

Complementary between HK & DUNE to complete 3-flavor PMNS picture & beyond

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1. Introduction

- Motivations for determination of mass ordering, CP phase
- Parameter degeneracy

2. Sensitivity of T2HK & DUNE to $N_\nu=3$ oscillation parameters

- Precision of Δm^2_{31} & θ_{23}
- Mass ordering
- Octant degeneracy
- CP

3. Sensitivity to scenarios beyond the 3-flavor PMNS picture

- ν_s , NSI, scalar NSI, long-range force, Non-unitarity, LIV

4. Conclusions

1. Introduction

Framework of 3 flavor ν oscillation

Mixing matrix

Functions of
mixing angles
 θ_{12} , θ_{23} , θ_{13} ,
and CP phase δ

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

All 3 mixing angles have been measured (2012):

ν_{solar} +KamLAND (reactor)

$$\theta_{12} \cong \frac{\pi}{6}, \Delta m_{21}^2 \cong 8 \times 10^{-5} \text{ eV}^2$$

ν_{atm} +T2K +MINOS+NOvA (accelerators)

$$\theta_{23} \cong \frac{\pi}{4}, |\Delta m_{32}^2| \cong 2.5 \times 10^{-3} \text{ eV}^2$$

DCHOOZ+Daya Bay+Reno (reactors),
T2K+MINOS+Nova etc

$$\theta_{13} \cong \pi / 20$$

Next things to do are to determine the following:

- $\text{sign}(\Delta m^2_{31})$
- $\pi/4 - \theta_{23}$
- δ

● Motivation for determining mass ordering

(i) Guidelines for models (GUT etc.)

(ii) Influence on the measurement of CP phase

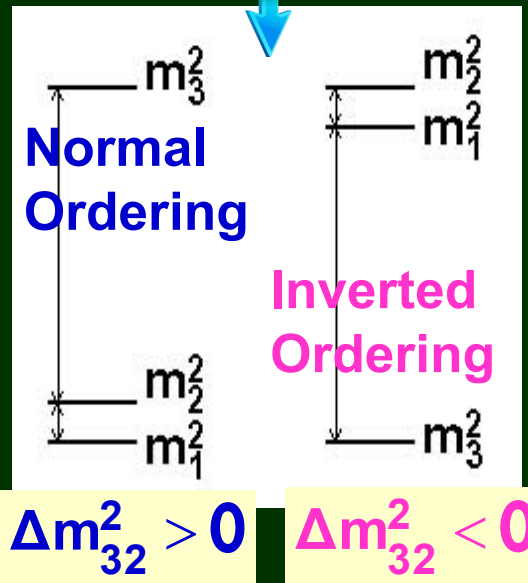
(iii) Implication for neutrinoless double β decay

● Motivation for determining the CP phase

(i) Guidelines for models (GUT etc.)

(ii) Leptogenesis?

Both mass patterns are allowed



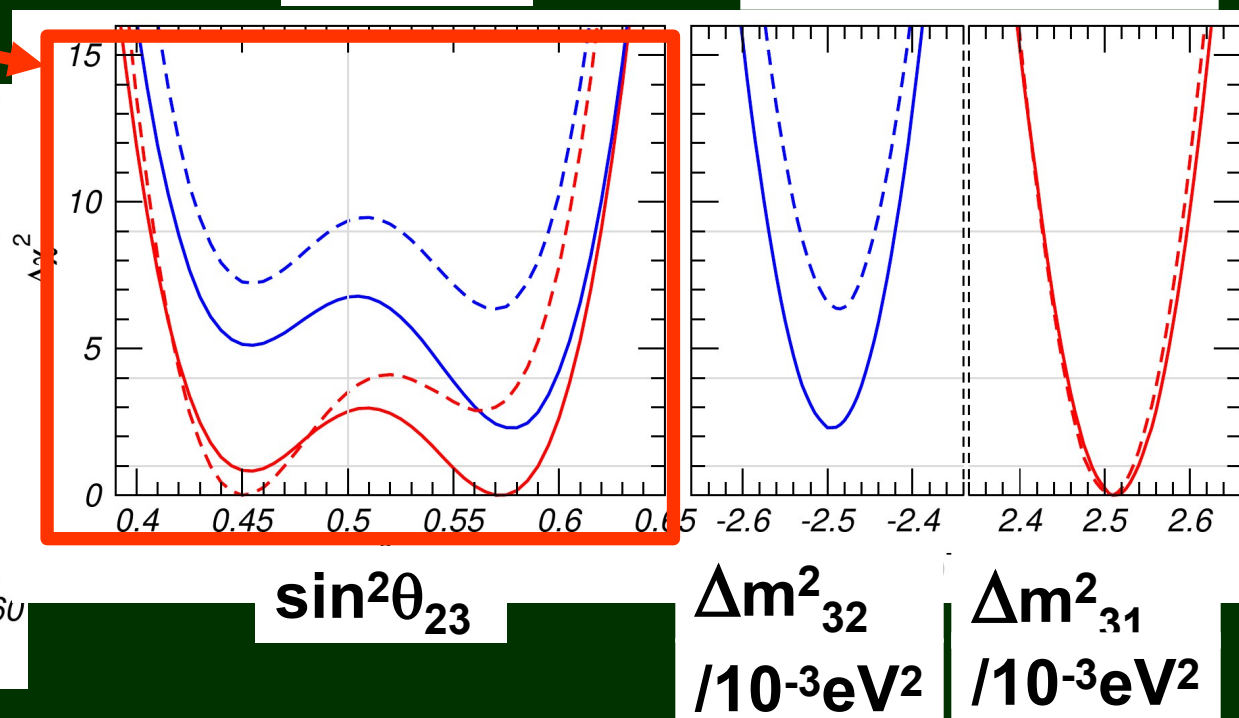
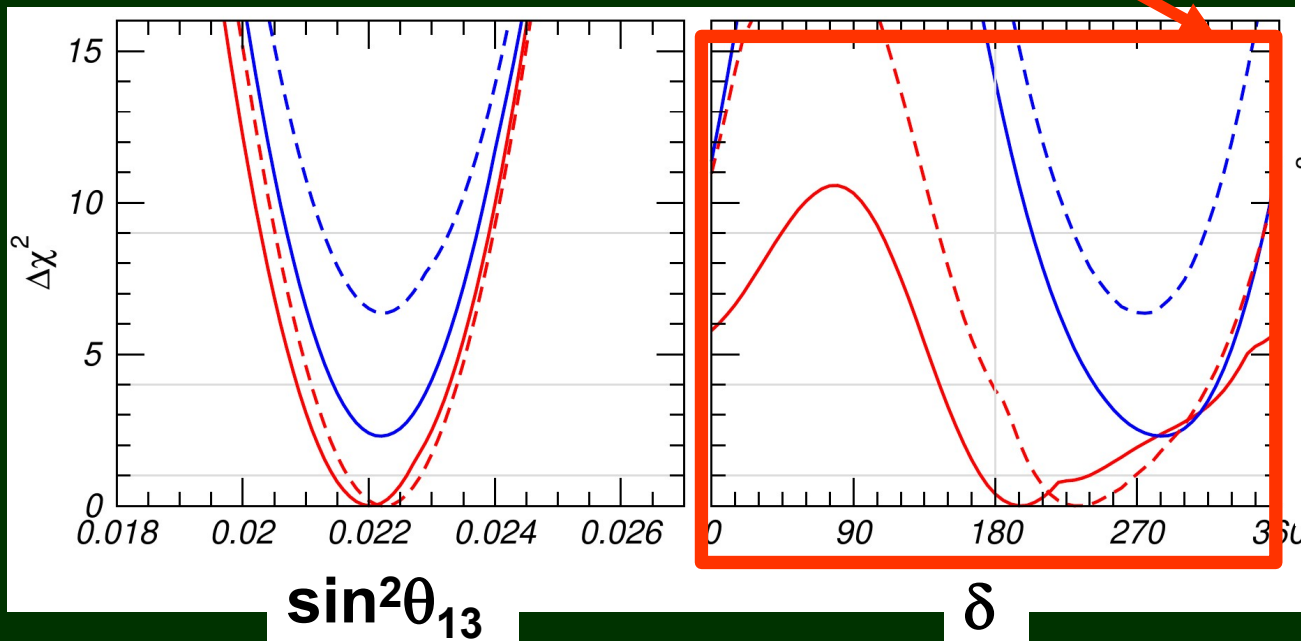
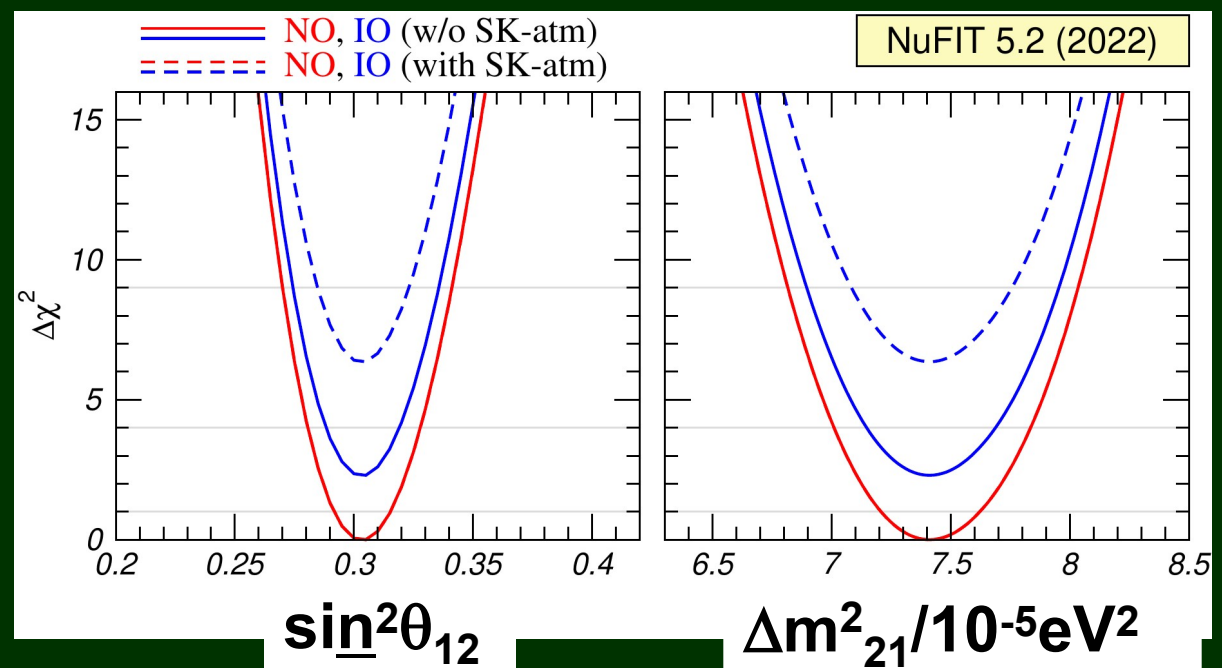
Status of 3ν fit (1)

www.nu-fit.org
v5.2 (Nov. 2022)

NO, $\delta \sim \pi$ seems to be preferred over **IO**, $\delta \sim 3\pi/2$

— NO, IO (w/o SK-atm)
- - - NO, IO (with SK-atm)

θ_{23} and δ have large errors

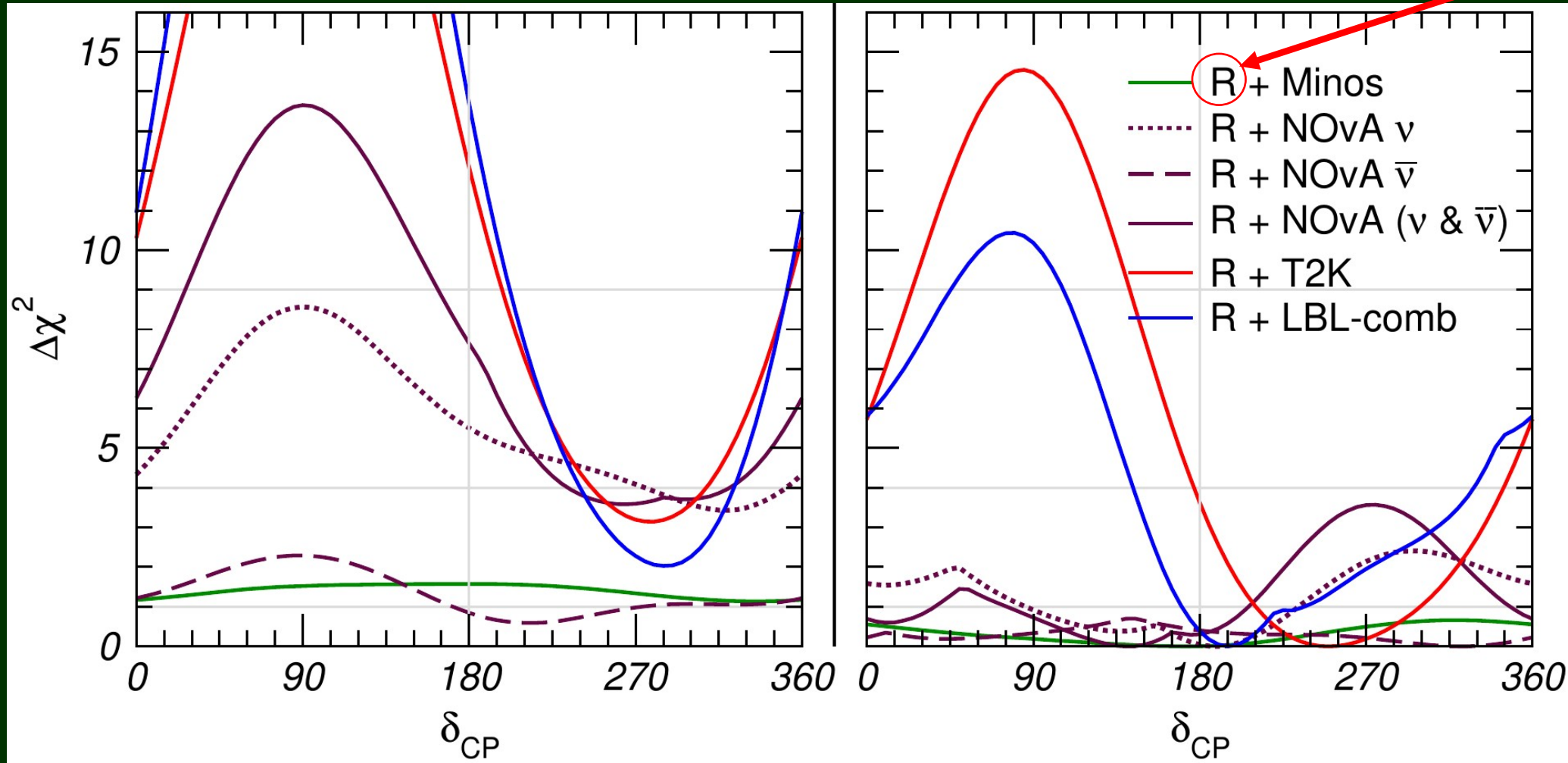


Status of 3ν fit (2)

Appearance data of LBL show us potential **tension** for NO, although T2K dominates over NOvA in statistics.

www.nu-fit.org v5.2 (Nov. 2022)

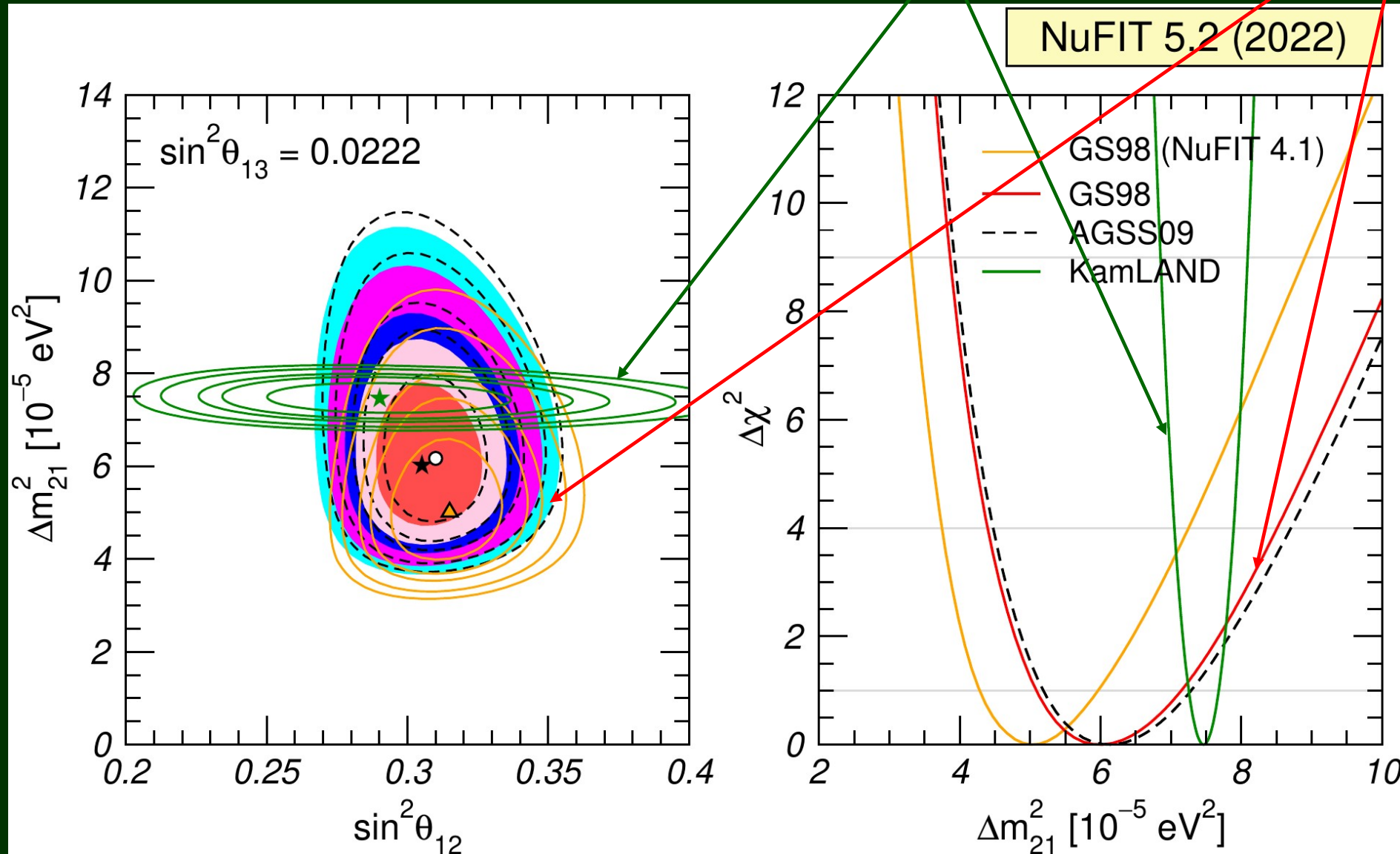
Reactor



Status of 3ν fit (3)

www.nu-fit.org v5.2 (Nov. 2022)

Tension of Δm^2_{21} between solar ν and KamLAND remains.



Tension may be a hint for **Non Standard Interactions** or **sterile neutrinos**.

● Parameter degeneracy

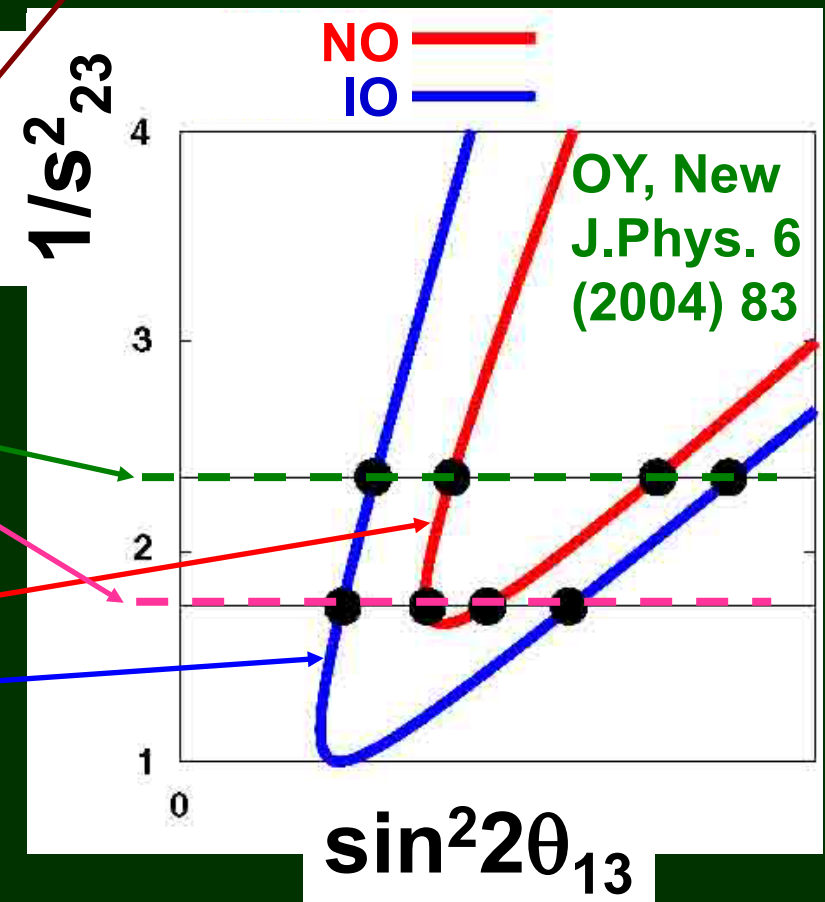
Even if we know $P \equiv P(\nu_\mu \rightarrow \nu_e)$ and $\bar{P} \equiv P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ in LBL experiments with energy E and baseline L , δ cannot be uniquely determined because of the 8-fold **parameter degeneracy**.

● **octant degeneracy** $\theta_{23} \Leftrightarrow \pi/2 - \theta_{23}$
(Fogli-Lisi, '96)

● **intrinsic degeneracy** (δ, θ_{13})
(Burguet-Castell et al, '01)

● **sign degeneracy** $\Delta m^2_{31} \Leftrightarrow -\Delta m^2_{31}$
(Minakata-Nunokawa, '01)

$(\sin^2 2\theta_{13}, 1/s^2_{23})$ plane
($P=\text{const}$ & $\bar{P}=\text{const}$ gives a **quadratic curve**)



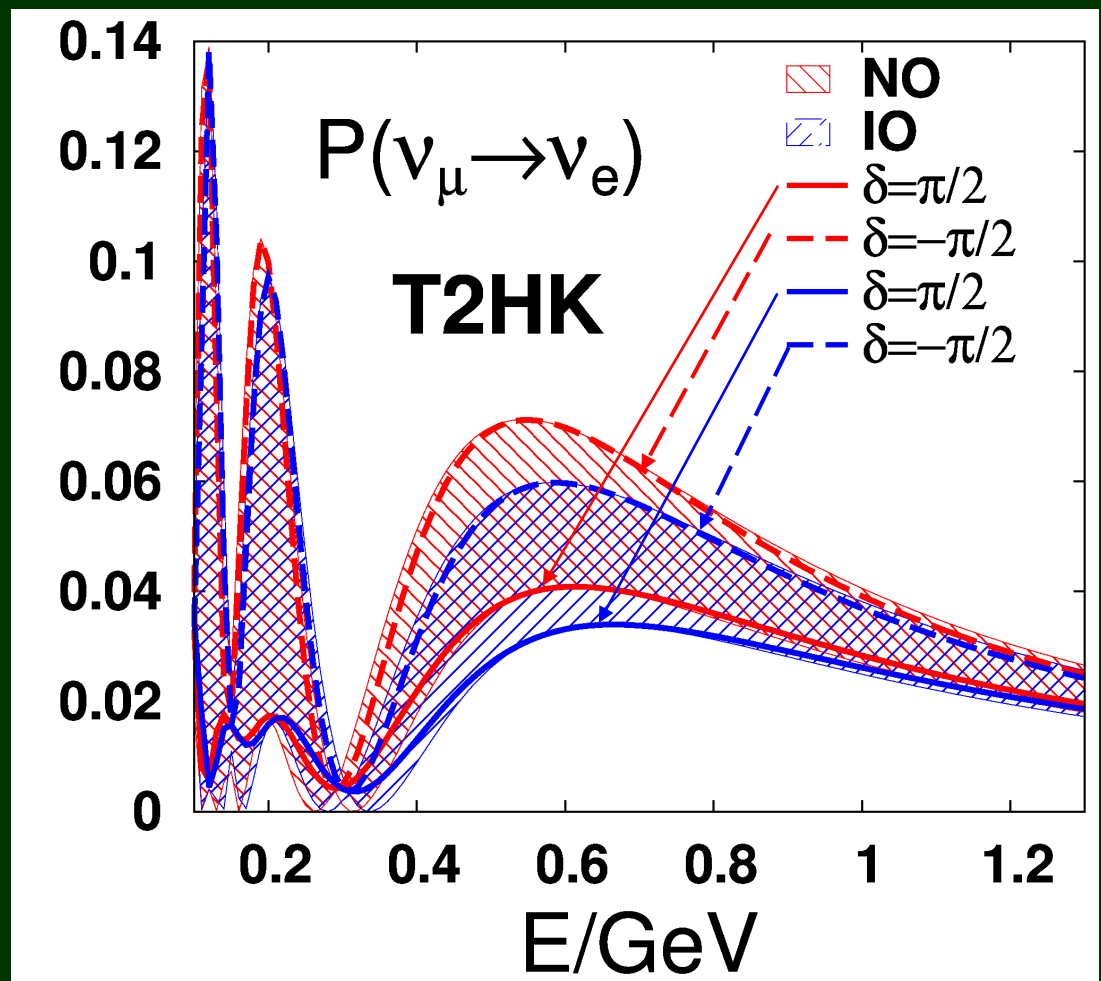
● Understanding degeneracy by appearance probabilities

hierarchy - δ

Prakash, Raut, Sankar, PRD 86, 033012 (2012)

octant - δ

Agarwalla, Prakash, Sankar, JHEP 1307, 131 (2013)

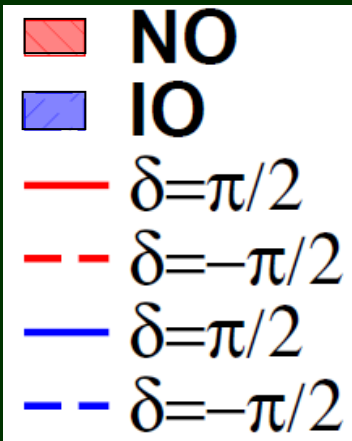


Due to uncertainty in δ , the appearance probabilities has finite width.

