity on systematic errors in (3+1)-scheme

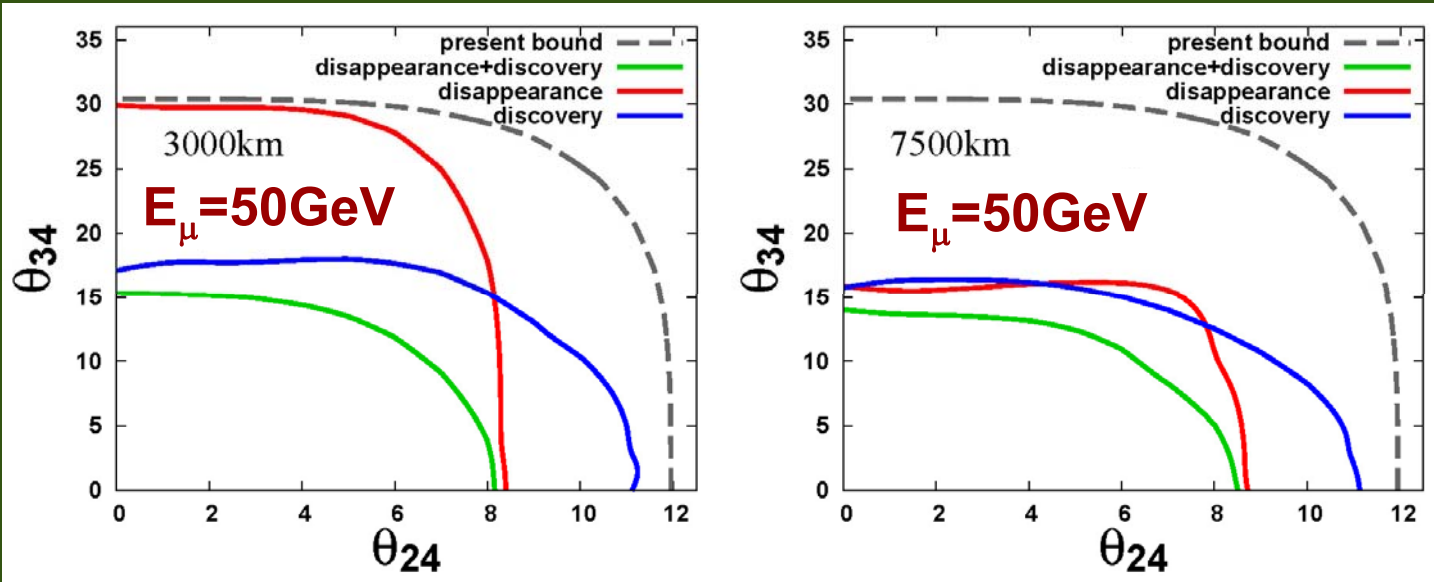
Donini, Fuki, Lopez-Pavon, Meloni, Yasuda, JHEP 0908:041,2009



Sensitivity to $U_{e4}U_{\mu 4}$ & $U_{e4}U_{\tau 4}$

Donini, Fuki, Lopez-Pavon, Meloni, Yasuda, JHEP 0908:041,2009



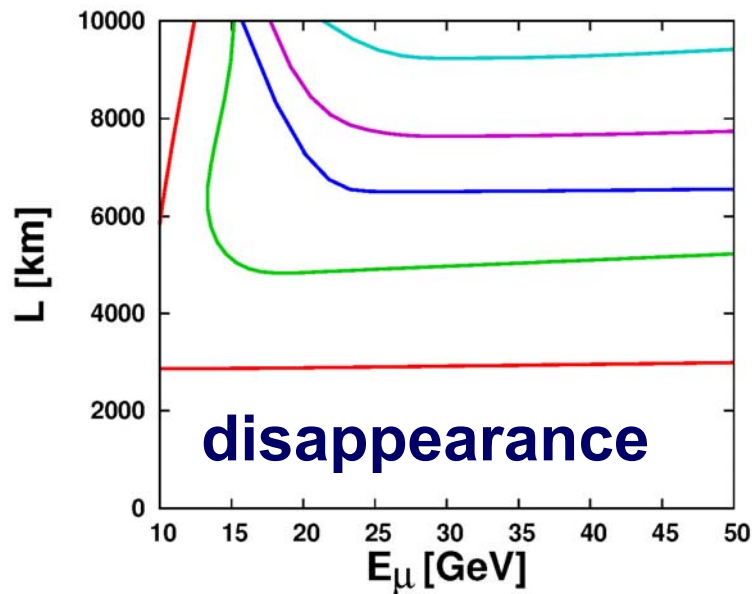


$$P(\nu_\alpha \rightarrow \nu_\beta) = 4\text{Re} \left[U_{\alpha 3} U_{\beta 3}^* (U_{\alpha 3}^* U_{\beta 3} + U_{\alpha 4}^* U_{\beta 4}) \right] \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right) + \dots,$$

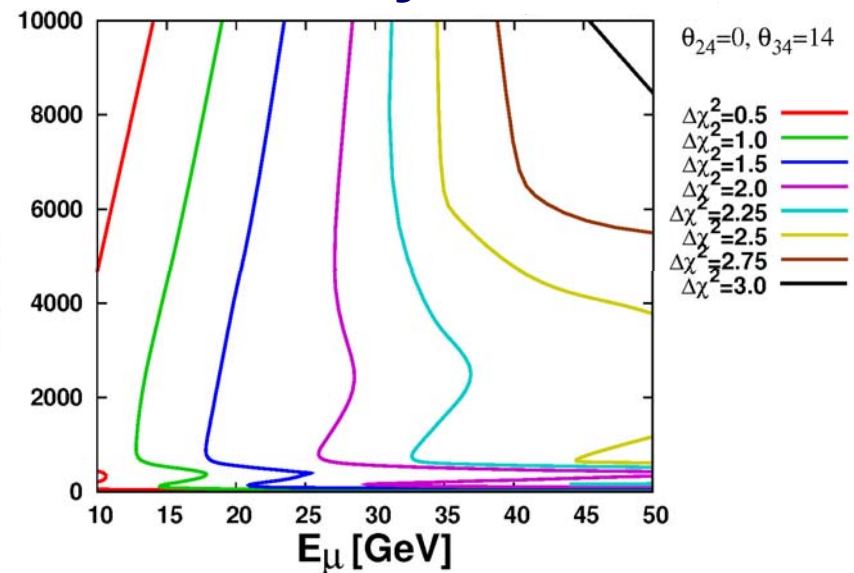
we can expect that the golden and silver channels have some sensitivity to $U_{e4}U_{\mu4}$ and $U_{e4}U_{\tau4}$. In the present parametrization (2) of the mixing matrix, we have $U_{e4}U_{\mu4} = s_{14}c_{14}s_{24} = s_{14}s_{24} + O(\epsilon^6)$ and $U_{e4}U_{\tau4} = s_{14}c_{14}c_{24}s_{34} = s_{14}s_{34} + O(\epsilon^5)$, where we have

Which τ detector is less important, @3000km or @7500km ?

Contour plot of significance for signal with $\theta_{24}=0, \theta_{34}=14$



discovery



For 50GeV (20GeV), $L=3000\text{km}$ ($L=4000\text{km}$) performs (slightly) poor. This is also the case w/ disappearance.

➔ One @7500km seems better in most cases