-> Each border is approximately realized for $\delta = +\pi/2$ or $-\pi/2$

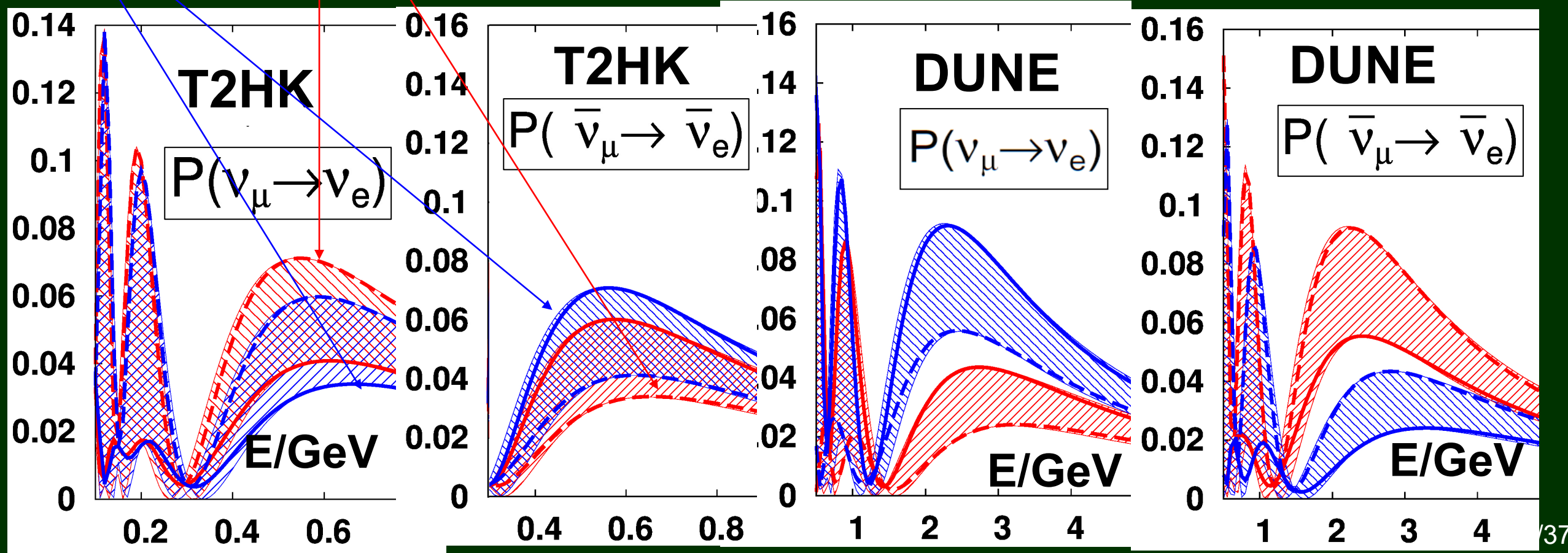
Mass Ordering

At T2HK, MO separation is good only for $\delta \sim -\pi/2$ (NO), $\delta \sim +\pi/2$ (IO)



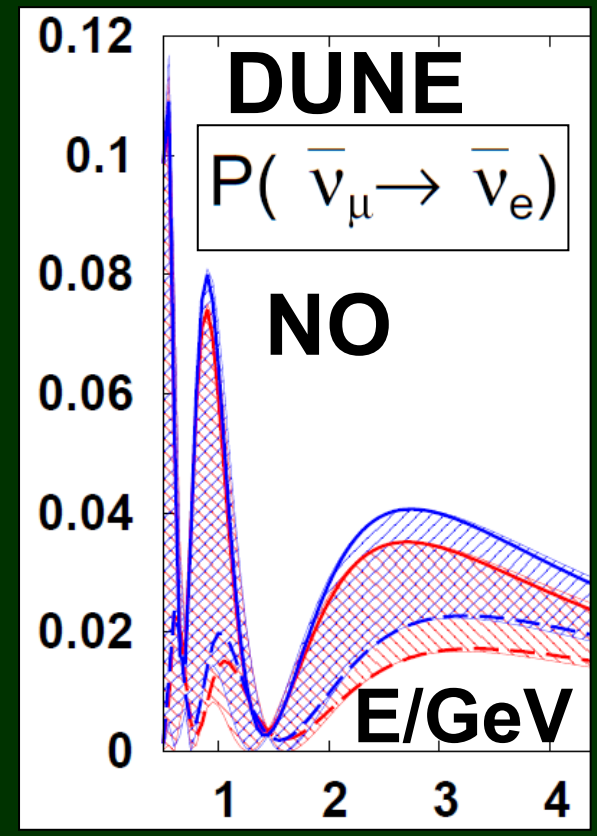
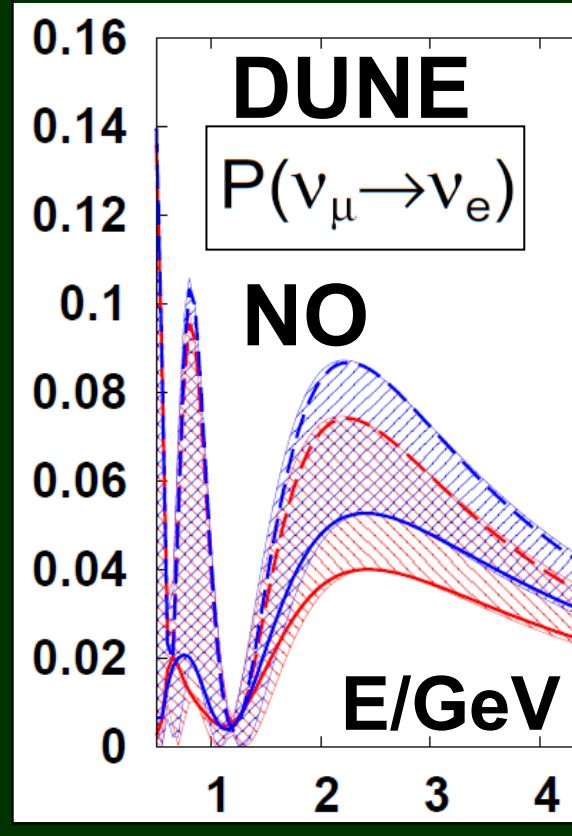
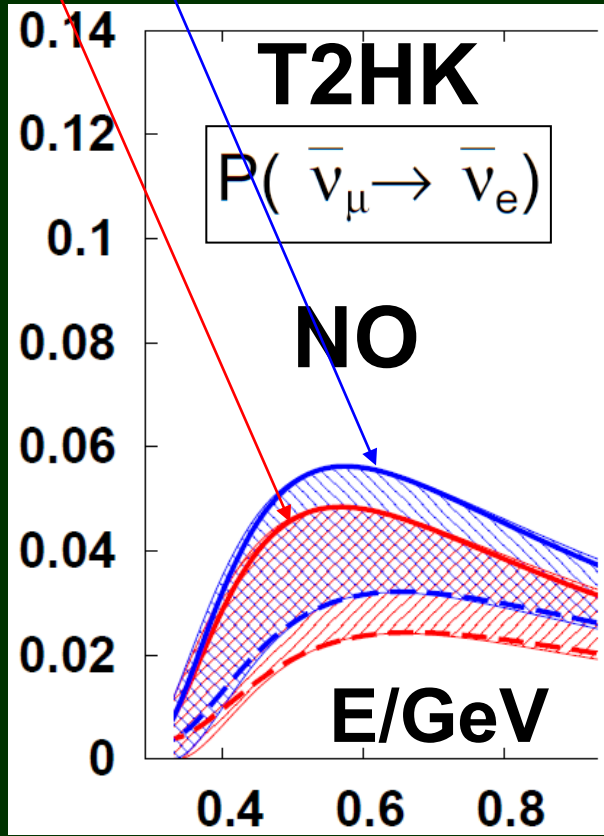
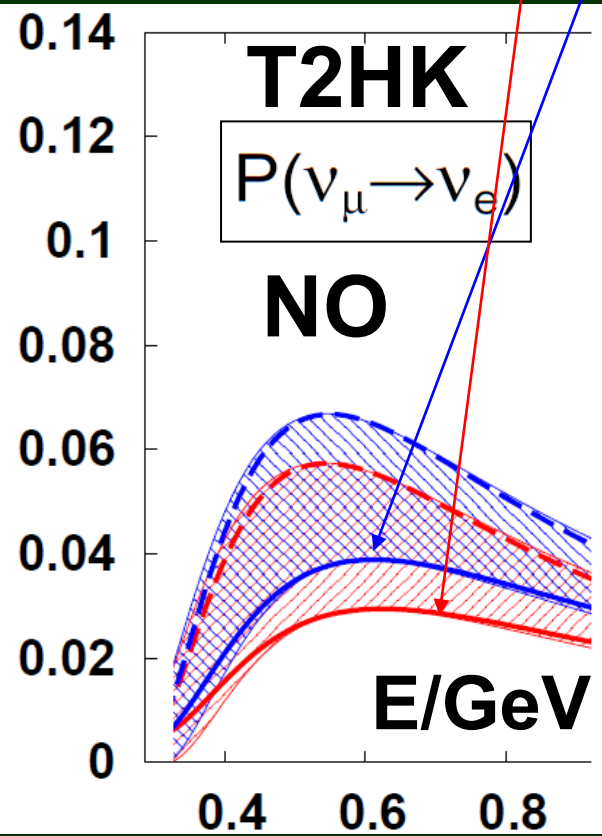
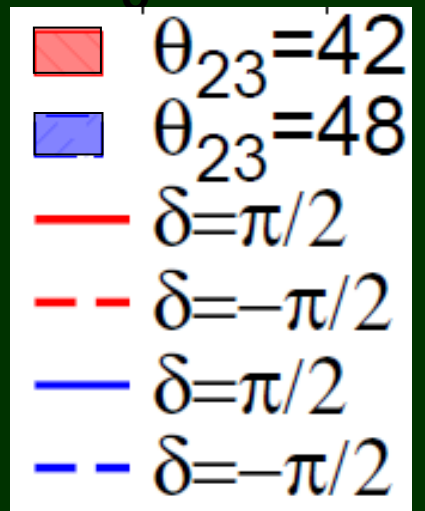
Fukasawa, Ghosh, OY, NPB 918 ('17) 337

At DUNE, NO-IO separation is good for any δ



At both T2HK & DUNE, HO-LO separation is possible w/ ν & $\bar{\nu}$ for most of δ

Unlike hierarchy degeneracy, $\delta=-\pi/2$ lies on the same side for ν & $\bar{\nu}$



Complementarity of T2HK and DUNE

T2HK

pro: #(events) ($\propto L^{-2}$) is large

con: little matter effect

DUNE

pro: #(events) ($\propto L^{-2}$) is smaller

con: large matter effect

Matter effect becomes most conspicuous if $\Delta E \cos 2\theta = A$ is satisfied.

$$P(\nu_\mu \rightarrow \nu_e) = \left(\frac{\Delta E \sin 2\theta}{\Delta \tilde{E}} \right)^2 \sin^2 \left(\frac{\Delta \tilde{E} L}{2} \right)$$

$$\tan 2\tilde{\theta} \equiv \frac{\Delta E \sin 2\theta}{\Delta E \cos 2\theta - A}$$

$$\Delta \tilde{E} \equiv \left[(\Delta E \cos 2\theta - A)^2 + (\Delta E \sin 2\theta)^2 \right]^{1/2}$$

$$A \equiv \sqrt{2} G_F n_e(x)$$

In this case, the baseline length L has to be large

$\rightarrow L > \pi/A \sim O(1000\text{km}) \rightarrow$ It is satisfied by DUNE but not by T2HK.

2. Sensitivity of T2HK & DUNE to $N_\nu=3$ oscillation parameters

Uncertainty in matter density taken into account

Ghosh-OY, NPB 989 ('23) 116142

The parameters assumed here:

T2HK

187 kton fiducial volume

$\nu:\bar{\nu} = 1:1$

Total exposure: 2.7×10^{22} POT

DUNE

40 kt LiAr detector,

$\nu:\bar{\nu} = 1:1$

Total exposure: 1.1×10^{21} POT

Reference value:

$\theta_{23} = 42^\circ$ or 48° ,

$\delta = -90^\circ$,

$\Delta m^2_{31} = 2.51 \times 10^{-3} \text{eV}^2$,

$\Delta\rho/\rho = 0, 5\%, 10\%$

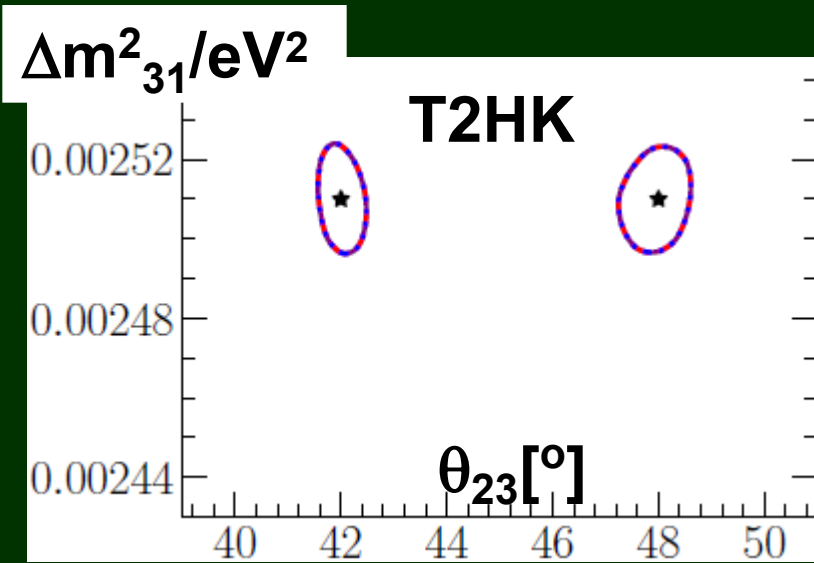
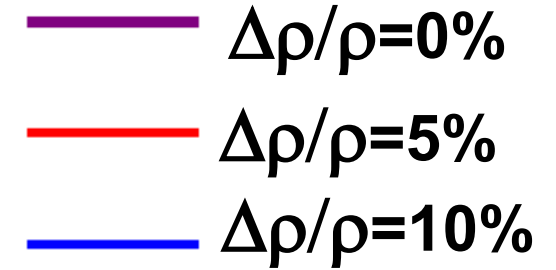
2.1 Precision to oscillation parameters

Ghosh-OY, NPB
989 ('23) 116142

Uncertainty in matter density
taken into account

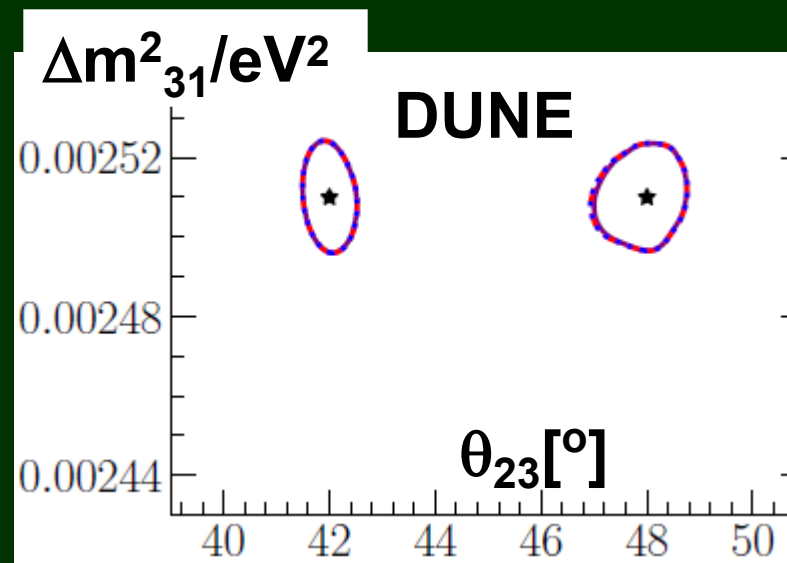
Precision of DUNE is slightly better than that of T2HK.
-> Combined precision is excellent.

Uncertainty in matter density has little effect
-> Major contribution comes from disappearance channel.



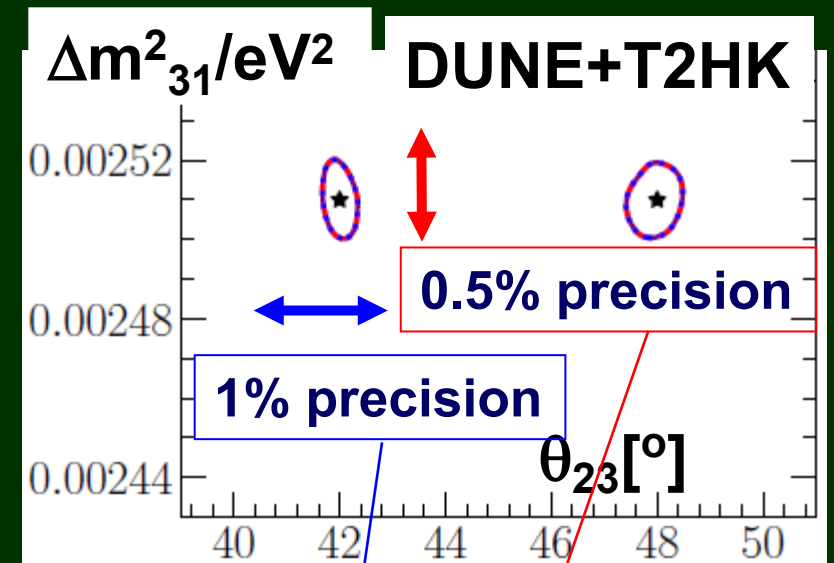
$$\Delta m^2_{31}/10^{-3}\text{eV}^2 = (2.51+0.013-0.014)$$

$$\theta_{23} = (42 \pm 0.5)^\circ$$



$$(2.51+0.015-0.014)$$

$$(42 \pm 0.5)^\circ$$



$$(2.51 \pm 0.01)$$

$$(42+0.4-0.3)^\circ$$

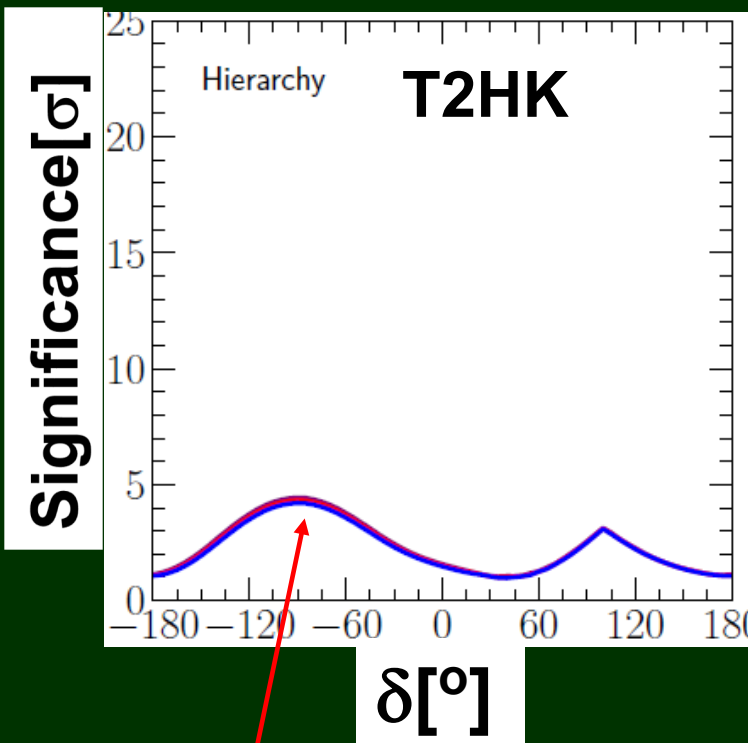
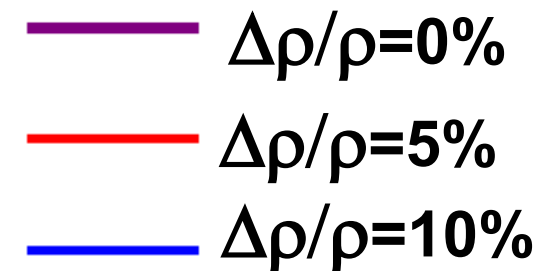
DUNE&T2HK vs Present status of global fit

	Ref	$\Delta m^2_{31}/10^{-3}eV^2$	$\theta_{23}[^{\circ}]$
Global fit	www.nu-fit.org v5.2 (Nov. 2022)	2.507+0.026-0.027	42.2+1.1-0.9
Future exp	T2HK Ghosh-OY('23)	2.510+0.013-0.014	42.0\pm0.5
	DUNE Ghosh-OY('23)	2.510+0.015-0.014	42.0\pm0.5
	DUNE+T2HK Ghosh-OY('23)	2.510\pm0.010	42.0+0.4-0.3

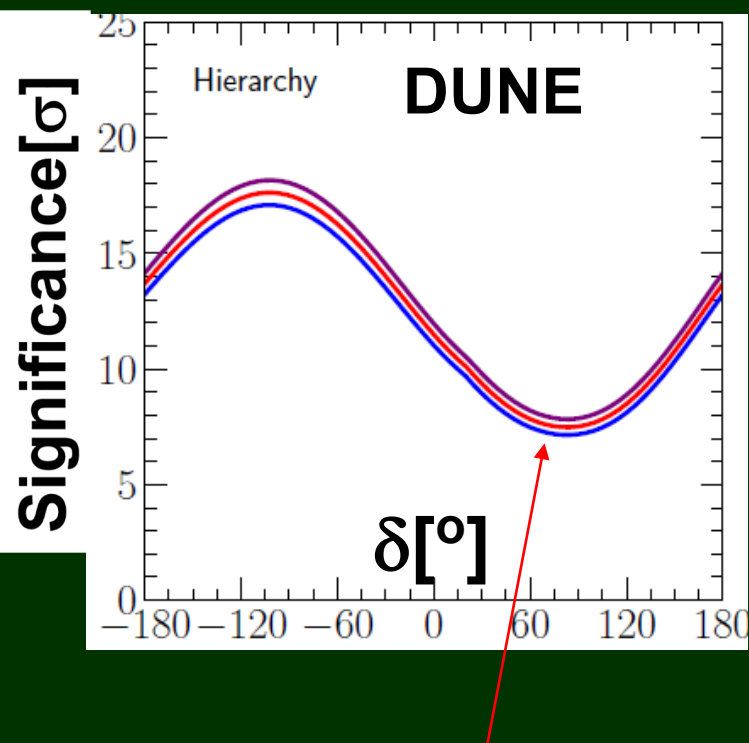
2.2 Sensitivity to Mass Ordering

Ghosh-OY, NPB 989 ('23) 116142

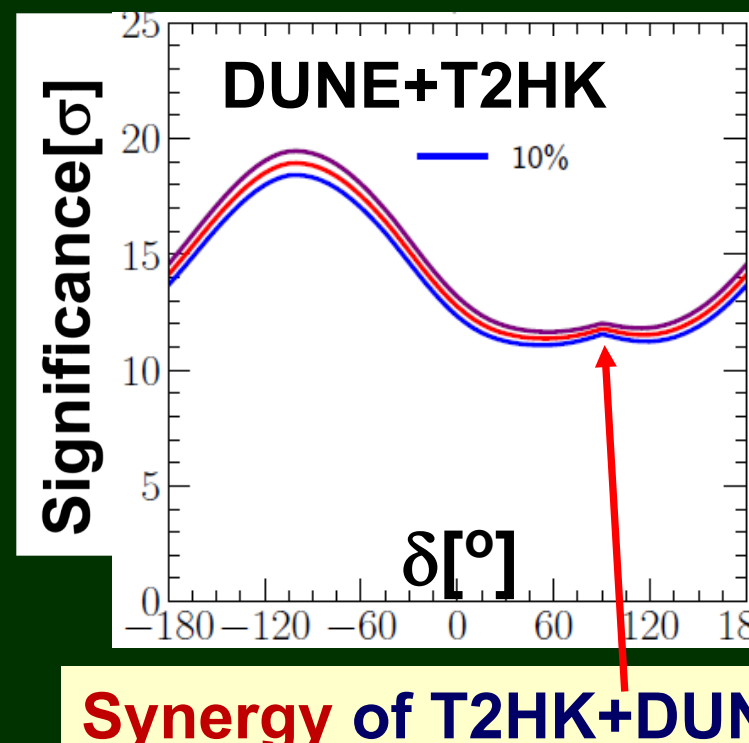
Uncertainty in matter density has some effect on DUNE
← DUNE has longer baseline $L=1300\text{km}$



Sensitivity of T2HK is poor except for $\delta \sim -\pi/2$



Sensitivity of DUNE is excellent for any δ



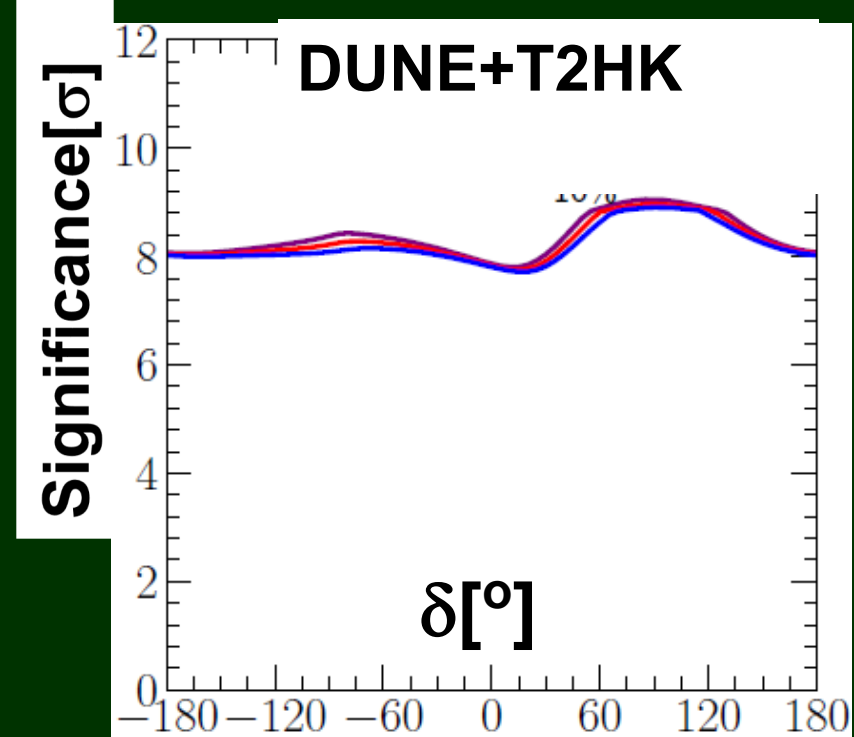
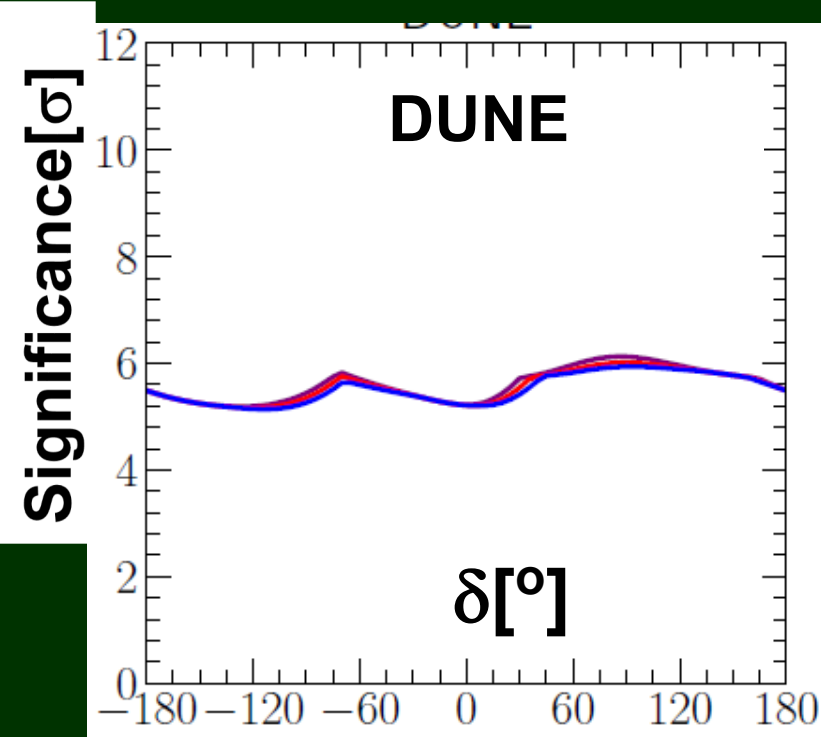
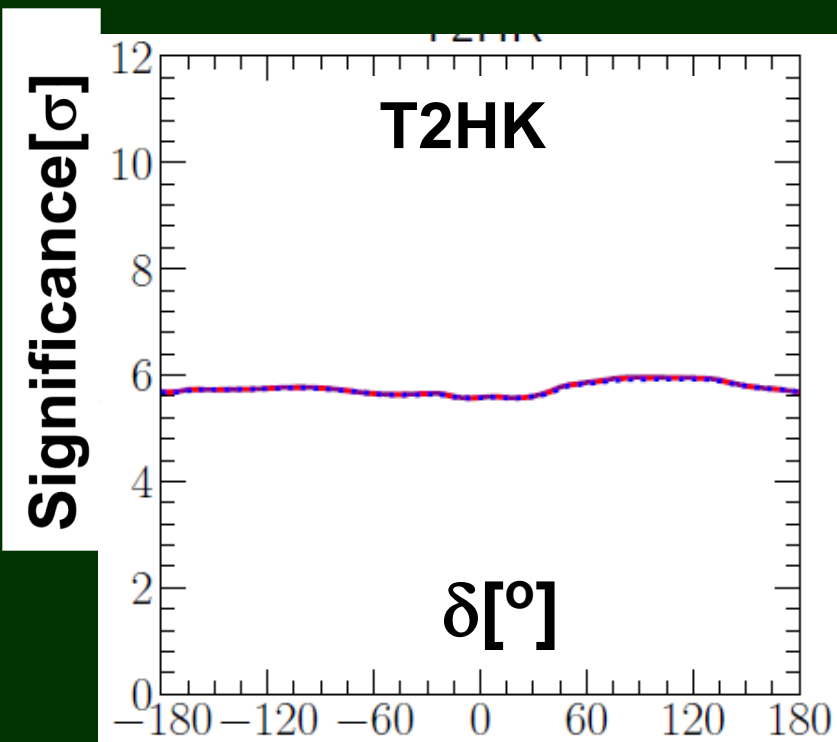
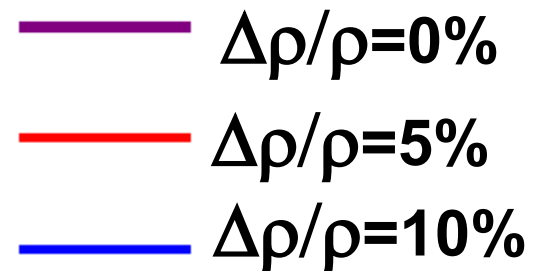
Synergy of T2HK+DUNE at $\delta = \pi/2$: If MO is known from DUNE \rightarrow Sensitivity of T2HK is improved

2.3 Sensitivity to Octant degeneracy

HO-LO Separation is possible for **T2HK** & **DUNE** w/ ν & $\bar{\nu}$ for most of δ

Ghosh-OY, NPB 989 ('23) 116142

$$\theta_{23} = 42^\circ$$



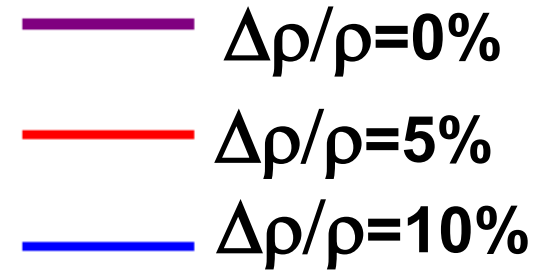
2.4 Sensitivity to CP(1)

Ghosh-OY, NPB 989 ('23) 116142

Uncertainty in matter density has some effect on DUNE.

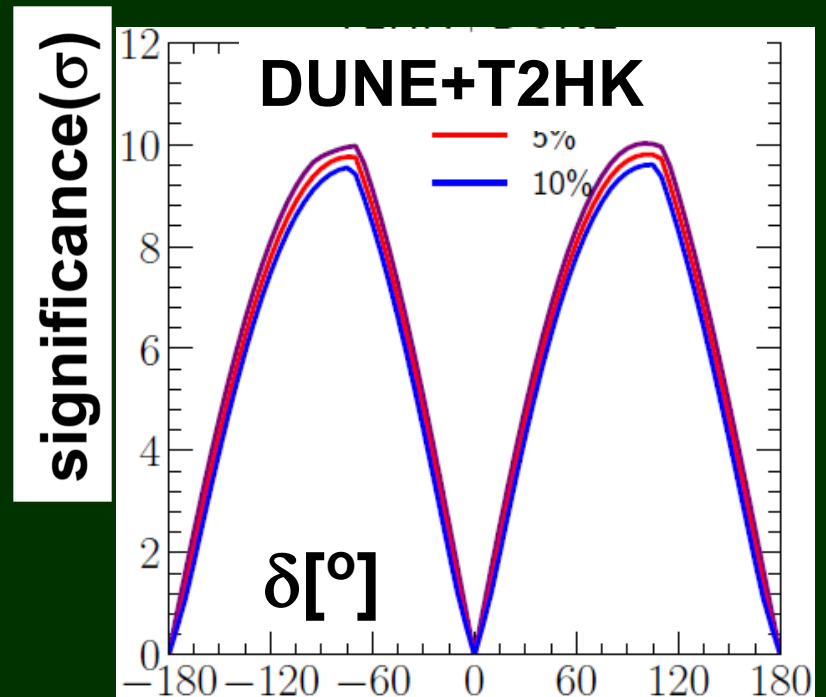
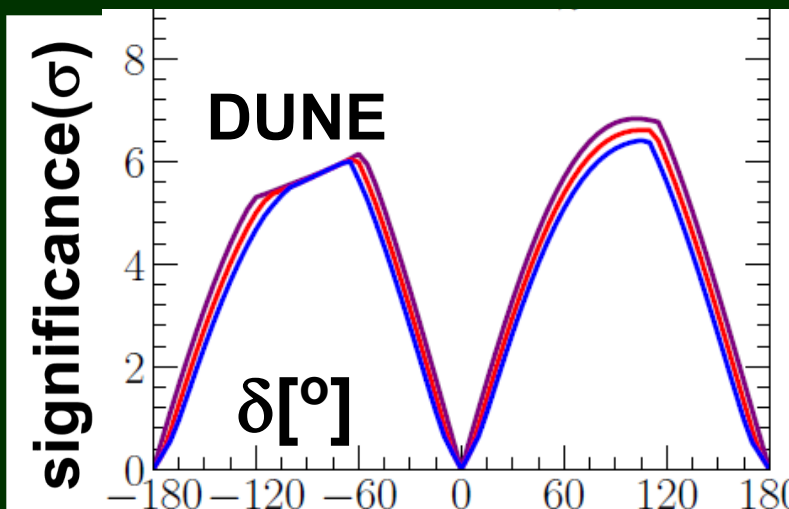
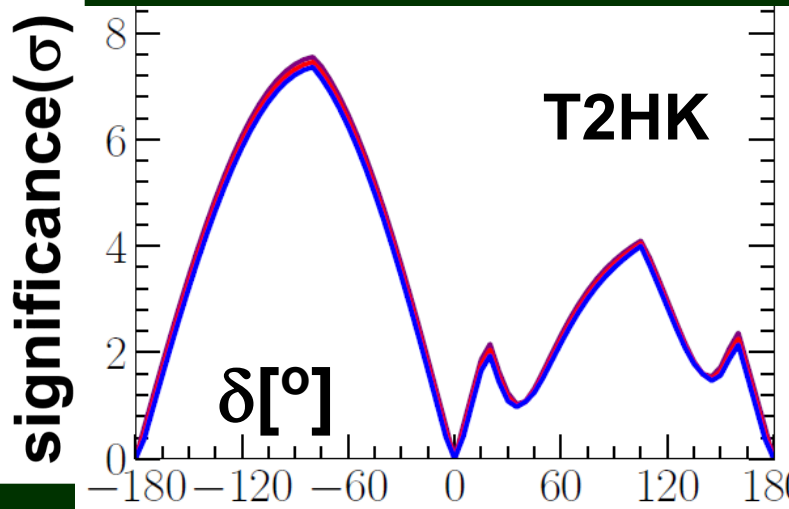
<- DUNE has longer baseline $L=1300\text{km}$

However even with $\Delta\rho/\rho=10\%$, the sensitivity is excellent.



Sensitivity of **T2HK** is poor for (NO, $\delta = +\pi/2$) & (IO, $\delta = -\pi/2$)

Sensitivity of **DUNE** is excellent for $|\sin\delta|=1$ for both mass hierarchy

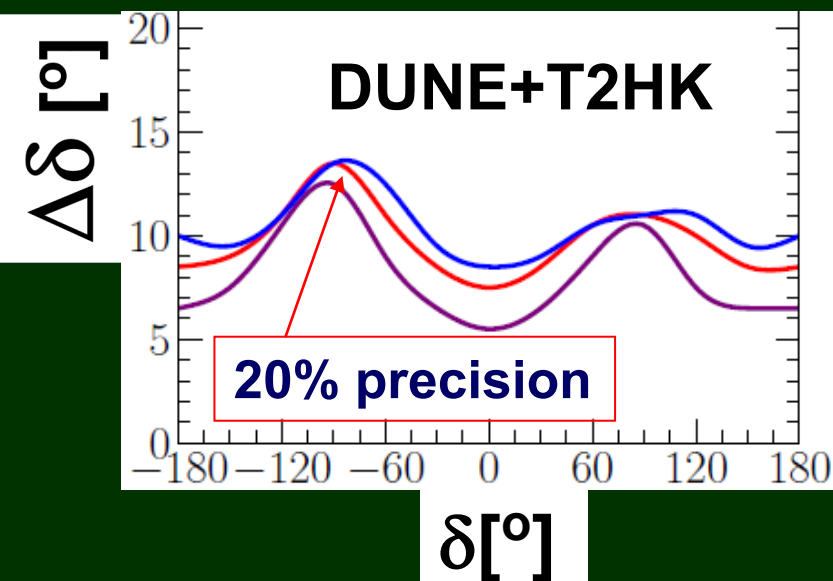
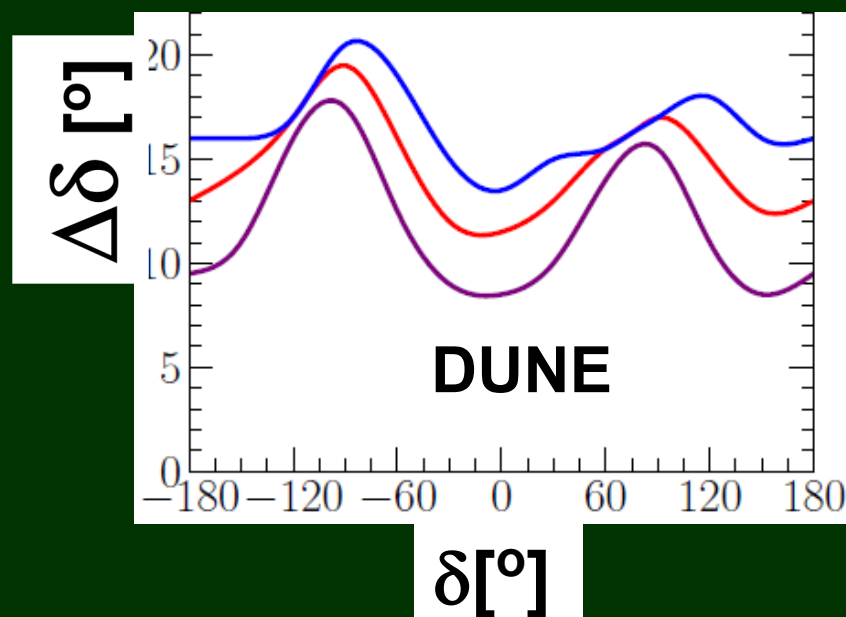
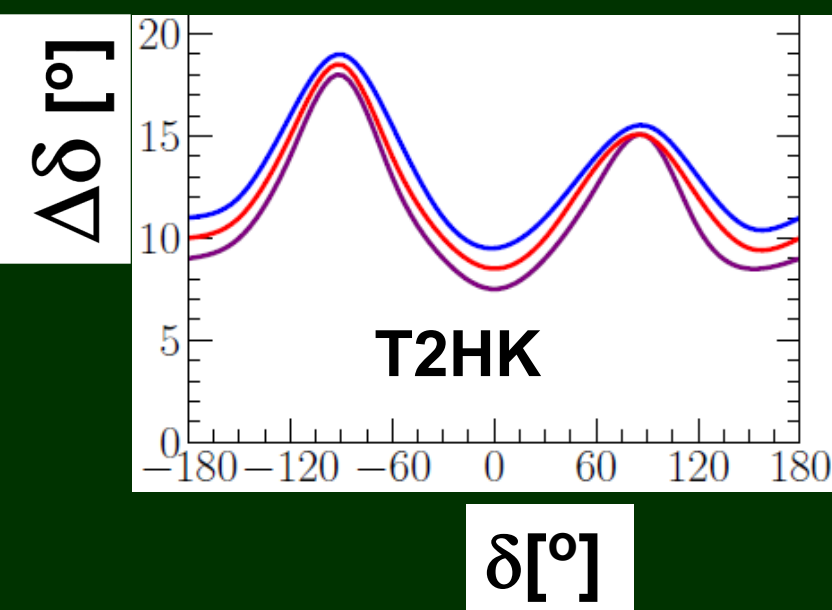
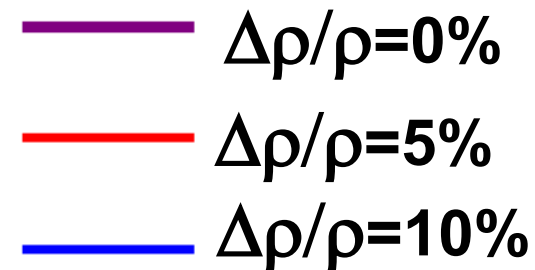


2.4 Sensitivity to CP(2)

Ghosh-OY, NPB 989 ('23) 116142

Uncertainty in matter density has some effect on the precision $\Delta\delta$ both for T2HK & DUNE.

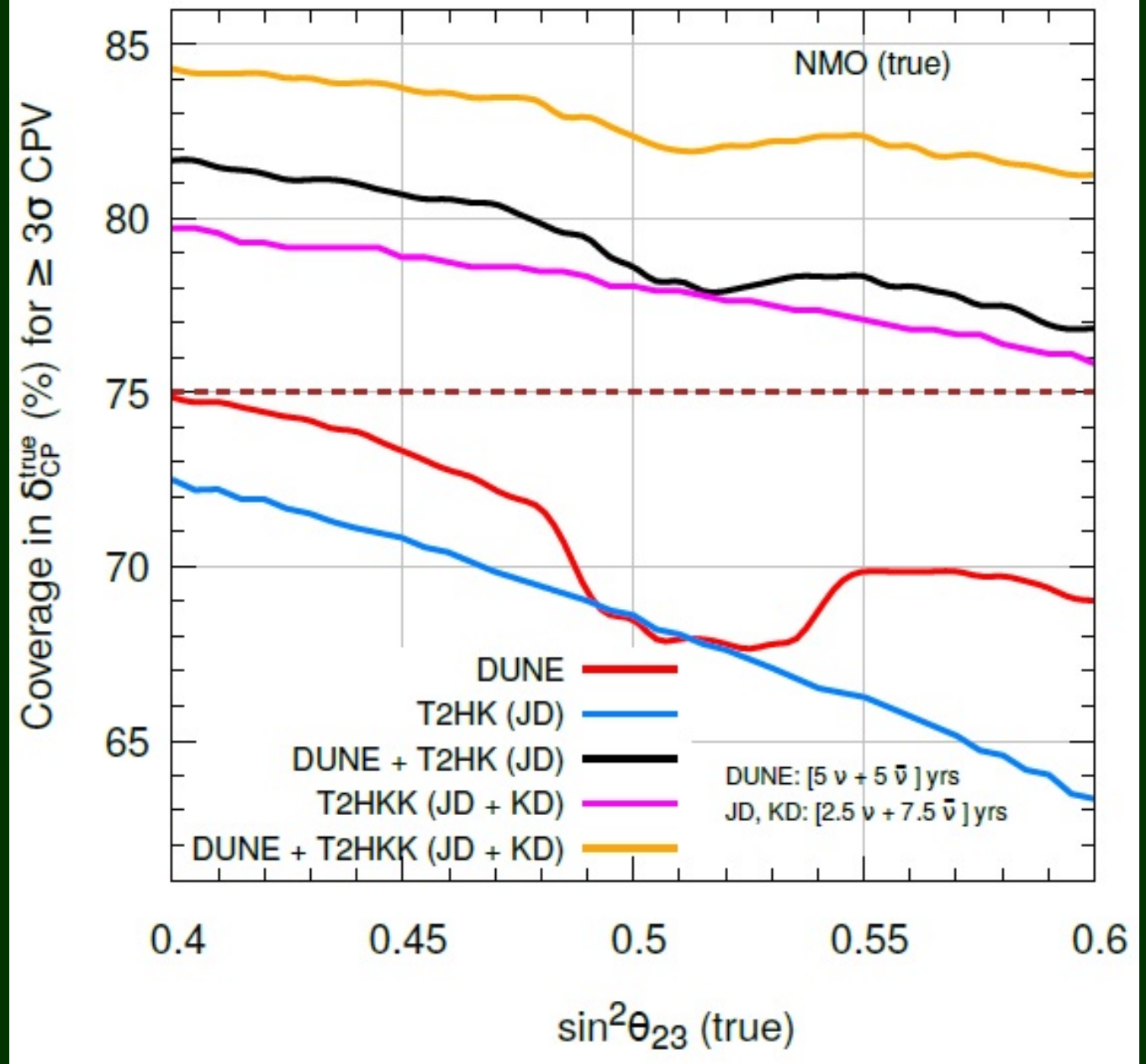
$\Delta\delta/\delta$ has mild dependence on δ but not much.



2.4 Sensitivity to CP(3)

Agarwalla, Das, Giarnetti, Meloni,
Singh, 2211.10620 [hep-ph]

Coverage in δ (= the range of δ for which leptonic CPV can be established at 3σ) is the largest by combining T2HKK & DUNE.



For details, see talk by Masoom Singh on July 19.

3. Sensitivity to scenarios beyond the 3-flavor PMNS picture

Scenarios suggested by experiments:

3.1 sterile ν  LSND/MiniBooNE, Reactor/Gallium ν **anomaly**

3.2 NSI  **Tension** of Δm^2_{21} between solar ν and KamLAND


Scenarios which are not suggested by experiments:

3.3 Scalar NSI

3.4 Flavor-dependent long-range interactions

3.5 Non-unitarity

3.6 Lorentz Invariance Violation

 No affirmative hints for these, but discovery of these would give a clue for Physics Beyond the Standard Model

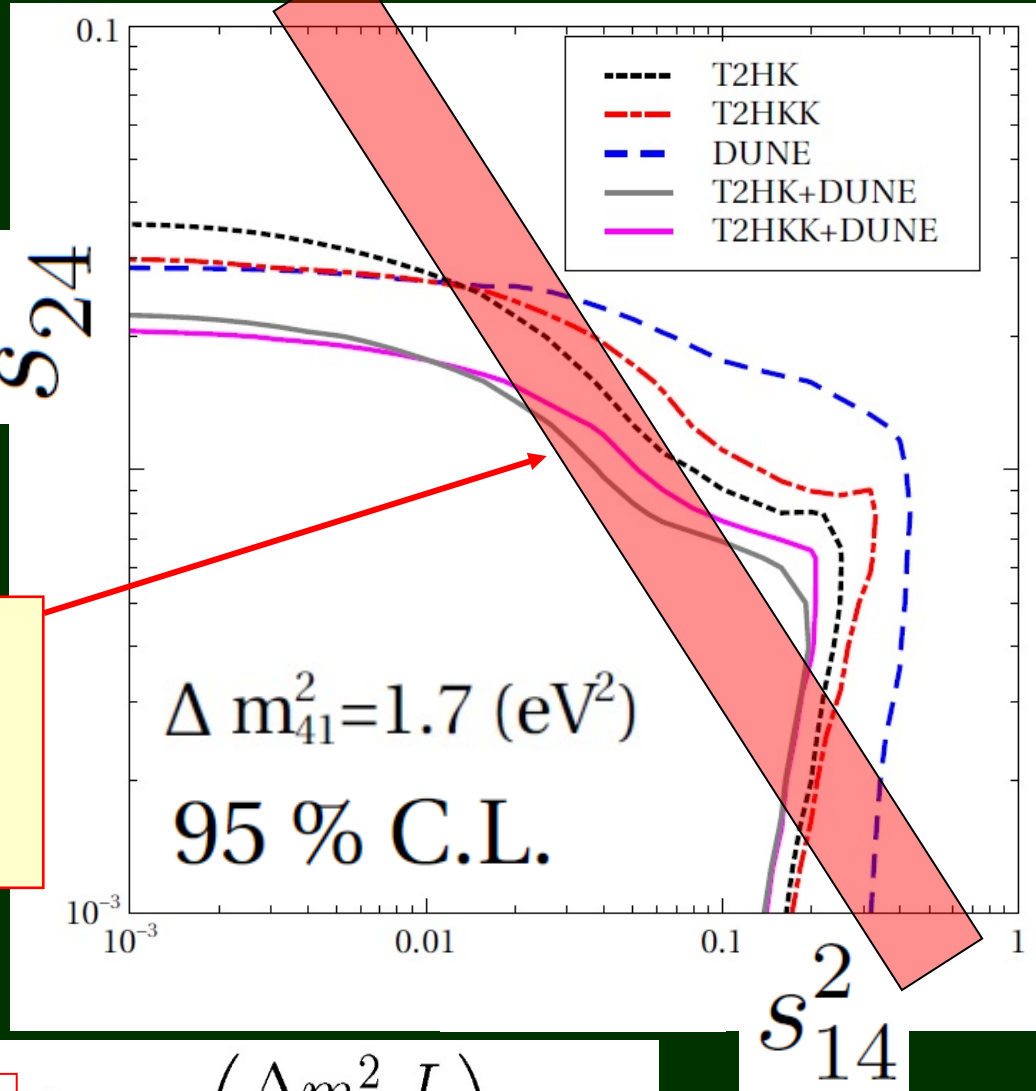
3.1 Sensitivity of T2HK & DUNE to Sterile ν

Choubey, Dutta, Pramanik,
Eur.Phys.J.C78('18)4, 339

(1) Accelerator ν (T2HK, T2HKK, DUNE)

Combined accelerator ν can cover some of the LSND region @ 90%CL

S_{24}



S_{14}


$$P(\nu_{\mu} \rightarrow \nu_e) = 4\text{Re} \left[U_{e3}U_{\mu 3}^* (U_{e3}^*U_{\mu 3} + U_{e4}^*U_{\mu 4}) \right] \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right) + \dots$$

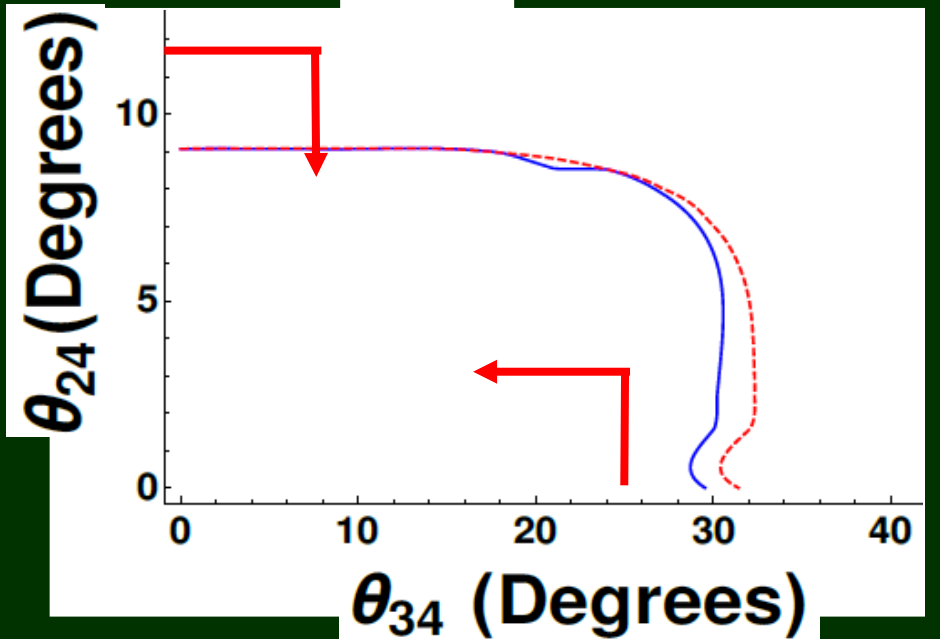
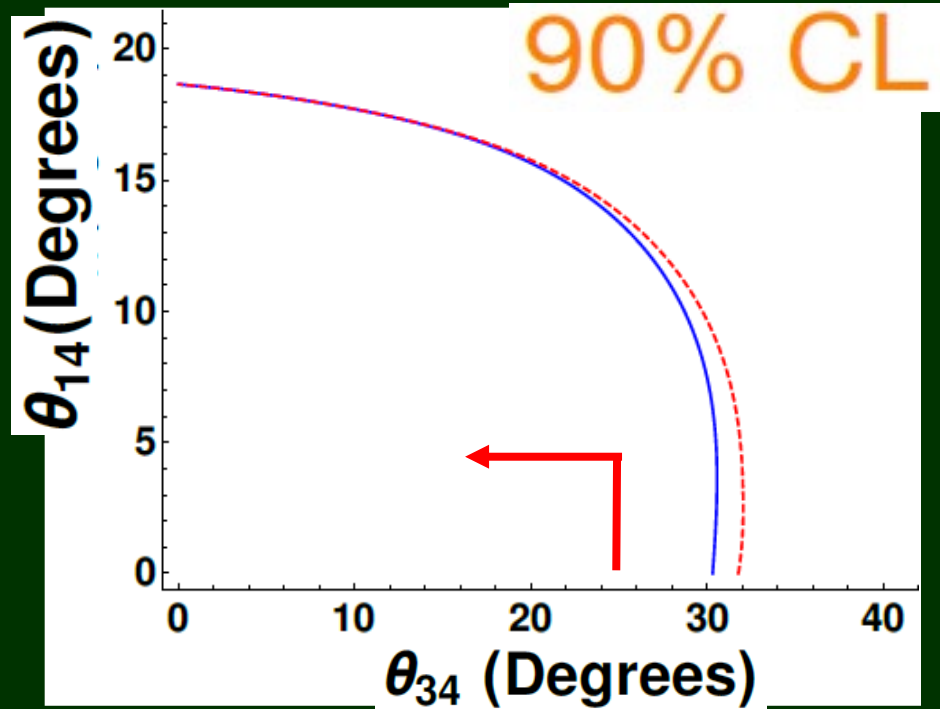
(2) Accelerator ν (DUNE w/ ν_τ)

Ghoshal-Giarnetti-Meloni,
JHEP 12 (2019) 126

$$\Delta m_{41}^2 = 1.0 \text{ eV}^2$$

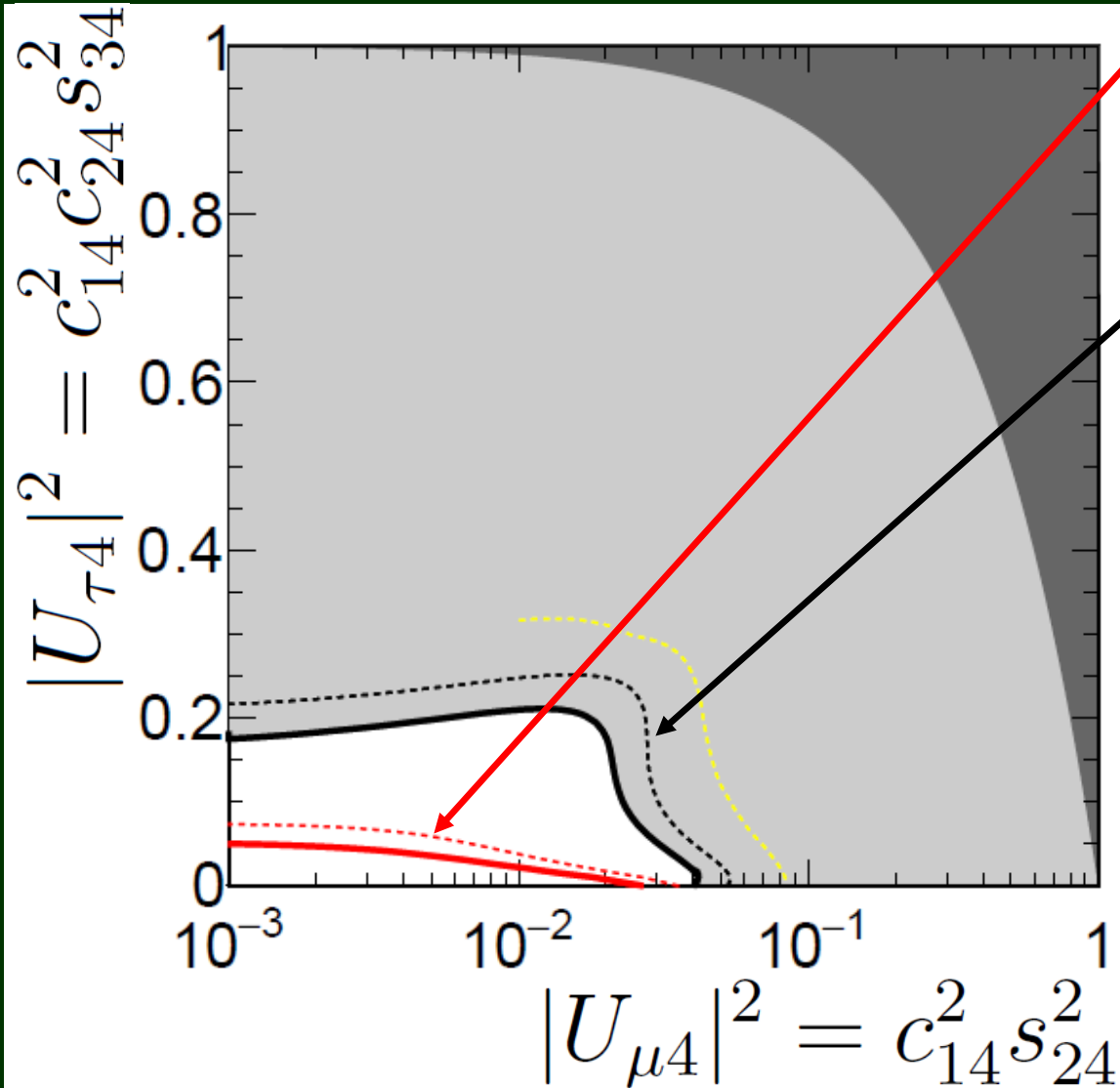
$$V_\tau + V_\mu + V_e \quad \text{— (solid blue line)}$$
$$V_\mu + V_e \quad \text{- - - (dashed red line)}$$

Accelerator ν has better
(worse) sensitivity to θ_{24} (θ_{34})
than ν_{atm} (depicted by )



(3) HK ν_{atm}

HK, arXiv:1805.04163v2



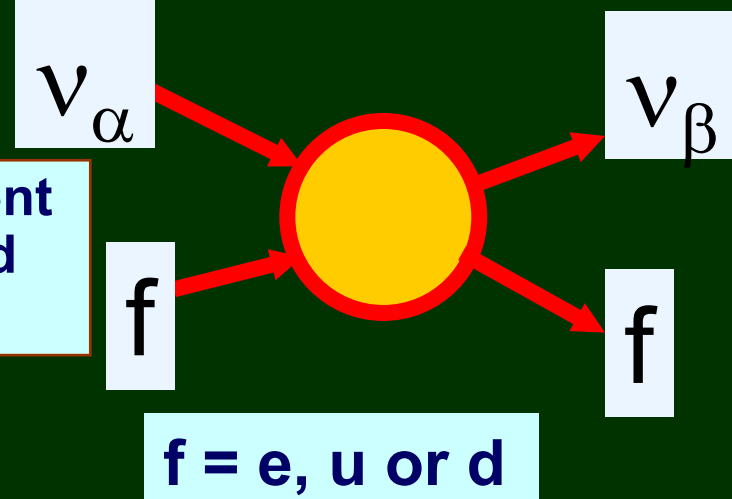
HK Sensitivity to θ_{34} (θ_{24}) is (is not much) improved compared to SK: 90% CL (solid) 99% CL (dashed)

3.2 Sensitivity of T2HK & DUNE to NSI

Phenomenological **New Physics** considered here: 4-fermi **Non Standard Interactions**:

$$\mathcal{L}_{eff} = G_{NP}^{\alpha\beta} \bar{\nu}_\alpha \gamma^\mu \nu_\beta \bar{f} \gamma_\mu f'$$

neutral current
non-standard
interaction



Modification of matter effect

$$A \equiv \sqrt{2}G_F N_e$$

$N_e \equiv$ electron density

$$\mathcal{A} \equiv \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + A \sum_{f=e,u,d} \frac{N_f}{N_e} \begin{pmatrix} \epsilon_{ee}^f & \epsilon_{e\mu}^f & \epsilon_{e\tau}^f \\ \epsilon_{\mu e}^f & \epsilon_{\mu\mu}^f & \epsilon_{\mu\tau}^f \\ \epsilon_{\tau e}^f & \epsilon_{\tau\mu}^f & \epsilon_{\tau\tau}^f \end{pmatrix}$$

NP

Tension of Δm_{31}^2 between solar ν and KamLAND may be accounted for by NSI.

CP-conserving Global fit (except T2K & NOvA appearance data) of NSI: Terrestrial ν basis

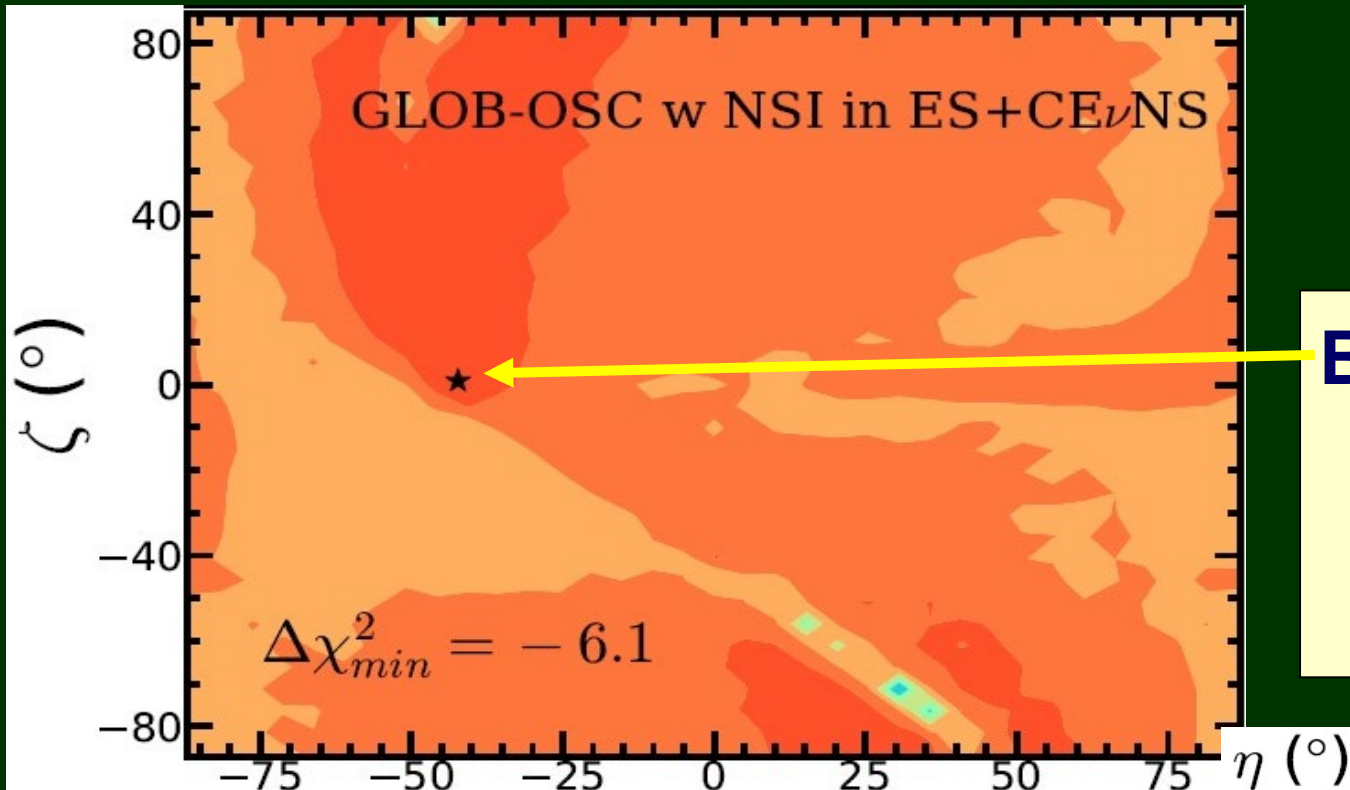
Coloma et al. arXiv:2305.07698v1 [hep-ph]

$$\mathcal{E}_{\alpha\beta}(x) = \varepsilon_{\alpha\beta} [\xi^e + \xi^p + Y_n(x)\xi^n] = \sqrt{5} [\cos \eta (\cos \zeta + \sin \zeta) + Y_n(x) \sin \eta]$$

$$= \sqrt{5} [\cos \eta' + Y_n(x) \sin \eta'] \varepsilon'_{\alpha\beta} \quad \tan \eta' \equiv \tan \eta / (\cos \zeta + \sin \zeta)$$

$$\varepsilon'_{\alpha\beta} \equiv \varepsilon_{\alpha\beta} \sqrt{1 + \cos^2 \eta \sin(2\zeta)}$$

To simplify the analysis, **no extra CP phase** are taken into account.



Best fit: $\zeta \sim 0$

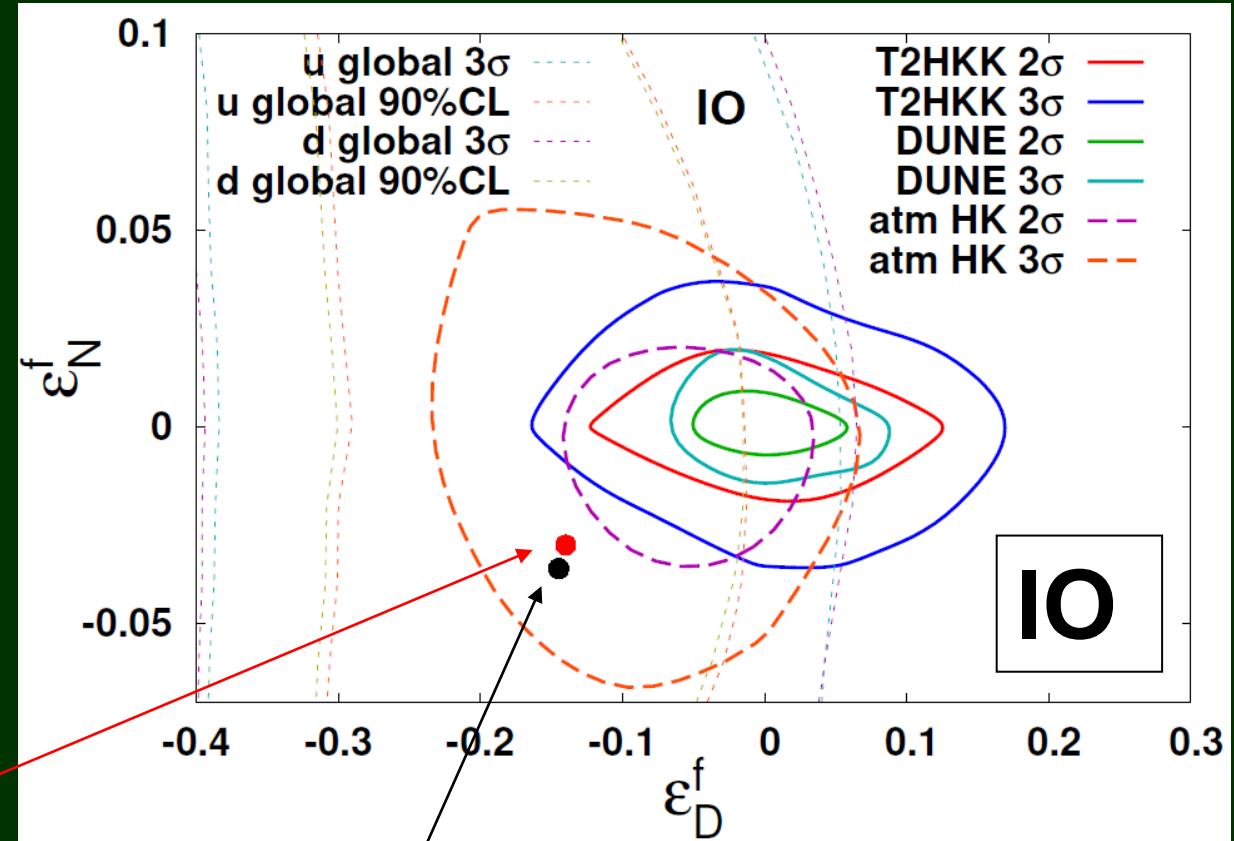
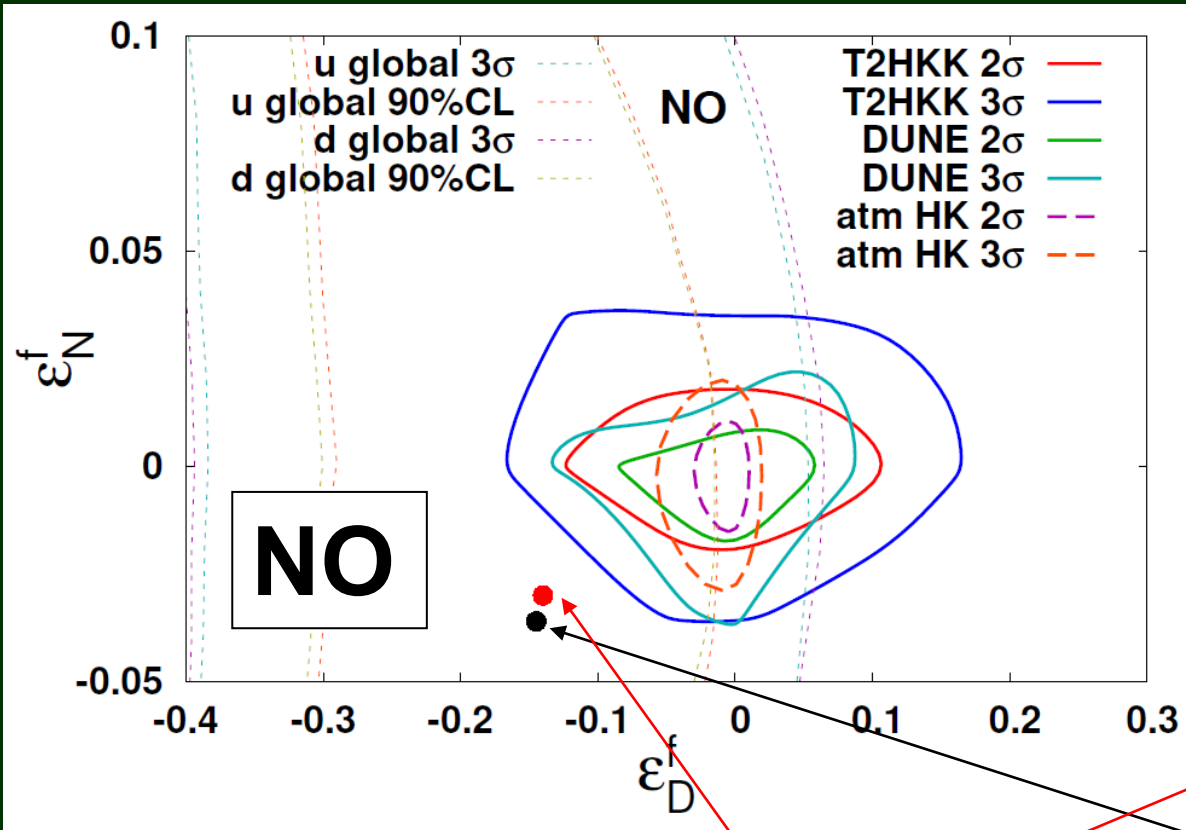
$$\eta \sim -43.6^\circ = \arctan(-1/Y_n)$$

$$\rightarrow \varepsilon_{\alpha\beta} \sim 0$$

- Before the work 1805.04530, fit only for the cases $\tan\eta=1/2$ ($\varepsilon_{\alpha\beta}^d=0$; $\eta=26.6^\circ$) and $\tan\eta=2$ ($\varepsilon_{\alpha\beta}^u=0$; $\eta=64.4^\circ$) were known.
- On the next page, significance of NSI at DUNE, $\nu_{\text{atm}}@HK$, T2HKK (possible extension of T2HK) is compared only the case for $\tan\eta=1/2$ ($\varepsilon^d=0$; $\eta=26.6^\circ$) and $\tan\eta=2$ ($\varepsilon^u=0$; $\eta=64.4^\circ$), assuming old design of HK (0.56 Mton fiducial volume x 12 yrs).
- After the work 1805.04530, the **best fit** is $\eta \sim \arctan(-1/Y_n) = -43.6^\circ$, and LBL experiments are useless for this value because $\varepsilon_{\alpha\beta} \sim 0$.
- Do the CP phases of $\varepsilon_{\alpha\beta}$, which were ignored in the analysis, change the best fit point $\eta \sim -43.6^\circ$?

● Comparison of sensitivity T2HKK, DUNE, $\nu_{\text{atm}}@HK$

Ghosh & OY, MPL A35 ('20) 17, 2050142



Best fit point of global analysis for $\tan\eta=1/2$ ($\epsilon^d=0$; $\eta=26.6^\circ$)

Best fit point of global analysis for $\tan\eta=2$ ($\epsilon^u=0$; $\eta=64.4^\circ$)

In the case of NO, $\nu_{\text{atm}}@HK$ is the best

In the case of IO, DUNE is the best

3.3 Sensitivity of T2HK & DUNE to Scalar NSI

Medhi, Devi, Dutta,
JHEP 01 (2023) 079

$$\mathcal{L}_{\text{eff}}^{\text{S}} = \frac{y_f y_{\alpha\beta}}{m_\phi^2} (\bar{\nu}_\alpha \nu_\beta) (\bar{f} f)$$

$$\mathcal{H} \approx E_\nu + \frac{(M + \delta M)(M + \delta M)^\dagger}{2E_\nu} \pm V_{\text{SI}}$$

$$\delta \widetilde{M} \equiv \sqrt{\Delta m_{31}^2} \begin{pmatrix} \eta_{ee} & \eta_{e\mu} & \eta_{e\tau} \\ \eta_{\mu e} & \eta_{\mu\mu} & \eta_{\mu\tau} \\ \eta_{\tau e} & \eta_{\tau\mu} & \eta_{\tau\tau} \end{pmatrix}$$

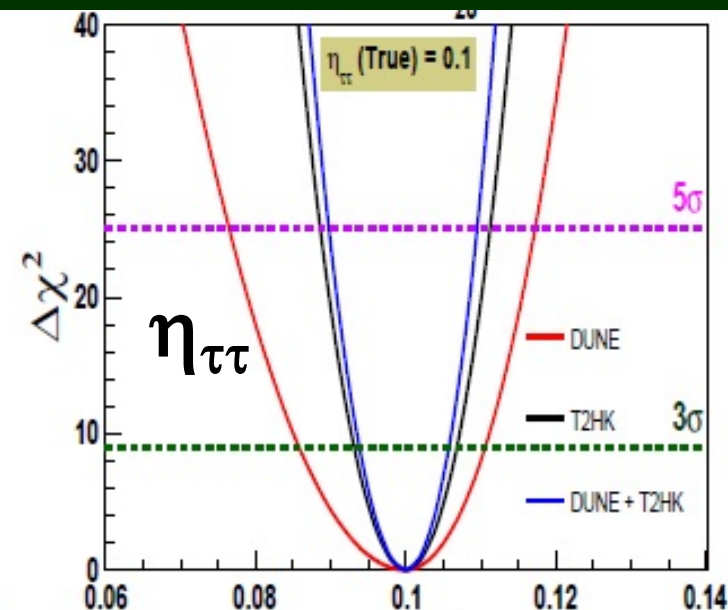
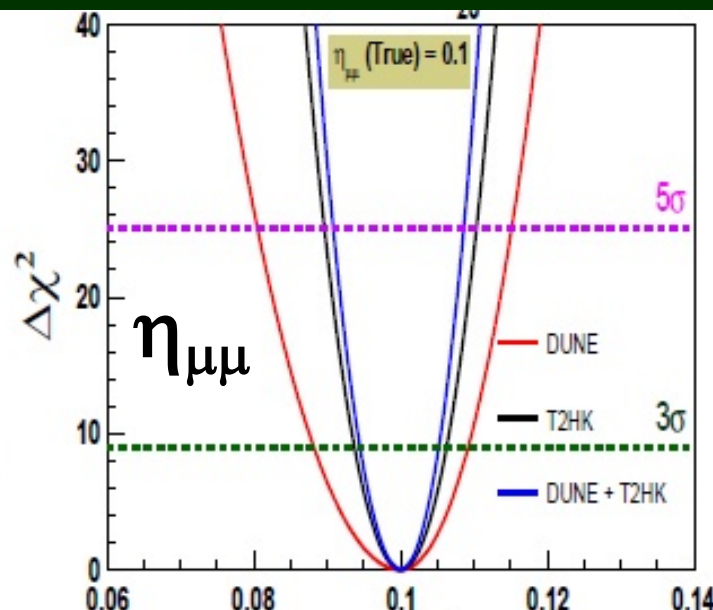
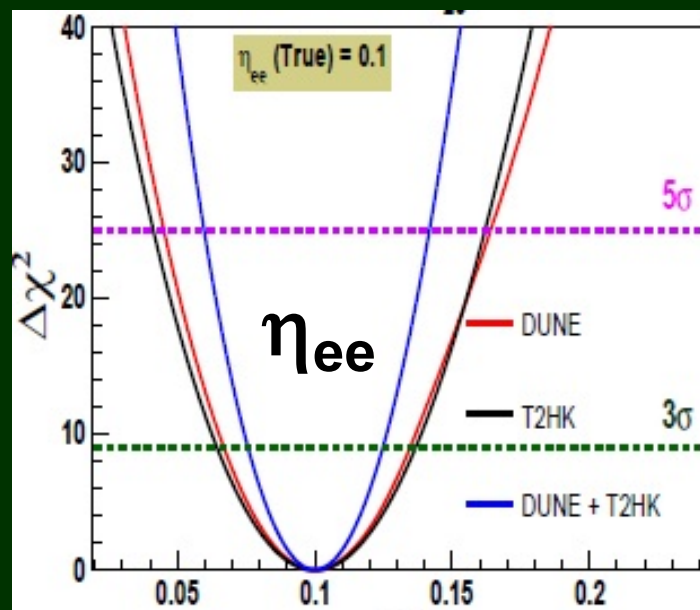
To simplify the analysis, only three cases were considered:

$$\delta \widetilde{M} \propto \text{diag}(\eta_{ee}, 0, 0), \text{diag}(0, \eta_{\mu\mu}, 0), \text{diag}(0, 0, \eta_{\tau\tau})$$

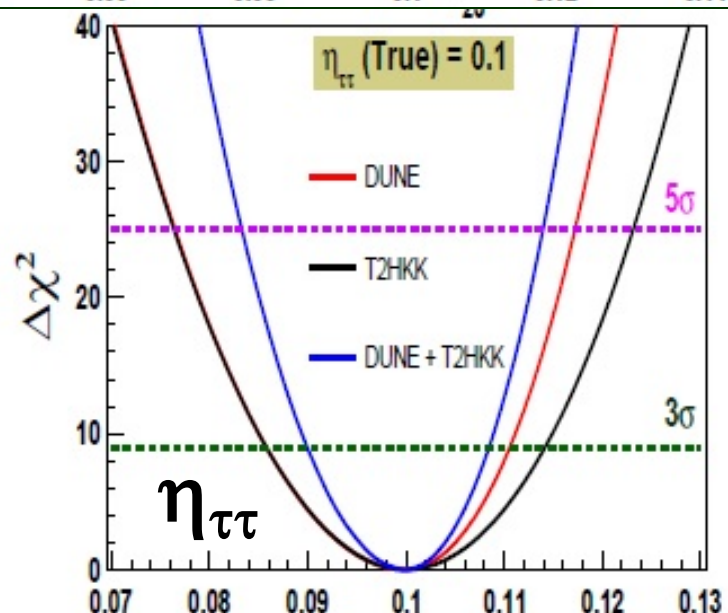
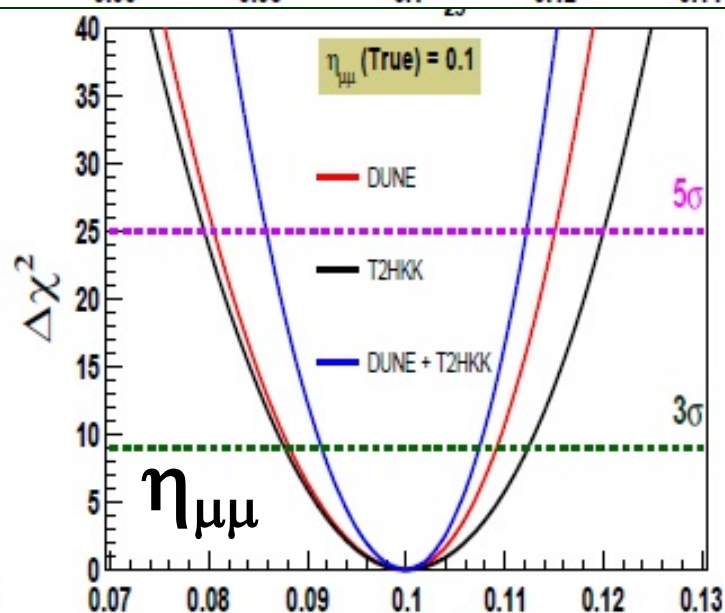
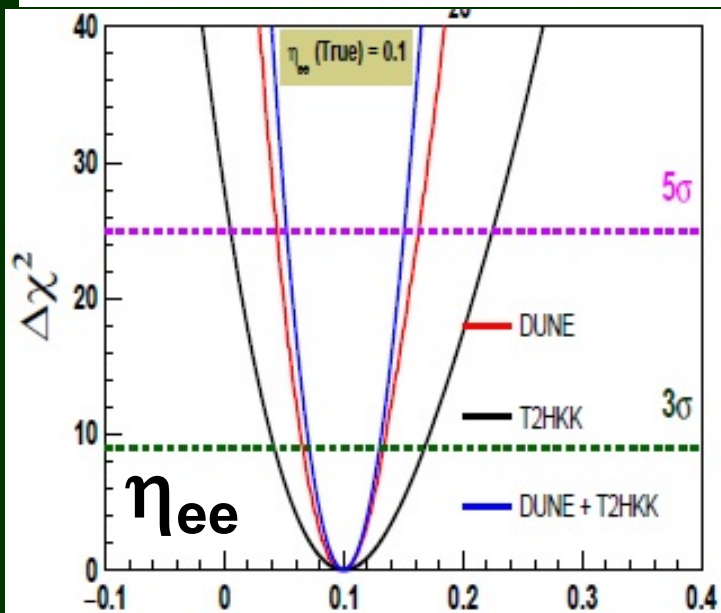
Reference value: NO, $\theta_{23} = 47^\circ$, $\delta = -90^\circ$, $\eta_{\alpha\alpha} = 0.1$ ($\alpha=e,\mu,\tau$)

Medhi, Devi, Dutta,
JHEP 01 (2023) 079

T2HK &
DUNE



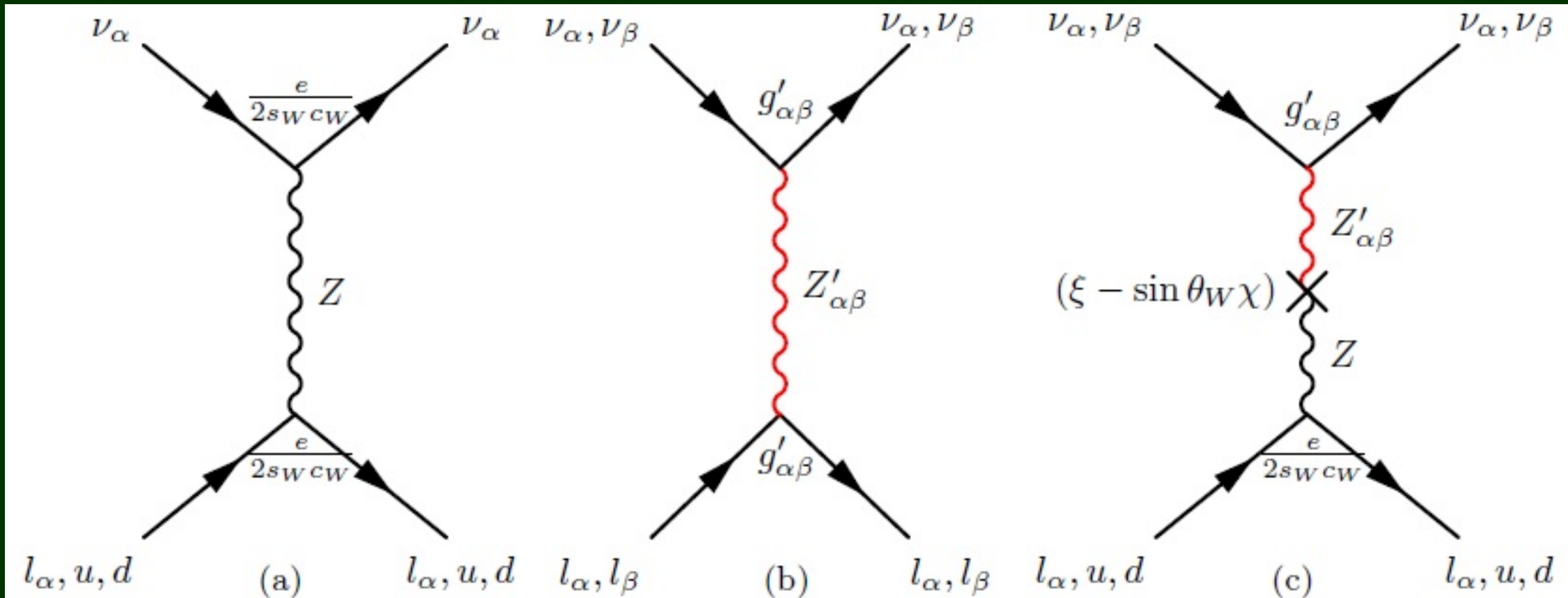
T2HKK
& DUNE



3.4 Sensitivity of T2HK & DUNE to Flavor-dependent long-range neutrino interactions

Singh, Bustamante, Agarwalla, 2305.05184 [hep-ph]

A possible flavor dependent long-range force, mediated by a very light gauge boson, between neutrinos and astrophysical objects (Earth, Moon, Sun, Milky Way, the cosmological distribution of matter in the local Universe) is considered.



$$\mathbf{H} = \mathbf{H}_{\text{vac}} + \mathbf{V}_{\text{mat}} + \mathbf{V}_{\alpha\beta}$$

$$\mathbf{H}_{\text{vac}} = \frac{1}{2E} \mathbf{U} \text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2) \mathbf{U}^\dagger$$

$$\mathbf{V}_{\text{mat}} = \text{diag}(V_{CC}, 0, 0)$$

$$\mathbf{V}_{\alpha\beta} = \begin{cases} \text{diag}(V_{e\mu}, -V_{e\mu}, 0), & \text{for } \alpha, \beta = e, \mu \\ \text{diag}(V_{e\tau}, 0, -V_{e\tau}), & \text{for } \alpha, \beta = e, \tau \\ \text{diag}(0, V_{\mu\tau}, -V_{\mu\tau}), & \text{for } \alpha, \beta = \mu, \tau \end{cases}$$

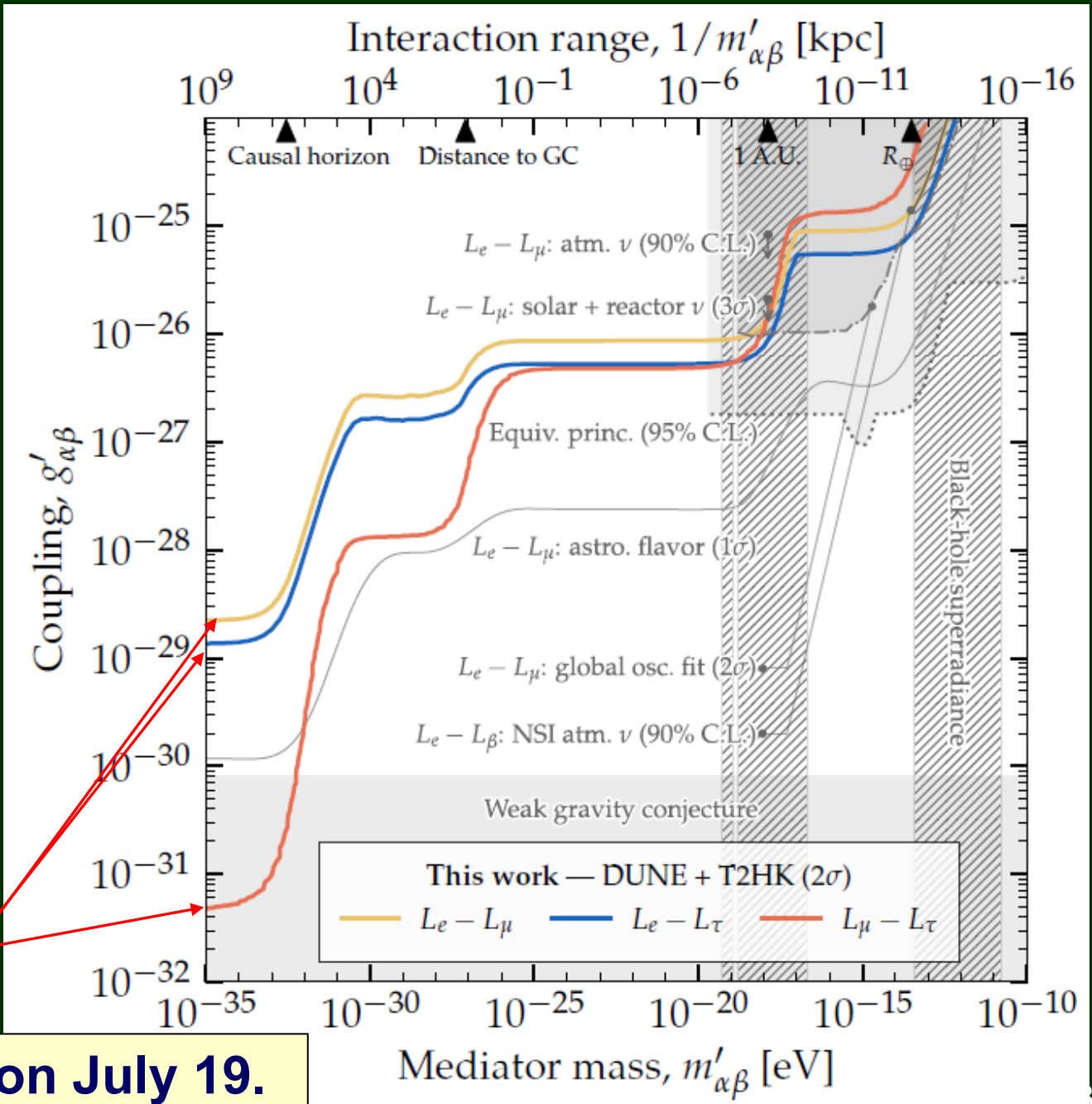
$$V_{\alpha\beta} = V_{\alpha\beta}^\oplus + V_{\alpha\beta}^\ominus + V_{\alpha\beta}^\odot + V_{\alpha\beta}^{\text{MW}} + V_{\alpha\beta}^{\text{cos}}$$

$$V_{\alpha\beta} \propto \frac{1}{r} \exp(-r m'_{\alpha\beta})$$

very light

T2HK + DUNE gives strong bound

For details, see talk by Masoom Singh on July 19.



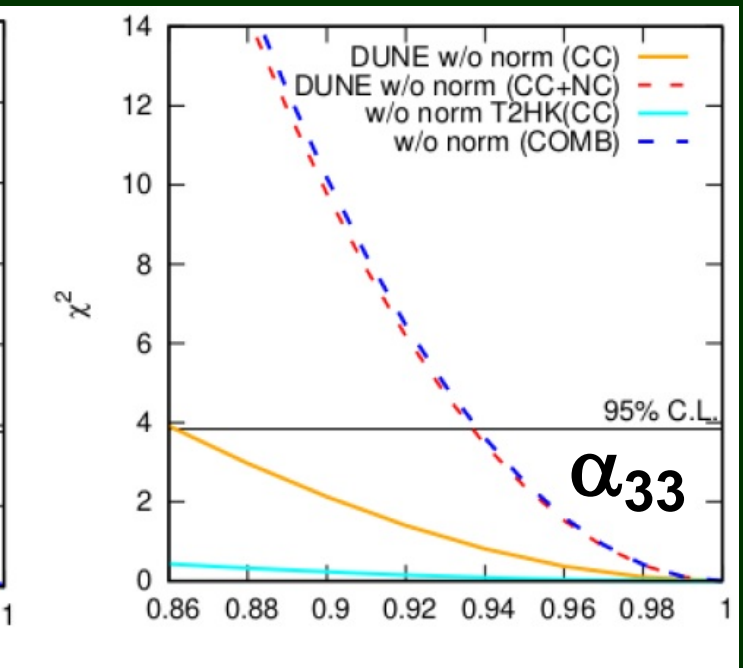
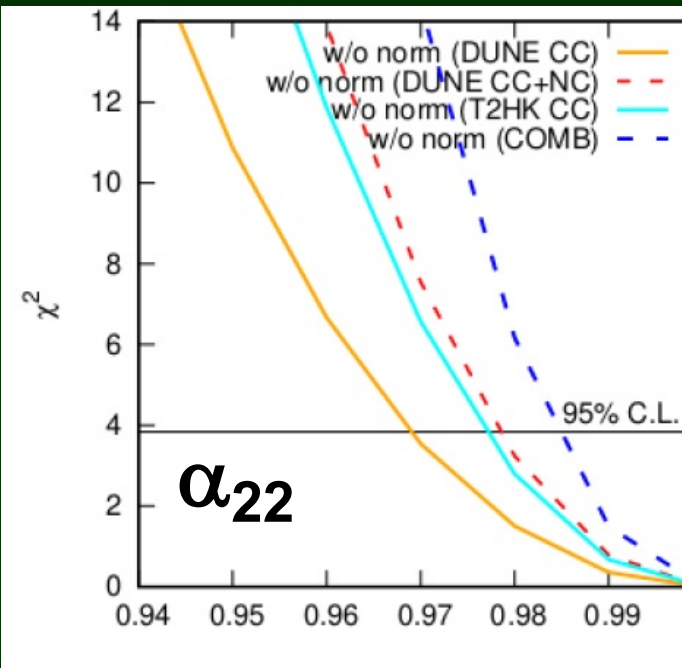
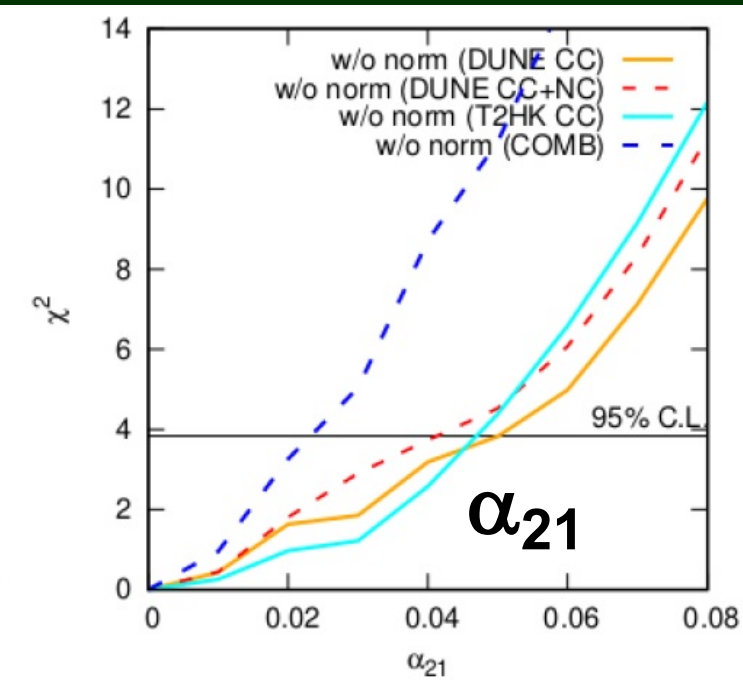
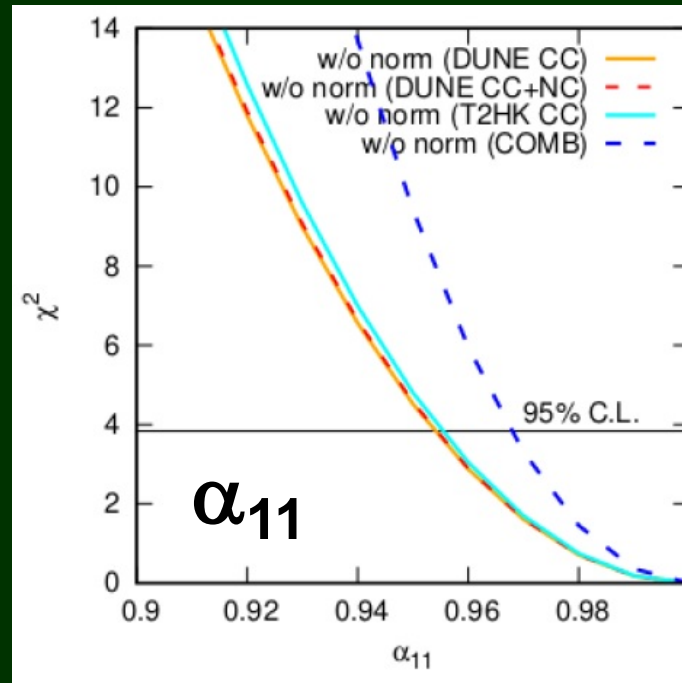
3.5 Sensitivity of T2HK & DUNE to Non-unitarity

Dutta, Roy, J.Phys.G 48 (2021) 4, 045004

$$N = N^{NU}U = \begin{bmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} U$$

Sensitivity of T2HK & DUNE is comparable except for α_{33} .

See also talk by Sudipta Das on July 19.



3.6 Sensitivity of T2HK & DUNE to Lorentz Invariance Violation

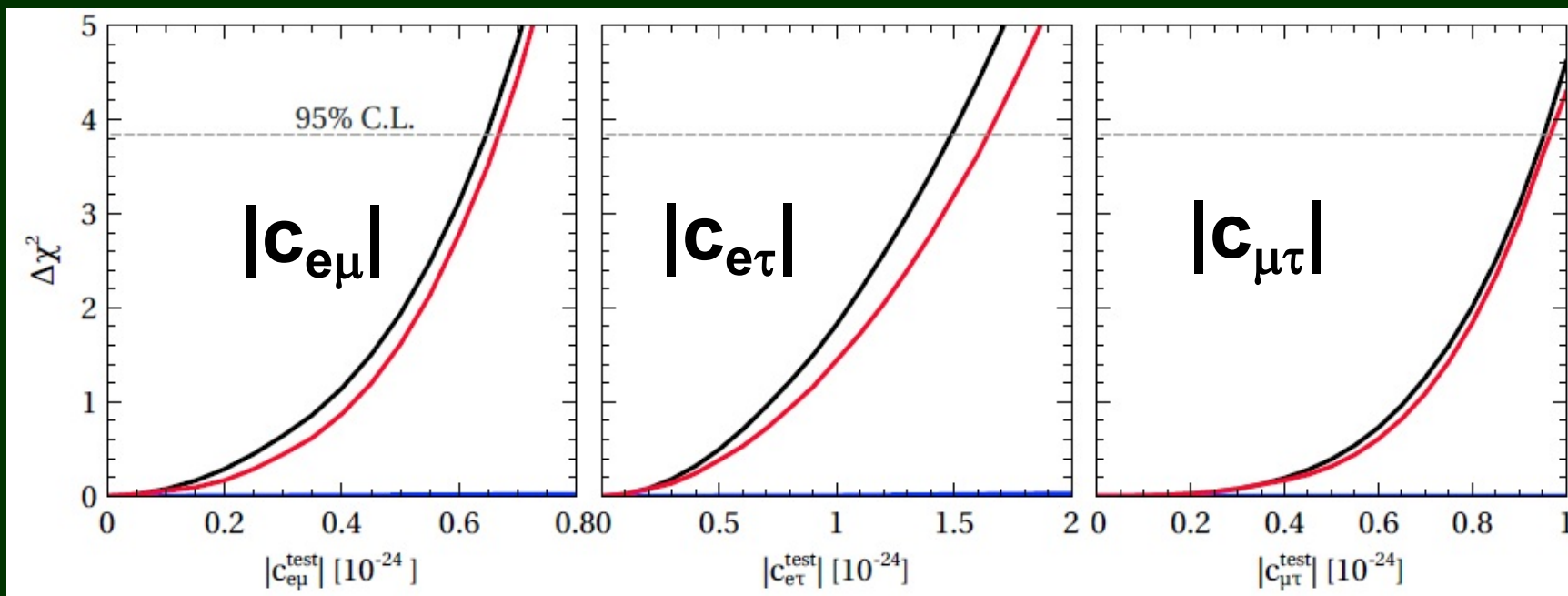
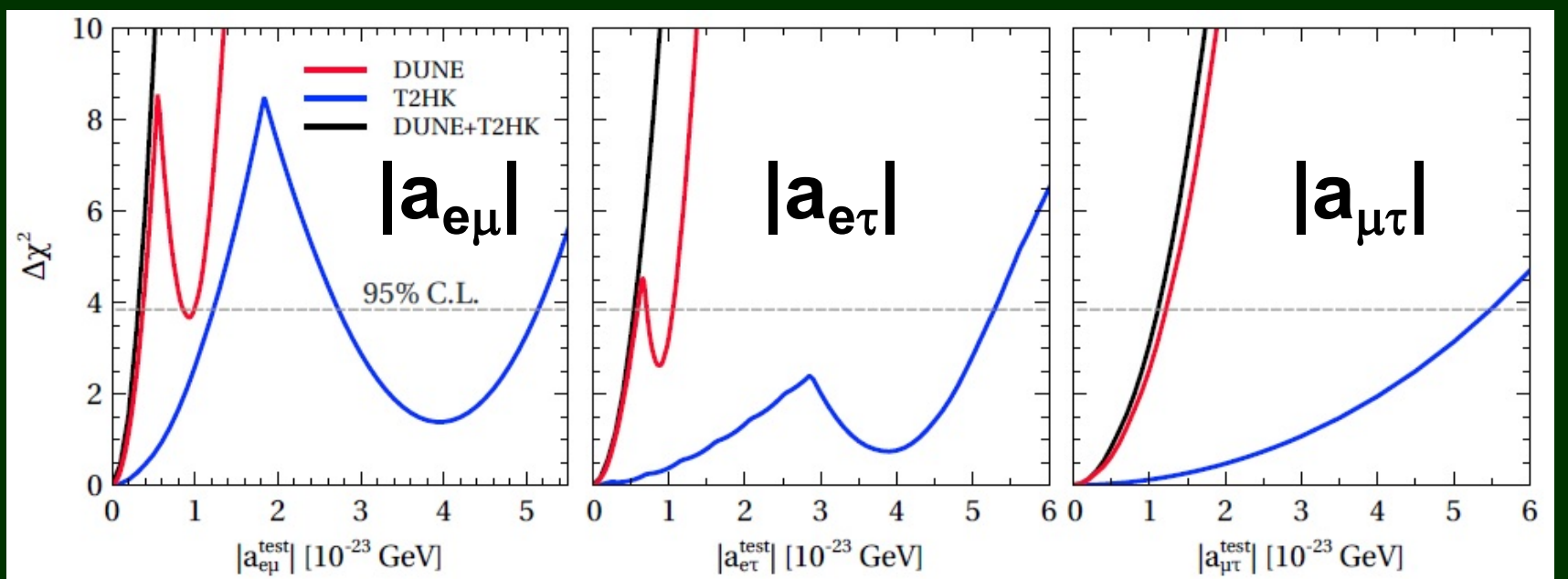
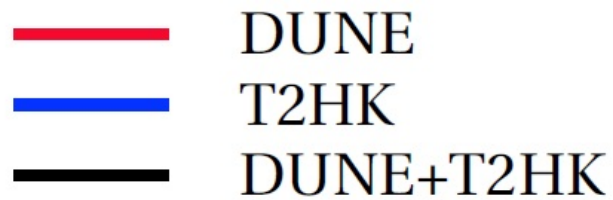
Agarwalla, Das, Sahoo, Swain,
2302.12005 [hep-ph]

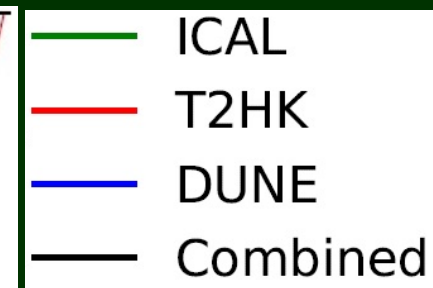
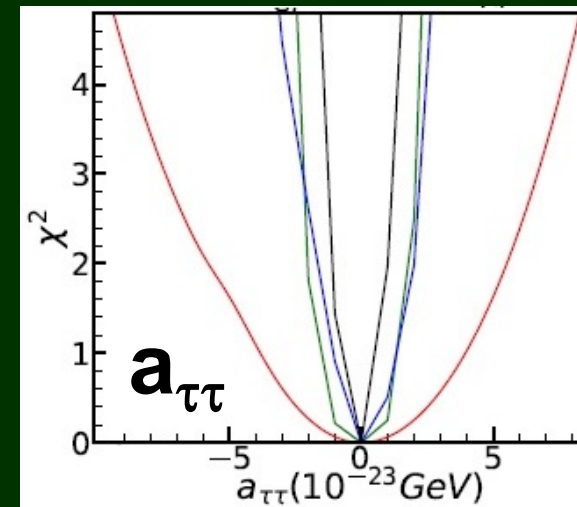
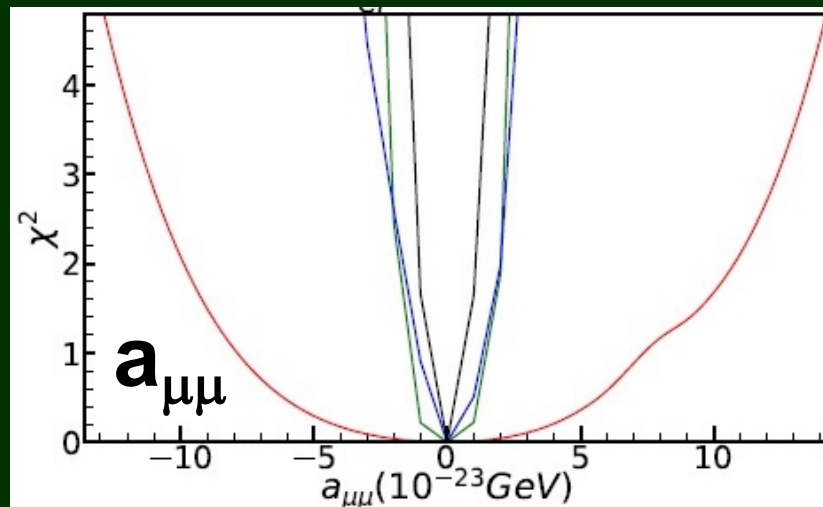
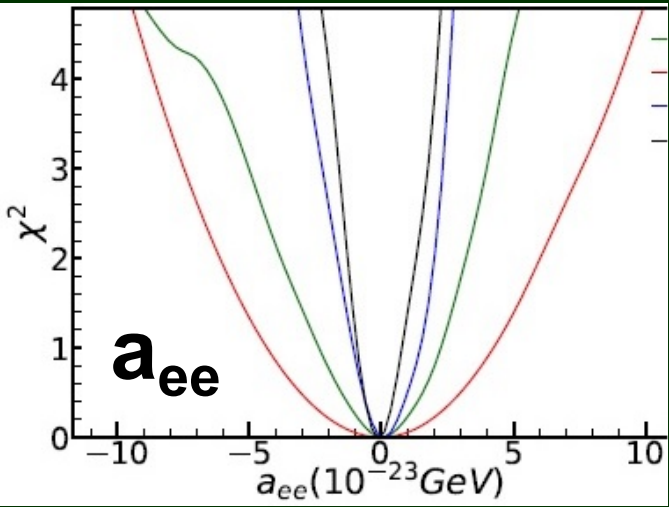
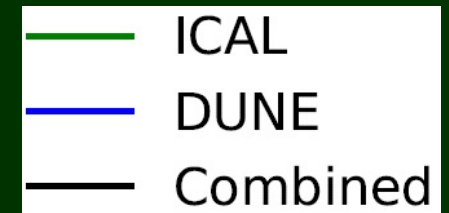
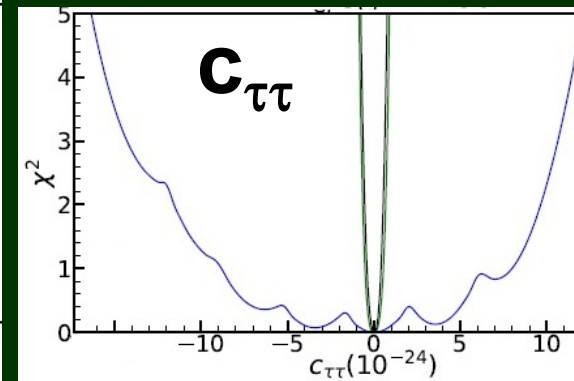
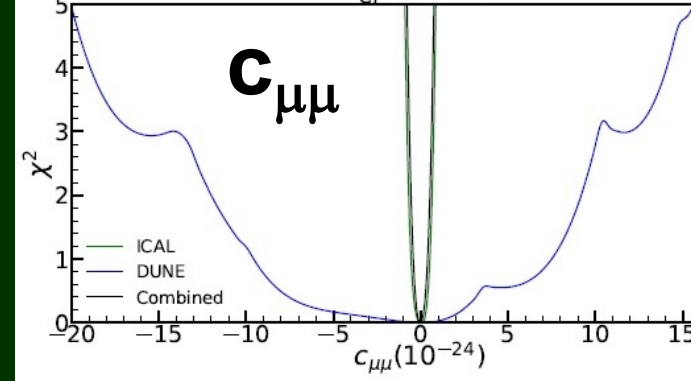
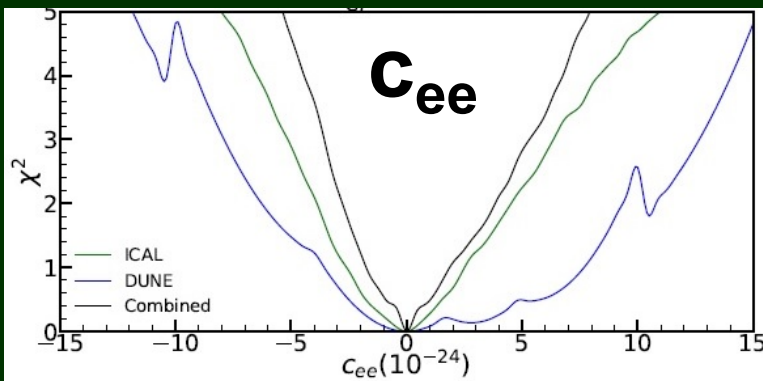
$$H = U M U^\dagger + V_e + H_{\text{LIV}}$$

$$H_{\text{LIV}} = \begin{pmatrix} a_{ee} & a_{e\mu} & a_{e\tau} \\ a_{e\mu}^* & a_{\mu\mu} & a_{\mu\tau} \\ a_{e\tau}^* & a_{\mu\tau}^* & a_{\tau\tau} \end{pmatrix} - \frac{4}{3} E \begin{pmatrix} c_{ee} & c_{e\mu} & c_{e\tau} \\ c_{e\mu}^* & c_{\mu\mu} & c_{\mu\tau} \\ c_{e\tau}^* & c_{\mu\tau}^* & c_{\tau\tau} \end{pmatrix}$$

independent of
energy & matter
density

proportional
to energy

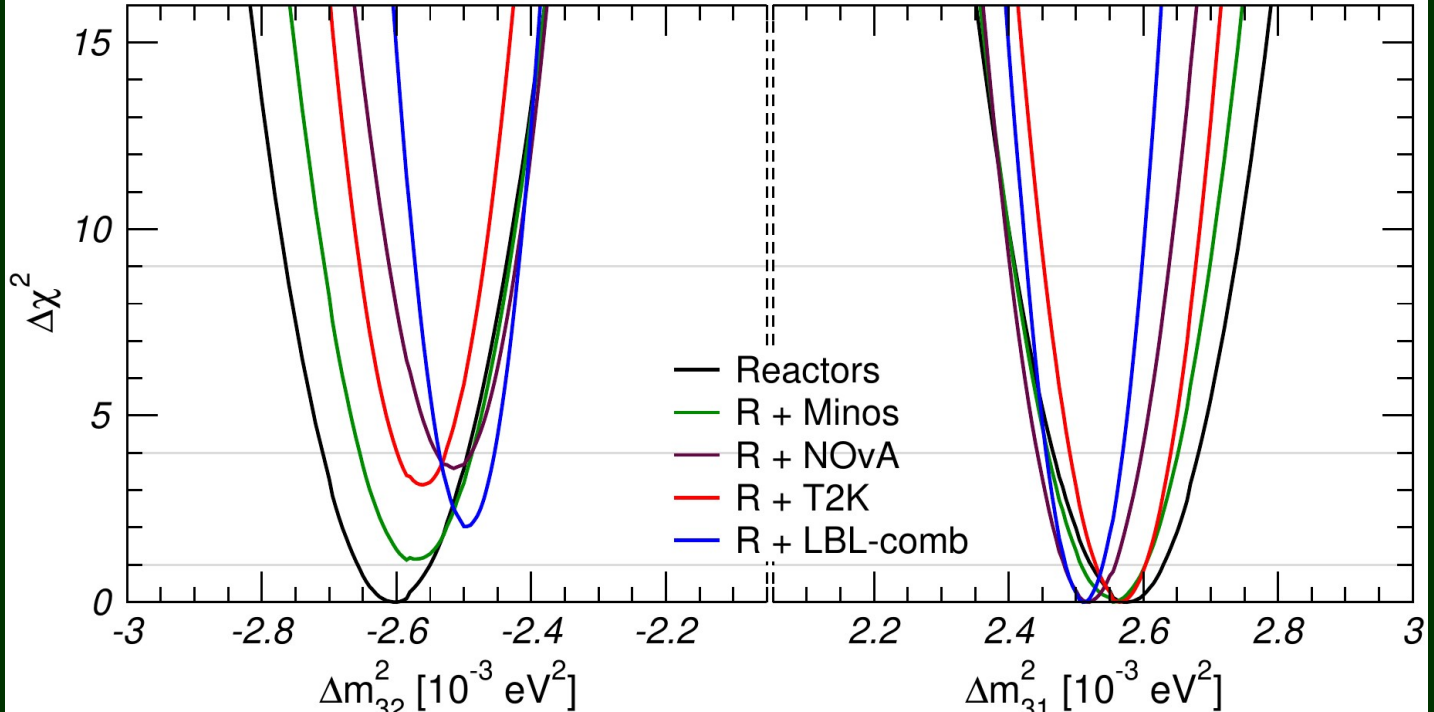
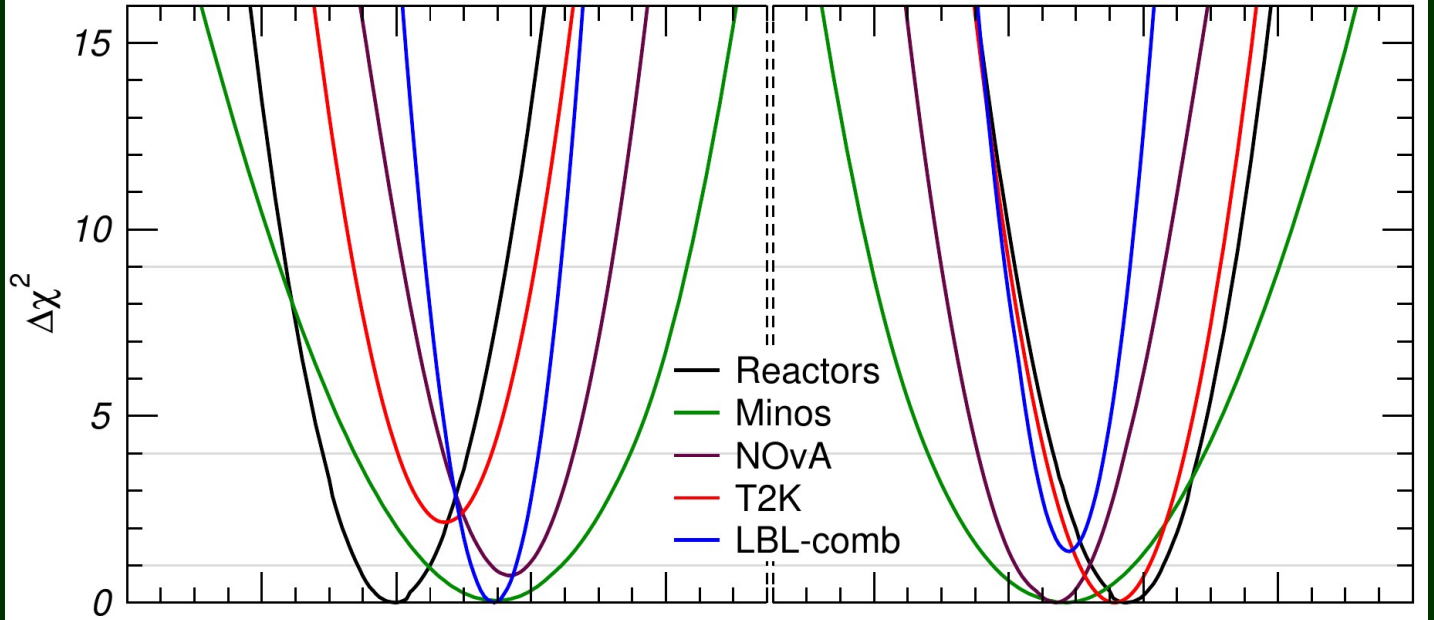


Sensitivity to $a_{\alpha\alpha}$ (independent of E): ICAL > DUNE > T2HKSensitivity to $c_{\alpha\alpha}$ (proportional to E): ICAL >> DUNE >> T2HK

4. Conclusions

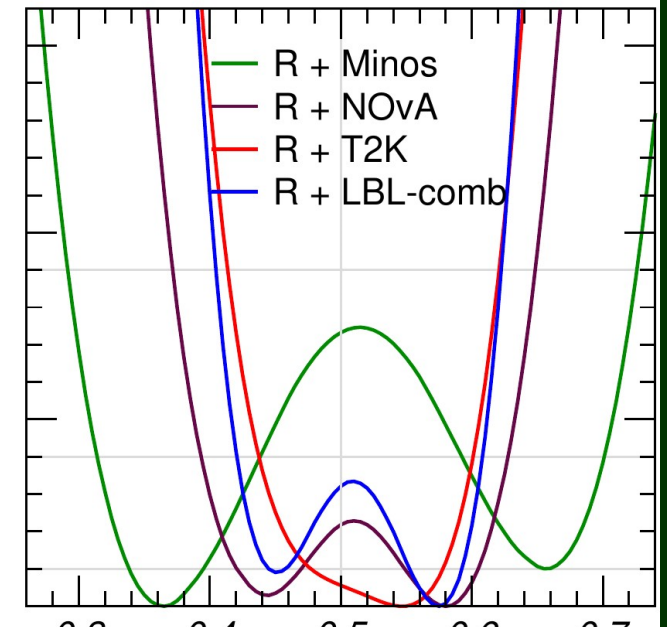
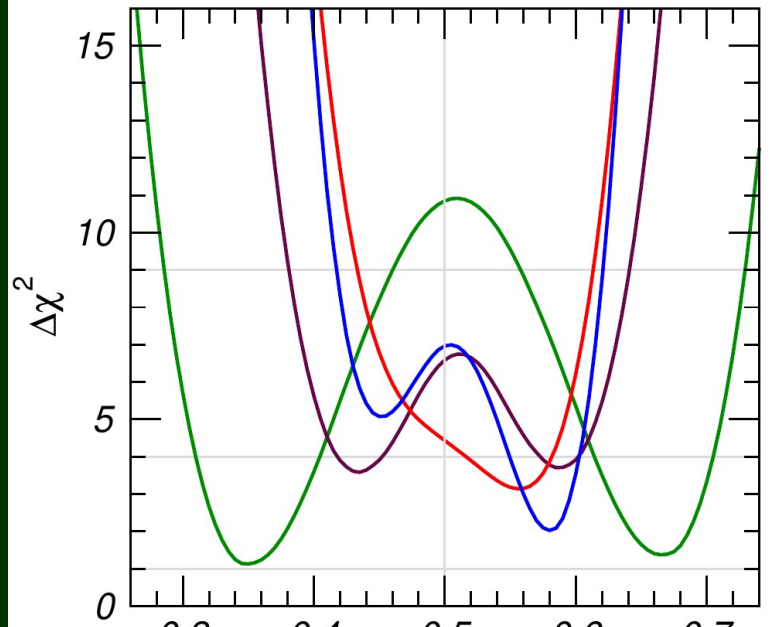
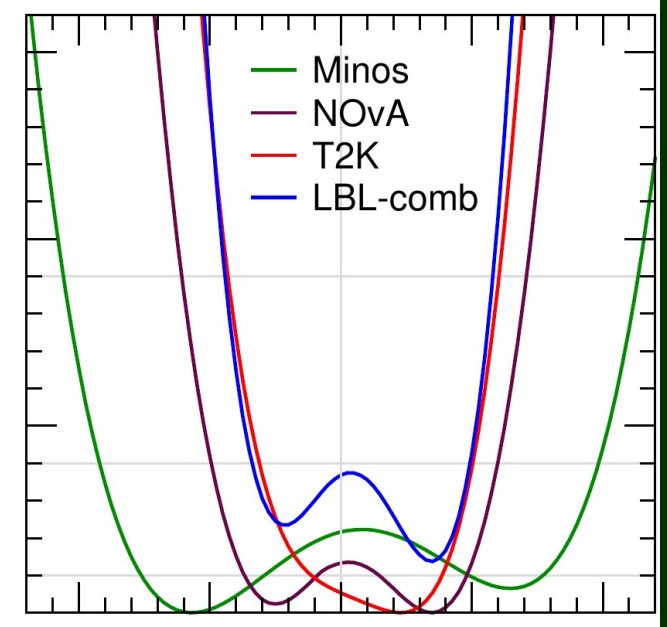
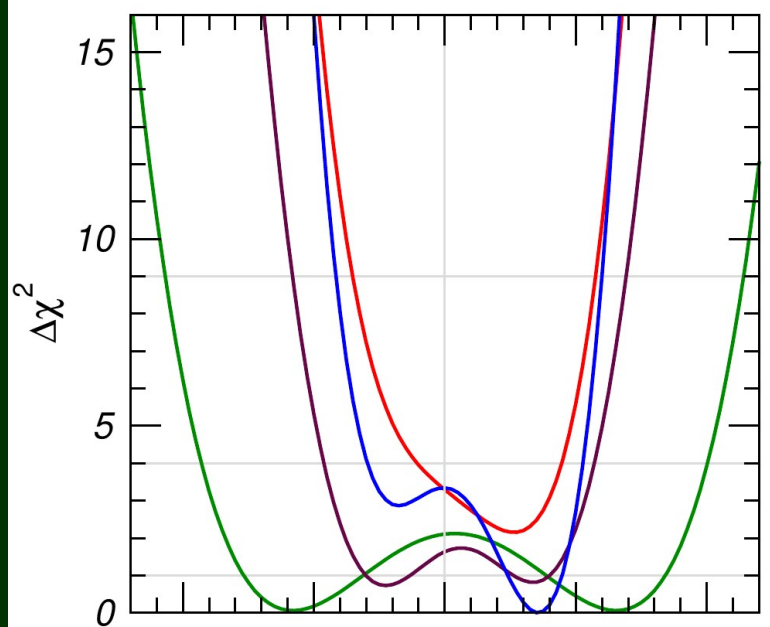
- Precision measurements of mixing angles, mass ordering and CP phase are important for research on Physics Beyond SM.
- Resolution of parameter degeneracy can be possible by combining ν & $\bar{\nu}$ at T2HK & DUNE.
- Measurements of anti-neutrino appearance are in general important to lift degeneracy.
- T2HK+DUNE gives us excellent precision in θ_{23} (1%), Δm_{32}^2 (0.5%), δ (20%).
- HK+DUNE have some sensitivity to scenarios beyond std PMNS (ν_s , scalar NSI, long-range force, Lorentz Invariance Violation, etc).
- For the best fit $\eta = -43.6^\circ$, LBL has no sensitivity to NSI, which is suggested by the solar ν tension.

Backup slides



IO

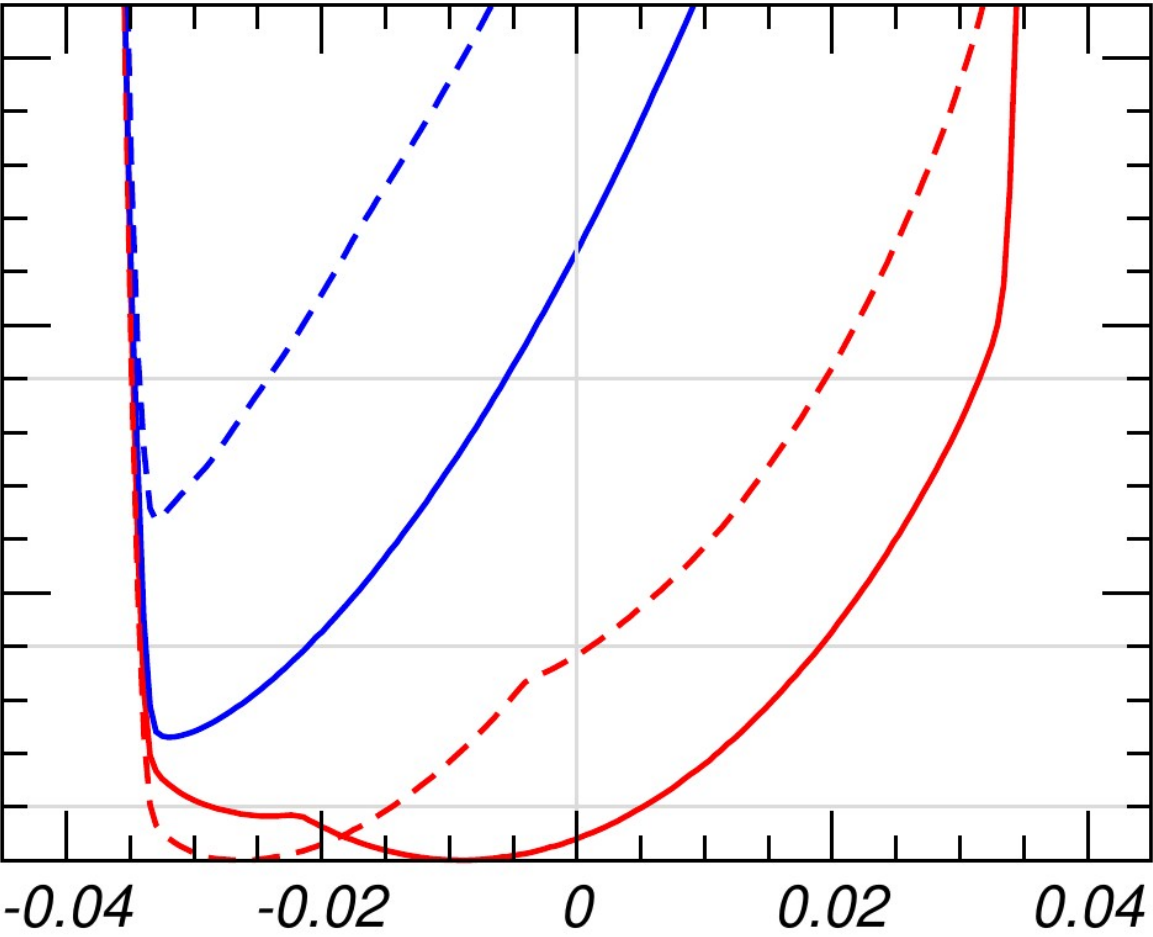
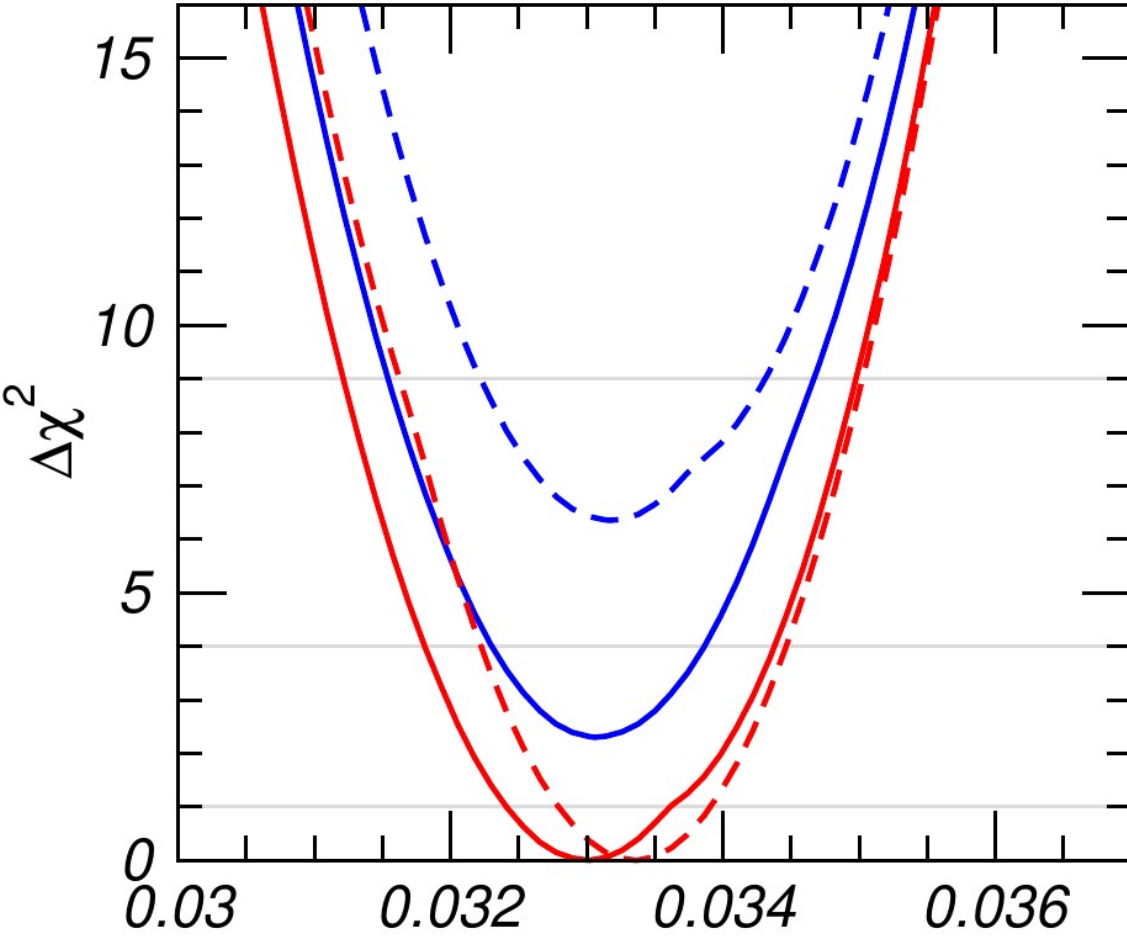
NO

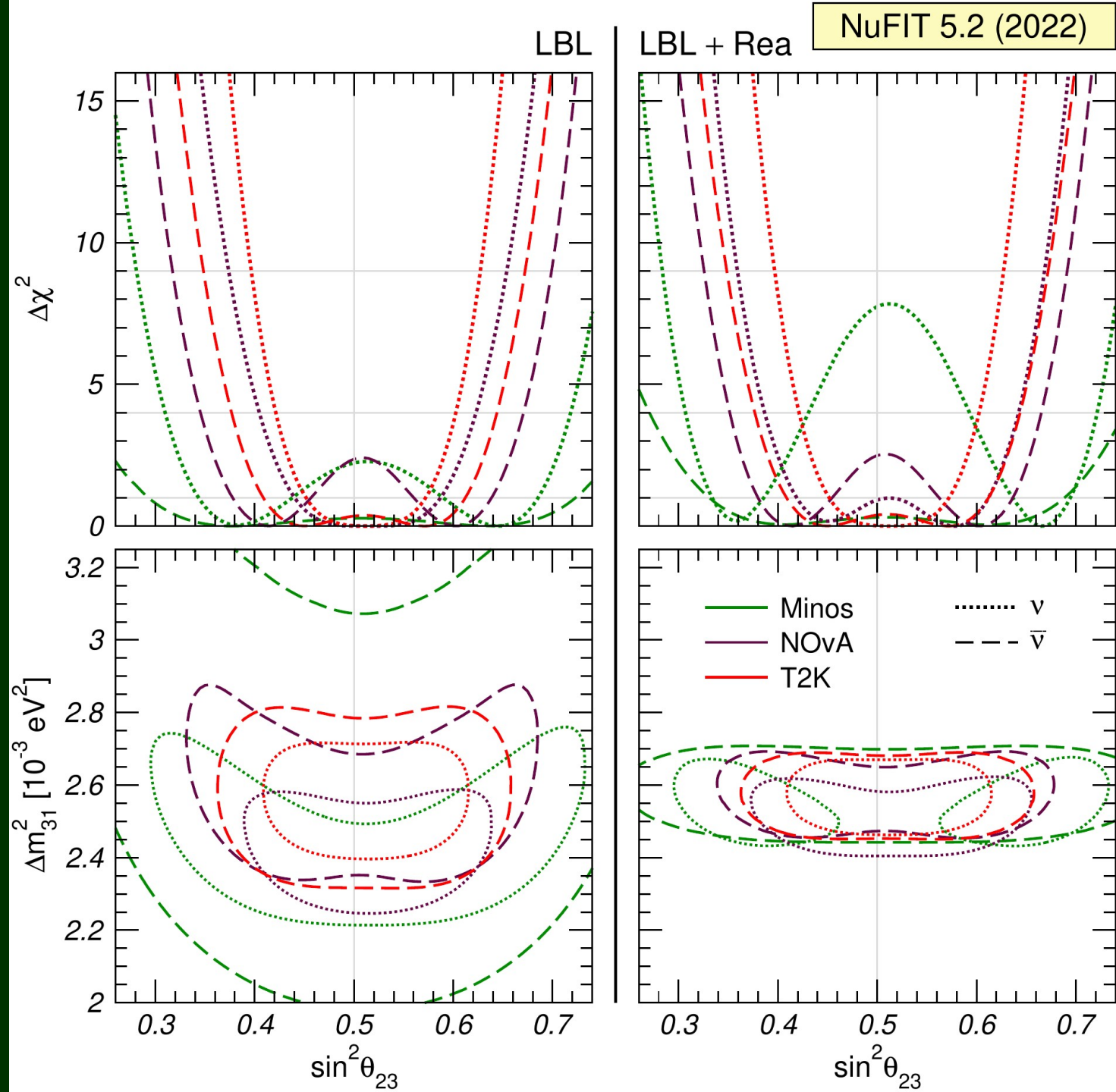


$\sin^2\theta_{23}$

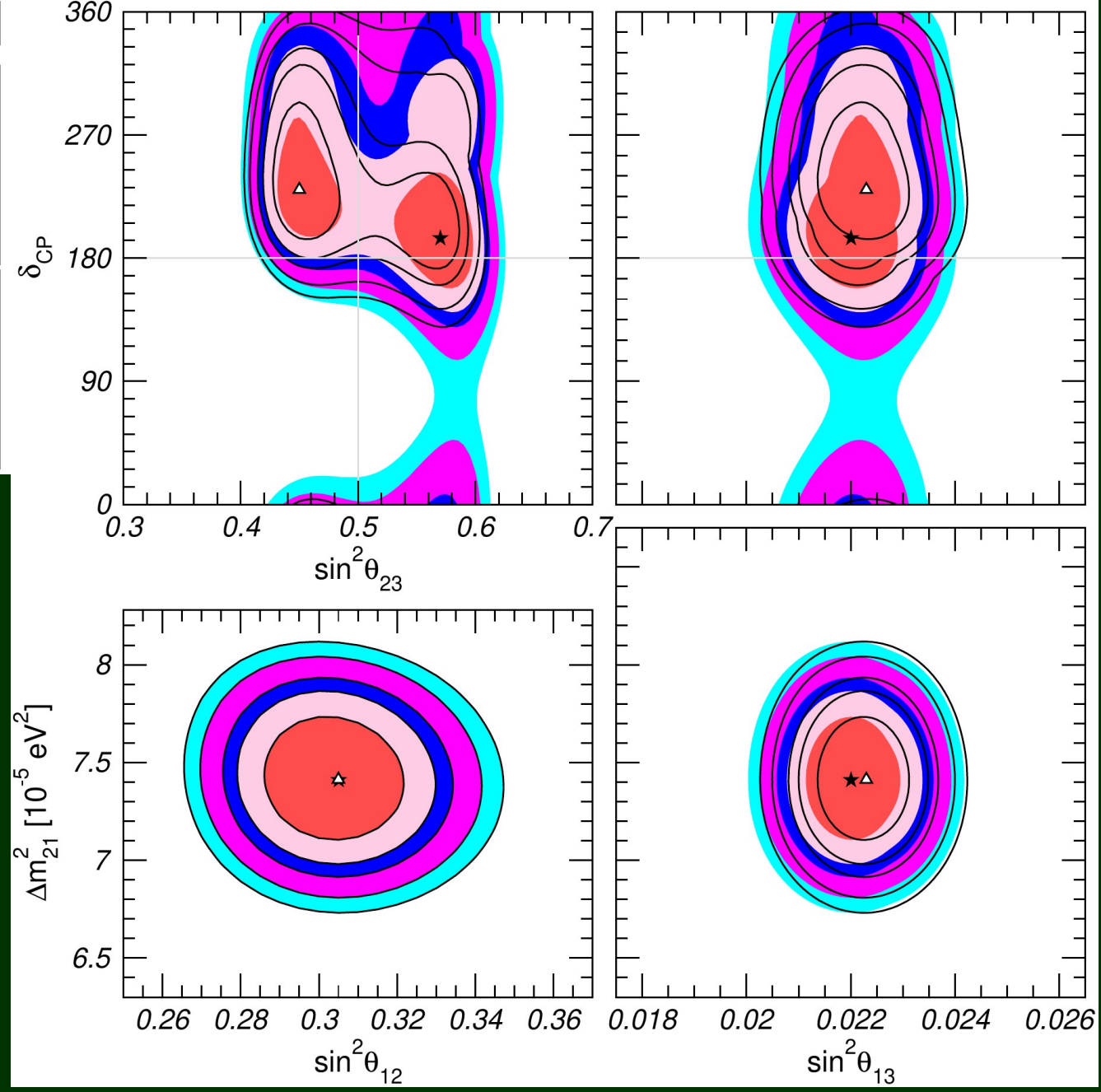
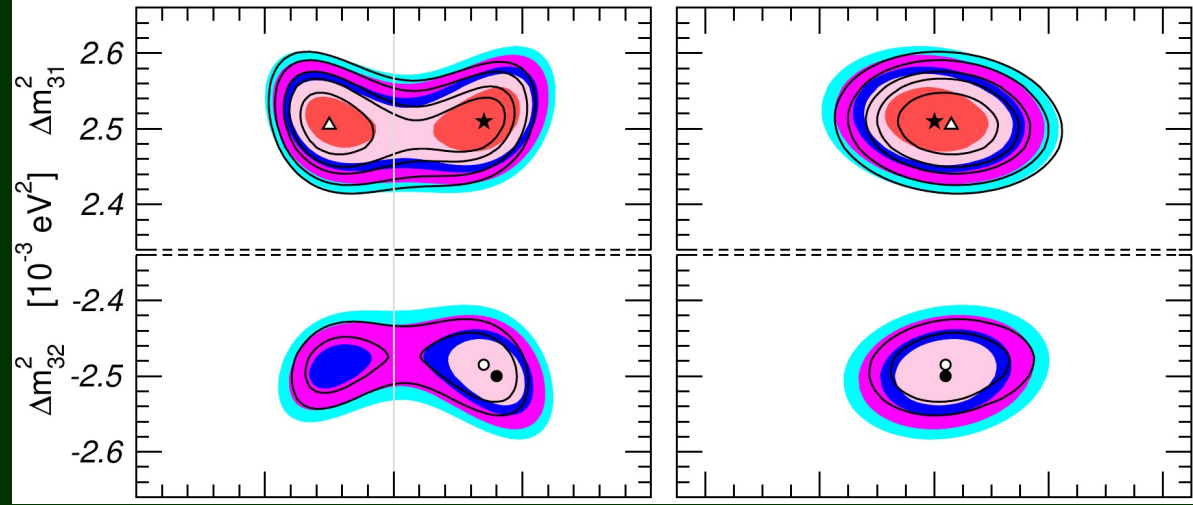
$\sin^2\theta_{23}$

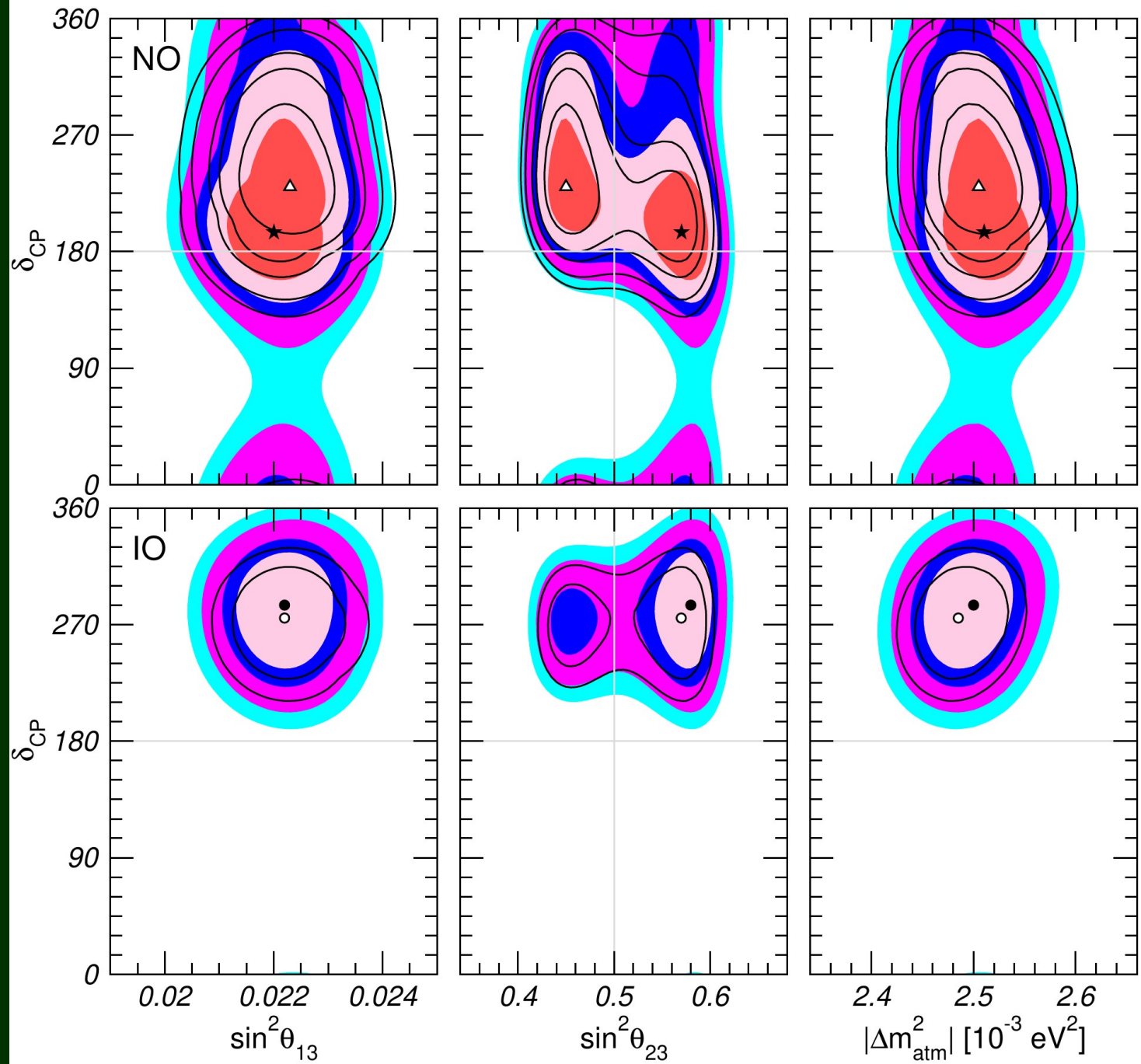
— NO, IO (w/o SK-atm)
- - - NO, IO (with SK-atm)

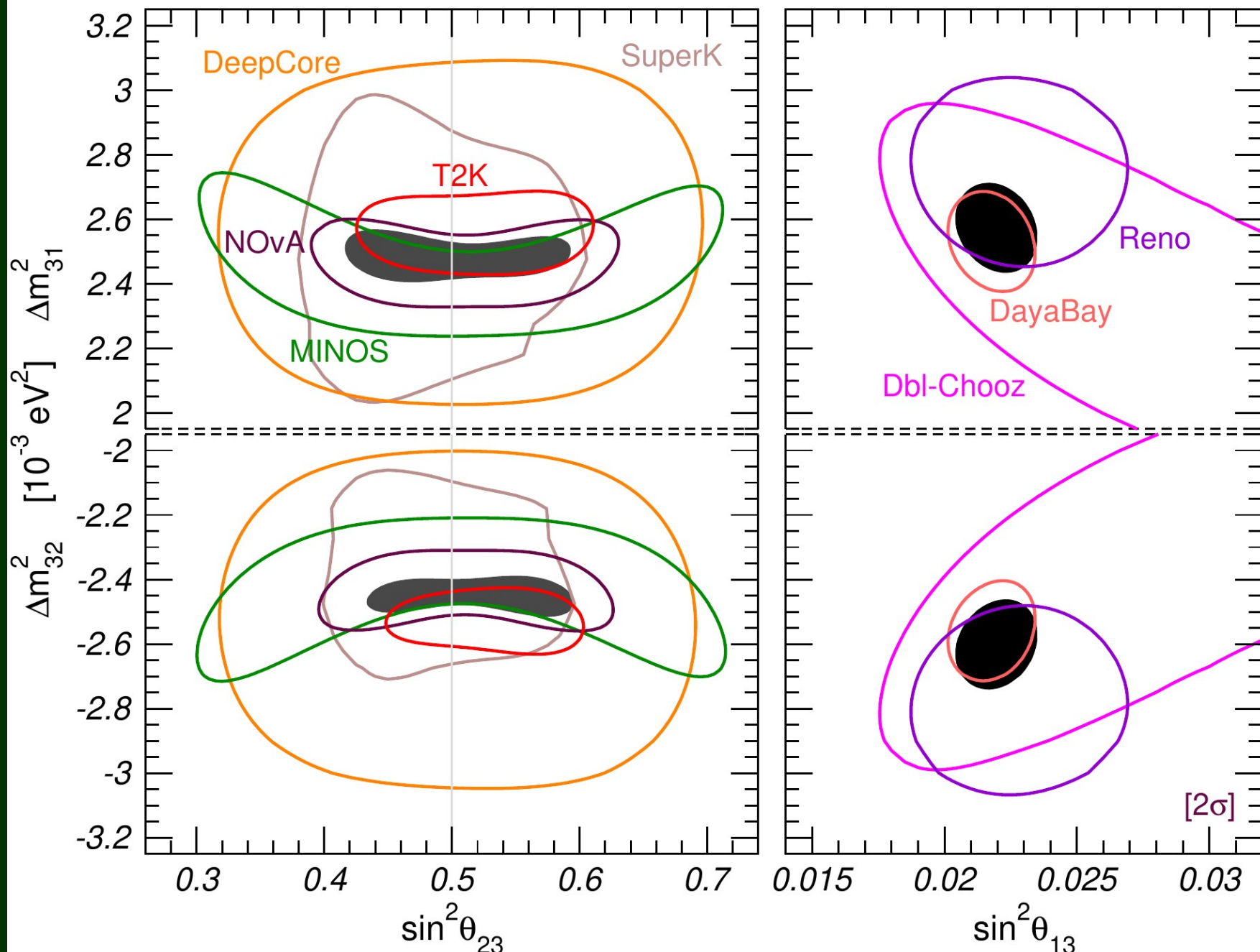


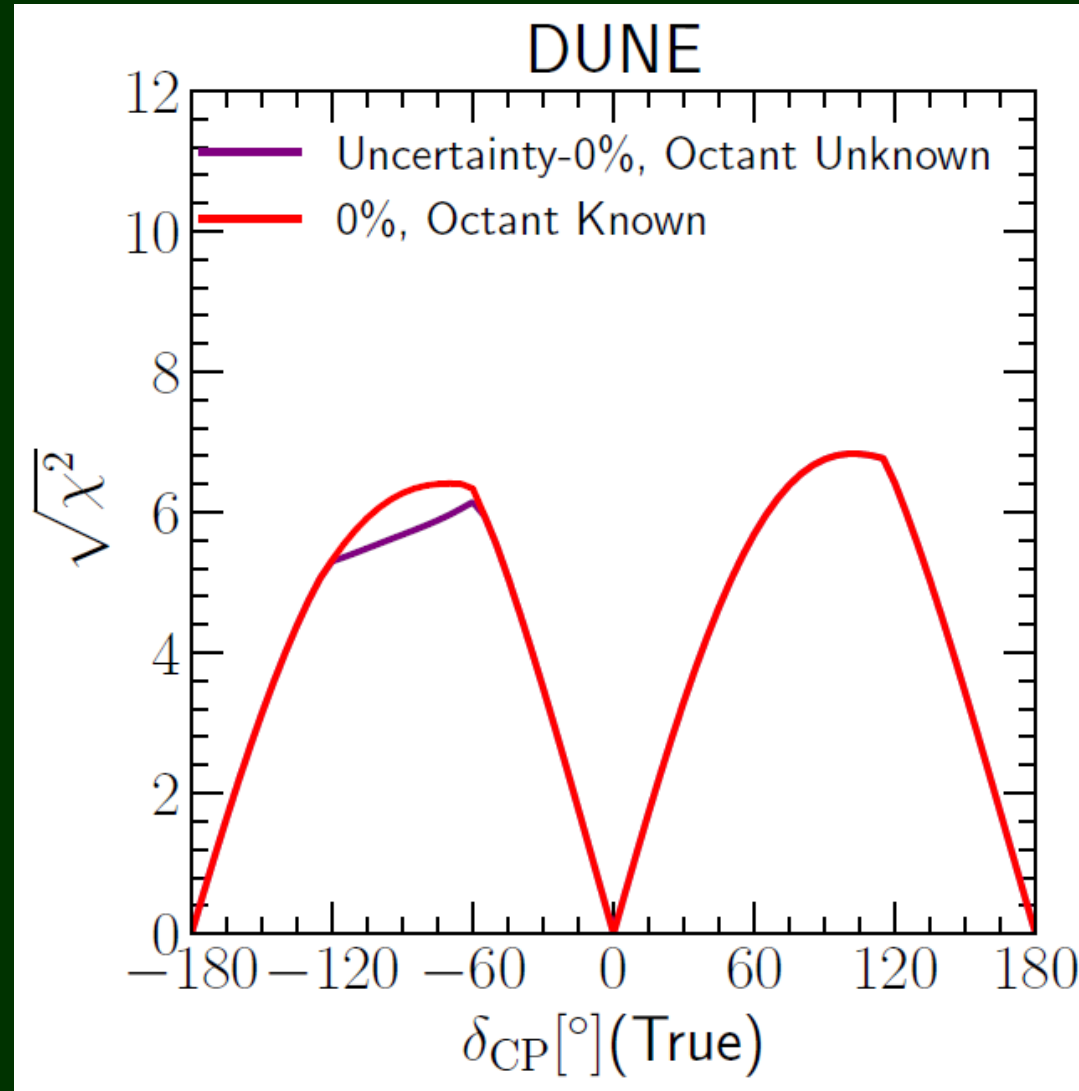


NuFIT 5.2 (2022)

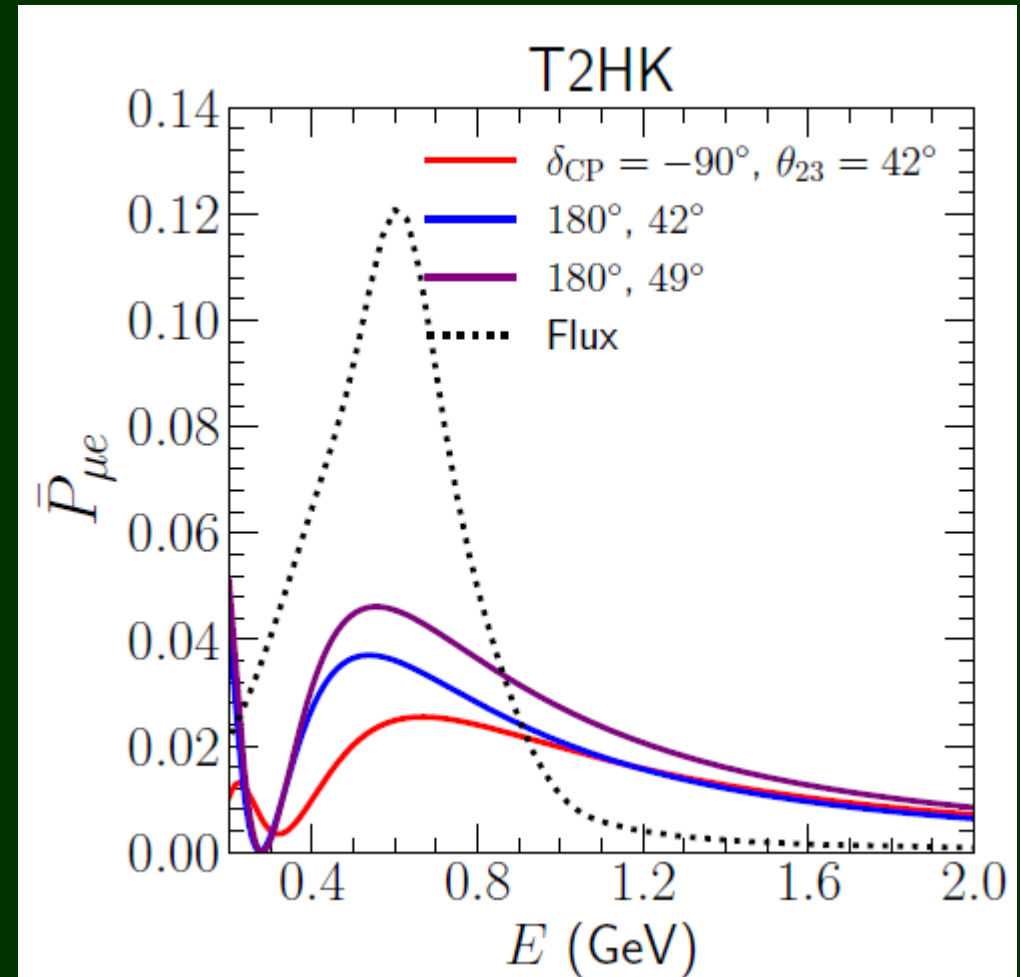
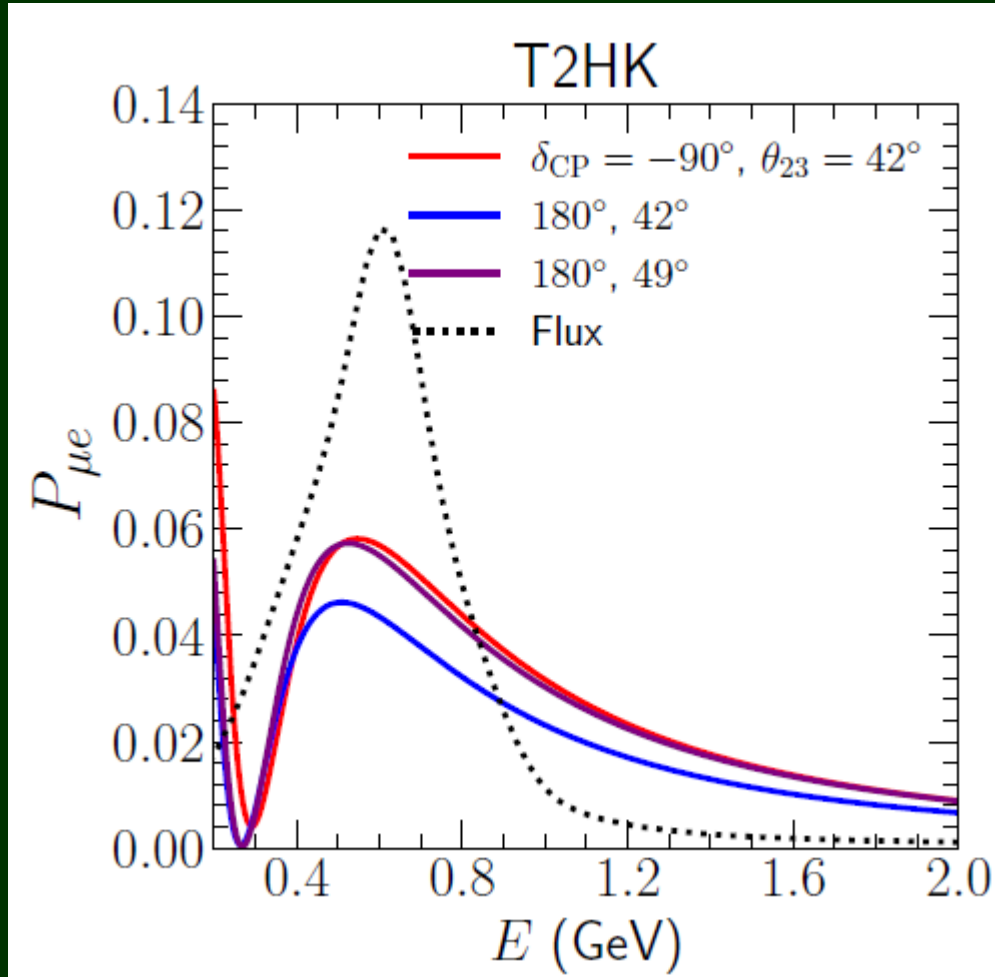




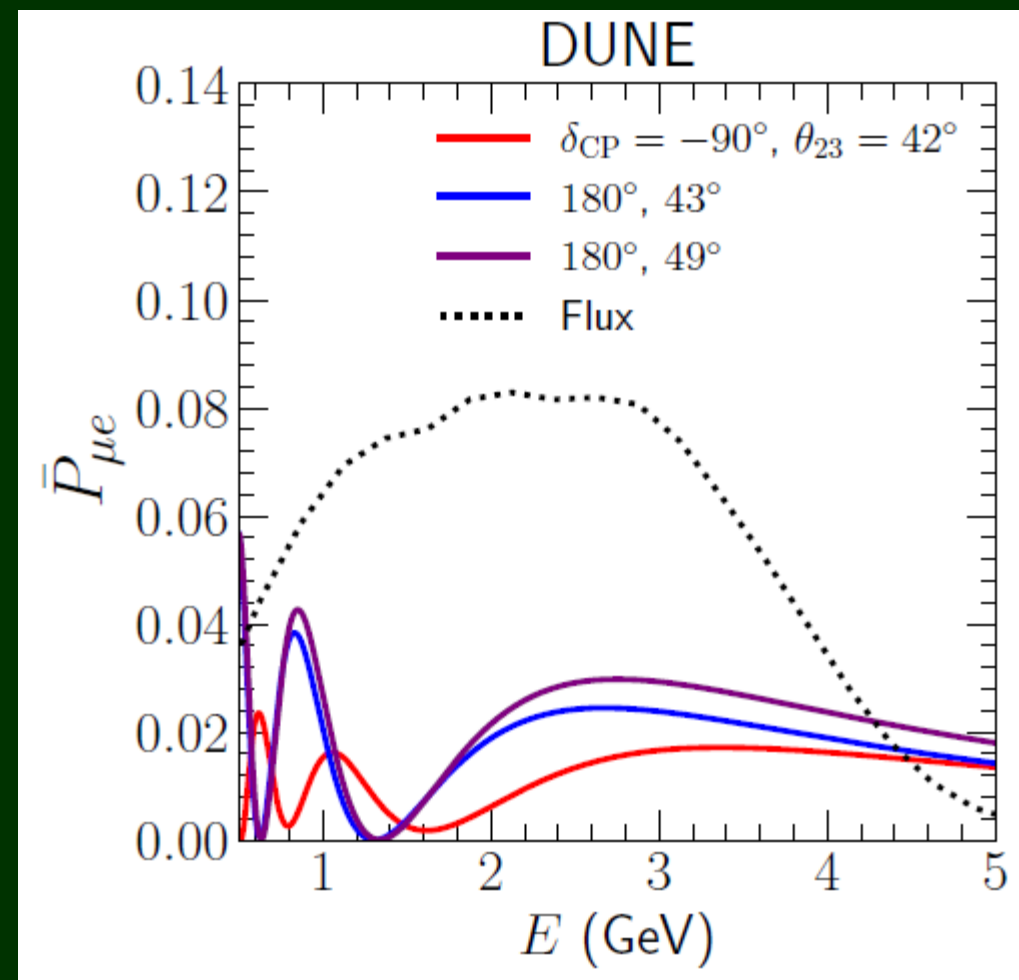
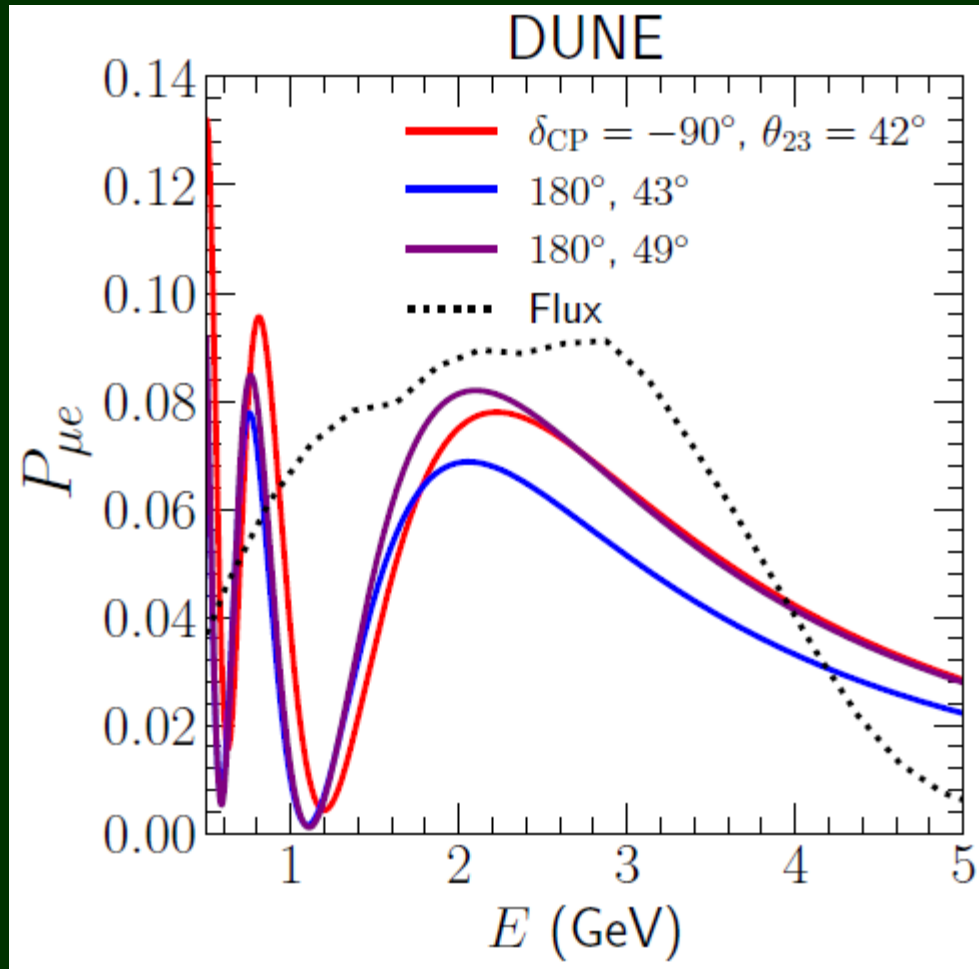




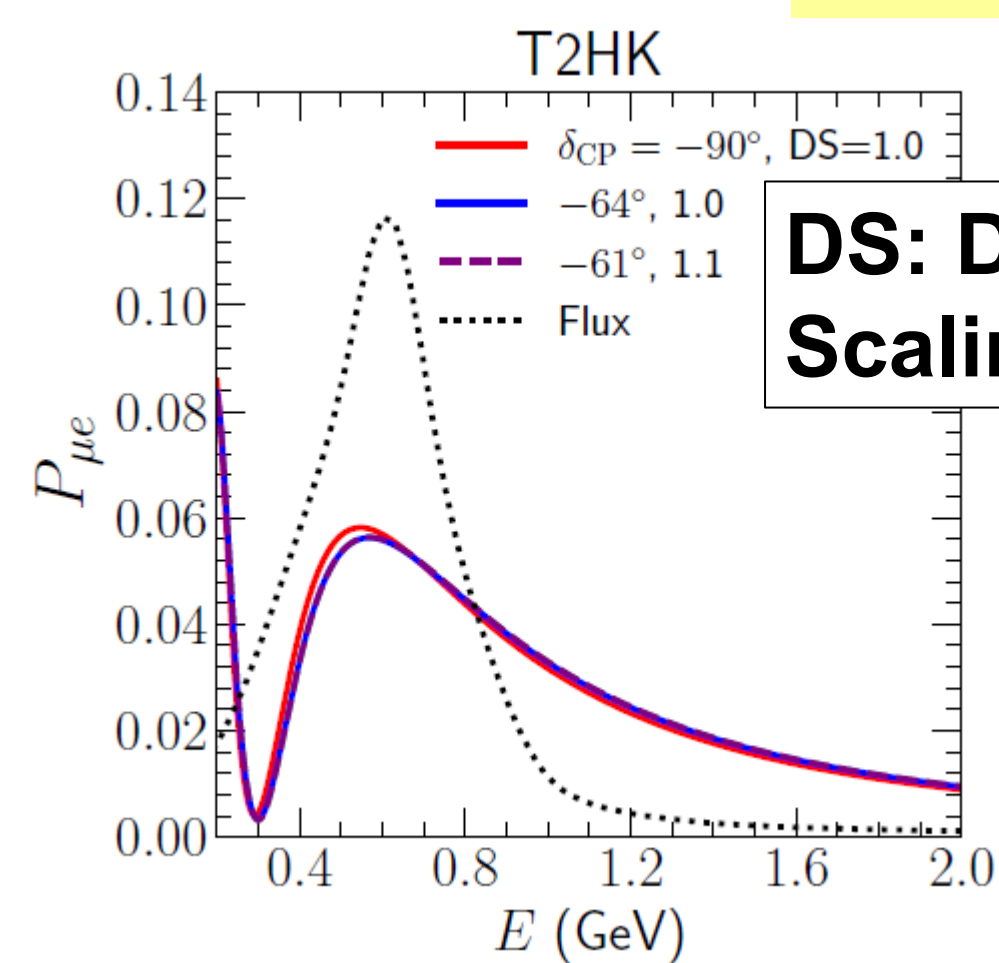
octant degeneracy at DUNE



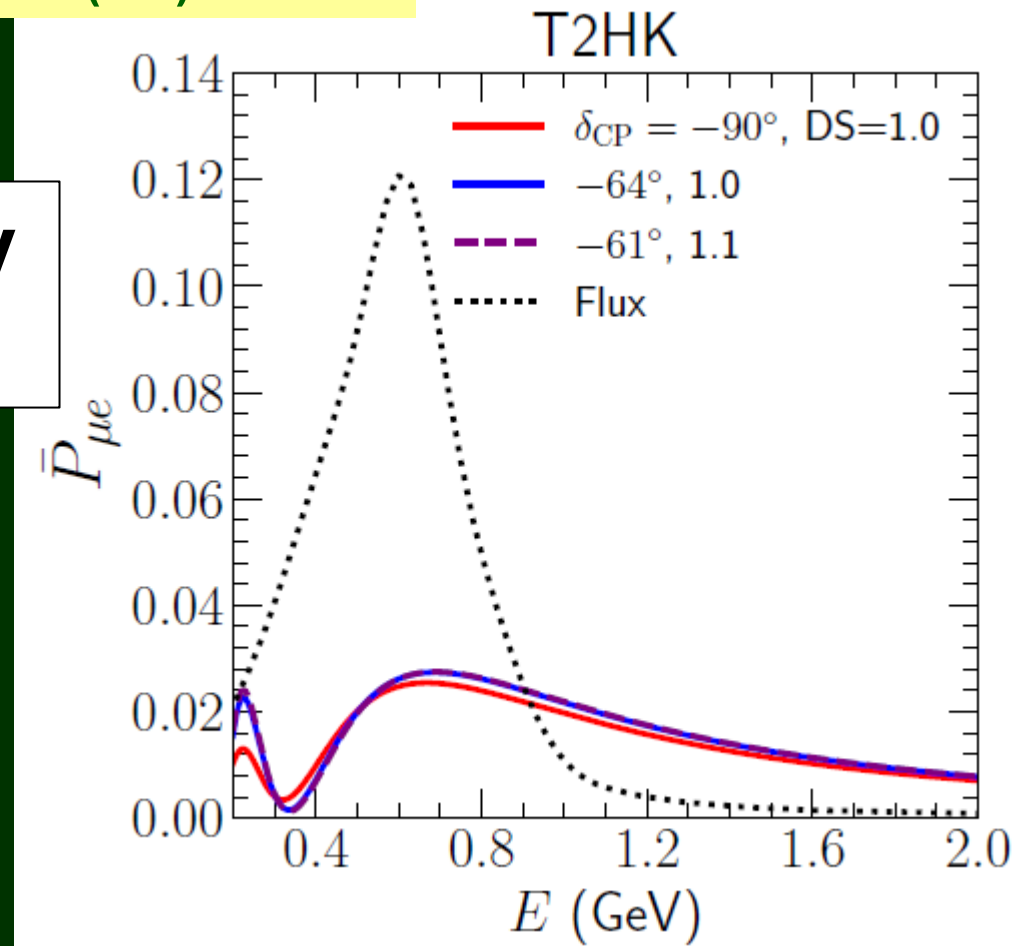
Probability vs octant degeneracy at T2HK



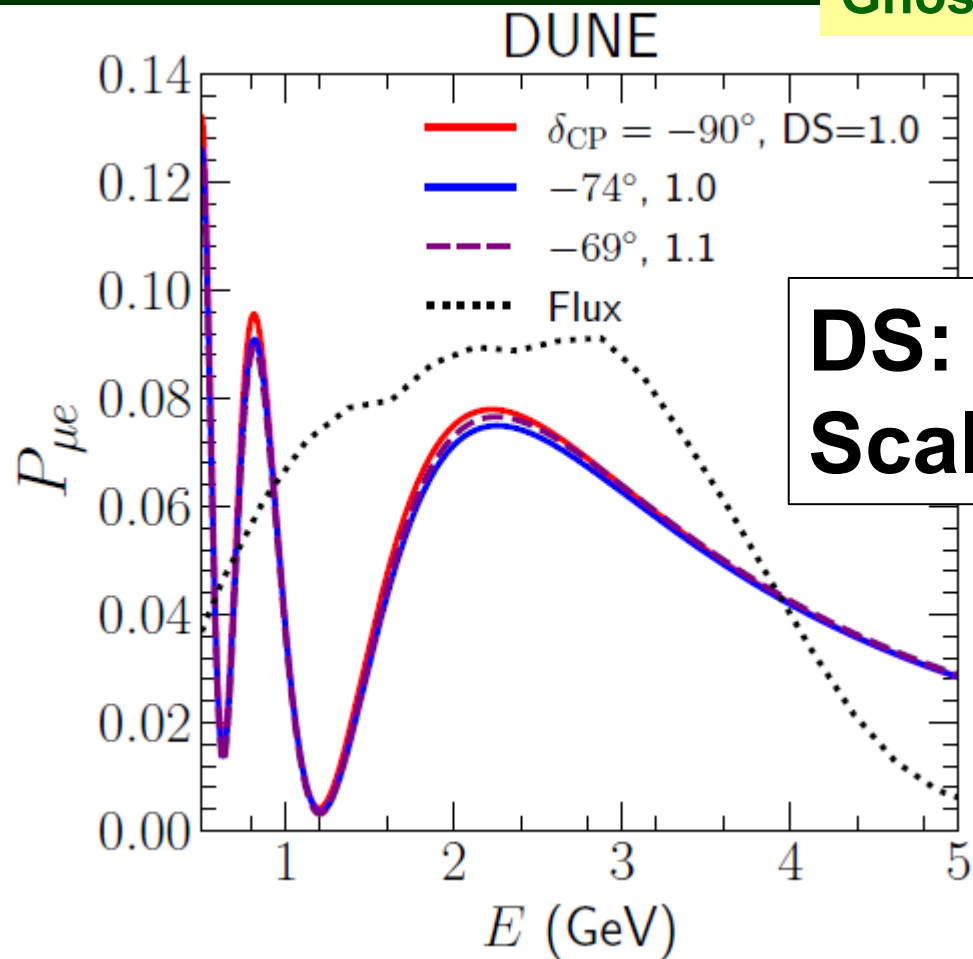
Probability vs octant degeneracy at DUNE



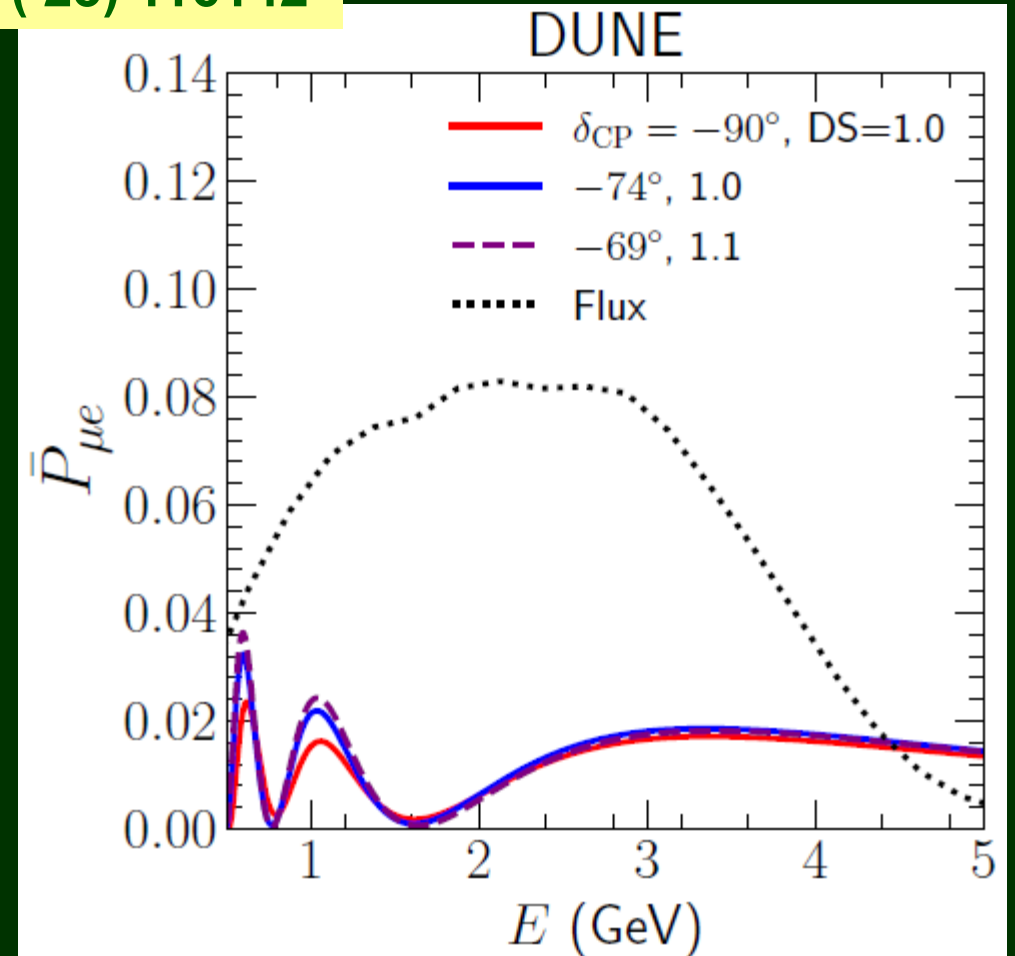
**DS: Density
Scaling**



For T2HK, $(\delta, DS)=(-64^\circ, 1.0)$ is degenerate with $(-61^\circ, 1.1)$.
 For $\delta(\text{true}) = -90^\circ$, $(\delta(\text{test}), DS(\text{test})) = (-61^\circ, 1.0)$ is excluded but $(\delta(\text{test}), DS(\text{test})) = (-61^\circ, 1.1)$ is allowed.



**DS: Density
Scaling**



For DUNE, $(\delta, \text{DS})=(-74^\circ, 1.0)$ is degenerate with $(-69^\circ, 1.1)$.
 For $\delta(\text{true}) = -90^\circ$, $(\delta(\text{test}), \text{DS}(\text{test})) = (-69^\circ, 1.0)$ is excluded but $(\delta(\text{test}), \text{DS}(\text{test})) = (-69^\circ, 1.1)$ is allowed.